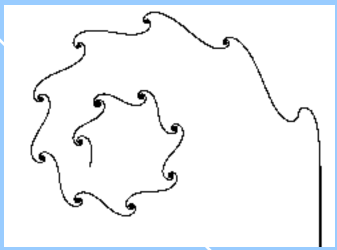
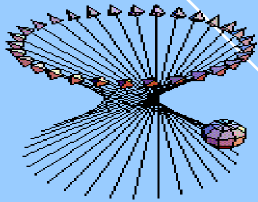
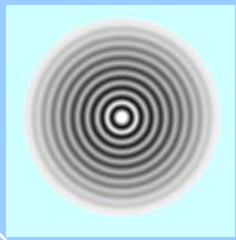

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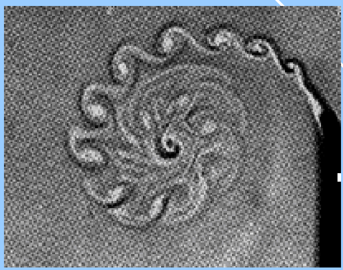
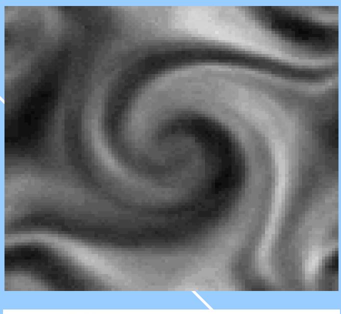

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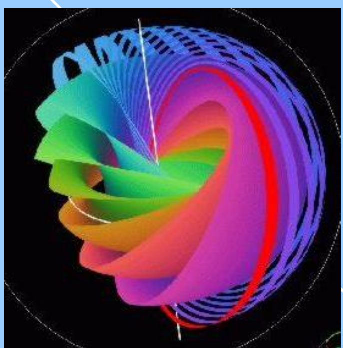
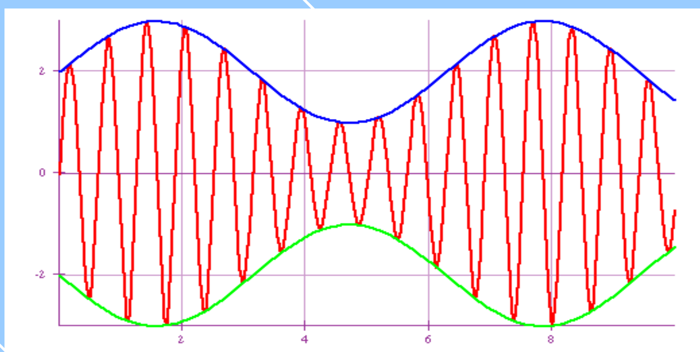
UNCERTAINTY of PARTICLE – WAVE DUALITY

Miodrag Prokic

1975 - 2020

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INNOVATIVE ASPECTS OF PARTICLE-WAVE DUALITY, GRAVITATION, AND ELECTROMAGNETIC THEORY ADDRESSED FROM THE POINT OF VIEW OF ELECTRIC AND MECHANICAL ANALOGIES

Author: Miodrag Prokic

Abstract:

This book is dealing with an innovative and generalized understanding of Wave-Particle Duality of matter, equally applicable to a micro and macro world of our Universe. An overall, multi-level system of physics and mathematics-related analogies is presented and exploited in this book. Based on the presented electromechanical analogies, number of hypothetical predictions are made regarding an innovative understanding of de Broglie matter-waves phenomenology (or particle-wave duality), generating elements for new conceptualization of Gravitation Quantum theory and directions for updating Maxwell-Faraday Electromagnetic theory. New aspects of gravitation are extended to (still hypothetical) phenomenology of masses and fields unity and coupling between electromagnetic and mechanical, linear, and rotational motions, and to different associated and dualistic, particle-wave states of involved motional energy members. Such approach, based on analogies, is suggesting that field of gravitation could have a complementary field (which has not been well understood until present, or still has not been considered or modeled as a complement to gravitation), in a similar fashion as electric and magnetic fields are mutually coupled and complementary fields. Masses are anyway internally composed of electromagnetic entities, such as atoms, electrons, protons, and neutrons (and all mentioned items have spinning, orbital and/or electromagnetic moments or dipoles). Electron and other electric charges are still partially and wrongly considered as being static parameters (or as numbers of Coulombs), but, electric charges belong to the family of dynamic parameters, like different mechanical moments are, and between electric charges should continuously exist certain Radiant, electromagnetic energy flow (as Nikola Tesla speculated). This is implicating that specific coupling and unity should exist between linear and torsional aspects of any mechanical motion (like intrinsic coupling between electric and magnetic fields). In other words, the field of gravitation, together with its (still hypothetical) torsional field complements should be a kind of electromagnetic field manifestation. It is also contemplated that the gravitational force could be closely related to formations of standing matter waves of the spatially and temporally resonating universe, acting towards nodal, or masses agglomerating zones of such standing waves (like in cases of acoustic or ultrasonic levitation in fluids), where the participation of electromagnetic forces or fields is still not appropriately considered.

Then, the understanding of probabilistic and other ontological roots and foundations of the contemporary Orthodox Quantum theory (which is presently the only widely and officially accepted theory dealing with particle-wave duality) is differently and innovatively addressed and updated showing that such theory can also be more realistic and deterministic than probabilistic, if certain more appropriate mathematical modeling will be applied. It is also shown why present Orthodox Quantum theory mathematically works well, but only in its own, self-defined boundaries and in an artificially assembled mathematical environment, but being with not sufficiently clear conceptual structure, not having enough causal and common-sense grounds. Innovative mathematical modeling and upgraded wave-particle duality concepts, as practiced in this book, are resulting in formulation of more general forms of Schrödinger and Dirac-like matter-waves equations than presently known, but without using assumptions of modern probabilistic Quantum theory, and being on the level of an elementary mathematics. The generalization of Schrödinger equation is achieved, first by using powerful mathematical modeling of physics-related wave-functions in the form of Complex and Hypercomplex Analytic Signal functions or Phasors (created based on Hilbert transformation), and by respecting specific and universally applicable relations between wave-packet, group, and phase velocity. In addition, the rest energy and rest mass are not considered being the part of the wave-packet model (resulting that de Broglie particle-wave phenomenology should only be related to states of motional energy). The meaning of stochastic and probabilistic quantum mechanical wavefunction is equally well-explained, void of most of its probability or statistics décor. Hidden grounds of probability-like behaviors (in modern Quantum theory) are coming from an "in-average" mathematical modeling of particle-wave duality (or unity) phenomenology, using the framework of Probability, Statistics and Signal Analysis on the way that basic conservation laws of Physics are effectively, and "in-average" satisfied. Such Orthodox Quantum theory is (on certain isomorphic way) respecting energy and moments conservation laws, while an instantaneous, real time-space-dependent signal phase is not considered, or not known, because of the nature and limitations of applied mathematical modeling, and because of presumed stochastic, ontological nature of mentioned phenomenology (without taking into account that elements of condensed matter in our Universe also have spontaneous tendency to

communicate and synchronize, internally and externally). This is producing “magical effects” that after certain time of playing with isolated particles (regarding diffraction experiments) we get results like experimenting with waves in fluids. It is also explained why present probabilistic wave function modeling works sufficiently well (since it is created as an isomorphic mathematical processing, that is conveniently hybridized and effectively guided by principles, concepts and laws already well known in Classical Mechanics, Mathematics and Signal Analysis). The new natural *fields unification platform*, proposed here, is initially based on exploiting united multi-level analogies and basic continuous symmetries between different natural couples of mutually original and spectral, or conjugate domains (such as Fourier-transform couples: time-frequency, momentum-position, electric-magnetic charges, angle-*orbital-momentum*, etc.), being followed by an appropriate conceptual and mathematical upgrading and integration into Physics.

Uncertainty relations (in this book) are also generalized and treated as the matching (or mismatching) absolute intervals relations between original and spectral signal durations (and between their elementary parts, similar as being treated in cases of signals synthesis) while explaining and extending this concept much further than presently exercised in Orthodox Quantum theory (since we can apply mentioned relations both to a micro and macro-Universe). Consequently, the particle-wave duality concept, *here presented more as the particle-wave unity,* is extended to any situation where motional or time-dependent *energy flow* is involved (regardless of its origin). This way, any change of motional (or driving) energy is immanently coupled with associated de Broglie matter waves, creating inertia-like waving effects, where mentioned inertial effects could manifest gravitational, mechanical, electromagnetic, or some other nature, depending on participants involved in an interaction. Something similar (regarding producing matter waves) is also valid for linear and spinning, or orbital moments' variations. In fact, in this book all kind of ordinary, mechanical, and electromagnetic oscillations and waves, known in Physics, are considered as forms of de Broglie matter waves.

Using an oversimplified formulation, in this book we will find that de Broglie matter waves always have their stationary, stable, or “packing”, standing waves places when rotating inside number of micro-domains inside of particles' structures (or inside atoms and elementary particles). Such internally captured matter-waves structure is also naturally externally extended, active and connected with other masses and atoms (this way explaining Gravitation), and it exist in all situations when particles are changing their previous energies or states of motions. In other words, particles are matter waves creations when involved matter waves establish kind of self-closed, stationary, and standing waves, rotating and spinning structures, this way becoming “self-localized and self-stabilized” in a limited space.

In this book (mostly formulated using the language of mathematics), we will avoid complicated mathematical analyses to facilitate the conceptual, intuitive, and common-sense understanding of Physics by a wide public interested in the same field. Thus, the author hopes, the initial scholastic and dogmatic critics of ideas presented in this book could be avoided, to grant the priority to conceptual, common sense, innovative and intuitive understanding of presented ideas (which can be later additionally and better developed). Another objective (of the author of this book) is that this book could serve, as much as possible, as a challenging and brainstorming, inspiring and indicative framework, and reminder for starting new projects and analyses that would be far-reaching and more rigorous then initiated here. Certain chapters of this book have origins from partially rewritten and updated version of the author's B. S. diploma work (cited literature under [3]).

Briefly summarizing, in this book will be elaborated and underlined that Modern Physics, and especially Quantum Theory, would be much different, more predictive, natural, and more causal if:

1. We consider Mobility system of electromechanical analogies (as presented in the first Chapter of this book), as a generally applicable, starting, and indicative platform for creating, modeling comparing and harmonizing relations between different domains in Physics.

2. We accept that all kind of matter-waves, oscillations and vibrations should always have certain ***carrier matter state***, fluidic medium, or spatial matrix, being especially important and necessary for electromagnetic waves and photons when propagating in a vacuum. Certain revitalized, fluidic ether should always exist. In a close relation with ether is A. Einstein Relativity theory, which should be remodeled and reestablished, starting only from (also innovated and better generalized) Maxwell-Faraday electromagnetic theory (see more in

Chapter 3.). Second Order, Classical, Differential Wave Equation is relevant and applicable to all kind of matter-waves known in micro and macro world of Physics, and it is very well connected with Schrödinger family of equations, which can be smoothly developed from the first one.

3. We should present all kind of matter-waves known in Physics using complex (or hypercomplex) waves, signals, and Phasors, which are created as Analytic Signal functions (based on Hilbert transform). See, for instance, in Chapter 4.0, under "**4.0.11. Generalized Wave Functions and Unified Field Theory** ", how Analytic Signal Phasors can be formulated and applied. See also in Chapter 10., similar ideas about wave-particle duality and matter-wave Phasors, being equally applicable and relevant in a micro and macro world of Physics, presented under 10.1 "Hypercomplex **Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"**. *Wave-Particle Duality, as elaborated in this book, has a lot of common-sense grounds, and it could be creatively united, with "de Broglie-Bohm" and "Both-particle-and-wave view", meaning that statistical and probabilistic Orthodox Quantum theory is not the best and only choice for describing microphysics world (see more here: https://en.wikipedia.org/wiki/De_Broglie%E2%80%93Bohm_theory).*

4. We consider quantizing in Physics as closely related to situations, structures and motions with intrinsic periodicities, resonance-related events, and structural standing waves formations. Generalized quantizing situations, relevant modeling and rules should be described by Kotelnikov-Shannon-Nyquist-Whittaker signal analysis and be equally well applicable in a micro and macro world of Physics (see more in Chapters 4.0 and 10. and in literature under [57, 58, 59, 117, 118 and 119]).

5. We need to significantly evolve with our conceptualization, understanding, modeling, and classification of fundamental Natural Forces. Couple of presently considered natural forces are too far from having good foundations.

6. We need to accept that certain Physics related theories and concepts can and should be reworked, redesigned, and reestablished in their fundaments, regardless how many publications and official recognitions are already waisted or invested there, until present time. This is especially applicable on Quantum and Relativistic theories.

7. We better apply Probability and Statistics in the final modeling and results-concluding states of certain deterministic and tangible conceptualization (related to Physics) for the purpose of better results presentation, than in its very first and introductory, ontological or founding steps. Statistics and Probability are and can be safely applied in Physics only if mathematical conditions for such accounting and modeling are satisfied (meaning to be applied on naturally tangible or measurable, sets with big number of mutually identical elements). Statistics and Probability should not have any relation to ontological meaning, modeling, and fundamental conceptualization in Physics, but could be easily connected to realize a perfect accounting and results reporting regarding global energy balance, and to an averaged quantitative satisfaction of conservation laws valid in Physics (see related thinking in Chapter 10.). Parseval theorem or identity is also the mathematical way expressing an exact energy balance or accounting between different, mutually conjugate spectral domains.

The short summary describing the foundations of Electromagnetism, Gravitation and Wave-Particle Duality, what is the content of this book, is an almost trivial statement that "***atoms are the origin, sources, and initial structures of everything what we find in a macro world of Physics***". Developing such concept, we can specify its meaning as follows.

1. There is a smooth and continual, omnidirectional natural fields and matter waves connection, communication, synchronization, and transition from atoms towards other atoms, masses, and all cosmic macro structures and vice versa (including biological species).

2. Atoms are stable packing formats or structures composed of electrons, protons, and neutrons, also presenting sets of energy states composed of spatial-temporal resonant structures.

3. Atomic, intrinsic oscillatory states, systems, or internal resonators, are characterized with self-closed and self-stabilized standing matter-waves, being also parts or structural resonant content of energy states of electrons, protons, and neutrons.

4. Resonant mechanical, electromagnetic, and atomic states are mutually, externally, and internally communicating and synchronizing by exchanging photons and other matter waves.

5. Mutually identical (by spectral overlapping), and spatially separated resonant states, systems or devices are susceptible to realizing fast resonant, mutual synchronization.

From such initial matter states of atoms, everything else what we know within our Universe is being smoothly and causally developed or created, as for instance:

a) Electrically charged atom constituents (electrons and protons) are creating electric fields, both inside and outside of atoms.
b) Any kind of spatial-temporal motion of electric charges is creating associated electric currents and magnetic fields, also both manifesting within atoms and externally (within a macro cosmos).
c) Electric and magnetic fields, currents, fluxes and charges manifest number of mutual relations and mutual derivatives presently described or modeled by Maxwell-Faraday and Lorentz electromagnetic theory.
d) Everything what we specify as Wave-Particle Duality and Matter-Waves phenomenology, has its initial roots, models and analog states within atomic states and structures, being valid equally for a micro and macro world of Physics.
e) Gravitation presents an external “electromagnetic extension” of forces and fields acting within atomic microstructure towards other atoms and masses, being presently scientifically described by Newton-Coulomb force law. In simple terms, Gravitation is a kind of electromagnetic, or rather magnetic, inductively coupled systems of mutually attracting masses. Gravitation is also creatively, imaginatively, and intuitively described by Rudjer Boskovic’s Universal Natural Force law, and by Nicola Tesla’s Dynamic Gravity theory, which are mutually complementing, and being still conceptually richer sources of conceptual inspiration compared to both Newton and A. Einstein Gravitation.
f) All other phenomenology and laws of Mechanics (and Physics) are creations, consequences, and properties of mass-energy-moments interactions, with deep electromagnetic and atomic origins.
g) Mathematical chapters related to complex and hypercomplex, analytic signal phasors, vector algebra, and differential geometry are fully applicable to overall phenomenology in Physics where we operate with motions, fields, and vectors.

This book is created and structured as an open-end, ongoing brainstorming draft, where a lot of imaginative, creative, unbounded, and positively colored goodwill should be implemented to read it and get its messages correctly. The book is still unfinished, and many of its chapters should be mutually better presented, united, and organized, because the author initially started to write it sporadically and without uniting strategy (as a hobby, and unprofessionally), from his early student days (about the year 1970), not really finalizing it until present (since this is still an evolving and very large problematic). Even in such unfinished, and not well-synchronized state, the book contains many challenging and indicative conclusions, messages, proposals, and possible innovations, that the author decided to prepare it in a form that will be understandable and motivating for others involved in the familiar kind of thinking (since such book cannot be finalized by one author, and for the duration of one human life). For strict, already fully convinced, too serious readers, who are officially certified and convinced experts in the field of contemporary Orthodox Quantum and Relativity theory, this book and its author would be an easy target (for all kind of critics), but for somebody who is creatively reading and catching (with an intellectual flexibility) only challenging concepts, ideas, and innovative proposals, this could be quite a different intellectual and motivating experience. Simply, such a large *problematic cannot be properly and fully elaborated by one person, only during an average human life, and this is the main reason why the book is given to the public in its present state. The author intends to make systematic improvements, upgrading, and unification of all chapters, as long the nature related to performing such activities would allow him. All suggestions and collaboration proposals are welcome.*

Miodrag Prokic

CONTENTS

Appendix:

INTRODUCTION

The main subject of this book is related to the formulation of an updated theory of particle-wave duality ***(or particle-wave unity)***, but indirectly this book also creates or opens the window towards a new platform regarding an innovative understanding of Gravitation, and to natural fields and forces unification theory. Deep roots of mentioned innovative understanding of Gravitation (and united fields' theory) can also be found in the heritage and publications from Nikola Tesla and Rudjer Boskovic (see [6] and [97]). The principal attempts in this book are presenting and exploring items and statements such as:

-**Gravity and Particle–Wave Duality** is based on specific couplings between linear and angular (or rotational) motions and associated electromagnetic complexity, initially originating, and extending from atoms towards other masses and atoms. In other words, the field of gravitation together with its (still hypothetical) torsional field components could be a kind of specific electromagnetic field structure, electromechanically coupled with different mass-energy-momentum states (within a global, spatial, and temporal standing-waves structure of an oscillating macro and microcosmic universe). Briefly summarized, we can say that de Broglie matter-waves and wave-particle duality conceptualization is applicable both to a micro and macro world of Physics.

-**Mass is specific matter-waves energy packing or formatting state of agglomerated atoms** (internally structured as number of mutually coupled micro domains and atoms, all of them being like ensembles of spatial standing waves or resonators, on a self-closed area). All mentioned micro domains dominantly or ontologically have an electromagnetic origin, based on self-stabilized, self-closed, mutually well synchronized matter-waves formations and couplings of moments-energy-mass spinning states.

-**Matter Waves,** are intermediary and transient states that are realizing unity, synchronizations, and couplings between atoms and masses in linear and angular or spinning motions within spatial-temporal structures, presenting dual, mutually orthogonal, spatial-temporal formations of waves-alike energy-momentum states (***participating in communications and self-organized resonant synchronizations***) between different matter entities and states (in a micro and macro world of Physics). Convenient superposition of matter waves is creating particles.

Presently (meaning from the beginning of the 21. century) official science is still considering the biggest achievements of Physics as belonging to several islands of knowledge: 1° **Mechanics** including **G**ravitation **T**heory (**GT**) and Einstein **R**elativity **T**heories (**RT**), 2° **Maxwell Electromagnetic Theory** (**EM**), and 3° **Quantum Theory** (**QT**), including all surrounding theories and mathematical tools in connection to them. Any new theory regarding particle-wave duality (including the one from this book) should also be placed in the same frames of **EM**, **GT**, **RT**, and **QT.** In addition, all new formulations, and updates of such (matter-waves) theory should start from already known works and results related to L. de Broglie matter waves hypothesis and M. Maric - A. Einstein, and Max Planck quantized energy concepts. *In this book, the renewed* ***P**article-**W**ave **D**uality **C**oncept (**PWDC**)* *is presented as the most important link (or necessary background) towards conceptualizing Universal and United Field theory. It is also shown that there could not be any* ***PWDC*** *manifestation without the presence of a certain two-body (or many-bodies) field (or energy and moments) interaction, meaning that* ***PWDC*** *is a product and a consequence of existing field couplings, interactions (and space-time evolving forces) between moving objects, when certain state of motion is advancing (while respecting all natural conservation laws). If we could imagine a moving particle in an "absolutely empty space", there would not be any* ***PWDC*** *manifestation. Of course, here we have in mind electromagnetic and mechanics (including gravitation) related field effects (because there is still nothing else easily comprehensible, explicitly and in details fully confirmable, conceptually, and logically tangible on a unique way, and experimentally and* *theoretically known and explicable with no doubts and ambiguities; -at least not in a macro world of Physics). In addition, we could say that without matter rotation and spinning (including the presence of different torsional matter-waves field components) associated with linear motions, particles cannot be created. This is kind of natural parallelism and intrinsic coupling between the mechanics of motional masses and analogical electromagnetic effects, where electric charges in linear motion are creating surrounding solenoid-shaped magnetic field lines. In fact, here, the use of the term* ***particle-wave duality****, presents only a temporarily given name, and in this book, it is shown that the much more convenient formulation would be the* ***particle-wave unity*** *(and it will be explained later how and why particles and wave properties are naturally and causally united). Any form of small and big masses, including living biological species is created thanks to universal tendency of matter and elementary mass ingredients to mutually communicate and synchronize, using intrinsically associated matter waves and surrounding natural fields.*

It is known that Einstein (and others) tried to make the unification of **EM** and **GT** in the frames of **RT**, without significant success, most probably because of, at least, two missing conceptual components that created the biggest and unsolvable problems to Einstein's efforts, as for instance:

1. **SRT** (**S**pec. **R**elativity **T**heory), besides having some of the challenging assumptions, and unusual or artificial conceptual aspects, as sporadically criticized from different sources, is also not addressing rotational, angular, and other accelerated motions. Such (accelerated) motional elements inseparably exist in different forms, associated with linear and angular motions, including other forms of inertial motions (still not established in Newtonian mechanics), and exist as intrinsic (structural) properties of all elementary particles and atoms (such as spin and orbital moment characteristics). In fact, the more modern (renewed and future) **SRT** and gravitation-related **GRT** should mostly be a kind of direct and unique theoretical consequence or mathematical product of an improved and renewed, better-established **EM** (since present **EM** theory still has a space to be better or more coherently, more symmetrically and more generally formulated, compared to present Faraday-Maxwell-Heaviside Electromagnetic Theory). It is also predictable that new **SRT** and **EM** would be radically modified, united, and upgraded (compared to the old, original ones), presenting a new and much more generally applicable, updated **EM** theory. Similar conclusions are also applicable regarding general Relativity theory **RT** or **GRT**, since spatial-temporal deformations around mass-energy agglomerations can also and dominantly be related to electromagnetic energy concentrations, and Gravitation could easily be a form of natural Electromagnetic activity or state around masses.

2. **EM** mathematical foundations (known within present forms of Maxwell equations), besides having few challenging and critical aspects (as sporadically announced from different authors), should also have "rotational or torsion field" components, as well as electromagnetic waves and fields in certain circumstances should have longitudinal electric and magnetic field components. As we know, **EM** fields and electric charges are omnipresent "inside and around" almost all constituents of our macro and micro universe (like in atoms, different masses, and solar systems), and it is the fact (maybe still not sufficiently and very explicitly exposed and recognized) that **Maxwell equations are originally assembled having in mind number of conceptual similarities between electromagnetic and fluid mechanics phenomenology**. Unfortunately, in the contemporary physics it is no more fashionable, acceptable, or defendable to deal with a space-texture, matter-matrix, or some other close to ideal, carrier-fluid (formerly, long time ago, named as an ether), where electromagnetic waves and fields are propagating like waves in fluids. Anyway, form of Maxwell equations analogically corresponds to fluid-mechanics equations (where we always know what the carrier-fluid is). We also know that physical vacuum has measurable, finite or fixed properties like dielectric and magnetic susceptibility (or permeability), which are mutually (at least mathematically) well coupled .

3. .
4. in a form of maximal light (or photons) speed c. Consequently, Maxwell equations should have some other, extended similarities with the phenomenology known in fluid mechanics, and should evolve towards formulating more (mutually symmetrical), united and more mutually dependent electric and magnetic field entities, being at the same time electromechanically coupled with mechanical waves and vibrations, since all mechanical and electromagnetic oscillators, resonators and waves are respecting or being mathematical solutions of the same Classical, second order, partial differential wave equation. Evolution of such (renewed and updated) generalized Maxwell equations will eventually generate the renewed **RT**. In fact, an innovated **EM** should take the higher practical and theoretical, leading position compared to a new **RT**, which would then remain as the simple, redundant consequence of such upgraded **EM**. *For instance, if we create transversal mechanical oscillations in solid wires, to be on some way analogical to electromagnetic waves, and to photons propagation (meaning being perpendicular to the direction of waves propagation), we will get acoustic or mechanically oscillating effects that are analogically associating on electric currents in metal conductors, or being like propagation of light in fiber-optic waveguides, regardless of wires length and shape (author's experimental experience in ultrasonic technology).*

Briefly saying, both **RT** and **EM** should be (firstly) upgraded and modified (or replaced by more advanced theories) up to the conceptual, mathematical, theoretical, and factual level when they would become much more mutually compatible for unification (otherwise, all efforts to combine them at present premature state will be largely meaningless, and like an artificial patchwork). Similar comments can be given regarding extended merging between **QT**, **RT**, and **EM.** Anyway, we cannot easily unite the theories that are mutually still not sufficiently compatible and comparable for unification, because such theories probably have (apart from all good sides) some of *only particularly valid*, missing, or wrong building elements, artificial or unnatural, assumptions and/or weak concepts, regardless how successful each of these theories is in its own domain. Such theories (as **QT** and **RT**) in the present state of the art are often useful mostly for explaining already known experimental facts, and could have some of not completely provable or verifiable, but mathematically well-functioning complexity (which, from the point of significance for Physics could be partially irrelevant or wrong, or just temporarily convenient for some local, "in-average" fittings and "targeted or forged" mathematical modeling). *There is also an example of seemingly successful hybridization, merging and fittings between **EM, RT,** and **QM**, known as **Q**uantum **E**lectrodynamics (**QED**), mostly realized without using Statistics and Probability mathematical tools. **QED** is in many cases experimentally verifiable with very high precision, but still without giving complet insight to profound and unique, ontological, conceptual, and qualitative understanding how **QT**, **RT**, **EM**, and Gravitation could be naturally and smoothly united.* Newtonian **GT** and A. Einstein **RT** address the same field of Gravitation on different ways, with similar practical success (where **RT** is much mathematically more complicated and impractical), also both having similar weak areas, and in some extreme astronomy-related cases, number of official mainstream science participants are arbitrarily proposing and generating almost impossible, improvable, or artificial assumptions regarding some dark mass and dark energy. In this book an upgraded, new **GT** is proposed to be related dominantly to specific manifestations of **EM**.

After reading recent publications from Mr. Thomas E. Phipps, Jr., [35], **regarding contemporary Relativity theory (versions published after the year 2000) it should become clear to everybody (from many fundamental points of view) that days of dominance of contemporary RT in Physics are counted. Also, the same is applicable until certain levels to present Electromagnetic and Quantum theory.** Of course, there are modern and challenging field-theories that are additionally addressing desired and expected united fields theory, as Superstrings or M-theory, but this book would deal mostly with common grounds of involved and basic **PWDC** concepts, for if grounds are well established, further theoretical advances based on such grounds should have much bigger significance and applicability, including transition to string theory concepts (see much more in Chapters 4.0 and 10.).

Most of well-established and already classical theories in Physics started from number of observations and empirical experiences, being gradually upgraded, rectified, and better understood by experimenting and using intuitive, analogical, inductive, and trial-and-error methods, until big quantity of repeatable and verifiable data was collected, and when the conditions for generalizations and deductions on a higher quality level appeared naturally. The most important and relevant are mathematical formulations and generalizations when certain natural theory will be formulated and smoothly connected to other surrounding theories and be considered as a well-established (especially when being additionally extended and confirmed from other theoretical platforms). This is, for instance, the case of Classical Mechanics and Thermodynamics. ***Using such Classical Physics' theories, we could safely make deductive and cross-platform, interdisciplinary or multidisciplinary comparisons and conclusions***. We cannot say that this is also and completely (and without some doubts) the case of Relativity and Quantum theory, since mentioned theories have structural elements with assumptions, postulations, artificially added mathematical elements, and somewhat abstract (not completely natural) operating rules. ***Deductive conclusions based on exploiting such theories are always until certain level challenging and suspicious, regardless how well being supported with abstract mathematics, and how well and vocally advertised by their famous promoters (since some "natural or conceptual ingredients" are simply missing there, or being replaced by an "artificial and abstract reality mapping", imaging and different functional transformations, which are not always verifiable, deterministic, reversible, and as 1:1 mappings).***

The new **p**article-**w**ave **d**uality **c**oncept (**PWDC**) in this book is presented highlighting the following step-stones:

A) First (as a relatively low and basic level of new ideas initiation), a multilevel system of electromechanical and other physics-related analogies is summarized and reestablished (in the first chapter of this book) indicating and generating conceptual elements, ideas, and hypothetical predictions regarding new **PWDC** and fields' unification framework. *The author of this book presents and defends the philosophical platform that well-established, mutually equivalent, symmetrical and/or similar, physics-relevant mathematical models (or mathematical structures) should describe the same already-united reality (only seen from different theoretical perspectives, from different spatial projections, or dimensionally presented using different sets of coordinates, or different cross-sections).* In fact, everything in nature or the Universe we are part of, is already united and cross-linked on many ways, like overlapping multiple layers of conceptualization and presentation, but in some cases of contemporary theories related to Physics, we are still not able to formulate a good unifying theory. In other words, just to give an example, *Mechanics is overlapping (or internally absorbing) underlaying Electromagnetic reality*. Starting deductively (or also philosophically) from the fact that **unity in our Universe already exists** (with or without our theoretical, practical, and conceptual approach to it), we could ask ourselves what the best and easiest strategy to make our specific and separated knowledgebases, and our physics-related theories, mutually more and better-united, is? The answer here is, to use multilevel analogies and symmetries, this way establishing an important indicative and "intellectual filtering process" for testing, correcting and better formulating old and new theories. Consequently, if we can show that today's "**probability framework"** of **QT**, or important concepts, assumptions, and models regarding **SRT** and **RT** are just specifically isolated cases among other (mutually equivalent, isomorphic, or better) mathematical concepts and structures, that are equally, or sufficiently well, or better describing the **QT** and **RT** world, we could also materialize other (non-probabilistic, or non-relativistic) pictures of the **QT** and **RT** world. In addition, we could explore other Space-Time and Relativity related concepts, without applying the same sets of initial assumptions, as presently practiced in **QT** and **RT**. Let us formulate the same opinion differently. For instance, we know that contemporary way of applying **QT**, **SRT**, and **RT** on the practical and mathematical level is producing number of useful and sufficiently correct results and predictions regarding certain set of experimental situations (or in some cases explaining what, on some different way already known facts are, which would be later retroactively assigned to unique achievements of **QT**, **RT**, **SRT**). This is a kind of semi-false and artificially forged confirmation that mathematical frames of mentioned theories are, maybe not mutually well united, and not the best, but being (in analyzed, separated cases) valid, and sufficiently applicable (of course, in their own, limited domains of definition), regardless how many initial assumptions and weak starting points such theories could have in their foundations. Of course, this would stay valid and correct unless one day we find a new experimental (and mathematical modelling) situation and facts that could not be explained using such older **QT**, **SRT**, and **RT**. It is also safe to say that mathematically we could have sufficiently correct final concepts (theories, results, or formulas) in modeling or explaining certain Physics-related event, even if we started from partially wrong assumptions, and on the way of developing our mathematical models, we introduce some other, maybe again partially wrong, but interactively, and iteratively correcting steps. This way, we are canceling errors linked to older erroneous assumptions, eventually creating new, sufficiently useful models and correct data fitting (after applying a few of such iterative, self-correcting steps), what could partially be the case of some here mentioned and contemporary Physics theories. Wrong, but working sufficiently well, is nothing new in science, and one of the great examples is the Ptolemaic geocentric system that lasted almost one millennium until certain more advanced thinkers succeeded to replace it with the much better and very correct heliocentric solar systems theory. The biggest mistake in such (essentially erroneous) theories and situations is that eventually, once *when we see that our (final) theory is producing some of useful results, we start glorifying its initial and in-process-made, mistakes-correcting assumptions, unintentionally and falsely presenting most of such steps and foundations as always being correct. Every new alternative theory, dealing with the same (old) problematic, producing most of the same, similar and including some additional results, is by default considered to be useless and worthless, or unnecessary, since old, published papers are already (on some way) describing this problematic and blocking access to others who are advancing with new ideas.* This way, we create situations that some of already established theoretical insights are forbidden and error-proof protected (by mainstream guardians of a contemporary physics) to be challenged or criticized. This could be the situation with certain segments of contemporary **QT**, **EM**, and **RT**. To test existence of such “structural and logical defects” of our contemporary fundamental theories, here favored is a method of using a screening based on the multilevel analogy platform (including generic, basic, Continuous Symmetries), as the most general and most neutral, indicative comparison and filtering framework applicable between different models and theories independently developed in Physics. This way (following analogical situations), new conclusions would be formulated (for instance) by creating and structuring certain mosaic of known and also hypothetical mathematical predictions, concepts, expressions, and equations that could be applied in making mutual and non-contradictory, cross-theoretical and interdisciplinary penetrations, and unifications. If certain crucial assumption exists only in **QT** or **RT** (or in another theory), making them operational, and if we would not be able to apply such assumptions in other naturally related or analogical domains and theories of Physics (where we already have other, well-operating arrangements), this will indicate that such an assumption could be at least partially wrong. It could also be only locally valid, serving the purpose of temporarily correcting some of earlier mistakes, or filling missing conceptualization gaps and spots with whatever fits and works the best within given circumstances. Since our knowledge about Nature is progressive, but advancing slowly, and often unpredictably and randomly (related to means and needs we have in certain historical period), we are always in a situation to process limited sets of data. Consequently, most of our theories related to Physics, or explaining our Universe, are limited and only temporarily, or locally valid, until we explore and explain a wider set of relevant facts and data and incorporate them into Physics by creating new or upgraded theories. The barriers for such natural process are older theories that are already officially established, well mathematically fitted, generalized, and almost dogmatized like assumed to be always valid, based on older limited sets of data and facts (erroneously serving as filtering or qualifying criteria for new attempts to introduce different and in some cases more advanced theories). One of the here-favored methods to facilitate an introduction and acceptance of new theories and concepts in Physics, which are directly competing, affecting, attacking, or remodeling older mainstream theories, is based on utilizing multilevel analogies and symmetries as a good method of screening and verification, as elaborated in this book. If we accept that our universe (including its forces and fields) is already intrinsically and naturally united, and most probably being multidimensional, we could say that whatever we are studying in physics are different layers, cross-sections, projections, analogies, and symmetries within such multidimensional universe. Consequently, the message of this book is that the power of multilevel analogical screening and cross-correlating of mutually comparable, analogical, and coherent results of different theories and accepted assumptions in Physics, should be exploited much more than we presently practice such strategies (of course, while satisfying all conservation laws and/or basic continuous symmetries known in Physics). *Such methodology or strategy is a kind of neutral, always-working, universally acceptable and easy applicable, and it can be progressive, self-guiding, self-adjusting and self-correcting, open-end process for Physics' theories harmonization and unification. Of course, analogies are only indicative and early precursors that are igniting new ideas and concepts, and often pointing towards possible imperfections in already established concepts and theories.* Not accepting mentioned strategy (related to respecting and satisfying analogies and symmetries) would mean that we are still not ready to face the possibility that some of our contemporary theories (in Physics) could be only locally valid, partially artificial and short-living, contrary to how their founders and followers would like to present them. In a process of understanding Physics, or our Universe, we already created such well or sufficiently well operating theories. Some of them we are considering as being fully correct (at least in certain domains where we can make experimental verifications of relevant theoretical predictions). Anyway, some of them are presenting *"limited sets of data and facts mathematical modeling",* without being really and mutually united and generally applicable, but we got used to treat such theories as fundamental, and as fully explaining certain phenomenology. For instance, **G**eneral **R**elativity **T**heory (**GRT**) is being considered as the very fundamental, significant, stable, and valid, regardless newly discovered challenging facts. Consequently, when astrophysicists started to observe discrepancies between predictions of **GRT** and unexpected astronomic facts, the first reflex, and intellectual inertia reaction, has been to say that **GRT** is still overwhelmingly valid, perfect and untouchable, but some new, not visible, not detectable, strange "Dark Matter or Dark Energy" or similar theory-correcting entities should be involved, invented, or imaginatively introduced in a space around us, and this will again keep the dominance of **GRT** as it was until present. Surprisingly, crowds of well obeying **GRT** disciples are still honestly and devotionally searching and writing about what this Dark Matter, missing mass or energy should be, and where it is. Of course, the second and better option could be to modify and update the skeleton and foundations of **GRT** (but this is still considered as a forbidden or a heretic voice in Physics). A similar situation and comments are applicable to **QT**. For instance. **QT** was artificially constructed, canonized, almost dogmatized, merged with Probability, Statistics and Signal analysis (and accepted by voting and consensus). Mass of well-obeying disciples and other faithful **QT** followers are "intellectually forging" surrounding theories to comply with prescriptions and postulates of such **QT**, and this is not always an easy task. Even when creating new, advanced and "should be revolutionary theories" like Superstrings theory, creators of such new theories are, just in case, frequently and almost frenetically, (and without big need and an explicit invitation) vocally and repeatedly underlining that grounds and step stones of such new theories (or advanced theoretical proposals) are not introducing any doubts in the full validity of contemporary **QT, RT** or **QED**.

B) It is also shown (in the third chapter of this book) that **EM** could be upgraded (generating Lorentz-like **EM** field transformations, without the use of any **SRT** methodology and assumptions), where the existence of **EM** angular field components becomes an explicit prediction. By formulating generalized and fully mutually dependent and symmetrical definitions and equations of electric and magnetic fields, charges, currents, and voltages, we can additionally upgrade present Maxwell-Faraday **EM** theory and extend the system of electromechanical analogies towards explaining Gravitation. In this book it is shown that such updated **EM** theory (when analogically and symmetrically extended towards mechanics) starts generating or reinventing the foundations of **SRT**, without a big need to have any support in A. Einstein **RT** (but in the time when Einstein established **RT**, such options were not known).

C) Later, it is shown obvious that every *linear and angular particle motion* is coupled with (complementary and conjugate) spinning motion (indicating that **SRT** should also be upgraded to cover different aspects of rotation ...). In addition, it is made clear that such associated rotational field components are strongly linked to de Broglie matter-wave phenomenology and to action-reaction, or to inertial forces (being an extension of Newton, mechanical action-reaction, and inertia concept to **EM**, **SRT**, Torsion phenomena, and to other domains of Physics). **Even in a purely linear and uniform (inertial) motion of an ordinary particle that has a rest mass, there is a unity and coupling of linear and rotational motions, but we usually do not see rotational aspects of such unity because different aspects of rotation and spinning are internally hidden or captured by a particle rest mass.** Much deeper background of mentioned unity of linear and spinning motion components is the (mathematical and phenomenological) symmetry and unity of mutually original and spectral, spatial, and temporal domains (see more in Chapter 10.).

D) **PWDC** phenomenology is also related to dynamic and transitory waving effects in all situations when or where interacting objects (particles, quasi-particles, matter waves...) are approaching each other, and separating from each other, and/or when the state of motion of a certain system is changing (being valid both in a micro and macro world of Physics). Without considering elements of rotation (within internal structures of moving objects) as a complement to linear motion, it is not possible to explain formation of real (space-time stable) masses, particles, atoms, and their constituents, because such (spinning, orbiting and torsion related) conditions are favorable for "***rest mass formations, particles and masses agglomerations and standing-waves structuring***". We already know that all elementary particles, subatomic particles, photons, and other natural quasi-particles have certain intrinsic spin and/or orbital moments' characteristics, and something similar appears to be also typical for planets, satellites, planetary systems, and galaxies, since all of them rotate in some ways relative to something. *Quantum or resonant and standing-waves nature* of the atom world should be related to energy packing, energy exchanges, couplings and **natural gearing or synchronization** between resonance-alike states with intrinsic periodicities, manifesting as matter waves interferences, superposition, and standing waves formations, among and within particles and matter-waves participants. Consequently, different **PWDC** objects (atom constituents, for instance) are mutually fitting (or packing) into certain unified, more complex, and relatively stable standing-waves object, which would eventually, after formation, obtain a stable rest mass. Even if we sometimes do not know what really rotates, oscillates, or makes matter-waves, certain, maybe still hidden, **PWDC** entity should exist within such background of mentioned matter-waves phenomena. Just categorically saying that everything in our universe, especially in a micro-world, respects conveniently and superficially invented, assumed or postulated nuclear forces, and exclusively follows the "channels and rivers" of mysterious and quantized probability waves, or probability distributions presentable as waving forms, is a common-sense absurd, or fogy and metaphysical statement; -like arbitrarily playing with conveniently invented names, without explicable meaning and content. ***It will be shown***

that de Broglie matter waves already exist inside particles, atoms, and other stable energy-moments states (creating intrinsic, internal particles structures, in their states of relative rest), even before we can externally detect something regarding such PWDC phenomenology, and this is becoming externally, directly, or indirectly, detectable in all situations when particles and other objects are changing their previous states of motions. **PWDC** framework elaborated in this book is equally applicable to micro-world of atoms and subatomic entities, as well as to the macro-world of planetary and galactic systems (being typically applicable within all systems with certain intrinsic, structural, and spatial-temporal periodicities). The meaning of uniform orbital motions, inertia and inertial systems can be well explained only if we consider that such motions are creating (or being part of) associated standing matter-waves. What we consider as quantization in physics (or as quantum world phenomenology) is dominantly related to discrete and/or finite energy-moments amounts, produced in many forms of discretized matter waves packing (like in resonant and standing-waves states and formations), and also related to relevant energy-moments exchanges, inside and between all matter forms and states, molecules, and atoms, where in most of cases electromagnetic photons are involved as quantum carriers of mentioned energy exchanges. In fact, every presently known form of wave motion (such as electromagnetic waves, light beams, electron, and other particle-beams, sound, and mechanical vibrations etc., including forces between them) present cases of matter waves. Mathematics that supports and explains such universal and natural quantizing is related to "Fourier-Kotelnikov-Shannon-Nyquist-Gabor" signal analysis, signals recovery and synthesis (see literature under [7, 8, 57, 58, 59]).

E) In this book, complex mathematical elaborations regarding Fields Unification Theory and modern Topology are avoided. After analogically formulating generalized forms of power and energy expressions, and developing universal wave equations, based on Analytic Signal modeling (almost without using methods practiced in the contemporary Quantum theory), and analogically applying them (deductively and backwards), towards all domains of physics, it will become more obvious what solutions or directions for upgrading of **EM**, **RT**, **GT**, and **QT** will be. In other words, we should be able to create the same wave equations (as known in **QT**), but now going step-by-step from the opposite end, starting from a much more general mathematical framework (which is the Analytic Signal, based on using Hilbert transform, first time introduced by D. Gabor). In this way, (by comparison and analogical screening) possible hidden, conceptual mistakes and missing elements of mentioned theories would become detectable and easy to correct.

F) In this book, foundations of Orthodox Quantum theory (**QT**) are conceptually, mathematically, and intuitively demystified, showing why, how, and when probabilistic **QT** works well regarding **PWDC**. It will become clear that the same results (or even richer analyses and conclusions) could be obtained using a bit different, more general, more deterministic, natural, and analogical mathematical modeling (which is well connected with a remaining body of surrounding physics and mathematics). Thanks to Hilbert transform and Analytic Signal functions, when used for waves modeling, all famous wave equations of **QT** can be developed easily, smoothly, naturally, and without inviting "divine" inspirations and "ad hoc, theory-saving, patchwork attachments" (but, unfortunately, when **QT** was created, formalized, and postulated, Complex Analytic signal functions where unknown). Stochastic philosophy and probabilistic modeling should be grounded on their proper and much less ontological place than the case in the current **QT** is. *Here, all references regarding **QT** are related only to Schrödinger's wave mechanics, since updated **PWDC** concept presented here is also developed around (mathematically) similar wave mechanics. The three remaining quantum theories (Heisenberg Matrix Mechanics, Dirac's Transformation theory and Feynman's Sum-over histories formulation of **QT**) are not analyzed (or mentioned) in this book (and the author of this book does not have necessary competence for such analyzes).* Obviously, since officially accepted, contemporary **QT** is experimentally and theoretically sufficiently well supported (mathematically producing correct outputs in number of situations), this book is explaining that involved modeling and predictions are correct because the probabilistic **QT** is also mathematically compatible and compliant (in average) with all conservation laws of Physics. In addition, we can say that present **QT** is being well generalized and formulated in its dimensionless normalized form, on the way similar or equivalent to the universally valid framework of Probability, Statistics, Signal and Spectrum Analysis theories. Of course, **QT** did not present its foundations using such simplified statements (as mentioned here), especially not by saying that probability-related strategy is compensating missing elements of certain essential, physics-related conceptuality, such as missing rotation and phase elements of involved particles and waves' motions. The proper key or code for creating physics-relevant and productive probability distributions (in the wave **QT**) was found (and in this book maximally exploited and extended) by incorporating the set of basic **PWDC** relations (originally established by L. De Broglie, M. Planck, M&A. Einstein, Schrödinger, Heisenberg...) in classical mechanics. The meaning of the basic set of **PWDC** relations will be explained in chapters 4.1, and 10. The rest of the results **QT** just creatively borrowed from mathematics (mostly from Statistics, Probability theory and Signal Analysis), while imaginatively hybridizing and complying with Conservation laws valid in Physics, this way also reinforcing its own weak areas. The understanding of famous Heisenberg Uncertainty relations will be rectified and more naturally explained (as being valid for micro and macro systems, based only on mathematics). In this book, an attempt is made to go beyond probabilistic **QT**, producing conceptually clearer, more causal, and richer picture about **PWDC** nature of our universe (than Orthodox and Probabilistic Quantum theory is currently presenting). This is realized by showing that an isomorphic, equivalent, and more deterministic modeling of **QT** world is possible, based on using Hilbert transform and Analytic signal functions (and being universally applicable, not only to a micro world of physics). We cannot neglect that existing **QT** mathematically works well (on its way, and in its self-predefined framework, which is somewhat unnaturally constructed). In this book is underlined that probabilistic **QT** should be considered mostly as a convenient "in-average-statistical-modeling", by merging and mutual fitting of different, healthy, and useful, theoretical, and generally valid platforms and already known facts, mixed with certain assumptions for filling some of still inexplicable or unknown and conceptually not captured gaps. **QT** ontologically formulated as a probabilistic theory is giving chances that everything we could imagine as a Physics or Cosmic Universe related event has a certain probability to happen (what is a very trivial and an always valid statement). This, on some way also means that virtual, mystical, metaphysical, ideological, and imaginary situations, creations, or events have always some chances and some probability to happen (even being exceedingly small). This way, everybody could be satisfied with such **QT**, including justifying imagination of people who are too far from any scientific thinking. Probabilistic grounds of **QT** could also be misleading for non-experienced disciples expecting one day to be mainstream recognized scientists. By its nature, Mathematics and Physics are originally established to describe logical, deterministic, natural, and realistic situations and structures in the world of Physics, even when the nature of certain phenomena is dominantly stochastic (like in thermodynamics of gases). We should not go further than that, meaning that Statistics and Probability are practical and efficient mathematical modeling tools for mainstream analyzes and presentations where a big number of mutually similar (or identical) entities are involved. The early signs of **QT** concepts eroding, for instance regarding alternative modeling of elementary particles, can be found in publications from Bergman, David L., and Lucas, Jr., Charles W; literature [16] to [20]; -http://www.commonsensescience.net/papers.html). ***In fact, present-days Quantum Theory will one day be known mostly as a creation of rich and imaginative mathematical toolbox, being also useful within any new, more natural, and better theory of Particle-Wave Duality.***

Contemporary theories *in a vicinity* of **QT**, established in the 20-th century of modern human history on the planet Earth, are in fact more of "**Patch-In** and **Fit-In**" theories, than theories really and strongly integrated into the solid and stable body of Natural Sciences. In some cases, applicable to modern Physics, we are simply being saturated, fascinated or blinded by overwhelming technological advances of powerful mathematical data processing, which are generally applicable mathematical methods and computing techniques (and not at all being an exclusive intellectual property and/or products of **QT**). Since such powerful mathematical processing is being used in Physics, there is a danger of attributing the power of it to specific theories (like **QT**, **RT**…), wrongly resulting in unconsciously claiming that specific theories in question are even more powerful than associated mathematical processing. For instance, well known “Uncertainty Relations” formulated in mathematics (mostly in signal analysis and theory of communications) and then applied in Physics by W. Heisenberg, are quite simple, universally valid mathematical relations between relevant signal durations of mutually conjugate original and spectral domains of any function (known in Signal Analysis independently from W. Heisenberg and **QT**), also being valid for any size of Universe. There are many of such examples that are simply products of generally applicable mathematics, but eventually, retroactively, and sometimes wrongly attributed as the unique achievements and cornerstones of certain Physics’ theories. Similar ***subject and framework replacement strategy*** is also being applied in many ideological teachings and practices (of human beings). For instance, existence of a supreme and untouchable leadership of “heavenly and virtual, metaphysical or iconic entities” is explained, supported, and masked by very tangible, amazing power of mixed, eternal, ritual and/or often superficial effects and messages of architecture, music, paintings, sculptures, associations to different “bibles”, improvable mystifications, and beauty of natural ambiences (and even being populated by some imaginative, virtual and artificial or symbolic members as being masters of different powers). In other words, whatever works well and creates an illusion and confusion of being on some way a supporting proof to divine, virtual, and untouchable creations is welcome in mentioned, artificially assembled theories and ideological teachings. Do we really need to accept or tolerate something like that in Physics is an important question for all of us? At least, now we can freely ask such questions and express our doubts without being severely punished. It was a time (in recent humans’ history) when expressing such heretic statements and doubts was severely punishable; -For instance, “the Solar system including the Sun was “*ideologically forced*” to rotate around planet Earth during many centuries”, and who said that this is not the case could experience a worst-case scenario affecting his life.

It is also the author's opinion that most of today's activities in the Fields Unification Theory are presenting various (sometimes arbitrarily assembled) mathematics and geometry-related combinations. For instance, number of participants in this process are creating or inventing different topological structures, symmetries, mathematical groups etc., while dogmatically and almost religiously defending the old framework of the Orthodox Quantum theory (because this is the membership price to stay inside the official mainstream crowd). It looks that most of contemporary science participants (in Physics) are copying, repeating, and following each other's works, competing in publishing, or presenting “looks like new ideas” (probably in some cases, only for the main purpose of maintaining their academic positions and privileges, or some personal power and existential stability). Many of mentioned publications dominantly and effectively belong to the same old-idea family, which are in most cases useless and forgotten short time after publishing. For instance, modeling of different geometry structures, and then making arbitrary “mathematical experiments”, starting from certain field vectors and their combinations, and testing what could be the consequences after satisfying certain equation, or mathematical structures... and if it would be possible to fit such results into some existing theoretical aspects of physics. In fact, what today’s physics needs are new conceptual breakthroughs, inventions, innovations, revisions, remodeling, refinements of Physics, and better unification of natural sciences theories produced until the end of the 20-th century (of the scientific and technological achievements experienced on the planet Earth).

Our (or humans’) advantage is that we are in a position “*to see*” (detect, measure, make modelling, conceptualize, think about ...) many of facts regarding experiments, interactions, secrets, and laws of Nature; -we see (or measure) elements that are entering into certain reaction, and we see (or measure) results after interaction happens. We can analyze an event (or experiment) inductively and based on analogies, knowing its important starting states, inputs, and other initial facts, or deductively, knowing only results, and going backward to discover initial elements that should create such results. If we apply proper methodology and creative intuition (based on multilevel and well-established analogies and basic continuous symmetries, while respecting Conservation Laws and other universally applicable principles), using language of mathematics, and if we are able to perform fruitful and multilevel "intellectual cross-correlation process", creatively combining well-known experimental and other theoretical facts, we do not need to create countless number of artificial and arbitrary "mathematical experiments", in order to search for explanation of certain natural phenomenology. In addition, the most global picture regarding our existence and our Universe is that all presently known, and maybe unknown fields and forces of Nature are already, intrinsically, structurally, and naturally united, regardless the fact that in some cases we still do not know how they are created and united. Of course, whatever we create in process of our Physics-unifying efforts should be well integrated with the background of the present Physics knowledge, mathematically generalized, and experimentally confirmed as the final step of the creation of every new theory. The big weakness of modern and official natural sciences interpretation is a tendency to show that certain theory (of big importance) is already constructed and optimized as the very convincing, well-established, very-stable, powerful, and being almost as a final knowledge (what is largely applicable to **EM**, **RT**, **GT**, and **QT**), and that only small improvements, and insignificant upgrades could be in front of us. ***Here, one of the objectives is to show that a lot of significant, unifying, and fundamental reconstruction work is in front of us regarding contemporary science interpretation of mentioned theories, and wider.***

In this book, most of the ideas, proposals, and hypotheses are established until the level of presenting conceptually, mathematically, and intuitively clear (common sense) physics-related picture, or easily understandable and sufficiently completed, preliminary project foundation. Such concepts are often over-simplified to serve as starting (intuitive or challenging) platforms for some future, more systematic, and more professional work (since the author of this book is only a *passionate and curious amateur* regarding most of the items discussed here). *In fact, the author's expectation here is, that eventually, somebody with similar imagination and interests would continue developing ideas from this book regarding challenging and evolving Physics-reconstruction projects (since almost everything, except Classical Mechanics and mathematics naturally developed based on Physics and Nature observations, should be significantly reestablished).*

The multilevel analogy platform, as here favored strategy, is chosen (by the author) because of a quite simple and obvious reason that could be explained as follows. Our contemporary Physics is composed of several complex and voluminous theories or *chapters*; -each of them formulated as almost self-sufficient and self-consistent theory, working well in its self-described backyard. In addition, each of them has been developed and mathematically modeled (often independently from other physics chapters), during a relatively long time, involving contributions of almost countless number of research workers and scientists. To contribute something to modern Physics (or to criticize something), we should devote our life and career only to a certain specific item or domain, often not seeing the global (much wider) picture and without knowing where that subject really belongs. For instance, it could happen that some parts of scientific establishment of leading professors, scientists, and some of their non-critical and well-obeying (in some cases existentially dependent) followers and students (behind each Physics *chapter*) could try to suppress, accuse, or eliminate any of non-professionally, or too originally formulated contributions, or critics. The usual answers of conservative science establishment (to new challenging ideas and concepts) are that existing scientific and theoretical concepts are already functioning well and being scientifically fully proven and established, and it is useless, not necessary, or it presents nothing significant to offer different, or alternative concepts and proposals to explain additionally and differently what is already known and explained (or published) earlier. It happens that the strongest opponents of new and challenging ideas already published a lot of books and papers to support their (old fashion or obsoleted) teachings that are formulated to be in full agreements with earlier publications of their teachers and theory founders (and why to challenge or change such already stable states). In most of the cases, mentioned opposition already has respectful professional positions to practice mainstream and officially legalized teachings, and why would they tolerate "attacks or perturbations on their foundations and property", or why to take a risk to be challenged or discredited by some still not certified and adventuristic, innovative incomers. We also know that any bigger waving against any kind of stationary mainstream has never been easily accepted in human history, life, ideology, politics, and science (and this is also part of the natural law of inertia regarding stationary and uniform motions both in mechanics and electromagnetic theory). To open new scientific perspectives and new ways of thinking, without creating big intellectual energy dissipating séances, the easy, common, and painless way (proposed here) to generate indicative and hypothetical or innovative ideas, is to apply "*analogical screening and rearrangements*" in physics. Later, we will be able to apply more efficiently, other "heavy weapons" of modern physics and mathematics, such as conclusions, predictions, and generalizations based on Symmetries and deductive insights.

Observing the same problematic (meaning Physics) from another (philosophical) platform, we already know that we are an integral part of a naturally well-united universe, and consequently, all Physics chapters (if correctly formulated) should intrinsically present well-united, mutually synchronized, and mutually complementary descriptions of the same reality. Unfortunately, we also know that this is still not the case with some "departments and chapters" of our contemporary Physics, and mainstream guardians of such old theories are still not ready to accept radical attempts for revisions.

What could be the most common and binding skeleton to search for important grounds and strategies for such unifications in Physics? Author of this book is proposing (only as a starting and an indicative platform) systematical formulation and utilization of *mutually compatible and coincidently applicable, multilevel analogies and basic, continuous symmetries* found between different chapters or domains of Physics (in this book addressed in the first chapter). Taking such analogies (especially missing and not-harmonized positions inside mentioned analogies), as an indicative argument to introduce or propose new ideas, hypothetical concepts, and unification platforms, is largely neutral, simple, and painless strategy to address a large and interdisciplinary set of mutually familiar phenomena and facts that will be additionally analyzed and updated. This way, we can avoid possible resistance and sanctions of some minor parts of officially recognized mainstream establishment and its administrative masters, or "guardians" of specific Physics *departments*. This is an easy and neutral strategy because analogical predictions are universally acceptable, neutral, impersonal, and mostly good as indicative starting spots for new hypothetical assumptions, but also, in too many cases shown as correct or particularly useful if properly established and applied. This way, we could always say that by following certain analogy platform (and understanding that Nature is always better and more united than we presently know), we would get new, potentially significant, interdisciplinary predictions in Physics. Of course, nobody is personally *culpable* if later it would be shown that some of such analogical predictions are for some other reasons not fully correct. Applying and developing systems of multilevel analogies, iteratively, systematically, and by testing them experimentally and theoretically, we shall be in every new step closer to a more-correctly unified Physics. This is exactly the greater part of the strategy applied in this book *(to make the Physics More Clear, Logical, and internally Analogical).* We also know that theories based on exploiting different Symmetries could bring even better unification platform in Physics (*since all conservation laws are "married" to continuous symmetries*). However, before we apply any strategy based on Symmetries, we should have a "*healthy and sufficiently large"* set of relevant elements, and good mathematical modeling frame for playing with Symmetries. Saying the same differently, science and new Physics will profit a lot from combining Multilevel Analogies and Continuous Symmetries with properly established mathematical framework, and Laws of Physics. This should be the best mutually complementary research tools. The theory of Symmetries is already well developed in mathematics and applied a lot in contemporary physics. It is covering very abstract spaces, where it is sometimes too difficult to make simple and clear conceptualizations and correlations between certain mathematically described symmetries and real-world situations. *We should also ask ourselves if there is a hierarchy among the number of different symmetries we are able to construct mathematically, because only certain symmetries are natural, stable, always applicable, and universally valid, like continuous symmetries (regarding the world of Physics), and all other have only some abstract and artificial level of applicability and stability (like in a games theory).* Since we are (most probably) living in a multidimensional universe, where we are still not able to conceptualize well the other, non-perceptible or still non-detectable dimensions, we are obviously making mistakes or experiencing problems when establishing symmetries (even without knowing that such problems exist). This is one of the reasons why in this book, multilevel analogies are considered as the indicative "guiding channels and platforms" for building every other higher-level conceptualization. *Of course, creating analogies and making predictions and conclusions based on analogies is also nothing new regarding any aspect of humans' activity (probably also valid for some of other species). There are already numbers of analogies we have been using in different scientific disciplines. Here, we will analyze only analogies relevant for physics and among the number of them it will be selected the most relevant and the most coherent set of multilevel analogies that are universally applicable from different points of view (this way creating a commonly acceptable consensus and toolbox for improving, uniting, and advancing fundamental theories in Physics). In addition, basic continuous symmetries that are applicable to such multilevel analogies will serve to increase the power of conceptualization, unification, and predictions in Physics. If we do not have such simple methodology, and "guiding channels" in formatting and addressing our knowledge about Nature, we could be (intellectually and creatively) lost among countless number of abstract options, dissipating our intellectual energy, and being not focused on our objectives and proper scientific activities (where Orthodox Quantum Theory could be an instructive example). Mathematics (combined with analogical perceptions) is offering the best indicative, screening, self-correcting and optimizing tools applicable in Physics. Mathematics is also the language of Physics originally discovered and developed by observing and explaining the Nature, and by solving life and technology related problems. Of course, such Physics language is always in process of developing and improving (evolving towards clearer and more general methods, pictures, and concepts for describing Nature). In many cases, Mathematics is the tool that can work deductively for making predictions, extrapolations, and interpolations to fill the missing spots in Physics related knowledge. Mathematics is (still) the language of Nature discovered and established by humans, since we know that dinosaurs, crocodiles, and other animals (known on the planet Earth) never evolved to a sufficiently high level of self-conscious and creative thinking to master mathematics. Humans are continuously developing Mathematics (from many different aspects), but the parallel process is that Mathematics is also evolving and becoming real, natural, humans-independent set of theories, tools, concepts, logic and language, and way of describing and thinking about Nature and Physics' matters. Such mathematics (based on mastering and describing Physics) should be learned and accepted by everybody who would like to participate in further understanding, discovering, and mastering the Nature. Obviously, **a quantity** of mathematics related works is already accumulated sufficiently to make transitions and generalizations towards new **qualities**. If Nature or Universe around us is the ultimate Reality to be discovered, Mathematics is our best tool, guiding framework, language, logic, and philosophy for addressing such objectives. How close our mathematical representations of an immediate (present situation) Reality, compared to an eternal Reality of Universe are, presents another question, but a permanently evolving trend is that mathematical Reality would (and should) continuously evolve closer to eternal Reality. Whenever we deviate too much from natural and realistic concepts and healthy mathematical modeling, we will enter a zone of "not-sufficiently clear Reality" or Virtual and artificial Reality (which could also be mathematically operational in its own frames), and this is like what contemporary Quantum theory followers are exercising and promoting. Regarding mathematics, we are already very close to the situation that we could start formulating rules and using Mathematical Logic, Artificial Intelligence algorithms, and standardized, symbolic mathematical language for replacing diversity of humans' languages, as presently used on our planet (serving as humans-assisting artificial and computerized intelligence and as a new universal language for all kind of communications). This way, our present linguistic creations, interpretations, and* *communications will be transformed into very clear, exact, and explicit mathematical formulations, even including philosophical domains.*

Briefly simplifying and summarizing, we could say that too many people being currently on some way involved in natural sciences are unnecessarily creating almost arbitrary, statistical, and probabilistic interpretations of our Universe. In fact, such quasi-scientists are intentionally, artificially, and superficially playing with some semi-false quantizing formulations, considering such wording as having a ticket to be accepted and professionally, socially, or formally recognized in a contemporary mainstream theater of Physics. Another group of non-scientific practices in contemporary sciences is related to verbal and descriptive mystifications. For instance, when we miss a good explanation of how some particles pass certain energy barrier, we simply invent a name "tunneling effects" and such name is now falsely serving as an explanation. Alternatively, when we notice that contemporary theories about Gravitation manifest some inexplicable difficulties regarding certain cosmic observations, we simply invent new names and labels, such as "dark energy, dark or hidden mass", serving as false explanations. Moreover, the problem looks as solved (or well addressed), but a later-created army of obeying disciples hopelessly consume and dissipate different economic means (of our planet), and invest their physical, intellectual, and creative energy, searching to find such virtual and most probably non-existent items. Modern Quantum theory has number of such virtual items, and nobody is officially allowed to criticize such practices, since some big masters and guardians of contemporary mainstream teachings in Physics theater initially established and legalized such shaky foundations, to be always taken as reasonable and acceptable explanations and references.

Contemporary scientific (and quasi-scientific) theater is already crowded with lots of new concepts, proposals, theories, and critical opinions about existing official science weak spots, coming from many of *mainstream-science* dissidents (most of them still non-recognized or being fully ignored and marginalized, but some of them having officially acceptable background and good scientific references). Even some of the most famous leaders of official science establishment are frequently self-allowed (when they do not see or find a better choice) to propose artificial, virtual, magic and like science-fiction concepts. For instance, as ideas about dark matter and dark energy, or some of mathematical concepts about multidimensional and parallel universes, or to introduce elementary particles with many of ad-hock imposed properties, or to invent some strange and new (inexplicable) names and relevant vocabulary, for something that is missing or still inexplicable in existing theories. What looks like kind of common characteristic of contemporary science dissidents is that many of them are promoting their visions, theories and concepts based on criticizing certain illogical, internally contradictory, ad-hock concepts, postulates, and assumptions, then explaining step-by-step how certain big scientific personality made mistakes, and how such mistakes could be corrected. In fact, such an approach is in many cases useless or wrong, regardless of its base in some isolated and proper thinking. The reason for such situation is that present physics was formulated through connecting piece by piece of mosaic-like knowledge elements during relatively long time, and in some cases involved inventors did not have sufficiently good, better or any answer regarding explanation of certain confusing phenomena (and they created what they had as the best guess, or as an optimal option at that time). Some of more curious, more ambitious, and faster-going scientific participants simply hybridized available exotic ideas and concepts with remaining scientific body of relevance, sometimes without having enough arguments and without fully respecting scientific logic and existing positive backgrounds (like Niels Bohr did with his atom model). Implementing challenging concepts in some cases was possible (often realized by chance, and by forging desired results to appear as we like to see them) to artificially establish missing factual links, making a theory in question sufficiently operational, and from certain points of view explicable. Later, some of unusually original scientific personalities, got by the "game of chances" some recognitions, since nobody else proposed any or better solution, and instead of being science dissidents they marked the history of physics engraving their names in officially accepted books or specific science foundations. The fact is that some of them introduced good, correct, and long-lasting concepts, and some of them introduced ad-hock postulates and virtual, imaginative creations, or whatever temporarily works sufficiently well in given circumstances. Almost nobody (in the frames of the relevant historical period) was able to distinguish completely who is who, what is what, and how much somebody is on the good or bad side. Later, the same science has been advancing by introducing new fitting and self-correcting ideas, and improvements generating new concepts (often without correcting or erasing obsolete or erroneous initial creations). After applying and combining many of such self-correcting and iteratively improving steps, and by completing symmetries that showed to be obvious, we got present-days physics that is operating sufficiently well (regarding how we are using it and what we are presently expecting from it). This way we also got the situation that some early and maybe suspicious science step-stones have been implicitly accepted as correct (because the final creation has been working sufficiently well), and therefore almost nobody (except some science dissidents) is asking if the early foundations were really and fully correct. Now, in such cases, the official science-establishment is jointly of effectively stating that since we already have something that is well-operational, already accepted and published, it looks unnecessary, time and resources wasting (even implicitly forbidden) to start revising old and early foundations that are no more of big use (what is not quite correct). This could easily pass as a diplomatic refusal for opening new research projects, which could produce different results, and initiate big changes in already established theories. *The author of this book abandoned the way of searching for isolated circumstantial mistakes, defects, and flaws in modern theories-foundations, because the much more effective and rational way is to start from what is already working well and what is generally, analogically and symmetrically applicable to whole Physics. Knowing that Nature or Universe is anyway united (even without our clear and complete knowledge and consensus about such unity), is giving us much weight and power to predictions and conclusions made using analogies and symmetries. Of course, the especially important condition (for enriching and improving Physics) is to establish the best possible and physics-realistic framework of analogies and symmetries, and the objective in writing this book has been to formulate such framework (see more about Analogies in the first chapter).*

Personal reflections of the author:

Most of achievements and eleborations in this book present something what is already on some way published, mentioned, defined using habitual terminology, but in this book many of such, more or less known items, arguments and facts are diffrently, innovatively and creatively rearanged, redefined, modified, and diffrently explained. Such work could be characterized (by very rigid mind warriors and convinced members of Orthodox Quantum theory) as something "déjà vu" (and consequently, scientifically worthless), but in reality, there is no other and better startegy to be used here, since our modern Physics-related theories are kind of hybridized, composite mixture of many scientific and empirical ingredients, and some of metaphisical, artificial, wrong and virtual (non-existent) items, and nobody is completely sure what of that matter is really on a good, fully scientific side, and what is an artificial and non-scientific part of such knowledge. That means, we could always and arbitrarily say that almost everyrhing of certain complicated theory is already known, published, mentioned, including scientific and non-scientific items, this way eliminating people who are trying to make some order and significant reorganizuation in modern Physics (since such people are in advance accused to repeat something what is old and known). The biggest problem in present-days science regarding creative amateurs and hobbyist somehow involved in scientific or quasi-scientific activities is not in personal and conflicting relations and misunderstandings between such "innovative" and not recognized debutants and some of science leaders that are officially recognized and admired within different domains of Physics. They anyway do not speak or use too much of the same language. Anyway, being well educated and mainstream recognized is nothing to do with being original, imaginative, innovative and creative. Diplomas and official recognitions are not always securing being creative and innovative. In addition, amateurs and hobbyist are sometimes too enthusiastic, somewhat naïve, excited, and often not sufficiently supported by their general Physics and Mathematics knowledge, but even such people could be intelligent and creative. Another side of the nature of such situations is that in both groups (professionals and amateurs or dissidents) we can find creative and non-creative, advanced and conservative, intelligent and less intelligent, curious and not curious, and differently educated elements. Anyway, group of creative participants in science is always exceptionally smaller when compared to other groups. For some of them, belonging to any kind of officially and mainstream recognized Science Theater this is only a question of maintaining existential, personal, and social position and prestige (without really and deeply being creatively, intellectually, and scientifically motivated or involved in natural sciences). Problems associated here are much more a kind of natural law of universal inertia and resistance to everything that makes sudden changes and big waves (equally valid in Physics, Biology, Economy, Sociology, Politics, Ideology… the same is also intrinsically integrated within other aspects of human nature and activities). Easy victims in such situations are the most courageous, most creatively advanced, or most naïve science children (if we exclude obvious and bottom-line, trivial cases). This natural inertia and resistance (against significant advances in officially established mainstream concepts) is well masked and supported by multi-layer rules and tricky diplomacy practiced in such situations. For instance, there are rules about how to write scientific papers and how to present new ideas, with ultimate requests to know and to respect all officially accepted (published) references, regardless some of such references are meaningless or essentially worthless, etc. Additional obstacles are in recognition and approval procedures, sometimes deformed towards favorizing particular interests of involved mentors and editors, or stimulating dishonest competition and favoring an economic interest of specific leaders and groups (too keep alive certain well-financed mega-projects, regardless of missing good and scientifically defendable reasons…), or simply related to exaggerated diplomacy, inactivity and bureaucratic prudence in such matters, favoring mostly what brings certain material profit. In most of the cases related to scientific projects, necessity to have certain workforce, and technical and financial means to participate in new and adventurous scientific activities are the biggest limiting factors. Most of such obstacles are not well known and not interesting to creative, courageous and impatient debutants and amateurs in science. Such creative and not very professional people are primarily and dominantly obsessed and guided only by their (often fantastic, not well supported, sometimes poorly presented) ideas and visions (not well understanding, or understanding but not accepting, why anything else of administrative and formal nature is more important than their innovative and logical thinking). In addition, the big secret of all scientific advances is that only freethinking and unconventional, creative minds (not too much saturated with current mainstream knowledge, assumptions, and conventions) are able to bring significant advances, novelties and to ask critical and good questions. *Usually, an old scientific theory is already able to explain correctly a number of experimental facts (on its old-fashioned way, or in some cases like Ptolemy's theory that erronously explained "<u>how the planet Earth is the center of Universe, or at least, center of our planetary system, and all other celestial objects and our sun are rotating around it</u>"). On a practical level (only regarding calculated planetary paths compared to observations from the planet Earth), Ptolemy's theory has been mathematically operational and sufficiently correct (thanks to involved and mutually related or linked periodical motions of planets), but in reality, it is conceptually completely wrong, and as we know, our planet Earth is not the center of our Solar system or our Universe. Anyway, Ptolemy geocentric theory resisted being in an official power almost thousand years… Contrary, a new theory (which should be in competition with an already existing, well established, working, and maybe being a "Ptolemy-type" theory, but we still do not know this), in its first steps, does just the same, producing the same or similar results as already known using older theories (and giving very little of new results). However, proposed new theory could be more general, presenting the same facts on a more elegant, simpler, more logical, and more scientifically acceptable way (maybe introducing new concepts, better mathematical models, and seeing reality in question from different points of view), although in its initial form, cannot be professional and completed enough.* It often happens that some parts of the officially mainstream-recognized scientific community are simply dismissing new and challenging theories saying (using very much elaborated rhetoric and during long-time, well perfected, diplomatic and sometimes hidden, inquisitor's style language) that proposed new theory does not bring something new to science. Anyway, new theory produces or repeats results and facts already known and well explained with an old theory. It is often comented that in such cases, it is useless, forbidden and heretic to introduce new theories and new explanations dealing with the same scientific subjects of old theories. Or, if not explicitly formulated like here (because of usual diplomacy in such situations), the resistance of "officiallyzed" scientific authorities manifests on some other (more hidden) highly negative way against new voices, what creates really painful situation, if somebody is persistent in promoting his new, non-matured, but creative and competing theory. To give an example, the opinion of the author of this book is that both **Q**uantum **T**heory and **R**elativity **T**heory, are operating mathematically well within their assumptions and domains of definition, producing sufficiently correct, or acceptable results (mostly in cases when something is already discovered experimentally, or by chance, independently from **QT** or **RT**, and then such discoveries are retroactivelly assigned to predictions of **QT** or **RT**). If a new, more common-sense, and conceptually richer, and promising (but still unaccepted and hypothetical) theory were given almost one century of development time of thousands of enthusiastic research workers (like the case is with **QT** and **RT**), we could only imagine the consequences of scientific results comparison between such new and old theories. If we were against giving a chances to new visions and concepts in science, it would be too difficult to promote new theories and to bring significant scientific contributions (that are usually coming much later after an initialization of any new theory). The experienced and officially recognized mainstream science veterans are often making formally perfect theories and "science bibles", mathematically well fitted within the texture of relevant historical and theoretical backgrounds. Since they are already well known in their fields of work, this way they are usually creating (almost) dead-end streets for others who are more original and unusually innovative, or some of them are simply making barriers for advances of incomers. Naturally, everybody (of educated human beings) has also a tendency to finalize and verify his work by making most general formulations that are practically not leaving a lot of space for future developments, or a space for contributions from creative, *naïve and* inexperienced incomers. Every officially established and well-structured system (here mainstream science-related establishment) is in its advanced stages creating self-protecting and structural theory reinforcements that are no more working for the primary and essential interest and goals of the same scientific system, as initially created. It is mostly working for some economic and administrative protection and longevity of its own structure. Practically, such systems are becoming stabilized, optimized and like dead-end streets for new creative ideas and scientific progress. The biggest problem with such matured systems is in already created, semi-false, obsolete, not-fully verifiable, or based on irrelevant data bank of different false references, achievements and publications. Rulers of the matured system are usually forcing all participants to respect all published, and officially verified or already glorified references. Well obeying members of such comunity are non-critically creating and publishing new semi-false references in a similar fashion (based on existing semi-false references), this way saturating the system with very much arbitrary, and sometimes scientifically almost worthless, or trivial information, just to satisfy needs of having publications. Certain old and obsolete theories and concepts, instead of being replaced with new, alternative and better concepts, and with new mathematical structures, are simply kept in power like being forever very relevant and ultimate achievements. Such, initially weak and erroneous theories, are gradually being updated, and on some way corrected, but the completed theory created on such way is becoming unnaturally bulky, complicated, with limiting filterring frames for new predictions, inventions and creative thinking (as contemporary **QT** and **RT** are).

Also *giving licenses and certificates (or diplomas)* to specifically chosen and specifically educated groups, regarding getting exclusivity and priority to be included in the officially supported scientific research, ant to produce and publish new scientific results (that are mainstream compatible and by default correct), and systematically neglecting and eliminating others (who have different ideas) is not giving expected results and significant scientific advances. This could agree with F. Nietzsche's philosophical statements that only extreme personalities, clearly separated from the mainstream, are producing extraordinary and exceptional results (both positive or negative, depending on which side and platform they are positioned, and how we qualify them).

The quite common and negative reality of today's scientific theatre could also be the fact that some of the officially title-recognized scientific participants are in permanent need to publish, where the bottom-line motivation is to maintain some position, power, or economy-related existence (and not a quality and originality of publication). Eventually, they are mastering the "mainstream-correct and diplomatically perfect language" of saying and writing good-sounding, scientific-alike papers, comments, and statements. Such processing is often realized by combining available, real, and correct scientific references and contributions with ingredients of semi-reality and virtual reality, or by inventing some artificial items (such as dark matter, mass, and energy). Some of irresponsible participants of such process are using supporting chain of similar semi-false and meaningless references (already published, but with statements and results not easy and directly verifiable), coming from certain number of similar, title-recognized, and irresponsible colleagues (where needs for publishing dominated the need to be scientifically correct, innovative, and creative). In most of the mentioned cases, involved participants even do not see or recognize that they are part of such a negative, science falsifying process. Later, some of such scientifically irresponsible personalities could take important positions in a system that oversees acceptance and filtering criteria for selecting others for taking new scientific positions. Moreover, mentioned irresponsible personalities and authorities could have preferences to "enrich" their operating environment producing similar and well-obeying colleagues and disciples, and instead of establishing a motivating, challenging and competitive science, merged with real and unbounded creativity, the self-degrading, self-defending, and conservative dead-end system would be unintentionally established. Of course, this worst possible or imaginable scenario, or negative vision should absolutely be avoided in practice, if we really would like to make important scientific advances.

The author of this book is a kind of passionate and (in this field) self-organized, professionally, and economically independent engineer, free-lance amateur (with all the above-mentioned weaknesses of courageous and naïve players), trying to promote his vision of innovated Physics, mostly regarding Particle-Wave Duality of matter. In addition, elaborations about physics and matter waves in this book are presented on an intuitive, simplified, neutral, and mostly brainstorming, common-sense way, without respecting usual limitations and prescriptions imposed to people working professionally in mentioned fields. Regardless all disadvantages of being a courageous, passionate, and self-supported amateur, the author of this book has been thinking that his science related **PWDC**, **EM**, **QT** and Gravitation concepts should be somehow documented (even unprofessionally, by writing this book). Eventually, somebody, well experienced in Physics, naturally creative and sufficiently open-minded could, one day, transform some of here presented ideas into much healthier and richer property of Natural Sciences.

The leading idea of the author of this book is that Physics should be formulated to be easily understandable, simple, clear, internally coherent theory, based on experimentally verified and mathematically described facts (without unnecessary artificial and very abstract mathematical modeling). Of course, this should be the goal in any domain of natural sciences, as much as possible (without introducing unverifiable assumptions and artificially constructed mathematical theories), to explain profound secrets of Nature (and we know that this is not always the case practiced in our contemporary Physics). Only talented, creative, and rare outsiders, debutants, and independent thinkers, or sufficiently educated amateurs faced with Physics' and proposed theoretical explanations can say how much simple, clear, common sense, and elementary certain theory they find in the available literature, is. The case is that matured professionals are well armed and bounded by what and how they learned and practiced becoming officially recognized professionals, and some of them are probably losing the ability to ask simple, elementary, obvious, and essential questions, and to see presentation-related problems on a simple and natural way. To put it differently, gifted "children and debutants" or amateurs, are sometimes able to ask particularly good, unusual, unbounded (or crazy) questions, and to initiate new and extraordinary insights. The biggest complexity and fundamental problems in modern Physics-house are in its foundations and starting assumptions. We are usually analyzing only what is the visible part of the house (or a floating and visible tip of the iceberg), only adding new "decorations" and new levels of complexity to its structure, avoiding touching the basements. The often-mentioned reasons defending such practices are that certain of the famous and untouchable predecessors (or the theory founder) invested there his professional and personal authority and his life-carrier, and left around a lot of convinced disciples, powerful followers, and guardians of his published heritage. This is maintaining the Physics-theater almost like a heritage and property of the long-lasting dynasty (or ideology), being charged with specific ideological dogmas, where the highest authority is certain "bible" (or basic reference book), which is written once in the past, made to stay valid forever, and it is not allowed to be changed or challenged. Everybody who starts introducing new ideas and doubts belonging to the world outside of the frames of existing and accepted *bibles,* by default has no rights to be mainstream accepted as a contributor to an official scientific knowledge base, because his arguments are not sufficiently in agreement with bible's prescriptions and existing teachings. In other words, some of the officially accepted "*bibles"* could have a well-hidden tendency of promoting themselves as unique and exclusive sources and regulatory acts of all officially acceptable knowledge, arguments, and frameworks for promoting, discussing, criticizing, dismissing, and punishing other (too much original and innovative) teachings that are differently addressing the spaces inside their intellectual property. Consequently, it could happen that somebody who takes a freedom to modify a "bible" (or write a new one) should be, as soon as possible, in a way punished or effectively eliminated, or totally discredited and classified as a trivial and foolish case. *Of course, all that kind of doings in the name of science well-being would be realized using much more elaborated, diplomatic, and sophisticated methods and language, than over-simplified formulations found here, what is still being known and practiced in all other domains of human life.* We could also safely say that until the end of the 20-th century of modern human history on the planet Earth, the human society (in all aspects of life, including science) was guided and mastered mostly by the rigid "*power of authority*". Here, an *authority* means a power of strong personalities and their operating bodies, and power associated to specific prescriptions, "bibles", politics, and ideology. It has been always somebody (in the past) who could say YES or NO, regardless of arguments and logic (even in natural sciences), but officially everything was packed in the frames of legalized mainstream correctness. The biggest obstacle for innovative freethinkers in that period of human history on the planet Earth (until the end of 20^{th} century) was the complexity of barriers, effectively disabling them to efficiently communicate, publish, propagate, realize, or spread their unconventional ideas. In a case when somebody persisted in promoting his "heretic" ideas, he took a risk of being punished or humiliated morally, economically, existentially, and on many of different perfidy ways, some of them being very cruel (just because such problems-making authors were opposing officially accepted concepts and ideologies). For everybody who would like to test and challenge or rectify his position regarding mainstream "*authorities and their bibles"* it is recommendable to read the following book: [50], Dawkins Richard, The God Delusion. The same method, good to mentally detoxicate and eliminate ideological, religious, dogmatic, and similar mind-barriers and brain viruses, elaborated by Dawkins, can easily be applied to mind barriers, inertia, and deviations in natural and other sciences …

If we take another independent approach towards free-lance creations, publishers and publications like this book is, we could imagine the existence of (at least) two families of their potential consumers and readers. One of them would present majority of people arbitrarily involved (until different levels) in some of compromising, apologetic, existential, political, economic, educational, servile, ideological, not sufficiently critical, and similar (short living) relations with an official mainstream science establishment, who are in some ways a priori more negative than positive regarding here mentioned free-lance, too much original, and unusual (not mainstream) creations. And the other, much smaller group, but more open towards unconventional and out-of-mainstream creations, would be populated by people who are critically oriented, searching for more realistic, independent, unbounded, original, neutral, and long-living insights which are "*Beyond Good and Evil" (as promoted by Friedrich Nietzsche).* In fact, nobody has been able (and would be able) to exactly quantify how much somebody belongs to one or the other family of consumers and followers of certain scientific (or quasi-scientific) platform. Most of the participants in such processes (except trivial and obvious cases) would also have different and evolving personal relations to such matters, and only a sufficiently neutral and existentially independent thinker, would be able to say about others, who is closer to one or to the other of here mentioned families. The essential differentiating situations (in exploring who belongs to certain family of mentioned consumers) will happen if we ask ourselves (without fulfilling any egocentric, existential, ideological, economic, or daily life interest) if certain scientific theory, model, or concept has really a chance to be long-lasting, generally applicable, convincing knowledge with enough neutral weight that is dominantly on a neutral side "Beyond Good and Evil". There is also the third, large and trivial community of ignorant, basically not properly educated (or not intelligent) members, who do not care, respect, or see anything worth in science or in here-presented elaborations. Of course, we could further elaborate here initiated ideas, but the important message is already formulated. Fortunately, with the contemporary developments of high-power computing and advanced, global communicational means (one of them Internet), we are now slowly and gradually getting free of old power management concepts, and entering the period where "**power of mathematics, arguments, unbounded imagination and creativity**" would (and should) be more and more dominant managing framework. In the present period of human history, almost everybody is getting able to find some channels and means to spread his ideas, regardless the opinions and positions of powerful and conservative mentors, editors, and authorities. It is interesting and challenging to analyze the evolving conflict between mentioned power management participants, being an integral part of it, since our present scientific period is still a transitional one (and maybe it would or should stay like that forever).

*All over this book are scattered small comments placed inside the squared brackets, such as, [♣ **COMMENTS & FREE-THINKING CORNER…** ♣]. The idea here has been to establish intuitive and brainstorming, maybe still not confirmed and freethinking corners for making fast comments, and presenting challenging ideas, that could be some other time developed towards something much more meaningful and more properly integrated into Physics. Without addressing such situations, many of potentially valuable ideas and concepts (or simply good proposals) would simply disappear. This is also a way to document the real, logical, and thinking background regarding generating new ideas, concepts, and hypotheses (to present how an author was really thinking in process of formulating his contributions…). We sometimes have situations that last judgments and explanations, regarding how somebody of founders of a certain theory was thinking (and what he/she had in mind), is given as an arbitrary, a posteriori interpretation by the present "untouchable rulers" of state-of-the-art theories that are "officially mainstream certified", but essentially wrong. In some cases (for instance in Relativity and Quantum theory) contemporary authors conveniently modify such retroactive visions, explanations, and interpretations (mostly unconsciously) to additionally support their own platforms, indirectly saying that they are direct successors and the best students of their famous teachers and theory founders, and maybe even greater than mentioned origins. The readers who are too busy, have not enough time, and not used to spend a time reading something like "[♣ **COMMENTS & FREE-THINKING CORNER…** ♣]", could neglect such (**blue color**) segments and advance faster in reading this book.*

Author of this book would also like to say that this book has been evolving, changing, and updating during a long period (when the author was able to catch a little bit of his freethinking *and mentally relaxing time; -during more than 40 years, starting around 1975.). There is a significant redundancy, scattering, and discontinuity regarding presenting similar or familiar concepts all over this book (written, rewritten, updated, or simply attached to old chapters in different times). Since this book presents mostly an amateurship creation, being still a time-evolving effort of the author to summarize many of his ideas roughly and briefly, it should be clear that a lot of additional work, editing and smoothing must be invested to make the real, well-organized, and scientifically meaningful book. Regardless of such weak sides, the author is convinced that the potential power and significance of here presented ideas, concepts and brainstorming thinking would survive negative spots scattered all over this book. The author also hopes that future readers of this book would not over-dissipate their mental power only on criticizing mentioned negative elements, instead of taking the most positive and creative approach that could result in developing new and more valuable scientific property based on their own thinking and on certain of ideas, concepts, and proposals taken from this book. Of course, the real and fully convinced members of the Community of present Orthodox Quantum and Relativity theory establishment are not advised to spend their precious time on such matters. Mostly unconventional science dissidents and suspicious and creative non-believers (tolerant to fuzzy, intuitive, and brainstorming freedom of expression) could find some pleasure when reading this book. Since the author realized relatively late that he cannot finalize this book, considering his present age, he decided to publish the book as it is (basically as a brainstorming open-ended draft, and challenging task list), without asking anybody for authorization, editing, and approvals. The activities on updating this book will continue as long the nature would allow such efforts.*

Miodrag Prokic, 2021, Le Locle, Switzerland.

(Works on this book started 1975. First time published in May 2006).

INTRODUCTION (in Serbian Language)

U analizama cesticno-talasnog dualizma materije, potrebno je pre svega imati u vidu fundamentalne situacije i pitanja, kao na primer:

1. Koji je to sadasnji kostur, metod ili struktura, raspolozivih puteva, naucnih oslonaca i teoretskih platformi odakle mozemo da delujemo, a da znamo da smo na stabilnom i plodotvornom terenu. Drugim recima, sta cemo usvojiti kao uvek vazece, univerzalno primenljive platforme u domenu matematike i fizike. Ovde se podrazumeva da ontoloski-probabilisticki koncepti, modeliranja i teorije ovde ne pripadaju, ali da se veoma uspesna modeliranja procesa sa velikim brojem identicnih ili slicnih clanova mogu bazirati na teoriji verovatnoce i statistici.

2. Koja je to makro obvojnica onoga sto se u svetu fizike dogadja, tj. koja je to gornja granica, ili aproksimacija koju cemo imati u vidu, kad nasa partikularna saznanja i nove teorije ukalupimo u neke sire naucne i teoretske okvire (u makro svetu fizike). To ce biti zakoni odrzanja, analogije, simetrije, principi koji definisu ponasanje svih sistema sa nekim kretanjem, matematicka logika i opste istine I principi sveta kontinuuma, ili makro sveta koji nas okruzuje. Neizbezna kreativna pozadina i neki od temelja svega toga su i dela nasledjena od Nikole Tesle i Rudjera Boskovica (vidi [6] i [97]).

3. Isto tako, mozemo se pitati koji su to neosporni, unutrasnji, bottom-line ili pozadinski mikro procesi, strukture i donje aproksimacije kojima sve mora da tezi kad polazeci od nekih makro struktura nesto svodimo na najelementarnije i minimalne strukture I elemente (ali da to bude nesto opipljivo, merljivo, vidljivo i matematicki uhvatljivo...), jer visi makro sistemi i teorije moraju u nekoj svojoj nultoj, osnovnoj aproksimaciji da se glatko i kontinualno spajaju sa teorijama i situacijama koje vaze u mikro svetu fizike. Na neki nacin nasa realnost se uvek nalazi u egzistencijalnom domenu izmedju dve beskonacnosti (tj. naucno i saznajno prodiranje u mikro i makro univerzuum je najverovatnije kao prodirenje u dve suprotno orijentisane beskonacnosti).

Strategija ili odgovor koji se ovde favorizuje je da su alati koji nam trebaju: izabrana poglavlja matematike, klasicna fizika i mehanika, teorija fluida, termodinamika i u velikoj meri Maxwell-ova teorija elektromagnetizma. Tu je skoro sve kauzalno, jasno, proverljivo, uhvatljivo opstim principima i ponovljivim eksperimentima, i podleze jasnoj, prirodno povezanoj matematici (a ne bazira se na apstraktnim, postuliranim i vestacki definisanim matematickim teorijama). Sada ide pitanje zasto je to tako:

1. ***Zato sto je matematika univerzalni jezik i logika prirode i formira se na bazi eksperimentalnih, induktivnih, deduktivnih, analognih i empirijskih zapazanja i generalizacija koje imaju svoje izvore i modele u prirodi (tj. u fizici).***

2. ***Zato sto je gradja mase ili matterije u svojoj dubokoj sustini elektromagnetske prirode***, gde se na specifican nacin uoblicena i upakovana elektromagnetska energija manifestuje kao masa (i moze biti u

raznim stanjima kretanja), pa posle dolazi sve ono sto se proucava u mehanici...

3. ***Zato sto prosirene, kontinualne simetrije i analogije (bazirane na « Mobility » analogijama koje su opisane u prvom poglavlju ove knjige) neosporno vaze iz vise uglova posmatranja, jer opisuju jedan isti univerzum (nas), i sve su to razne vrste prostornih i intelektualnih preseka, projekcija i korelacija iz razlicitih uglova, a zajednicko im je da ih opisuju iste ili slicne matematicke jednacine ili modeli, jer se odnose na isti i vec ujedinjeni univerzuum u kome zivimo.*** Sto mi jos uvek nismo u stanju da formulisemo univerzalnu teoriju polja, nije ni bitno, jer je ta (nasa) priroda vec ujedinjena (pre nase civilizacijske pojave) i ne mora bas nas da ceka, prati, slusa i da uvazava sve nase teorije kako bi po tim teorijama bila ujedinjena... To prirodno jedinstvo (kao sto bi ga opisivala neka nova jedinstvena teorija polja i sila) vec postoji nezavisno od nasih metoda opisivanja prirode i ostalih teorija i koncepata kojima masemo i to je izvor svih simetrija i analogija u prirodi. Mnogi se slazu da Gravitacija kao poglavlje fizike nije jos uvek u svojoj zavrsnoj formi i da nije povezana sa kvantnom teorijom (a verovatno je i bespredmetno i nerealno ocekivati da moze biti povezana sa sadasnjom Kvantnom Teorijom). ***Prema univerzalno vazecim elektromehanickim analogijama (sto je kao neki prirodni zakon ili kostur koji se mora uvaziti), mi mozemo da vidimo da silu gravitacije ne izaziva sama masa, nego masa u kretanju ili oscilovanju, sa nekoliko pridruzenih atributa, tj. izvori gravitacije su: medjusobno spregnuti linearni i obrtni momenti i njima pridruzeni (spoljasnji i unutrasnji) dipolni i magnetni, ili elektromagnetni momenti, tj. masi pridruzeni elektricni naboj i magnetni fluks. Mase sa takvim pridruzenim i medjusobno spregnutim atributima mogu na razne nacine da se medjusobno privalace ili odbijaju, a jedan od tih nacina privlacenja (koji ima neku svoju specificnu kombinaciju pomenutih atributa) rezira ono sto zovemo Gravitacijom. Mi isto tako znamo da se u nekoj meri gravitacija i elektromagnetizam « medjusobno osecaju » i intreaguju (jer putanje fotona bivaju savijene u jakim gravitacionim zonama), a znamo i da gravitacija i elektromegnetsko polje interaguju skoro sa svim ostalim konstituentima naseg univerzuuma.*** Ovo glediste o gravitaciji (da sama masa nije primarni izvor gravitacije) se razlikuje od gledista armije onih koji dosledno slede Njutna i Ajnstajna i Kvantnu teoriju (gde vazi da samo masa predstavlja glavni gravitacioni naboj ili izvor), ali kako Fizika nije politika, tu nadglasavanje i kvorum istomisljenika i njihovih poslusnih sledbenika nista ne znace. Uprosceno receno: neki dobar djak koji poseduje sva akademska priznanja ovoga sveta i koji je napisao mnogo radova o gravitaciji, nije ustvari mnogo dao i razumeo gravitaciju ako kroz svoja razmisljanja i radove pokazuje da ne postuje Mobility system elektromehanickih analogija... Tako je i Ptolomejev geocentricni sistem ostao na snazi oko 1000 godina, jer religiozni mudraci nisu zeleli da vide i uvaze neosporne astronomske cinjenice... I tada (u vreme vazenja Ptolomejevog geocentricnog koncepta) je bilo kao i danas: veoma mnogo uvazenih i visoko-pozicioniranih mainstream autoriteta su se dogovorili i rekli da njihov Ptolomejev model mora da vazi po svaku cenu, a ostali nesrecno rodjeni koji su dokazivali i videli to isto drugacije su spaljivani... Sada barem

znamo da necemo biti spaljeni zbog ovakvih pisanja (ali da budemo kritikovani i ismejani zbog ovakvih misljenja, to je najmanje sto se moze realno ocekivati, sto i nije tako strasno ako znamo da se borimo za cilj kao sto je naucna istina o nasem univerzumu...).

4. Naravno, tu treba dodati apsolutno postovanje svih kontinualnih simetrija i neospornih zakona odrzanja koji su univerzalno primenljivi u fizici, mehanici i elektromagnetizmu... (bez uvodjenja probabilistickih i statistickih izuzetaka).

5. Na bazi deduktivnog pristupa mi imamo sansu da (hipoteticki i analogno) dodjemo do indikativnih elementarnih principa, pa da posle toga sve to ispitujemo i usaglasavamo induktivnim i iterativnim naucnim I matematickim metodama...

Sada dolazimo do pitanja kako startovati od takvih generalizovanih, osnovnih i okvirnih platformi i principa i sve to primeniti na svet fizike. Odgovor koji se ovde favorizuje je da je najbolje da se u najvecoj meri drzimo samo dobro selektovanih, proverenih i univerzalnih matematickih teorija, principa, zakonitosti, modela, jednacina, analogija i logike... jer ako se toga drzimo onda necemo imati sanse da mnogo pogresimo. Koji su to univerzalno primenljivi matematicki blokovi koje mozemo koristiti i koje moramo uvek postovati:

1. Parsevalova teorema koja je neka vrsta prezentacije univerzalno vazeceg zakona o odrzanju energije u vremenskom I frekventnom domenu... Na primer: Ako se toga ne drzimo, imacemo kvantno-mehanicku i ontoloski-probabilisticku talasnu funkciju i jedan apstraktni i vestacki matematicki aparat koji ce sve to podrzavati na konceptualno nedovoljno jasan nacin(sto je sadasnji slucaj sa kvantnom teorijom).

2. Talasi matterije su realni i merljivi i analiziramo ih preko univerzalnih talasnih jednacina (gde podrazumevamo da sve talasne jednacine matematicke fizike imaju korene u d'Alambert-ovoj i klasicnoj talasnoj, parcijalnoj diferencijanoj jednacini drugog reda)... Bez obzira sto je mehanika i teorija fluida izvorno proizvela te klasicne talasne jednacine, one mora na odredjeni analogan nacin da vaze i u elektromagnetizmu uz odgovarajuce dorade modela i, kako znamo, takve analogne jednacine elektromagnetskih talasa su vec izvedene u izvornoj Maxwell-ovoj teoriji... ***Cesticno-talasni dualizam matterije zapravo mora da obuhvata sve realne, merljive talasne, vibracione i oscilatorne fenomene u fizici... (a ne samo one iz domena mikro sveta koji su prirodno probabilisticki, direktno nemerljivi i egzoticni). Ako talasnu funkciju klasicne talasne, diferencijalne jednacine drugog reda tretiramo kao funkciju koja je u formi kompleksnog analitickog signala (formiranog koristeci Hilbertovu transformaciju), mozemo direktno i lako izvesti sve talasne jednacine kvantne i mikro fizike (bez ubacivanja novih elemenata u takve jednacine, kao sto je to radjeno u kvantnoj teoriji tokom formiranja Schrödingerove jednacine).*** Posto postoji neka linearna i direktna veza izmedju prostornih i vremenskih dimenzija (kao sto se vidi u faznom clanu talasne funkcije (ωt - kx), odakle se izvode grupna i fazna brzina svih talasa matterije), odatle mi dobijamo sve ono sto je

Schrödinger vestacki nakalemio, pa posle toga dobijamo talasne jednacine kvantne mehanike... Dakle, Schrödingerov rezultat je tacan, ali metoda kako je do toga dosao je neko intuitivno, vestacko i neprirodno kalemljenje (ili dodavanje novih clanova), sto se ne sme legalizovati kao nesto univerzalno preporucljivo, dobro i naucno prihvatljivo.

3. Na raspolaganju nam stoji i Analiza I Sinteza Signala u vremenskom i frekventnom domenu, bazirana na univerzalnom matematickom modelu svih kretanja (ukljucujuci i talasna) kao sto je Analiticki signal (sto je generalizacija fazora na bazi Hilbertove transformacije).

4. Tu su i ostale vazne grane matematike kao sto je vektorska analiza itd.

Dakle, kao zakljucak, trebalo bi da se drzimo vrhunskog principa koji glasi: ***Ne pocinjimo nista drugo ili novo (prilikom modeliranja i formiranja, ili inoviranja teorija fizike) dok postojecu ili prethodnu situaciju ne razcistimo i stabilisemo na ovim, gore pomenutim, okvirima, koji treba da budu nasi osnovni i polazni nivoi, tj. temelji novih teorija…***

Normalno je da cemo u sadasnjoj fizici naci i neke kontradikcije i praznine, jer je istorijski mnogo toga zapoceto bez postojanja zajednicke osnove i formirano iz mnogo razlicitih uglova… Bez jasnih generalnih vidjenja i matematickih struktura i putokaza, mi mozemo samo da budemo izgubljeni u nizu mogucnosti sa razlicitim verovatnocama izbora, ili da budemo nesigurni i sa lose formulisanim ciljevima… Ne moze se nesto novo u fizici stvarati bez cvrstih, neospornih, zajednickih, univerzalnih temelja i bez jedinstvene matematike, logike i filozofije prirode… (sto u ranim danima formiranja Fizike i Matematike nije bio slucaj).

Kada se, na primer, uhvatimo modela Analitickog signala (koji je generalizacija FAZORA za sve sto se krece), videcemo da svaki signal ili funkcija nekog kretanja ima svoj Hilbertov, ortogonalni par, tj. drugu, komplementarnu ili pridruzenu (fazno pomerenu) funkciju kretanja… To je slicno situaciji u vezi sa elektromagnetskim talasima (i verovatno primenljivo mnogo sire), gde su elektricno i magnetno polje medjusobno povezani, tj. uvek nastaju u spregnutim parovima medjusobno komplementarnih polja… Radi se o fazno pomerenim talasnim funkcijama (za $\frac{\pi}{2}$), ali jednu od tih funkcija mi zovemo elektricnim poljem a drugu magnetnim jer im je prostorna geometrijska ili vektorska forma drugacija (jedno polje je laminarno a drugo spiralno…).

Najbolje je da uvek postujemo generalno vazece i veoma primenljive matematicke principe, modele i structure (kao sto su, least action, Euler-Lagrange equations, and Lagrangians and Hamiltonians) i da se sto manje prilagodjavamo parcijalnim, usko vazecim modeliranjima, abstraktno I vestacki definisanim teorijama i raznim fitovanjima na bazi statistike I teorije verovatnoce… Jedan od takvih generalno primenljivih koncepata na sva kretanja je i Doplerov efekat, ali je problematika strukture i kretanja matterije ukljucujuci cesticno-talasni dualizam jos sadrzajnija nego sama primena Doplerovih formula.

Najverovatnije i najprirodnije je da su ***svi talasi koji se spominju i fizici (pod raznim obicnim i neobicnim imenima kreativnih autora, ili nazvani prema***

sredini u kojoj se prostiru) zapravo deo iste univerzalne talasne, vibracione i oscilatorne stvarnosti koja je jedinstvena u fizici (tj. jedinstvena za nas univerzuum) i to su zapravo talasi matterije koje mozemo na neki nacin da merimo, vidimo, cujemo, kao na primer, zvuk, oscilacije cvrstih i fluidnih masa, svetlost, razni elektromagnetski talasi u raznim sredinama, talasi praskastih formacija, toplotni talasi, talasi subatomskih cestica i kvazicestica (tj. sve sto se da na neki nacin meriti ili detektovati kao talasna ili oscilatorna pojava). ***Zato je ista klasicna differencijalna talasna jednacina primenljiva u nizu razlicitih talasnih fenomena (a u ovom radu je pokazano da je skoro univerzalno primenljiva i u kvantnoj teoriji, tj. da se odatle dobija Schrödinger-ova jednacina i druge talasne jednacine mikro fizike, kada se talasna funkcija tretira kao kompleksni analiticki signal).***

Nema niceg drugog u ovom nasem talasajucem univerzuumu kao sto bi bili neki misticni, transformabilni, ontoloski-probabilisticki i na druge nacine neuhvatljivi talasni oblici. To sto smo mi uveli u fiziku neka nova imena i opise neobicnih ili egzoticnih talasa je deo nase nepotpune i nedovoljno tacne konceptualizacije te fenomenologije (zanemareni su neki vazni faktori u vezi sa prostornom i vremenskom prezentacijom signala, ili jos uvek nismo u stanju da sve to sto nedostaje u nasem teorijskom opisivanju empirijski i eksperimentalno merimo i sagledamo u realnom vremenu).

Razna imena koja su davana raznim talasima matterije u publikacijama iz fizike su ponekad samo matematicke i konceptualne komplikacije i mistifikacije, ili vestacke (fitting i ad-hock postulirane) tvorevine da bi se podrzala ili spasila necija hipoteza, model ili teorija, u nedostatku boljih objasnjenja i matematickih modela, tj. to su apstrakcije koje cesto ne uzimaju sve vazne elemente u obzir, ili krace receno to su losi, ili nekompletni modeli.

Dakle, de Broglievi talasi su talasi matterije kojom smo okruzeni i opipljivi su, merljivi itd., ali mi smo u sadasnjoj kvantnoj teoriji nasli nacine kako da ih normalizujemo, usrednjimo i izomorfno predstavimo kao neke ontoloski-probabilisticke, bezdimenzione funkcije koje imaju formu talasa (sto je mozda korisno i upotrebljivo u mirkofizici, jer tamo imamo ogromne skupove ili ansamble slicnih, ili identicnih entiteta, pa statistika i verovatnoca tamo produktivno rade, kao sto je slucaj u termodinamici fluida, ali to ne sme da se generalizuje na sve ostale domene fizike)... Sve ostalo (sto nisu direktno merljivi talasni fenomeni) su nekompletna i delimicno pogresna sagledavanja i modeliranja talasnih pojava (gde ce se jednog dana pokazati da tu nesto nedostaje i bice evolucije u pogledu stvaranja boljih modela)... To je poruka koja se nalazi u pozadini ovakvih elaboracija. Uloga statistike i verovatnoce kao dobrih teorija za modeliranje i obradu masovnih podataka i klasifikaciju fenomena, tendencija i kretanja je veoma znacajna i mocna, ali treba da se primenjuje kao zavrsna faza veoma sadrzajnog prikazivanja rezultata koji prvo treba da se dobiju na sve druge deterministicke, empirijske, logicke, analogne i matematicke nacine. Srtatistika i verovatnoca ne mogu i ne smeju da budu implementiranje kao ontoloska, konceptualna i izvorno-modelarna baza novih teorija, vec kao neka vrsta mocne zavrsnice i donosenja dobrih generalizovanih zakljucaka.

Drugi deo gledista koje se ovde favorizuje je da ***svi ranije pomenuti realni, matterijalni talasi (koji su merljivi, vidljivi i poznati iz iskustva) mora da***

se na neki nacin manifestuju u nekakvim komplementarnim, spregnutim talasnim parovima, kao sto je sprega elektricnog i magnetskog polja koja cini elektromagnetske talase. Dakle, nesto slicno (kao kod analitickog signala) mora da postoji i kod svih ostalih, vec pomenutih talasnih i oscilatornih fenomena (u akustici, gravitaciji, vibracijama cvrstih tela i fluida...). Osnova za talasanje matterije je da dolazi do dinamickog sprezanja kineticke i potencijalne energije kada se stvara neki oscilujuci ili talasni proces, gde se ukupna energija oscilatorno preliva iz kineticke u potencijalnu i vice-versa... To oscilatorno prelivanje kineticke u potencijalnu energiju mora i moze da bude prezentirano nekom spregom dva fizicka polja ili dva fenomena koji su medjusobno komplementarni (sto je za sada poznato i primenjeno, tj. modelirano samo u elektromagnetizmu, ali je to i sastavni deo modela analitickog signala). Druga je stvar sto mi do sada jos nismo nasli kakve su to fizicke i talasne sprege u gravitaciji ili u mnogim drugim talasnim fenomenima, silama i fizickim poljima. Zapravo, u mnogim slucajevima nismo jos uvek modelirali ili koncipirali tu drugu komponentu talasnog polja (osim u slucaju elektromagnetskih talasa gde obe komponente polja poznajemo). Hipoteza koja se ovde favorizuje je da ***svi talasi matterije mora da imaju pomenutu spregu izmedju dva medjusobno komplementarna fenomena, ili polja, ili talasanja***, ali da nase matematicke konceptualizacije (i eksperimentalna praksa) jos uvek nisu dosle do tog nivoa sagledavanja (ili modeliranja) jer u nizu slucajeva nemamo, tj. neznamo tu pomenutu, komplementarnu i/ili fazno pomerenu, drugu talasnu komponentu. U ovom radu se objasnjava da je ***jedna od fundamentalnih talasnih sprega u kretanju i talasanju materije, sprega izmedju elemenata linearnog i rotacionog kretanja, ukljucujuci spinning (i oko takvog modela gravitiraju sve ostale konceptualizacije u ovoj knjizi)***... a to u nekoj daljoj konsekvenci najverovatnije evoluira do sprege elektricnog i magnetnog polja, ili do neke druge familijarne sprege dva medjusobno komplementarna fenomena (i u najvecem broju slucajeva talasa matterije i oscilacija masa, elektromagnetska sustina se najverovatnije nalazi u pozadini svih takvih talasnih fenomena, samo sto to ponekad moze da bude dobro skrivena situacija).

Dakle ***problem je u tome sto mi ustvari neznamo, ili nismo matematicki, konceptualno i empirijski uhvatili jos jednu komponentu pomenutih talasa matterije (kao sto je slucaj u elektromagnetici, gde barem znamo obe talasne komponente)... i tu dolazi do izrazaja de Broglieva filozofija, koja hipoteticki (i analogno sa fotonima) nadomesta ili uvodi tu neuhvacenu komplementarnu talasnu komponentu*** i daje tacne rezultate (najcesce tamo gde su analizirane eksperimentalne interakcije izmedju entiteta sa nekakvom elektromagnetskom prirodom). Dakle, nema ni Schrödinger-ovih, ni Heisenberg-ovih, ni de Broglie-vih talasa... Sve je to deo iste talasne stvarnosti (sve su to talasi matterije i treba da se opisuju istim matematickim modelima i odnose se na isti univerzum). Sto mi jos nismo dosli do takvog nivoa konceptualizacije (analogicnog sa elektromagnetizmom), to je druga prica... ali treba da tezimo ka tome i jednom cemo sve to sagledati kako treba. Ovaj rad o tome govori... ili barem pocinje tu pricu i otvara mali prozor u takav svet...

Kada se stvore stojeci talasi matterije koji su samo-zatvorenog tipa (zatvaraju se prostorno sami-u-sebe na nekoj zatvorenoj liniji, krugu, povrsini, zapremini...) tada nastaju talasne grupe koje pocinju da primaju osobine cestica... a grupacije takvih elementarnih cestica stvaraju vece mase... Za sada mozemo zamisliti da ovde mora da se radi najvise o talasima koji izvorno (mozda na nekin nacin dobro skriveno) imaju neku vezu sa elektromagnetizmom. Naravno, postoje i drugi talasi neelektromagnetske prirode koji mogu da formiraju grupacije stojecih talasa, ali je najverovatnije da samo elektromagnetske stojece formacije talasa stvaraju talasne grupe koje pod odredjenim uslovima postaju ono sto zovemo elementarnim cesticama. ***Da bi jedna sama-u-sebe-zatvorena, stojeca talasna grupa mogla da stvori cesticu, nije dovoljno da su njene (unutrasnje) talasne komponente jednog tipa tj. iste prirode. Mora da postoji neka interakcija ili dinamicka sprega izmedju dva tipa (medjusobno komplementarnih i spektralno konjugovanih) fizickih polja ili talasa koji se tu pojavljuju.*** Ovo tvrdjenje je bazirano na poznatim i vazecim elektromehanickim analogijama i simetrijama u fizici, kao sto su: sprega elektricnog i magnetnog polja kod elektromagnetskih talasa (ili neki oblik sprege kineticke i potencijalne energije, ili neka forma sprege linearnog i rotacionog kretanja), da bi se od takvih medjusobno komplementarnih i spregnutih stojecih talasa stvorio kompleksniji i sadrzajniji stojeci matterijalni talas i da bi se kasnije taj talas zatvorio sam u sebe (na neki nacin), a time se stvara relativno stabilna talasna grupa koja moze da vodi formiranju cestica koje ce da imaju stabilnu masu mirovanja.

U prirodi postoje i talasi koji nisu zatvoreni sami u sebe (koji nisu tipicni stojeci talasi), a to su propagirajuci, ili progresivni, ili slobodni talasi matterije... a zajednicko im je da se svi ti talasi (i slobodni i stojeci) opisuju slicnim matematickim modelima i jednacinama... ***i to se odnosi na sve talase i vibracije koje poznajemo u prirodi, jer je sve to medjusobno spregnuto i deo iste stvarnosti... pa se zbog toga i manifestuju razne analogije u opisivanju talasnih kretanja. Najverovatnije da je sve to sustinski prozeto ili komponovano (u svojoj dubokoj ontoloskoj pozadini) nekim specificnim pakovanjem elektromagnetskih polja, koja se talasaju, ili osciluju***, sto sve jos treba mnogo bolje razraditi...

Na primer, kod elektrona imamo niz direktnih indicija da su elektroni bas takve (pomenute) samo-zatvorene talasne grupacije fotona ili elektromagnetskih talasa, a protoni bi trebalo da budu nesto slicno, ali sa mnogo vise upakovane energije (jer se opet radi o naelektrisanju koje stvara elektricno polje, a kretanje svih naelektrisanih entiteta stvara magnetno polje...). Na primer, fotoni izbacuju elektrone iz atoma ili ulaze u njih i podizu energiju unutrasnjih elektrona u atomu, pa kasnije takav ekscitovani elektron opet izbaci foton. Elektron i pozitron se anihiliraju stvarajuci fotone, a postoji i inverzni proces... Komptonov i Fotoelektricni efekat govore o medjusobno slicnim interakcijama... (i nije bas slucajno da se sve to na neki nacin vezuje za fotone, ili naelektrisanja, ili za neku formu elektromagnetske energije).

Skoro da je dokazano, ali eksplicitno nepriznato u fizici, da su neutroni neki specificni parovi jednog protona i jednog elektrona i to neutron cini elektricno neutralnim, jer kad se analizira njegova struktura, nadju se jake indicije da su unutar neutrona jedan proton i jedan elektron (kao u atomu vodonika, ali drugacije upakovani ili vezani). Dakle, sada imamo sve sastojke materije: elektrone, protone, neutrone, fotone (a znamo da postoje i njihove anti-cestice). Svakako, ovde jos nedostaje i neutrino (da se tu negde smesti) i da se lepo ukomponije i objasni (jer i to je eksperimentalno uhvatljiva talasna grupa, ili neka vrsta kvazi-cestice) ... A to se nekako opet sve intuitivno svodi na pakovanje fotona ili pakovanje elektromagnetske energije (ako se udje u dublju istoriju procesa). Naravno, ovo sto je ovde izlozeno je mnogo uprosceno, ali sa svrhom da se bolje i brze razume, nije lose za pocetak diskusije i za stvaranje novih koncepata i temelja o talasima i strukturi matterije.

Sto se kretanja elektrona tice tu postoje neke znacajnije razlike u kretanju pojedinacnih, relativno izolovanih i slobodnih elektrona od kretanja mase slabo vezanih elektrona kroz metalne provodnike. Dakle, bice tesko da ih opisujemo ili koncipiramo na veoma slican nacin, jer usamljeni elektron u slobodnom prostoru predstavlja neku talasnu grupu specificno upakovanih fotona (dakle predstavlja talasnu elektromagnetsku strukturu koja ima neku svoju prostorno-vremensku lokalizovanost ili koncentraciju, ili nesto sto asocira na cesticu, ili talasnu grupu i ima svoju rezultantnu grupnu i faznu brzinu koje zavise od osobina prostora u kome se takva talasna grupa krece, kao sto i blizina okolnih objekata moze da utice na grupnu i faznu brzinu elektrona).

Po konceptima favorizovanim u ovoj knjizi, slobodni, ili izolovani elektron koji se krece linijski stvara oko sebe (oko ose svog linearnog kretanja) helikoidalno, spiralno, ili vrtlozno magnetno polje *(helikoidna linija se ovde spominje vise kao neka zamisljena vodeca, solenoidna linija sile magnetnog polja, ili kao obvojnica koju prati stvoreno, komplementarno magnetno polje, tj. ta obvojnica je vezana za svoju talasnu grupu. Ta asocijacija sa helikoidom nam pruza neku vrstu vizualizacije talasne duzine i frekvencije matterijalnog elektronskog talasa, jer je intuitivno ocigledno da pomenuta talasna duzina mora da bude jednaka rastojanju izmedju dva sukcesivna zavojka helikoidne obvojnice).* Elektron u kretanju podleze zakljuccima hipoteze matterijalnih talasa (Luis de Broglie) i talasna duzina tog pridruzenog, solenoidnog, elektromagnetnog polja ili talasa je jednaka $h/p = h/mv$ (gde je m masa elektrona, a v je njegova linearna brzina). Ta (de Broglie-va) talasna duzina, kao sto je vec pomenuto, je jednaka razmaku koji obuhvata jedan zavoj zamisljenog solenoida koji okruzuje i prati elektromagnetsko polje oko elektrona u kretanju (tj. ta razdaljina odgovara jednoj periodi pomenute helikoidalne obvojnice). Takav elektronski talas (koji ima elektricne i magnetne komponente polja) neminovno ima i svoju de Broglie-vu talasnu duzinu i frekvenciju i faznu i grupnu brzinu (a mi iz drugih analiza vec poznajemo jednacine univerzalnih veza izmedju grupne i fazne brzine, koje vaze za sve talase, pa sve to mozemo i ovde primeniti, bez ikakve potrebe da pozivamo verovatnocu i statistiku u pomoc). Grupna brzina elektrona je jednaka stvarnoj, mehanickoj ili cesticnoj, linearnoj brzini elektrona (prema modelu talasne grupe). Dakle, moguce je pridruziti elektronu koji se krece neku frekvenciju (zavisno od njegove brzine kretanja, tj. zavisno od njegove kineticke energije). Ova situacija postaje jos kompleksnija i ocigledniјa ako uzmemo u obzir da svi elektroni imaju svoj spin (jer je to modus postojanja elektrona, tj. bez spinskog momenta elektron nebi postojao kao cestica, sto znaci da je koncentracija i grupisanje specificno formiranog, stojeceg elektromagnetskog talasa ostvareno time sto je linearna komponenta kretanja energetski pogodnog elektromagnetskog talasa (ili fotona) harmonijski spregnuta sa pogodnom spinskom, tj. rotacionom komponentom tog istog talasnog kretanja i tako nastaje elektron).

To isto postaje komplikovanije ili teze opisati za veliki broj slabo vezanih elektrona koji se krecu unutar metalnog provodnika jer ne mozemo da na jednostavan i slican nacin izdvojimo neku usamljenu, pridruzenu solenoidalnu putanju vektora magnetnog polja i da joj odredimo talasnu duzinu ili pripadajucu frekvenciju (posto ima bezboroj elektrona koji uzajamno interaguju, interferiraju, mesaju se i nisu na istom mestu, tj. nema izolovanih helikoidalnih matterijalnih ili magnetskih linija sila). U metalnim provodnicima, elektroni se krecu ili struje, ili se talasaju na neki nacin koji pomalo moze da asocira na kretanja cestica i talasa u fluidima gde ima bezbroj malih cestica (atoma i molekula i vecih grupacija) koje su medjusobno povezane Van der Vaals-ovim i elektromagnetskim silama, pa zbog toga u neposrednoj okolini provodnika kroz koji prolazi elektricna struja necemo prepoznati izolovane helikoidalne, magnetne linije polja pojedinacnih elektrona. Tu se radi o ogromnoj masi elektrona sa slucajnim prostornim i faznim raspodelama, pa cemo imati utisak da se u okolini provodnika kroz koji tece struja nalazi kontinualno, homogeno, solenoidno magnetsko polje koje opisuje Biot-Savart-ov zakon (jer ovde u sustini imamo superponiranu masu helikoidalnih magnetnih linija sila, koje imaju svoje slucajno-raspodeljene talasne duzine, faze i frekvencije, sa zajednickom centralnom ili osnom linijom kroz koju kazemo da prolazi elektricna struja).

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1. PRESENTATION OF BASIC ELECTROMECHANICAL ANALOGIES

In this book, we will formulate predictions and conclusions based on well-known electromechanical analogies, to create new and hypothetical concepts and theories related to still challenging domains of Physics, and to make the screening, filtering, and initiate updating of already existing theories and natural sciences concepts towards a higher level of unification. Since there are many analogies' platforms developed and used in Physics, and especially in Electrical sciences, we will first select the most promising one that looks generally applicable from different points of view. Among the simplest, well-known, and very much operational are several of electromechanical analogies. Here, we will mostly address one group of such analogies (known as Mobility type of Analogies) useful to understand, create, and intellectually visualize already existing, innovative, and other unification concepts regarding Electromagnetism, Gravitation, Matter Waves and Wave-Particle Duality, what will be the main subjects of this book.

We know from Fourier and Analytic Signals Analysis (also in relation to "Kotelnikov-Shannon, Whittaker-Nyquist" works), that different signals, motions, and matter states related functions, can be decomposed, or synthesized using elementary, simple-harmonic (or sinusoidal) functions. *This way conceptualizing, we could say that our Universe, or diversity of all matter states, are on some structured way composed of elementary physical (electromagnetic and electromechanical) oscillators, resonators, or resonant states and circuits, all of them producing simple harmonic waves, and narrow-banded wave-groups composed of simple-harmonic wave components. Such ensembles of physical oscillators are (by mutual, resonant, temporal, and spatial synchronization, coupling, superposition, and interferences) creating other, more complex wave-groups, or matter forms and states, respecting Fourier and Analytic Signal theory, and quantizing concepts embedded in Kotelnikov-Shannon, Whittaker-Nyquist signal analysis and signals reconstruction*. See much more about such problematic in [57, 58, 59, 117, 118 and 119], and in Chapters 8., 9., and 10. of this book. This is the reason why we will start the foundation of analogies based on the comparison between simple electric and mechanical resonant or oscillating circuits, as presented in Fig. 1.1. Mentioned oscillating and resonant configurations, including analogical resonant structures in atoms and molecules (considered as elementary building blocks of matter) are anyway mutually and naturally coupled and synchronized, creating all matter or mass states in our universe. ***Conceptually, analogically, and mathematically concluding, if something is oscillating, there is always certain rotation-related phenomena behind, and vice versa.***

Well-established analogies are the first indicative (and brainstorming) step saying that certain seemingly different theories, concepts, fields, or electromechanical systems, have the same background, and could be (conceptually and mathematically) united by creating a unique common theory, or common explanation for all of them. For instance, atomic, molecular and mechanical systems, different oscillators, and resonant circuits, certainly have even deeper and essential structural "*set-of-micro-resonators" nature*, based on specifically, spatially formatted (or packed) electromagnetic (electromechanical, piezoelectric, magnetostrictive ...) resonant or oscillatory matter states. In other words, matter (or "energy-masses-moments") states are internally composed as spatial or structural combinations of more elementary physical resonators and oscillators, which are, after number of superposition and interference events, taking forms of elementary particles, atoms, molecules and other masses.

*Connections and forces (attractive, repulsive, adhesion, cohesion, Van der Waals ...) between mentioned elementary resonating states are only superficially of an electromechanical nature, but essentially or ontologically being of an electromagnetic nature. Mentioned resonant states mutually communicate by exchanging different matter-waves, photons, mechanical and electromagnetic vibrations, what is macroscopically creating mechanical properties of masses, and producing associated heat or thermodynamics-related phenomenology. Resonant states, circuits and oscillators, which are mutually very similar or identical (topologically, physically and by mathematical, temporal and spatial spectral contents) are also susceptible to enter mutual relations of couplings, synchronizations and entanglements (for instance, Casimir effect could be an example of such coupling and synchronization between cosmic radiative energy and two almost identical, capacitive mirror plates, including number of known entanglement situations among photons and elementary particles … -see more in **[103] , [104]** and **[130]).***

Let us first establish or summarize the basic set of electromechanical analogies that will be used later to formulate some of the most important ideas and messages in this book. We can compare different configurations of electric circuits in Fig.1.1 a), b), c), d), consisting of electrical elements (resistance **R**, inductance **L**, and capacitance **C**), to different configurations of mechanical circuits, consisting of mechanical elements (mechanical resistance or damper $\mathbf{R_m}$, spring **s**, and mass **m**). From literature it is known that we can create six possibilities of different electromechanical analogies (see **[1],** pages 9-18**, [2]** and **[3]**). Here, we shall not present or analyze the way of getting mentioned analogies (since this can be found in an enormous number of publications). As a general conclusion, we shall state that *all the known electromechanical analogies have been created after noticing **similarity of the mathematical forms of corresponding differential equations** (between selected, dual or mutually similar electric and mechanical networks), which are describing currents, voltages, forces and velocities in/trough/across involved electric or mechanical networks and belonging elements. We are thus establishing mathematical analogies (on a basic and simplest way that presents the **first** criteria in creating analogies). The mathematical conclusion here is that all possible six analogy situations (literature **[1]** and **[2]**) are mutually equal, or equally useful.*

We shall now introduce the ***second*** analogy criteria (**which is not mathematical**) to give more weight and power to certain analogies (from the existing six). Let us say that besides *mathematical* analogies we would like to have (visually) mutually *equivalent electric and mechanical circuits, meaning to have **the same circuit, geometric configurations*** (or topology) of their elements (if we look at how they are mutually connected in corresponding electric and mechanical circuits, or where they are physically placed). *By introducing this **second** analogy criterion, the previous set of six analogies (which is systematically analyzed in **[1], [2]** and **[3]**) is reduced to the set of only two dual analogy situations (as presented on Fig.1.1 a), b), c), d)). Such mathematical and **topographic analogy** or situation can be represented by velocity-to-voltage, and force-to-current analogy (known in literature as the **Mobility** type of analogies).*

Fig. 1.1c

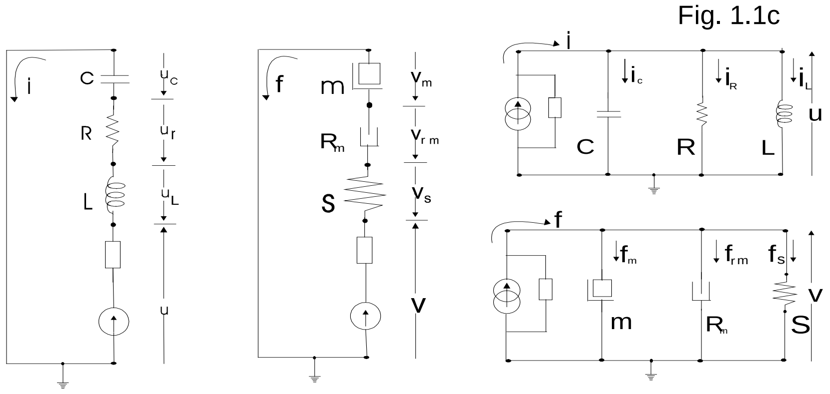

Fig.1.1a Fig. 1.1b Fig. 1.1d

Fig.1.1. Equivalent electric and mechanical circuits

The content and message that results from this *double level analogy platform* is shown in T.1.1. In this book, it will be demonstrated that the analogies from T.1.1 are presenting the most important, most predictive, most practical, and most natural (electro-mechanical) analogy platform in physics (and later, the same Mobility type analogy platform from T.1.1 will be widely extended; -see T.1.2 until T.1.7). Here we are creating grounds for productive analogical and conceptual thinking, conclusions and predictions making, to understand and describe world of Physics on a more intuitive, analogical, and better conceptual way.

T.1.1

Electric parameter / [unit]	**Mechanical parameter / [unit]**
Voltage (=) u (=) [V = volt]	**Velocity (=) v (=) [m/s]**
Current (=) i = dq/dt (=) [A = ampere]	**Force (=) f = dp/dt (=) [N = kg m/s^2 = newton]**
Resistance (=) R (=) [Ω = ohm]	**Mech. Resistance (=) R_m (=) [m /N s = s/kg]**
Inductance (=) L (=) [H = Henry]	**Spring Stiffness (=) S (=) [m/N = s^2/kg]**
Capacitance (=) C (=) [F = farad]	**Mass (=) m (=) [kg]**
Charge (=) q = Cu (=) [C = coulomb]	**Momentum (=) p = mv (=) [kg m/s]**
Magn. Flux Φ (=) [Wb = V s = Weber]	**Displacement (=) x (=) [m]**

To establish full (1:1) analogy and symmetry between idealized elements of electrical and mechanical circuits, as given in T.1.1 (**L ⇔ S** and **R ⇔ R_m**), it was necessary to redefine the unit of spring stiffness and unit of mechanical resistance or damping. In literature we find for spring stiffness **S (=) [N/m]**, and here we use **S (=) [m/N]**, and for mechanical resistance in today's literature is very usual to find **R_m (=) [Ns/m]**, and here we use **R_m (=) [m/Ns]**. The consequences of these parameter redefinitions are that usual (traditional) meaning of **Mechanical Impedance** = **Force/Velocity** should also be redefined into **"New Mechanical Impedance"** = **Velocity/Force** (presently known in Mechanics as **Mobility**), to fully support the **VOLTAGE-VELOCITY** and **CURRENT-FORCE** analogies. Similar redefinitions will be also applied to mechanical resistance (or friction constant) and spring stiffness valid for rotation (see all equations from (1.1) until (1.9)).

All the other analogies are equally useful (but only on the mathematical, formal level of analogies), and when we consider certain (additional) rules, correct use of any of these analogies will lead to the same result (but often in a more complicated, more difficult and not so obvious and intuitively clear way).

Let us come back to the equivalent (and idealized) models given in Fig.1.1 and introduce rotational motion, yet another level of study for the creation of additional analogies. Let us imagine two more (mechanical), analogous, oscillating circuit models added to this situation, presenting the case of rotation (in series and parallel configuration of oscillatory rotating mass having a moment of inertia $\mathbf{J}$, angular velocity ω, angular momentum $\mathbf{L} = \mathbf{J}\omega$, torque $\tau = d\mathbf{L}/dt$, rotational friction $\mathbf{R_R}$, and spring stiffness for rotational oscillations $\mathbf{S_R}$). In such situation (analogous to cases shown in Fig.1.1 a), b), c), d)) we can establish all the relations between the parameters from T.1.1, adding to them similar relations concerning the rotational oscillatory motions (here not presented with any additional figure, but easy to understand and visualize).

Generalized Ohm's Laws and Generalized Kirchoff's Laws are directly applicable to the circuit situation on Fig.1.1 (see **[2], Vol. 2**).

Kirchoff's Voltage Law states: "**The sum of all the *across element voltage differences* in a loop is equal to zero.**" Based on the situation in Fig.1.1 a), we shall have:

$$\sum \mathbf{u}_i = 0,\ \mathbf{u} = \mathbf{u}_L + \mathbf{u}_R + \mathbf{u}_C = \mathbf{L}\frac{\mathbf{di}}{\mathbf{dt}} + \mathbf{Ri} + \frac{1}{\mathbf{C}}\int \mathbf{idt} = \mathbf{L}\frac{\mathbf{d^2q}}{\mathbf{dt}^2} + \mathbf{R}\frac{\mathbf{dq}}{\mathbf{dt}} + \frac{\mathbf{q}}{\mathbf{C}},\quad (\mathbf{i} = \mathbf{i}_L = \mathbf{i}_R = \mathbf{i}_C = \frac{\mathbf{dq}}{\mathbf{dt}}), \tag{1.1}$$

For mechanical oscillating circuits, the analogy to Kirchoff's Voltage Law will state, "**The sum of all the *across element velocity differences* in a loop is equal to zero.**" Based on the situation in Fig.1.1 b), we shall have:

$$\sum \mathbf{v}_i = 0,\ \mathbf{v} = \mathbf{v}_S + \mathbf{v}_{Rm} + \mathbf{v}_m = \mathbf{s}\frac{\mathbf{df}}{\mathbf{dt}} + \mathbf{R}_m\mathbf{f} + \frac{1}{\mathbf{m}}\int \mathbf{fdt} = \mathbf{s}\frac{\mathbf{d^2p}}{\mathbf{dt}^2} + \mathbf{R}_m\frac{\mathbf{dp}}{\mathbf{dt}} + \frac{\mathbf{p}}{\mathbf{m}},\quad (\mathbf{f} = \mathbf{f}_S = \mathbf{f}_{Rm} = \mathbf{f}_m = \frac{\mathbf{dp}}{\mathbf{dt}}). \tag{1.2}$$

For a series, rotational resonant elements circuit (like Fig.1.1 b)), the analogy to Kirchoff's Voltage Law will state, **"The sum of all the angular *across element velocity differences* in a loop is equal to zero"**,

$$\sum \omega_i = 0,\quad \omega = \omega_{SR} + \omega_{RR} + \omega_J = s_R\frac{d\tau}{dt} + R_R\tau + \frac{1}{\mathbf{J}}\int \tau dt = s_R\frac{d^2\mathbf{L}}{dt^2} + R_R\frac{d\mathbf{L}}{dt} + \frac{\mathbf{L}}{\mathbf{J}},\ (\tau = \tau_{SR} = \tau_{RR} = \tau_J = \frac{d\mathbf{L}}{dt}). \tag{1.3}$$

Kirchoff's Current Law states: **"The sum of the entire *through element currents* entering into a junction is equal to the sum of the *through element currents* out of the junction".** Given the situation in Fig.1.1 c), we shall have:

$$\sum \mathbf{i}_{inp.} = \sum \mathbf{i}_{outp.},\ \mathbf{i} = \mathbf{i}_C + \mathbf{i}_R + \mathbf{i}_L = \mathbf{C}\frac{\mathbf{du}}{\mathbf{dt}} + \frac{\mathbf{u}}{\mathbf{R}} + \frac{1}{\mathbf{L}}\int \mathbf{udt} =$$
$$= \mathbf{C}\frac{\mathbf{d}^2\mathbf{\Phi}}{\mathbf{dt}^2} + \frac{1}{\mathbf{R}}\frac{\mathbf{d\Phi}}{\mathbf{dt}} + \frac{\mathbf{\Phi}}{\mathbf{L}},\ (\mathbf{u} = \mathbf{u}_C = \mathbf{u}_R = \mathbf{u}_L = \frac{\mathbf{d\Phi}}{\mathbf{dt}}). \tag{1.4}$$

For similar mechanical circuits, the analogy to Kirchoff’s Current Law will state, **"The sum of all the input *__through element forces__* is equal to the sum of the output *__through element forces__*"**. Looking at the situation in Fig.1.1 d), we shall have:

$$\sum \mathbf{f}_{inp.} = \sum \mathbf{f}_{outp.},\ \mathbf{f} = \mathbf{f}_m + \mathbf{f}_{Rm} + \mathbf{f}_S = \mathbf{m}\frac{\mathbf{dv}}{\mathbf{dt}} + \frac{\mathbf{v}}{\mathbf{R}_m} + \frac{1}{\mathbf{s}}\int \mathbf{vdt} =$$
$$= \mathbf{m}\frac{\mathbf{d}^2\mathbf{x}}{\mathbf{dt}^2} + \frac{1}{\mathbf{R}_m}\frac{\mathbf{dx}}{\mathbf{dt}} + \frac{\mathbf{x}}{\mathbf{s}},\ (\mathbf{v} = \mathbf{v}_m = \mathbf{v}_{Rm} = \mathbf{v}_S = \frac{\mathbf{dx}}{\mathbf{dt}}). \tag{1.5}$$

For an analogical rotational element being in parallel-configuration circuit (like on Fig.1.1 d)), the analogy to Kirchoff’s Current Law will state, **“The sum of all the input *__through element torques__* is equal to the sum of the output *__through element torques__*”**,

$$\sum \tau_{inp.} = \sum \tau_{outp.}\ ,\quad \tau = \tau_J + \tau_{RR} + \tau_{SR} = \mathbf{J}\frac{d\omega}{dt} + \frac{\omega}{R_R} + \frac{1}{s_R}\int \omega dt =$$
$$= \mathbf{J}\frac{d^2\theta}{dt^2} + \frac{1}{R_R}\frac{d\theta}{dt} + \frac{\theta}{s_R}\ ,\ \ (\omega = \omega_J = \omega_{RR} = \omega_{SR} = \frac{d\theta}{dt})\ . \tag{1.6}$$

To avoid possible (terminological) confusion in understanding the previous system of analogies, we must make a difference between the electric potential and voltage, which is a difference of potentials between the two points ($u = \varphi_2 - \varphi_1 = \Delta\varphi$). The same is valid for velocities concerning certain reference level, and for the velocity «on» a certain mechanical element (measured ***__across the element__***), which is equal to the difference of two referential velocities. On the same way ***__through-element__*** variables are Current, Torque, Force, since such variables are transmitted through (or measured when passing through) certain element (see chapter 2, reference [55], Modern Control Systems, 12th Edition, Richard C. Dorf, Davis Robert H. Bishop, University of Texas at Austin, Pearson Education). A next important difference between the electric and mechanical analog elements is in the fact that all electric elements from T.1.1 are scalars, and some of the analogical mechanical elements can be vectors (**v, p, f**).

On the other hand, if we now start with the mechanical conservation law of momentum, we shall have:

$$\sum \mathbf{p}_{inp.} = \sum \mathbf{p}_{outp.} \Rightarrow \frac{\mathbf{d}}{\mathbf{dt}}\sum \mathbf{p}_{inp.} = \frac{\mathbf{d}}{\mathbf{dt}}\sum \mathbf{p}_{outp.} \Leftrightarrow \sum \mathbf{f}_{inp.} = \sum \mathbf{f}_{outp.}\ . \tag{1.7}$$

Clearly, momentum conservation (1.7) directly corresponds (or leads) to "Kirchoff's Force/Current Laws" (1.5). Following the same pattern of conclusion in (1.7), we can return to the electric system and develop the total electric charge conservation law:

$$\sum \mathbf{i}_{inp.} = \sum \mathbf{i}_{outp.} \Rightarrow \frac{\mathbf{d}}{\mathbf{dt}} \sum \mathbf{i}_{inp.} = \frac{\mathbf{d}}{\mathbf{dt}} \sum \mathbf{i}_{outp.} \Leftrightarrow \sum \mathbf{q}_{inp.} = \sum \mathbf{q}_{outp.} . \qquad (1.8)$$

If we now take the mechanical conservation law of angular momentum, we shall have:

$$\sum L_{inp.} = \sum L_{outp.} \Rightarrow \frac{d}{dt} \sum L_{inp.} = \frac{d}{dt} \sum L_{outp.} \Leftrightarrow \sum \tau_{inp.} = \sum \tau_{outp.} . \qquad (1.9)$$

Obviously, angular momentum conservation (1.9) directly corresponds (or leads) to "Kirchoff's Force/Current Laws" for rotational motion (1.6), or, in other words, this is just a torque conservation law.

*In a previous way, step-by-step, we introduced new elements for supporting the **third** level of Mobility-type of electro-mechanical analogies, connecting them to the well-known and proven conservation laws in physics ((1.1) to (1.9)). This task must be completed by considering all other, known conservation laws in physics in such a way as to obtain multilevel parallelism and analogies between electric and mechanical formulas and relevant closed circuits, as before. By using such method, we could maybe formulate or predict some new, up to present not explicitly and distinctively expressed, conservation laws.*

Now we can summarize all the additionally exposed analogies (comparing corresponding equations from (1.1) to (1.9)) and extend the table of Mobility-type analogies T.1.1 to T.1.2 (by adding the elements originating from “rotational” circuit situations, (see **[3]**)).

T.1.2

Electric parameter / [unit]	Mechanical parameter / [unit]
Voltage (=) u (=) [V = volt]	Velocity (=) v (=) [m/s]
	Angular Velocity (=) ω (=) [rad/s]
Current (=) i = dq/dt (=) [A = ampere]	Force (=) F = dp/dt = f (=) [N = kg m/s^2 = Newton]
	Torque (=) τ = dL/dt (=) [kg m^2/s^2]
Resistance (=) R (=) [Ω = ohm]	Mech. Resistance (=) R_m (=) [m /N s = s/kg]
	Mech. tors. Resistance (=) R_R (=) [s/kg m^2]
Inductance (=) L (=) [H = henry]	Spring Stiffness (=) S (=) [m/N = s^2/kg]
	Torsion. Spring Stiffness (=) S_R (=) [s^2/kgm^2]
Capacitance (=) C (=) [F = farad]	Mass (=) m (=) [kg]
	Moment of Inertia (=) J (=) [kg m^2]
Charge (=) q = Cu (=) [q = coulomb]	Momentum (=) p = mv (=) [kg m/s]
	Angular Momentum (=) L = J ω (=) [kg m^2/s]
Magn. Flux Φ (=) [Wb = V s = weber]	Displacement (=) x (=) [m]
	Angle (=) θ (=) [rad] (=) [1]

Along with T.1.2, let us **dimensionally** compare relevant expressions for linear motion in a gravitational field, the situation in electromagnetic field, and the situation related to the rotation of masses, as summarized in T.1.3, T.1.4, and T.1.5. For the common and generic names of the corresponding (internally analog) columns in the following tables, we shall use the names and symbols of relevant electromagnetic parameters (see **[2], Vol. 1**).

T.1.3	[W] = [ENERGIES]	[Q] = [CHARGES] (Integrated Through Variable)	[C] =[CAPACITANCES] (Capacitive Storage elements)
Electro-Magnetic Field	$[Cu^2]$ (=) $[qu^1]$ $[Li^2]$ (=) $[\Phi i^1]$	$[Cu^1]$ (=) $[qu^0]$ (=) [q] $[Li^1]$ (=) $[\Phi i^0]$ (=) $[\Phi]$	$[Cu^0]$ (=) $[qu^{-1}]$ (=) [C] $[Li^0]$ (=) [L]
Gravitation	$[mv^2]$ (=) $[pv^1]$	$[mv^1]$ (=) $[pv^0]$ (=) [p]	$[mv^0]$ (=) $[pv^{-1}]$ (=) [m]
Rotation	$[J\omega^2]$ (=) $[L\omega^1]$	$[J\omega^1]$ (=) $[L\omega^0]$ (=) [L]	$[J\omega^0]$ (=) $[L\omega^{-1}]$ (=) [J]

T.1.4	[U] = [VOLTAGES] (Variable across Element)	[I] = [CURRENTS] = d[Q]/dt (Variable Through Element)	[Z] = [IMPEDANCES] (=[mobility] in mechanics)
Electromagnetic Field	[u] (=) $[d\Phi/dt]$	[i] (=) [dq/dt]	$[Z_e]$ (=) [u] / [i]
Gravitation	[v] (=) [dx/dt]	[F] (=) [dp/dt]	$[Z_m]$ (=) [v] / [F]
Rotation	$[\omega]$ (=) $[d\omega = d\theta/dt\ /dt]$	$[\tau]$ (=) [dL/dt]	$[Z_R]$ (=) $[\omega]$ / $[\tau]$

T.1.5	[L] = [INDUCTANCES] (Inductive Storage elements)	[R] = [RESISTANCES] (Energy dissipaters)	[Φ] =[DISPLACEMENTS] (Integrated Across Variable)
Electromagnetic Field	[L]	[R]	$[\Phi]$ (=) [Li]
Gravitation	[S]	$[R_m]$	[x] (=) [SF]
Rotation	$[S_R]$	$[R_R]$	$[\alpha]$ (=) $[S_R\tau]$

The most interesting and most significant set of Mobility analogies in T.1.3 is related to different natural fields' charges, or fields sources. We can see that the source of electric field is electric charge q, and that for magnetic field the most representative is magnetic flux Φ. We could also introduce an imaginative, and innovative way of seeing mutually analogical items from T.1.3. For instance, under "[C] = [CAPACITANCES]" we find items that are on certain specific way able to accumulate electromagnetic and mechanical energy, or, at least, able to capture different field-charges that are analogically summarized as "[Q] = [CHARGES]". Here we also need to understand or to notice that mass m always presents kind of energy accumulation.

We also know that internal mass constituents are atoms and molecules, or states with some of different field-charges, dynamic or motional elements such as electric and magnetic charges (including relevant dipoles and multipoles), magnetic fluxes, and mechanical and electromagnetic moments, meaning that mass (or better to say gravitational mass) is a kind of storage or accumulator of mentioned dynamic entities. ***That also means that real origins and sources of gravitation are dynamic entities being inside masses, or that standstill (rest) masses are not the primary and dominant sources of gravitation, but oscillating masses are.***

As we see in T.1.3, for gravitation, rotation and mechanics related fields, the most important are linear moment p and angular moment L (which are anyway coupled with electromagnetic charges and moments). Surprisingly, here we do not find an analogical indication that a static mass m should be the only, primary, and unique source of gravitation (as expected based on analogy between Newton and Coulomb force laws), ***but we could now analogically conclude that just mentioned dynamic elements of vibrating and motional masses are the principal sources of gravitation, or influencing gravitation, since here (in this Chapter) initially we refer to analogies between oscillatory or resonant circuits (and anyway oscillating masses have associated mechanical moments)****.*

Practical predictive meaning of such analogical and indicative revelations is that I. Newton and A. Einstein theories about Gravitation would be one day significantly updated. The other brainstorming and indicative insights here are that electric charges (or electrons and protons) should always be kind of dynamic, motional energy states (behaving like mechanical and electromagnetic moments and matter-wave packets), since all other mutually analogical mechanical and electromagnetic moments and flux entities are also kind of moving or dynamic states and vectors (as we find in T.1.3). About gravitation and familiar elaborations, see much more in Chapters 2., 8., 9. and 10.

As we know, in Physics, presently we still wrongly consider electric charges as something stable or being with fixed and static parameters (quantified as number of Coulombs). Consequently, we could expect certain electromagnetic energy exchange (currents, flux, or electromagnetic energy flow) between any of positive and negative electric charges, or between conveniently aligned electromagnetic dipoles, especially in presence of oscillations, this way creating an intuitive background for explanation of Gravitation electromagnetically. In electromagnetic theory such phenomena are also related to alternating, dielectric currents, and to electric induction.

*We also know that electric charge and magnetic flux are naturally bipolar entities (able to create dipoles or other polarized structures). Something similar should be valid for linear and angular moments, what is already known and related to action and reaction forces, electromagnetic induction, Lorenz forces and different inertial (electromagnetic and mechanical) effects. In other words, since gravitational force is known only as an attractive force, to satisfy mentioned bipolarity, real mass-energy-momentum flow should exist as a reaction force complement to gravitation, what N. Tesla conceptualized as a "**radiant energy**", and this is also kind of effective mass flow from all masses towards other masses, [97]. Such oscillating and radiating masses (practically most of masses in our Universe) are, most probably, creating standing matter waves between them, as Nikola Tesla speculated in his Dynamic Gravity theory. Rudjer Boskovic also created similar concept of Universal Natural Force*

Analogical predictions and expectations, until here, are still relatively weak and not sufficiently supported, but very much indicative. Later in this book, we will create much more of supporting background to show that on some way electric charges, magnetic fluxes, linear and angular moments are often (or most probably always) mutually coupled, complementary, and conveniently integrated or packed (characterized as matter properties), presenting the most important source (or the most significant ingredients) of all natural fields and forces. This is already kind of foundation for an innovative General Field Unification Platform. Mentioned analogies are also in a harmony with Rudjer Boskovic's "Universal Natural Force", [6], and Nikola Tesla's "Dynamic Force of Gravity", [97]. The far reaching, analogically indicative and predictive consequence here would be that present understanding of four fundamental, natural forces should be fundamentally modified and upgraded, what will dramatically change our modern Physics.

*Citation from PowerPedia, on Internet; -Tesla's Dynamic Theory of Gravity: The **Dynamic Theory of Gravity** of Nikola Tesla explains the relation between gravitation and electromagnetic force as a unified field theory (a model over matter, the aether, and energy). It is a unified field theory to unify all the fundamental forces (such as the force between all masses) and particle responses into a single theoretical framework.*

[♣ COMMENTS & FREE-THINKING CORNER: *Let us make one digression towards the Special Relativity Theory. In T.1.2, we find that mass* $\mathbf{m}$ *is analog to the electric capacitance* $\mathbf{C}$ *(*$\mathbf{m_0}$ *- rest mass), meaning that this analogy can possibly be extended in the following way:*

$$\mathbf{m} = \frac{\mathbf{m}_0}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} \Rightarrow \mathbf{C} = \frac{\mathbf{C}_0}{\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}} \tag{1.10}$$

The analogy (1.10) can be strongly supported by the following example. Let us imagine the plane electrodes capacitance, where the surface of a single electrode is $\mathbf{S}$*, and the distance between the electrodes is* $\mathbf{d}$*. In that case, electric capacitance is:*

$$\mathbf{C} = \frac{\mathbf{q}}{\mathbf{u}} = \varepsilon_0 \frac{\mathbf{S}}{\mathbf{d}}. \tag{1.11}$$

Let us suppose that the electric capacitance is moving by velocity $\mathbf{v}$ *in the direction of its electrode distance (perpendicular to the electrode surface). In this case, the distance between the electrodes is described by the relativistic contraction formula:*

$$\mathbf{d} = \mathbf{d}_o\sqrt{1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}}. \tag{1.12}$$

Introducing (1.12) in (1.11), we get the formula for capacitance (1.13) that is equal to (1.10):

$$\mathbf{C} = \frac{\mathbf{q}}{\mathbf{u}} = \frac{\varepsilon_0 \frac{\mathbf{S}}{\mathbf{d}_0}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} = \frac{\mathbf{C}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} = \frac{\frac{\mathbf{q}_0}{\mathbf{u}_0}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}. \qquad (1.13)$$

From (1.10) and (1.13) we can get the relativistic formula for the electric charge (on its own capacitance electrodes), and mass-charge direct proportionality relations,

$$q = q_0 \frac{\frac{u}{u_0}}{\sqrt{1-\frac{v^2}{c^2}}}\ ,\ \frac{q}{m} = \frac{q_0}{m_0}\frac{u}{u_0},\ \frac{q}{q_0} = \frac{m}{m_0}\frac{u}{u_0},\ m = q\frac{m_0 u_0}{q_0 u} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}. \qquad (1.14)$$

We know that a mass is velocity dependent. Starting from (1.14), we see that an electric charge is also velocity dependent, and that mass and its charge are mutually proportional, as follows.

$$\boxed{\begin{array}{l} q = m\cdot \text{const.} \\ (q \neq q_0) = q(v) \end{array}} \Rightarrow q = m\cdot\frac{q_0}{m_0}\frac{u}{u_0} = q_0\frac{\frac{u}{u_0}}{\sqrt{1-\frac{v^2}{c^2}}} = q(v) \Leftrightarrow \frac{q}{m} = \frac{q_0}{m_0}\frac{u}{u_0} = \text{const.}, u = u_0\frac{q}{q_0}\sqrt{1-\frac{v^2}{c^2}}. \qquad (1.15)$$

In (1.15), voltage **u** *is equal to the potential difference between the capacitance electrodes,*

$$u = \phi_2 - \phi_1 = \Delta\phi = u_0\frac{q}{q_0}\sqrt{1-\frac{v^2}{c^2}} = (\Delta\phi_o)\sqrt{1-\frac{v^2}{c^2}}. \qquad (1.16)$$

Also, velocities «on» analogical mechanical elements are equal to the difference of their corresponding referential velocities (if we want to treat the previous analogies correctly). Charge-to-Mass relation (or ratio) is also important from the point of view related to the direct analogy and proportionality between a mass and an electric charge (associated to the same mass). If this is valid, we could say that mass and certain associated electric charge are mutually analog (in relation to $1/r^2$, central force laws), meaning that Newton force law of universal Gravitation and Coulomb force law between equivalent electric charges are describing essentially or ontologically the same phenomenology.

Another interesting aspect of analogies can be developed if we compare the relativistic formulas for additions of the referential velocities with electrical voltages and potentials (but this might be somewhat premature and complicated at this stage).

Following the same analogy and the same supporting description as in (1.10) - (1.16), it is possible to conclude that the moment of inertia (for spinning particle in rectilinear motion), most probably, could be analogically presented as (see T.1.2):

$$\mathbf{m} = \frac{\mathbf{m}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \Rightarrow \mathrm{J} = \frac{\mathrm{L}}{\boldsymbol{\omega}} = \frac{\mathrm{J}_0}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}}\ \left\{ \Rightarrow \mathrm{L} = \frac{\mathrm{J}_0\boldsymbol{\omega}}{\sqrt{1-\frac{\mathbf{v}^2}{\mathbf{c}^2}}} \right\}. \qquad (1.17)$$

The analogical prediction of the formula (1.17) is almost obvious as a correct result since, definition of the moment of inertia is (dimensionally) the product of a certain mass and certain surface. Of course, a spinning mass, which has the moment of inertia J (described by formula (1.17)), should be moving along its axis of rotation, having a linear (axial) velocity v. In the same case, combined, non-relativistic, motional (kinetic) energy can be expressed as ($v << \mathbf{c}$):

$$E_k = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2, \qquad (1.18)$$

which is well applicable for observations in a center of gravity system. In relativistic motion situations (using (1.17)), when certain mass is in linear motion and make a spinning, we can generalize (1.18) into (1.19), a little bit hypothetically, assuming the applicability of used analogies:

$$\mathbf{E_k = (m - m_0)c^2} + (J - J_0)\omega_c^{\ 2} = \frac{pv}{\mathbf{1+\sqrt{1-(\frac{v}{c})^2}}} + \frac{L\omega}{\mathbf{1+\sqrt{1-(\frac{v}{c})^2}}} \qquad (1.19)$$

$$(\Rightarrow \mathbf{pv + L\omega = E_k}\left[\mathbf{1+\sqrt{1-(\frac{v}{c})^2}}\right],\ (\frac{\boldsymbol{\omega}}{\boldsymbol{\omega}_c})^2 = (\frac{v}{c})^2(1-\frac{v^2}{c^2})^{-1/2}).$$

From (1.19) we could try to go back to analogous electromagnetic equivalents, using again (1.13) and T.1.2 - T.1.5, (with an objective to create corresponding and meaningful relativistic expressions, in the frame of the system of analogies presented here).

*It would be interesting to make a comparison between electric charge **q**, and momentum **p** (regarding its "Mobility-type" analog match in Mechanics). As we already know, electric charge is invariant (always stays the same, constant amount), regardless of its velocity. By analogy, the same could be valid for Linear Momentum (or quantity of motion), **p** = **m**v. If we want to make them formally analogous (regardless of this is correct or not), then such mathematical exercise would produce:*

$$\left\{\begin{array}{l}\{q = Cu = inv.\} \Rightarrow \{p = mv^* = inv.\},\ m = m_0/\sqrt{1-v^2/c^2}, \\ C = C_0/\sqrt{1-v^2/c^2},\ \left\{u = u_0\sqrt{1-v^2/c^2}\right\} \Rightarrow \left\{v^* = v_0^*\sqrt{1-v^2/c^2}\right\}\end{array}\right\} \Rightarrow \qquad (1.20)$$

$$\Rightarrow p = mv^* = (m_0/\sqrt{1-v^2/c^2}) \cdot (v_0^*\sqrt{1-v^2/c^2}) = m_0 v_0^* = inv..$$

*Velocity v_0^* in (1.20) could belong to certain spatial matrix, which has its own background velocity, as something intrinsically associated with any particle, which has certain non-zero rest mass. In other words, rest mass in a certain system of reference is not at all in the state of rest if we could see it from an observer fixed to mentioned spatial matrix. A similar concept (based on analogies) should be extendible to Angular Momentum **L** = **J**ω (for rotational or spinning motion). At present, nothing confirms the possibility that momentum **p** and **L** could be invariant regarding velocity, meaning that we still miss some important elements (regarding analogy between electric and mechanical charges) to make (1.20) possible. **The real challenge here is that Newton theory about Gravitation considers only a mass as the real source of gravitation, but if we follow analogical conclusions (see T1.3, T1.6 – T1.8, and elaborations in Chapter 2.), real gravitational charges or sources should be oscillating masses, linear and angular moments coupled with electromagnetic charges, and moments originating from atoms.** In Chapter 2. and 10. of this book, we can find the more complete explanation of the same situation regarding hidden or background velocity parameters and Newtonian attraction between relevant linear and angular moments (see (10.1.4) - (10.1.7)). Since many contributions and discussions regarding boundary areas and basic assumptions of Einstein Relativity Theory are currently on rising, it would be better to leave aside (for a certain time) all premature and analogy-based conclusions (as for instance (1.10) - (1.20)) that could be in a direct connection with the Relativity Theory. ♣]*

A careful analysis of the previously established analogies, T.1.1 - T.1.5, shows that all electric and mechanical entities can be very conveniently arranged and classified into two groups, according to their definition or nature, such as: **Spatial/Geometry parameters, and Action parameters**, T.1.6.

T.1.6

Spatial/Geometry parameters	Linear Motion Gravitation	Electromagnetism	Rotation	*Spatial/Geometry parameters*
Velocities and voltages	$v = dx/dt$	$u = d\Phi/dt$	$\omega = d\theta / dt$	Velocities and voltages
Displacements	$x = sF$	$\Phi = Li$	$\theta = s_R \tau$	Displacements
Reactances (+)	s	L	s_R	Reactances (+)
Resistances and Impedances	(*) $R_m = (v / F)_{Real}$ $Z_m = (v / F)_{Complex}$	(*) $R = (u / i)_{Real}$ $Z = (u / i)_{Complex}$	(*) $R_R = (\omega / \tau)_{Real}$ $Z_R = (\omega/\tau)_{Complex}$	Resistances and Impedances
Reactances (-)	m	C	J	Reactances (-)
Charges	$p = mv$	$q = Cu$	$L = J\omega$	Charges
Forces and Currents	$F = dp/dt$	$i = dq/dt$	$\tau = dL/dt$	Forces and Currents
Action Parameters	Linear Motion Gravitation	Electromagnetism	Rotation	*Action Parameters*

(*) $R = (u / i)_{Real} \Leftrightarrow$ u(t) and i(t) in phase (=) Real, Active Impedance or Resistance,
$Z = (u / i)_{Complex} \Leftrightarrow$ u(t) and i(t) out of phase (=) Complex Impedance (=) Resistance + Reactance.

As we can see, in the table T.1.6 we do not have an explicit separation regarding parameters relative only to electric and only to magnetic fields. In the next chapters of this book, it will be shown that such parameters separation (by using analogy criteria) can be realized in the following way (see table T.1.7):

T.1.7

Spatial/Geometry parameters	Electric field	Magnetic field	*Spatial/Geometry parameters*
Velocities and voltages	$u = d\Phi/dt$	$i = dq/dt$	Velocities and voltages
Displacements	$\Phi = Li$	$q = Cu$	Displacements
Reactances (+)	L	C	Reactances (+)
Resistances and Impedances	(*) $R_{el.} = (u / i)_{Real}$ $Z_{el.} = (u / i)_{Complex}$	(*) $R_{mag.} = (i / u)_{Real}$ $Z_{mag.} = (i / u)_{Complex}$	Resistances and Impedances
Reactances (-)	C	L	Reactances (-)
Charges	$q = Cu$	$\Phi = Li$	Charges
Forces and Currents	$i = dq/dt$	$u = d\Phi/dt$	Forces and Currents
Action Parameters	Electric field	Magnetic field	*Action Parameters*

The classification given in T.1.6 and T.1.7 represents an almost complete and basic picture of electromechanical analogies, structure, and order of important physics-relevant entities known at present. Based on the above-systematized analogies, several hypothetical statements and predictions will be formulated later, regarding new aspects of Gravitation, Faraday-Maxwell Electromagnetic Theory, and Quantum Mechanics (see (2.1) - (3.5), (4.1) - (4.4), (5.15) and (5.16)).

At the same time, the specific unification platform will be proposed (in later chapters of the same book), based on generic ***Symmetries*** *(as assembled in T.1.8), that will underline possible and most probable unification areas of Gravitation, Electromagnetism, and Quantum Mechanics. The concept of basic* ***Symmetries*** *is additionally integrating and connecting all analogies found in T.1.3 – T.1.8, formulating the message that everything being on mentioned way analogical and symmetrical also presents the first indicative sign and step towards creation of the Unified Physics or Unified Field theory.*

In 1905, a mathematician **Amalie Nether** proved the following theorem (regarding universal laws of Symmetries):

-For every continuous symmetry of the laws of physics, there must exist a conservation law.
-For every conservation law, there should exist a continuous symmetry.

Let us only summarize already-known conservation laws and basic symmetries in Physics, by creating the table T.1.7.1 (without entering a more profound argumentation, since in later chapters of this paper we will again discuss basic continuous symmetries, introducing much wider background).

T.1.7.1 Symmetries of the Laws of Physics (mutually conjugate variables)

Original Domains $\leftrightarrow$ (spatial, geometry or static parameters)	$\leftrightarrow$ **Spectral Domains** (motional, dynamic parameters) Spectral domains
Simple symmetries of spacetime	
Time $= t$	**Energy** $= E$ **(or frequency** $= f$**, or mass** $= \mathbf{m}$**)**
Time Translational Symmetry	**Law of Energy Conservation**
Displacement $= x = S\dot{p} = SF, (F = \text{Force})$	**Momentum** $= \mathbf{p} = \mathbf{m}\dot{\mathbf{x}} = \mathbf{mv}$
Space Translational Symmetry	**Law of Conservation of Momentum**
Angle $= \theta = S_R\dot{L} = S_R\tau$	**Angular momentum** $= L = J\dot{\alpha} = J\omega$
Rotational Symmetry	**Law of Conservation of Angular Momentum**
Extended spacetime symmetry (Gauge theory symmetry)	
Electric Charge $=$ $\mathbf{q}_{el.} = \Phi_{el.} = \mathbf{C}\dot{\mathbf{q}}_{mag.} = \mathbf{C}\,\mathbf{i}_{mag.}$	**Magn. Charge** $= \mathbf{q}_{mag.} = \Phi_{mag.} = \mathbf{L}\dot{\mathbf{q}}_{el.} = \mathbf{L}\,\mathbf{i}_{el.}$
Law of Total Electric Charge Conservation	**The Electric Charge-reversal Symmetry**
The Magnetic Charge-reversal Symmetry	**"Total Magnetic Charge" Conservation**
(Magnetic monopole or charge is not a free and self-standing physical entity)	

In the T.1.7.1, mutually conjugate variables are presenting Fourier integral transformations (in both directions) between all values from one side of the table to the opposite side of the same table. Obviously, there are deeper connections and symmetry between conditionally labeled spatial or static parameters of our universe (on the left side of the table T.1.7.1), and their motion-related, or dynamically conjugated couples (on the right side of the table T.1.7.1). We can see from T.1.7.1 that couples of the conjugate, Original-to-Spectral domains can be extended on both sides, by creating T.1.8, using simple analogies already present in T.1.3 – T.1.7.1. *In fact, the proper merging between Analogies and basic or generic Symmetries (related to most important conservation laws, generally valid, as shown in T.1.8) should be the strongest mathematically and intuitively predictive and unifying platform of future Physics* (very much applied all over this book).

T.1.8 Generic Symmetries and Analogies of the Laws of Physics

Velocity, Voltage analogies ↓	Original Domains ↔ (spatial, static parameters)	↔ Spectral Domains (motional, dynamic parameters)	Force, Current analogies↓
	Time = t	**Energy** = E	
	Time Translational Symmetry	Law of Energy Conservation	
$v = \frac{dx}{dt} = \dot{x}$ (=) Linear Velocity	**Displacement** = x	**Momentum** = p	$F = \frac{dp}{dt} = \dot{p} = m\dot{v}$ (=) Force
	Space Translational Symmetry	Law of Conservation of Momentum	
$\omega = \frac{d\theta}{dt} = \dot{\theta}$ (=) Angular velocity	**Angle** = θ	**Angular momentum** = L	$\tau = \frac{dL}{dt} = \dot{L} = J\dot{\omega}$ (=) Torque
	Rotational Symmetry	Law of Conservation of Angular Momentum	
	Electric field	**Magnetic field**	
↑ $u = \frac{d\Phi_{mag.}}{dt} = \dot{\Phi}_{mag.}$ (=) Voltage	**Electric Charge** = $q_{el.} = \Phi_{el.} = C\dot{q}_{mag.} = C\, i_{mag.}$	**Magnetic Charge** = $q_{mag.} = \Phi_{mag.} = L\dot{q}_{el.} = L\, i_{el.}$	↑ $i = \frac{d\Phi_{el.}}{dt} = \dot{\Phi}_{el.}$ (=) Electric Current
	Law of Total Electric Charge Conservation	**The Electric Charge-reversal Symmetry**	
	The Magnetic Charge-reversal Symmetry	**“Total Magnetic Charge” Conservation**	

Spatial/Geometry parameters	Linear Motion (Gravitation)	Electromagnetism	Rotation	Spatial/Geometry parameters
Displacements	$x = sF$	$\Phi = Li = q_{mag.} = \Phi_{mag.}$ $q = Cu = q_{el.} = \Phi_{el.}$	$\theta = s_R \cdot \tau$	Displacements
Velocities and voltages	$v = dx/dt$	$u = d\Phi_{mag.}/dt = u_{el.} = i_{mag.}$ $(i = d\Phi_{el.}/dt = u_{mag.} = i_{el.})$	$\omega = d\theta/dt$	Velocities and voltages
Acceleration	$a = d^2x/dt^2 = \dot{v}$	$du/dt = d^2\Phi_{mag.}/dt^2 = du_{el.}/dt = di_{mag.}/dt$ $(di/dt = d^2\Phi_{el.}/dt^2 = du_{mag.}/dt = di_{el.}/dt)$	$\dot{\omega} = d^2\theta/dt^2 = \alpha$	Acceleration
Reactances (+)	s	$L = q_{mag.}/i = \Phi_{mag.}/i_{el.} (=) \Phi_{el.}/i_{mag.}$	s_R	Reactances (+)
Resistances and Impedances	(*) $R_m = (v/F)_{Real}$ $Z_m = (v/F)_{Complex}$	(*) $R = (u/i)_{Real}$ $Z = (u/i)_{Complex}$	(*) $R_R = (\omega/\tau)_{Real}$ $Z_R = (\omega/\tau)_{Complex}$	Resistances and Impedances
Reactances (-)	$m = p/v$	$C = q/u = \Phi_{el.}/u_{el.} (=) \Phi_{mag.}/i_{el.}$	$J = L/\omega$	Reactances (-)
Charges	$p = mv$	$q = Cu = q_{el.} = \Phi_{el.}$ $\Phi = Li = q_{mag.} = \Phi_{mag.}$	$L = J\omega$	Charges
Forces and Currents	$F = dp/dt = ma$	$i = d\Phi_{el.}/dt = u_{mag.} = i_{el.}$ $(u = d\Phi_{mag.}/dt = u_{el.} = i_{mag.})$	$\tau = dL/dt = J\dot{\omega} = J\alpha$	Forces and Currents
Power	$P = Fv$	$P = iu = i_{el.}u_{el.} = u_{mag.}i_{mag.}$	$P = \tau\omega$	Power
Work	$Fx = P \cdot \Delta t$	$\Phi i = Li^2 = q_{mag.}i = \Phi_{mag.}i$ $qu = Cu^2 = q_{el.}u = \Phi_{el.}u$	$\tau\theta = P \cdot \Delta t$	Work
Kinetic or field energy	$\frac{1}{2}mv^2$	$(\Phi i = Li^2 = q_{mag.}i = \Phi_{mag.}i)$ $(qu = Cu^2 = q_{el.}u = \Phi_{el.}u)$	$\frac{1}{2}J\omega^2$	Kinetic or field energy
Action Parameters	Linear Motion Gravitation	Electromagnetism	Rotation	Action Parameters

(*) $R = (u/i)_{Real}$ ⇔ u(t) and i(t) in phase (=) Real, Active Impedance or Resistance,
$Z = (u/i)_{Complex}$ ⇔ u(t) and i(t) out of phase (=) Complex Impedance (=) Resistance + Reactance.
(Comment: static and permanent magnetic monopoles do not exist)

Of course, the concept of Symmetries could be extended much more than shown in T.1.8, but in this book, we will consider having the strongest predictive platform when using unified analogies and continuous symmetries, as presented in T.1.8 (based on conservation laws in Physics, and such symmetries are very closely related to Translational and Rotational symmetries). In fact, from T.1.8 we can (analogically thinking) also conclude that **Electromagnetism** should have its natural mechanical couple (or equivalent) appropriately and naturally united **Linear Motion** and **Rotation** field (what is manifesting as Wave-Particle Duality in physics).

For many of symmetries known in physics, mathematics, and natural sciences, we could say that those are binary (or bipolar) types of symmetries. For instance, positive and negative values or signs of certain property, mirror-type symmetries, or existence of particles and anti-particles, positively and negatively, or mutually opposed poles of (electrically and magnetically) charged particles, real and imaginary values, left-right and up-down symmetric situations…, and we often implicitly combine Continuous symmetries with mentioned Binary or Bipolar symmetries. Somehow intuitively, by intellectual inertia and rigidity we are too often considering binary types of Symmetries as the most relevant, but maybe we should also search for more exotic symmetries based on higher numbers of participants (higher than 2; -see chapter 6, Multidimensionality, in relation to *Hypercomplex functions and n-dimensional Hypercomplex and Minkowski spaces*). For instance, Quantum theory mathematical concept of Quarks and neutrinos could be better addressed in a framework of "three-nary, triadic or three-polar" symmetries (where an extended "*Minkowski-Hypercomplex space*" has three imaginary units; -see more in Chapter 10.).

Consequently, we can also conclude from established analogies and symmetries that linear motion and spinning should be mutually complementing, coupled and united on a similar way as electric and magnetic fields (currents and voltages) are coupled and united. Here is the starting platform where new theory of Gravitation and Wave-Particle Duality should be established *(and such elaborations and objectives will be the dominant content of this book). All of that is also and essentially based on Translational and Rotational Symmetry, resulting in (or from) laws of linear, angular moments, and energy conservation. Effectively, all stable, linear, uniform, and inertial motions are mutually relative motions, but similar (relative motions) concept is also applicable to stable rotational or orbital motions that are accelerated motions. Wave-Particle Matter Duality phenomenology also presents kind of a phenomenological and conceptual "bridge" between the* ***linear motions with spatial translational symmetry****, and always and naturally associated* ***rotational motions with spatial rotational symmetry****. In addition, under linear motions of masses, we can analogically and phenomenologically associate linear motions of electrically charged particles being in certain electric field. The same way, under rotational motions of masses, we can analogically and phenomenologically associate rotational (spinning and helix) motions of electrically charged particles being in certain magnetic field.*

The existence of analogies and symmetries in physics is also indicating that fields, forces, and motions governing our universe are naturally interrelated and well united, essentially having an electromagnetic origin.

1.1. Inertia, Inertial Systems, and Inertial Motions

At the beginning of this chapter, we started comparing and creating analogies between closed electric and mechanical (oscillatory or resonant) circuits with passive (and idealized) components or elements (see Fig.1.1.). Now, since we know how to, mutually and analogically, associate electric to mechanical components, and analogically compare circuits with such components (with analogies summarized in T.1.8), we could imagine many of different configurations of electrical and mechanical circuits and networks, which are mutually analog circuits to develop advanced conceptualization and modeling of real electromechanical systems, forces and motions acting in our Universe. What mentioned physical circuits and networks have in common, regarding how we presently analyze them is:

- Most of them are kind of planar, 2-dimensional circuits (until present). We still and very seldom analyze more complex 3-dimensional or multidimensional cases since we are not producing or exploiting such technology or devices.

- Most of them (as simplest cases) can be presented as 4-pole networks, where we will have certain ***front-end, primary source*** (like voltage, current, velocity or force generators), certain network or black box (or processing body) with some components, which is distributing, circulating, or dissipating ***primary source or front-end input***, and certain load impedance which is ***the last-end, or principal power, or energy consumer***.

- We can also imagine that in addition to passive electric and mechanical components in number of 2-dimensional, planar configurations, our electric and mechanical circuits will also have some of active electromagnetic and mechanical components, like amplifiers, transistors, different current, voltage, force and velocity sources, regulators, sensors, piezoelectric and magnetostrictive components, motors, actuators etc. Of course, it will be again interesting to establish an extended list or comparative table of mutually equivalent or analog active elements between typically electromagnetic devices and systems, and typically mechanical devices and systems.

- Now we can easily imagine that mentioned 2-dimensional (planar) networks and circuits could naturally evolve towards more general 3-dimensional, spatially distributed circuits with passive and active elements, with certain set of ***front-end energy sources*** (like current, voltage, force, or velocity generators), and one or more ***last-end load impedance consumers***, or sinks, what is much closer to reality of forces and motions existing in our Universe. We still do not have sufficiently well-developed practices, technologies, devices, and theories to handle such 3-dimensional networks.

- We should underline that here-mentioned networking concept is always presenting certain mutually connected (or coupled), fully closed system (or spatial matrix) of electric, and/or mechanical, or mixed circuits and elements. Of course, in addition to mentioned passive and active circuit elements, we could also have networks and circuits where different matter waves, fields, forces, and radiation will be on some ways involved (see such illustration and explanations around Fig. 4.1.6 in chapter 4.1). This is how our world or universe is structured and working: We should never have open electric and/or mechanical circuits without ***front-end sources*** and ***last-end consumers*** *(as loads or sinks)*. All the mentioned circuits (in the world of Physics) are typically closed (nothing is hanging or flying without mounting fixtures, being fixed to something, and nothing freely and divergently disappears towards unknown infinity, without having front and last ends, being mutually connected, or coupled). *Natural laws and rules governing functionality of such circuits and networking are Momentum and Energy conservation laws, Generalized Ohm's, and Kirchoff's Laws, generalized charges conservation laws, assisted with Norton's and Thevenin's theorem (of course, all of them analogically extended and applied to mechanics).*

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From Wikipedia, the free encyclopedia:

Thévenin's theorem holds, to illustrate in DC circuit theory terms, that (see image):

1. Any linear electrical network with voltage and current sources and resistances can be replaced at terminals A-B by an equivalent voltage source V_{th} in series connection with an equivalent resistance R_{th}.
2. This equivalent voltage V_{th} is the voltage obtained at terminals A-B of the network with terminals A-B open circuited.
3. This equivalent resistance R_{th} is the resistance obtained at terminals A-B of the network with all its independent current sources open circuited and all its independent voltage sources short-circuited.
4. For AC systems, the theorem can be applied to reactive impedances as well as resistances.
5. The theorem was independently derived in 1853 by the German scientist Hermann von Helmholtz and in 1883 by Léon Charles Thévenin (1857–1926), an electrical engineer with France's national Postes et Télégraphes telecommunications organization.[1][2][3][4][5][6]
6. Thévenin's theorem and its dual, Norton's theorem, are widely used for circuit analysis simplification and to study circuit's initial-condition and steady-state response.[7][8] Thévenin's theorem can be used to convert any circuit's sources and impedances to a Thévenin equivalent; use of the theorem may in some cases be more convenient than use of Kirchhoff's circuit laws.[9][6]

In Electronics, electro technique and Electric circuit's theory, concepts regarding closed electric circuits with certain front and last end sources and loads involving different electric and electromechanical components are something natural, normal, clear, obligatory, and always satisfied situations. *Freely hanging, or randomly floating, not-fixed, open-ends electric circuits do not have any real meaning (practically do not exist in Physics and in our Universe).*

In this chapter (related to overall electro-mechanical analogies) we started to understand that in principle, everything that can be realized within electric circuits and with electric components, could be analogically presented or realized within mechanical circuits (respecting analogies summarized in T.1.8).

Let us now address contemporary Physics, Mechanics, Relativity and Quantum theory modeling, dealing with motions of masses, fluids, waves, light beams… In many cases (regarding published references), there we do not see where mentioned closed mechanical and energy-momentum circuits (or networks) are, with well-defined front and last ends (or sources and sinks). Instead of such (closed-circuit) concepts, we can find much more complex (and often unnatural, or philosophic, intuitive and anthropomorphic) concepts about Inertia and Inertial motions, about different reference systems and frames, about Lorentz transformations of energy-momentum, motional, and geometry-related parameters, with number of associated rules, postulates, transformations and conveniently invented, or postulated mathematics… and we are basically lost in a number of imaginative and often only particularly (or very specifically) applicable options, mutually not well connected and not completely explicable (but also often being intellectually seducing and amazing and maybe in some cases misleading).

In this book we will briefly accept that: ***All mechanical, electric and matter waves phenomenology can and should be conceptualized with fully closed circuits and networks (with real front and last-end elements).*** Within such closed (well defined) circuits we should make revisions and updates of everything what (in contemporary physics) directly or analogically belongs to Inertia, Inertial motions, Inertial forces, Lorentz transformations, Wave motions, Electromagnetic phenomenology, Relativity, and Quantum theory, including Action-Reaction and Induction laws in Mechanics and Electromagnetism.

Taken from: http://en.wikipedia.org/wiki/Inertia: "**Inertia** is the resistance of any physical object to a change in its state of motion or rest, or the tendency of an object to resist any change in its motion (including a change in direction). The principle of inertia is one of the fundamental principles of classical physics which are used to describe the motion of matter and how it is affected by applied forces. Inertia comes from the Latin word, *iners*, meaning idle, or lazy. Isaac Newton defined inertia as his first law in his *Philosophiæ Naturalis Principia Mathematica*, which states:[1]

The *vis insita*, or innate force of matter, is a power of resisting by which every body, as much as in it lies, endeavors to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.

In common usage the term "inertia" may refer to an object's "amount of resistance to change in velocity" (which is quantified by its mass), or sometimes to its momentum, depending on the context. The term "inertia" is more properly understood as shorthand for "the principle of inertia" as described by Newton in his First Law of Motion; that an object not subject to any net external force moves at a constant velocity. Thus an object will continue moving at its current velocity until some force causes its speed or direction to change.

Taken from: http://en.wikipedia.org/wiki/Inertial_frame_of_reference: In physics, an **inertial frame of reference** (also **inertial reference frame** or **inertial frame** or **Galilean reference frame**) is a frame of reference that describes time and space homogeneously, isotropically, and in a time-independent manner.[1]

All inertial frames are in a state of constant, rectilinear motion with respect to one another; an accelerometer moving with any of them would detect zero acceleration. Measurements in one inertial frame can be converted to measurements in another by a simple transformation (the Galilean transformation in Newtonian physics and the Lorentz transformation in special relativity). In general relativity, in any region small enough for the curvature of space-time to be negligible, one can find a set of inertial frames that approximately describe that region.[2][3]

Physical laws take the same form in all inertial frames.[4] By contrast, in a non-inertial reference frame, the laws of physics vary depending on the acceleration of that frame with respect to an inertial frame, and the usual physical forces must be supplemented by fictitious forces.[5][6] For example, a ball dropped towards the ground does not go exactly straight down because the Earth is rotating. Someone rotating with the Earth must account for the Coriolis effect—in this case, thought of as a force—to predict the horizontal motion. Another example of such a fictitious force associated with rotating reference frames is the centrifugal effect or centrifugal force.

Taken from: http://academickids.com/encyclopedia/index.php/Inertial_frame_of_reference: When there is no force being exerted on an object then the object will move inertially. This is also called 'free motion'. For example, a space module that is not firing any thrusters. (If this space module is located in intergalactic space in a region of space where gravitational influences of surrounding galaxies cancel then that is effectively a zero-gravity environment.) A frame of reference that is defined as co-moving with that object is an **inertial frame of reference**. This definition also covers rotation. A spinning gyroscope will maintain its orientation. To change the orientation of a spinning gyroscope a torque must be applied. When this torque is applied inertia manifests itself, it is inertia that maintains the gyroscope's orientation. A gyroscope that is suspended friction-free allows an observer to maintain zero rotation with respect to the co-moving inertial frame of reference.

All reference frames that move with constant velocity and in a constant direction with respect to any inertial frame of reference are members of the group of inertial reference frames.

[♣ COMMENTS & FREE-THINKING CORNER:

Let us consider having a mass in an inertial motion. Such mass should have a constant velocity (at least stable or stationary). Newton introduced the meaning of inertia and inertial motions only in relation to linear motions. In present days of Physics, it is almost obvious and an implicit consensus that inertial motions also exist for rotating, orbital or circular and spinning motions, and in this book, we will work on even larger and more general conceptualization of Inertia and Inertial motions since initial Newtonian definition is limited and obsolete.

Electric Mobility analogy to a mass and its velocity is certain capacitance and voltage across such capacitance. For instance, in a certain closed-ends mechanical circuit where the system of masses is moving like in our, or any other stable planetary system, every of planets has certain, close to inertial, state of periodical motion (meaning, it has sufficiently stable orbital velocity around its local sun). Analogically conceptualizing and concluding, such planetary system dominantly corresponds to a system of mutually connected capacitors (since capacitors and masses are mutually analog items in the Mobility system of analogies). Planetary masses have angular and tangential orbital velocities in relation to a common center of mass, which is (almost) where the sun is, meaning that equivalent, particular capacitors replacing such masses are also related to a common grounding point where the center of mass, or the sun is. In other words, the sun is one electrode, and the second electrode of every capacitance is the surface of the corresponding planet. Mass and capacitor are reactive, non-dissipative impedances (in ideal cases, without resistive and friction related losses). Since a stable planetary system is behaving approximately as frictionless, non-dissipative system during an exceptionally long time-interval, at least for us humans, we can apply mass-capacitance analogy and consider that planetary orbital motions are dominantly (and almost ideal) inertial motions. Here, the leading (and simplified) idea is that Inertia or Inertial motion largely corresponds to keeping constant mass velocity, what in analog (Mobility-analogy) terms corresponds to having constant voltage on certain capacitance (of course, in average and with negligible deviations). In addition, continuing analogical conceptualization between any of planet and all other planets, there are also many of time-evolving capacitive (or gravitational) couplings with much smaller, non-dominant capacitors. Now we should consider that planets are rotating around local sun following elliptic orbits, meaning that orbital velocities and involved (dominant) capacitances will be slightly affected by such motions, and mutual interplanetary (non-dominant) capacitive couplings will be significantly affected with permanently changing positions between any of two planets (valid for moons and other satellites, too). Such kind of interplanetary and variable capacitive couplings (in relation to periodical planetary motions) will introduce certain periodical voltage or velocity modulations to principal orbital velocities of planets, telling us that our definitions and understanding of inertial motions should be better defined (not only as constant-velocities motions). We also know that every planet in certain planetary system has a constant and very stable orbital moment, and this could create better grounds for updated (more general) definition of an inertial motion…

Anyway, the bottom line (on some way generalized, averaged and sufficiently reliable) initial and analogical conceptualization here is that constant orbital velocities are analogically equivalent to corresponding constant voltages within an equivalent network with capacitors. Constant voltages (on capacitors) in a certain capacitive network can be realized both with DC voltage sources and with AC voltage sources. In a case when the source voltage is DC (meaning stable, constant voltage), distributed planetary voltages (in analogical terms) will also be stable DC voltages, and immediate, temporal communication between such capacitors configuration can only be realized in a framework of short-living, transient situations. In a case when (front-end) source voltage is AC (meaning alternating and RMS-stable, effective voltage), distributed planetary voltages (in analogical terms) will also be stable AC, RMS-voltages (meaning stable RMS planetary velocities), and communications and couplings within such capacitors' network will be permanently active, periodical, synchronized, continuous and real-time evolving. Of course, here we are introducing new and hypothetical ideas about structural stability of planetary systems (in relation to inertial states), basically saying that there is certain front-end oscillatory signal-source or carrier (like voltage or velocity generator, or an energy storage created very long time ago), which is keeping in motion and synchronizing all planets (somewhat similar to an equivalent rotating disk or flywheel energy storage, where complete solar or planetary system presents mentioned flywheel energy storage).

In case of planetary systems, the much more probable situation is that we should have an oscillating matter-waves field carrier, which is presenting a driving front-end (oscillating voltage-velocity, or force-current) source (being most probably of an electromagnetic nature or origin). As we know from wave-particle duality phenomenology, every particle motion is on some way presentable as a matter-wave motion (defined with de Broglie wavelength…), and something similar should also be valid for inertial motions, regardless our idealized feelings and descriptions that such motions are very uniform and stable. In fact, certain stable and uniform (inertial) motion of many particles or planets belonging to the same solar system, should also present matter-waves-synchronized and stabilized motion, which is structurally shaping as standing waves structures. A similar concept is elaborated in the second chapter of this book, dealing with Gravitation (see 2.3.3. Macro-Cosmological Matter-Waves and Gravitation).

Briefly concluding, Inertia and inertial states should also be linked to standing and mutually synchronized matter waves structures since such matter waves are linked to stable, uniform motions (see more in the chapters 2., 4.1. and 10.). If we do not properly understand, conceptualize, and recognize what real inertial systems and inertial states are, what relevant, dominant, and locally superior reference systems are, our discussions, analyses, and conclusions based on such non-clarified foundations will be full of weak areas. We will arbitrarily dive into a world of different functional transformations with different names like Lorentz, covariant, invariant etc., and be satisfied if certain of results obtained with such (in some cases multiple errors-compensating) methodology is in certain situations experimentally productive and verifiable… Of course, after almost a century of performing such (on some way trial and error) practices, we could say that certain fruitful procedures, concepts, and assumptions in mathematical and theoretical physics are surfacing and showing useful, but it would be better to take a freedom and critically and creatively address, redesign, and generalize mentioned foundations (regarding reference and inertial frames, matter states and motions etc., based on analogies and symmetries).

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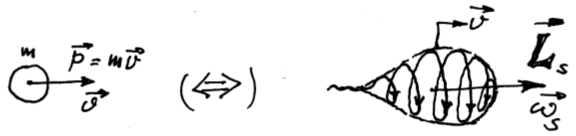
Later (in this book; -chapter 4.1 and chapter 10) we will show that every particle in linear motion effectively has certain associated, angular momentum $\mathbf{L}_s$ *that corresponds to certain kind of spinning, where particle's linear moment* **p** *and its spinning moment* $\mathbf{L}_s$ *are mutually collinear.*



$$E_k = \begin{Bmatrix} \frac{1}{2}mv^2 \\ \frac{1}{2}J\omega_s^2 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \dfrac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \dfrac{J\omega_s^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \dfrac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \dfrac{L_s\omega_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \dfrac{\vec{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \dfrac{\vec{L}_s\vec{\omega}_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Rightarrow$$

$$pv = L_s\omega_s,\ \omega_s = 2\pi f_s \Rightarrow \vec{p} = \frac{\omega_s}{v}\vec{L}_s,\ \vec{L}_s = \frac{v}{\omega_s}\vec{p}$$

Developing specific (or a new) theory in physics (separately from other fields) is usually not an easy job and can sometimes be an arbitrary and meaningless process if we do not have a general picture of the natural place and most probable, basic structure of the new theory in question. The author's position, and the main idea of this paper, is to show that multi-level analogies and symmetries, if correctly established (while respecting all conservation laws), create a common, simple, and very much acceptable (intuitive, indicative, conceptual and theoretical), self-regulatory and self-arranging platform for further generalizations in physics, and present an easy guide to new scientific discoveries. Also, in the process of creating analogies we could be able to notice possible irregularities, missing links or missing positions, or weak points in already known and well-accepted theories, or to transfer positive achievements known (only) in one field to its analogous domains. Often analogies are helping us to initiate and motivate a creative and intuitive thinking, supporting valuable brainstorming ideas.

Most of the concepts, theories, and philosophical platforms related to Physics have been initially and independently established or formulated based on limited sets of data and facts known in the time when such achievements are created. The process of creation was nothing else than best possible facts fitting, data interpolation and extrapolation, followed by convenient mathematical modeling. This is the reason that presently we have Classical Mechanics, Relativistic Theory, Electrodynamics, Thermodynamics, Quantum Theory etc. that are in some cases dealing differently within the same universe of physics. To make practical and useful mathematical modeling based on limited data fittings, every of here mentioned theories introduced certain fitting constants (theorems, postulates, assumptions, too), whenever that was necessary. There is no way to be sure that such theories are giving the most general and final picture valid for our universe because certain reasonable fitting on limited database/s (with a freedom to introduce missing links and adjusting constants) will sooner or later produce something locally operational and practical. The most common networking and structural links between theories established in such ways are conservation laws and multilevel analogies and symmetries since all of them are describing the same universe, which is a priory united (without asking us why we still do not have the United Fields theory). By applying systematically, the strategy that everything we know about nature is belonging to multilayer analogical structures, we should be able to rectify, improve, complete, and unite all present Physics knowledge and create the most stable, large, and unique platform from where we can make new excursions and explorations towards unknown domains of Physics.

Another feeling of the author of this book is that some of contemporary Physics' theories and models are originally established and promoted by their founders, which were very strong and authoritative personalities, and later being dogmatized by the army of their faithful (but often not sufficiently critical) followers, and professionally dependent assistants, or in some aspects also existentially dependent co-workers and students, becoming successors and defenders of the theory founders, and presenting barriers for penetration of new and different ideas and theories.

To painlessly introduce new ideas and concepts into such environment of old and "well established theories", the simplest and commonly acceptable strategy, proposed by the author of this book, is to test them creating here explained multilevel analogy and symmetries platform (and to see which elements of the larger Physics picture are missing or contradicting to previously established elements).

Modern science realized its biggest achievements (during the 20th century) based on the concept that nature prefers symmetry in a physics theory and that symmetry is the key to constructing physics laws without disastrous anomalies and divergences (see [10] and [11]). The objective of this book is to show that certain symmetry concept should be supported and united with wide (and multilevel) background analogy-platform, to become the strongest framework in describing and mastering natural phenomena.

One can argue that analogy platform itself is a very weak theoretical and practical platform (for making relevant conclusions and predictions) in comparison with modern Topology, but this is not correct since we are obviously living in an already (spatially-temporally) united universe, where all forces and fields are coincidentally and harmonically present and mutually compatible in every sequence of our existence (internally and externally). Nature is not asking for our permission to be united and it is staying united regardless of how well we describe, accept, or understand that unity. The omnipresent signs and projections of that unity are analogies and symmetries, and we need to follow them properly, creatively and with sufficient level of intellectual flexibility to discover the ultimate natural laws.

Most probably that our field theories are in some cases too primitive, too simple, or incorrectly formulated (based on incomplete and oversimplified, or too much artificial and complicated mathematical models), that we cannot see (or describe well) the areas of their unification. If we just conclude that (today's) Maxwell Theory and Gravitation cannot be easily united because of problems with some second rank and curvature tensors, this is maybe temporarily and conditionally correct, but it also indicates that most probably some important topology components are missing both in Maxwell and in Gravitation Theory. The consequence is that

we should continue our search for better and natural mathematical models and for missing field components (as for instance, to find them by including rotational, or torsion field components and motions in Maxwell Theory, Relativity Theory, and Gravitation). Really, to realize something like that, we do not have better starting points than using well-established and multilevel analogies, and later to test them experimentally and theoretically using modern Topology, Symmetries, Group Theory... Mathematics is the best language and logic of Nature and non-separately associated to the structure of Nature (or Universe), and mathematics itself, if properly used, can be an excellent mining and building tool for explaining and mastering the Nature inductively and deductively. ♣]

Another, much more **fundamental approach to analogies, generalized forces, and fields unification** platform we can find in our modern Physics, Mathematics, and Mathematical Physics, as follows,

1. For instance, *Vector analysis, Vector calculus, Vector Field Analysis, Linear Vector and Analytical geometry* is the part of Mathematics that is universally related to everything what in Physics belongs to vectors, pulses, forces, fields, and different moments, applicable on the same or analogical ways in Mechanics, Fluid motions, Electromagnetism and everywhere else where we characterize something with vectors. This is an enormously large unification and analogical conclusions-based platform, initially being created based on Physics, and later generalized and completed by Mathematics, but again and anyway, universally (this time deductively) being applicable to all vectors known in Physics (where we apply universally valid, natural conservation laws, including coincidental emanation of Action and Reaction forces, and electromagnetic and electromechanical induction laws).

2. Another domain of known Natural Conservation Laws is everything (in Physics) causally related to a total energy conservation. The simplest form of energy conservation law is that all input (or initial), total energy states, after certain transformation or interaction are producing number of output energy states having the same total energy. Much more sophisticated options to address energy conservation laws in Mathematics and Physics are laws and modeling belonging to Probability and Statistics, and different aspects of Parseval's identity (known from Signal and Spectral Analysis). Again, to underline, mentioned mathematical processing is universally and analogically applicable to complete (micro and macro) Physics, Mechanics, Field theories, Gravitation, Electromagnetism, Wave motions etc.

3. In cases where we have coupled rest-energy states and states of different, by vectors-characterized moments, fields, and forces (of an electromagnetic and/or mechanical nature), again we have the universally (and analogically) applicable framework of Minkowski 4-vectors concepts and modeling.

4. Universally valid laws of "**Action-equal-to-Reaction**", **Induction**, and **Inertia**, are mutually analogical or related, and known in Mechanics, Gravitation, Electromagnetism, and within everything else what has "**mass-energy-moments**" properties. There is a lot of (analogical) space to additionally develop such concepts and situations, which are clear manifestation of the united, natural fields theory (ontologically having the same origin).

5. Everything (from the world of Physics) what has certain shape in its temporal, spatial and spectral domains, including different matter-waves, can be decomposed (recomposed and later reconstructed), using Fourier and Analytic Signal analysis, on number of elementary simple-harmonic (or sinusoidal) waveforms. This is really universally applicable and essentially valid, ontological and mathematical background of everything we know in the world of Physics (see much more about such options in relation to Shannon-Nyquist-Kotelnikov-Whittaker and Denis Gabor Analytic signal analysis in [7, 57, 58, and 59]), Uncertainty relations also belong here regardless of a size of objects and matter states we address, and without need to use methods and mathematics of Statistics and Probability theory (see more in Chapter 5.). From such concept of signals and spectral analysis we can deduce understanding of a present, real time, as a resulting effect (or superposition) of past and future time events (or involved matter-waves).

6. All kind of matter-waves and wave-particle duality situations (in Physics) are universally and analogically explicable or can be naturally modeled using the same concept and mathematical modeling of Complex Analytic Signal, established by Denis Gabor (see much more in Chapter 4.0 and 10.). Classical wave equation, when formulated using the Complex, Analytic Signal Wavefunction is universally representing all wave phenomenology in micro and macro-Physics

(producing Schrödinger and other wave equations known in Quantum theory; -see more in Chapter 4.3 and Chapter 10.). Even mechanical and ultrasonic waves in solid wires can be analogically created as "ultrasonic currents" on a similar way as electromagnetic waves and photons are propagating in an open space.

7. The **variational principle of least or stationary action**, which is (in Mechanics) generating Newtonian, Lagrangian and Hamiltonian equations of motion, is also, equally and universally (or analogically) valid and applicable to all other domains of Physics (when conveniently structured).

8. Universally valid coupling and synchronization effects between resonant systems, resonant circuits, identical acoustic, electromagnetic, atomic resonators, and entities with mutually overlapping spectral characteristics, are well known (being on many ways experimentally confirmed). We know that all atoms, electrons, protons, and neutrons… can be treated as sets or ensembles of electromagnetic and mechanical resonators, and all mutually identical resonators are naturally communicating. The consequences of such universal coupling and synchronization are that all objects, masses, and atoms in our Universe are mutually connected. The second indicative and intuitive, conceptual (and still somewhat hypothetical) situation is that all kind of natural resonators in our Universe should have ontological roots or essential background in specific formation and packing of electromagnetic waves. Complete Periodic System of Elements could be (innovatively, creatively, and imaginatively) synthetized, composed, or decomposed based only on hydrogen atom.

9. Concepts of dimensionality of our Universe (until 4-dimensional spatial-temporal world), and later with mathematical extension to higher levels, or towards multidimensional spaces is basically related to the idea that all of involved natural (basic and extended) dimensions should be on some way mutually perpendicular or orthogonal (since that way mater-states in different worlds or dimensions are mutually independent and not interfering, meaning cross-correlation integrals between mentioned states are equal to zero). Until three spatial dimensions we can easily explain and visualize such concept based only on Geometry and Vector analysis. We should not forget that all three of presently known spatial dimensions (in our world) are presenting mutually equivalent spatial lengths. When we extend such three-dimensional spatial world to capture a time dimension, we are again (analogically, directly, or indirectly) using the same concept of dimensions mutual orthogonality, practically placing the time dimension on an imaginary-numbers axis, as elaborated within Minkowski 4-vectors modeling. On the same way, analogically introducing Hypercomplex or Quaternion based complex numbers and Hypercomplex Minkowski vectors (with three imaginary units), we can create spatial-temporal world with three spatial and three temporal dimensions. Or, if we operate with Hypercomplex numbers with many imaginary units (more than three), we could create many of multidimensional spaces (see more in Chapters 4.0 and 10.). Concept or matter-states functional and integral orthogonality is even much more developed and generalized in Signal Analysis, giving chances to have number of analogically constructed combinations of mutually independent multidimensional worlds and coincidently existing matter states that are mutually not interfering.

10. As we can see, Mathematics based on ideas, concepts and empirical facts coming only from naturally accessible Physics (as it is), ontologically and originally developed on a tangible and an experimentally verifiable way, is the best and universally applicable philosophy, logic, and theoretical framework of our Universe. Statistics and Probability theory are also universally and analogically applicable to all natural and other sciences (and domains of Physics and life), being exceptionally correct when we are dealing with sufficiently large numbers of identical items. When we are describing something from our Universe, using mentioned "*natural Mathematics*", we will always have clear, non-contradicting, deterministic explanations, analogic conclusions, and modeling related to all events and matter states from our Universe. Universally applicable ***Set theory*** and ***Mathematical Logic*** are gradually being accepted as the best presentations, computations and communications-related language, methods and tools of our Universe, Physics, Information and Computing related technologies (pretending that one day such Mathematical theory would be conveniently upgraded to be useful for all our daily-life needs, ordinary, technical, scientific, literal, and philosophic communications, and presentations). Such Mathematical language and logic, when completely integrated within an artificial intelligence software, and implemented on Internet communications, will help us to create the united and advanced planetary civilization, and powerful and creative supercomputing and creativity-assisting planetary brain, ready for cosmic expansion. When analyzing philosophical, ontological, imaginative and spiritual items, we should also and always try to give relevant answers and formulate valid concepts, primarily based on described "*natural mathematics*" and already proven analogies (as much as possible), instead of formulating

and practicing some “free-lance and arbitrary or artificial game-theory concepts”, or promoting some ideological philosophical statements with arbitrary conclusions, since our spoken language and available conceptual foundations (based only on our perceptual, natural and intellectual means) are still too weak, primitive and usually not well developed for addressing multidisciplinary and multiparameter items (such as the nature of space and time is).

In modern science or physics, we have number of examples of ad hock and imaginative assumptions and arbitrary labelling when explaining new and challenging phenomenology. For instance, in Physics we accepted “tunnelling effect”, as an explanation, but this is still a superficial and arbitrary assumption, and too fast created name, or statement. Assumption about probabilistic nature of wave functions is also kind of seducing platform, but we should not forget that probability and statistics are only mathematical methods and tools (universally applicable in all sciences) for summarizing behaviors and results of sets with big number of participants. We cannot give ontological and essential meaning to probability and statistics.

Assumptions and labelling of still inexplicable, weak, and strong nuclear forces are only ad hock and too fast created names, without strong, tangible, and verifiable grounds, and without good mathematical modelling. Giving names to something still unknown are not explanations and reasons to consider something as a solved problem.

Every time, before we, imaginatively and arbitrarily, by inspirative feelings give some well sounding names to new and still non explained phenomenology, we first need to consider (and respect) already existing facts, laws and frames that are known (and universally good) from mathematics and physics. Then, when we are aware of natural, physical, and mathematical, always valid, tangible, and obligatory framework, we can implement new assumptions, hypotheses, and concepts. For instance, from Signal Analysis we know that Parseval identity in different forms must be always satisfied. Also, uncertainty relations (between mutually conjugated spectral domains), should always be respected (not at all to link such relations only to Heisenberg, and to micro-world physics). All methods, laws and facts regarding Infinitesimal Calculus should also be respected (when applicable). All rules and equations related to analytic, differential and vector analysis (when relevant and applicable) should also be considered as universally valid. Then, Conservation laws of Physics should always be respected. ***We also have number of Mathematical and other Analogies that are indicative and respectful guidelines for creating and supporting new insights and concepts.*** The aforementioned foundation is the most natural and tangible platform where we need to implement any of our new, visionary, and challenging assumptions and concepts related to newly discovered phenomenology. An arbitrary, ad hock, and inspirative labelling in Physics could be very much naïve and wrong approach. *Theories, ideas, concepts, and elaborations where we are not able to create natural mathematical models and tangible, clear, repeatable verifications, are in a domain of inspirative, imaginative, ideological, artificial and/or virtual (often arbitrary, or multi-meaning) creations, even if some of such constructions could be sufficiently practical and operational (like in the case of Orthodox, Probabilistic Wave Mechanics).*

2. UNDERSTANDING GRAVITATION

This second Chapter could be better accepted and completely understood if it were placed at the end of this book, instead being here, at the very beginning, because to understand Gravitation, we really need to understand matter waves and wave-particle duality of matter, or how atoms and masses are being created from matter waves, and how masses and atoms are mutually connected and communicating using or creating matter waves. This is the reason why we will frequently refer to other chapters of this book as sources for supporting visions regarding Gravitation. To introduce new conceptual platform regarding deeper and innovative understanding of Gravitation let us first construct certain kind of suitable "**intellectually creative correlations and framework of new ideas, concepts, indicative facts and imaginative proposals"** using multi-level analogies, symmetries and phenomenological similarities between different domains and matter manifestations in physics that are in some relation to Gravitation. According to (1.1) - (1.9), T.1.6 - T.1.8, from the first chapter of this book, and T.2.1 from this chapter (but not exclusively), we could make several starting-platform, brainstorming statements, and intuitive, analogical, and hypothetical remarks, regarding gravitation. See also publications with familiar items under [63], Arbab I. Arbab, [64], Marçal de Oliveira Neto, [36] Anthony D. Osborne and N. Vivian Pope, [3], M. Prokic, [89], Charles W. Lucas Jr., [6], Rudjer Boskovic, [97], Nikola Tesla, [99], Konstantin Meyl), and [117], Jean de Climont:

1. *Gravity or field of gravitation, or gravitational force (as explained in contemporary physics) are our or Newton and A. Einstein concepts and assumptions that such force and field should exist around masses, deduced from our experience that masses around us are mutually attracting. Modeling based on such assumption (or observation) concerning planetary or solar systems is successfully, creatively, and somewhat hypothetically merged with another situation known in mechanics, described by Newton as the balance between centrifugal and centripetal forces. Also, it has been postulated, and somewhat superficially, experimentally, and mathematically elaborated an equivalence of (heavy) mass-weight in the assumed field of gravitation with similar consequences of mass acceleration. What is not considered is that masses in complex (combined oscillatory, linear, and torsional, or accelerated) holistic motions could have kind of weak, internal electromagnetic dipoles polarizations that will also create attractive electromagnetic forces (because masses internally have molecules, atoms, and electrically charged particles and dipoles with magnetic and spin moments). This way (thanks to created and conveniently aligned electromagnetic dipoles) an extended conceptualization of gravitation is becoming the same as Coulomb-type attraction between electromagnetically charged macro masses. Since all mases are in relative motions, gravitation can also be an attraction between two parallel (colinear) wires with electric currents having the same direction, because relative motions of internally created electromagnetic dipoles are creating effects of electric currents.*

Based on the mathematical similarity of Coulomb and Newton force laws we know that gravitation on some analogical way could belong to an electromagnetic phenomenology. Also, we know that matter in our universe has atoms and molecules as its building elements, and atoms have other, internal, electrically charged building elements that are electrons, protons, and neutrons (with associated linear, angular, and magnetic moments). In stationary, stable,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

inertial, or states of relative rest, it only looks that electromagnetically neutral macro masses are mutually experiencing only attractive gravitational forces (because we assume that electromagnetic charges, fields, and forces around atoms and molecules are randomly distributed and mutually, and omnidirectionally, internally compensated, neutralized, or canceled). Here is probably the mistake of our intellectual perception and conceptualization. Masses and energies of electrons and protons (including neutrons) are mutually too much different (1836 times), and everything in our universe is in some relative motion with oscillatory, linear, and angular motional components. Practically all motions in our world have some aspects of linear, angular, and other accelerated motions (including oscillatory and number of resonant states). This has an influence on creating (local, internal, volumetric, and microscopic) electromagnetic dipoles polarized formations, because of spatial separations of electrons and protons inside macro masses and atoms. Also, both, an electron, and a proton (and other elementary particles, or matter waves formations) have magnetic, orbital, angular, and spinning moments, this way creating magnetic dipoles and micro currents. Any kind of relative motion of mentioned electromagnetic entities, atoms, charges, and dipoles (including oscillations) is also creating effects of electromagnetic currents, potentials, fields, and forces around, and most probably that within such environment (of mentioned electromagnetic entities) we should search for what we consider as Gravitation.

Anyway, we still do not know in all details what the gravitation really is. However, we know that such attraction between macro masses exist, and that it could mainly be related to attractions between internal electric and magnetic dipoles being active (as an electromagnetic field) inside and around mutually attracting masses, thanks to assumed intrinsic and holistic, macro, and micro scale global rotations, orbital motions, spinning, vibrating, and associated electromagnetic complexity of matter in our Universe. Briefly summarizing, we can say that our Universe is holistically (and locally or microscopically) vibrating, rotating, and spinning on many ways (what presents certain specifically organized unity of synchronized accelerated motions). This is producing some internal effects of centrifugal mass separations and specifically oriented polarizations of associated electric and magnetic dipoles, being spatially organized on a way that forces between such dipoles are Coulomb (or Newton) attractive forces (since masses of electrons and protons are enormously different, and atoms in accelerated motions are spatially deformable and polarizable). Here, we should not forget that electric charges (based on the analogies from the first chapter) are dynamic, motional entities like different mechanical and electromagnetic moments are, but we still treat electric charges as constant, fixed, or stable parameters (or as a quantity of coulombs). That means, between electric charges (or electric dipoles) always exist certain continuous streaming or flux of radiative electromagnetic energy, which is producing external effects of Gravitation. Internal (or interatomic) attractive aspects of the same natural force are keeping attracting masses and atoms as stable objects (very much in agreement with Rudjer Boskovic Universal Natural Force and N. Tesla Dynamic Gravity descriptions). Here we have two options regarding assumed, global, accelerated and mutually synchronized motions of all masses in our Universe. For instance, masses' formations are, either globally (and mechanically) moving or rotating about something, or something else (like an electromagnetic field, flux of elementary particles and other cosmic rays of matter-waves and unspecified particles, some exotic fluidic medium, or we could say ether), is on some way streaming around and through mentioned masses (since all motions in our Universe are anyway mutually relative and interconnected). We need to have accelerated relative motion situations to benefit from electromagnetic and other matter-waves force-like effects (since forces are created around energy gradients, or around effective atoms and mass-energy agglomerations and deagglomerations).

We also empirically know that gravitation relates only to attractions above certain (minimal) size of macro-masses agglomerations. If we go deeper, below certain small, threshold-size of masses (where distances between involved mass-energy-momentum states are minimal, like inside and around atoms), there is no more meaning to speculate with Gravitation related *phenomenology (since in such cases gravitation disappears as a meaningful, measurable, or detectable phenomena and what remains are Coulomb and Electromagnetic interactions). Such threshold distance limits are (most probably) in the range, or below 100 micrometers. On the certain, associated, spectral point of view, radiative and matter-waves effects of gravitation are overlapping with red, infrared, microwaves, and lower frequency electromagnetic radiations (including possible mechanical and electromechanical vibrating effects in the same spectral domain, covering ultrasonic and exceptionally low frequencies).*

*Familiar, imaginative, and interesting concepts about gravitation (related to radiative electromagnetic energy couplings, forces, and exchanges between masses) we can find in publications of Rudjer Boskovic, [6], Konstantin Meyl, [99], "Reginald T. Cahill (Dynamical 3D-Space, Emergent Gravity, [73]" and in [117], Jean de Climont), or in remaining heritage of Nikola Tesla, [97]. See more on Internet, about "**Tesla's Dynamic Theory of Gravity**", and in the Chapters 8. and 9., of this book.*

We are still practicing Newton's gravitational force concepts between involved masses, because we still do not have a better, simpler, more straightforward, more comfortable, and practically operational concept. Of course, A. Einstein introduced an equivalent concept saying that based on specific, curved spatial-temporal deformed geometry appearing around masses, identic effects of the same Newtonian gravitation field, or force are being created (what could also be analogically, creatively, and conveniently applied on mathematical mastering of any other, natural force or field). A. Einstein conceptualization of gravitation is very much close to saying that force effects (of any kind) are following gradients of relevant potential energy, which depends on spatial-temporal shape or configuration of mutually attracting objects. What we know for sure is (citation from [90]): "There exist fields acting on both a moving body and a body at rest. They are, for example, gravitational and electric fields. A magnetic field acts only on a moving charge. An inertial force may be compared with a magnetic force..." Anyway, the essential and ontological origins of gravitation are atoms and atoms agglomerations, being mutually synchronized and connected to our Universe, internally and externally, with $1/r^2$ -dependent central, electromagnetic forces. This is an obvious and almost trivial statement that should replace the oversimplified concept that only masses are sources of gravitation. Atoms are internally structured and stabilized thanks to specific self-closed standing matter waves, or thanks to specific structure of electromagnetic fields and forces. The extension of mentioned internal atomic field structure (of electromagnetic and mechanical moments and dipoles) towards external environment presents or creates force of Gravitation (see more in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces").

We also know that gravitation inside atoms and subatomic states is inexistent or very much meaningless, and most probably attractive effects between macro masses are (at least partially) the resulting "macro-Coulomb-force effect" of superposition of many internal electromagnetic micro-dipole states. We still do not master how to shield (or neutralize) gravitation, and how to create anti-gravitation repulsive effects, because gravitation originates from an internal (spatially distributed) mass content of electromagnetically polarized entities, specifically coupled with an external, surrounding matter (and electromagnetic field) content. Most probably, that an external magnetic field could have some measurable influence (or shielding and displacements manifestations) on gravitational attraction between two (of electromagnetically neutral) masses, since atoms are formations of many spinning and orbiting constituents with magnetic-moments properties.

2. *The motion of electrically charged particles in an electrostatic field is in certain aspects analogous to the rectilinear motion of electrically neutral particles in a gravitational field (at least mathematically presentable with mutually analog expressions, such as given in T.2.1 and T.1.8, based on the established analogies from the first chapter). When a charged particle q is placed in a uniform electric field, E, in the absence of all other forces, it will experience an acceleration in the direction of the electrical field lines: $F = ma$, $F = qE = ma$, $a = qE/m$. Here, all laws of Kinematics can be applied to the motion of the charged particle. The same, seemingly (and locally) analogical situation between electric field and gravitation will become much more challenging when we realize that electric charge and mass are not mutually analog items (implicating that only a static mass cannot be the source of gravitation, but vibrating or resonating masses are especially important for understanding Gravitation).*

[♣ **The parallelism between gravity and electrostatics**.
Taken from http://physics.bu.edu/~duffy/PY106/Charge.html

An electric field describes how an electric charge affects the region around it. It is a powerful concept because it allows you to determine ahead of time how a charge will be affected if it is brought into the region. Many people have trouble with the concept of a field, though, because it is something that is hard to get a real feel for. The fact is, though, that you are already familiar with a field. We have talked about gravity, and we have even used a gravitational field; we just did not call it a field.

When talking about gravity, we got into the (probably wrong) habit of calling g "the acceleration due to gravity". It's more accurate to call g the gravitational field produced by the Earth at the surface of the Earth. If you understand gravity, you can appreciate electric forces and fields because the equations that govern both have the same form.

The gravitational force between two masses (m and M) separated by a distance r is given by Newton's law of universal gravitation:

$F = - G m M / r^2$, where the constant $G = 6.67 \times 10^{-11}\ N m^2 / kg^2$

A similar equation applies to the force between two charges (q and Q) separated by a distance r:

Coulomb's Law: $F = k q Q / r^2$, where the constant $k = 8.99 \times 10^9\ N m^2 / C^2$

The force equations are identical, so the behavior of interacting masses is identical to that of interacting charges, and related analysis methods can be used. The main difference is that gravitational forces are always attractive, while electrostatic forces can be attractive or repulsive. The charge (q or Q) plays the same role in the electrostatic case that the mass (m or M) plays in the case of the gravity.

A good example of a question involving two interacting masses is a projectile motion problem, where there is one mass m, the projectile, interacting with a much larger mass M, the Earth. If we throw the projectile (at some random launch angle) off a 40-meter-high cliff, the force on the projectile is given by:

$F = mg$.

This is the same equation as the more complicated equation above, with G, M, and the radius of the Earth, squared, incorporated into g, the gravitational field.

So, you have seen a field before, in the form of g. Electric fields operate similarly. An equivalent electrostatics problem is to launch a charge q (again, at some random angle) into a uniform electric field E, as we did for m in the Earth's gravitational field g. The force on the charge is given by **F = qE**, the same way the force on the mass m is given by **F = mg.**

We can extend the parallel between gravity and electrostatics to energy, but we will deal with that later. The bottom line is that if you can do projectile motion questions using gravity, you should be able to do them using electrostatics. In some cases, you¹ll need to apply both; in other cases, one force will be so much larger than the other that you can ignore one (generally if you can ignore one, it will be the gravitational force). ♣**]**

3. *Linear motion of electrically charged particle in a magnetic field can be transformed in rotating helix-path motion, this way becoming like combined linear and spinning motion (of electrically neutral particles or planets) in the field of gravitation (explicable, at least, by analogy in relevant mathematical forms regarding moments and energies; -see T.2.1 and T.1.8).*

4. *Electric and magnetic fields are mutually complementary, or intrinsically coupled fields with mutual interactions and transformations; -the two fields also create an electromagnetic field in the form of electromagnetic waves. Something similar is still not known regarding Gravitation, but it should exist (at least based on analogies and coupling relations between linear and rotational motions and associated magnetic moments). If the gravitation is (naturally and intrinsically) an electromagnetic manifestation, we already have an intuitive and conceptual explanation about fields and motions complementarity in case of gravitation (such as complementarity and coupling between linear and spinning motions, and complementarity between associated electric and magnetic fields).* ***Another hypothetical (or analogical) consequence here, is that gravitational attraction could be the manifestation of attractive and agglomerating effects in the close vicinity of spatial nodal-zones of stationary and standing matter waves between masses (like in cases of acoustic or ultrasonic levitation;*** *-see more in [150] and [151]), with the possibility that mentioned matter-waves are specific electromagnetic, standing waves. Internal dipole electromagnetic polarizations could be the consequence of mentioned standing waves oscillations. The meaning underlined here is that our universe is synchronously oscillating (electromagnetically, mechanically, and electromechanically). See familiar and even much more explicit and direct statements in [99] from Konstantin Meyl.*

5. *Expressions for Coulomb and Newton force laws (first between charged, and second between electrically neutral particles, or masses) are mathematically the same, but applicable within two different fields: Coulomb force in the Electric or Electrostatic field, and Newton force in the field of Gravitation. Force law between two permanent magnets can also be formulated by mathematically identical Coulomb or Newton force expression. Indeed, there must be some more profound analogy and the connection between all of them (than only mathematical one). Missing link for completing this analogy is most probably related to rotational or spinning motion components (or fields), which should be associated to every linear motion (also associated to de Broglie matter waves), and to the fact that both Gravitation and Electromagnetism could have common origins.*

6. *Rotation, spinning, and orbital motions, including oscillatory motions are an omnipresent natural phenomenology (or dominant, physical types of all movements in our universe). For instance, we see rotation of planets around their suns, rotation of moons around their planets, rotation of solar systems around galaxy centers, spinning, orbiting, and associated mutually coupled, angular and magnetic moments in the world of atoms. All of that are also (rotation related) characteristics of all subatomic microparticles, quasi-particles, other astronomic objects, and galactic formations. The origin of rotation, orbiting and spinning in Nature cannot be just a hazard, random movement distribution, or it does not happen only by pure chance (but it can be by statistics and probability quantifiable phenomena when conditions for such modeling are met). In fact, every motion (or mathematical function describing the same motion) that can be characterized by specific spectral distribution, frequency, wavelength, oscillating process, waving, etc. should be linked to, visible or hidden, rotation or spinning phenomena (when conveniently presented in its original or transformation domain). Also, every linear motion belongs to a specific case of curvilinear motion (just a question of how long the radius of rotation is). Pure straight line, constant parameters of certain uniform motion is only a mathematical idealization (or a laboratory approximation) and does not exist in the world of physics (because of omnipresent multi-body interactions trough coupled fields, energy propagation and energy-moments exchange in forms of matter waves). Micro-world rotational, orbital and spinning motions should be synchronously connected or coupled with macro-world (and astronomic) rotations and spinning, all of them being mutually coupled with oscillatory and linear movements of all participants, and here is the starting point to search for foundations of the new theory of Gravitation. Practically, all micro and macro particles and energy states in our universe should have mutually coupled linear and angular moments (including synchronous "entanglement couplings"; -see more in Chapter 10.). In [36] we can find the statement: "**The natural (i.e., forceless) state of motion is orbital, not rectilinear, i.e., bodies orbit one another freely unless an external force is applied to prevent them**". Even if we do not see how to associate orbital and spinning moments to certain mass or two-body system, we could imagine that some global, holistic, substantial orbiting-radius motion, about some dominant center of mass, exist for an adequately positioned observer (meaning that matter is always in states of relative linear and angular motions). Gravitation is the **central-force** phenomena that is the consequence of mentioned global orbiting (what is the well-supported concept in [36]). Of course, in our Universe, we also observe many cosmic-masses impacts, scattering, explosions, and associated plasma matter-states, what is producing non-rotational and non-spinning, pulsating motions, but anyway, the evolution of such events is that additional angular and spinning motional field components are being created, following mentioned random movements.*

It could also be that the field of Gravitation related to linear motions is coupled with certain, specific complementary field (so far unknown, or better to say omnipresent and only partially known under different phenomenology, but in a contemporary physics theoretically still unrecognized as such separate key-field identity). Gravitation and its complementary spinning field couple most probably have specific "field, force and moments interactions", and they are likely to create a complex "gravitational-rotational" field (presently still unrecognized, as formulated *here, and behaving like complementarities between electric and magnetic fields). Mentioned coupling between linear and spinning (or torsional) fields can be mathematically described by using the complex Analytic Signal model, where two mutually phase-shifted wave functions are creating a complex Analytic Signal wave function (see more in Chapters 4.0 and 10., and in literature references under [7], [57] and [107]). This also has big similarity with the coupling between electric and magnetic field vectors when electromagnetic waves and photons are being created (see more about Analytic Signals and matter waves in the chapters 4.0, 4.1 and 10).* ***Experiments (of Japanese research workers) are already confirming the change of spinning-mass weight in the field of gravitation of our planet. "**<u>Spinning mass that is spinning in the same direction and same plane as that of the Earth weighs little more than when at rest on the Earth's surface, and when spinning in the opposite direction weigh a little less</u>**" (cited information taken from [36]). In addition to mechanical mass spinning, one of the probable rotating field candidates, being a complement to linear mass motions and gravitation, <u>could be certain kind of associated and spinning matter wave (familiar to de Broglie, matter waves concept</u>).** Also, various phenomena of inertia and Gravito-magnetic induction, Gyromagnetic and charge-to-mass ratios, pendulum, mass-spring oscillatory systems, and gyroscopic motions should be in certain relationship with above-proposed complementarity platform (but obviously, the appropriate terminology and generally applicable conceptualization are still missing here). About "Origins of rotation of celestial and other astronomic objects", we can find still challenging sources of information in the works cited under [51]. In other words, the field of gravitation, together with its (still hypothetical) rotational, orbiting, spinning and torsional matter-waves, and other oscillatory field complements, could be certain specifically structured, electromagnetic field manifestation.*

*Presently, it looks to us that Newton's Law of Gravitation is addressing only the attractive central force between stationary or rest masses. There is nothing explicitly and directly visible regarding velocities and moments' dependent members in Newton's law. We also know that all masses in our universe are in mutually relative motions (the same mass could be in a relative state-of-rest regarding certain system of reference, and in many other states of relative movements regarding other reference systems, including oscillatory and resonant states). <u>Since all masses are always in specific kind of motion (relative to something), the new, upgraded law of Gravitation should also have dynamic motional members such as linear and orbital moments and velocities, including associated electromagnetic dipoles, moments, and charges (for instance, in reference to its dominant "**center of total energy, coordinates system**"</u>).* ***Only a mass with mentioned motional and electromagnetic attributes (including oscillating masses) can be the source of gravitation (not a static and neutral rest mass).*** *Such major conceptual platform regarding Gravitation will be addressed in this chapter. Briefly, here we intend to show that* ***pure electromagnetically neutral and static mass is not the real gravitational field charge, or source, contrary to electric charges that are real sources of electric field, also meaning that the formal and superficial analogy between Coulomb and Newton laws is partially misleading (since a static mass and electric charge are not mutually analog items, as shown in the first chapter of this book).*** *The leading idea in exploring new (hypothetical) aspects of gravitation and electromagnetism in this book (serving only as a starting platform) is based on the extended Mobility analogies chart (see Chapter 1., T.1.1 and T.1.2). There, we can clearly* *notice that some analogical positions in the chart are either empty, or undefined (see later T.2.1), or not in the full agreement with present formulations of the gravitational force, field, and energy expressions. To complete empty places of mentioned analogy chart, using again conclusion process based on analogies, we can formulate new (and hypothetical) force and field relations (as given in T.2.2). This way, (in this book) we are practically generating new ideas, mostly based on our confidence in extended Mobility-type analogies (since "Mobility system" of analogies satisfies three analogy levels, such as:* ***Mathematical level, Topology level, Similarity of different force expressions and Energy-Momentum conservation laws****). How far we could go in this process (while producing correct predictions) is still an unanswered question. At least we can notice where the already-established system of analogies is not fully coherent with other analogies and analogical predictions (for instance, related to electric charge, and mechanical and magnetic moments analogies). An initial form of such ideas is first time formulated by the author of this book in* **[3],** Prokic Miodrag; -"Energies Parallelism", Diploma Work (B.S. Thesis), November 1975. The University of Nis, Faculty of Electronic Engineering, Yugoslavia**.**

7. *Favorable opinion of the author of this book is that* ***gravitational attraction we could also explain as an (experimentally easily detectable) force within nodes of standing matter waves (what are well-known resonant effects around nodal zones of half-wavelength, standing waves ultrasonic resonators and as effects of acoustic levitation;*** *-see more in [150] and [151]). Since temporal and spatial domains (of all motional states and corresponding wave functions) are mutually linked and proportional (see more in Chapter 10.), it is reasonable to speak about temporal and spatial periodicity and temporal and spatial resonant states. One of special and important resonant states is combined temporal-spatial, resonant, and standing waves situation, when, at the same time, mutually synchronous, temporal, and spatial resonance happens. Such standing-waves' states can also create stable particles with non-zero rest masses when mentioned standing waves are on some way self-closed, or could be sources or emitters of (mechanical, electro-mechanical and electromagnetic) mater-waves when having open-ends. Nodal and antinode zones of standing waves are locally manifesting effects of attractive and repulsive forces (respectively) because this way gradients of vibrational energy are being created towards (or from) nodal and/or antinode zones. This could be an alternative conceptual explanation for natural forces we meet in the world of Physics (see much more of similar ideas in [99] from Konstantin Meyl). Masses of our universe behave as being submerged in a specific matter-waves field of resonant, standing-waves structure (acting between gravitationally attracting masses), covering an interval of low frequencies, what is again pointing to N. Tesla and R. Boskovic concepts about natural and gravitational forces; -see more in [99], [97]. Matter-waves evoked here should have mixed (or combined) mechanical, acoustic, electromechanical, and electromagnetic nature.*

*The most general and unavoidable foundations and definitions of any natural force, including Gravitation and Nuclear forces are elaborated in Chapter 10. of this book, under "*10.02 MEANING OF NATURAL FORCES". *All forces are simply spatial and temporal gradients, or first derivations of relevant energy (or mass density) distributions and involved mechanical and electromagnetic moments, being ontologically related to globally valid Uncertainty relations and omnipresent cosmic, structural, and spatial standing-waves formations. This is the only good and most* *general platform for explaining the force of Gravitation (including all other forces). Everything else (what we presently exercise about natural forces) are experimental, analogical, intuitive, and brainstorming, or indicative observations, modeling, and until certain level reasonable hypothetical assumptions.*

....

To correct or harmonize intriguing spots about real gravitational charges, we shall be able to make new hypothetical predictions (again based on analogies and symmetries, as shown in the first chapter, "T.1.8 Generic Symmetries and Analogies of the Laws of Physics"). By direct comparison, we see that Coulomb and Newton force expressions are mathematically the same, but respecting facts from T.1.8, we see that static masses and electric charges are not mutually analogous. If we consider Coulomb forces as something of primary relevance, being dominant, tangible and most realistic (as a model for understanding other forces), by applying analogy with laws of mechanics and Gravitation (as we see in T.1.8), we will find that electric (and magnetic) charges (or fluxes), and linear and angular mechanical moments are mutually analogous. Consequently, gravitational attraction (in the form of Coulomb-Newton force law) should exist only between mutually coupled (still on some way internal and hidden, spatially distributed) linear and angular (mechanical and electromagnetic) moments, dipoles and charges internally associated to masses in question. We formulate such forces using Coulomb-Newton force law, what is, at the same time, an attraction between internal, intrinsically coupled and associated electromagnetic charges, moments, and dipoles. We could also conclude that there should be certain intrinsic unity and coupling between linear and angular or spinning (mechanical and electromagnetic) moments of the same motional macro particle (familiar to de Broglie matter waves concept). Here, meaning of motion, if not detectable externally as some macro-motion of particles, could be an internal spinning, oscillatory, electromagnetic, thermal, and micro-motion situation within atoms and molecules, effectively presenting kind of spatial currents and electromagnetic charges streaming and oscillating between masses (related to certain dominant reference system, like a local galactic center of mass).

We also know that magnetic and angular (or spinning) moments are often mutually coupled in a micro and macro world of Physics (keeping constant gyro-magnetic ratios, both valid for the world of atoms, planetary systems, and galaxies). Also, we will try to properly conceptualize and understand linear and angular motions (coupled with associated electromagnetic charges and dipoles), as being fundamentally relevant for a deeper understanding of Gravitation.

On a similar way as superposition of number of micro-spinning magnetic domains is creating total, macro magnetic-moment, we should be able to, analogically present a total orbital or angular, macro moment of specific mass (and vice versa, meaning to analogically decompose a macro angular-moment into number of elementary spinning moments, as vectors).

In this book, an intrinsic unity of linear and spinning (or angular), electromagnetic and mechanical motions, with coupled moments and charges (of certain mass) is considered as the primary source of gravitation, and associated matter waves (see Chapter 4.1). Our present concepts and understanding of mass should also evolve because mass cannot be something static, totally electromagnetically neutral, and with only locally stable and fixed properties and geometry (because it is internally, externally, fundamentally, and holistically linked to associated linear and angular (or spinning) moments and being

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER...** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

electromagnetically coupled with a surrounding Universe). This is the platform for new understanding of Gravitation, favored in this book. Even in cases of sufficiently stable rest masses (within inertial and orbital states), which are macroscopically electromagnetically compensated and neutral, there is still certain minimal (residual) level of non-compensated, electromagnetically polarized, dipoles-attracting effect. Such polarizable electromagnetic charges (or dipoles and different electromagnetic and mechanical moments) produced as consequences of permanently coupled, global, holistic, and mutually relative, linear, orbital and spinning motions (including oscillations) in our Universe are creating gravitational attraction (like attraction between two wires with electric currents acting in the same direction).

We (and everything around us) are submersed entities within such global motional and specifically distributed, standing matter waves structure of mass-energy-moments and electromagnetic charges, where stable masses are localized in some relatively stable nodal positions (analogical to acoustic levitation effects; -see more in [150] and [151]). In our laboratory frames, we do not see such structured holistic couplings and motions, but Gravitation reminds us that mentioned globalized holistic movements (referenced to the specific dominant center of masses or centers of inertia) are omnipresent. In case if our Universe has more dimensions than 4 [more than (x,y,z,t)], we could imagine that mentioned holistic motion could also be recognizable from, for us not detectible, higher dimensional spaces. In [36] we can find an excellent theoretical background about gravitational central force related to orbital motions. The existence of mentioned phenomenology and associated, electromagnetically charged and polarized entities (experimentally proven, and sufficiently well conceptualized) can also be found in the number of publications of Dr. Jovan Djuric, [71].

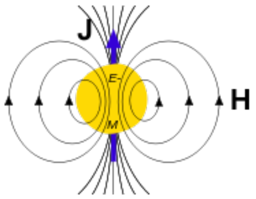
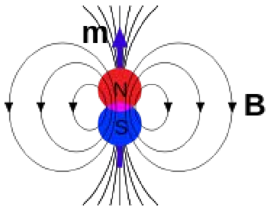
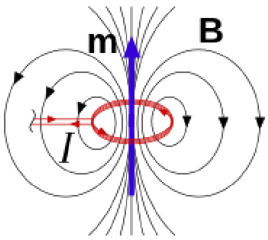
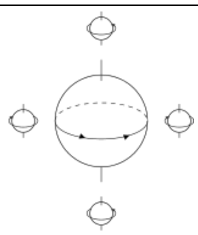
There are many theoretical attempts to create gravitation field equations following an analogy and symmetry with Maxwell electromagnetic field equations, known as equations of Gravitoelectromagnetism; -for instance, see here, https://en.wikipedia.org/wiki/Gravitoelectromagnetism.

In all such analogies-based attempts, only mass (or mass density) is taken as a direct analogical property comparable to electric charge (since, by default, we consider a mass as the real and principal source of gravitation, what is wrong, but useful and mathematically working assumption). Electric charge is indeed the source of electric field, but electric charge and mass are not directly and mutually analog items. Electric charges, when in motion, are producing electrical currents and magnetic field effects, but static mass cannot be the real and unique source of gravitation, since the well proven analogy exist only between an electric charge and oscillatory, linear or angular mass-moments, which are directly related to associated electromagnetic moments (see more about mentioned electromechanical analogies in the first chapter of this book).

Consequently, all theories where some new "Maxwell + Gravitation" elaborated equations are directly analogically formulated based on static masses attraction, are essentially or fundamentally wrong, since instead of masses, it is necessary to consider mass moments (or masses in motions, including associated electromagnetic complexity as internal electric and magnetic dipoles, currents, and potentials, what could also be parameters of vibrating masses). Mass and electric charge are not mutually analog or directly replaceable items, but anyway, mass should have some intrinsic electromagnetic properties and internal polarizations thanks to an omnipresent, global, multi-axial, linear *and angular cosmic motions (see more in Chapter 3. of this book). Real ontological sources of gravitation are atoms, connected and synchronized with other atoms, mases, and with our Universe (respecting 1/r² central forces). This is the reason why mass and gravitation cannot be locally electromagnetically shielded and neutralized because any kind of (externally applied) shield also belongs to mentioned global, omnidirectional, and multiaxial, linear, and angular motions, and cosmic couplings and synchronizations.*

Later, we will find out that Newton gravitational constant G has certain hidden, cosmic, angular speed component, what makes possible that "mass-electric-charge" analogies are still on some way defendable, understandable, and useful. Here is convenient to mention N. Tesla's ideas about Dynamic Gravity Theory, where he is imaginatively assuming that certain "radiant energy flow" should always exist between masses, this way creating effects of gravitational attraction, and saying that masses are not static entities, [97].

In conclusion, we can say that the most promising innovative (future) gravitation theory will be kind of reformulated Gravitomagnetism theory, as an analogical extension and appropriate modification of Maxwell equations, where gravitation related charges (or sources) are different subatomic and mass-spinning moments and associated electromagnetic moments, charges and dipoles (see examples of such analogies below; -Physical analogs between different fields [7], taken from https://en.wikipedia.org/wiki/Gravitoelectromagnetism).

 **Gravitomagnetism** — gravitomagnetic field **H** due to (total) angular momentum **J**.	 **Electromagnetism** — magnetic field **B** due to a dipole moment **m**...
 ... or, equivalently, current *I*, same field profile, and field generation due to the rotation.	 **Fluid mechanics** — rotational fluid drag of a solid sphere immersed in fluid, analogous directions and senses of rotation as magnetism, similar interaction to frame dragging for the gravitomagnetic interaction.

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[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Later, in the same chapter, familiar ideas are additionally elaborated (see equations under (2.11), and Figs. 2.2 and 2.3).

Every electromagnetically neutral (or by opposed-charges-compensated) macro-mass is created (once in its past) from elements that are electromagnetically charged (with magnetic and electric charge properties of its internal, specifically polarized, and dipole- and multipole-behaving constituents). In a process of creation electromagnetically neutral (non-charged) macro masses, involved electromagnetic charges are spatially creating dipoles or bipolar electromagnetic entities, presenting a process of mass compacting and/or optimal energy packing, resulting in electromagnetic charges neutralization because of mutual compensations). Thus, neutral macro-mass becomes externally, electrically, and magnetically, self-compensated and regarding electromagnetic charges (macroscopically) neutralized (but still being electromagnetically polarizable, internally). Elementary matter domains (atoms, different electric, and magnetic dipoles, etc., mostly composed of charged elementary particles in motion) present forms of self-sustaining and auto-stabilized, internally self-closed matter-waves, with rotating and/or spinning states. Such internal, elementary matter and energy-momentum states are also kind of electric-dipoles, and kind of elementary magnets (or magnetic-dipoles), because all of them are in mutually coupled motions and with an intrinsic spinning. ***Since the difference between masses of an electron and proton is enormous (1836 times), this is additionally contributing to creation of polarized electromagnetic dipoles, especially if our Universe has certain intrinsic (holistic or global) macro-rotation, and such countless number of polarized micro electromagnetic states are mutually superimposing and contributing to a macro force of gravity.*** Furthermore, from the equivalence relation between mass and energy, it is obvious that such diversity of mutually coupled rotating and oscillating matter-wave forms has its effective masses and mechanical moments. Since mass and Gravitation are mutually linked (Newton law), and mass is a "modus of energy packing" ($E = mc^2$), this implicates that Gravitation should be part or form of an electromagnetic field, because electrons, protons, and neutrons are creating atoms and other larger masses. Neutron also separates on an electron and a proton, including all known relations and interactions between photons, electrons, and other elementary particles (all of that contributing the coupling between Gravitation and electromagnetic entities, since matter or macro-mass internally and dominantly has an ensemble of electromagnetically charged and moving entities). *Since electrons (and other charged particles) always have their intrinsic magnetic, spinning, and orbital moments being mutually coupled, where gyromagnetic ratios are constant (as well as electric charge-to-mass ratios are also constant), we should conclude that mechanical rotation and electromagnetic properties (of elementary matter constituents) are also mutually dependent and coupled. Moreover, most of the phenomena known in one field (for instance in mechanics of particles motions) should be presentable or (at least mathematically and analogically) convertible to equivalent states in electromagnetism, and vice versa (see literature under [31], Chapter 4, T4.0 of this book, and comments around equations (10.1.4) - (10.1.7) in chapter 10; -See also **[131]** **"**Francisco Cabral⊠ and Francisco S. N. Lobo. Gravitational waves and electrodynamics").*

The fact is that *in our universe different centers of masses, centers of moments of inertia, centers of electric and magnetic charges and centers of gravity (where self-gravity is zero), are never, and coincidently, covering the same space (or the same spot) since all masses are in mutually relative linear and angular motions (including* *possible vibratory and resonant states). This is creating intrinsic, spatial mechanical couple or torque and bipolar and multipolar tension and stress effects (between mentioned mutually displaced centers), additionally producing dipoles, multipoles, and electromagnetic polarization effects. The same situation is analogically valid and coincidently present within involved electric and magnetic elements and charges, meaning that it is impossible to have totally electromagnetically neutral mass, but it is always the case that certain (even small), resulting, internal, bipolar, or multipolar, or dipole polarization exist in all cases of masses*. Such mutually displaced centers of opposed couples, moments and dipoles are also mutually related and coupled. An attractive coupling force always exists between mentioned internal, mechanical, and electromagnetic constituents, moments, and charges of a certain mass, but, ***since mass constituents are atoms and molecules, the sources of gravitation should be causally and strongly linked to atoms and forces emanating from atoms*** (see more about such hypothetical ideas in Chapter 8). This is the area where we should search for electromagnetic and atomic origins of gravitational force. Very indicative experimental verification of such (displaced centers and associated moments and torques) situations are described and theoretically sufficiently well supported in papers from Prof. Dr. Jovan Djuric, [71]. The existence of mentioned effects is additionally confirming that our Universe has certain level of (hidden) inherent, intrinsic, holistic rotation, which is coupled with linear-motion elements, creating number of intrinsic resonant states (being the meaning of Wave-Particle duality and Matter Waves phenomenology). Similar and richer concepts about the real nature of gravitation can be found in [89], from Charles W. Lucas Jr.

8. In 1916, A. Einstein published his General Relativity theory that is describing gravity as certain space-time geometry deformation around masses. Nevertheless, here we are still and mostly addressing Newtonian gravity, since General Relativity is not at all presenting and producing any of dramatically different, much better, and clearly verifiable, unique, and distinctive predictions and results. Applicability of Newtonian mechanics and old gravity laws is still very practical and sufficient (at least in the frames of our solar system). In 1919, General Relativity was considered officially validated by the observation of the bending of light during a solar eclipse, as the sun's gravitational pull warped space-time (but mentioned verification was not enough precise, it was realized too fast, and based on few facts and assumptions). For instance, possible and typically electromagnetic interaction between light and gravitation-field explicable as the manifestation of an electromagnetic reality is neglected because Gravitation is considered as an independent and distinct natural force. Another success of Einstein General Relativity was in resolving an anomaly in the orbit of Mercury (see [60]; John W. Moffat). "*Like all planets, Mercury traces an ellipse in its orbit around the sun, and the position of its closest approach to the sun called the "perihelion" advances with successive revolutions. The pattern of the planet's orbit thus looked like a complex rosette over time. Newton theory was not able to explain such revolutions of Mercury orbit. Einstein performed a calculation investigating the anomalous advance of the perihelion of Mercury. He discovered that when he inserted the mass of the sun and Newton's gravitational constant into his equations, his theory of general relativity correctly predicted the strange precession of Mercury's orbit" (but later it was demonstrated by others that most of such mathematical processing has too little to do with supporting Relativity Theory and its predictions. It was demonstrated that similar results can be mathematically drawn without using General Relativity theory concepts).* Einstein's General

Relativity theory was a long time (close to the end of 20th century) considered and believed to be in perfect and generally applicable agreement with all astronomic observations (that are already known, and even with future, still unknown data, and facts). However, there is increasing evidence, based on new astronomic observations, that this could be different. Growing amount of observational data are also revealing that both Newtonian and Einstein's gravity have similar weak sides. There is now overwhelming evidence for much stronger gravity in remote spinning galaxies than what is expected by Newton and Einstein's theories. To keep Einstein and Newton's theories in power, it was postulated (and never observed, measured, or proven) that there must exist a large amount of "dark, invisible matter, mass and/or energy" in remote galaxies and galaxies clusters (and everywhere around us). Such postulated dark mass and/or energy could strengthen the pull of gravity and lead to an agreement of presently practiced theories with the observed data, as much as shows necessary (see [60]; John W. Moffat). Similar effects of stronger gravitational attractions (in some spinning parts of galactic motions) could also be understandable by assuming the (still hypothetical) existence of stronger electromagnetic-dipoles polarization conveniently distributed. Obviously, Newtonian and Einstein's gravity should be updated, preferably not by postulating the presence of some new, exotic dark, invisible and improvable, artificial energy-mass entities. Relativity theories concerning A. Einstein, who creatively placed all relevant elements in the same envelope, will most probably be reorganized and evolve a lot and soon (see [73] Reginald T. Cahill).

Concerning A. Einstein Relativity, we could be a little bit more creatively curious, critical, suspicious, or intellectually open towards analyzing of not-main-stream and heretic voices, new contradicting experimental results, and publications saying, and maybe proving, that:

1° There are indications that the light speed C, or maximal velocity of electromagnetic waves, or maximal possible speed in our Universe is not always constant, and not as absolute as postulated or promoted in Relativity theories.

2° We really need, and we must conceptualize, the existence of certain material texture, fluid, specific matter state, or new state of an empty vacuum, historically named as ether, as being a carrier of electromagnetic and other matter waves. In other words, old Michelson-Morley interferometer results are most probably wrongly and incompletely organized, conducted, and interpreted, maybe because light beams or photons are oscillating only transversally and an expected streaming, or flow of ether should be kind of laminar or linear motion. Of course, as long dielectric permittivity and magnetic permeability of an ideal vacuum state are stable and verifiable constants, we can be convinced that maximal light speed C in a vacuum is also constant ($\mathbf{C = 1/\sqrt{\varepsilon_o \mu_o}}$). Since ideal vacuum state has such stable, durable, and measurable electromagnetic properties (as $\boldsymbol{\varepsilon_o}$, $\boldsymbol{\mu_o}$), how we could say that mentioned constants are not parameters of a specific, realistic, and tangible substance that also has elastic properties. We call it an ether. If we measure the speed of light in some much less exotic transparent medium like water, we already know that such light speed also depends on medium motion (or water flow). The same (slightly modified) formula for light speed (in a static case) will again be valid as $\mathbf{C = 1/\sqrt{\varepsilon \mu}}$. Consequently, electromagnetic properties of space and matter, and involved relative motions are decisive or influential for the nature and amount of the light speed C. From different, independent measurements, we already know that speed of light is not always constant, and not perfectly absolute as postulated in Relativity theory. We should consider specific importance of the constant $\mathbf{C = 1/\sqrt{\varepsilon_o \mu_o}}$ for Physics, and have another, separate and independent interpretation of light speed in different conditions. Something similar is valid for

Newton gravitational constant G (it is never the same and never constant, when measured in different laboratories, and in different spatial locations). By its definition, ether is a texture, matrix or fluidic part of any material substance and space, meaning that ether motion (when effectively exist) should have an influence on the speed of light.

3° Experimental and non-doubtful evidence about the existence of relativistic length contraction, time dilatation, and mass distortion is only, partially, and approximately, confirmed, as conceptualized in Relativity Theory (but it is verified that such effects of mutual temporal-spatial couplings, dependence and flexibility really exist). GPS satellites time-synchronization and related distances measurements are not fully proving, disproving, nor entirely using Relativity Theory concepts and Lorentz time-length transformations. GPS also operates well without any involvement of calculations based on Relativity theory.

What works very well, and what is creatively and correctly fitted or associated to A. Einstein Special and General Relativity theory (but it also exists independently, and it would exist even if Relativity Theory was never created) is Minkowski extension of Euclidean and Newtonian space (where time is considered as something stable and absolute). The most useful products of mentioned association (or integration between Minkowski Space, Classical Mechanics, and Relativity Theory concepts) are energy-momentum 4-vectors. It is often misleading and wrongly interpreted that 4-vectors are exclusive products and achievements of Relativity Theory. A. Einstein, or probably it is more correct to say, his wife (who was Minkowski's student) realized that when using Minkowski space and 4-Vectors he can elaborate, rectify, and reinforce his theory much better (than what was the simpler case when he postulated it initially). This way analyzes of different interactions in Mechanics are becoming practically very elegant, or mathematically very operational and productive (at least in analyzing and predicting the number of impact and scattering interactions in microphysics). Without proper merging of Minkowski's and Relativity Theory concepts, we would still have complicated, problematic, and confusing Relativity-theory ideas. In fact, only what works very well, and what is practically useful and valid in experimental physics (concerning Relativity theory) are different aspects and applications of energy-momentum (Minkowski space) 4-vectors. All other Relativity theory 4-vectors are well fitted, mutually complementing formulas, conveniently arranged to show that Relativity theory (including Lorentz transformations) really works as postulated by A. Einstein. Just saying, we can also find challenging publications, voices and experimental results showing that light speed C may not always be absolute and constant, that time and length extension or contraction cannot be fully experimentally proven on the way prescribed using Lorentz transformations, etc. *In the last chapter of this book (Chapter 10.), it will be demonstrated that existence and understanding of mass can be significantly extended based on specific elaborations around Minkowski energy-momentum 4-vector and corresponding Phasors expressed as Analytic Signal forms (see "*10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality*"). Briefly concluding, in this book, the objective is to show that sources of gravitation are not static masses, but on some way, oscillating masses and mass-associated mechanical and electromagnetic moments and electromagnetic charges are. Effectively, force of Gravitation is essentially originating from atoms and molecules, being electromagnetic formations and mass constituents.*

9. *Here, we will attempt to explain, update (and conceptually redesign) gravity as specific dynamically stable and standing matter-waves structured field formation between masses (or between atoms that are creating interacting masses). Gravitation is an effect based on specific unity and mutual interactions of linear and rotational motions (including oscillatory and resonant situations) that are intrinsically coupled with associated magnetic and electric complexity of motional masses (see later in this chapter about gravitational standing matter waves in "*2.3.3. Macro-Cosmological

Matter-Waves and Gravitation"*). This will be on some way familiar to micro world wave-particle dualism concepts, while respecting Mobility system of electromechanical analogies (see chapters 8., 9., and 10. of this book)***.**

10. *Gravitation (as presented in this book) is the natural consequence of mutually interacting internal states of matter (meaning states of atoms and other masses) concerning intrinsic orbiting and spinning motions (see familiar conceptualization in [36]) and concerning strongly associated electromagnetic complexity. Newtonian gravitation, obviously omitting internal and external, linear, orbital and spin moments and associated electromagnetic properties (of interacting atoms and masses), is still using well-operating and simple Newton formula for gravitational force, involving only masses. A. Einstein G.R. effectively (on a complicated and impractical, but an elegant way) replaced Newtonian Gravitation force with certain mathematically defined, curved-space geometry that is representing fields and forces around masses, (this way being close to concepts elaborated in [36]). In this book, favorable framework (based on exploiting Mobility system of analogies) is that only static masses (without other dynamic parameters, including oscillations) are not enough to create field or force of gravitation (see more in the first chapter of this book dealing with Analogies), but material fields acting within vibrating atoms, and outside atoms, are creating $1/r^2$ dependent forces (see more in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces"). Mutually interacting, and electromagnetically and mechanically, by standing matter waves coupled motional masses and involved atoms with internal electromagnetic charges, having different magnetic moments and internally distributed electric dipoles, and associated linear and angular or spinning mechanical moments, are real sources of gravitation (see more in Chapter 3. about electromagnetism). Effectively, all macro masses, atoms, and astronomic objects in our universe are in relative motions and mutually communicating by creating synchronized standing matter waves and vortices of Radiant electromagnetic energy, or flow of certain fluid (of exceptionally fine matter or "ether" particles in any form of matter-waves). This is creating effects of gravitational attraction, being explicable like in N. Tesla "Dynamic Gravity Theory" (see much more about familiar concepts in chapters 8. and 9., and in [6], [97], [99] and [117]). Every linear "mass-energy-momentum motion" always has an associated motional-energy component (or matter wave) that is behaving like a spinning or helix wave-packet motion around the path of linear motion (being like a couple of wave functions that create an Analytic Signal function; -see much more about Matter Waves and Analytic Signals properties in Chapters 4.0, 4.1 and 10.). An Analytic Signal couple (of mutually orthogonal wave functions) is on some significant (mathematical modeling) way very convenient for proper conceptualizing of matter waves and effects of gravitation (and linked to an associated electromagnetic field manifesting between attracting masses).*

The complexity or difficulty of understanding gravitation is related to several of coincidently existing states, effects, or situations regarding atoms and masses, contributing to the same force, such as:

a) *Gravitation essentially originates from forces acting around atoms, manifesting internally and externally (see more in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces"). Within our cosmos, there is certain (low frequency) macro standing-matter-waves structured field between gravitationally attracting masses, where masses (solar systems and galaxies) are in spatial nodal-zones, and where we detect only attractive Newtonian forces (what is the concept familiar to N. Tesla Dynamic Gravity). The field effects around such nodal, standing-waves zones, are producing only attractive forces. Balancing, and effects of repulsive forces (opposite to gravitation) are on some way inside of atoms, masses, or masses agglomerations, what is familiar concept to universal Natural Force, described by Rudjer Boskovic. Here is good to underline that $1/r^2$ force law is mathematically the same for Coulomb and Newton force, and for R. Boskovic "external", long-distance universal natural force part. The "internal" part of alternating R. Boskovic force keeps atoms and mases compact or agglomerated, being coincidently balanced with an "external" R. Boskovic $1/r^2$ force.*

b) *All masses in our Universe are permanently in some holistic, omnidirectional oscillatory, angular, and helical motions, and in multiaxial rotation and spinning (regarding different potential-forces centers). This is creating certain level of spontaneous electromagnetic, dipole and multipole tensions and polarizations within involved macro masses (where mentioned electromagnetic dipoles are conveniently aligned and mutually attracting, respecting Coulomb forces). Electric charges in any form (including electric dipoles) are still wrongly conceptualized as fixed and static parameters (or as number of Coulombs). In fact, electric charges are dynamic entities such as linear and angular* *moments in Mechanics are, meaning that between electric charges always exist certain Radiant, electromagnetic energy flow (as speculated by Nikola Tesla), and acting Coulomb type of $1/r^2$ force, which has the same mathematical form as Newton gravitation force, also being an "external", R. Boskovic natural force. Dominant intrinsic mass ingredients relevant for creating force of Gravitation are internal and external spinning (rotating and orbiting) states, and associated magnetic moments or domains, which are mutually attracting with opposite magnetic moments from neighbor's masses. Synchronized, global, cosmic, linear, rotational, orbital, and spinning motions of elementary particles, electric charges, and dipoles (which are inside macro masses) is effectively producing effects of electric currents and associated magnetic field components (being mutually aligned in the same direction), this way producing an attractive electromagnetic force between any of two masses. Such forces are contributing to, or creating Newton force of gravity, thanks to complexity of involved electromagnetic interactions, as presented in Chapter 3., within equations (3.1) – (3.4). See more of supporting information in the works of Eric Roberts Laithwaite (14 June 1921 – 27 November 1997) who was a British electrical engineer, known as the "Father of Maglev" for his development of the linear induction motor and maglev rail system (ref. [102]).*

c) *For an effective presence, and an active contribution of electromagnetic forces in creating gravitation, we also need to accept that an exceptionally fine fluid or an "ether" exist in the space between atoms and masses, serving as the carrier of electromagnetic field and waves. Gravitational, electromagnetic, quantum-mechanical, and mechanical, or acoustic matter waves, are all respecting the same Classical Wave Equation (being best presentable and modeled when using Complex Analytic Signal model).*

2.1.1 More about Newtonian Gravitation based on analogies

By continuing the same process practiced in this book regarding creating and contemplating analogies and fundamental symmetries (like given in the first chapter by T.1.8), we can combine and compare mathematical expressions for energies and forces in different domains of physics, including the cases of mass rotation (see **[3]**). At the same time, we shall remain in accordance with the already established analogies presented in T.1.2 - T.1.8. The data for the following step are given in T.2.1.

T.2.1	Electric Field	Linear Motion and Gravitation	Magnetic Field	Rotation
Energy (Motional, kinetic, field...)	$\frac{1}{2}Cu^2 = \frac{1}{2}qu = \frac{1}{2}\frac{q^2}{C}$	$\frac{1}{2}mv^2 = \frac{1}{2}pv = \frac{1}{2}\frac{p^2}{m}(= \frac{1}{2}Sf^2)$	$\frac{1}{2}Li^2 = \frac{1}{2}\Phi i = \frac{1}{2}\frac{\Phi^2}{L}$	$\frac{1}{2}\mathbf{J}\omega^2 = \frac{1}{2}L\omega = \frac{1}{2}\frac{L^2}{\mathbf{J}}(= \frac{1}{2}S_R\tau^2)$
Energy Density	$\frac{1}{2}\varepsilon E^2$	?	$\frac{1}{2}\mu H^2$	?
Coulomb-Newton-J. Mitchell $1/r^2$ Forces	$\frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$ (Coulomb)	$-G\frac{m_1 m_2}{r^2}$ (Newton)	$\mathbf{F_{1,2} = \frac{\mu_0}{4\pi}\frac{q_{m1} \cdot q_{m2}}{r^2}}$ (John Michell)	?

Table T.2.1 is filled with the known (and mutually analog) formulas related to energy and corresponding forces (in an electrostatic field, in linear motion related to Gravitation, in magnetic field, and related to rotational-motion forces). Empty boxes with a question mark (**?**) are left for developing hypothetical ideas that will "reconstruct" missing mathematical forms, based on Mobility type analogies (from T.1.1 to T.1.8).

After (conditionally) accepting previous analogical, brainstorming, and possible conclusions and/or statements (see points from 1. to 10. from the introductory statements of this chapter), to additionally support them, we can fill the table T.2.1 with missing energy and force expressions, by using electromechanical analogies from T.1.2 – T.1.8. This way (producing new table, T.2.2) we are creating and exercising a more complete symmetry of indicative, mutually corresponding and analogous (energy and force) expressions, some of them obviously, highly hypothetical, but good enough for initiating challenging thinking and insights. Anyway, explaining, updating, and redefining Gravitation from the level where Kepler and Newton established its foundations is not an easy and straightforward task (since what we already have from mentioned authors, still works very well). To do this meaningfully we need to create a specific creative environment with new ideas, and to develop new, probable, or possible directions (here initially based on applying analogies), while mathematical options and tools from contemporary mathematical physics are not missing. Newtonian Gravitation force intensity, when considered only as static masses attraction, is negligible compared to any other force known in Physics. Naturally, we are not in a comfortable position to deal with such almost negligible effects (within our planet as a laboratory), if some new and still unknown gravitational force component is also involved. For instance, if we succeed to extend the framework of Gravitation to other interactions between involved "mass-energy-moments" entities (not only to assume that gravitation acts only between static masses), and again to make richer modeling of solar systems, we should have bigger chances to upgrade and verify certain new vision of Gravitation, and this is the strategy being promoted here (as an example of mentioned innovative conceptualization, see later in this chapter, 2.3.3. Macro-Cosmological Matter-Waves and Gravitation).

T.2.2	**Electric Field**	**Linear Motion** and Gravitation	**Magnetic Field**	**Rotation**
Energy	$\frac{1}{2}Cu^2 = \frac{1}{2}\frac{q^2}{C} \Downarrow$	$\frac{1}{2}mv^2 = \frac{1}{2}\frac{p^2}{m} \Downarrow$	$\frac{1}{2}Li^2 = \frac{1}{2}\frac{\Phi^2}{L} \Downarrow$	$\frac{1}{2}\mathbf{J}\omega^2 = \frac{1}{2}\frac{\mathbf{L}^2}{\mathbf{J}} \Downarrow$
Energy Density	$\frac{1}{2}\varepsilon E^2 \Rightarrow$	$(\frac{1}{2}g_p \mathfrak{G}^2)^*$	$\Leftarrow \frac{1}{2}\mu H^2 \Rightarrow$	$(\frac{1}{2}g_L \mathfrak{R}^2)^*$
Coulomb Force types	$F_{ed} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} \Rightarrow$	$(F_{gd} = \frac{1}{4\pi g_p}\frac{p_1p_2}{r^2})^*$	$(F_{md} = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2})^*$	$(F_{Rd} = \frac{1}{4\pi g_L}\frac{L_1L_2}{r^2})^*$
Newton Force types	$(F_{es} = (?)\frac{C_1C_2}{r^2})^*$	$F_{gs} = -G\frac{m_1m_2}{r^2}$ $\Leftarrow \quad \Rightarrow$	$(F_{ms} = (?)\frac{L_1L_2}{r^2})^*$	$(F_{Rs} = (?)\frac{J_1J_2}{r^2})^*$

(…)* -Hypothetical formulas; $\mathfrak{G}$ -Gravitation field, g_G -"Gravitational-permeability"; $\mathfrak{R}$ -"Field of Rotation", g_R -"Rotational-permeability"; Indexing: ed = electro-dynamic, gd = gravito-dynamic, md = magneto-dynamic, Rd = rotational-dynamic, es = electro-static, gs = gravito-static, ms = magneto-static, Rs = rotational-static (most of them are here invented, temporary formulations for the purpose of making initial analogical expressions)

All energy and forces expressions in T.2.2 are intentionally formulated, suggested, chosen, or modified (or analogically reinvented to formally comply Mobility system of analogies), to have mutually similar mathematical forms regarding relevant constants (such as: $\frac{1}{4\pi\varepsilon}, \frac{1}{4\pi\mu}, \frac{1}{4\pi g_p}, \frac{1}{4\pi g_L}$). Of course, we are already used to present the constant of gravitational force as, G = 6.67428 (+/- 0.00067) $\cdot 10^{-11}$ [m^3 kg^{-1} s^{-2}] (= $\frac{1}{4\pi g_p}$ ...). All of that is serving to obtain maximal formal symmetry and parallelism of mutually analog and corresponding force and energy expressions, and to implicate that an analogical force member related to certain kind of *linear motion associated rotation* (or spinning) could also exist with its own force constant $\frac{1}{4\pi g_L}$. What could be particularly interesting in here introduced strategy is to find different, mutual and cross-platform, analogical links of relevant constants, such as speed of light expressed as the function of electric permittivity and magnetic permeability, $c = \frac{1}{\sqrt{\mu\varepsilon}}$, or in cases of electric transmission lines as $c = \frac{1}{\sqrt{LC}}$, where **L** and **C** are inductance and capacitance per unit length. Analogically concluding, the speed of light or universal constant **c** could have another form as, $c = \frac{1}{\sqrt{g_L g_p}} = \sqrt{\frac{4\pi G}{g_L}} (= \frac{1}{\sqrt{\mu\varepsilon}}) \Rightarrow g_L = \frac{4\pi G}{c^2}$ (see similar thinking in **[70],** Guido Zbiral; "*Does Gravitation Have an Influence on*

Electromagnetism"). Such relations are implicating possible existence of much more profound unity of different natural fields and forces, and if we continue that way, something similar could be applicable to old and new, analogically introduced constants regarding mechanics and gravitation, as by brainstorming exercised in T.2.2.

Citation (from the Internet): "Self-consistent gravitational constants are complete sets of fundamental constants, which are self-consistent and define various physical quantities associated with gravitation. These constants are calculated in the same way as electromagnetic constants in electrodynamics. This is possible because in the weak field equations of general relativity are simplified into equations of gravitomagnetism, similar in form to Maxwell's Equations. Similarly, in the weak field approximation equations of the covariant theory of gravitation [1] turn into equations of the Lorentz-invariant theory of gravitation (LITG). LITG equations are Maxwell-like gravitational equations, which are the same as equations of gravitomagnetism. If these equations are written with the help of self-consistent gravitational constants, there is the best similarity of equations of gravitational and electromagnetic fields". Taken from:
http://en.wikiversity.org/wiki/Nonstandard_physics/Selfconsistent_gravitational_constants .

At this time, it is still better to postpone arbitrary discussions about the terminology and meaning of new expressions, constants and symbols introduced in T.2.2, because it is clear (from obvious analogies) how certain mathematical forms are created, and what meaning they could have (here temporarily figuring mostly for purposes of easy introduction and simplified presentation based on similarities). In any case, the importance of the previous brainstorming step would be mostly in sparking possible original ideas about connections and symmetries that could exist between different natural fields and forces. In addition, we will be able to rethink if we are really explaining the same known items (regarding fields and forces) in some different way, or maybe we are still not used to seeing the same facts from the analogical point of view as previously initiated. All expressions (in T.2.2) with the white background, marked by an asterisk (…) *, are newly created and hypothetical. Of course, on this level, there is still no commitment as to which conditional expression in T.2.2 has some real value, or a chance to be transformed towards something with Physics-related meaning (which is a task to be realized later). Based on the data in T.1.6, T.2.1, and T.2.2, we also see that (by analogy) magnetic flux Φ belongs to "spatial or geometry parameters," like rectilinear displacement or angle. At the very least, we can also conclude that (at this time) a full analogy (in all directions) is not satisfied for magnetic field phenomena (cf. T.1.6 and T.2.2) in the same way as it is satisfied for all other electromechanical parameters and entities. Later, we shall attempt to establish a more complete symmetry in electromagnetism; - see (3.1) to (3.5) in the third Chapter.

As we can see in T.2.1 and T.2.2, Newton and/or Coulomb "$\mathbf{1/r^2}$" force laws could be extended (presently analogically and hypothetically) to rotational, orbital, and spinning motions involving linear, angular, and orbital moments.

It should be mentioned (what it is already experimentally known) that attractive and repulsive forces between two permanent magnets (between the opposite or same magnet poles (q_{m1}, q_{m2})) are also satisfying the same Coulomb-Newton force-law expression analog to electrical charges situations, $F_m = \mu \frac{q_{m1} \cdot q_{m2}}{4\pi \, r^2}$ (first time discovered by John Michel in 1750).

Also, Ampère in 1822 introduced his first (empirically and intuitively fitted) law of electrodynamics, addressing the force between two electric current-elements, which is also analog or familiar to Newton-Coulomb "**1/r²**" force laws. Ampère's law (modernized on different ways, by number of authors) is increasingly receiving recognition (almost 200 years after its first formulation) in predicting or complying with experimental results related to forces between current elements or other electrically conductive channels (see literature references [28], [29], [30] and [36]). The significance of Ampère law is in documenting that specific experimentally known phenomenology, which cannot be explained or predicted by contemporary Maxwell Electrodynamics, is explicable by Ampère's law. In fact, Ampère's law shows the existence of some additional structural issues of modern Maxwell Electromagnetic Theory (for instance, related to Lorentz force, and to some still non-counted force manifestations).

There is another "**1/r²**" force law (see later (2.4-4), and [45]) *between equidistant, parallel paths moving electrical charges* (q_1 , q_2), having velocities (v_1 , v_2), where magnetic force between them is found to be, $F_{1,2} = \frac{\mu}{4\pi} \frac{(q_1 v_1) \cdot (q_2 v_2)}{r^2}$.

Obviously, there is certain potential in further elaborating and generalizations of Newton-Coulomb "**1/r²**" force laws because we find them applicable in gravitational or electromagnetically neutral masses interactions, in electric charges and magnetic poles interactions, as well as in electrodynamics' related situations between current elements and/or moving electric charges. Here, we will devote specific space to different analogical and unifying, also hypothetical aspects of "**1/r²**" force laws. *We should strongly underline that an analogy based only on "1/r²" force laws is not at all what will be taken as the strongest, single and unique argument that Gravitation has an electromagnetic nature (since we will have number of additional options for such conclusion later).*

In electromagnetic theory, it is also evident that a total symmetry, parallelism, mathematical and analogical presentations can be elaborated for electric and magnetic fields, moments, and charges (see the third chapter of this book and [99], from Konstantin Meyl).

We can also demonstrate the existence of significant analogical structure between Bohr atom model and solar or planetary systems (see chapter 8. and T.2.8. N. Bohr hydrogen atom and planetary system analogies) intuitively indicating that gravitation should have direct links to matter structure and electromagnetic phenomenology. *Static masses are not real and unique charges or sources of gravity, but masses in all kinds of motions, including vibrations, with linear and angular or spin moments, and with electromagnetic properties, are real sources of Gravitation.*

Later, we will realize that every linear motion, when presented using the Analytic Signal model, has an associated complementary spinning motion (and spinning anyway has or creates elements of an oscillatory motion). From here, we can determine de Broglie matter wave frequency, wavelength, group and phase velocity, *relevant phase functions, and connect such conceptualization with Wave-Particle duality of matter, this way effectively upgrading Gravitation. If we consider gravitational force as a part of standing-matter-waves phenomenology (between attracting masses) we will come closer to N. Tesla and R. Boskovic concepts about gravitation (see more in the chapters 4.0, 4.1, 8., 9. and 10., and in [97] and [117] from Jean de Climont).*

In the process of searching for new insights about gravitation, we should have in mind all here mentioned facts and analogies, since this can later facilitate our conceptual understanding of gravity.

For instance, high Power Mechanical, Ultrasonic or Acoustical energy, moments, forces, oscillations and vibrations, or audio signals and music, can be created and transferred by applying different signal-modulating techniques on laser beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); - See relations from Chapter 10. under (10.2-2.4) and literature references from [133] until [139].

[♣ COMMENTS & FREE-THINKING CORNER (brainstorming, while exploring analogies)

Calculating the attractive or repulsive force between two magnets is, in a general case, an extremely complex operation, as it depends on the shape, magnetization, orientation, and separation of the magnets. The John Michell force between two magnetic poles is given by: $F_m = \mu \frac{q_{m1} \cdot q_{m2}}{4\pi r^2}$, where: F_m is force (SI unit: Newton), q_{m1} and q_{m2} are the pole strengths (SI unit: ampere-meter), μ is the permeability of the intervening medium (SI unit: Tesla, meter per ampere or Henry per meter) and r is the separation (SI unit: meter). The pole description is useful to practicing "magneticians" who design real-world magnets, but real magnets have a pole distribution more complicated than a single north and south. Therefore, implementation of the poles idea is not simple (taken from the Internet: http://en.wikipedia.org/wiki/Magnet).

By analogy (from T.1.2 and T.1.6), as the electric capacitance **C** is a kind of a static reservoir or storage for the electric-charge elements, **q**, the same is (hypothetically) valid for the mass **m**, which should be a static storage of (somehow internally packed and stabilized) "momentum elements" **p**. Also, a moment of inertia **J** could be understood as a "static storage" for (somehow internally packed) "angular momentum elements" **L**, what is going back to a mass as storage of rotating entities. Above statements and terminology are only temporarily established and conditionally suitable for the purpose of intriguing and analogical ideas generation, until some more convenient formulations are found. In fact, nothing very original and new has been claimed here since we already know that matter (or mass) elements are atoms with all their complexity of internally synchronized and well packed motional states. The author introduced such ideas first time in [3].

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

2.1.2 Some Hypothetical Force Laws

If we give a freedom to our hypothetical brainstorming, again based on analogies, (see [3]), from the table T.2.2 we could exercise that gravitational type of attractive force between the two moving masses (in their center-of-mass coordinate system) is not only given by original Newton's formula, but rather by (2.1), as a superposition of two forces. One, which represents a static case of Newtonian attraction between two static masses, prefix "stat. = s", and the other, for the time being very hypothetical, which represents a dynamic interaction between corresponding moments, prefix "dyn. = d", which could be repulsive or attractive:

$$F_g = F_{g-dyn.} + F_{g-stat.} = +\frac{1}{4\pi g_p} \cdot \frac{p_1 p_2}{r^2} - G \cdot \frac{m_1 m_2}{r^2}. \quad (2.1)$$

Similar analogical thinking (and new "hypothetical innovation") can be applied to the situation with spinning masses (taken from T.2.2), in which case there could exist (attractive and/or repulsive or non-central) force between them. Such force could analogically be expressed as in (2.1), as static and dynamic force members' addition (with corresponding angular and spinning moments, about the same center of rotation):

$$F_R = F_{R-dyn.} + F_{R-stat.} = +\frac{1}{4\pi g_L} \cdot \frac{\mathbf{L}_1 \mathbf{L}_2}{r^2} - G_R \cdot \frac{\mathbf{J}_1 \mathbf{J}_2}{r^2}, \quad (2.2)$$

In cases when we have the presence of combined rectilinear and spinning elements of certain complex motion of two objects, we could appropriately combine (2.1) and (2.2), to describe the Newtonian resulting force between them. As we know, linear motion of certain particle could be conceptualized as a case of rotation, where the relevant radius is arbitrarily long, by accepting that all motions in our universe are in their nature rotational or angular motions. This would produce that (2.1) and (2.2) are mutually equivalent. *We also need to address the proper vector forms of (2.1) and (2.2) to conceptualize the 3-dimensional picture regarding such forces* (but presently this is not the primary objective here).

In fact, we still do not have good conceptualization about involved (internal and external) linear and angular moments, and we also know that mentioned mechanical moments are often coupled or interacting with naturally associated electromagnetic and dipole moments. Additional theoretical and conceptual grounds to the idea about Newton-type forces originating from orbital moments and spinning can be found in chapter 10. of this book (see comments around equations (10.1.4) - (10.1.7)), as well as in [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

Of course, it is an open question if formulas such as (2.1) and (2.2) can be experimentally proven and how (since the most realistic approach should be that all elements of linear and rotational motions are coincidently present and mutually coupled in the motion of the same mass or object. Also, we know that masses are kind of agglomerations of internally spinning, more elementary domains (since almost all subatomic entities have spin characteristics). When electrically charged particles have spin characteristics, coincidently there we also find associated magnetic moments, magnetic fluxes, and magnetic dipoles. Since masses of electrons and protons are enormously different, in accelerated and non-uniform motional situations of masses, we can expect to have many electric dipoles (all of them internally integrated and well packed). Consequently, (2.1) and (2.2) can also be united in a form that is more general. In fact, the unifying imperative produces that static members found in (2.1) and (2.2) could be mutually identical, $G \cdot \frac{m_1 m_2}{r^2} = G_R \cdot \frac{\mathbf{J}_1 \mathbf{J}_2}{r^2} = F_{stat.} \Rightarrow G \cdot m^2 = G_R \cdot \mathbf{J}^2 \Rightarrow \mathbf{J} = m\sqrt{\frac{G}{G_R}} \Rightarrow r = \sqrt{\frac{G}{G_R}}$, and that force elements responsible for (possible) dynamic interaction between two moving objects could have both linear and/or orbital or spin moments, $\frac{1}{4\pi g_p} \cdot \frac{p_1 p_2}{r^2} = \frac{1}{4\pi g_L} \cdot \frac{\mathbf{L}_1 \mathbf{L}_2}{r^2} = F_{dyn.} \Rightarrow \frac{g_L}{g_p} = \frac{\mathbf{L}_1 \mathbf{L}_2}{p_1 p_2} = \frac{\mathbf{L}^2}{p^2} = r^2$, this way generalizing the force law between two moving objects as: $F_{1\text{-}2} = F_{dyn.} + F_{stat.}$. Of course, all relevant orbital and/or spin moments should be linked to the same center of rotation (or to their common center of mass), as for instance: $\frac{1}{2} pv = \frac{1}{2} \mathbf{L}\omega \Leftrightarrow \frac{\mathbf{L}}{p} = \frac{v}{\omega} = \frac{\omega r}{\omega} = r = \sqrt{\frac{G}{G_R}} = \sqrt{\frac{g_L}{g_p}}$. The situation addressed here (by applying analogical thinking) is still some kind of simple and easy introduction, and familiarization with the ideas that gravitation should not be only an attraction between static masses, and that other vibrating, dynamic, especially rotational and spinning properties of masses constituents (including electromagnetic moments and dipoles) can have the vital importance. We will come back to the same situation several times more in this chapter (see "2.3.2. Macro-Cosmological Matter-Waves and Gravitation"), and we will see that the reality behind here introduced ideas is not as simple as expressed by (2.1) and (2.2), but also not too far from the relevant insights. See later, in the chapter 4.1, around equations (4.3-0), (4.3-0)-a,b,c,d,e,f,g,h,i,j,k..., an extended conceptualization of familiar ideas (regarding spinning). See also illustrations on Fig.4.1.1 and Fig.4.1.1a. ***In chapter 10. of this book, we can find the most complete and simple explanation of the same situation regarding Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7)).***

The essential question here would be to ask if we really have any simple and clear candidate-examples for a better and deeper understanding of Gravitation as for instance, examples of repulsive and other unusual gravitation-related forces, where we could start establishing a new experimentally and scientifically valid theory. Some of the possible answers are:

A) The most probable candidates for repulsive and/or torsional gravitation-force-related components could be investigated concerning centrifugal forces, gyroscope and pendulum behaviors, and Coriolis forces, all of them being coupled with internally structured magnetic and electric field moments and dipoles. For instance, planets of specific Solar System would collapse and unite masses with their Sun if there are no orbital rotations and associated centrifugal forces. Here, centrifugal forces are on some way reacting as repulsive gravitational forces, balancing in the same amount relevant gravitational attraction, and keeping every planet on a stable orbit. If the repulsive gravitational force (as formulated here) is something inherently related to rotation (as centrifugal force), most probably that the nature of Gravitation itself is on some similar effective way (intrinsically) also related to specific (equivalent and hidden) rotation and oscillations inside of gravitational masses. As we know, ordinary mass internal constituents are molecules, atoms, electrons, protons, and neutrons. There is an enormous difference between the masses of electrons and protons (almost 2000 times), and neutron is presenting specific coupling between an electron and a proton. When certain mass is in a state of relative rest, its internal content could be considered as self-stabilized and electromagnetically neutral, or non-polarized, but if such mass (externally) starts getting elements of linear and rotational motions and spinning, specific internal, electric, and magnetic dipoles polarization would happen (because electrons and protons have quite different masses and different mobility). The appearance of such, slightly polarized, electric, and magnetic dipoles will be the source of different electromagnetic attractions and repulsions between mutually interacting masses (and associated dipoles), and this could be a part of the explanation of hidden electromagnetic nature of Gravitation. We know from Classical Mechanics concepts that certain object, mass, or system of particles has its center of mass, or center of gravity, and/or center of inertia, regarding linear motions, and something similar can be, on a remarkably similar way, defined for all rotational and spinning movements of the same system participants. We can extend the same idea to establishing centers of (neutrality of) electric and magnetic charges and moments. An ideal case of electromagnetic and mechanical neutrality, stability, and inactivity (of certain mass or system with many participants) would be when all centers of inertia, gravity, and involved field charges (regarding linear and rotational motions, including Gravitation and Electromagnetic fields and charges) are linked to the same point in space. Since all motions in our Universe are mutually relative and curvilinear motions (meaning, accelerated), such idealized concept of a stable center does not exist, or it is merely impossible. The consequence of such reality is that mass in motion (which internally has electrons, protons, and neutrons) is always, until center level, multiple electric and magnetic dipoles-polarized state (see [71], from Jovan Djuric; "Magnetism as Manifestation of Gravitation"). There is no real and total electromagnetic neutrality and fully compensated (or neutralized) state of mass, without electric and magnetic charges somewhat spatially separated. Such non-compensated electromagnetic dipoles (or multi-poles) are most probably creating the force of gravity or dominantly contributing to such phenomenology. Effectively we can find significant contributions to similar ideas in the publications under [63], Arbab I. Arbab.

B) It could be that (2.2) predicts that fast rotating or spinning objects would mutually create additional and measurable gravitation-related forces, which are making significant difference compared to ordinary Newton-Coulomb static forces between them (see [36], Anthony D. Osborne, & N. Vivian Pope, An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces*).*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER... ♣**]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

C) It is essential to notice that common in all here mentioned, hypothetical phenomena candidates (regarding different aspects of gravitation-related forces) is that all of them are in some ways combining linear and rotational motions. Later (see chapters 4.xx and 10.) we will show that particle-wave duality and matter waves are always a consequence of specific fields coupling between linear and rotational or spinning motions (where oscillatory and rotating motions are strongly coupled).

D) It will be interesting to explore if diffractions of micro-particles, photons, electrically charged or neutral particles (when beam of such particles or quasi-particles is passing through a small diffraction hole) have something to do with here mentioned repulsive forces, since diffraction basically means that a rectilinear stream (or flow) of matter (with momentum-energy-mass content) is getting divergently dispersed. Such dispersion could also be conceptualized as the two or many-body interaction involving repulsive forces caused by intrinsic elements of particle rotation (the same proposal is introduced 1975 in [3]*). Particle-Wave dualistic aspects of mentioned diffraction events are making this situation relatively complicated. We could argue whether wave or particle related effects are dominant since by applying wave concepts we are able to explain diffraction patterns and pictures on a screen behind diffraction holes. Still, there is also a place to ask questions about the appearance of a certain kind of associated particles repulsion. Then we may present such situations as equidistant beams of particles streaming, where mentioned particles (or quasi-particles) are guided by a common interaction field (which is created between them and diffraction holes) while passing diffraction holes. To be bottom-line clear (or to try to visualize and maximally simplify this situation), we could say that the diffraction hole here is a kind of a "bottleneck", channeling the particles in coincident mutually parallel (or mutually synchronized) motion. This way, particles are forced to create mutually closer, higher density paths. Under such conditions, hypothetical repulsive gravitation forces (for instance as forces (2.1) and (2.2)) have a chance to work, eventually dispersing (or diffracting) the particles after passing the hole. The probable reason is that such situation is followed by associated matter-wave effects of spinning fields, primarily appearing, and becoming dominant in a space-time vicinity of any two-body interactions; -see additional explanations in Chapter 10.*

E) Continuing the same brainstorming process, we could also say that static and dynamic forces between different objects should also have elements of electromagnetic nature (again complying Coulomb-Newton-J. Mitchell force laws). Even further, we may find a way to show that electromagnetic force elements are effectively producing or influencing all other gravitation and rotation related forces and field effects (being in the conceptual background as the primary sources of all mentioned forces, where masses are considered as agglomerations of atoms with numbers of internally spinning states). Magnetic and spin moments of elementary particles, as well as of astronomic formations are always mutually and proportionally coupled. It is almost evident that, with an artificially created, external magnetic field and/or rotation of various objects, including convenient action of strong electric fields, in a vicinity of certain conveniently-structured mass, we should be able to influence, modify or shield the local force of Gravitation (for instance to create a repulsive Gravitation-like force; - see [66] -Dr. Evgeny Podkletnov). Somewhat familiar concept of combining electrostatic and dynamic force elements between two moving and electrically charged particles, was a long time ago introduced by Wilhelm Weber's force law (see literature [28], [29], [36]). The matter and all natural forces, fields, and conservation laws in our universe are anyway naturally united (regardless of our present state of knowledge about the same subject). By noticing or discovering different analogies and continuous symmetries, we are seeing the most probable connections between such analogous entities and interpretations of the same, already united universe, meaning that future (updated) explanation of Gravitation would most probably consider additional mass properties influencing masses attraction (not only an attraction between static, rest masses). See table of analogies T.1.8, from the first chapter, as the support to here elaborated conclusions.

The challenging ideas and possibilities radiating from T.2.2 and (1.19) - (2.9) could be intuitively addressed as a unity and couplings between rectilinear and rotational motions. We usually relate gravitational field to a specific space deformation around a certain mass. However, from T.2.2, from similar expressions for energy density, we can see that gravitational and rotation-related forces could be differently introduced, in two ways:

-in static cases (prefix "stat."), as certain spatial deformation resulting as a force proportional to the product of involved masses m_1, m_2 and/or moments of inertia J_1, J_2; and

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

- In a dynamic case (prefix "dyn."), as the force proportional to products between relevant linear and/or orbital moments $\mathbf{p}$ and $\mathbf{L}$. We could also create a vector addition of both (when shows relevant), combining (2.1) and (2.2) with (2.4)). Later (see (2.11.14-20), in "2.3.3-3 Standing-Waves Resonators and Gravitation"), after developing such ideas, we will notice the possibility to determine gravitational force (in the Center of Mass coordinates) as, $F_g = v\frac{(L_1 + L_2)}{r^2} = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2}$, which could present certain progress in explaining Newton law of gravitation.

Just to give an idea about possible experimental evaluation, we can exercise (2.1) and (2.2) as an alternative explanation of elementary micro particles and quasi-particles diffraction, as well as to analyze different interaction phenomena between micro particles with spin characteristics, in the atom world (see **[3]**). For instance, the Pauli Exclusion Principle (“Only two electrons which possess mutually opposite spin can occupy any given quantum, stationary orbit”, in the conceptual frames of Bohr’s planetary atom model) could be in a relation with (2.2). Spinning electrons (or electron wave packets with torsional field components) are creating mutually coupled magnetic and mechanical angular moments making that electromagnetic (or electro-dynamic) Coulomb forces and attraction or repulsion between their magnet-field poles are coincidently present. All such (mutually coupled) forces with direct or indirect energy-mass rotation origins are balancing with all other (here associated) electromagnetic interactions, giving the possibility of proving interactions between rotating objects. It is almost obvious that electrons in stationary atom orbits should be in couples of two, thus creating a couples of two elementary magnets with mutually opposite spins, enabling an attraction between two opposite magnet poles. It is also known that protons and neutrons have the same spin as electrons and obey the Pauli Principle (inside the atom nucleus). If a new type of hypothetical forces (2.2) exists between rotating objects, it would be sufficient to make just a small creative step towards using such forces in explaining the nature of nuclear forces (also related to de Broglie matter waves’ phenomenology (see (4.18)). **Since magnetic and orbital and/or spinning moments of elementary particles are mutually strongly related (having constant gyromagnetic ratios), it could eventually happen that we will find that in the background of all such “new and hypothetical” forces are well known electromagnetic forces** *(see [71], from Jovan Djuric; “Magnetism as Manifestation of Gravitation”)***.**

The most exciting idea, coming from (2.1) and (2.2), would be the possibility of producing and controlling a kind of non-stationary, transient, dynamic antigravity force, in a similar form of the repulsive force known between the same magnetic poles of two magnets. This has chances to happen after exploiting vibrating, pulsing, dynamic time-variable effects of rotating masses, and various angular momentum interactions, combined within a specific rigidly coupled system (or frame), on the way to generate resulting linear and torque forces acting against the local force of gravitation.

Like (2.1) and (2.2) we could also exercise creating the mixed force formulas for electric and magnetic fields (combining static and dynamic cases), using force formulas from T.2.2 (see also (3.1) - (3.5) and (4.18)). However, this would be too simplified mathematical modeling process since we already have much more complex and more general Maxwell electromagnetic theory for managing electromagnetic fields and forces. Moreover, there is a recent extension of Maxwell’s Electromagnetic Theory, and a different approach to Relativity Theory, made by revitalizing and upgrading of Wilhelm Weber’s force law generating new ideas about connections between electromagnetism and gravitation. This presents an attempt to realize natural unification of fundamental laws of classical electrodynamics, such as Gauss’s laws, Coulomb’s law, Ampere’s generalized force law, Faraday’s law, Lorentz force laws and Lenz’s law (see literature [28], [29], [34] and [36]).

Mass in the steady state of relative rest presents (macroscopically) neutral electromagnetic balance between its positive and negative electric constituents and mutually opposite magnetic charges or moments. Contrary, mass in motion including mass oscillations (because of accelerations, effects of inertia and dynamic transitory states) would tend to create and/or increase internal electrical and magnetic dipoles polarization and associated linear and orbital moments since there is a big mass difference between electrons and protons (1836 times). Of course, here given statements and ideas are only the starting and brainstorming points that should initiate a creative process to formulate specific new and valuable force laws. At this level of analogical ideas generation (regarding Gravitation), the most important would be to initiate and start supporting the idea that what we presently know about Gravitation (Newton law) is something limited to Classical Mechanics, planetary motions, Kepler laws, *and that future theory about Gravitation would be significantly upgraded after considering more complex situations related to electromagnetism. See examples of such attempts in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces" and later in this chapter under "2.3.3. Macro-Cosmological Matter-Waves and Gravitation".*

Another, original and exciting method of generating velocity dependent, static and dynamic, force-field formulas (starting from expressions for "Newton-Coulomb" force types) is presented in [4]. Similar method can also be applied to all force formulas from T.2.2, as well as to all possible combinations like in (2.1) and (2.2). This will additionally support here presented hypothetical thinking (since the author of [4] presented certain experimental evidence of the general validity of such force formulas). For instance, in [4], we can find for all velocity-dependent "Newton-Coulomb" types of fields and forces the following expression, $E_{moving} = E_{dyn.} = E(v) = E_{stat.}(1-\frac{v}{c}\cos\vartheta)$, *where: E(v) is the intensity of certain velocity-dependent, "Newton-Coulomb" field/force type;* $E_{stat.}$ *corresponds to field/force formulas from T.2.2 and to force expressions like in (2.1) and (2.2); v is the speed of the (relevant) field charges; c is the speed of light; and* ϑ *is the angle between the direction of the charge motion (the speed v) and the direction of involved field intensity propagation.* ♣]

In this book, we will gradually continue exploring gravitation towards the understanding of:

-***Mass as specific energy packing or formatting state*** based on "self-closed and stabilized standing matter waves agglomerations" (including associated vibrating, spinning and electromagnetic states, such as electromagnetic charges moments and dipoles).

-***Conclusively, it will be shown that only mass with associated linear and angular moments and electromagnetic dipoles, charges, and moments, including oscillating masses is the real source of gravitation (see [36])***. Even in a state of relative rest, every mass is still coupled with specific "***velocity field***" belonging to a surrounding spatial and global (background) motion, making that mass is always "***velocity charged***" with associated linear and angular velocity components and moments (see [73], Reginald T. Cahill). The bottom line is that **natural tendency of all motions in our micro and macro universe is to be, or to become oscillatory, orbital, curvilinear, or circular** (meaning angular and accelerated), and coupled with self-spinning or helix fields phenomenology. Such complex motion is creating effects of central forces and gravitation (based on individual level of spontaneous electromagnetic-dipoles, or masses and charges polarizations). ***Since all macro-masses have as building blocks different electromagnetic entities, being coupled, in mutual relative motions, and in some universal cosmological movement, this is supporting the idea that gravitation should ontologically have an electromagnetic nature.***

-**Extended foundations of inertial states** (where linear, rotational, spinning, oscillatory and other motions are included), towards self-stabilized, stationary and standing-waves spatial formations (as atoms and planetary systems). Effectively, different quantizing known in the world of Physics are consequences of mentioned self-stabilized, resonant, or standing waves, of spatial-temporal energy-momentum states formations.

-***United Gravity and Particle–Wave Duality concepts*** based on specific couplings between linear, angular, and spinning motions with associated electromagnetic fields, extended from the micro-world of subatomic structures towards macro-universe of astronomic objects. This could open new communicating channels between distant orbiting and spinning states, where information could propagate much faster than the speed of light (or instantly), if angular, or orbital and spinning moments conservation is globally (holistically) satisfied. Such instantaneous spinning and orbital-moments global couplings (if really exist) may belong to entanglement effects known in Quantum theory. ***There are innovative thinking, proposals, and indications that all orbital and spinning moments could be, universally, intrinsically, holistically, and synchronously coupled as an entanglement-type of local and distant cosmic communications (see more in [36], and in Chapter 10).*** *We already know that solar systems with planets are respecting certain kind of standing matter-waves arrangements (see more at the end of this chapter). There is also an astonishing analogical parallelism between atom models and planetary systems (see more in Chapters 8. and 10. of this book). Modeling of macrocosmic matter-waves and planetary motions can also be realized using complex Analytic Signal functions and* *this way we should be able to use the same mathematical explanation or modeling for explaining macrocosmic entanglements, on the analogical way as known in microphysics (see more, later in this chapter, under "2.3.3. Macro-Cosmological Matter-Waves and Gravitation").*

-We should not forget that contemporary Newton and A. Einstein interpretations of gravitation assume that force or field of gravity exists by itself (virtually without other reasons), or only looks like a force because of specifically structured (and curved) spatial-temporal geometry around masses. Such an assumption is conveniently hybridized with circular motions, centrifugal and centripetal forces conceptualization, and with the empirical fact that mass acceleration creates similar effects of mass weight as being in an assumed field of gravitation. Apparently, such hybridization (between different assumptions, provable physics, and obviously correct mathematics) works well until certain limits. Now we know that we already exhausted the limits of such somewhat arbitrary modeling of Gravitation, and we need to create more extensive and more realistic foundations for a new theory of gravitation (see, for instance [71], from Jovan Djuric, "Magnetism as Manifestation of Gravitation", and about *orbital motions produced effects* of gravity in [36]).

-Presently, still the best ideas and concepts about gravitation are sparking from Rudjer Boskovic's universal Natural Force theory, [6] and from complementing ideas of N. Tesla about Dynamic Theory of Gravity, [97], indicating that Gravitation should also be a certain specific manifestation of universal electromagnetic force. Since Coulomb and Newton force laws are mathematically identical, and also identical to the right-side (external) part of R. Boskovic universal, natural force law, except that Newton force of gravitation is very much attenuated when compared to Coulomb force, we can intuitively and analogically conclude that the Gravitation is certain residual electromagnetic (or Coulomb) force manifestation which remains after being heavily balanced and compensated (or reduced) with the alternating elements of the universal R. Boskovic force from the left (or central) part of mentioned force. Here, R. Boskovic' part of natural force curve is serving to agglomerate and stabilize atoms, solid masses, and what remains on the right side is what we consider being Gravitation. To keep such dynamic force balance, it is also necessary to have properly distributed standing waves (between and among structural elements of masses). Since Coulomb forces are acting between electrically charged particles, and from the first chapter of this book we know that (based on analogies), electric charges are kind of dynamic matter properties (like mechanical moments), consequently, certain natural and continuous flow of N. Tesla Radiant, electromagnetic energy should always exist between masses in mutual gravitational attraction. The opinion of the author of this book is that in our Universe (microscopically and macroscopically) we have only two families of acting forces (not four, as superficially and arbitrarily speculated in the contemporary Physics), such as electromagnetic forces, and forces related to standing waves structured matter waves. If all matter-waves are, essentially (intrinsically and ontologically), related to an electromagnetic nature or origin, in that case, our Universe has only one acting family of forces, meaning only electromagnetic forces. See much more of familiar ideas in [117], Jean de Climont.

2.2. Generalized Coulomb-Newton Force Laws

Regardless indications that Newtonian Gravitation should be replaced by the more conceptually advanced theory (like in [36]) let us still explore the consequences and limits of already well operating Newtonian Gravitation. There is an easy way to upgrade Newton gravity force law in combination with relativistic understanding of mass and energy. We start from Newton attractive, gravitational force between two masses, $\mathbf{m_{01}}$ and $\mathbf{m_{02}}$, ($m = m_0/\sqrt{1-v^2/c^2} = \gamma m_0$, $m_0 = const.$). Now, let us imagine that mentioned masses are in linear motion, and let us replace the two (static or standstill) masses ($\mathbf{m_{01}}$ and $\mathbf{m_{02}}$) with their relativistic, total energy equivalents, using well-known relations from Relativity Theory,

$$\mathbf{E_{t1} = m_1c^2, E_{t2} = m_2c^2, (E_t = E_0 + E_k} = m_0c^2/\sqrt{1-v^2/c^2} = mc^2 = \gamma m_0c^2,\ E_0 = m_0c^2,$$

$$E_k = (m - m_0)c^2 = \mathbf{pv / (1 + \sqrt{1 - v^2 / c^2}\,)} = (\gamma - 1)m_0c^2,\ p = mv = \gamma m_0 v\ ,$$

$$m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{c^2(1+\sqrt{1-v^2/c^2})} = m(m_0, p) = m(m_0, E_k),$$

$$p = \gamma m_0 v = \frac{E}{c^2\sqrt{1-v^2/c^2}} v = \frac{E_0 + E_k}{c^2\sqrt{1-v^2/c^2}} v = p(E_0, E_k), \qquad ,$$

$$E_k = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_k(p).$$

If masses (in addition) have spinning motion elements, we will need to add spinning energy amount, $\mathbf{E_{ks}} = \frac{\mathbf{L\omega_s}}{\mathbf{1+\sqrt{1-\frac{v^2}{c^2}}}}$ and this spinning energy is effectively increasing a total particle mass for the amount $\mathbf{m_s = \frac{E_{ks}}{c^2} = \frac{L_s\omega_s}{c^2\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)}}$ producing that the new total mass will be (see later (2.11.3-1)),

$$\mathbf{m = m_0 + \frac{E_k}{c^2} + \frac{E_s}{c^2} = m_0 + \frac{pv + L_s\omega_s}{c^2\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)} = m(p, L_s) = m_0 + \tilde{m}}\cdot$$

Practically, spinning particle, in a state of relative rest (regarding linear motion, meaning v = 0) has a mass, $\mathbf{m = m_0 + \frac{L_s\omega_s}{2c^2} = m_{rest}}$. Here we see that resulting and effective internal spinning of mass constituents $\mathbf{L_{s-internal}}$ (as well as external, macro spinning $\mathbf{L_{s-ext.}}$, if exist) have a direct influence on a total mass amount (since $\mathbf{L_s = L_{s-internal} + L_{s-ext.}}$.

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We could even (hypothetically) consider that a real, standstill or rest mass $\mathbf{m_0}$ is only a kind of stabilized, resulting agglomerate or condensate of internal spinning states $\mathbf{L_{s-internal}}$, this way conceptualizing rest mas as, $\mathbf{m_{rest} = m_0(L_{s-internal}) + \frac{L_{s-ext.}\omega_{s-ext.}}{2c^2}}$. See also familiar elaborations around equations (2.11), Fig.2.4. Tables T.2.4 and T.2.5., and equations (4.41-1) to (4.41-4) in chapter 4.3. **Eventually, we will have some reasonable grounds to say that meaning of mass, $\mathbf{m = m(p, L_s)}$, is always closely related to associated linear and spinning moments, and the intention here is to show that real sources of gravitation can only be masses with associated linear and angular momenta, including associated mass vibrations. On a similar way, we could also hypothetically exercise that additional mass contributors could come from associated electromagnetic energy participants, this way specifying mass as $\mathbf{m = m(p, L_s, q, \phi)}$, where $\mathbf{q}$ is a certain amount of electric charge, and $\boldsymbol{\phi}$ is specific magnetic flux. Here, we are respecting the Mobility system of analogies where we have the clear analogical prediction that sources of gravitation should only be active-charge elements such as $\mathbf{(p, L_s, q, \phi)}$.**

Newton's force law between two (almost static) masses presents the traditional understanding of gravitation. Now we assume that every object or mass $\mathbf{m = m(p, L_s, q, \phi)}$ should have associated gravity-related charge attributes $\mathbf{(p, L_s, q, \phi)}$. Here we should appropriately include matter waves, different acting fields, photons, etc. All the mentioned energy-momentum entities can be represented as having an equivalent (relativistic) mass or total energy content based on Einstein energy-mass relation $E = mc^2$, and vice versa, and consequently, we are able to address Newton law within a much broader meaning than usually, as follows:

$$\left(F_g = -G\frac{m_{01}m_{02}}{r^2}\right) \rightarrow \left(F_g = -G\frac{m_1 m_2}{r^2},\ m_{1,2} = m_{0-1,2} / \sqrt{1 - v_{1,2}^2 / c^2}\right) \Leftrightarrow$$

$$\mathbf{F_g = -G \cdot \frac{m_1 \cdot m_2}{r^2} = -\frac{G}{c^4} \cdot \frac{(m_1c^2) \cdot (m_2c^2)}{r^2} = G* \cdot \frac{E_{t1} \cdot E_{t2}}{r^2}, (G* = -\frac{G}{c^4} = const.)}$$
$$\mathbf{F_g = G* \cdot \frac{1}{r^2}(E_{k1} + E_{01}) \cdot (E_{k2} + E_{02}) = G* \cdot \frac{1}{r^2}(E_{k1}E_{k2} + E_{k1}E_{02} + E_{01}E_{k2} + E_{01}E_{02})}. \quad (2.3)$$

Here, the most general gravity force expression (2.3), and later (2.4), (2.4)-a, (2.4)-b, are on some indicative way approaching to A. Einstein General Relativity, or curved space-time conceptualization of Gravitation, if we present involved energies in terms of relevant charges, moments, and forces.

It is immediately clear that in the force law (2.3) there are only relations (products) between motional (kinetic or dynamic) energies, $\mathbf{E_{k1} = E_{1dyn.}}$, $\mathbf{E_{k2} = E_{2dyn.}}$, and state-of-rest energies, $\mathbf{E_{01} = E_{1stat.}}$, and $\mathbf{E_{02} = E_{2stat.}}$, of the interacting objects. Since total energy members, $\mathbf{E_{t1} = E_{k1} + E_{01} = E_{1dyn.} + E_{1stat.}}$, $\mathbf{E_{t2} = E_{k2} + E_{02} = E_{2dyn.} + E_{2stat.}}$, could have different origins, composed from many total energy-contributing elements (electromagnetic, gravitational, rotation, kinetic and potential energies, etc.), we could generalize Newton force law (for instance, between two complex energy states, here marked with **"i ↔ 1"** and **"j ↔ 2"**) creating the following force formula, $\mathbf{F_g} = \mathbf{F_{1,2}}$:

$$\begin{aligned} \mathbf{F_{1,2}} &= \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} [(\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{k2j}}) + (\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{02j}}) + \\ &+ (\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{k2j}}) + (\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{02j}})] = \\ &= \sum_{i,j} \mathbf{F_{(1-2)dyn.}} + \sum_{i,j} \mathbf{F_{(1)dyn.(2)stat.}} + \sum_{i,j} \mathbf{F_{(1)stat(2)dyn.}} + \sum_{i,j} \mathbf{F_{(1-2)stat.}} \ . \end{aligned} \tag{2.4}$$

It should be underlined that in (2.4) there is only one purely static (or constant, velocity independent) force member involving only rest masses interaction or interaction between static, stable and constant field charges (if mass in its wider meaning, as introduced here, could be characterized as a real, unique, self-standing and fully representative gravitation field-charge):

$$\sum_{i,j} \mathbf{F_{(1-2)stat.}} = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{02j}}) = -\ \mathbf{G} \cdot \frac{\mathbf{m_{01} \cdot m_{02}}}{\mathbf{r^2}} \ . \tag{2.4)-a}$$

Here favored concept is that a static or rest mass (if really exist) is just a part of the gravitation related picture. The other significant part (of gravitational force components between two objects) should be referred to involved linear and orbital moments and intrinsic mass vibrations (including coupled electromagnetic moments, dipoles, and charges), or related to dynamic, motional energy parts $(\mathbf{p}, \mathbf{L_s}, \mathbf{q}, \boldsymbol{\phi})$***,*** like ones we can find in (2.1), (2.2), and (2.4),

$$\begin{aligned} &\sum_{i,j} \mathbf{F_{(1-2)dyn.}} = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{k2j}}) = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \int_{[0,r]} \mathbf{F_{(1\text{-}2)i}} \mathbf{dr_{(1-2)i}})(\sum_{i} \int_{[0,r]} \mathbf{F_{(2\text{-}1)i}} \mathbf{dr_{(2-1)i}}) \ , \\ &\sum_{i,j} \mathbf{F_{(1)dyn.(2)stat.}} = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{k1i}})(\sum_{j} \mathbf{E_{02j}}) = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \int_{[0,r]} \mathbf{F_{(1\text{-}2)i}} \mathbf{dr_{(1-2)i}})(\sum_{j} \mathbf{E_{02j}}) \ , \\ &\sum_{i,j} \mathbf{F_{(1)stat(2)dyn.}} = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{01i}})(\sum_{j} \mathbf{E_{k2j}}) = \mathbf{G^*} \cdot \frac{\mathbf{1}}{\mathbf{r^2}} (\sum_{i} \mathbf{E_{01i}})(\sum_{i} \int_{[0,r]} \mathbf{F_{(2\text{-}1)i}} \mathbf{dr_{(2-1)i}}) \ . \end{aligned} \tag{2.4)-b}$$

We should have in mind that the total motional energy (of both interacting objects) could have linear (or rectilinear) kinetic energy components $E_k = pv/(1+\sqrt{1-v^2/c^2}) \cong \frac{1}{2}pv$ and certain amount of rotating (or spinning) motion energy $E_{kr} = L\omega/(1+\sqrt{1-v^2/c^2}) \cong \frac{1}{2}L\omega$ (in case if any of interacting objects is spinning), including embedded electromagnetic and other energy forms. *The point here is that (using conclusions based on analogies) we are coming closer to understanding that Newton law of gravitation, initially defined as an attraction of masses, is also presentable as an attraction between linear and/or angular moments (* p_i *and* L_i *) of involved mass-energy-momentum states, since,*

$$E_k = \sum_{(i)} (E_{ki} + E_{kri}) \cong \frac{1}{2} \sum_{(i)} (p_i v_i + L_i \omega_i) \ ,$$

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$$\left(m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{c^2(1+\sqrt{1-v^2/c^2})}\right)\left[\begin{array}{c}\Leftrightarrow \\ \text{analog to}\end{array}\right]\left(\mathbf{J} = \mathbf{J}_0 + \frac{E_{kr}}{c^2} = \mathbf{J}_0 + \frac{L\omega}{c^2(1+\sqrt{1-v^2/c^2})}\right),$$

(See the table of analogies T.1.8 from the first chapter).

For instance, the force component $\sum_{i,j}\mathbf{F}_{(1-2)dyn.} = \mathbf{G^*}\cdot\frac{\mathbf{1}}{\mathbf{r^2}}(\sum_i \mathbf{E}_{k1i})(\sum_j \mathbf{E}_{k2j})$ from (2.4)-b can be transformed into, $\sum_{i,j}\mathbf{F}_{(1-2)dyn.} = \frac{\mathbf{G^*}}{\mathbf{4}}\cdot\frac{\mathbf{1}}{\mathbf{r^2}}(\sum_i \mathbf{p}_i\mathbf{v}_i)(\sum_j \mathbf{p}_j\mathbf{v}_j)$. If we now consider (of course, as a specific or exceptional case) within Mobility system of analogies (from T.1.8), that electric charge analogically corresponds to linear momentum, $\mathbf{q}_i \Leftrightarrow \mathbf{p}_i$, we can create another (already known) expression that should be a force between *equidistant, parallel paths uniformly moving electrical charges* (q_1 , q_2), while at the same height (see later (2.4-4), and [45]),

$$\sum_{i,j}\mathbf{F}_{(1-2)dyn.} = \frac{\mathbf{G^*}}{\mathbf{4}}\cdot\frac{\mathbf{1}}{\mathbf{r^2}}(\sum_i \mathbf{p}_i\mathbf{v}_i)(\sum_j \mathbf{p}_j\mathbf{v}_j) \quad \left(\begin{array}{c}\Leftrightarrow \\ (\mathbf{p}\Leftrightarrow\mathbf{q})\end{array}\right)$$

$$\Leftrightarrow \frac{\mathbf{G^*}}{\mathbf{4}}\cdot\frac{(\sum_i \mathbf{q}_i\mathbf{v}_i)(\sum_j \mathbf{q}_j\mathbf{v}_j)}{\mathbf{r^2}}(\Leftrightarrow)\frac{\mu}{\mathbf{4}\pi}\frac{(\mathbf{q}_1\mathbf{v}_1)\cdot(\mathbf{q}_2\mathbf{v}_2)}{\mathbf{r^2}}. \qquad (2.4)\text{-c}$$

$$\left[\begin{array}{l} a\cdot G^* = \mu/\pi = a\cdot G/c^4 \Leftrightarrow \mu c^4 = \pi\cdot a\cdot G = \mu/\varepsilon^2\mu^2 = 1/\varepsilon^2\mu \Rightarrow \pi\cdot a\cdot G\varepsilon\mu = 1/\varepsilon \Rightarrow \boxed{\pi\cdot a\cdot Gc^2\varepsilon = 1}\ ?? \\ a = \text{constant}\end{array}\right].$$

Of course, such fragmented analogical constructions and involved conclusion process as in (2.4)-c is oversimplified, not well supported, and still presents an intuitive and brainstorming product, but it is on some indicative way saying that gravitational attraction could also be an attraction between some effective electric charges or dipoles, as it will be exercised later (see elaborations around equations from (2.4-7) until (2.4-10)). Here, we are also becoming familiar with the ideas that Newtonian or $1/r^2$ gravitational central force could have dynamic or active interacting elements like mechanical and electromagnetic moments, and corresponding velocities (and not only static masses).

Direct analogical (and still hypothetical) force expression between two moving masses, based on (2.4)-c and on Mobility analogies from the first chapter, is again similar or equivalent to (2.4), as follows,

$$F_{1,2} = K'\frac{(p_1v_1)\cdot(p_2v_2)}{r^2} = K''\frac{E_{k1}\cdot E_{k2}}{r^2} = K''\frac{(E_1-E_{01})\cdot(E_2-E_{02})}{r^2} =$$

$$= \frac{K''}{r^2}(E_1E_2 - E_1E_{02} - E_{01}E_2 + E_{01}E_{02}) = \frac{K''}{r^2}(\gamma_1\gamma_2 m_1m_2C^4 - \gamma_1 m_1m_2C^4 - \gamma_2 m_1m_2C^4 + m_1m_2C^4) =$$

$$= K'''\left(\gamma_1\gamma_2\frac{m_1m_2}{r^2} - \gamma_1\frac{m_1m_2}{r^2} - \gamma_2\frac{m_1m_2}{r^2} + \frac{m_1m_2}{r^2}\right) = \boxed{K'''\frac{m_1m_2}{r^2}}\cdot(\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1),$$

$$(K', K'', K''') = \text{constants}$$

(2.4)-c-1

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To support the existence of such attractive force, that will be equal to Newton force of gravitation, it will be necessary that masses m_1 and m_2 are in permanent (holistic) motion, including rotation, as follows,

$$\boxed{K''' \frac{m_1 m_2}{r^2}} \cdot (\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) \cong G \frac{m_1 m_2}{r^2} \Rightarrow (\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) \cong \text{Const.},$$

$$G = G(\gamma_1, \gamma_2) = G(v_1, v_2) = K''' \cdot (\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) \cong \text{const.}, \quad \gamma_{1,2} = (1 - v_{1,2}^2 / C^2)^{-0.5} ,$$

$$(v_1 = v_2 = v), (\gamma_1 = \gamma_2 = \gamma) \Rightarrow K'''(\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) = K'''(\gamma^2 - 2\gamma + 1) = G \Rightarrow v = \underline{\text{const}}.$$

(2.4)-c-2

In our solar system and within our part of cosmic space, Newton's gravitational constant G is equal to G ≅ 6.67408 × 11^{-11} m^3 kg^{-1} s^{-2}, but in some different parts of cosmic space, it could be different, since involved holistic rotations and background velocities, and electromagnetic fields could be different. The logical consequence of (2.4)-c-2 is that we could imagine that our Universe is in perpetual, global macrocosmic motion, including rotation, to support the existence of Newtonian gravitation. See similar analogical elaborations later around (2.4-4), (2.4-5), (2.4-5.1), T.2.2-1 and T.2.2-1. Mentioned rotation (being an accelerated motion) is producing spatially organized dipoles, or electric and magnetic polarizations, and such electromagnetic entities are mutually experiencing effects of attractive forces, and creating rotating, spiral, elliptic and circular, galactic, and planetary systems (as speculated later in this chapter around equations (2.4-7) and (2.4-8)).

We also know that an internal rest mass structure has its natural elements (molecules, atoms, and subatomic constituents) composed of electromagnetically or dipole-balanced, charged states. These are in perpetual motion, oscillating, spinning, and rotating in their stationary energy states, being somehow auto compensated, and stabilized like self-closed standing waves, resonant structures, or as mutually coupled elementary magnets and positive and negative charges. The characteristic property of rest masses in our universe is that all of them have specific latent (internal), rest-state energy (equal to $\mathbf{m_0 c^2}$). The force between rest masses would still exist, even if they look (macroscopically, or externally) as fully electrically and magnetically neutral. This should be the real nature of gravitational force (because rest masses are at the same time (or dominantly) agglomerates and ensembles of different internal spinning states). Of course, when rest masses are in relative motion interacting with other energy states and other masses, the force of Gravitation would become only more complex, like in (2.4). There are only mutually relative motions and relative rest states among all masses and energy states in our universe and involved forces should also be mutually relative. The challenge here would be to find the essential, generalizing relations and connections between electromagnetic and gravitation forces, since mainly, all of them (when isolated) are obeying the same form of Coulomb-Newton's force law, like shown in T.2.2.

It is clear that what we are presently considering as Newton law of gravitation force is only the most static and maybe the weakest part of the generalized force expression (2.4): $\sum_{i,j} \mathbf{F}_{(1-2)stat.} = \mathbf{G} * \cdot \frac{\mathbf{1}}{\mathbf{r}^2} (\sum_i \mathbf{E}_{01i})(\sum_j \mathbf{E}_{02j}) = -\mathbf{G} \cdot \frac{\mathbf{m}_{01} \cdot \mathbf{m}_{02}}{\mathbf{r}^2}$. Obviously, that (2.4) can be modeled or mathematically transformed as in (2.1) and (2.2), or also to include linear and orbital moments, electromagnetic and other force, and energy elements. Here, we can pose the question what real, elementary, and essential sources or charges of different fields and forces in our Universe are. We can also try to model them deductively, starting from (2.4), taking the energy as the most common quantifying property for all of them (avoiding starting from specific objects and other field charges, as usually practiced in Physics). Until present, official mainstream science still considers that mass is an exclusive source of gravity, as the source of an electric field is an electrical charge. Based on an extended understanding of Newton-Coulomb force law (2.4), and on respecting Mobility-type Analogies (from the first chapter), we could find or re-invent more of essential *force charges* (most of that matter being still hypothetical and based on analogical thinking). If our present knowledge regarding mentioned analogical situations about natural forces is essentially correct, the same, already known facts regarding field sources and charges should be re-confirmable from different conceptual, symmetries and analogies related platforms, becoming mutually compatible for generalizations, and being uniquely treated regarding all possible field charges, starting from (2.4), by going back to elementary relations between different field charges. As we know, this is still not the case in the contemporary Physics. For instance, if we know that objects $\mathbf{m_1}$ and $\mathbf{m_2}$, from (2.3) are not only neutral masses, but also have certain (non-compensated) electrical charges and specific magnetic properties (being like permanent magnets and/or electric dipoles), we can extend the Newton-Coulomb force law (2.4) by adding two more force members. However, we need to remember that associated electromagnetic forces in many cases could be enormously stronger than any attraction caused by gravitation and that in such cases the force member belonging to static mass attraction, $-G\frac{m_{01}m_{02}}{r^2}$, could be negligible, especially in instances when masses are sufficiently small). At the same time, we still do not have any quantitative argument to say how big other (hypothetical) gravity related force contributing members (found in expressions (2.1), (2.2) and (2.4)) could be, compared to electromagnetic forces. Most probably it will be explained that all of such "old and new force members" are essentially belonging to electromagnetic forces or directly originating from them (see [71], from Jovan Djuric, "Magnetism as Manifestation of Gravitation").

[♣ COMMENTS & FREE-THINKING CORNER (brainstorming)

Since both, Newton and Coulomb force laws have the same mathematical form, for developing more general (for the time being only qualitative) understanding of force expressions between two objects "charged electrically ($q_{el.} = q$), magnetically, ($q_{mag.} = \Phi$) and gravitationally, (m)"; -we could hypothetically transform (2.4 into):

$$\mathbf{F_{1,2}} = \mathbf{F_g} \Rightarrow \mathbf{F_{1,2}}\textbf{(combined, multiple charges)} = \mathbf{F_g} + \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} + \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2} + \ldots .$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$F_{1,2} = \sum_{i,j} F_{(1\text{-}2)stat} + \sum_{i,j} F_{(1\text{-}2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn} =$$
$$= \left[-G\frac{m_1 m_2}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} + \frac{1}{4\pi\,\mu}\frac{\Phi_1\Phi_2}{r^2} + ... \right] + \qquad (2.4\text{-}1)$$
$$+ \sum_{i,j} F_{(1\text{-}2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn}$$
$$\left(F_{1,2} = F_m = \mu\frac{q_{m1}\cdot q_{m2}}{4\,\pi\;r^2} = \frac{1}{4\pi\,\mu}\frac{\Phi_1\Phi_2}{r^2}, \Phi_{1,2} = \mu q_{m1,2} \right),$$

however, being modified for new motional energy contributions coming from interactions between electrical, magnetic and all other presently known or maybe some unknown field charges (if any of such still hidden charges really exist). The reason for having such situation is not only that Newton and Coulomb's laws have the same mathematical forms, but in the fact that every electrical or magnetic entity has its equivalent mass (or energy content). This means that the logic applied in developing (2.4-1) can be extended to any Coulomb-Newton central force situation between different field charges. At the same time, qualitatively and conceptually, (2.4-1) could present a kind of simple field unification platform, but with many associated problems such as, that active centers of masses, electric charges and magnetic dipoles or fluxes are never completely overlapping the same areas. Those distances between mentioned charged entities (between relevant charged object centers) are always different. We anyway know that motional electric and magnetic objects and states are always mutually related, electromagnetically coupled and embedded in a mass as a state of energy packing. Also, force laws (2.4) – (2.4-1) are non-linear, $1/r^2$ central force laws, and simple (linear) superposition and spatial homogeneity is not applicable here. Different field charges (mass, electrical charge, magnetic flux …) are only looking mutually separate and distinct to us (especially if we start comparing them quantitatively and dimensionally), but maybe we just have different "packing and energy-atomizing formats" of the same universal field (meaning that (2.4-1) presents an impossible situation). A similar concept will be additionally elaborated later; -see equations (2.4-6) – (2.4-10), just to explore how far we could advance using conclusions based on analogies.

Consequently, we could imagine that every mass internally has combination of mass-energy elements coming from its electromagnetically neutral rest mass, and from its electromagnetically active (not compensated) elements. This way conceptualizing, neutral rest masses $m_{1,2-n}$ will mutually attract respecting Newton law (of Gravitation), and electromagnetic elements $\frac{E_{1,2-em}}{c^2}$ will mutually interact respecting Coulomb law, as follows,

$$F_{1,2} = G\frac{m_1 m_2}{r^2} = G\frac{(m_{1-n} + \frac{E_{1-em}}{c^2})(m_{2-n} + \frac{E_{2-em}}{c^2})}{r^2} \; .$$

If at the same time, certain mass has spinning, rotating, torsional or angular motions (which are not electromagnetic), we could extend Newton-Coulomb law as,

$$F_{1,2} = G\frac{m_1 m_2}{r^2} = G\frac{(m_{1-n} + \frac{E_{1-spinning}}{c^2} + \frac{E_{1-em}}{c^2})(m_{2-n} + \frac{E_{2-spinning}}{c^2} + \frac{E_{2-em}}{c^2})}{r^2}$$

The well-known particle-wave **duality** (or better to say **unity**) concepts and their origins, known from Quantum Theory (first introduced by L. De Broglie, Max Planck, A. Einstein, and N. Bohr), should also be (qualitatively) recognizable in (2.4-1) as for instance,

$$\begin{aligned} & F_{1,2} = \sum_{i,j} F_{(1\text{-}2)stat} + \sum_{i,j} F_{(1\text{-}2)dyn.} + \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn} = \\ & = F(\text{Particles-related}) + F(\text{Waves \& Fields-related}) + \\ & + F(\text{mixed-particles-waves-related}), \\ & F(\text{Particles-related}) = \sum_{i,j} F_{(1\text{-}2)stat} = \frac{\Omega_1 \Omega_2}{r^2}, \\ & F(\text{Waves \& Fields-related}) = \sum_{i,j} F_{(1\text{-}2)dyn.}, \\ & F(\text{mixed-particles-waves-related}) = \sum_{i,j} F_{(1)dyn.(2)stat} + \sum_{i,j} F_{(1)stat(2)dyn}. \end{aligned} \quad (2.4\text{-}3)$$

$\Omega_{1,2}$ (=) here introduced symbol for generalized universal field charges.

Familiar ideas regarding unique origins of Newton-Coulomb force laws (as consequences of orbital and spin moments conservation) can also be found in [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

Gravitation explicable as a weak magnetic fields attraction

There is another $\frac{1}{r^2}$ force law *between equidistant, parallel-paths uniformly moving electrical charges* (q_1, q_2), while at the same height, where the force is given by (see (2.4)-c and [45]),

$$F_{1,2} = \frac{\mu}{4\pi} \frac{(q_1 v_1) \cdot (q_2 v_2)}{r^2} = K \frac{(q_1 v_1) \cdot (q_2 v_2)}{r^2}, \ K = \frac{\mu}{4\pi} = \text{const.} \quad (2.4\text{-}4)$$

The magnetic force between two moving charges in the more general case is,

$$\vec{\mathbf{F}}_{1,2} = \frac{\mu}{4p} \frac{q_1 q_2}{r^2} \vec{\mathbf{v}}_1 \times (\vec{\mathbf{v}}_2 \times \hat{\mathbf{r}}).$$

Here, original Coulomb's law (like in cases of static electric charges) cannot directly explain magnetic-field force between them. For the same sign on the products $(q_1 v_1)$ and $(q_2 v_2)$ the charges are drawn closer (presenting an attractive force, like between two parallel wires with currents in the same direction), and for opposite signs, the charges are brought apart (introducing repulsive force, like between two parallel wires with currents in mutually opposed directions). The most likely, still an intuitive explanation is that moving charged particles are creating helicoidally spinning electromagnetic fields around paths of motion (or at least helicoidally spinning magnetic field). Where such electromagnetic fluxes are mutually not canceling, motional electric charges will experience an attractive force, and in cases of mutually opposed fields, a repulsive force will appear. *The same idea about specific "field spinning" around any moving particle will later be shown applicable to all matter waves (see chapter 4.1), as an extended understanding of de Broglie, matter-waves hypothesis.*

Let us continue generating more of intuitive, analogical, and brainstorming or hypothetical ideas regarding Gravitation, since it is evident that from the time when I. Newton created his foundations of gravitation, modern physics did not move too much (regardless A. Einstein elaborated relatively complex and seducing mathematics within General Relativity theory), and as we know, very new ideas and concepts are still missing. If we apply already known analogies (see tables of analogies T.1.2 until T.1.8, from the first chapter), where electrical charge q is analog to linear, $p = mv$, ($m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{c^2(1+\sqrt{1-v^2/c^2})}$), or spin moment $L = J\omega$, or to a magnetic flux Φ, we will be able to transform (2.4-4) into another couple of interesting (and hypothetical) $\frac{1}{r^2}$ force expressions (2.4-5). Instead of different field charges we will (based on mentioned Mobility analogies) have products of corresponding motional, kinetic, or field energy members, like in (2.3), (2.4) and (2.4)-c,

$$F_{1,2} = K\frac{(q_1v_1)\cdot(q_2v_2)}{r^2}\begin{pmatrix}\Rightarrow \\ \text{by analogy}\end{pmatrix} F_{1,2} = \begin{Bmatrix} K_p\frac{(p_1v_1)\cdot(p_2v_2)}{r^2}, \text{ for neutral particles} \\ K_L\frac{(L_1v_1)\cdot(L_2v_2)}{r^2}, \text{for spinning particles} \\ K_\Phi\frac{(\Phi_1v_1)\cdot(\Phi_2v_2)}{r^2}, \text{ for magnetic dipoles}\end{Bmatrix}$$

$$\begin{pmatrix}\Rightarrow \\ \text{by analogy} \\ =\end{pmatrix}\begin{Bmatrix} K_p\frac{(p_1v_1)\cdot(p_2v_2)}{r^2} \\ K_L\frac{(L_1\omega_1)\cdot(L_2\omega_2)}{r^2} \\ K_\Phi\frac{(\Phi_1u_1)\cdot(\Phi_2u_2)}{r^2}\end{Bmatrix}\begin{pmatrix}\Rightarrow \\ \text{by analogy} \\ =\end{pmatrix}\begin{Bmatrix} K_p^{'}\frac{(E_{k1})\cdot(E_{k2})}{r^2} \\ K_L^{'}\frac{(E_{r1})\cdot(E_{r2})}{r^2} \\ K_\Phi^{'}\frac{(E_{m1})\cdot(E_{m2})}{r^2}\end{Bmatrix}, \quad (2.4\text{-}5)$$

$(K, K_p, K_L, K_\Phi, K_p^{'}, K_L^{'}, K_\Phi^{'}) = \text{Constants}, v_i (=)\text{velocity}, u_i (=)\text{voltage}, \omega_i (=)\text{angular velocity}$

Of course, (2.4-5) could have some analogical predictive power under similar conditions when (2.4-4) is applicable (between equidistant, parallel paths moving charges). ***In cases of electric charges, corresponding voltages could replace velocities (using Mobility system of analogies).***

Another, revitalized and emerging force law between electric current elements, also familiar and analogical to (2.4-4), (2.4-5), and to other Newton-Coulomb ($1/r^2$)-force laws, is based on the old Ampère's force law. Such force law is showing promising results and introduces competitive modeling compared to similar conclusions from Maxwell-Faraday and Quantum Electrodynamics (see [36], Anthony D. Osborne, & N. Vivian Pope; -Immediate Distant Action and Correlation in Modern Physics).

The real challenge in applying analogies for creating possible and brainstorming options regarding what gravitation field and force could be (in addition to what we already know) is related to the fact that only a static mass is not the entity that should be the essential, and unique source of gravitation. Of course, if we start only from the Newton and Kepler Laws, we can get a strong impression that mass is the only source of gravity. Since mobility-type analogies of mass are electric capacitance **C** and moment of inertia **J** (see the chapter 1.), it is difficult to imagine how such direct analogical approach could produce Coulomb law $F_{1,2} = \frac{1}{4\pi\varepsilon}\frac{q_1 \cdot q_2}{r^2}$ from Newton law of gravitational attraction between two masses, $F_{1,2} = G\frac{m_1 \cdot m_2}{r^2}$.

The reason is that mass and electric charge are not mutually analogical or equivalent (meaning not directly replaceable in the mobility-type of analogies). What analogically corresponds to electric charges (in mobility type of analogies) is a linear moment, $p = mv$, and angular moment, $L = J\omega$ (as already exercised in the first chapter and in T.2.2), but presently we do not know enough about gravitational interactions between linear and/or angular moments. Also, since mass analogically corresponds to specific capacitance, analogically created a law of (hypothetical) attraction between two capacitors $F_{1,2} = K\frac{C_1 \cdot C_2}{r^2}$ is meaningless, except when capacitors in question are electrically charged (and when we could imagine the existence of certain surrounding electric fields attraction). Obviously, to resolve this challenge and contradictions based on analogical conclusions, masses should have some hidden velocity components or attributes. On a similar way as in cases of capacitors, masses could be conveniently "velocity-charged" or activated to be like electrically charged capacitors. What makes masses mechanically and electromagnetically "charged" is certain kind of combined linear and rotational motion (including spinning and associated vibrations), meaning that masses should have linear and angular velocity components and moments to be analogically comparable to charged capacitors… As we know, Newton law is still not addressing any of dynamic or motional parameters like velocities, but it is anyway mathematically like Coulomb law. We could ask ourselves, where missing (angular and linear) velocity components are? Most probably (and still hypothetically), something (still invisible and not properly conceptualized in physics) is permanently embedded, moving or oscillating everywhere around and inside gravitational masses, also creating certain intrinsic, background velocity field. This way conceptualizing, we know that Newton Law is not explicitly considering mentioned ***velocity-characterized***, and (should be) omnipresent background field, but anyway, all masses (in our Universe) are on some natural way always "velocity-charged" regarding such hidden and moving, spatial texture (see more of familiar and challenging ideas in [73] and [74]). However, if we now start from the Coulomb law and familiar "$1/r^2$" (electromagnetic) force laws, such as:

T.2.2-1

$F_{1,2} = \frac{1}{4\pi\varepsilon}\frac{q_1 \cdot q_2}{r^2}$	(=) Coulomb force between electric charges
$F_{1,2} = F_m = \mu\frac{q_{m1} \cdot q_{m2}}{4\pi r^2} = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2}$, $\Phi_{1,2} = \mu \cdot q_{m1,2}$	(=) Coulomb-type force between two magnets (first time discovered by John Michell in 1750, see [36]).
$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2}$, $\Phi_{1,2} = \mu \cdot q_{m1,2} = \mu \cdot q_{1,2} \cdot v_{1,2}$ $q_{m1,2} = q_{1,2} \cdot v_{1,2}$	(=) $1/r^2$ force law between equidistant, parallel-paths, uniformly moving electrical charges (q_1 , q_2); (2.4-1), (2.4-4), (2.4)-c (see [45])

and if we consider that electric charge is (in mobility-type of analogies) replaceable with linear, orbital and/or angular moments, $p = mv$, and/or $L = J\omega$ (see Analogies in the chapter 1.), we could apply such mobility-type analogies (on T.2.2-1) and formulate the following (hypothetical) mechanical-world (would be gravitational force) analogies,

T.2.2-2

1/r² force laws	Possible, mobility-type analogies (still hypothetical)
$F_{1,2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 \cdot q_2}{r^2}$	$F_{1,2} = G_p\frac{p_1 \cdot p_2}{r^2}(=)G_L\frac{L_1 \cdot L_2}{r^2}$
$F_{1,2} = F_m = \frac{1}{4\pi\mu}\frac{\Phi_1\Phi_2}{r^2}$	$F_{1,2} = G_p\frac{p_1 \cdot p_2}{r^2}(=)G_L\frac{L_1 \cdot L_2}{r^2}$
$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2}$	$\cdots \to F_{1,2} = G_p\frac{p_1 \cdot p_2}{r^2}(=)G_L\frac{L_1 \cdot L_2}{r^2}$ (?!)

How to transform $F_{1,2} = G_p \frac{p_1 \cdot p_2}{r^2} (=) G_L \frac{L_1 \cdot L_2}{r^2}$ into an expression of Newton gravitation force law $F_{1,2} = G \frac{m_1 \cdot m_2}{r^2}$, considering that specific "***background velocity field***" is intrinsically linked to a locally surrounding spatial structure of our Universe (see [73] and [74]), is possible to imagine and analogically exercise on the following way,

$$\left\{ \begin{array}{l} F_{1,2} = G_p \frac{p_1 \cdot p_2}{r^2} = G_p \frac{m_1 v_1 \cdot m_2 v_2}{r^2} = G_p v^2 \frac{m_1 m_2}{r^2} = G_{pv} \frac{m_1 m_2}{r^2} = G \frac{m_1 m_2}{r^2} \Rightarrow G_{pv} = G = G_p v^2, \\ F_{1,2} = G_L \frac{L_1 \cdot L_2}{r^2} = G_L \frac{J_1 \omega_1 \cdot J_2 \omega_2}{r^2} = G_L \omega^2 \frac{J_1 J_2}{r^2} = G_{L\omega} \frac{J_1 J_2}{r^2} = ... = G \frac{m_1 m_2}{r^2} \Rightarrow G_{L\omega} = G = G_L \omega^2, \\ \left(v = \sqrt{v_1 v_2}, \omega = \sqrt{\omega_1 \omega_2}, G = G_p v^2 = G_L \omega^2 = v\omega \sqrt{G_p G_L} \right) \end{array} \right\} \Rightarrow$$

$$\begin{array}{l} v / \omega = \sqrt{G_L / G_p} \, (=) \, r \, (=) [m] \, (=) \text{certain radius, or length} \Rightarrow \\ \text{If: } \left((G, G_p, G_L) = \text{constants} \right) \Rightarrow (v, \omega) = \text{Constants}, \\ \text{If: } \left(G_p = (1/4\pi g_p) \; \& \; G_L = (1/4\pi g_L) \right) \Rightarrow c = (1/\sqrt{\varepsilon\mu}) = (1/\sqrt{g_p g_L}) = 4\pi\sqrt{G_p G_L} = 4\pi G / v\omega \Rightarrow \\ v\omega = 4\pi G / c = G / \sqrt{G_p G_L} \, (=) \, v^2 / r = r\omega^2 (=) \text{central or radial acceleration} \, (=) \text{constant}, \sqrt{G_p G_L} = c / 4\pi, \\ \frac{G_L}{G_p} = \frac{g_p}{g_L} = \frac{L_1 \cdot L_2}{p_1 \cdot p_2} (=) \frac{L^2}{p^2} (=) \frac{v^2}{\omega^2} (=) r^2 (=) [m^2], \left(\frac{1}{2} pv = \frac{1}{2} L\omega \right) \Leftrightarrow \left(\frac{L}{p} = \frac{v}{\omega} (=) \frac{\omega r}{\omega} = r = \sqrt{\frac{G_L}{G_p}} = \sqrt{\frac{g_p}{g_L}} \right), \\ \left(m = m_0 + \frac{E_k}{c^2} = m_0 + \frac{pv}{c^2 (1 + \sqrt{1 - v^2/c^2})} \right) \left[\begin{array}{c} \Leftrightarrow \\ \text{analog to} \end{array} \right] \left(J = J_0 + \frac{E_{kr}}{c^2} = J_0 + \frac{L\omega}{c^2 (1 + \sqrt{1 - v^2/c^2})} \right), \end{array} \quad (2.4\text{-}5.1)$$

where $v = \sqrt{v_1 v_2}$ and $\omega = \sqrt{\omega_1 \omega_2}$ are mentioned, intrinsic and mutually coupled velocities of "*spatial matrix*", creating locally relevant "***background velocity field***" (to avoid using often criticized item like "***ether***").

Of course, we already know that variety of all involved linear, mechanical, and electromagnetic moments, dipoles and fluxes, or relevant fields charges (of gravitationally interacting masses) is originating from interactions between involved electrons and protons (being inside atoms and masses). Electrons and protons (including atoms and molecules) are oscillating, spinning, mutually interacting electrodynamically, and by synchronizing, extending, or radiating mentioned internal, atomic, and electromagnetic fields and properties towards other atoms and masses externally. Neutron is also effectively composed of one electron and one proton, and it has spinning properties. This way conceptualizing, all other (external) mechanical properties and mechanical moments of masses (including gravitation) are being created. In addition, all masses are still behaving as being slightly magnetized, and most probably that gravitation can also be conceptualized as an attraction between mutually opposed magnets (respecting Coulomb or John Mitchel laws), as promoted by Dr. Jovan Djuric in [31, 71]. We should not forget that the force of gravitation is $1/r^2$ central force. All of that is also familiar to N. Tesla`s Dynamic Gravity and R. Boskovic Universal Natural Force. See more (about closely related problematic) in Chapters 8., 9., and 10.

We already know that in the gravitational field of a big mass, if we neglect possible friction with surrounding space, two different (smaller) masses will fall with equal speed (being synchronized and arriving at the surface of a big mass at the same time), meaning that it could be $v^2 = v_1 v_2$, $v_1 = v_2 = v$. Later, we will realize that there is no linear motion without certain associated field rotation (or spinning), leading to a conclusion, that it could exist another, "***background velocity field***" where, by analogy, is valid, $\omega^2 = \omega_1 \omega_2$, $\omega_1 = \omega_2 = \omega$. Familiar concepts about force of gravitation, effectively equal to the product of involved moments will be exercised and extended later (in the same chapter) within equations (2.4-13), (2.4-5.1), (2.11.13-1) - (2.11.13-5), (2.11.23), (2.11.24), (2.11.14-4), and table of analogies T.2.8. In Chapter 10. of this book, we can find the most complete explanation of the familiar situation regarding hidden or background velocity parameters, complex character of mass, and Newtonian attraction between relevant linear and angular moments (see more in Chapter 10, around equations (10.1.4) - (10.1.8)). Briefly, our contemporary understanding of mass (as something constant, or as a stable building block of matter) should and will evolve significantly, becoming much more linked to Wave-Particle duality of matter.

Since mutually interacting masses and their linear and orbital or spinning moments are creating what we describe as gravitation (as an interaction of relevant, standing matter waves), we could also exercise that any two of such states are effectively creating specific "half-wavelength resonant dipole". Such electro-mechanical dipoles are in standing-waves resonance, where we can recognize the existence of an attractive force in nodal zones. Such resonant effects with the appearance of attractive force acting towards resonant nodal zone are well known in ultrasonic technique when half-wavelength sonotrode or resonator is oscillating on its resonant frequency, or in cases of acoustic levitation; -see more in [150] and [151]. It is highly probable that the effect of gravitation is a similar local state of an attraction within a resonating Universe. The question here is to discover what the “divine source” of such (still unknown) oscillatory energy is (see more of similar elaborations in [99] from Konstantin Meyl).

To illustrate larger spectrum of such (or familiar) ideas about *spatial, background* ***“velocity-field”***, or “***linear and orbital moments linked***” mass and electromagnetic charges and dipoles understanding, or concepts about gravitation and mutual communication (and couplings) between space, matter, and electromagnetic phenomenology, let us first read the following abstracts and citations (see next pages): Taken from [73]; -Reginald T. Cahill, Dynamical 3-Space: Emergent Gravity:

Dynamical 3-Space

An Expanding Earth Mechanism

Extended Abstracts Book, p.5

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Extended Abstract

In remarkably prescient work Hilgenberg in the 1930’s (Scalera and Braun, 2003) proposed that the expansion of the Earth, driven by an increasing mass content, might be explained by a dynamical space that caused the generation of the new matter, and that the acceleration of gravity was nothing but the effect of an accelerated flow of that space. This was in direct conflict with the prevailing belief that space and time were integral aspects of a geometrical entity – spacetime and that gravitational acceleration was a spacetime curvature effect. This Einstein worldview had its origin in the supposedly null results from the 1887 Michelson-Morley interferometer experiment designed to detect the anisotropy of the speed of light, which would otherwise have indicated that a dynamical space, or an ether in the terminology of that era, was flowing through the detector Despite a few successes, the spacetime paradigm has faced an ever-increasing list of inexplicable failures, including the need for dark matter and dark energy. However, in 2002 (Cahill and Kitto, 2003), it was discovered that the Michelson-Morley experiment was not null, and the published 1887 data, using a new calibration theory for the device, showed a space-speed to 500km/s (Fig.1). Miller’s 1925-1926 experiment showed even more detailed confirming results (see also Fig.1), and recently Doppler shifts from spacecraft earth-flybys have confirmed those early results (Cahill, 2009), revealing the galactic speed of the solar system to be some 490km/s in the direction RA=4.3h, Dec=-75deg, and within 5deg of the direction that Miller had determined. As well, the flyby data also revealed an inflow of space into the Earth, confirming the expected speed of some 11km/s at the surface, as well as the sun inflow speed at 1AU of 42km/s. The Sun's surface inflow of 615km/s now follows from a new account of the deflection of starlight by the Sun (Cahill, 2009b). These developments change all of physics,

and now provide a mechanism to explain the expanding earth, precisely along the lines suggested by Hilgenberg, and in accord with later developments (Carey, 1989, Scalera, 2003, Maxlow, 2005.

Dynamical 3-Space: Emergent Gravity

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Invited contribution to “Should the Laws of Gravitation be reconsidered”. H´ector A. Mu´nera, ed. (Montreal: Apeiron 2011)

The laws of gravitation devised by Newton, and by Hilbert and Einstein, have failed many experimental and observational tests, namely the borehole g anomaly, flat rotation curves for spiral galaxies, supermassive black hole mass spectrum, uniformly expanding universe, cosmic filaments, laboratory G measurements, galactic EM bending, precocious galaxy formation,.. The response has been the introduction of the new epicycles: "dark matter", "dark energy", and others. To understand gravity, we must restart with the experimental discoveries by Galileo, and following a heuristic argument, we are led to a uniquely determined theory of a dynamical 3-space. That 3-space exists has been missing from the beginning of physics, although it was 1st directly detected by Michelson and Morley in 1887. Uniquely generalizing the quantum theory to include this dynamical 3-space we deduce the response of quantum matter and show that it results in a new account of gravity and explains the above anomalies and others. The dynamical theory for this 3-space involves G, which determines the dissipation rate of space by matter, and α, which experiments and observation reveal to be the fine structure constant. For the 1st time, we have a comprehensive account of space and matter and their interaction - gravity.

Citations from [122], LING JUN WANG, UNIFICATION OF GRAVITATIONAL AND ELECTROMAGNETIC FORCES.

"Newton's law of gravitation is strikingly similar to Coulomb's law; (2) Newton's law differs from Coulomb's law, equally striking, by lacking a dynamic term dependent on the velocity of the gravitating source. As a result, Newton's law is essentially a static theory unable to describe the propagation of the interaction. This is the theoretical origin of the historical problem of action at a distance. Natural questions arise: Is the lacking of a dynamic term in Newton's law a real manifestation of the law of Nature, or a theoretical miss due to the weakness of the dynamic part of the gravitational force that escaped the detection by observational astronomers and experimental physicists? If the latter is the case, what kind of dynamic term should be added to Newton's law?

……………..

In the following sections, we will present a generalized theory of gravitation by adding a dynamic term similar to the Lorentz force. It turns out that the inclusion of a dynamic Lorentz force alone is enough to develop the whole theory of dynamic gravitation and the wave equations. It has been shown that the inverse square law is a result of Gauss’ Law and Wang’s Law. The newly discovered Wang’s Law shows that the total linear momentum propagated into space is conserved.

……………………..

We generalize Newton’s Law of gravitation to include a dynamic term like the Lorentz force in Eq. (1):

$$\mathbf{F} = k_1 \frac{qq'}{r^2}\left[\hat{r} + \frac{1}{c^2}\mathbf{v}' \times (\mathbf{v} \times \hat{r})\right], \qquad (1)$$

……………………….

$$\mathbf{F} = -G\frac{mm'}{r^2}\left[\hat{r} + \frac{1}{c^2}\mathbf{v}' \times (\mathbf{v} \times \hat{r})\right], \qquad (2)$$

where v and v¢ are the velocities of the masses m and m¢, respectively".

…………………………

The background or natural texture and spatial matrix of our universe is also remarkably close to ether concepts.

Meaning and existence of ether within our universe is still not wholly addressed (solved or decided) in modern Physics. For A. Einstein Relativity theory, ether does not exist, or it is not necessary to consider it. It looks that ether is also not an essential item for electromagnetic theory (based on contemporary mainstream physics). Anyway, Maxwell created his equations very much based on analogies with real fluid-flow theory. Also, light, or electromagnetic waves speed (in different material media, including vacuum) strongly depends on measurable parameters like dielectric and magnetic permeability, since $c = 1/\sqrt{\varepsilon\mu}$, meaning that matter waves carrier media should always exist ; - see familiar relations (3.7-1) and (3.7-2) from the third chapter of this book.

Modern Quantum theory is attributing, sporadically and not consistently, different matter-waves and particles properties, and vacuum energy stochastic events fluctuations, to an absolute vacuum (*whenever necessary and useful to explain something strange*, but not always, and not on the same way).

Nikola Tesla also was very much convinced, descriptive, and clear regarding ether existence (but on some way based only on his experiments, without mathematical modeling).

What is typical for different ether-understanding concepts should be that this is a fluidic state of matter, somewhat analogical to ideal gas states. Such intrinsic, background matter state (or spatial matrix), should exist in an absolute vacuum, even without masses, particles, and waves (presenting material carrier of electromagnetic and other matter waves). Anyway, we still do not have better and more precise explanation what ether is, but we can read below how the same problematics is conceptualized and explained in contemporary physics, and get some challenging ideas:

Ether and gravitation [Citation from https://en.wikipedia.org/wiki/Aether_(classical_element)]

Ether has been used in various gravitational theories as a medium to help explain gravitation and what causes it. It was used in one of Sir Isaac Newton's first published theories of gravity, *Philosophiæ Naturalis Principia Mathematica* (the *Principia*). He based the whole description of planetary motions on a theoretical law of dynamic interactions. He renounced standing attempts at accounting for this particular form of interaction between distant bodies by introducing a mechanism of propagation through an intervening medium.[24] He calls this intervening medium aether. In his aether model, Newton describes aether as a medium that "flows" continually downward toward the Earth's surface and is partially absorbed and partially diffused. This "circulation" of aether is what he associated the force of gravity with to help explain the action of gravity in a non-mechanical fashion.[24] This theory described different aether densities, creating an aether density gradient. His theory also explains that aether was dense within objects and rare without them. As particles of denser aether interacted with the rare aether, they were attracted back to the dense aether much like cooling vapors of water are drawn back to each other to form water.[25] In the *Principia*, he attempts to explain the elasticity and movement of aether by relating aether to his static model of fluids. This elastic interaction is what caused the pull of gravity to take place, according to this early theory, and allowed an explanation for action at a distance instead of action through direct contact. Newton also explained this changing rarity and density of aether in his letter to Robert Boyle in 1679.[25] He illustrated aether and its field around objects in this letter as well and used this as a way to inform Robert Boyle about his theory.[26] Although Newton eventually changed his theory of gravitation to one involving force and the laws of

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motion, his starting point for the modern understanding and explanation of gravity came from his original aether model on gravitation.[27]

The Dirac sea [Citation from https://en.wikipedia.org/wiki/Dirac_sea]
The **Dirac sea** is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930[1] to explain the anomalous negative-energy quantum states predicted by the Dirac equation for relativistic electrons.[2] The positron, the antimatter counterpart of the electron, was initially conceived of like a hole in the Dirac sea, well before its experimental discovery in 1932.[nb 1]

Vacuum energy [Citation from https://en.wikipedia.org/wiki/Vacuum_energy]
Vacuum energy is underlying background energy that exists in space throughout the entire Universe. One contribution to the vacuum energy may be from virtual particles, which are thought to be particle pairs that blink into existence and then annihilate in a timespan too short to observe. Their behavior is codified in Heisenberg's energy-time uncertainty principle. Still, the exact effect of such fleeting bits of energy is difficult to quantify. The vacuum energy is a particular case of zero-point energy that relates to the quantum vacuum.[1]

The effects of vacuum energy can be experimentally observed in various phenomena such as spontaneous emission, the Casimir effect, and the Lamb shift, and are thought to influence the behavior of the Universe on cosmological scales. Using the upper limit of the cosmological constant, the vacuum energy of free space has been estimated to be 10^{-9} joules (10^{-2} ergs) per cubic meter.[2] However, in both quantum electrodynamics (QED) and stochastic electrodynamics (SED), consistency with the principle of Lorentz covariance and with the magnitude of the Planck constant requires it to have a much larger value of 10^{113} joules per cubic meter.[3][4] This vast discrepancy is known as the vacuum catastrophe.

Implications: Vacuum energy has many consequences. In 1948, Dutch physicists Hendrik B. G. Casimir and Dirk Polder predicted the existence of a tiny attractive force between closely placed metal plates due to resonances in the vacuum energy in the space between them. This is now known as the Casimir effect and has since been extensively experimentally verified. It is therefore believed that the vacuum energy is "real" in the same sense that more familiar conceptual objects such as electrons, magnetic fields, etc., are real. However, alternative explanations for the Casimir effect have since been proposed.[5]

Other predictions are harder to verify. Vacuum fluctuations are always created as particle-antiparticle pairs. The creation of these virtual particles near the event horizon of a black hole has been hypothesized by physicist Stephen Hawking to be a mechanism for the eventual "evaporation" of black holes.[6] If one of the pairs is pulled into the black hole before this, then the other particle becomes "real" and energy/mass is essentially radiated into space from the black hole. This loss is cumulative and could result in the black hole's disappearance over time. The time required is dependent on the mass of the black hole (the equations indicate that the smaller the black hole, the more rapidly it evaporates) but could be on the order of 10^{100} years for large solar-mass black holes.[6]

The vacuum energy also has important consequences for physical cosmology. General relativity predicts that energy is equivalent to mass, and therefore if the vacuum energy is "really there", it should exert a gravitational force. Essentially, non-zero vacuum energy is expected to contribute to the cosmological constant, which affects the expansion of the universe.[citation needed] In the special case of vacuum energy, general relativity stipulates that the gravitational field is proportional to ρ+3p (where ρ is the mass-energy density, and p is the pressure). Quantum theory of the vacuum further stipulates that the pressure of the zero-state vacuum energy is always negative and equal in magnitude to ρ. Thus, the total is ρ+3p = ρ-3ρ = -2ρ, a negative value. If indeed the vacuum ground state has non-zero energy, the calculation implies a repulsive gravitational field, giving rise to acceleration of the expansion of the universe,[citation needed]. However, the vacuum energy is mathematically infinite without renormalization, which is based on the assumption that we can only measure energy in a relative sense, which is not true if we can observe it indirectly via the cosmological constant.[citation needed]

The existence of vacuum energy is also sometimes used as a theoretical justification for the possibility of free-energy machines. It has been argued that due to the broken symmetry (in QED), free energy does not violate conservation of energy since the laws of thermodynamics only apply to equilibrium systems. However, the consensus amongst physicists is that this is unknown, as the nature of vacuum energy remains an unsolved problem.[7] In particular, the second law of thermodynamics is unaffected by the existence of vacuum energy.[citation needed] However, in Stochastic Electrodynamics, the energy density is taken to be a classical random noise wave field which consists of real electromagnetic noise waves propagating isotopically in all directions. The energy in such a wave field would seem to be accessible, e.g., with nothing more complicated than a directional coupler.[citation needed] The most obvious difficulty appears to be the spectral distribution of the energy, which compatibility with Lorentz invariance requires to take the form Kf^3, where K is a constant and f denotes frequency.[3][8] It follows that the energy and momentum flux in this wave field only becomes significant at extremely short wavelengths where directional coupler technology is currently lacking.[citation needed]

Since we know that (2.4-4) is correct force law, and since we have numerous indications that mass and surrounding velocity field are mutually coupled, we can (hypothetically) imagine that all masses in our Universe have certain (somewhat hidden), internal ***electric-charges-weak-dipole-polarization*** produced by global, holistic, and universal, always present orbital motions around dominant center of masses (or galactic center...). Mentioned dipole polarization is propagating or spreading from a local dominant center of mass (or galaxy center) over all surrounding and orbiting bodies and other astronomic objects, making that all the captured masses are mutually attracting (because of omnidirectional, multi-helical and properly aligned or arranged dipole polarizations of mutually opposed electromagnetic dipoles; -see more in Chapter 3. especially around equations (3.5-a), and in Chapter 8.).

For instance, if $\mathbf{v_1 \cong v_2 \cong v = \frac{1}{\sqrt{\varepsilon\mu}} = c}$, we could evaluate and exploit the validity of the following force relations between two masses that are electrically polarized like mutually opposed dipoles,

$$\mathrm{Lim}\left(F_{1,2}\right)_{v\to c} = \mathrm{Lim}\left[\frac{\mu}{4\pi}\frac{(q_1v_1)(q_2v_2)}{r^2}\right]_{v\to c} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} = G\frac{m_1m_2}{r^2},$$

$$\left[\begin{array}{l} F_{1,2} = \frac{\mu}{4\pi}\frac{q_1q_2v^2}{r^2} = \frac{\mu}{4\pi}\frac{k^2m_1m_2v^2}{r^2} = G\frac{m_1m_2}{r^2} \Rightarrow \frac{\mu}{4\pi}k^2v^2 = G \Rightarrow \\ v = \frac{2}{k}\sqrt{\frac{\pi G}{\mu}} = \frac{2}{k}\sqrt{\frac{\pi\varepsilon G}{\varepsilon\mu}} = c\frac{2\sqrt{\pi\varepsilon G}}{k} = \frac{1}{\sqrt{\varepsilon\mu}} = c \Rightarrow k = 2\sqrt{\pi\varepsilon G} = \text{const.}, \\ q_{1,2} = km_{1,2} = m_{1,2}\sqrt{4\pi\varepsilon G} \Rightarrow q = km = m\sqrt{4\pi\varepsilon G} \Rightarrow \boxed{(qv) = m\sqrt{\frac{4\pi G}{\mu}}} \end{array}\right]. \qquad (2.4\text{-}4.1)$$

Here we assume that ***one-side-electric-dipole-charge*** is directly proportional to the mass of the object in question, $\mathbf{q_{1,2} = km_{1,2}}$, and this way we can equally use Newton or Coulomb force

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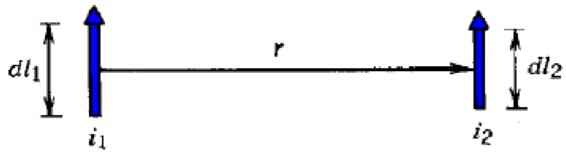
law for presenting the same attractive force between two masses (see more in Chapter 3.). This is on some way (with a lot of creative imagination) insinuating or evoking memories about exotic concepts and speculations in relation to Nikola Tesla's ***Ether***, ***Radiant Energy*** flow and his ***Dynamic Gravity*** theory, since $v_1 \cong v_2 \cong v = \frac{1}{\sqrt{\varepsilon\mu}} = c$ could be the speed of some closely related electromagnetic waves (see more of similar elaborations around equations from (2.4-4) to (2.4-10)).

Another indicative $1/r^2$ force law applicable on familiar or analogically extended situations, showing that Coulomb and Newton laws are mutually equivalent, both having only an electromagnetic nature (see the picture below), here composed of two electric charges (creating two elementary current elements, i_1 and i_2) coincidently moving as mutually parallel, and in the same direction is,

$$\left|dF_{1,2}\right| = \frac{\mu}{4\pi}\frac{(i_1dl_1)\cdot(i_2dl_2)}{r^2} (=) G\frac{dm_1\cdot dm_2}{r^2} \Rightarrow dm_1\cdot dm_2 = \frac{\mu}{4\pi G} i_1\cdot i_2\cdot dl_1\cdot dl_2 \qquad (2.4\text{-}4.2)$$



Now, we can demonstrate the intrinsic relations between certain mass, its electric charge, and its velocity (see more of familiar elaborations in Chapter 3.). Again, like in (2.4-4.1), but now starting from (2.4-4.2), we will get the same result $\boxed{(qv) = m\sqrt{\frac{4\pi G}{\mu}}}$ as below (in (2.4-4.3)),

$$\left(dm_1\cdot dm_2 = \frac{\mu}{4\pi G} i_1\cdot i_2\cdot dl_1\cdot dl_2\right) \Rightarrow$$

$$\text{for}\begin{pmatrix} m_1 = m_2 = m \\ l_1 = l_2 = l\end{pmatrix} \Rightarrow (dm)^2 = \frac{\mu}{4\pi G} i^2\cdot(dl)^2$$

$$dm = \sqrt{\frac{\mu}{4\pi G}} idl = \sqrt{\frac{\mu}{4\pi G}}\frac{dq}{dt}dl = \sqrt{\frac{\mu}{4\pi G}}\frac{dl}{dt}dq =$$

$$= \sqrt{\frac{\mu}{4\pi G}} vdq \le \sqrt{\frac{\mu}{4\pi G}} cdq = \frac{dq}{\sqrt{4\pi\varepsilon G}} \Rightarrow$$

$$\Rightarrow m = (qv)\sqrt{\frac{\mu}{4\pi G}} \le \frac{q}{\sqrt{4\pi\varepsilon G}} \Rightarrow \boxed{(qv) = m\sqrt{\frac{4\pi G}{\mu}}}, G = \frac{\mu}{4\pi}\left(\frac{qv}{m}\right)^2 \le \frac{1}{4\pi\varepsilon}\left(\frac{q}{m}\right)^2 \qquad (2.4\text{-}4.3)$$

$$\Rightarrow \frac{q}{m} = \frac{1}{v}\sqrt{\frac{4\pi G}{\mu}} \ge \sqrt{4\pi\varepsilon G} = 8.616032252\cdot 10^{-11}\left[\frac{C}{kg}\right], \; v \le \frac{1}{\sqrt{\varepsilon\mu}} = c,$$

$$q \ge m\cdot\sqrt{4\pi\varepsilon G}.$$

Anyway, mass, and *electric charges direct proportionality* (as in (2.4-4.3)) should also be the consequence of the total electric (or electromagnetic) charge conservation, as well as it should be (indicatively and intuitively) directly related to energy and mass conservation (of course not to understand everything literally). This is in the background regarding analogy between Newton and Coulomb force.

Citation from: https://energyeducation.ca/encyclopedia/Law_of_conservation_of_charge

Law of conservation of charge

Law of conservation of charge says that the net charge of an isolated system will always remain constant.[1] This means that any system that is not exchanging mass or energy with its surroundings will never have a different total charge at any two times. For example, if two objects in an isolated system have a net charge of zero, and one object exchanges one million electrons to the other, the object with the excess electrons will be negatively charged and the object with the reduced number of electrons will have a positive charge of the same magnitude. The total charge of the system has not and will never change.[1]
This concept is important for all nuclear reactions—alpha decay, beta decay, gamma decay, etc.— because it allows scientists to predict the composition of the final product in the reaction, shown in Figure 1.[2]
Charged particles are allowed to be created or destroyed, as long as the net charge before and after the creation/destruction stays the same. Therefore this must happen with oppositely charged pairs of matter and anti-matter.[1]

As we know from electromagnetics, parallel wires with current flowing in the same direction attract each other. This is also, on some way analogically valid for attractions between two masses, meaning that any of two (or many) neighbor's masses moving in the same direction, as a group, or mechanical system, having certain center-of-mass velocity, will mutually attract. Often, we do not see, measure, or conceptualize mentioned center-of-mass velocity and related linear motion, since this should be, for instance, specified concerning an external, dominant, larger inertial frame (like local galaxy center), and we are just dependent observers, or part of the two-masses reference system. Such hidden or external motion is producing internal electric and magnetic dipoles polarizations, creating electrical currents in the same direction, and such currents are creating magnetic fields that are mutually attracting, as presented in (2.4-4.1) - (2.4-4.3). Consequently, all masses in our universe are in some holistic synchronized motion, oscillating, orbiting, and spinning about some local or global centers of cosmic masses (or galaxies).

The significant conclusion here is that matter, and mass in our Universe is in perpetual motion (most probably in global helical, holistic rotation and spinning), and that mass and specific electric charge, and associated spin effects or electromagnetic angular moments are always mutually coupled and interrelated. Mentioned electromagnetic and electromechanical couplings are on some way exchangeable and reciprocally proportional, depending on associated matter-waves communications, involved energy, and different mechanical and electromagnetic moments, dipole and multipole polarizations, and energy-mass (standing waves) packing and formatting (see more in Chapters 3. And 8.). Similar or identical remarks and conclusions about matter content and its nature we will have later in this chapter, around equations from (2.4-4) until (2.4-8), and in the chapter 5., around equations (5.2) and (5.2.2). This is, on some imaginative way, implicitly and creatively giving grounds to Nikola Tesla's visions about ***Ether***, ***Radiant Energy, Death Ray Gun,*** *and* ***Dynamic Gravity*** concepts (see more in [97] and [117]). Tesla performed experiments with remarkably high alternating voltages between specially assembled electrodes, where he was able to evacuate gas (or mass of a gas content) from an open-end glass tube, thanks to the *mass-electric-charge coupling-relations* like in (2.4-4.3).

There is also a big chance that Casimir effect (or force) presents a manifestation of Tesla's radiant energy flow (see [103] and [104]).

Citation [121]; -What is gravity? From Ralph Sansbury: *http://www.holoscience.com/wp/electric-gravity-in-an-electric-universe/ :*

"Gravity is due to radially oriented electrostatic dipoles inside the Earth's protons, neutrons, and electrons. [18] The force between any two aligned electrostatic dipoles varies inversely as the fourth power of the distance between them and the combined force of similarly aligned electrostatic dipoles over a given surface is squared. The result is that the dipole-dipole force, which varies inversely as the fourth power between co-linear dipoles, becomes the familiar inverse square force of gravity for extended bodies. The gravitational and inertial response

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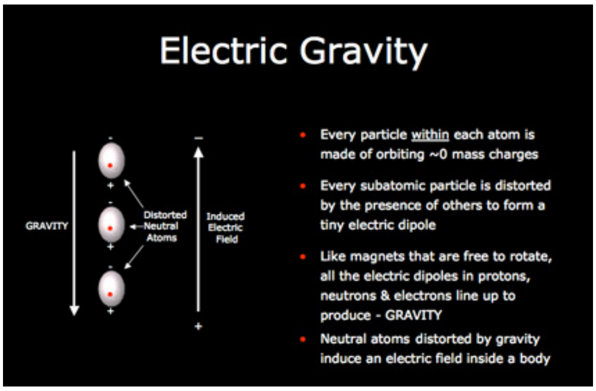
of matter can be seen to be due to the same cause. The puzzling extreme weakness of gravity (one thousand trillion trillion trillion trillion times less than the electrostatic force) is a measure of the minute distortion of subatomic particles in a gravitational field.



Celestial bodies are born electrically polarized from a plasma z-pinch or by core expulsion from a larger body.

The 2,000-fold difference in mass of the proton and neutron in the nucleus versus the electron means that gravity will maintain charge polarization by offsetting the core within each atom (as shown). The mass of a body is an electrical variable—just like a proton in a particle accelerator. Therefore, the so-called gravitational constant—'G' with the peculiar dimension $[L]^3/[M][T]^2$, is a variable! That is why 'G' is so difficult to pin down.

Antigravity?

Conducting metals will shield electric fields. However, the lack of movement of electrons in response to gravity explains why we cannot shield against gravity by simply standing on a metal sheet. As an electrical engineer wrote, "we [don't] have to worry about gravity affecting the electrons inside the wire leading to our coffee pot." [19] If gravity is an electric dipole force between subatomic particles, it is clear that the force "daisy chains" through matter regardless of whether it is conducting or non-conducting. Sansbury explains:

"...electrostatic dipoles within all atomic nuclei are very small, but all have a common orientation. Hence their effect on a conductive piece of metal is less to pull the free electrons in the metal to one side toward the center of the earth but to equally attract the similarly oriented electrostatic dipoles inside the nuclei and free electrons of the conductive piece of metal." [20]

This offers a clue to the reported 'gravity shielding' effects of a spinning, superconducting disk.[21] *Electrons in a superconductor exhibit a 'connectedness,' which means that their inertia is increased. Anything that interferes with the ability of the subatomic particles within the spinning disk to align their gravitationally induced dipoles with those of the earth will exhibit antigravity effects.*

Despite many experiments demonstrating antigravity effects, no one has been able to convince scientists attached to general relativity that they have been able to modify gravity. This seems to be a case of turning a blind eye to unwelcome evidence. Support for antigravity implicitly undermines Einstein's theory".[22]

Citation (below): See Ref. [121], Raymond HV Gallucci: *http://www.holoscience.com/wp/electric-gravity-in-an-electric-universe/*, "Electric Gravity in an Electric Universe." https://principia-scientific.org/electromagnetic-gravity-examination-of-the-electric-universe-theory/

One of the prime tenets of Electric Universe Theory is that electromagnetism dominates over gravity throughout the universe, given that 99% of all matter (ignoring the fictitious Dark Matter and Energy) is plasma and that electromagnetism is 39 orders of magnitude stronger than gravity.

Nonetheless, gravity is far from dismissed. In fact, Wal Thornhill, lead physicist for The Electric Universe, has developed a theory for gravity as a manifestation of an electromagnetic phenomenon that ever so slightly causes distortion within atoms such that a dipole is created that could account for gravitational force.

This paper summarizes Thornhill's theory and examines it mathematically, concluding that it is at least plausible. The Electric Universe (EU) theory postulates that gravity is just another manifestation of electromagnetism, albeit at an almost inconceivably lower force (~10^{-39} as strong).

This paper examines the EU conjecture about an electromagnetic basis for gravity based on simplified mathematical analysis for an idealized arrangement of three hydrogen atoms. Results suggest that the possibility of an electromagnetically induced distortion of a hydrogen atom to create an atomic dipole is at least plausible.

"Gravity is due to radially oriented electrostatic dipoles inside the Earth's protons, neutrons, and electrons. The force between any two aligned electrostatic dipoles varies inversely as the fourth power of the distance between them and the combined force of similarly aligned electrostatic dipoles over a given surface is squared. The result is that the dipole-dipole force, which varies inversely as the fourth power between co-linear dipoles, becomes the familiar inverse square force of gravity for extended bodies. The gravitational and inertial response of matter can be seen to be due to the same cause. The puzzling extreme weakness of gravity (one thousand trillion trillion trillion times less than the electrostatic force) is a measure of the minute distortion of subatomic particles in a gravitational field. Celestial bodies are born electrically polarized from a plasma z-pinch or by core expulsion from a larger body. The 2,000-fold difference in mass of the proton and neutron in the nucleus versus the electron means that gravity will maintain charge polarization by offsetting the nucleus within each atom (as shown). The mass of a body is an electrical variable — just like a proton in a particle accelerator. Therefore, the so-called gravitational constant — 'G' with the peculiar dimension $[L]^3/[M][T]^2$, is a variable! That is why 'G' is so difficult to pin down."

This exercise attempted to interject some mathematics, greatly simplified, into the paradigm of the EU theory that gravity can be attributed to an electromagnetic effect, albeit almost inconceivably smaller, due to the distortion of atoms by their neighbors into electric dipoles. While we have not attempted to address the mathematics that would be involved in explaining the 10^{39} factor difference between the respective strengths of these forces, the possibility of an electromagnetically induced distortion to create an atomic dipole appears at least plausible.

For gravitation, rotation and mechanics related fields and forces the most important are linear moment p and angular moment L. Surprisingly, we do not find an analogical indication that a static mass m should be the primary and unique source of gravitation, but somehow intrinsically vibrating mass should produce effects of Gravitation (see more in the first chapter about analogies). Practical predictive meaning of such analogical revelations is that I. Newton and A. Einstein theories about Gravitation should be one day significantly updated. The other brainstorming insight here is that electric charges (or electrons and protons) should anyway be kind of dynamic, motional energy states (like mechanical and electromagnetic moments), since all other mutually analogical mechanical and electromagnetic moments and flux entities are also kind of moving or dynamic states (see T.1.3 in the first chapter). In Physics, presently we still wrongly consider electric charges as stable, or fixed and static parameters (as number of Coulombs). Consequently, we could expect certain continuous electromagnetic energy-exchange (or natural flow of N. Tesla Radiant energy) between any of positive and negative charges, or between conveniently aligned electromagnetic dipoles, this way creating necessary background for explanation of Gravitation, especially if we revitalize the existence and meaning of ether as fine particles fluid, which is the carrier of electromagnetic waves and energy.

Another point of view is giving more weight to relations between gravitation and internal spatially distributed magnetic moments. Every macro mass (meaning its content, such as molecules, atoms, elementary particles, interatomic states), beside electrically charged entities, has many spinning and magnetic moment states internally, spatially, or volumetrically distributed. We know that the nature of electric and electrostatic charges is to tend to be on an external surface distributed, but internal magnetic entities of masses do not have such tendency (and stay captured by the internal mass structure). If we now consider that all masses in our universe are globally (macrocosmically) rotating around galactic and other dominant centers of masses, what presents accelerated motions, we will come again to the conclusion that mentioned electromagnetic moments (or some elementary magnets) would get a certain level of organized or properly aligned magnetic dipoles polarization. Such magnetic elements, obviously having (in average) very weak alignment, are mutually experiencing the same Coulomb force as analogically known in electrostatics and Newton gravity theory. This is directly indicating that gravitational force (between seemingly neutral, or non-charged masses) could be dominantly related to the very weak Coulomb force between mentioned masses with internally (and conveniently) distributed magnetic elements, meaning gravitation is being misinterpreted as certain new natural force (see more in T.2.2-1, T.2.8. , [33] and [36]).

Citation took from the European Space Agency. "Anti-gravity Effect? Gravitational Equivalent Of A Magnetic Field Measured In Lab." ScienceDaily. ScienceDaily, 26 March 2006. <www.sciencedaily.com/releases/2006/03/060325232140.htm>

Just as a moving electrical charge creates a magnetic field, so a moving mass generates a gravitomagnetic field. According to Einstein's Theory of General Relativity, the effect is virtually negligible. However, Martin Tajmar, ARC Seibersdorf Research GmbH, Austria; Clovis de Matos, ESA-HQ, Paris; and colleagues have measured the effect in a laboratory.

....

It demonstrates that a superconductive gyroscope can generate a powerful gravitomagnetic field, and is, therefore, the gravitational counterpart of the magnetic coil. Depending on further confirmation, this effect could form the basis for a new technological domain, which would have numerous applications in space and other high-tech sectors" says de Matos. Although just 100 millionths of the acceleration due to the Earth's gravitational field, the measured field is a surprising one hundred million trillion times larger than Einstein's General Relativity predicts. Initially, the researchers were reluctant to believe their own results. ....

Since moving electric charges are creating electrical currents magnetic fields, and anyway electric and magnetic entities with spinning and dipole (or multi-pole) moments are coincidently populating every macro-mass internal content, we should conclude that masses are on some way directly proportional to their internal content of electromagnetic entities (meaning to electric and magnetic micro domains' fluxes and/or charges). Here, we are on some descriptive and intuitive way justifying that mass product in Newton force law is directly proportional to involved electric charges product, or to the outcome of involved, internally distributed micro-magnetic domains of magnetic field fluxes. However, at the same time, we should have specific background, global and holistic mass rotation, or mass-energy flow, since mass and electric charges are not directly analogical, but the mass in motion and electrical charges are mutually analogical. This is explaining why and how it is possible to involve typically electromagnetic Coulomb forces in explaining force of gravitation. Very much familiar with such concepts are conclusions of William Hooper, [112], regarding natural diversity of electric and magnetic fields, related to masses in different relative motions, what is effectively manifesting as what we consider as gravitational force (and showing that gravitation has an electromagnetic origin. See more in [89]).

....

Citation from: (4) Quora , Viktor T. Toth, IT pro, part-time physicist 2y
Can you explain dark matter to a non-physicist?

Galaxies are collections of stars that are held together by their mutual gravity.

However, galaxies spin. And they spin too fast. When we calculate how much gravity is there, given the stuff that we see, there just isn't enough. Newton's law of gravitation tells us that there is not enough gravitational force to keep the stars together. Galaxies should fly apart.

They don't.

Unless we got everything completely wrong, there can be two fundamental reasons for this: Either we misunderstand the law of gravity, or there is more matter in a galaxy than the eye can see.

We are very reluctant to conclude that we misunderstand the law of gravity, because it works so well everywhere else. It is more likely, then, that there is simply stuff that we do not see.

We call this stuff "dark matter" (dark, because we do not see it; but it really should be called invisible or transparent matter, as it doesn't exactly cast a shadow either, it has no effect on light whatsoever.) And we keep looking for it, and theorists keep inventing new theories as to what this dark matter might be.

So far, we have not been able to establish the existence of dark matter independently from their gravity. So the door is still open for the alternative, modified theories of gravity. On the other hand, it so happens that presuming the existence of dark matter also improves our mathematical models of the cosmos as a whole. That is a strong point in favor of the dark matter idea.

But we won't know until and unless we have direct observational evidence or, alternatively, until we have a convincing modified theory of gravity, confirmed through testable predictions.

Citation: Dark matter. *From Wikipedia, the free encyclopedia; - https://en.wikipedia.org/wiki/Dark_matter*
*"**Dark matter** is a theorized form of matter that is thought to account for approximately 80% of the matter in the universe, and about a quarter of its total energy density. The majority of dark matter is thought to be non-baryonic in nature, possibly being composed of some as-yet-undiscovered subatomic particles.[note 1] Dark matter has not been directly observed, but its presence is implied in a variety of astrophysical observations, including gravitational effects that cannot be explained unless more matter is present than can be seen. For this reason, most experts think dark matter to be ubiquitous in the universe and to have had a strong influence on its structure and evolution. The name dark matter refers to the fact that it does not appear to interact with visible electromagnetic radiation, such as light, and is thus invisible (or 'dark') to the entire electromagnetic spectrum, making it extremely difficult to detect using normal astronomical equipment.[1] The primary evidence for dark matter is that calculations show that many galaxies would fly apart instead of rotating, or would not have formed or move as they do if they did not contain a large amount of dark matter.[2] Other lines of evidence include observations in gravitational lensing,[3] from the cosmic microwave background, from astronomical observations of the observable universe's current structure, from the formation and evolution of galaxies, from the mass location during galactic collisions,[4] and from the motion of galaxies within galaxy clusters".*

....

For instance, based on assumptions and concepts from (2.4-1) - (2.4-5), (2.4-4.1), (2.4-4.2) and (2.4-4.3), we could hypothetically exercise that within spiral and rotating (or spinning) galaxies, rotating messes are getting additionally electrically or electromagnetically charged and polarized, meaning that involved central force between two masses (considered as a binary system) would have one force component being the Newton force of gravitation, and an additional force component being the Coulomb type of attraction, based on involved (free-standing, non-compensated) electric charges (or dipoles), such as,

$$\mathbf{F_{1,2} = G\frac{m_1 m_2}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}}$$

Since, based on the system of analogies established in the first chapter of this book, electric charges are analogue to linear mechanical moments, we will have,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$F_{1,2} = G\frac{m_1m_2}{r^2} + g\frac{m_1v_1 \cdot m_2v_2}{r^2} = G\frac{m_1m_2}{r^2} + g\frac{m_1\omega_1r_1 \cdot m_2\omega_2r_2}{r^2} \cong$$
$$\cong G\frac{m_1m_2}{r^2} + g\frac{m_1m_2\omega^2r^2}{r^2} \cong G\frac{m_1m_2}{r^2} + g\omega^2m_1m_2 \cong m_1m_2\left(\frac{G}{r^2} + g\omega^2\right),$$
$$G, g\ (=)\ \text{constants}$$

$$\left[\begin{array}{l} v = \frac{2}{k}\sqrt{\frac{\pi G}{\mu}} = \frac{2}{k}\sqrt{\frac{\pi\varepsilon G}{\varepsilon\mu}} = c\frac{2\sqrt{\pi\varepsilon G}}{k} = \frac{1}{\sqrt{\varepsilon\mu}} = c \Rightarrow k = 2\sqrt{\pi\varepsilon G} = \text{const.}, \\ q_{1,2} = km_{1,2} = m_{1,2}\sqrt{4\pi\varepsilon G} \Rightarrow q = km = m\sqrt{4\pi\varepsilon G} \Rightarrow \boxed{(qv) = m\sqrt{\frac{4\pi G}{\mu}}} \\ \omega_1r_1 \cdot \omega_2r_2 \cong \omega^2r^2 = v^2,\ v = \omega r \end{array}\right].$$

This new, rotation-dependent force $F_{1,2} \cong m_1m_2\left(\frac{G}{r^2} + g\omega^2\right)$, for small rotating velocity, will be equal to the Newton force of gravitation. Conceptualizing this way, we do not need to assume or invent any dark matter, mass, or energy. Instead of such "dark and naive speculations", it would be recommendable to conceptualize better the meaning of electromagnetic phenomenology and interactions within mass-energy formatting and packing. Of course, we could additionally improve and refine here hypothesised ideas, but the concept about the electromagnetic reasons for increased central (gravitational or Coulomb-Newton) force is already paved on a simplified, approximated, and certainly an indicative way.

"https://cosmosmagazine.com/science/introducing-the-amazing-concept-of-gravito-electromagnetism/ , 30 August 2021/, Robyn Arianrhod, **Amazing gravito-electromagnetism!**

How can mathematics analogies shed light on reality?

Originally published by Cosmos as Amazing gravito-electromagnetism!

Today this so-called "gravito-electromagnetism", or GEM for short, is generally treated mathematically via the "weak field" approximation to the full GR equations – simpler versions that work well in weak fields such as that of the earth.

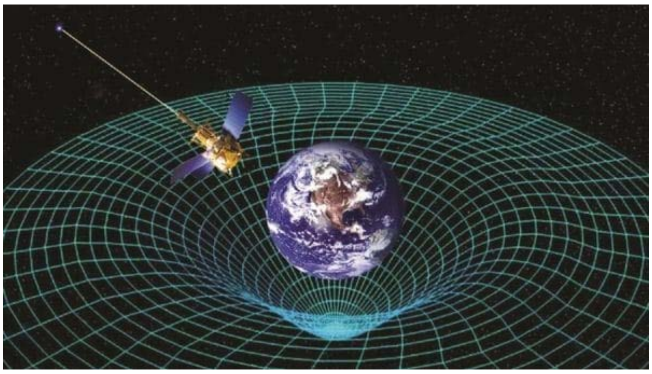
It turns out that the mathematics of weak fields includes quantities satisfying equations that look remarkably similar to Maxwell's. The "gravito-electric" part can be readily identified with the everyday Newtonian downward force that keeps us anchored to the earth. The "gravito-magnetic" part, however, is something entirely unfamiliar – a new force apparently due to the rotation of the earth (or any large mass).

It's analogous to the way a spinning electron produces a magnetic field via electromagnetic induction, except that mathematically, a massive spinning object mathematically "induces" a "dragging" of space-time itself – as if space-time were like a viscous fluid that's dragged around a rotating ball. (Einstein first identified "frame-dragging", a consequence of general relativity elaborated by Lense and Thirring.)

But how far can such mathematical analogies be pushed?

Is "gravito-magnetic induction" real? If it is, it should show up as a tiny wobble in the orbit of satellites, and – thanks also to the "geodetic" effect, the curving of space-time by matter – as a change in the direction of the axis of an orbiting gyroscope. (The latter is analogous to the way a magnetic field generated by an electric current changes the orientation of a magnetic dipole.)

Finally, after a century of speculation, answers are unfolding. Independent results from several satellite missions – notably Gravity Probe B, LAGEOS, LARES, and GRACE – have confirmed the earth's geodetic and frame-dragging effects to varying degrees of precision. For frame-dragging, the best agreement with GR has been within 0.2%, with an accuracy of 5%, but astronomers expect that a new satellite (LARES 2), to be launched in 2022, will, with data from LAGEOS, give an accuracy of 0.2%.



Results from satellite missions such as Gravity Probe B have confirmed the Earth's geodetic and frame-dragging effects. Credit: Gravity Probe B team/Stanford/NASA

More accurate results will provide more stringent tests of GR, but astrophysicists have already taken gravito-magnetism on board. For instance, it suggests a mechanism to explain the mysterious jets of gas that have been observed spewing out of quasars and active galactic nuclei. Rotating supermassive black holes at the heart of these cosmic powerhouses would produce enormous frame-dragging and geodetic effects. A resulting gravito-magnetic field analogous to the magnetic field surrounding the two poles of a magnet would explain the alignment of the jets with the source's north-south axis of rotation." ....

Author's comments:

If we consider that only a force of gravitation, based on masses attraction, is the dominant force that should keep rotating galaxies structurally compact, inventing or postulating an invisible dark matter (or its equivalent mass) will help to increase hypothetically missing gravitational central force, until a necessary level to balance centrifugal forces of rotation with such imaginative and postulated, total gravitational attraction.

We could also merely consider that gravitation is the manifestation of electromagnetic interactions (related to convenient electromagnetic dipoles polarization, thanks to the global and holistic rotation within our universe). Electromagnetic Coulomb and Lorentz forces are anyway significantly stronger than the force of gravitation, and this could be an explanation of missing central force component (see more about mutually coupled and self-regenerative electromagnetic fields and forces in Chapter 3.). This effectively means that an additional, or needed central attractive force, that is missing to explain observed structural integrity of rotating spiral galaxies, is not coming from an additional, non-detectable and invisible dark energy and/or dark mass. It is merely coming from conventional, electromagnetic, Coulomb and Lorentz type of electromagnetic forces acting within the configuration of mentioned galaxy, being specifically and naturally structured, or electromagnetically polarized, this way creating convenient equivalent, macro electric circuits, produced by the same galaxy rotation, and by other kinematic factors, eventually delivering necessary or missing part of a central force (see more in [121], Raymond HV Gallucci, Electromagnetic Gravity? Examination of the Electric Universe Theory).

The intuitive idea favored here is that everything valid for static and moving electric charges and magnetic and electric dipoles (like Coulomb law, Lorentz force, induction laws…) could be (within the first steps of better conceptual understanding) analogically extended to familiar effects between masses and mass moments, indicating that Gravitation essentially has an electromagnetic origin. The other stepstone ingredient here is that mass and electric charge are not mutually and directly or analogically replaceable, since an electric charge is analogically and directly comparable to linear, angular, and magnetic moments (meaning that electric charges are not at all static and fixed entities). Effectively, macro masses, or agglomerations of atoms and molecules, are dominantly presenting specific and well-packed (flexible and deformable), dynamic and motional electromagnetic entities with magnetic moments properties. Such internal structure of rotating and spinning constituents of atoms and macro masses is creating external effects of gravitation (see more in Chapters 3. and 8.).

In addition, it is good to know that High Power Mechanical, ultrasonic or acoustical energy, moments, forces, oscillations and vibrations, or audio signals and music, can be created and transferred by applying different signal-modulating techniques on laser beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); - See more in Chapter 10. under (10.2-2.4), and in literature references from [133] until [139].

2.2.1. WHAT THE GRAVITATION REALLY IS

In this book, favorable concept (based on Mobility system of analogies from the first chapter) is that only static ***masses are not enough to create field and force of gravitation. Mutually rotating, and electromagnetically and mechanically, or by standing matter waves coupled and interacting, motional masses with internal dipoles and free-standing electromagnetic charges, electromechanical moments, and associated linear and angular or spinning moments, are real sources of gravitation.*** *In other words, all atoms, masses, and astronomic objects in our universe are communicating by exchanging mutually synchronized matter-waves and balancing (or conserving) involved electromechanical moments. The real sources of such communications and interactions are atoms or specific fields and forces inside atoms, extending outside of atoms. This is producing effects of gravitational attraction in spatial nodal zones of created standing matter-waves formations, where masses naturally have their stable places or orbits. States of linear, angular, oscillatory, spinning, and orbital motions are anyway causally connected and interacting with surrounding electric and magnetic fields, dipoles, and electric and mechanical moments. Linear and angular forces and torques are interfering or superimposing with the gravitational force.*

Consequently, gravitational attraction we could alternatively explain as the mass-agglomerating force around nodal zones of standing matter waves, what are also analogically well-known resonant effects around nodal zones of half-wavelength resonators in cases of acoustic levitation (as known in acoustics and ultrasonic technique; *-see more in [150] and [151]****).*** *Practically, masses, particles, matter-states (or self-stabilized matter-wave packets) composed of (mechanical and electromagnetic) energy-moments states, are fixed to nodal zones of involved cosmic, standing matter-waves. Such standing-wave formations around and between masses are effectively creating gravitational field and force effects (see [99], from Konstantin Meyl). We can mathematically explain and formalize on several of well-known ways (that are not-contradictory to here favoured concept of standing waves) all kind of natural forces, including Gravitation, as we can find in Chapter 10. Under "10.02 Meaning of natural forces".*

Let us first address question ***how we can influence, increase, or decrease already existing gravitational force*** *of certain big object (or big mass). If a small object, mass, mechanical system, or certain small body is in a gravitational-field zone of a much bigger object (or planet), then we already know how we can act against such gravitational force. It is enough to create certain non-uniform, accelerated, non-inertial motional conditions (being linear or angular, rotational, or spinning motions) of a small object (placed there), and existing external force of gravitation (from a small object towards a big object) would be affected (in a positive or negative direction) during the activity of mentioned combined acceleration applied on a small object.*

We should know that inertial, stable, uniform, and stationary motions (including stable periodical motions), can be analogically related both to linear and angular (or circular, rotational, and spinning) motions. ***From Newtonian mechanics we know much more about linear and uniform inertial motions, but something analogical also exist in cases of other stable, periodical, angular, and uniform motions.*** *Mentioned stable, uniform, and stationary motions (which are not producing transitory dynamic force and torque effects, meaning being not accelerated) will bot influence or*

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*modify a preexisting force of gravitation of a big body. That means inertial (non-accelerated), stable and uniform linear, periodical, orbital, and spinning motions will not make perturbations of preexisting (external) gravitational force, but if we create elements of variable accelerations in mentioned motions (of a small body), we will create new, dynamically active force and torque components, which will interfere and superimpose with an external, preexisting gravitational force. Or (for example, we intend to assemble a flying object), we could simplify the same explanation, first considering that any stable and uniform gyroscopic, spinning motion of certain disk tends to stabilize its own spatial position (and orientation) in gravitational conditions around any massive object or planet (and this can be demonstrated on many ways experimentally). That means, any spinning torque (or angular moments) of involved gyroscopes is on some way synchronously interfering with similar vortex or spinning Radiant energy flow emanating from atoms. Then, if we combine such inertial, stationary and stable spinning of a massive disk, and superimpose it with spinning moments of several surrounding, smaller spinning disks (all of them fixed to the same platform or solid frame), massive central disk will serve as a spatial "**level and position stabilizer**" (of such complex object), and other smaller spinning disks (fixed to the same bigger disk), if periodically modulated (producing variable acceleration of rotational speed), and if small gyroscopes are conveniently fixed and oriented, this will produce manageable linear motions (of the total object) in any selected (resulting) direction, based on mechanical moments and involved forces and torques superposition. This way we can create moving and flying objects based on here described gravitational* technology (of course, this is still too simplified and brief description).

*Universally valid laws of "**Action-equal-to-Reaction**", **Induction**, and **Inertia**, are mutually related and analogical, as already known in Mechanics, Gravitation, Electromagnetism, and within everything else what creates dynamic "mass-energy-moments", currents, voltages, linear forces, torques, and energy-flow properties (see about such analogies in the first chapter of this book). There is a lot of space to develop such concepts and situations additionally and analogically (having the same origin), which are the clear manifestations of the united, natural fields theory, being also very much challenging and indicative for understanding, explaining, and manipulating Gravitation.*

*Similar thinking and conclusions can be formulated if we create electromagnetic, electromechanical, electrostatic, and magnetic preconditions, associated to mentioned small, spinning object (regarding relations between linear and spinning or rotating motions). Coulomb and Lorentz forces are always acting in such cases and can be attractive or repulsive. Such forces and torques would be superimposed as vectors with preexisting force of gravitation (of a big mass). Also, every time **when we create evolving non-stationary, dynamic or accelerated electromagnetic conditions for producing effects related to induction laws, or electric current-voltage generation, and for generating Lorentz force effects, we will be able to affect existing external field of Gravitation** (see more about induction laws in Chapter 3.).*

*All our vehicles, airplanes, rockets, electromotors and flying objects are acting against preexisting gravitation on here described way (until reaching some uniform, and inertial motional state), and in such cases, only natural "friction energy dissipation" should be continuously compensated to maintain achieved uniform and inertial motion. That means, **any initial, transient and startup driving of small objects should have linear force and/or torque components (which are accelerated motions), this way being able to change an external gravitational force** (of a big body), until reaching certain stable, uniform, and new inertial, motional state. Of course, here we need to reestablish our understanding of angular inertial motions (including uniform rotations, orbiting, and spinning). **If we have other, electromagnetically, or electromechanically, conveniently***

created, non-uniform, startup conditions, related to a motion of the small object, this way we can create new linear force and torque (vector elements) that are superimposing with the preexisting gravitational force of a big object. *See supporting background about relations between linear and rotational or spinning moments, and between linear force and torque (of the same motional object) around (2.4-5.1), (2,5.1-7), (2.9.1), (2.9.5-7), (2.9.5-8), (2.9.5-9), and T.2.5., from this chapter, and in Chapter 4.1, starting from (4.3), (4.3-0)-d,e,f,g,..,t, (4.3-1), T.4.2.1, Fig.4.1.2…, and later, such as,* $\mathbf{pv = L\omega}$, $\mathbf{p = (\omega / v)\cdot L}$, $\mathbf{F = dp/dt}$, $\mathbf{\tau = dL/dt}$, $\mathbf{p\cdot(dv/dt) + v\cdot F = L\cdot(d\omega/dt) + \omega\cdot\tau}$, $\mathbf{v\cdot F = \omega\cdot\tau}$, $\mathbf{pdv = Ld\omega}$. *To explain this situation better let us imagine that certain motional particle has its kinetic energy,*

$$\mathbf{E_k = \tilde{E} = \frac{pv}{1+\sqrt{1-v^2/c^2}}\Bigg|_{v<<c} \cong \frac{pv}{2} = \frac{mv^2}{2}}. \tag{2.4-4.4}$$

If we now replace the same kinetic energy with an equivalent spinning energy (of the same mass), we will have,

$$\mathbf{E_k = \tilde{E} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{L_s\omega_s}{1+\sqrt{1-v^2/c^2}}\Bigg|_{v<<c} \cong \frac{L_s\omega_s}{2} = \frac{J\omega_s^2}{2} \Rightarrow L_s\omega_s = pv}, \tag{2.4-4.5}$$

where $\mathbf{L_s, \omega_s}$ *are particle spinning moment and angular spinning velocity, and* $\mathbf{p, v}$ *are particle linear moment and linear velocity (this way effectively describing certain spiral or helix-line motion around the axis or line of linear particle propagation). Such spinning moment and spinning velocity could be externally visible and measurable parameters, but here we will conceptualize that mas m internally has many of atomic micro-spinning and orbiting states that are under favorable conditions mutually superimposing, orienting, and aligning. Of course, certain resulting spinning is introduced here mathematically, but we will upgrade such concept to the hypothetical statement saying that elements of Radiant energy spinning and created vortices (emanating from atoms) would (on some wave-particle dualistic way) really appear around any motional mass. This way conceptualizing, we get,*

$$\mathbf{pv = L_s\omega_s \Rightarrow pdv + vdp = \omega_s dL_s + L_s d\omega_s \Leftrightarrow p\frac{dv}{dt} + v\frac{dp}{dt} = L_s\frac{d\omega_s}{dt} + \omega_s\frac{dL_s}{dt} \Rightarrow}$$
$$\mathbf{\Rightarrow \begin{bmatrix} p\frac{dv}{dt} + vF = L_s\frac{d\omega_s}{dt} + \omega_s\tau_s \\ F = \frac{dp}{dt}, \tau_s = \frac{dL_s}{dt} \end{bmatrix} \Rightarrow \begin{bmatrix} p\frac{dv}{dt} = L_s\frac{d\omega_s}{dt} \\ vF = \omega_s\tau_s \end{bmatrix}}. \tag{2.4-4.6}$$

The hypothetical consequence here is that if certain linear force is applied on a moving particle, such particle can also get some angular spinning speed $\mathbf{\omega_s}$, *and certain spinning torque* $\mathbf{\tau_s}$, *aligned with the particle velocity* $\mathbf{v}$ *and with mentioned linear force* $\mathbf{F}$ (and vice – versa). *Now we can generate the following relations,*

$$\frac{vdp}{pdv}=\frac{\omega_s dL_s}{\omega_s dL_s}\Rightarrow\left\{\frac{\left(\frac{dp}{p}\right)}{\left(\frac{dv}{v}\right)}=\frac{\left(\frac{dL_s}{L_s}\right)}{\left(\frac{d\omega_s}{\omega_s}\right)}\Rightarrow\frac{\ln\left|\frac{p}{p_0}\right|}{\ln\left|\frac{v}{v_0}\right|}=\frac{\ln\left|\frac{L_s}{L_{s0}}\right|}{\ln\left|\frac{\omega_s}{\omega_{s0}}\right|}\right\}\Rightarrow \tag{2.4-4.7}$$

$$\Rightarrow\begin{bmatrix}\left|\frac{p}{p_0}\right|=\left|\frac{L_s}{L_{s0}}\right|\\ \left|\frac{v}{v_0}\right|=\left|\frac{\omega_s}{\omega_{s0}}\right|\end{bmatrix}\Rightarrow\begin{bmatrix}p=\left|\frac{p_0}{L_{s0}}\right|L_s\\ v=\left|\frac{v_0}{\omega_{s0}}\right|\omega_s\end{bmatrix},[v_0,\omega_{s0},p_0,L_{s0}](=)\begin{bmatrix}\text{constants presenting}\\ \text{initial conditions}\end{bmatrix}.$$

The most important, significant, and (gravitation related) predictive relations, here are,

$$p=\left|\frac{p_0}{L_{s0}}\right|L_s\ ,v=\left|\frac{v_0}{\omega_{s0}}\right|\omega_s\ ,vF=\omega_s\tau_s\ ,F=\left|\frac{p_0}{L_{s0}}\right|\tau_s\ , \tag{2.4-4.8}$$

indicating conditions how the force of gravitation can be influenced and connected with linear and angular forces, either using a reactive rocket drive, or effects created with accelerated spinning disks. Of course, in the background of just drawn conclusions are laws of conservation of linear and angular moments, including total energy conservation (synchronously, instantly, and coincidently applicable to any motion, meaning that linear, angular, and force of gravitation, should also be on a similar way interdependent and connected). In a case of an ideal, uniform circular or orbiting motion, with orbiting radius R, and orbiting angular velocity $\omega = v / R$, *we can on a similar way add the orbital moment conservation to* (2.4-4.4) - (2.4-4.6), *producing* $L_s\omega_s = pv = p\omega R = m\omega^2 R^2$. *In other cases of combined (but not ideal circular) linear and angular motions, it will be necessary to apply a total angular momentum conservation, which would be a sum of mechanical spinning and orbital moments (of course, based on Lagrangian and Hamiltonian Mechanics). Cases of entanglement connections or couplings should be causally related to the same problematic (as couplings between linear and angular motions; -see more in [36], and in Chapter 10.).*

We also need to specify ***ontological or initial and basic conditions and properties of certain group of atoms, mass or object, relevant for existence of Gravitational force****. Experimental facts, theoretical, imaginative, indicative, and promising concepts concerning understanding of gravitational coupling of masses are:*

1. *Nikola Tesla,* ***[97]****, made number of patents and well operating devices showing existence of* ***remote, wireless electromagnetic couplings and resonance effects*** *between tuned electric (or electromagnetic and electromechanical) resonant circuits, where one of such circuits could be an active source (or an initial driving energy oscillator and emitter), and the other one is only a passive receiver. Later, many Tesla followers, research workers and scientists repeated mentioned experiments, and optimized, generalized, and explained electromagnetic and electromechanical couplings using different resonant circuits configurations, often assuming existence of a specific fluidic coupling medium named as an ether (see more in [98], [99], [144]). N. Tesla was originally inspired by R. Boskovic's "Universal Natural Force" description that is on some indicative and challenging way considering different interatomic and subatomic fields and forces, being smoothly linked (or extended outside atoms) to Newtonian force of gravitation. Submerged in such ideas and concepts, and based on his experimental results with electromagnetic fields, N. Tesla imaginatively described his (many times announced and never finalized or*

published) "Dynamic Gravity theory", which is still challenging, creatively indicative, and exciting (because we know that many of Tesla's visions are proven being correct, producing realistic, correct, and practical results). The most important references regarding Tesla's Dynamic Gravity and R. Boskovic Universal Natural Force can be found on Internet under the following links:
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/

2. *Mechanical (also electromechanical and electromagnetic), **remote resonant couplings and synchronizations are experimentally known in acoustics and ultrasonic technology, between two, or among many (electrically neutral) objects, when being properly in-resonance-tuned**. Such mechanical resonators (mutually coupled while synchronizing) should be submersed in certain fluid (analogically as submersible ultrasonic transducers are communicating in water). It is enough to have only one active solid-body resonator (meaning oscillatory driven resonator or vibrating source) and all other passive, on the same frequency-tuned resonators, submersed somewhere around, in the same fluid (without having any of active driving elements and mutual solid-state connections), will start synchronously resonating, following the active or source-resonator. The same or fully analogical situation is valid for resonant couplings of electromagnetic resonators. As we see, here we also need to have certain coupling fluid (which is in cases of electromagnetic couplings a hypothetical fluid named as an ether). Of course, the very first or essential (divine, cosmic) driving and the oscillatory source could be something else from an external space.*

3. *Similar electromagnetic (and associated electromechanical) couplings, and resonant, tuned, and quantized energy exchanges are assumed to exist within atomic energy states of electrons and atom nucleus. This is what contemporary atom models are explaining and proving on many ways, starting from N. Bohr atom model until Quantum Wave-Mechanics models, meaning that stable electron states (which are sets or resonators) are bi-directionally (or even omni-directionally) communicating, externally and internally, by exchanging photons and producing waving; -see more of similar comments in Chapters 8., 9., and 10. of this book). Naturally, mutually coupled (and electromagnetically polarized) atoms and masses, are creating electromagnetic fields around, respecting $1/r^2$, Coulomb forces (since involved, radiated energy density is diluting with a distance by $1/r^2$).*

4. *Consequently, now we can say that electromagnetic, mechanical, electromechanical, acoustic and any other "moments-energy communications" between all atoms, masses, and objects within our universe exist based on resonant electromagnetic and electromechanical coupling and synchronization effects between mechanical and electromagnetic moments and field charges, and based on standing waves created between and inside mentioned objects, masses, and atoms (see more, later in the same chapter, under "**2.3.3. Macro-Cosmological Matter-Waves and Gravitation**"). Static masses without mechanical linear, angular, and electromagnetic moments cannot be sources of gravitation (see more in the first chapter of this book). Also, here we should not forget intrinsic entanglement effects of immediate synchronous connections*

*between directly coupled atoms, matter-waves and other "mass-energy-moments" entities (see more about such mathematical background of entanglement in Chapter 10.). This way, we could conceptualize gravitational force, matter-waves, and other outcomes as globally and spatially induced electrostrictive, magnetostrictive and entanglement effects (while associated matter waves will be created in all cases of stationary, periodical, or mass-distribution oscillations and disturbances). Since all matter constituents are mutually and synchronously coupled and communicating as more or less tuned resonant circuits (or resonating objects), electromagnetically, mechanically, acoustically, and electromechanically, we could also advance with philosophical statement that all organic and inorganic matter, objects, alive beings, plants, and animals are until certain level also mutually synchronized, connected, and coupled with resonating matter waves (see more in Chapter 8. under "8.3. Structure of the Field of Subatomic and Gravitation related forces). Some of the human beings could also be very much sensitive to such electromagnetic resonant couplings. It is known that Nikola Tesla, **[97]**, was one of such extraordinary sensitive and unique species.*

5. *We also know that macro-masses can be macroscopically (in average) almost electromagnetically neutral, but internally, masses have spinning and vibrating atoms, electrons, protons, neutrons... Consequently, micro-spatial distribution of related internal electric and magnetic fields, forces moments and dipoles are not globally electromagnetically compensated or neutralized in every point of mass distribution. We could always search where centers of electric and magnetic neutrality of internal electric and magnetic dipoles inside certain mass are (or also where centers of neutrality of positive and negative electrical charges or magnetic fluxes are). Such centers of electromagnetic neutrality are not exactly overlapping with relevant center of mass, a center of inertia, or center of self-gravitation. **This situation is naturally creating certain tension, torque, angular moments, and vibrations (between involved masses), indicatively supporting the existence of global rotation united with linear motions within our Universe (what effectively presents or creates wave-particle dualistic manifestations).** This looks like effects of motor-vibrators where an eccentric-mass-disk is being rotated this way producing vibrations. The future theoretical advances regarding similar situations (related to macroscopically or electromagnetically compensated masses) will be to redefine specific, universal, effective center of energy or masses, including centers of belonging linear and angular moments, and centers of electromagnetic neutrality. If some masses have non-compensated electromagnetic charges, fluxes, angular and spin moments macroscopically, effectively behaving like weak permanent magnets, the same situation is becoming more complex, but easily understandable as the Coulomb (or John Mitchel) attraction between magnets. The existence of mentioned, not-completely spatially overlapping dipoles, moments and mass centers is, most probably influencing matter waves and gravitation. See similar ideas in [33], from Jovan Djuric. To simplify here described situation, we can say that a mass in **linear accelerated motion**, anyway, has number of internal spinning, atomic states that can be gradually aligned with the total liner mass moment, becoming rotationally accelerated, and this way creating, by superposition, its resulting or total macro-mass-spinning moment and torque.*

6. *Eventually, we can warmly accept A. Einstein General Relativity, or the equivalence between Gravitation and "Spatial-temporal geometry generalizations", since the force of Gravitation looks as fully equivalent to what we feel and measure inside certain uniformly accelerated object, (box, airplane, or elevator), and motions within a curved-geometry space, especially self-closed circular motions, have elements of accelerated and central force motions. Practically, motions and attractions of all gravitational masses (being in a vicinity of other masses and gravitational sources) mathematically and imaginatively correspond to continuous motions along "**spatially-temporally curved slopes or curved surfaces**" naturally existing around all masses, this way connecting Gravitation and space-time geometry around masses, providing that we can fully and correctly describe and model such curved spaces (which are not at all static situations). General Relativity concept of gravitation is philosophically amazing and seducing but mathematically it could be complicated, impractical, and not easily considering possible dynamic effects of (from atoms) evolving fields, forces matter waves and variety of natural fields distributions between masses and other energy-moments states. For instance, this (about General Relativity and Gravitation) is especially interesting and challenging when being compared to N. Tesla's Dynamic Gravity concepts, since natural property of masses (or energy concentrations) is to create (all-around and omnidirectionally) certain uniformly accelerated, like a fluid-flow motion, what effectively corresponds to N. Tesla Radiant energy and A. Einstein accelerating elevator. Anyway, A. Einstein General Relativity and Gravitation conceptualization can also be deduced and developed from generally valid natural forces definitions, as presented in Chapter 10. of this book, under "10.02 Meaning of natural forces", and from "2.2. Generalized Coulomb-Newton Force Laws" in this chapter, meaning that there is still a lot of creative and modelling space to update and unite Newton and General Relativity Gravitation.*

In the Chapter 8. of this book (8.3. Structure of the Field of Subatomic and Gravitation related Forces) we can find familiar elaborations about Gravitation explicable as matter-waves exchange effects between atoms, all masses, and surrounding Universe. The condition of a balance of the potential energy of all attractive and all repulsive forces (see (8.74)) within an atom may be added on by a hypothesis about the existence of permanent, omnidirectional (in wards and outwards) communication by an exchange of electromagnetic quanta between stationary states of a nucleus and stationary states of *electron-waves cloud* of an atom. This is happening synchronously and coincidently, in all directions (radially and axially, inwards, and outwards), so that such photons exchange is always captured by the internal field of an atom, i.e., could not be noticed in the external atom space, if an atom is really and totally neutral, isolated, self-standing, not-connected to other atoms, and not-excited (what is the impossible case). Here we could profit from the same conceptualization, hypothetically saying that mentioned (bidirectional) electromagnetic, (quantized or resonant) exchanges between atom nucleus and electrons shell are extendable by the force of gravitation, almost endlessly towards infinity of an outer, or external atom space (outside of atoms, towards other atoms, masses, and cosmic constellations). In other words, all atoms are on such way connected to our Universe, continuously radiating, and receiving electromagnetic waves (and other cosmic rays). This is like having certain action (meaning force), or source of radiative energy, and consequently, in such cases always exist an equivalent reaction of the Universe to mentioned radiation (since action and reaction forces are always balanced, working synchronously). On a similar intuitive way, Nikola Tesla, **[97]**, conceptualized existence of "Radiant energy", or radiant fluid flow from all atoms and masses towards universe (and vice versa). Outside of neutral atoms and other masses, we have a dominant presence of the force we qualify as gravitational attraction (being a derivative of existing electromagnetic phenomenology). Every mechanical force should be a time derivative of certain momentum, F = dp/dt, or Torque (=) dL/dt (see about other force meaning options in Chapter 10., under "10.02 Meaning of natural forces"). Since action and reaction and electromagnetic induction laws, including Lorentz forces are always mutually compensating and synchronously acting, we could imagine a continuous steady flow of some "Radiant fluid", where microelements of such fluid have linear moments, p = mv, and angular moments L = Iω, to be able to conceptualize existence of gravitational force (evoking A. Einstein accelerated elevator). This way we described continuous and closed-circuits of ***energy-momentum-radiant-energy-states flow*** from atoms towards Universe in both directions (inwards and outwards), and this is a similar concept to N. Tesla Dynamic Gravity Theory, and R. Boskovic Universal Natural Force ideas, [6], [97]. Most probably, Casimir effect is also a familiar and measurable manifestation of mentioned "radiant fluid" flow (see [103] and [104]).

The additional understanding of Gravitation, in relation to considering atoms as real sources of gravitation, can be supported by the following facts, comments, and concepts:

a) *We need to start from the Newton's cannonball thought experiment or explanation, showing how we can launch and place certain small mass m in a stable or permanent, circular orbit around much bigger mass M (where r is the distance between centres of m and M, and v is the orbital or tangential velocity of the mass m). Here we limit our attention only on circular orbits since it is easier to make important conclusions for what we need later. This is also the case when the mass m is in a stationary orbit, having its orbital velocity as,*

$$v = \sqrt{G\frac{M}{r}} .$$

b) *Then we need to consider that translatory and rotational symmetry, or conservation of linear and orbital (or angular) moments are always satisfied, this way implicating that small mass m, being already in a stable circular orbit around a much bigger mass M, in addition, can make some of translatory and rotating or spinning motions, while keeping constant its radius of rotation around mass M, without disturbing its stable circular orbit. Of course, here we also need to satisfy a total balance of relevant energy members (kinetic, potential, spinning ...).*

c) *Now we need to consider that rotation of the mass m around much bigger mass M (within the same stable, circular, and stationary orbit) can be considered literally as just stated (meaning one mass is rotating around the other), but since all motions are mutually relative, we could also imagine that m and M are not moving, especially when m is in a stationary orbit (keeping always the same distance, r, between them). Then, a surrounding spatial content, background matrix, or certain (still unknown) fluidic ether, which belongs to the same "big picture background" is rotating (around M, or better to say around common centre of mass of the m-M system) on a way that around small mass m, circular or tangential velocity v (of mentioned spatial and fluidic background, or an etheric fluid) is the same as mass m had before. Here we are exploring the meaning of <u>mutually relative and on some way equivalent circular motions</u>, that are accelerated motions, and should be mutually kinematically or mathematically replaceable. Now we can (mathematically) consider that both masses m and M are in the same centre of slightly bigger mass equal to m + M, and their spatial, joint-mass distribution cowers the space which starts from mentioned centre of bigger mass until the radius where it was the* *small mass m. This is at the same time as a case of rotation of certain disk or ring with an eccentric mass m. Naturally, eccentric mass m (or m + M) will produce standing matter waves around its stable circular orbit (* $\lambda = \frac{H}{mv} = \frac{2\pi r}{n}$, $H = \text{const.}$, *n = 1, 2, 3, ...). Now we can summarize what will be relevant parameters of such stable, circular, and stationary, orbital motion,*

$$\frac{mv^2}{r} = G\frac{mM}{r^2} \Rightarrow \boxed{v = \sqrt{G\frac{M}{r}}}, \quad 0 < v < c \Rightarrow$$

$$\Rightarrow 0 < \sqrt{G\frac{M}{r}} < c \Rightarrow r = \frac{n\lambda}{2\pi} = \frac{GM}{v^2} > \boxed{r_{min} > \frac{G(M+m)}{c^2}} > 0 \, .$$

We still do not discuss what could be the nature of a surrounding spatial fluid, which (hypothetically) rotates around centre of mass M, (or m + M), but we could deduce its properties later.

*d) Now, we could imaginatively consider that every stable mass **m** in a state of rest, effectively belongs to some mathematically equivalent (and hidden) background motion, which is as certain spatial and etheric fluid rotation about a centre of that static mass. Mentioned fluidic matter rotates around mass **m** with certain speed v. Here, small mass element **dm** already belongs to a mas **m**, and this could be the mass of an atom. We also consider that mass **m** is spherical, having radius **r**. Speculated rotation of **dm** around the centre of mass **m** is on some way securing overall stability of the mass **m** (considering that involved centrifugal force, acting on **dm** is balanced with the self-gravitation force of the mass **m**), as follows,*

$$\frac{dm \cdot v^2}{r} = G\frac{m \cdot dm}{r^2} \Rightarrow v = \sqrt{\frac{Gm}{r}} \, .$$

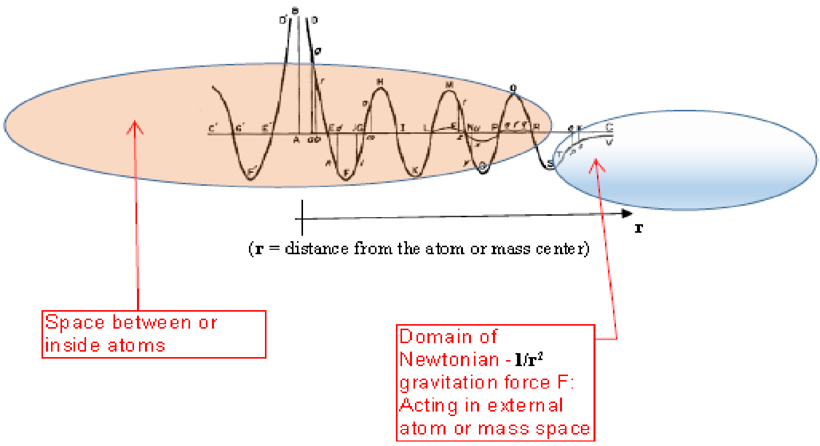
*Since we know that Gravitation (as a force or field, in here mathematically idealized conditions) is homogenous, isotropic, and omnidirectional, acting on the same way all around spherical mass m, what means that every single atom inside mass m is on some way like an elementary vortex-type source (or sink) of gravitation related phenomenology. This is supporting the argument that Gravitation is directly proportional to mutually attracting masses (since sum of all atoms of certain mass, is creating such total mass). That means, very specific and complex atomic field (or force) is created as a spatial vector's superposition of all atomic forces of internal mass constituents (meaning atoms), eventually creating a resulting force of Gravitation (in all angular or radial directions around mass **m**). See more about one of possible conceptualizations of such atomic force in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces".*

Planetary, or solar systems, are analogically structured as their (mutually synchronized) atom constituents, respecting Coulomb-Newton 1/r² force law, and standing matter-waves resonant packing and couplings; -See more in "2.3.3. Macro-Cosmological Matter-Waves and Gravitation, "2.8. N. Bohr hydrogen atom and planetary system analogies", and "2.3.3-6, Rudjer Boskovic and Nikola Tesla's theory of Gravitation", at the end of this chapter. Such facts are directly indicating and implicating that the essential nature of Gravitation should be electromagnetic. *More of familiar ideas regarding mutual coupling between mechanical and electromagnetic forces and moments in can be found in Chapters 3., 8., 10., and in [117], [144].*

External forces in question (outside of atoms and masses) belong to attractive effects of Gravitation (being essentially and primarily of an electromagnetic nature since atoms are communicating internally and externally by exchanging photons and by receiving cosmic rays and other types of cosmic radiation). Practically, all atoms, different particles, more significant objects, and our Universe are communicating bi-directionally (or omnidirectionally) by radiating electromagnetic energy, and by *receiving an echo of the attractive Gravitational force (or by receiving Tesla's cosmic Radiant energy consisting of matter-wave rays). See familiar ideas and elaborations in chapter 9. of this book (9. BLACK BODY RADIATION, GRAVITATION & PHOTONS).*

Embryonic roots of such interpretation of an atomic (as well interplanetary systems) force and field are found in the publications of Rudjer Boskovic, **[6]**, about universal natural force (in [6], Principles of the Natural Philosophy), as well as, much later, in certain papers published in «Herald of Serbian Royal Academy of Science» between 1924 and 1940 (J. Goldberg 1924; V. Žardecki 1940). Nikola Tesla's, **[97]**, Dynamic theory of Gravitation is also close to Rudjer Boskovic's unified natural force (see the picture below), and to the here-elaborated concept, summarized with (8.74) in chapter 8. of this book (see "8.3. Structure of the Field of Subatomic Forces"). Another source of familiar ideas we can find in [73], Reginald T. Cahill, Dynamical 3-Space; -Emergent Gravity and in [117], Jean de Climont."

$$\oint\!\!\!\iiint_{\left\{R\in[0,\infty],\ \theta\in[0,2\pi],\ \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right\}} \mathbf{F}d\mathbf{R} = \mathbf{0}, \qquad (8.74)$$



Rudjer Boskovic's Universal Natural Force Function

If we consider that force of gravitation is kind of electromagnetic force, naturally such forces are both respecting the same Coulomb-Newton $1/r^2$ force law (since this is the same force). In cases we focus our attention on what we name as gravitation, we can see that this is only an attractive-force part of the R. Boskovic universal natural force curve (on the right picture side, towards infinity). Usually, we expect that natural forces have mutually balanced positive and negative values or sides, and (still surprisingly) for Newtonian gravitation we cannot find where the repulsive-force part is. In case of considering Rudjer Boskovic universal natural force curve as qualitatively and conceptually relevant, it is obvious that repulsive, balancing forces are in the "alternating forces-amplitudes part" regarding masses and atoms structure (placed on the left or central side of the Universal Natural Force curve). See more in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces"). We can also experimentally demonstrate existence of such attractive and repulsive forces with "half-wavelength ultrasonic resonators" and with effects of acoustic or ultrasonic levitation (see more in [150] and [151]).

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Here we can add that mentioned standing waves, spatially structured as R. Boskovic's Natural-Force-field, should also have radial, angular and longitudinal components (since what we are addressing is at least a three-dimensional space or field). Also, associated matter-waves-carriers of (mechanical and electromagnetic) "energy-momentum" flow participants are always described with second order Classical Differential Wave Equations, as elaborated in Chapter 4.3 of this book (see equations starting from (4.9) until (4.11), and around). ***Solutions of such wave equations (including Schrödinger equations) always have two waves, or we can represent them with two wavefunctions propagating in mutually opposite directions, both temporally and spatially (one inward, and the other outwards, or internally and externally), synchronously capturing past and future time of an event, associated with effects of micro and macrocosmic entanglements and resonant synchronizations.***

*Nikola Tesla, **[97]**, made several patents (and presented affirmative and successful experiments) showing that his "radiant energy fluid" or steady radiant mass-flow really exist and has an electromagnetic nature (being able to carry positive and negative electric charges and mechanical moments). Consequently, we can draw conclusions that all atoms and masses in our universe are mutually communicating on some way conceptually respecting Rudjer Boskovic Universal Natural Force, **[6]**, being mutually coupled as frequency tuned, electromagnetic resonators. Mentioned attracting masses or agglomerated atoms can be equally considered as being of electromagnetic or mechanical nature, thanks to coupling by a surrounding fluidic medium named as ether (useful until we find another more convenient name and better conceptualization). Properties of such ether (based on Nikola Tesla concepts) have both electromagnetic and mechanical nature (having extremely fine fluidic elements that are carriers of mechanical and electromagnetic moments and charges, as well as having dielectric and magnetic properties). In fact, we should have in mind that all electrical, electromechanical, electromagnetic, acoustical, mechanical, fluidic, vibrational, and gravitational circuits, devices, and systems should always be completely closed circuits or systems (see more about closed circuits in the first chapter). Such systems should still have certain front-end or source inputs, and last-end outputs, consumers, or loads. In the contemporary Physics and Cosmology, we can still find mathematical analyses based on non-closed, or arbitrarily open, hanging or flying structures, formations, motions, fields, and forces descriptions, what are our (sufficiently mathematically acceptable) simplifications and signs that we still operate with incomplete models, theories, and concepts.*

To illustrate the broader spectrum of Ruđer Bošković's ideas about universal, united, Natural Force, let us read the following Quote:

Citation from the Internet: **Ruđer Bošković, (English: Roger Boscovich): full biography.** Home > About Croatia > Culture > 300th anniversary of the birth of Ruđer Josip Bošković > *Ruđer Bošković: full biography.*

..."In his work Bošković investigated various fields of science, making his most profound contribution to the understanding of the structure of matter. His theory of forces and the structure of matter is now widely accepted, making him a scientist two centuries ahead of his time. His theory was postulated on the principles of simplicity and analogy within nature, and on the principle of continuity. The practical

purpose of the theory was to develop the then-topical scientific problem of collision analysis. According to Bošković, the matter is composed of points (puncta), which are simple, indivisible, non-extended, impenetrable, discrete, and homogenous, and which are sources of forces that act remotely. These points differ from mathematical points in that they possess the property of inertia, and in that, there is a force – Bošković's force – acting between them, which is represented by the Bošković curve (lat. curva Boscovichiana). At close distances, the force is repulsive. As distances increase, it reaches the point of neutrality, then becomes attractive, then reaches neutrality again, and finally becomes repulsive again. At farther distances, the force is attractive, in accordance with Newton's theory of gravity. B. proposed a modification in Newton's law of gravity concerning very long distances. The Bošković curve is uninterrupted, and it has two asymptotic ends (the repulsive and the attractive). It crosses the x-axis at the points of neutrality, called the points of cohesion and non-cohesion. Bošković's force is very akin to the force between atoms in a molecule or solid matter as well as to the nuclear force between nucleons (protons and neutrons). Hence, Herzfeld described it as "potential energy according to Bošković". A single law of forces existing in nature (lat. lex unica virium in natura existentium), i.e., the idea that one law can explain all of reality, constitutes Bošković's main contribution to science. The same idea has been entertained by A. Einstein, W. Heisenberg, and more contemporary scientists, but the four forces in nature (gravitational, electromagnetic, weak, and strong nuclear energy forces) have yet to be described by a unified theory. Bošković's single law is a framework for a unified theory of fields or, even more so, for a theory of everything. As a result of the unconditional assumption that the law of continuity must be observed, it followed that there can be no direct contact between particles because of the repulsive force (until then nobody had challenged the idea that there was contact between particles of matter). Modern scientists now agree with Bošković's conception of the basic elements of matter. Bošković's puncta are the most basic particles of matter and are as such comparable to quarks and leptons in modern science. Since matter consists of points, it follows that it contains a lot of empty space. This idea disproved the materialistic-corpuscular theory of matter, set foundations for a real dynamic-atomic theory, and provided a new perspective on the perception of reality. Just as the work of Copernicus resulted in the idea of the Copernican Turn, this breakthrough should be recognized as the "Boscovichan Turn" since it constitutes "the greatest triumph over the senses achieved on Earth to this time", and since Bošković and Copernicus "have been the greatest and the most victorious opponents of appearances" (F. Nietzsche, 1882., 1886).

Allowing for multiple repulsive areas in its potential, B. built the "pre-model" of "quark confinement", which is one of the central points of interest in modern elementary particle physics. Non-extended aspects of the matter are the building blocks of bigger particles, which in turn build up even bigger masses. B. speaks of them as the particles of the first, second, third, etc. order. This reflects the modern understanding of the structure of matter: quarks and antiquarks correspond with the particles of the first order, nucleons of the second, atomic nuclei of the third, atoms of the fourth, and molecules of the fifth. The properties of these particles and the distinctions between them are the result of their internal structures. B. was one of the champions of this idea, though the concept of the interconnection between the property and composition of matter was not accepted until the 19th century. (J.J. Berzelius, 1830).

The application of Bošković's law of forces to three points, two of which are placed in the foci of ellipses, is known as Bošković's "model of the atom" (1748). Long before the advent of quantum physics, this model identified the concept of "allowed" and "forbidden" orbits in nature, i.e., it quantifies the trajectory of the particle. J.J. Thomson was directly inspired by Bošković in formulating that idea (1907), which was central to the Bohr Model of the Atom (1913). »The Bohr model of the atom is a direct successor of Bošković's law of forces between microscopically removed particles"... "Where B. sowed 200 years ago, the others have reaped" (H.V. Gill, 1941). Bošković can also be considered the forerunner of thermodynamics, the kinetic theory of matter, the theory of elasticity of solid objects, and the explanation of the form of the crystal.

B. criticized Newton's conception of absolute space and time, and he construed the understanding of spatial and temporal relations as inextricable from point-like atoms and the forces between them. The extended matter is discrete rather than continual and, as such, entails a dynamic configuration of a *finite number of centers of force. According to Bošković's pure dynamic atomism, the matter is not only endowed with forces (dynamic system), but it is composed of forces (dynamic system). Forces flow out of the atom and permeate empty space. This idea led to the concept of the field, much later formulated by M. Faraday (1844), who together with J.C. Maxwell introduced this idea into science. Bošković's conception of spatially and temporally variable modes of existence (modi existendi) had ramifications, which despite all of the differences bring him into connection with Einstein's theory of relativity. Bošković can be regarded as the forerunner to the theory of relativity in three respects. First, he embraced the principle of relativity (one and a half centuries before E. Mach and A. Einstein) by proposing that direct observation and experimentation can neither distinguish between real space, relative space, time and motion, nor prove the principle of inertia. Secondly, he advocated the idea that the dimensions of an object change as its location changes. However, Bošković did not offer a quantitative measure of that change. Finally, he suggested that space might have four dimensions.*

In Philosophiae naturalis theoria… Bošković proposed the idea of an omniscient "spirit" that, based on Newton's laws and on the knowledge of all of the forces and initial positions at one moment, would have complete knowledge of the past and the future. Following essentially identical postulates, the French scientist P.S. Laplace formulated the classical determinism principle nearly half a century later (1814). That "spirit", that "intelligent entity", was termed by E. Du Bois-Reymond "Laplace's spirit" or "Laplace's demon", although it should have been named "Bošković's spirit" (S. Hondl)..."

……………………..

Citation from: [132] Dragoslav Stoiljkovic, Abstract: "In 1758 Roger Boscovich (1711-1787) published his monumental work "A Theory of natural philosophy reduced to a one unique law of forces that exist in nature". The Theory has had a major impact on Boscovich's contemporaries and resulted in many followers in the 19th and at the beginning of the 20th century. Today it is no longer present in the curricula of schools and colleges. Apart from the few individuals, our contemporaries, even highly educated people, know almost nothing about Boscovich. His life, scientific activity and philosophical views, as well as his influence on contemporaries and followers are dealt in this monograph. His Theory is the very first quantum theory. He was the first one to draw the orbitals by which a particle moves around particles located in a center and explain that by transition from one orbital to another a particle either gains or loses a certain amount (quantum) of energy. His primary contribution was to the discovery of the structure of atoms of chemical elements. He pointed the possibility of existence of macromolecules (i.e., polymers) and nanotubes, described the structure of these materials and their basic properties, and also the structure of diamond and graphite. Following his line of thought, the ideas of neutrino, quarks and gluons can be reached. The foundation of his theory is Boscovich's curve that describes the change in force between the particles of matter depending on the distance between them. We listed a dozen examples that confirm the validity of Boscovich's curve at several levels in the hierarchy of matter - of nucleons in atomic nucleus to the colloidal particles. The value of Boscovich's Theory is reflected in the multitude of ideas that sprout from it and that can be used to solve some of the problems of modern science. With some adaptation of Savich-Kashanin theory, we can obtain significantly more accurate calculations of solar planets densities, the volumes of matter at critical point conditions and in some other characteristic states of matter and the more correct interpretation of the mechanism and kinetics of the polymerization of ethylene and methyl methacrylate. A comparison of Boscovich's understanding of attractions and repulsions, as the essence of matter, with the understandings of Hegel and Engels are presented, too".

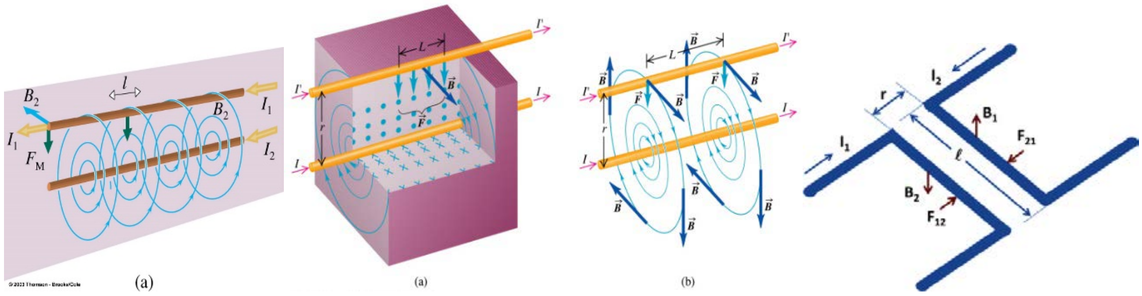
♣ COMMENTS & FREE-THINKING CORNER (brainstorming):

Existence of a hidden velocity-related parameters (or some of associated linear and angular moments) that are intrinsically incorporated in the Newton force law (for instance, as parts of the universal gravitational constant) can be intuitively and creatively explored knowing (for instance) that ***two parallel electric current elements (or wires) with currents in the same direction are mutually attracting***. If our universe is holistically rotating, and if this is producing effects of electromagnetic dipoles creation, we could imagine that two electrically neutral masses (close to each other or being parts of the same center of mass system, and the same mainstream angular motion), are anyway, until a certain level, electrically, internally charged and slightly polarized with electric and magnetic dipoles (appropriately oriented or aligned). This could be the case because they belong to the same, joint holistic motion concerning certain dominant reference system, for instance, being like in relation to a local galactic center of mass. Depending on an observer (meaning being reference system related), there is spatial flow or circulation of micro-electric currents inside such masses (within the mainstream of their joint, global motion). This way, any two masses could be treated as segments of two electric wires (or two electro-conductive bodies) with mutually colinear or parallel electric micro-currents flowing in the same direction (that is a direction of mentioned natural and holistic, background angular motion; -see (2.4-4.1) - (2.4-4.1)). We know that such currents or wires (or internal electric and magnetic dipoles) are mutually attracting by creating attractive magnetic fields (what locally and effectively manifests as Gravitation). Situation is probably much more spatially and electromagnetically complex (because we should imagine spatially distributed currents produced by motions and oscillations of internally distributed electric and magnetic dipoles around dominant centers of masses), but the effects of associated electric and magnetic field forces (including Lorentz forces) are still there, and this is most probably the dominant part of what we consider as the force of gravitation (and inertial effects). Mathematics to show that such forces are again $1/r^2$ dependent will be an additional mental (theoretical and mathematical) effort to make because here we have the influence of several force components on resulting "gravitational" force (such as magnetic and electrostatic ones). Intuitively and qualitatively, here we have a specific, imaginative background explaining why masses are only attracting (because they are part of certain uniform, inertial, and holistic angular motion within the same mainstream). If we pay our attention to the $1/r^2$ force-law (2.4-4), the same attractive force situation will be additionally supported (again based on the same intrinsic and holistic motion and internal micro currents of involved dipole charges). Anyway, all our speculations here are supporting the platform that mass-concept and gravitational-charges should be something else (with an intrinsic, electromagnetic nature), more complex than presently conceptualized in classical Mechanics, and Newton and Einstein Gravitation (as conclusions based on predictions from Mobility system of analogies; -see the first Chapter of this book).

Theoretical reminder (taken from Internet); -Force between Two Parallel Wires

- Two long parallel wires suspended next to each other will either attract or repel depending on the direction of the current in each wire.
- B-field produced by each wire interacts with current in the other wire.
- Produces magnetic deflecting force on other wire.
- Wires exert equal and opposite forces on each other.

$$F_{1,2} = -F_{2,1} = \frac{\mu_o I_1 I_2}{2\pi r} \cdot L$$ (=) Force between two parallel wires (L = l = l is the active length of wires)



1. Parallel wires with current flowing in the same direction attract each other.
2. Parallel wires with current flowing in the opposite direction repel each other ($d = r$).

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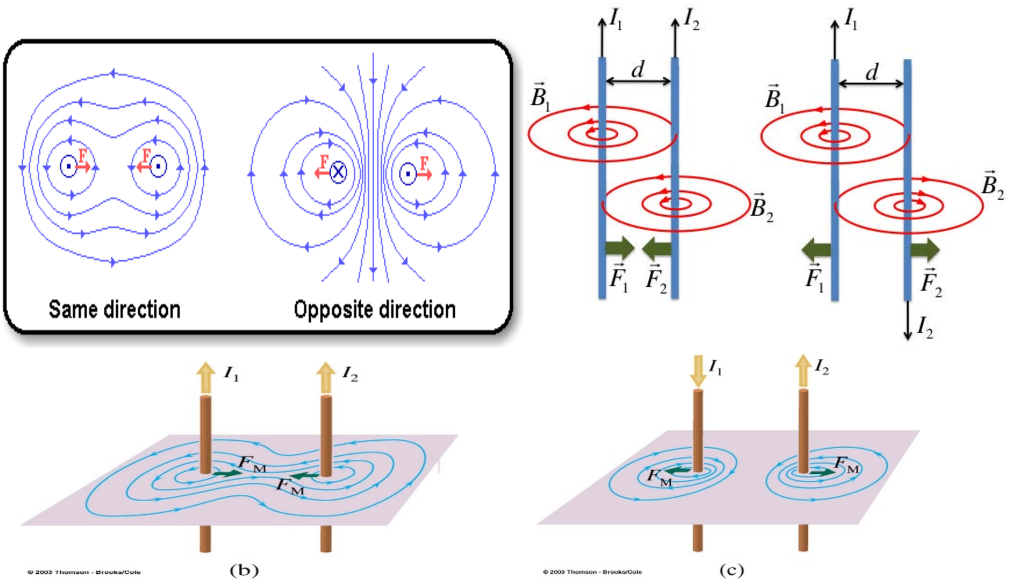
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.



- Note that the force exerted on I_2 by I_1 is equal but opposite to the force exerted on I_1 by I_2.

Here is an attempt to show how force between two parallel wires with currents I_1 and I_2 could be presentable as Newtonian attraction between two effective masses m_1, m_2 of wires in question. By respecting Mobility system of analogies (see T.2.2 in the first chapter), we know that there should be some direct correspondence or similarity between current and certain force. Of course, force here ($f_{1,2(axial)} = m_{1,2} \cdot a_{1,2}$) should act on electrically charged particles that are creating currents I_1 and I_2, meaning that this is not a gravity force. Let us speculate how the force between two wires, $F_{1,2}$, could be equivalent to the Newtonian force of gravitational attraction, as for example,

$$F_{1,2} = -F_{2,1} = \frac{\mu_o I_1 I_2}{2\pi r} \cdot L$$

$$\left\{ \begin{array}{l} I_{1,2} \Leftrightarrow k \cdot f_{1,2(axial)} = k \cdot m_{1,2} \cdot a_{1,2} \\ f_{1,2(axial)} = m_{1,2} \cdot a_{1,2}, k = const\,(=)\left[\frac{A}{N}\right] \end{array} \right\} \Rightarrow \begin{array}{l} F_{1,2} = \frac{\mu_o k^2 m_1 m_2}{2\pi r} a_1 a_2 \cdot L = G\frac{m_1 m_2}{r^2} \Rightarrow \\ \Rightarrow G = \frac{\mu_o k^2 a_1 a_2 \cdot L \cdot r}{2\pi} = \frac{\mu_o k^2 a^2 \cdot L \cdot r}{2\pi} \Leftrightarrow ka = \sqrt{\frac{2\pi G}{\mu_o \cdot L \cdot r}} \end{array}$$

$$F_{1,2} = \frac{\mu_o k^2 m_1 m_2}{2\pi r} a_1 a_2 \cdot L$$

Here we assume that axial electric charge acceleration in both wires is the same and constant, $(a_1 \cong a_2) \cong a = \text{Const.}$.

Of course, here we have a very indicative result, (2.4-4.1) - (2.4-4.3), showing that in this case we already have Newtonian or Coulomb attraction of involved masses, or currents produced by motions of internal electric dipoles, supported by the following, approximated and analogical comparisons,

$$\mathbf{dm = \sqrt{\frac{\mu_0}{4\pi G}} \cdot idl \Rightarrow m = \sqrt{\frac{\mu_0}{4\pi G}} \cdot I \cdot L + m_0 = m_0 + \Delta m,\ m_0 = const. \Rightarrow}$$

$$\begin{pmatrix} \mathbf{F}_{1,2} = \frac{\mu_0 \mathbf{I}_1 \mathbf{I}_2}{2\pi \mathbf{r}} \cdot \mathbf{L} \\ \mathbf{I}_1 = \mathbf{I}_2 = \mathbf{I} \\ \mathbf{F}_{1,2} = \mathbf{G}\frac{\mathbf{m}_1 \mathbf{m}_2}{\mathbf{r}^2} \\ \mathbf{m}_1 = \mathbf{m}_2 = \mathbf{m} \end{pmatrix} \Rightarrow \begin{pmatrix} \mathbf{F}_{1,2} = \frac{\mu_0 \mathbf{I}^2}{2\pi \mathbf{r}} \cdot \mathbf{L} \\ (=) \\ \mathbf{F}_{1,2} = \mathbf{G}\frac{\mathbf{m}^2}{\mathbf{r}^2} \end{pmatrix} \Rightarrow \mathbf{r} = \frac{1}{2}\mathbf{L}$$

Since $\mathbf{r} = 0.5\mathbf{L}\ (= \mathbf{d})$ *and* $\mathbf{L}$ *are directly comparable lengths (being the same order of magnitude), the force between two parallel wires with electric currents should be roughly presentable or effectively convertible into a kind of Newton or Coulomb force.*

In conclusion, attraction between masses is effectively presented by an attraction between currents being effectively developed inside masses, based on motions of internal electric and magnetic dipoles. In other words, without having certain intrinsic, global or background motion of masses (internally and externally), what could be some cosmic resonance effects with standing waves, assuming radiant energy circulation between masses, we cannot explain gravitation. Very much familiar and well supported elaborations about "New Horizons In Electric, Magnetic & Gravitational Field Theory" and "Similarities of the Motional Electric & Gravitational Forces" we can find in publications from William J. Hooper, [152], and in Chapter 3. of this book.

Natural Forces based on Standing Waves, Resonating Universe

In fact, all here presented mutually analogical ($1/r^2$)-forces (from (2.1) to (2.4-5.1)) is only an exercise showing that generalized $1/r^2$-Coulomb-Newton forces could have static and dynamic force members, and that in all such cases "energy-moments" content of mutually interacting participants has significant relevance, like in (2.4). In addition, everything that exists in our universe is in relative motions to its environment, and an internal structure of all particles (and atoms) has its intrinsic fields and standing-waves nature with some spinning and orbital moments. All Newton-Coulomb force laws and predictions based on analogies are implicating that properties, like electric charges, masses, magnetic fluxes, orbital and spin moments, Gyromagnetic and Charge-to-mass ratios ... are mutually dependent, and anyway managed by the same conservation laws. The next consequence is that gravitation and other natural forces in our universe could effectively be presentable from several mutually equivalent, synchronized, convertible, and mutually analogical platforms. *Here we could also speculate with existence of an immediate action on all distances, to maintain such global "energy-moments" balance, synchronization, couplings, and validity of conservation laws on micro and macro-cosmological distances (see more in [36]).*

Coulomb force laws (one for electric charges and other for magnets) are mathematical forms where attractive or repulsive forces between static electromagnetic charges are considered. This is because we are usually taking mass or electric charge, or permanent magnet flux, as something fixed and static. If we start analyzing what is internally creating a mass, an electric charge, or a magnet, we will find different dynamic, or by resonance and standing waves structured field formations, behind all of them. Matter is composed of molecules, atoms, fields and elementary particles (also being electric charges, magnets, having intrinsic spin and orbital moment attributes, and oscillating). Here should be the place to search for essential sources and effects influencing gravitation. Let us conceptualize certain very much simplified, intuitively, and analogically founded picture about gravitational attraction between identical

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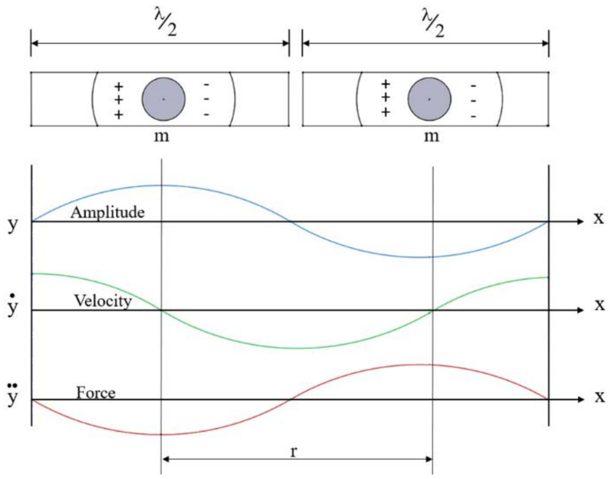
masses m and m, based on induced electromagnetic forces within structurally resonating Universe. Here, we are also addressing an analogy of attracting masses with equivalent half-wavelength mechanical (or ultrasonic) resonators, where masses m and m are presented as half-wave mechanical resonators, this way effectively being parts of standing matter-waves created between them (see upper part of the picture below, with rectangular bars, presenting mechanical half-wavelength resonators). We could consider the picture below as a kind of over-simplified imaginative conceptualization, where masses (m and m) are also treated as "electrically or electromagnetically polarized half-wave resonators", on a way to mutually attract (because of convenient internal electromagnetic dipoles polarization).



This picture also presents a simple concept of Gravitational or Coulomb attraction based on electrostatic (also magnetic) dipoles attraction (being in a form of standing waves) between oscillating masses m and m. First, upper sinusoidal curve, $\mathbf{y}$ *, symbolizes oscillating amplitude of involved matter waves (like being an oscillatory displacement) between masses m and m. Middle sinusoidal curve* $\dot{\mathbf{y}} = \mathbf{v}$ *signifies velocity of matter waves between masses m and m. Zones where sinusoidal velocity curve has minimal (or zero) values, are zones of agglomerated (and polarized) masses and/or electromagnetic charges. Velocity curve between m and m could also be analogically compared with certain voltage or potential (respecting Mobility system of analogies, as presented in Chapter 1.). Second derivation of oscillating amplitude* $\ddot{\mathbf{y}}$ *corresponds to spatially distributed acceleration or force* $\ddot{\mathbf{y}} \sim \mathbf{F}$ *between masses m and m, here having an indicative and analogical meaning of gravitational force, also on some way being analogical to the attractive force between "magnetized and/or to electric dipole polarized masses" m and m. Maximal attractive forces are in zones where masses are agglomerated or placed, and maximal repulsive forces are where force or acceleration curve* $\ddot{\mathbf{y}} \sim \mathbf{F}$ *has minimal or zero values. Oscillatory,* ***Kinetic energy*** *spatial distribution between involved masses is proportional to a squared velocity curve (*$\mathbf{v}^2$*), since* $\mathbf{E_k} = \mathbf{0.5mv^2}$*. Similar imaginative meaning of here involved* ***Potential (gravitational, or central force) energy-distribution*** *also corresponds to acceleration, or to a force-curve* $\ddot{\mathbf{y}} \sim \mathbf{F}$ *between m and m, meaning that such maximal* ***Potential energy*** *is just around mases m and m, where absolute value of the force or acceleration curve has maximal values. From thermodynamics we know that if certain gas (or body) is being heated, then its temperature is directly proportional to the kinetic energy of involved gas particles, or to* $\mathbf{0.5mv^2}$*, meaning that maximal value of the square of mentioned velocity curve is the zone where temperature of involved gas particles*

will also be maximal. This is the very indicative, or almost direct proof that in any solar system (where a Sun is much bigger compared to masses of all orbiting planets) temperature around it will not be maximal on the Sun's surface (where standing waves velocity is almost zero). If we consider the Sun (being a part of the described two body, or two-mass, oscillating system), as a heat energy radiator, and also as a radiator of ionized and other gas particles and matter-waves, the increasing velocity curve **v** *of mutually resonating masses will start from zero (or from certain minimal value on the Sun surface), and it will rise in a direction of the second mass, this way also influencing motions and streaming of gaseous state around the Sun surface (as much as conditions around the Sun support such motions). We already know that maximal temperature about our Sun (which is between 1 to 2 million °C) is in its* ***Corona zone*** *(where velocity of resonating standing waves between involved masses is still rising). Where the same oscillating velocity is minimal (or zero), our Sun has only around 6000°C on its surface (about 300 times less compared to Corona zone temperature). Of course, the Sun (as a blackbody and electromagnetic plasma radiator) in its near-to-surface zone also has certain increased density of gasses and other particles and matter-waves, which is gradually decreasing (with a distance) until arriving to a state of vacuum, meaning that temperature and kinetic energy of such gaseous state will naturally start increasing until its Corona zone, and then it will start decreasing in a high vacuum and low mass-energy-density conditions, toward the second mass. Solar winds or streaming of charged and ionized plasma particles will be created within Corona zone (continuing to flow after Corona zone, towards high vacuum zone, in a direction of the other mass). Such (low density) solar wind particles are being additionally accelerated until the velocity curve maximal values, what we know from astronomic measurements (see more about solar winds, corona effects and blackbody radiation in Chapter 9.). Anyway, Corona zone should also be the effect of high voltage electric field or electric plasma discharges, meaning that what we here consider as a velocity curve, analogically corresponds to standing-waves voltage distribution between masses m and m. In other words, this about Corona zone temperature, followed by accelerated motion of plasma or solar-wind particles, directly translates to the concept of* ***Resonant Universe and formations of standing waves between gravitational masses,*** *including participation of other energy agglomerations; -see citation below). This is also in a conceptual compliance with Electromagnetic Plasma phenomenology, Blackbody radiation, N. Tesla Dynamic Gravity, Radiant Energy, and Rudjer Boskovic's Universal, Natural Force Law. See more in Chapter 9. of this book, and here:*
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/ .

Citation from: https://www.space.com/17137-how-hot-is-the-sun.html :
"Temperatures in the sun's atmosphere also vary considerably between the layers. In the photosphere, temperatures reach about 10,000 degrees F (5,500 degrees C) according to the educational website The Sun Today. It is here that the sun's radiation is detected as visible light. Sunspots on the photosphere appear dark because they are cooler than the other parts of the sun's surface. The temperature of sunspots can be as low as 5,400 to 8,100 degrees F (3,000 to 4,500 degrees C) according to the University Corporation of Atmospheric Research (UCAR).
The chromosphere lies above the photosphere and temperatures range from approximately 11,000 degrees F (6,000 degrees C) nearest the photosphere to about 7,200 degrees F (4,000 degrees C) a couple of hundred miles higher up.

Now here is where things get a little bit strange. Above the chromosphere lies the corona — the outermost layer of the sun's atmosphere. The sun's corona extends thousands of miles above the visible "surface" (photosphere) of the sun. Now you might think that temperatures here must be the lowest here since we are the farthest away from the heat-generating core… but that isn't the case. At all.

The sun's corona can reach temperatures of around 1.8 million degrees F to 3.6 million degrees F (1 to 2 million degrees C), that's up to 500 times hotter than the photosphere. But how is the sun's upper atmosphere hotter than the surface? It's a great question, and one that has scientists rather stumped. ***There are some ideas about where the energy comes from that heats the corona, but a definitive conclusion is yet to be made***. If you'd like to read more about this solar mystery check out this article on "Why is the sun's atmosphere hotter than its surface?"."

*Of course, this is a very simple, visual, and analogical, standing waves conceptualization (like in experiments with **ultrasonic levitation**; -see more in [150] and [151]), useful to give an idea how attracting and oscillating masses could mutually interact (based on properly oriented electrostatic and magnetic moments or dipoles, and on some way created electromagnetic standing waves). Masses m and m are macroscopically neutral (but internal electric dipoles, or electric field spatial distributions are producing electrostatic and magnetic attraction between m and m, equivalent to gravitational attraction). Atoms and subatomic constituents of masses also (and naturally) have magnetic and spin moments. In cases of planetary systems, masses are also in orbital and spinning motions, this way producing effects of electric currents with associated magnetic field standing waves.*

Along with here introduced standing waves conceptualization of gravitation, we should not forget that cosmic and galactic formations are also structurally hosting large scale of strong magnetic fields formations, and we know that magnetic fields are being created around electric current loops. Essentially, this means that we are the part of an Electric Universe. Consequently, we should understand mentioned cosmic standing waves as being the part of electromagnetic waves formations, including streaming of involved plasma states and other electroconductive phenomenology, all of that having influence on creating and explaining Gravitation (see relevant conceptualization about fully symmetrical and perpetually inter-related electric and magnetic fields in Chapter 3. of this book).

Citation: "How Magnetic Fields Challenge Gravity-Centric Cosmology | Space News: ThunderboltsProject How Magnetic Fields Challenge Gravity-Centric Cosmology | Space News - YouTubeHow Magnetic Fields Challenge Gravity-Centric Cosmology | Space News - YouTube.
It has been one of the greatest surprises of the Space Age – powerful magnetic fields pervade the cosmos. Mainstream astronomers and astrophysicists do indeed acknowledge pervasive cosmic magnetism, but they did NOT predict it, and the realization has come begrudgingly. Here we explore why powerful cosmic-scale magnetic fields associated with countless celestial objects is the clearest indication that we live in an Electric Universe".

Contemporary attempts to create a Quantum field theory of Gravitation are searching for a non-existent meaning and nature of Gravitation (as presently treated), just for the purpose to create a theory that should be entirely Quantum Mechanics symmetrical, analogical, and compatible. This is unrealistic, since the field of gravitation is profoundly linked to electromagnetic phenomenology, based on structural, spatial standing-waves phenomenology of involved electromagnetic fields, currents, and electromagnetic dipoles-polarizations of involved masses. All of that (as conceptualized here) looks on some way familiar to String theory foundations, to N. Tesla Dynamic Gravity descriptions, and to Rudjer Boskovic Universal Natural Force concepts (see more in [6], [97], [117] and [144]).

Quantum nature of our Universe are manifestations of a multi-resonant assembly of different (mutually coupled) resonant states and standing matter-waves formations. All *stable matter states* in our Universe are structural (spatial-temporal) combinations of more elementary, vibrating, and resonant states with dualistic wave-particle properties, equally applicable to a micro and macro world of Physics. This way we are coming closer to "***String theory" concepts***. Since the ***String theory is the most promising, universal, natural fields and forces unification platform, or concept***, it would be very much beneficial to merge and unify here favoured mater structure concepts based on resonant and standing matter waves (***originating from atoms***) with the contemporary String theory achievements (where the most important, or background mathematical and modelling support should comply with Kotelnikov-Shannon-Nyquist-Whittaker and Complex Analytic Signal analysis theory).

We also know that High Power mechanical, ultrasonic or acoustical energy, moments, forces, oscillations and vibrations, or audio signals, can be created and transferred by applying different signal-modulating techniques on laser beams and/or dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations; - See in Chapter 10., (10.2-2.4), and literature references from [133] until [139].

....

The simplest, little bit naive, and descriptive aspect of explaining Gravitation is what we already know from our experience, and from Newton and Einstein theories (and this still works well, but offered relevant explanations are still superficial and incomplete). Understanding of Gravitation is on the way to significantly evolve becoming a better theory, compared to what Newton and Einstein established (on different ways). A deeper and more sophisticated explanation of Gravitation (which is also opening and supporting antigravity options) is related to:

1. *Wave Particle Duality ... and to universal applicability of Uncertainty relations to any micro and macro cosmic situation ...*
2. *Electromagnetism and effects of synchronized electromagnetic polarisations, are effectively creating Coulomb attractive forces, resembling on Gravitation. Gravitation is anyway certain subtle integration between Electromagnetic theory and overlaying Mechanics ...*
3. *Understanding of atoms and extending atomic fields and forces towards other masses and atoms of our Universe is what creates force of Gravitation.*
4. *Structural vibrations of our Universe and effects of associated attractive and repulsive forces in nodes and anti-nodes of standing waves (between masses) are anyway creating effects of Gravitation (as well as effects of other fundamental natural forces, and this could be on some way familiar to String theory concepts).*
5. *Conceptual revelations coming from Nikola Tesla (Dynamic Gravity theory), and Rudjer Boskovic (Universal Natural Force), are essentially correct (but still not formulated mathematically, and properly explained). For instance, when a big mass is attracting a small mass, something should permanently radiate between a big and small mass (or from their atoms), presenting Tesla's radiant energy flow. Since force of gravitation (in average and roughly) is the same around any spherical mass (being on certain stable orbital level), that means that masses are almost uniformly and omnidirectionally emitting mentioned radiant energy, or saying this differently, between mutually attracting masses there is always certain bidirectional radiant energy flow (proportional to involved masses), producing the force of gravitation. Such radiant energy flow and force of Gravitation have much more chances to belong to electromagnetic phenomenology (then to conveniently invented Gravitation), since internal structure of matter is electromagnetic.*
6. *Translational and Rotational Symmetry, linear, angular moments, and energy conservation is the important background of Gravitation. All stable, linear, uniform, and inertial motions are mutually relative motions, but similar (relative motions) concept is also applicable to stable rotational or orbital motions that are accelerated motions.*
7. *Universal Entanglement and "spatial-temporal-spectral" Synchronization effects are also supporting Gravitation and creation of standing matter waves between mutually attracting masses.*

8. *Based on universally valid and applicable Analogies (as presented in the first Chapter of this book), we can conclude that real sources of Gravitation are not only stable and static masses, but atoms belonging to masses in motions (including vibrations), with mechanical and electromagnetic fields, moments, and charges, are.*
9. *For proper and complete understanding of Gravitation (or electromagnetism) we need to revitalize the concept of ether, that should be some very fine, almost ideal fluid, being in the background of all matter in our Universe ...*
10. *Spatial-Temporal unity, proportionality and mutual domains transformability is also an important part necessary for understanding Gravitation.*

All here mentioned aspects relevant for deeper understanding of Gravitation (from 1. to 10.) are interrelated, united, and synchronized. It is not at all easy to extract certain simplified meaning of Gravitation from its larger context of all here mentioned items. In other words, Gravitation only looks as being one of fundamental natural and easy explicable forces, but essentially it has an electromagnetic nature and high complexity. As a short summary we could also say,

a) *That Quantum theory and Wave-Particle Duality concepts can and should be significantly updated and replaced by new and much better theories and concepts (where probabilistic ontological grounds of present Quantum theory will become only useful mathematical processing tools when conditions for applying Probability and Statistics are mathematically satisfied). In this book we can find number of ideas how to make Quantum theory revision.*

b) *That concept of four fundamental natural forces will also significantly evolve, showing present forces classification as being obsolete, oversimplified and very much naive concepts, or almost meaningless (regardless of having certain mathematical complexity). If, or when something like that happens, some chapters of modern Physics would either disappear or evolve and change a lot (such as Standard Model of matter, Black holes concepts, Black or Dark mass and energy ...), requesting much better and more united, new conceptualization. ♣]*

....

The position of the author of this book is that Particle-Wave Duality framework of modern Physics should be related to the complex nature of (already known and maybe still unknown or unexposed) fields and forces of stationary and standing matter waves between interacting objects, equally applicable to a micro and macro universe. Here, linear motion and associated spinning are presented as mutually coupled or conjugate pair of motional entities (behaving similar as when an electric and magnetic field couple is creating electromagnetic waves; - See also equations (4.1), (4.2), (4.3), (4.33.1), (4.5-1) – (4.5-3), T.4.2 and T.4.3, which are supporting the unity of linear motion and spinning). Anyway, eventually, the final picture and explanations of Particle-Wave Duality (as favored in this book) should become clearer, more straightforward, and much more obvious, compared to the present state of the art of Quantum theory (free of unnecessary "probabilistic framework", except in mathematically reasonable cases related to mass data processing; -see more in Chapter 10.).

The intention of the author of this book is also to show (presently only on the conceptual level of understanding) that, regarding gravity and "*linear-motion and rotation-related phenomenology*", the questions about their essential field sources and charges are not entirely or clearly answered. This is unlike as we have in electromagnetic theory, even though General Relativity Theory, from the topology platform, in many cases is correctly describing spatial deformations and consequences caused by the presence of gravitational participants. It is also clear that present, simple forms of Newton-Coulomb force laws are applicable only in a limited framework, but still showing that an electromechanical analogy and different forces and fields coupling exist between all relevant participants.

Here, we are trying to show (on an easiest possible way; -by addressing Newton-Coulomb force laws combined with new hypothetical proposals regarding vibrating and resonating Universe) that space between two interacting objects in mutually relative motion, has multi-component, mutually coupled, electromagnetic field structure, like presented in (2.4-3), and in Chapter 3. of this book. Also, *specific field, and force components between interacting objects (intimately involved in particle-wave duality phenomenology) are still not completely conceptualized, or maybe not known in the contemporary physics.* Relations from (2.4) until (2.4-3) are also too far from being generally applicable, but here serving to give a limited brainstorming insight about possible complexity of fields and forces between two interacting objects, at least to show directions where we could search for new interactions, and new ideas about *universal field theory* upgrading.

It is already clear that the most relevant set of field-charges would evolve from present grounds (effectively being reestablished or reinvented), becoming more general and internally more coherent. The conceptual platform regarding gravitation related forces, introduced by (2.3) - (2.4-3) and elsewhere in this book, is based on exploiting interactions between "energy-momentum" content of mutually coupled, internally resonating, and mutually reacting matter domains, having associated elements of spinning and other angular movements. Energy-momentum content and distribution of such coupled-domains should be the starting position for any new charge/field/force foundation. Consequently, static mass that is presently considered as the principal cause, or source of Gravitation, will probably be effectively replaced by interactions between involved (mutually coupled) linear and angular moments (and other natural fields charges), manifesting as spinning and electromagnetic properties of internal matter-domains, including effects between oscillating or resonating masses (implicating that gravitation could be something more than a simple, conservative field, or central force).

Later, in chapter 5, we will see that mass understanding could be conceptualized differently involving certain kind of mutually coupled electric and magnetic dipole charges, and/or elements of linear motion and spinning, as for instance, (see (5.2) and (5.2.2)),

$$m=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{c^2\Delta t}=\frac{\Phi\cdot q}{c^2\Delta t}=\frac{\Phi\cdot i_{el.}}{c^2}=\frac{\Delta\alpha\cdot\Delta L}{c^2\Delta t}=\frac{\Delta\alpha\cdot\tau}{c^2}=\frac{\Delta x\cdot\Delta p}{c^2\Delta t}=\frac{\Delta x\cdot F}{c^2}=\frac{\Delta E}{c^2},$$
$$\left(\begin{array}{l} i_{el.}=\frac{\Delta q}{\Delta t},\Delta q_{mag.}=\Phi=\text{magnetic flux},\ q_{el.}=q=\text{electric charge},\\ \tau=\frac{\Delta L}{\Delta t}=\frac{\Delta E}{\Delta\alpha},F=\frac{\Delta p}{\Delta t}=\frac{\Delta E}{\Delta x},\ \frac{\Delta E}{\Delta p}=\frac{\Delta x}{\Delta t},\ \Delta E=\tau\Delta\alpha=F\Delta x.\end{array}\right) \tag{2.4-6}$$

Based on (2.4-6) we should also be able to explain gravitational attraction of neighbor's masses as kind of attraction between their electrically and/or magnetically, "partially and weakly polarized dipoles", thanks to certain level of spontaneous (mutually induced and opposite) polarization of corresponding electromagnetic charges (see [121]). For instance, if such (presently hypothetical polarization) is really happening, we would have the situation that Newton masses attraction should be equal to certain Coulomb effective-charges attraction, as follows,

$$\left\{\begin{array}{l} F_{12} = G\frac{m_1 m_2}{r^2} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}, p_1 = \frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = p_2 \\ \Rightarrow m_1 m_2 = \frac{1}{4\pi\varepsilon G} q_1 q_2,\ m^2 = \frac{1}{4\pi\varepsilon G} q^2, \\ \frac{q_1}{m_1} = \frac{q_2}{m_2} = \sqrt{4\pi\varepsilon G} = 8.616032252 \cdot 10^{-11}\left[\frac{C}{kg}\right] = \text{const.}, \\ \text{valid for vacuum}. \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{m_1}{m_2} = \frac{q_1}{q_2} = \frac{v_2}{v_1}\frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}} (\cong \frac{v_2}{v_1}, v_{1,2} << c) \\ \frac{q_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{q_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{q_1 p_1}{m_1} = \frac{q_2 p_2}{m_2} \end{array}\right\}.$$

(2.4-7)

What we see from (2.4-7) is that the "*one-sided charge-to-mass ratio"* (of slightly polarized, bat macroscopically electrically neutral masses) is velocity independent and constant ($\frac{q}{m} = \sqrt{4\pi\varepsilon G} = 8.616032252 \cdot 10^{-11}\frac{C}{kg}$, in free space and vacuum). This could be the principal reason that Newton force between two masses (which are anyway in relative motion) is equal or proportional to Coulomb force between two corresponding effective electric or magnetic charges (of mutually opposed electric and magnetic dipoles) since Newton and Coulomb's laws have the same mathematical forms. Here, we should not forget that relevant, effective charges (in (2.4-7)) are not freestanding charges (because they belong to corresponding electrical dipoles embedded in relevant masses). Since one-sided charge-to-mass ratio (of electrically neutral and slightly polarized masses) is a tiny number ($8.616032252 \cdot 10^{-11}\frac{C}{kg}$), this is the explanation why such effects of week spontaneous dipole (or multi-pole) polarization are still not experimentally detected. In cases of significant astronomical objects (where dipole charges in question could be significantly stronger), again there is a measurement problem since it is not possible or easy to apply direct measurements. Anyway, we know that the freestanding charge-to-mass ratio of an electron is also velocity-independent and constant (as in (2.4-7), but a much more significant number) and that it can be found experimentally, or calculated theoretically as,

$$\frac{e}{m_e} = 1.759 \cdot 10^{11}\frac{C}{kg}.$$

Of course, the electron is not a dipole or an electrically neutral mass, and this is the reason why its charge-to-mass ratio is enormously more significant, compared to the similar ratio of electrically neutral masses. The "$q/m=e/m_e$" of an electron was successfully measured and calculated by J.J. Thomson in 1897, and more successfully by Dunnington's Method, which involves the angular momentum and deflection due to a perpendicular magnetic field.

The same way, we could calculate the gravitational attraction between two electrically neutral atoms (considering that some very small or weak electric-dipole polarization, *"(+q)↔(-q)"* would for some natural reason appear between them). In the case of two hydrogen atoms, a charge amount of such spontaneous polarization between them (here expressed as q) will be:

$$\left\{\begin{array}{l} \frac{q}{(m_e + m_p)} = \sqrt{4\pi\varepsilon G} = 8.616032252 \cdot 10^{-11} \frac{C}{kg}, \\ \frac{e}{m_e} = 1.759 \cdot 10^{11} \frac{C}{kg},\ m_p = 1836.152701(37) \cdot m_e, \\ \frac{e}{m_e + m_p} = 957459.8775 \frac{C}{kg} \end{array}\right\} \Leftrightarrow q = 8.998844186 \cdot 10^{-15} e.$$

Calculated polarized (one side) charge is small ($q = 8.998844186 \cdot 10^{-15} e$) to be seriously considered for hydrogen atoms attraction, but in cases of astronomic objects, a similar situation could be much different.

Analog to (2.4-7), we can extend the same chain of conclusions considering relevant dynamic "magnetic polarization charges" or (one-sided) magnetic fluxes, as for instance,

$$\left\{\begin{array}{l} F_{12} = G\frac{m_1 m_2}{r^2} = \frac{1}{4\pi\ \mu}\frac{\Phi_1\Phi_2}{r^2}, p_1 = \frac{m_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{m_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = p_2 \\ \Rightarrow \Phi_1\Phi_2 = 4\pi\ \mu G \cdot m_1 m_2,\ m^2 4\pi\mu G = \Phi^2, \\ \frac{\Phi_1}{m_1} = \frac{\Phi_2}{m_2} = \sqrt{4\pi\mu G} = 32.45920531 \cdot 10^{-9}\left[\frac{W_b}{kg}\right], \\ \frac{q_1}{m_1} = \frac{q_2}{m_2} = \sqrt{4\pi\varepsilon G} = 8.616032252 \cdot 10^{-11}\left[\frac{C}{kg}\right] \\ c = \frac{1}{\sqrt{\varepsilon\mu}} = 2.99792458 \cdot 10^8 \frac{m}{s}\ (\text{in vacuum}), \\ G = (6.67428\ \pm\ 0.00067) \cdot\ 10^{11}\left[\frac{m^3}{kg \cdot s^2}\right]. \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{\Phi_1 v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{\Phi_2 v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{\Phi_1 p_1}{m_1} = \frac{\Phi_2 p_2}{m_2}, \\ \frac{m_1}{m_2} = \frac{\Phi_1}{\Phi_2} = \frac{q_1}{q_2} = \frac{v_2}{v_1}\frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}} (\cong \frac{v_2}{v_1}, v_{1,2} << c), \\ \frac{\Phi_1}{q_1} = \frac{\Phi_2}{q_2} = \sqrt{\frac{\mu}{\varepsilon}} (= 377\,\Omega,\ \text{for vacuum}), \\ \Phi = q\sqrt{\frac{\mu}{\varepsilon}} = m\sqrt{4\pi\mu G} = 32.45920531 \cdot 10^{-9} \cdot m\,(=)\left[W_b\right], \\ q = m\sqrt{4\pi\varepsilon G} = 8.616032252 \cdot 10^{-11} \cdot m\,(=)\left[C\right], \\ \frac{\Phi q}{m^2} = 4\pi G\sqrt{\varepsilon\mu} = \frac{4\pi G}{c} = 4191.690044\left[\frac{m^2}{kg \cdot s}\right] \end{array}\right\}$$

$$\Rightarrow m = \frac{\Phi}{\sqrt{4\pi\mu G}} = \frac{q}{\sqrt{4\pi\varepsilon G}} = \sqrt{\frac{c\Phi q}{4\pi G}} = \begin{pmatrix} 1.160627039 \cdot 10^{10} \cdot q \\ 3.08079015 \cdot 10^7 \cdot \Phi \end{pmatrix} = 597967252.4\sqrt{\Phi q}\,(=)\left[kg\right] \tag{2.4-8}$$

Analog to Charge-to-mass ratio we also know that Gyromagnetic ratios for many of elementary particles are always constant numbers, and why not to have a similar situation for any other mass including astronomic objects (like we see in (2.4-7) and (2.4-8)). Also, planets that have their own magnetic field, like our planet Earth, should have specific, quantifiable relation ($\Phi = q\sqrt{\frac{\mu}{\varepsilon}} = m\sqrt{4\pi\mu G} = 32.45920531 \cdot 10^{-9} \cdot m\,(=)\left[W_b\right]$) between planet's mass and its total (one-sided) magnetic flux. In the case of the planet Earth, $m\ =\ 5.976 \cdot 10^{24}\ kg$, $\Phi = 32.45920531 \cdot 10^{-9} \cdot 5.976 \cdot 10^{24} = 193.9762109 \cdot 10^{15}\ W_b$. Here we should imagine an equivalent bar-magnet and consider the total magnetic flux from one side (single pole) of the magnet. Of course, in cases of such planetary magnetic fields, we could have a superposition of some other, spinning, and internal electric currents related magnetic field components (for instance, electric dipoles rotation under favorable conditions will also produce magnetic fields). Anyway, since electric and magnetic phenomena and fields are always mutually coupled (do not exist as isolated entities), and since all matter states motions in our universe are relative linear and angular motions (including oscillatory and resonant states), it should be clear that gravitation and electromagnetism are very much connected regardless of what our contemporary theories and concepts are presenting.

Again, in the case of our planet Earth, the total one-sided, dipole electric charge (2.4-8) should be $q = 8.616032252 \cdot 10^{-11} \cdot m = 8.616032252 \cdot 10^{-11} \cdot 5.976 \cdot 10^{24} = 51.48940874 \cdot 10^{13} C$. For comparison, the negative electron charge is $1.6021892 \cdot 10^{-19} C$, producing that one “Earth-electric-dipole” side could capture $32.13690914 \cdot 10^{32}$ electrons. Of course, such displaced electrons are distributed on a large surface. All that conceptualization (here over-simplified) looks, sooner or later, on some ways verifiable, and of course, it is mixed with familiar electromagnetic and fluid dynamics complexity (see successful, more advanced, but familiar concepts published under [63], Arbab I. Arbab). The author of this book also introduced in [3] similar ideas about effective electric masses-polarizations and equivalency or analogy between masses and electrostatic charges attraction.

*Let us imagine that **q** presents a single electron, **e =1.602176565 x 10⁻¹⁹C**. The corresponding (minimal) magnetic flux of permanent spinning, belonging to an electron charge (in a vacuum), will be* $\Phi_0 = e\sqrt{\frac{\mu_0}{\varepsilon_0}}$ *= **6.03588480 x 10⁻¹⁷ Wb**. But, since the relation* $\Phi = q\sqrt{\frac{\mu}{\varepsilon}}$ *is formulated for electrically neutral particles that are for specific reason electrically and magnetically polarized as dipoles (2.4-8), the one-side magnetic flux value of **6.03588480 x 10⁻¹⁷ Wb** is more likely to be valid for a certain hydrogen atom state.*

Most probably, Northern and Southern Polar Lights (or Auroras) are not only related to solar winds (of electrically charged particles) that are trapped in the Earth's magnetic field. Here-hypothesized “electric-dipoles, or **Earth and Sun mutually induced polarization**” should have its significant guiding or attracting contribution to such polar light and solar wind effects (or to currents and winds of electrically charged particles between the Sun and Earth). Since solar winds are real and detectable, another reality, still under probation, would be to show that mentioned electromagnetic dipoles polarization has an essential place in explaining currents of electrically charged particles between the Sun and Earth. Of course, atmospheric thunderclouds’ lightning (between electrically charged clouds and Earth) could also be in close relation to here hypothesized electrostatic polarization and solar winds. All of that, if shown correct, will give another challenging contribution to understanding electromagnetic fields and Gravitation (see later Fig.2.1 and equations (2.4-9) and (2.4-10)).

It could also be that Casimir Effect, known experimentally in Quantum Physics (see [103] and [104]), is not too far from here hypothesized attraction based on “spontaneous electromagnetic polarization” between two neighbors’ masses (The Casimir force is an attractive force between two flat conductive surfaces in vacuum).

The far-reaching consequences related to here-hypothesized electromagnetic polarizations intrinsically coupled with mechanical motions and gravitational attraction of involved objects could lead to an explanation of gravitation as the specific manifestation of electromagnetic interactions (see much more of familiar or very indicative ideas in Chapter 3. of this book, and in [72], Dr. László Körtvélyessy; -The

Electric Universe, as well as in papers from Prof. Dr. Jovan Djuric [71], and in [121] and [144]). Of course, the meaning of mass (such as inertial, heavy, gravitational, rest, relativistic…) will also evolve.

[♣ COMMENTS & FREE-THINKING CORNER:

Now, after correlating and combining results and relations from (2.4-6) until (2.4-8) we could start getting another challenging insight into (2.4-4), which is an empirically verified $\frac{1}{r^2}$ force law between equidistant, parallel paths moving electrical charges (q_1 , q_2). The arising message here should be that matter particles have mutually correlated, coupled and dependent intrinsic properties such as mass, relevant dipole-related electric charges, and associated magnetic moments and fluxes (as well as stable Gyromagnetic and charge-to-mass ratios). Of course, it is still too early to draw generally valid conclusions, and most probably, that lot of new conceptual and mathematical fittings and managing efforts should be implemented here, but the direction towards an innovative conceptualization of matter structure and involved forces is already paved. The following set of mutually analog relations and results, extracted from (2.4-4), (2.4-6), (2.4-7) and (2.4-8), could serve only as a brainstorming reminder about some of remaining challenging problems in connection with mentioned innovative conceptualization about Gravitation. This is a part of objectives in front of us (not to mention many other challenging issues initiated by [35], Thomas E. Phipps, Jr., Old Physics for New, and [121], Raymond HV Gallucci, Electromagnetic Gravity? Examination of the Electric Universe Theory).

$$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1v_1)\cdot(q_2v_2)}{r^2} = K\frac{(q_1v_1)\cdot(q_2v_2)}{r^2},\ K = \text{const.},$$

$$m = \frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{c^2\Delta t} = \frac{\Phi\cdot q}{c^2\Delta t} = \frac{\Phi\cdot i_{el.}}{c^2} = \frac{\Delta\alpha\cdot\Delta L}{c^2\Delta t} = \frac{\Delta\alpha\cdot\tau}{c^2} = \frac{\Delta x\cdot\Delta p}{c^2\Delta t} = \frac{\Delta E}{c^2},$$

$$\frac{q_1v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{q_2v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{q_1p_1}{m_1} = \frac{q_2p_2}{m_2}\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix}\frac{\Phi_1v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{\Phi_2v_2}{\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{\Phi_1p_1}{m_1} = \frac{\Phi_2p_2}{m_2},$$

$$\frac{q_1}{m_1} = \frac{q_2}{m_2} = \sqrt{4\pi\varepsilon G} = 8.616032252\cdot10^{-11}\frac{C}{kg}\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix}\frac{\Phi_1}{m_1} = \frac{\Phi_2}{m_2} = \sqrt{4\pi\mu G} = 32.45920531\cdot10^{-9}\frac{W}{m},$$

$$\frac{q_1q_2}{m_1m_2} = 4\pi\varepsilon G = 74.23601177\cdot10^{-22}(\frac{C}{kg})^2\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix}\frac{\Phi_1\Phi_2}{m_1m_2} = 4\pi\mu G = 1053.600009\cdot10^{-18}(\frac{W}{m})^2,$$

$$\Phi = q\sqrt{\frac{\mu}{\varepsilon}} = m\sqrt{4\pi\mu G} = 32.45920531\cdot10^{-9}\cdot m\,(=)[W_b]\begin{pmatrix}\leftrightarrow\\ \text{analog}\end{pmatrix} q = m\sqrt{4\pi\varepsilon G} = 8.616032252\cdot10^{-11}\cdot m\,(=)[C],$$

$$\frac{m_1}{m_2} = \frac{\Phi_1}{\Phi_2} = \frac{q_1}{q_2} = \frac{v_2}{v_1}\frac{\sqrt{1-\frac{v_1^2}{c^2}}}{\sqrt{1-\frac{v_2^2}{c^2}}}(\cong\frac{v_2}{v_1}, v_{1,2} << c),\ \frac{\Phi_1}{q_1} = \frac{\Phi_2}{q_2} = \sqrt{\frac{\mu}{\varepsilon}}(=377\,\Omega,\text{ for vacuum}),$$

$$\frac{\Phi q}{m^2} = 4\pi G\sqrt{\varepsilon\mu} = \frac{4\pi G}{c}, m = \frac{\Phi}{\sqrt{4\pi\mu G}} = \frac{q}{\sqrt{4\pi\varepsilon G}} = \sqrt{\frac{c\Phi q}{4\pi G}} = \begin{pmatrix}1.160627039\cdot10^{10}\cdot q\\ 3.08079015\cdot10^{7}\cdot\Phi\end{pmatrix} = 597967252.4\sqrt{\Phi q}\,(=)[kg],$$

(2.4-4), (2.4-6), (2.4-7), (2.4-8)

$$c = \frac{1}{\sqrt{\varepsilon\mu}} = 2.99792458\cdot10^8\frac{m}{s}\ (\text{in vacuum}).$$

…………………………………… ♣]

Let us now try to explore the simplified modeling of the possible partial electrical dipole polarizations between two electrically neutral masses, which are related to (2.4-7), as illustrated on the Fig.2.1. Relatively isolated and mutually distant masses **m = m_1** and **M = m_2** are attracting each other by Newton gravitation force, and at the same time both masses are (hypothetically) becoming slightly electrically polarized, aligning, and orienting their electrical dipoles in a mutually opposite positions that the force between them will only be attractive (as presented on Fig.2.1). The dipole belonging to a mass **m** has electrical charges **+q*** and **–q***

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and the dipole belonging to a mass **M** has electrical charges **+Q*** and –**Q*** (**q = q_1**, **Q = q_2**). The dipole length or distance between **+q*** and **–q*** is **d**, and between +**Q*** and –**Q*** is **D**, and distance which is separating centers of masses **m** and **M** is **r**.

After we represent masses attraction by equivalent and effective electrostatic dipoles attraction (using Coulomb force law, based on data and configuration from Fig.2.1, where polarized electrical charges are presented as couples of smaller circles inside of bigger circles), we will get results as shown in (2.4-9) which are equivalent to (2.4-7). What is extremely interesting in (2.4-9) is the fact that the force between relevant electrical dipoles (which corresponds to Newton masses attraction) has a negative sign, and here, the first time we have a clear conceptual picture and mathematical explanation why masses are experiencing only a mutually attractive force. In cases of several masses (attracting each other gravitationally), we should imagine an equivalent multi-poles electrical polarization. Of course, we still do not know why and how, at least two neighboring, electrically neutral masses could get certain electrical dipole polarizations. Since "macro-mass and atoms embedded" carriers of electrical charges (electrons and protons) have very much different masses they should be susceptible to the specific level of masses and electric dipoles separation caused by associated rotation, centrifugal forces and accelerated or curvilinear motions, producing rotation-related coupling forces between them. If this would be the case, we will be able to explain Newton law and Gravitation from the platform of electromagnetic theory as in (2.4-9) or (2.4-10).

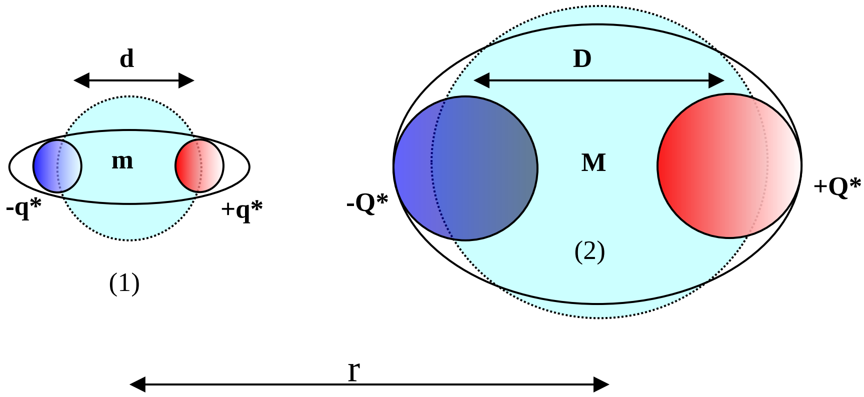

Fig.2.1. Conceptualization of electrical dipoles polarization between two masses

$$F_{12} = -G\frac{mM}{r^2} \cong \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2} + \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{(r+\frac{d}{2}+\frac{D}{2})^2} - \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{(r+\frac{d}{2}-\frac{D}{2})^2} - \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{(r-\frac{d}{2}+\frac{D}{2})^2} =$$

$$= \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2}\left[1+\frac{r^2}{(r+\frac{d}{2}+\frac{D}{2})^2} - \frac{r^2}{(r+\frac{d}{2}-\frac{D}{2})^2} - \frac{r^2}{(r-\frac{d}{2}+\frac{D}{2})^2}\right] = -\frac{1}{4\pi\varepsilon}\frac{qQ}{r^2} = -G\frac{mM}{r^2}, \quad (2.4\text{-}9)$$

$$qQ = q^*Q^*\left[1+\frac{r^2}{(r+\frac{d}{2}+\frac{D}{2})^2} - \frac{r^2}{(r+\frac{d}{2}-\frac{D}{2})^2} - \frac{r^2}{(r-\frac{d}{2}+\frac{D}{2})^2}\right] \cong q^*Q^*\left[\frac{r^2}{(r+\sqrt{d\cdot D})^2}-1\right] < 0.$$

We can also find couple more of similar forms of attractive forces applicable to the situation from Fig.2.1, and make similar conclusions like in (2.4-9),

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$$F_{12} = -G\frac{mM}{r^2} \cong \frac{1}{4\pi\varepsilon}\left[\frac{q^*Q^*}{r^2} - \frac{q^*Q^*}{(r+D)^2} - \frac{(q^*)^2}{d^2}\right] = \frac{1}{4\pi\varepsilon}\left[\frac{q^*Q^*}{r^2} - \frac{q^*Q^*}{(r+d)^2} - \frac{(Q^*)^2}{D^2}\right] =$$
$$= \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2}\left[1 - \frac{r^2}{(r+D)^2} - \frac{q^*}{Q^*}(\frac{r}{D})^2\right] = \frac{1}{4\pi\varepsilon}\frac{q^*Q^*}{r^2}\left[1 - \frac{r^2}{(r+d)^2} - \frac{Q^*}{q^*}(\frac{r}{d})^2\right] = \quad (2.4\text{-}10)$$
$$= -\frac{1}{4\pi\varepsilon}\frac{qQ}{r^2} = -G\frac{mM}{r^2},\ qQ = q^*Q^*\left[1 - \frac{r^2}{(r+D)^2} - \frac{q^*}{Q^*}(\frac{r}{D})^2\right] = q^*Q^*\left[1 - \frac{r^2}{(r+d)^2} - \frac{Q^*}{q^*}(\frac{r}{d})^2\right] < 0.$$

Like in (2.4-9) and situation from Fig.2.1, we should be able to make another conceptualization that corresponds to (2.4-8), but now taking into consideration attraction between certain kind of associated and equivalent “magnetic dipoles”, or weekly-magnetized permanent magnets (mutually aligned on the way that only attractive force between them is possible). For ordinary, discrete permanent magnets, it is already known that force between them is respecting similar mathematical form like in cases of Coulomb and Newton force laws. All kind of electric or magnetic phenomena, here analyzed separately in a limited framework are anyway interconnected, what we can see if we extend the scope of such analysis to a broader context. The simplified electrostatic conceptualization presented on Fig.2.1 can be naturally extended covering associated and mutually coupled electric and magnetic fields. *The main reasons or sources of mentioned electromagnetic dipole (or multi-pole) mutually induced polarizations are most probably coming from the fact that for specific body its center of mass, center of inertia, center of gravitation, equivalent center of electric charge/s distribution, and its magnetic poles centers are separated (not in the same point, or not geometrically identical). This could be particularly significant phenomenology in cases of rotations because of the enormous masses difference between positive and negative electric charges (proton mass is almost 2000 times bigger than an electron mass), where centrifugal mass and electromagnetic entities separation (or some other inertial and transitory effect) will produce electric (and magnetic) charges separation or dipoles creation. The last significant argument we can add here is that rotational, circular, and other curvilinear motions are most probably a dominant property of all matter in our universe. The universe is keeping its global macro-cosmological orbital momentum in equilibrium (differently saying that total orbital momentum of our universe is conserved), and if some perturbation on the level of orbital momentum of specific small object happens, the rest of world will immediately feel it and recreate the global orbital momentum equilibrium, what presents macrocosmic entanglement effects. Since global Gyromagnetic and global electric-charge-to-mass ratios should also (analogically) be conserved on the Universe level, and since all of them are mutually coupled (belonging to the same body of the Universe), we should be able to make equivalent and analogical interpretations of gravitational forces from different platforms, as already elaborated in this book.*

Consequently, every electromagnetic, rotational, or spinning perturbation in a specific part of our universe will penetrate our universe, keeping global balances of all respective properties conserved. As we know, circular, elliptic, pendulum and other curvilinear motions are creating accelerated movements towards their centers of rotation, and we feel or express such effects like effects of gravitation. Formerly and historically, gravity is associated to an attraction between masses because empirical and direct manifestations of such forces are giving us the false impression that only masses are significant in such an attraction, but angular, circular, and other rotational motions (including oscillatory and resonant states) are real sources of gravitation.

Here, we are coming much closer to understanding why forces between electrical charges, between permanent magnets, masses, etc. are fully analogical, respecting the same mathematical form of Newton-Coulomb force laws. The answer is, because all of them are globally conserved, mutually coupled, and balanced on the level of the whole universe, as well as combined and balanced with their associated orbital moments (see more complete explanation in chapter 10, equations (10.1.4)- (10.1.7)).

Based on (2.4-6) to (2.4-10) we will be able to conclude that Gravitation could effectively (and analogically) present a kind of weak electromagnetic-dipoles (or multi-poles) interaction (or attraction), mostly being much more a direct manifestation of central forces belonging to associated, overwhelming, and intrinsic rotational motions in our universe, which are coupled with relevant electromagnetic charges and electromagnetic field complexity. Analogical, and even more profoundly elaborated concepts (concerning Wilhelm Weber's electrodynamics), about electric dipoles interactions as real sources of gravitation, can be found in [69], A.K.T. Assis; "Gravitation as a Fourth Order Electromagnetic Effect". Other mass-dipoles related, gravitation force origin is presented in [71], Jovan Djuric, "Magnetism as Manifestation of Gravitation". Number of revolutionary (and verifiable) ideas, showing or constructing rich and well-operating analogy between masses, electric charges, electromagnetic field, gravitational and Coulomb force, applied to planetary systems and N. Bohr atom model can be found in [63], Arbab I. Arbab, and in [64], Marçal de Oliveira Neto.

If electromagnetic forces and charges (including mechanical and electromagnetic moments) are essential sources of gravitation (originating from atoms of involved masses), consequently, what we could expect to see as gravitational waves should be very low-frequency electromagnetic waves coupled with similar cosmic macro-oscillations. In cases of stable solar or planetary systems, we should be able to find such standing and stationary, macro electromagnetic field structures between planets and a local sun.

Since such "electromagnetic-gravitation" cannot be shielded and stopped using some metal (or solid-state) plates and envelopes, because such shields (consisting of atoms) will also produce similar radiant energy flow and integrate in the overall radiant energy flow (between attracting masses), the more promising way to shield and attenuate or reflect effects of gravitation could be to use specific cellular or with "spatial-periodicity-assembled resonant shielding envelopes". This way we could manipulate effects of gravitation and even produce effects of antigravitation.

We can also find another extraordinary and complementary approach in explaining Newton-Coulomb forces because of global (holistic) conservation of orbital and spin moments in [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces". See also [121].

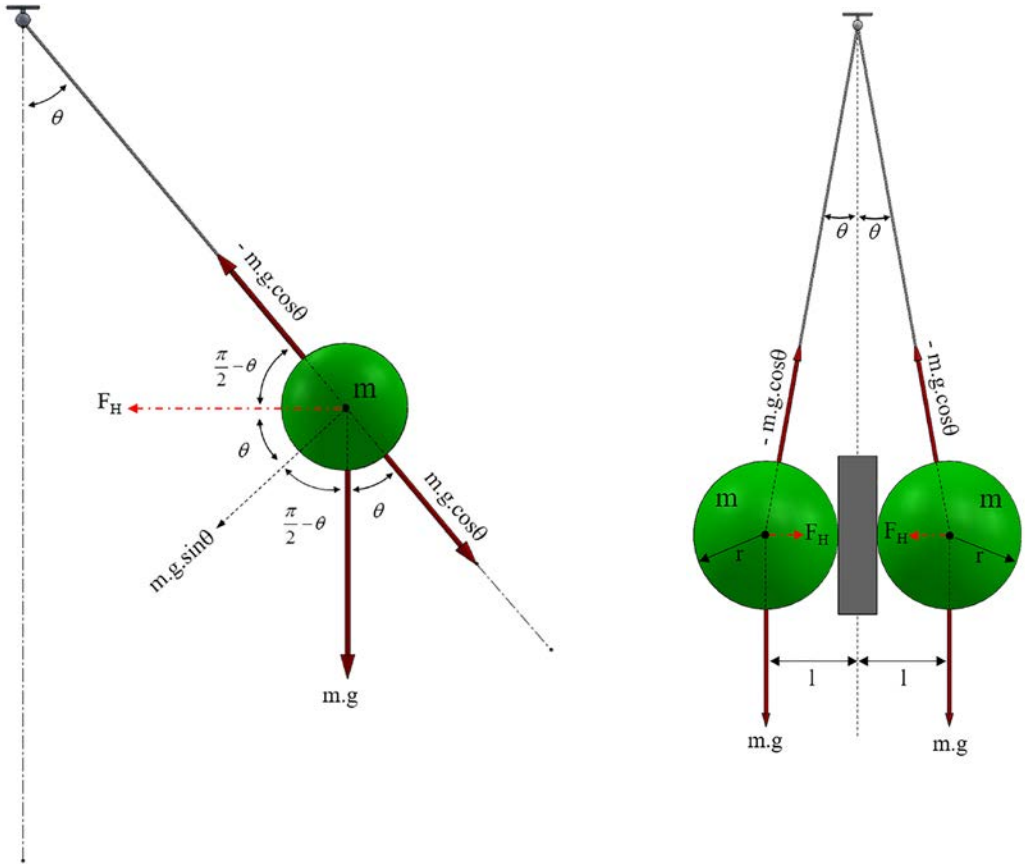
There is specific (astronomic) observational and experimental evidence that orbiting and spinning bodies or systems could be strongly coupled, on a way that their orbital and spin moments are mutually communicating with almost infinite speed, or at least for 10 orders of magnitude higher speed compared to the speed of light (see [65], T. Van Flandern, J.P. Vigier). Such facts (if shown entirely correct) are introducing critical doubts in most of the foundations of Relativity Theory.

About "Origins of rotation of celestial and astronomic objects", we can find still challenging sources of information in the works cited under [51], Pavle Savic / R. Kasanin / V. Celebonovic.

Many familiar and other challenging ideas and relations between fundamental constants, masses, and forces of our universe, can be found in the papers published by:

-Anatoly V. Rykov (see literature under [32]), as well as in [3] , [33] and [34]: 1°, Journal of Theoretics, PACS: 03.50.Kk, "Magnetism as Manifestation of Gravitation",
-Dr. Jovan Djuric, [71], Balkanska 28, 11000 Belgrade, Serbia, http://www.journaloftheoretics.com/, oliverdj@eunet.yu , http://jovandjuric.tripod.com/, 2°, GRAVITATION FIELD AS ANNULED ELECTROMAGNETIC FIELD, I.E., GRAVITONS AS ENTANGLED PHOTONS, and Magnetism as Manifestation of Gravitation. -Dipl.-Ing. Andrija S. Radović, Address: Nike Strugara 13a, 11030 Beograd, Serbia & 3°, ESSENCE OF INERTIA AND GRAVITATION, Andrija S. Radović, http://www.andrijar.com/.
-Dr. László Körtvélyessy, [72]. The Electric Universe.

[♣ *COMMENTS & FREE-THINKING CORNER: How to detect* hypothetical oscillatory force of Gravitation



Single and two masses pendulum

Let us try to explore the effects of gravitational attraction F_H between two masses, based on pendulum forces analysis (as presented on the picture, above).

If we consider only single mass pendulum (left side picture), horizontal force component, F_H will be,

$\mathbf{F_H = mg \cdot \sin\theta \cdot \cos\theta}$.

If we create similar two-mass pendulum (as presented on the right-side picture), the attractive horizontal force between two masses, F_H, will be at least two times stronger compared to a single mass pendulum,

$\mathbf{F_H = 2mg \cdot \sin\theta \cdot \cos\theta}$.

However, here we also need to consider force of horizontal gravitational attraction between two masses, and this will produce,

$$\mathbf{F_H = 2mg \cdot \sin\theta \cdot \cos\theta + \delta F_H = 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m \cdot m}{R^2} =}$$

$$\mathbf{= 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m^2}{R^2}\ ,\ (R = 2l)\ .}$$

Another questionable assumption here is that two-mass pendulum will intrinsically present specific half-wave dipole or mechanical resonator. Consequently, mentioned mechanical resonator will experience certain (probably exceptionally low amplitude) oscillatory or resonant motion, and if this could be the case, we will have, either

$$\mathbf{F_H = 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m^2}{R^2}\cos\omega_H t\ ,}$$

or more probably,

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$$\mathbf{F_H = 2mg \cdot \sin\theta \cdot \cos\theta + G\frac{m^2}{R^2} + \Delta F_H \cdot \cos\omega_H t} \ .$$

Now, let us imagine that we place one piezoelectric element, or sensor (like an accelerometer), between two masses, as already presented on the right-side picture with a gray-color rectangular block. New experimental and analytical options, starting from here, are:

1. If the oscillatory force component ($\cos\omega_H t$) really exist, we should be able to detect specific oscillatory-voltage signal on the piezoelectric element between two masses. Moreover, we could discover or listen some kind of "*breathing and pulsating*" of our Universe, by analyzing voltage-signal detected with a piezoelectric element.

2. Or we can measure complex electrical Impedance-Phase-Frequency curves of the two-mass system (directly on the piezoelectric element) and try to find sensitive and "*active*" impedance-frequency zones and draw some exciting conclusions,

3. Alternatively, we could externally bring specific oscillatory voltage signal on electrodes of a piezoelectric transducer and analyze the response of the two-mass pendulum system, and its interaction with our Universe and involved forces.

4. Also, we could place a permanent magnet (in different positions) in the vicinity of the two-masses-pendulum and try to detect if masses are being influenced by a magnetic field, and mutually interacting on a different way (of course, experimenting with masses that are not ferromagnetic or magnetically active).

5. Two-mass pendulum with the piezoelectric sensor could also serve as a detector of vibrations, and detector of seismic and gravitational perturbations.

The main idea (or objective) here is to verify if the oscillatory force component $\Delta F_H \cdot \cos\omega_H t$, or maybe $G\frac{m^2}{R^2}\cos\omega_H t$ (or something similar) really exist.

Later, based on geometrical relations (such as $R = 2l$, relationships for half-wavelength resonant dipoles, matter waves, etc.), we could unveil certain, still hidden, nature of gravitational forces, and show that gravitation is an attractive force only between moving or oscillating masses (or between electromechanical dipoles) with associated angular and linear mechanical moments, and most probably with some coupled electromagnetic moments and charges.

Another intention here is to show that gravitational attraction, only between neutral, not-charged, standstill (or static) masses, is not realistic. In other words, we may conclude and prove that we are living in a continuously moving (rotating and oscillating) Universe, where all motions are mutually coupled, and have linear and angular components (combined with complementary electromagnetic moments and charges).

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*[♣ **Free thinking corner... brainstorming.***

Macro mass state (or solid, liquid, and gaseous states) is something that internally has atoms and molecules. In cases of electromagnetically (and macroscopically) neutral masses, internal mass configuration and structure of internal electric and magnetic items, inside and between atoms and molecules (like electrons, protons and on some way neutrons), are mutually compensated, neutralized, and stabilized.

In cases when an external macro-mass state of motion is changing, evolving, or oscillating, internal electromagnetic masses states (of electrons, protons, spinning and magnetic moments states…) are also sensing external macro motion, and internally readjusting and electromagnetically polarizing, like dipoles and "multi-poles". Since carriers of positive and negative electric charges have enormously different masses, and linear and spinning motions are mutually united and complementary, the creation of internal, non-compensated electric and magnetic dipoles is an expected consequence. Such internal polarizations are also creating macroscopically measurable effects, fields, and forces (between masses), and in some instances, we are wrongly interpreting such forces as typically mechanical, and related only to gravitation. The existence of spontaneous dipole (electromagnetic) polarizations (if proven) is essentially indicating that all motions in our universe are non-linear, meaning belonging to curvilinear, torsional, angular, rotational, and spinning movements, which are motions with accelerations. In some way the whole world is rotating relative to some centers of rotations (for instance to galaxies centers, to black holes' centers, etc.). Since positive and negative carriers of electric charges have very much different masses, thanks to the overwhelming presence of motions with elements of rotations, we can expect the appearance of effects of centrifugal electromagnetic and mass separations, meaning the creation of electric and magnetic, micro and macro dipoles. This way is possible to have an electromagnetic or Coulomb-J. Michel attraction between big masses, which is by mistake characterized only as a gravitational attraction. In fact, here we always have manifestations of unity between electromagnetic and mechanical effects, motions and forces, and integration of linear, spinning, and rotational movements (since internal mass constituents have electric charges, spinning states, magnetic moments, and electrical and magnetic polarizations as dipoles). Expected (external) consequences of such situations (responsible for energy-momentum couplings, communications, and exchanges) are manifestations of matter waves (associated to moving masses), which, also, should have dominant electromagnetic nature. We also know that High Power, mechanical, ultrasonic or acoustical energy, moments, forces, oscillations and vibrations, or audio signals and music, can be created and transferred by applying different signal-modulating techniques on laser beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); - See relations from Chapter 10. under (10.2-2.4) and literature references from [133] until [139].

For instance, if we apply (external) mechanical oscillations on specific photovoltaic, solar cell (on a silicon chip), we will interfere with its internal electromagnetic dipoles and spinning states, by creating new, modified, internal resonant states, and this way we could enable specific photovoltaic cell to produce electric current excited with enlarged, wideband spectrum of low and high-energy photons.

It is also evident that gravitation should be causally related to the "two-body" problem, being the force between two bodies that are in a (mutually) relative motion. If bodies in question (in a Laboratory reference system) have masses m_1 and m_2, velocities v_1 and v_2, from classical mechanics we know that kinetic energy balance of such system can be presented as (2.4-11); -See Fig.4.1.5, chapter 4.1, and usual mathematical treatment of two-body problem, easily found in physics and/or mechanics books, is as follows,

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_cv_c^2+\frac{1}{2}m_rv_r^2,$$
$$m_c=m_1+m_2\,,\ m_r=\frac{m_1m_2}{m_1+m_2},\ \vec{v}_r=\vec{v}_1-\vec{v}_2,\vec{v}_c=\frac{m_1\vec{v}_1+m_1\vec{v}_1}{m_1+m_2},\qquad (2.4\text{-}11)$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

or it could also be valid (by applying a reversed analogy and most probable approximations, passing from non-relativistic motions to relativistic ones, where all velocities are related to the same laboratory system),

$$\left\{\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2=\frac{1}{2}m_cv_c^2+\frac{1}{2}m_rv_r^2\right\}\Rightarrow\left\{(\gamma_1-1)m_1c^2+(\gamma_2-1)m_2c^2=(\gamma_c-1)m_cc^2+(\gamma_r-1)m_rc^2\right\}\Leftrightarrow$$
$$\Leftrightarrow\left\{\begin{array}{l}\frac{\gamma_1m_1v_1^2}{1+\sqrt{1-v_1^2/c^2}}+\frac{\gamma_2m_2v_2^2}{1+\sqrt{1-v_2^2/c^2}}=\frac{\gamma_cm_cv_c^2}{1+\sqrt{1-v_c^2/c^2}}+\frac{\gamma_rm_rv_r^2}{1+\sqrt{1-v_r^2/c^2}},\\ \gamma_r=1/\sqrt{1-v_r^2/c^2},\ \gamma_i=1/\sqrt{1-v_i^2/c^2}\cong\left(1+\frac{1}{2}\frac{v_i^2}{c^2}\right)_{v<<c}\end{array}\right\}\Rightarrow \tag{2.4-12}$$
$$\Rightarrow\left\{\begin{array}{l}m_1v_1dv_1+m_2v_2dv_2=m_cv_cdv_c+m_rv_rdv_r=dE_{k1}+dE_{k2}=dE_{kr}+dE_{kc}=dE_k,\\ v_1dp_1+v_2dp_2=v_cdp_c+v_rdp_r=dE_k=v_cdp,\ p_i=\gamma_im_iv_i,\vec{p}=\vec{p}_1+\vec{p}_2=\gamma_1m_1\vec{v}_1+\gamma_2m_2\vec{v}_2=\gamma_cm_c\vec{v}_c\end{array}\right\}$$

In (2.4-12) we are faced with the imaginative and challenging possibility to understand the new kinematic reality of an artificial (mathematical) object that has mass m_r and velocity v_r , but also has an equivalent zero linear momentum ($\sum\vec{p}_r=\vec{0}$), since $\vec{p}=\vec{p}_1+\vec{p}_2=\gamma_1m_1\vec{v}_1+\gamma_2m_2\vec{v}_2=\gamma_cm_c\vec{v}_c$. Apparently, such m_r should be composed of at least two objects (effectively rotating around m_c of local center of mass) with mutually canceling moments. We can additionally address the nature of such m_r by applying Newton gravitation force law between m_1 and m_2 (including Kepler laws and Binary Systems relations). We will see that the same gravitational force is acting between masses m_c and m_r, and between m_1 and m_2 when compared in the same laboratory system of coordinates,

$$F_{12}=G\frac{m_1m_2}{r_{12}^2}=G\frac{m_cm_r}{r_{12}^2}\Leftrightarrow m_1m_2=m_cm_r,$$
$$E_{kr}=\left[\left(\frac{\gamma_rm_rv_r^2}{1+\sqrt{1-v_r^2/c^2}}\right)_{v_r<<c}\cong\frac{1}{2}m_rv_r^2\right]=\int_0^{r_{12}}F_{12}dr_{12}\Rightarrow \tag{2.4-13}$$
$$dE_{kr}=m_rv_rdv_r=F_{12}dr_{12}\Rightarrow F_{12}=m_r\frac{dv_r}{dt}=m_ra_r=m_r\frac{v_r^2}{r_{12}}=G\frac{m_1m_2}{r_{12}^2}=G\frac{m_cm_r}{r_{12}^2}\Rightarrow$$
$$\Rightarrow G=\frac{r_{12}v_r^2}{(m_1+m_2)}=\frac{r_{12}v_r^2}{m_c}\Rightarrow F_{12}=\frac{r_{12}v_r^2}{m_c}\frac{m_1m_2}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{(m_1v_r)(m_2v_r)}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{p_{1r}p_{2r}}{r_{12}^2}=$$
$$=\frac{r_{12}v_r^2}{m_c}\frac{m_cm_r}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{(m_cv_r)(m_rv_r)}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{p_{cr}p_{rr}}{r_{12}^2},\ p_{cr}=m_cv_r,p_{rr}=m_rv_r.$$

From (2.4-13) we could also conclude that force of gravitation between two moving masses is directly proportional to the product between involved moments (at least dimensionally), $F_{12}=\left(\frac{r_{12}}{m_c}\right)\frac{p_{1r}p_{2r}}{r_{12}^2}=\left(\frac{r_{12}}{m_c}\right)\frac{p_{cr}p_{rr}}{r_{12}^2}$, on a way that should be better analyzed and completely understood. We know that all masses in our universe are anyway in mutual relative motions, and such intrinsic or holistic movements are bringing hidden, linear, and angular moments in the same framework, regarding gravitation (see more complete explanation about matter-wave complexity of motional messes in Chapter 10., equations (10.1.1)- (10.1.8)).

Consequently, Newtonian force presented initially as depending on the product between involved masses, could be a lucky coincidence and circumstantial result or acceptable mistake, meaning that real or effective sources of gravitation are involved as linear and angular, mechanical, and electromagnetic moments including associated, intrinsic vibration related states (not only static masses). Of course, specific intrinsic (hidden, or background) rotation (with angular and linear velocity components) should also be associated to all masses in our universe to give more practical meaning and weight to (2.4-13). Here, we are implicitly introducing the concept that any two masses in linear,

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mutually relative motion (in the process of approaching), are naturally being enriched (or complemented) with elements of angular motion, which in some cases could generate orbital (and spinning) motions (or planetary and satellite motions). See also similar exercising around equations (2.4-5.1). Later in the same chapter, with elaborations around equations (2.11.13-1) - (2.11.13-5), (2.11.23) and (2.11.24), we will additionally address similar analogical and challenging options. In Chapter 10. of this book, we can find the most complete explanation of the same situation regarding hidden or background velocity parameters and Newtonian attraction between relevant linear and angular moments (see (10.1.1) - (10.1.7)).

By considering validity of (2.4-13) in connection with Kepler laws, we can conclude that (from the point of view of a relevant laboratory system) reduced mass m_r tends to be "symmetrically self-balancing distributed", and to rotate around m_c , or it should be effectively (mathematically) presentable as such rotation (later in this chapter proven by (2.11.13-1) - (2.11.13-5)). Such hidden, rotating-like situations can be exploited as the background for explaining the origins of de Broglie matter waves; -see chapter 4.1. If there were no rotation and spinning motions, including associated centrifugal forces, we would have only attractive forces between masses, with a permanent tendency of masses to agglomerate. We already know that this is not a generally valid case in our universe (at least not concerning stable planetary systems), and the same could easily and analogically be extended to orbital situations in atom models, considering Coulomb forces.

***Laboratory Coordinate System*, observes what is happening between two mutually moving bodies with masses m_1 and m_2 , and it belongs to much less general and divine "kitchen", compared to what is happening between m_r and m_c in the same system, or in a dominant local *Center of Mass System* in relation to matter-waves, orbital and spinning motions, and particles creation, or disintegration …**

Let us now try to roughly formulate (in the same Laboratory system of coordinates) the same mass-product relation $m_1 m_2 = m_c m_r$ from (2.4-13), considering that masses are velocity-dependent (as in (2.4.12)). This will produce the following options,

$$\begin{pmatrix} m_1 m_2 = m_c m_r \,,\; m_1 \to \gamma_1 m_1, m_2 \to \gamma_2 m_2 \\ m_c \to \gamma_c m_c = \gamma_1 m_1 + \gamma_2 m_2 \,, m_r \to \gamma_r m_r = \dfrac{\gamma_1 m_1 \cdot \gamma_2 m_2}{\gamma_1 m_1 + \gamma_2 m_2} \end{pmatrix} \Rightarrow \gamma_1 m_1 \cdot \gamma_2 m_2 = \gamma_c m_c \cdot \gamma_r m_r \Leftrightarrow$$

$$\Leftrightarrow \gamma_1 \cdot \gamma_2 = \gamma_c \cdot \gamma_r \Leftrightarrow \left\{ \begin{array}{l} \gamma_r = \dfrac{\gamma_1 \cdot \gamma_2}{\gamma_c} = \sqrt{\dfrac{1 - v_c^2/c^2}{(1 - v_1^2/c^2)(1 - v_2^2/c^2)}} \\ v_1^2 + v_2^2 - \left(\dfrac{v_1 v_2}{c}\right)^2 = v_r^2 + v_c^2 - \left(\dfrac{v_r v_c}{c}\right)^2 \end{array} \right\}. \tag{2.4-14}$$

Since, in (2.4-14) we are presenting mutually similar situations in the same reference system, time meaning, and/or the time-flow relevant for all associated objects should be the same. Of course, we should additionally verify if γ_r in (2.4-12) and (2.4-14) are mutually identical since the unifying, and more general case would be,

$$E_{k1} + E_{k2} = E_{kc} + E_{kr} \Leftrightarrow (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 = (\gamma_c - 1)m_cc^2 + (\gamma_r - 1)m_rc^2 \Leftrightarrow$$
$$\Leftrightarrow (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 = (\gamma_c - 1)m_cc^2 + E_{kr}$$
$$\Leftrightarrow (\gamma_1 m_1 + \gamma_2 m_2)c^2 - (m_1 + m_2)c^2 = (\gamma_c m_c + \gamma_r m_r)c^2 - (m_c + m_r)c^2 \Rightarrow$$
$$\Rightarrow E_{kr} = (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 - (\gamma_c - 1)m_cc^2 =$$
$$= (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 - (\gamma_c - 1)m_1c^2 - (\gamma_c - 1)m_2c^2 =$$
$$= (\gamma_1 - \gamma_c)m_1c^2 + (\gamma_2 - \gamma_c)m_2c^2 = (\gamma_r - 1)m_rc^2 \left(\cong \frac{1}{2} m_r v_r^2 \right)_{v_r << c} \Leftrightarrow$$

$$\Rightarrow \gamma_r = \frac{1}{\sqrt{1 - v_r^2/c^2}} = \begin{Bmatrix} 1 + (\gamma_1 - \gamma_c)\frac{m_1 + m_2}{m_2} + (\gamma_2 - \gamma_c)\frac{m_1 + m_2}{m_1} = 1 + \frac{\gamma_c m_c^2}{m_1 m_2} = 1 + \frac{\gamma_c m_c}{m_r} \\ (\text{and / or}) \\ \frac{\gamma_1 \cdot \gamma_2}{\gamma_c} = \sqrt{\frac{1 - v_c^2/c^2}{(1 - v_1^2/c^2)(1 - v_2^2/c^2)}} \end{Bmatrix}. \tag{2.4-15}$$

In fact, here (with (2.4-11) – (2.4-15)) we are roughly trying to construct or elaborate conditions when a two-particle system (with particles m_1 and m_2) is evolving or transforming into another two-particle system with particles m_c and m_r. The characteristic property of both two-particle systems (observed in the same Laboratory system) is that both have the same (total) linear momentum, $\vec{p} = \vec{p}_1 + \vec{p}_2 = \gamma_1 m_1 \vec{v}_1 + \gamma_2 m_2 \vec{v}_2 = \gamma_c m_c \vec{v}_c$, the same (total) kinetic energy $E_k = (\gamma_1 m_1 + \gamma_2 m_2)c^2 - (m_1 + m_2)c^2 = (\gamma_c m_c + \gamma_r m_r)c^2 - (m_c + m_r)c^2$, and the same total energy $E = E_c = (\gamma_1 m_1 + \gamma_2 m_2)c^2 = (\gamma_c m_c + \gamma_r m_r)c^2$. This is giving a chance to formulate two 4-vectors representing such systems, $\overline{P}_4 = (p, \frac{E}{c})$, $\overline{P}_{4c} = (p, \frac{E_c}{c})$,

$$\begin{Bmatrix} \begin{bmatrix} \overline{P}_4^2 = (p, \frac{E}{c})^2 = \text{inv.}, \quad \overline{P}_{4c}^2 = (p, \frac{E_c}{c})^2 = \text{inv.} \\ \vec{p} = \vec{p}_1 + \vec{p}_2 = \gamma_1 m_1 \vec{v}_1 + \gamma_2 m_2 \vec{v}_2 = \gamma_c m_c \vec{v}_c \end{bmatrix} \Rightarrow \\ (\gamma_1 m_1 + \gamma_2 m_2)c^2 - (m_1 + m_2)c^2 = E_{k1} + E_{k2} = E - E_0 = E_c - E_{0c} = E_{kc} + E_{kr} = (\gamma_c m_c + \gamma_r m_r)c^2 - (m_c + m_r)c^2 \\ E = (\gamma_1 m_1 + \gamma_2 m_2)c^2 = E_1 + E_2 = E_0 + E_k, E_0 = (m_1 + m_2)c^2 = m_cc^2 = E_{01} + E_{02} \\ E_c = (\gamma_c m_c + \gamma_r m_r)c^2 = E_{0c} + E_{kc}, E_{0c} = (m_c + m_r)c^2, E_{kc} = (\gamma_c - 1)m_cc^2, E_{kr} = (\gamma_r - 1)m_rc^2 \\ E = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{0c} + E_{kc} + E_{0r} + E_{kr}, E_{0r} = (m_c + m_r)c^2 = E_{0c} + E_{0r} \end{Bmatrix} \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} \Leftrightarrow p^2c^2 - E^2 = -E_0^2 \Leftrightarrow p^2c^2 + E_0^2 = E^2 \\ p^2 - \frac{E_c^2}{c^2} = -\frac{E_{0c}^2}{c^2} \Leftrightarrow p^2c^2 - E_c^2 = -E_{0c}^2 \Leftrightarrow p^2c^2 + E_{0c}^2 = E_c^2 \end{Bmatrix} \Rightarrow \tag{2.4-16}$$
$$\Rightarrow \begin{Bmatrix} E^2 - E_0^2 = E_c^2 - E_{0c}^2 = p^2c^2 = (E - E_0)(E + E_0) = (E_c - E_{0c})(E_c + E_{0c}), E - E_0 = E_c - E_{0c} \\ p^2 = (\gamma_1 m_1 + \gamma_2 m_2)^2 c^2 - (m_1 + m_2)^2 c^2 = (\gamma_c m_c + \gamma_r m_r)^2 c^2 - (m_c + m_r)^2 c^2 \end{Bmatrix} \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} E + E_0 = E_c + E_{0c} \\ E - E_0 = E_c - E_{0c} \end{Bmatrix} \Leftrightarrow E = E_c = (\gamma_1 m_1 + \gamma_2 m_2)c^2 = (\gamma_c m_c + \gamma_r m_r)c^2 \Rightarrow$$
$$\Rightarrow \gamma_r m_r = \gamma_1 m_1 + \gamma_2 m_2 - \gamma_c m_c, \gamma_r = \gamma_1 \frac{m_1}{m_r} + \gamma_2 \frac{m_2}{m_r} - \gamma_c \frac{m_c}{m_r}.$$

Here (in (2.4-12) – (2.4-16)) we are merely speculating or hypothesizing with analogical applications and rough approximations of relativistic-like (velocity dependent) mass-energy formulations (but presently we do not take too seriously what would the same expressions be if somebody formulates them using strictly mathematical Relativity theory methods and Lorentz transformations). At least, we know for sure that kinetic energy balance (2.4-12) will generate non-relativistic kinetic energy balance (2.4-11) in cases when all relevant velocities are much smaller than the light speed c. It will be interesting to find which γ_r formulation, summarized in (2.4-17), is closest to a correct and generally valid case that will satisfy (2.4-13), (2.4-14), $m_1m_2 = m_cm_r$, as well as to satisfy energy and momentum conservation laws ((2.4-11) – (2.4-16)) for any relative speed of mutually interacting objects, since until here we found the following options,

$$\gamma_r = \frac{1}{\sqrt{1-v_r^2/c^2}} = \begin{cases} \gamma_1 \frac{m_1}{m_r} + \gamma_2 \frac{m_2}{m_r} - \gamma_c \frac{m_c}{m_r}, & \text{or} \\ \frac{\gamma_1 \cdot \gamma_2}{\gamma_c} = \sqrt{\frac{1-v_c^2/c^2}{(1-v_1^2/c^2)(1-v_2^2/c^2)}}, & \text{or} \\ 1 - \frac{m_c}{m_r}\left[\sqrt{1+(\gamma_c \frac{v_c}{c})^2} - \gamma_c\right], & \text{or} \\ 1+(\gamma_1-\gamma_c)\frac{m_1+m_2}{m_2} + (\gamma_2-\gamma_c)\frac{m_1+m_2}{m_1} = 1+\gamma_c \frac{m_c}{m_r}. \end{cases} \quad (2.4\text{-}17)$$

The consequences of such (little bit lose) analytical testing and speculations will contribute to showing that Lorentz contraction factor $\gamma = (1-\frac{v^2}{c^2})^{-0.5}$ and Lorentz transformations could be differently addressed or modified regarding effective particle velocity or velocity-momentum influence on Newtonian masses attraction. In addition, any mass in motion has elements of particle-wave duality, or presents a matter wave group, and can be interpreted as a complex, Analytic Signal function (see more in Chapter 10.; -equations (10.1.1) - (10.1.7)). For instance, by considering accelerated curvilinear motions, involved angular and spinning motions, gravitational potential and/or other forces and fields present during particles interactions, we may find another, more accurate understanding, or replacement for current Lorentz transformations. At the same time, we are also challenging the meaning, nature, and universality of contemporary Lorentz transformations (since we will attempt to show later (in the third chapter), that real origin of Lorentz transformations should be in Maxwell electromagnetic theory and not so much in mechanics, optics, and contemporary relativity theory based on imaginative, mental experiments). Here ((2.4-11) to (2.4-17)), the *temporal-nature* and *time-flow,* for all participants are the same (since everything is presented in the same Laboratory reference frame, and the same Newtonian force is acting between mutually corresponding masses). See similar hypothetical approach around equations (2.11.24).

In this book is shown that particle-wave duality of matter in motion is causally related to the fields and forces manifesting in two (or many) bodies interactions, or more specifically to motional energy associated to reduced mass m_r. Since kinetic energy E_{kr} of reduced mass m_r cannot be treated merely as an energy of a particle in a purely linear motion, the remaining, imaginative, and most probable, or most promising case (regarding understanding formations of solar, planetary systems, atoms' structure, and origins of particle-wave duality) is that E_{kr} is a form of rotational energy (or energy which has an angular motion nature). Meaning of such speculations is to conclude that reduced mass m_r has a tendency towards establishing rotational (closed line) motion around a mass m_c . All of that should also be in relation with Kepler and Coulomb laws, in case of electrically charged masses, because Newton (or Coulomb) attractive force should be balanced with a similar, centrifugal, and repulsive force. See much more about familiar situations in the same chapter under "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"; -equations (2.11.13-5) - (2.11.13-5), and from (2.11.10) until (2.11.21), as well as number of supporting elements in chapter 4.1).

The approach here, regarding relativistic-like scaling, Lorentz, or gamma contraction factors γ_i , (and Lorentz transformations) is essentially non-relativistic. Briefly, what is proposed here is:

First: Let us forget or neglect for a moment that Relativity theory and Lorentz transformations exist in a mutual context of such somewhat hybridized, partially speculative, and probably incomplete theory (*see extremely relevant, exceptional, and unavoidable references [35], [80] and [81] for understanding such serious statements much better*). Physics anyway evolved (even before Einstein, independently from any form of modern Relativity theory) towards understanding that the total rest energy of a particle can be quantified as $E_0 = mc^2$. The extension of such mass-energy relation is that total particle energy in motion is increasing multiplied by "scaling, or gamma factor" $E = \gamma mc^2 = \gamma E_0$, $\gamma = (1-v^2/c^2)^{-0.5}$, where particle kinetic energy is $E_k = (\gamma-1)mc^2 = E - E_0$ (what is experimentally verifiable). This is

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additionally reinforced by the fact that for non-relativistic velocities we will get kinetic energy expression known from Classical Mechanics, $E_k = (\gamma - 1)mc^2 /_{(v<<c)} \cong \frac{1}{2}mv^2$. In other words, transitions, and connections between Classical and Relativistic Mechanics should be smoother and more natural, without conceptual, logical, and mathematical discontinuities created by postulating and inventing artificial and non-realistic missing links and challenging concepts, as sporadically practiced in Relativity theory.

*When presenting specific particle in motion, the most significant and probably only relevant links to Relativity theory (without covariant or Lorentz invariant misrepresentations), are definitions of particle "**proper time** τ", "**proper mass** m_0" and "**proper energy** E_0", such as,*

$$dt = \gamma d\tau,\ d\tau^2 = dt^2 - dr^2/c^2 = \text{invariant},\ (dr^2 = dx^2 + dy^2 + dz^2,\ v = dr/dt),$$

$$\begin{bmatrix} m = \gamma m_0 \\ E = \gamma E_0 = \gamma m_0 c^2 \\ p = \gamma p_0 = mv = \gamma m_0 v \\ \gamma = (1 - v^2/c^2)^{1/2} \end{bmatrix} \Rightarrow p^2 - \frac{E^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2}. \qquad (2.4\text{-}17.1)$$

*Proper time τ (measured with a single co-moving clock, linked to the particle in question) has the same meaning as well elaborated and clarified by Thomas E. Phipps, Jr. in [35], and proper mass and proper energy, m_0, E_0, are only analogical names introduced here to underline the analogy with a proper time ($dt = \gamma d\tau \leftrightarrow m = \gamma m_0, E = \gamma E_0$), but effectively meaning rest mass and rest energy. Everything else what we know from Einstein-Minkowski 4-vectors formalism should be explicable based on "**proper parameters**". Inertial "frame time" t is the time measured by a spatially extended set of clocks at rest in that inertial frame (and something similar is valid for inertial "frame mass" and inertial "frame energy", m and E.*

***<u>Second</u>**: Since number of situations in relation to motions and energy and momentum conservation laws are anyway experimentally verifiable, here we merely utilize only obvious and no-doubts, well-operating energy-momentum, velocity dependent relativistic formulas, without making too many of glorifying, apologetic and generalizing links to A. Einstein Relativity theory, until we rich the next, more general level of deeper understanding. The fact is that A. Einstein Relativity theory, besides establishing challenging or imaginative (some of them maybe not verifiable) concepts and practices, also has few of convenient and useful building blocks, such as concepts related to "inertial frame time" and particle "proper time", and Einstein-Riemann-Minkowski 4-vectors, which are very much productive in analyzing the world of interactions in microphysics. Regardless objections and critics of Relativity Theory, we still cannot avoid using such concepts and mathematical prescriptions, but we should not forget that much broader and more general understanding of the same problematic is still in front of us (see similar and more profound elaborations in Chapter 10.). For instance, based on (2.4-17.1) we can easily create the following table of analogies (such as introducing new complex 4-vectors and 4-scalars) that are on some way simulating Minkowski-Einstein 4-vectors formalism, being entirely correct and applicable,*

T.2.2-3

	Ref. Frame Values	Symbolic Vectors	Invariant Expressions
Time	$dt = \gamma d\tau$	$\overline{\tau} = (dt, \frac{dr}{c})$	$d\tau^2 = dt^2 - \frac{dr^2}{c^2} = inv.$
Mass	$m = \gamma m_0$	$\overline{M}_4 = (M, \frac{E/c}{c}) = (M, \frac{p}{c})$	$\overline{M}_4^2 = M^2 - \frac{p^2}{c^2} = M_0^2 - \frac{p_0^2}{c^2} = inv.$
Energy	$E = \gamma E_0$ $E_0 = m_0 c^2$	$\overline{\varepsilon} = (E, cp)$	$\overline{\varepsilon}^2 = E^2 - c^2p^2 = E_0^2 = inv.$
Momentum	$p = \gamma p_0$ $p_0 = mv$	$\overline{P}_4 = (p, \frac{E}{c})$	$\overline{P}_4^2 = p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} = inv.$
Velocity	$v = \frac{dr}{dt} = \frac{1}{\gamma^2}\frac{d\rho}{d\tau}$	$\overline{V} = (\gamma v, \gamma c)$	$\overline{V}^2 = \gamma^2 v^2 - \gamma^2 c^2 = -c^2 = inv.$
Distance (space interval)	$d\rho = \gamma v dt = \gamma dr$, $dr \cdot dt = d\rho \cdot d\tau$	$\overline{\tau} \cdot \overline{V}$	$\overline{\tau} \cdot \overline{V} = \gamma v dt - \gamma dr = 0 = inv.$

We could now start establishing new 4-vectors or 4-scalars, Phasors, complex spaces, introduce new coordinate systems with imaginary units, and develop new "Super or Hyper-Complex Relativity" theory based on such exciting mathematical combinations based on T.2.2-3 (see more in Chapter 10). Of course, such creative imagination and further mathematical action steps should be guided by something experimentally verifiable, with predictive power, and be well integrated with the texture of surrounding Physics, avoiding creating artificial, non-natural, conventions and axioms-based theories.

[Thanks to Lorentz, Poincaré, Minkowski, A. Einstein, and most probably to Einstein's first wife Mileva Maric (Serb by origin, which was good in mathematics and physics), and to complexity of involved circumstances, Relativity theory is constructed, producing, or suggesting challenging results, some of them particularly useful, and some of them maybe never fully verifiable. If the history took a different path, without Einstein, we would still have above-mentioned energy-momentum related expressions formulated by others (but, most probably, not all of assumptions, postulates and concepts as known in modern Relativity theory). The message here is that we are not obliged to take all of such (energy and velocity related) results exclusively from the Relativity theory. Present Relativity theory is at the same time offering particularly useful and logical results or formulations regarding masses, velocities, moments and energies, and a package of challenging, certain of them improvable, somewhat suspicious, or unnecessarily complicated options (as elaborated by Thomas E. Phipps in [35], by Milutin Milankovic in [80], and by Velimir Abramovic in [81]). Consequently, to stay on a good side, we could pragmatically select and use everything that is working well and being proven correct from many platforms (including contemporary Relativity theory), indirectly paving the road to say that modern Relativity theory should still evolve towards more advanced and realistic concepts (becoming a direct derivate of the upgraded Maxwell electromagnetic theory). Anyway, A. Einstein expressed part of his respect and obligations (among other complicated feelings and different reactions) towards his first wife Mileva Maric, by generously promising to give her the money he would (most probably) receive as the future Nobel-prize laureate. This really materialized later (but only after Mileva reminded A. Einstein that she could write his biography to survive). Mileva left the officially known part of the Relativity-related history without leaving missing segments of the Relativity theory picture, if any, since somebody of Einstein's friends removed all remaining Relativity and Einstein related documents from Mileva's apartment after she died (see relevant supporting information in [78], Christopher Jon Bjerknes. -The Manufacture and Sale of Saint Einstein). Until present, most science historians are, only sporadically and marginally, mentioning Mileva Maric as somebody who could be everything, just not a Serb. Anyway, science historians and reporters are very often giving to Mileva the geographic and newly created ethnic and national identity by the name of the geographic area where she was born (Vojvodina, in Serbia, which temporarily was the part of Austro-Hungarian empire in that time), from Serbian parents (this way indirectly insinuating that she could maybe have Croatian, or at least only Austro-Hungarian origin). Even until the beginning of the 21st century, the grave of Mileva Maric–Einstein was not known, maybe because she did not pay for her resting place for long enough time. Eventually, her tomb is recently discovered in the middle of Switzerland, unfortunately by unilateral action of some marginal Serbs, and thanks to a certain collaboration with relevant communal and cemetery authorities. Maybe, one-day, Mileva's tomb could be transformed in a valuable touristic item of Switzerland. We should also not be surprised if Mileva Maric is already on the way to get Vatican and EU-certified Croatian identity, like the cases of famous Serb and cosmopolitan scientists and inventors Nikola Tesla and Rudjer Boskovic are, complying with geopolitical and ideological trends and activities in Vatican-elaborated and Germans-admired kitchen. Of course, offhand comments, like given here, are also showing the universally applicable power of mainstream inertia, manipulations, and indoctrinations, even in cases when the subject is not very much relevant for the scientific achievements we are addressing. Humans are often dissipating their mental energy, even in areas where this is almost meaningless, and not necessary, like in modern natural sciences]. Complete Physics, starting from 20th century until present is still polluted with such semi-false or inaccurate, or problematic and challenging conceptualizations and theories.

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

After reading publications from Mr. Thomas E. Phipps, [35], Jr., and from Reginald T. Cahill, [73], regarding contemporary Relativity theory, it would become imaginable to everybody that days of dominance of the contemporary Relativity theory in Physics are counted. This is also applicable until certain (much smaller level) to present Electromagnetic theory.

The complexity in the background of (2.4-17) is related to the following, almost philosophical platform: We (humans), as good mathematicians, are equally able to analyze and see certain two-body or multiple-body system (with number of internal interactions) from many of our Laboratory (and inertial) systems, and from the common platform of the unified Center-of-mass reference system. From our specific observers' perspective, Laboratory systems are something more realistic and tangible, and Center of Mass System is a kind of virtual, mathematically generated reference system. Contrary to a physical observer position (laboratory system), from Nature or Physics (or from a common and most neutral and more significant observer position), much more relevant, dominant, and tangible is the reality related to Center of Mass System. All complexity of particle-wave duality and interactions between members of a specific multi-body system is internally coupled relative to their common Center of Mass System (valid also for atoms, planetary, solar, and galactic systems). Almost needless to add that all objects, energy states, and masses in our universe are in respective states of relative motions, and in relative states of rest, when we start analyzing such states concerning their Center of Mass Systems. For instance, in cases of solar systems with many orbiting planets (see equations from (2.11.10) to (2.11.21)), or just in cases when comparing two-body-problems with N-body-problems, the natural conceptualizing approach would be kind of analogical reduction to many instances of two-body systems, as for example,

$$\begin{aligned}&\left(m_c \to \gamma_c m_c = \gamma_1 m_1 + \gamma_2 m_2\right) \Rightarrow \gamma_c m_c = \sum_{(i)} \gamma_i m_i = \gamma_c \sum_{(i)} m_i \\ &\left(m_r \to \frac{\gamma_1 m_1 \cdot \gamma_2 m_2}{\gamma_1 m_1 + \gamma_2 m_2} = \gamma_r m_r, \gamma_r = \frac{\gamma_1 \cdot \gamma_2}{\gamma_c}\right) \Rightarrow \gamma_{r-i,j} = \frac{\gamma_i \gamma_j}{\gamma_c}.\end{aligned} \tag{2.4-18}$$

In addition, we could say that the significant differences between Classical Mechanics, Quantum theory and Relativity theory should mostly be related to conceptually incomplete Particle-Wave Duality formulations, and to its present, incomplete mathematical modeling in Physics (among other problems in relation with Quantum and Relativity theory, since both should originate from specifically updated Electromagnetic theory). Naturally, in mentioned theories, some of the particles and fields binding components, present in (2.4-3), are missing, or not being adequately considered (here also being accountable hypothetically). For instance, adverse results of famous Michelson-Morley interferometer-related experiments are pointing to the exceptional and universal place of the maximal speed of light in a vacuum (which is based on Michelson-Morley test, considered being independent of the speed of its source and from its receiver). Michelson-Morley interferometer is using the same (single) light source, which is conveniently split or converted in two mutually orthogonal light beams (which are anyway mutually electromagnetically coupled and synchronized). Both light beams are also electromechanically coupled to a laboratory (and to our planet) where we make measurements. The goal has been to measure, if "global laboratory speed" (Earth motion and hypothetical ether wind around Earth) would produce specific time-delay or time-advance (at least) on one of such light beams. The old Michelson-Morley testing and results of such experiment were incompletely organized and interpreted as negative (regarding detecting ether winds), but it is, much later, until present days, repeated with better-organized equipment and measurements that are giving different results. Anyway, early Michelson-Morley interferometer results were too much affirmatively (and wrongly) interpreted as a clear evidence of no-influence of Earth and ether motion on crossed light beams, leading to the wrong conclusion that <u>ETHER does not exist</u>. We should also take into consideration that such measurements, performed in a laboratory on our planet Earth are influenced by environmental, geomagnetic, and other electromagnetic fields, including influences of surrounding masses. Michelson-Morley interferometer was too small, and too much (locally) coupled and temporally-spatially synchronized between two light beams and with our planet and laboratory where measurements took place, and practically the involved field medium or ether was synchronously moving with the same speed as the interferometer and laboratory (including our planet). Since electric and magnetic fields are also strongly coupled to relevant sources such as electric and magnetic charges and dipoles, if mentioned charges are moving, surrounding fields are also moving synchronously, meaning that in such cases expected ether, being between electromagnetic charges, is not freely and independently streaming. Also, photons and light beams cannot exist completely isolated and separately from their *electromagnetic sources, charges, and dipoles (since all electric, magnetic, electromagnetic, mechanical, fluidic, and electromechanical circuits in our Universe should present closed loops, having front-end sources, and last-end consumers, as elaborated in the first chapter of this book). Consequently, in a vicinity of Michelson-Morley interferometer, Earth spinning and rotation around our Sun is not creating sufficiently freely moving currents, waves, streaming, or flow of the fluid, or matter state we named as an ether. To get clear interferometric results, showing that ether winds and streaming have an influence on propagating light beams, we need to construct different and much larger interferometer (on the scale of certain solar system), with two of independent light sources, where mentioned direct and strong light beams couplings, and synchronizations will be significantly attenuated or eliminated. Another reason for confusion produced by old Michelson-Morley experiments is maybe related to the situation that light beams or photons are oscillating only transversally and supposed streaming or flow of ether should be kind of laminar or linear motion.*

Also, as we know, a big part of Relativity Theory foundations is the consequence of mentioned (early and incomplete) Michelson-Morley conclusions (without considering that light beams, or photons, besides linear moments, also have specific spinning, and that all rotational and spinning motions in our universe are globally coupled and synchronized). We also know that present satellite GPS systems are operating very well while using specifically developed and simple spatial-temporal calculations (for space, time, and velocity related synchronizations), and this is showing that mathematics of Relativity Theory is still not necessary, not practical, or not fully applicable here. Nevertheless, we should not conclude that contemporary Relativity Theory is wrong. It is only incomplete (and partially artificially assembled), and it should be primarily and essentially developed starting from Maxwell-Faraday electromagnetic theory. It should be appropriately updated, to become more suitable for dealing with linear, angular, relative, and absolute motions. See more profound and fully clarifying elaborations about the same problem in publications from Thomas E. Phipps in [35], Milutin Milankovic in [80], and Velimir Abramovic in [81].

Certain particle-wave interactions, diffractions, interference, and scattering phenomena, are still incompletely analyzed in modern Physics, replacing missing motion components by "theory-saving" probabilistic modeling (putting in question the real place of causality and objectivity in natural sciences). Of course, we need to admit that Quantum theory works well in its own frames, by performing its mathematical and "theory-saving job" very successfully (mostly because statistics and probability theory are very well mathematically established and universally applicable in all natural and life sciences). Quantum theory is until present also successful in convincing generations of scientist that what we see is only a certain level of probability of what this could be (what is anyway equally and trivially valid for everything around us in all other domains). Quantum theory is, also, elaborating that different observer will coincidently see (ontologically, empirically, intellectually, mathematically, and conceptually) the same situation differently, what in some cases is philosophically correct, but still, it needs to be deterministically explicable (including statistically and probabilistically). It is also evident that measurements method and measuring instruments (in some cases, more or less) could interfere with results of specific measurements in micro physics, but this is not giving legitimacy to say that everything in microphysics is fundamentally and ontologically stochastic. Such dogmatic positions are sometimes resembling a little bit to certain ideological teachings found in the known part of human history (all of that being in some cases applicable to present Relativity and Quantum theory, and to some aspects of Maxwell Electromagnetism). Also, it should be mentioned that no new theoretical approach regarding particle-wave duality would be able to completely neglect and undermine many of mathematically well-elaborated concepts and models, presently used in Quantum theory, since a lot of successful mathematical fittings, modeling, and arrangements (mostly on mathematical level, and based on satisfying and completing obvious Symmetries) is already made there. So well operating mathematical structures (like practiced in present Quantum, Relativity, and Electromagnetic theory) could appropriately be modified, upgraded, and used again (until certain limits and inside new frameworks) in formulating new models, concepts, and new approaches regarding an innovated Wave-Particle Duality theory. We should never forget that powerful mathematical tools are not (immediately, automatically, completely and non-doubtfully) giving the same power and significance to theories and authors using such devices (or tools). While using particularly good tools, somebody can also make bad or wrong constructions. This what we presently have within Quantum and Relativity theory could have some aspects of a significant subject replacement with something trivial, what always works well, to support not well-established concepts. Similar subject replacements we can find in many ideological and political concepts and movements. For instance, the power of good music, painting, architecture, pleasant ambiance and other unusual and time penetrating creations are often being used to support imaginary, virtual, metaphysical, ideological practices, and teachings (attaching the tangible and exiting content to something what in reality has not very substantial and logical grounds). Looks like our human race is still not learning well from our history that is full of such examples, and such an irrational tendency is in some level still populating and propagating within our officially recognized, mainstream natural sciences (often being implemented on a well-hidden way). Anyway, glorious days of dominance of contemporary Relativity and Orthodox Quantum theory are already belonging to the past; -see similar statements in [27] J. M. T. Thompson, Visions of the Future. ♣]

2.3. How to account Rotation concerning Gravitation?

The next idea, for the time being also hypothetical, and still not being wholly formulated opinion of the author of this book, is that every mass, composed of atoms and molecules naturally has number of spinning states inside, since almost all elementary particles, electrons, protons, and neutrons have spin and magnetic properties (of course this is kind of an effective or equivalent spinning of matter-waves formations, where we do not insist that some solid particles are really spinning). Under certain favorable conditions, such internal spinning and associated magnetic states could be specifically oriented, polarized, united as vectors, and react as a macro system with certain resulting angular moment and permanent magnet field (spatially distributed around the mass in question). Every macro mass in linear motion is also creating external, matter-waves and gravity related field structure, again because mass (composed of atoms) has internal and polarizable spinning, electromagnetic and electromechanical elements (with angular and magnetic moments), externally manifesting as Gravitation. The same motional mass is effectively in a permanent rotation regarding its center of rotation (or regarding certain dominant center of mass). Consequently, described linear motion of masses (even on an exceptionally large circular-orbit diameter) also tends to create associated macro spinning field or spiral wave motion around direction of motion, and at the same time it is keeping a balance between attractive Newton-Coulomb's and repulsive centrifugal forces. In case of stable, periodical, and orbital mass motions we will also see creation of orbital standing matter waves. As the support to such statements, we have Kepler and Newton Laws on a macro-cosmological scale regarding solar or planetary systems. Also, many of supporting examples are related to modeling of atoms and elementary particles, where spinning, helix, oscillatory and orbital motions with standing waves structures are omnipresent (and usually coupled with associated electromagnetic complexity). Obviously, there is some universal tendency of mass and energy-formations in our universe towards creating self-closed orbital motions (where a radius of such rotation can be arbitrarily long). Local and global orbital and spin momentum conservation is anyway universally applicable to such situations, also involving macrocosmic entanglement effects (because all atoms and subatomic entities in any part of Universe are sets of resonators, and all resonators tend to mutually synchronize. All of that (just mentioned) should be valid both on micro and macro cosmological scales, even in cases when we do not see rotational and spinning motions (but we should assume that certain motional or oscillatory phenomena, equivalent to a rotation, should exist somewhere in the background). Masses in motions are effectively presenting specific "*energy packing formats*" captured (or agglomerated) respecting global energy-momentum conservation laws (and we will see later how this is working; -for instance as a good background regarding familiar situations it is recommendable to read "4.3.8. Mass, Particle-Wave Duality and Real Sources of Gravitation", from Chapter 4.3, and in Chapter 10. "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality").

0***As we know, every closed-path stationary motion (which can be circular, elliptic, etc.) is also an accelerated motion, and has the vector of orbital acceleration towards its center of rotation, very much analog to the force of Gravitation.*** See much more about such phenomenology later in this chapter, under “2.3.3. Macro-Cosmological Matter-Waves and Gravitation”, and in Chapter 8. Mass is internally composed of atoms with a countless number of elements (subatomic and elementary particles) that are also (on some way) rotating, having a mutually coupled orbital, linear, and magnetic moments, spin, and other motional manifestations (like having many mutually coupled micro gyroscopes). In fact, we can safely say that certain level of equivalent rotational (mostly spinning) energy is permanently accumulated or blocked inside masses and atoms, being also “field-extended” outside, in a space surrounding external shapes of macro particles. The *resulting action* of such, by mass “*internally captured and stabilized rotation*”, of randomly distributed, and mutually coupled states with electromagnetic and angular moments, which are also combined with similar external, *macro-orbital, mechanical and electromagnetic moments,* should present the most important sources of gravitation. An accelerated linear mass motion will also produce polarizations or alignment effects on its internal mass-spinning domains, this way creating an external macro mass angular momentum and torque, what is influencing the external gravitational force. (See chapter 4.3; -equations (4.41-1) until (4.45), and chapter 10; -equations (10.1.4) - (10.1.7)).

Intuitively advancing (to support above-given ideas), we could start with an oversimplified *analogical and dimensional* “manipulation” of the formula for a centripetal force applied on a rotating particle (that is in some way analogically representing the force of Gravitation). Let us create several of mutually equivalent, only dimensionally and analogically comparable expressions (2.5), and explore the concept that gravitation could present an external manifestation of a kind of resulting *angular moments’ related force* of countless number of internal, mutually coupled, energy-carrying, and spinning micro-mass domains, also combined with similar external orbital and electromagnetic moments. Such “***internally packed quantity of spinning***” is, when observed externally, creating a property, we consider as a total mass that has its mechanical-angular and coupled electromagnetic moments, being the origin of Gravitation.

$$F_c = \frac{mv^2}{r} (=) \left[\frac{E_k}{r}\right] (=) \left[\frac{L\omega}{r}\right] (=) \left[\frac{\tau}{r}\right] (=) \left[\frac{Lv}{r^2}\right] (=) \left[m\frac{dv}{dt}\right] (=) \left[\frac{kg\times m}{s^2}\right] (=) [\text{Force}] \ ,$$

$$\left(\begin{array}{l} \underline{[E_k](=)[mv^2](=)[J\omega^2](=)[pv](=)[L\omega]\ (=)\left[\frac{kg\times m^2}{s^2}\right]}(=)[\text{Motional energy}](=)[J]\ , \\ [L](=)[J\omega](=)[mr^2\omega](=)\left[\frac{kg\times m^2}{s}\right](=)[\text{Orbital moments}]\ , \\ \underline{[\tau](=)\left[\frac{dL}{dt}\right](=)\left[\frac{kg\times m^2}{s^2}\right]}(=)[\text{Torque}](=)[\text{Energy}](=)[J]\ ,(\text{when multiplied by an angle in radians}), \\ [v](=)[\omega r](=)\left[\frac{m}{s}\right](=)[\text{orbital velocity}],\ [\omega](=)\left[\frac{1}{s}\right](=)[\text{angular velocity}]. \end{array}\right) \quad (2.5)$$

By making only dimensional comparisons within members of (2.5), we can spark new indicative ideas regarding what kind of physics-related parameters or values (such as: $m,\ J,\ p,\ L, \tau,\ v,\ \omega,\ E_k ...$) could be involved in the expressions for a centripetal force, and forces related to orbital moments, and their (mathematical) derivatives. This way we are introducing an oversimplified, analogical, indicative and conceptually enough-clear starting platform (that should also stay compatible with (2.1) - (2.4-3)) for formulating still hypothetical force laws, originating from "Newton-Coulomb" force laws. ***If we, creatively and imaginatively, combine (2.1) - (2.4-3), (2.5) and analogies from T.1.2 and T.1.8, we could again indicatively conclude that some stable rest mass should be a kind of its own (internal) orbital and spin moments storage, or packing format*** *(see later (2.10), (2.11))*. Briefly, the objective here is to show the probable existence of missing force-links causally related to fields created by rotation or spinning that are complementarily coupled to linear motions and (externally) manifesting as gravitation, or also as magnetic field effects. This could be very much analogical like coupling between an electric and magnetic field in electromagnetism (such as electric dipoles and permanent magnets attraction and repulsion), or as two wave-functions creating an Analytic Complex Signal, or Phasor function (using Hilbert transform; -see much more in chapter 4.0). In fact, atoms are sets of resonators and we know that all resonators having the same resonant frequencies are mutually synchronizing. The intrinsic coupling between electric and magnetic fields, and material properties (as known in electromagnetism) shows, for instance, that electric charges (besides respecting Coulomb-Newton force laws) can perform in electric and magnetic fields almost any kind of linear and rotational movements (because of interactions based on different forms of attractive and repulsive forces. See additional supporting background in [144]). We also know how to associate properties of mass and different moments, to electromagnetic waves (as in analyses of Compton and Photo Electric Effects). Since mass in motion (internally and externally) is an essential ingredient of Gravitation, one of the first confirmations of Relativity theory (at least qualitatively, and in the limits of measurements errors) was in astronomy, confirming that big masses are bending light beams (or attracting photons, respecting the Newton Law of Gravitation). In fact, this primarily confirms that photons (as purely electromagnetic and dynamic, or motional entities, without a rest mass) have certain equivalent dynamic-mass and orbital or spin moments that behave similarly as every other mass concerning conservation laws, mechanical moments, and forces of gravitation. *Here we could add that there are some different opinions regarding gravitational attraction between photons and big masses, related to purely electromagnetic interactions, such as in [89].*

The challenging situation here is to conceptualize a *mass meaning*, since presently we understand gravitation only as the force between (almost static) macro masses, what is not correct. There is always certain minimal size and distance between two masses when, for smaller sizes of involved masses and smaller distances between them, forces of gravity (as currently known in physics) have no meaning or applicability. Also, we know that macro masses are agglomerations or compositions of atoms and molecules. We also know that atoms, elementary particles, and other interatomic entities usually have electric and magnetic charges and related fields properties, including angular, orbital, and spinning electric and mechanical moments. Widespread case (within the world of atoms and elementary particles) is that mechanical spinning and orbiting is coupled with magnetic moments properties. Also, all dynamic cases of magnetic field manifestations are always combined with corresponding and complementary electric charges and electric field manifestations.

Here, we are establishing the platform that forces of gravitation should be closely related to mechanical spinning, orbiting and torque effects, mutually coupled (and synchronized) with similar electromagnetic manifestations of atoms and involved field charges. This is the natural background supporting a direct and striking analogy between Newton and Coulomb force laws.

Another strong backing for "*reinventing the origin of gravitation*" is in the fact that based on electromechanical analogies and comparison between Coulomb and Newton laws (as elaborated in the first chapter of this book about Analogies), electric charge and static mass are not mutually analog items. What analogically corresponds to electric charges are electromagnetic, and mechanical, linear, and angular moments including vibrational states of masses (and not at all static masses). Consequently, real sources of gravitation should be electromagnetic and mechanical moments and charges (including relevant currents, as analogically predicted in the first chapter).

[♣ COMMENTS & FREE-THINKING CORNER,

What, could be a little bit intriguing and indicative (in (2.5) and later) is that ***dimensionally***, the same unit (Joule) measures energy and torque,

$$\left[\frac{kg \cdot m^2}{s^2}\right] (=) [\tau] (=) [Torque] (=) [Energy] (=) [J] (=) \text{Joule} \cdot \tag{2.10}$$

Any torque, naturally having the dimension of energy, can be effectively presented (at least dimensionally) as certain equivalent linear motion, axial force $\mathbf{F}^*_{\mathbf{linear}}$ (collinear with a torque-vector τ) acting along a linear particle path Δs, as for instance,

$$\tau = F_{angular} \cdot \theta = F_{linear} \cdot \Delta s \ (=) [Nm] \cdot \tag{2.10.1}$$

Since dimensionally a torque (multiplied by relevant acting angle of rotation θ, expressed in non-dimensional radians) presents an energy amount, and since we know the relation between rest-mass and its internal energy content $\mathbf{E} = \mathbf{mc^2}$, we could imaginatively say that the energy of every elementary rest mass presents an equivalent to certain (full circle, $\theta = 2\pi$) work of its internally captured torque elements $\tau_{int.} \cdot 2\pi = mc^2$. Thus, we associate an internally accumulated, resulting torque, to many omni-directionally, and randomly distributed, "spinning mass-energy micro-domains", all of them having certain micro-torque amount (or having relevant orbital and spin moments, not really insisting that something inside masses is mechanically rotating like micro gyroscopes). In fact, (following the same concept) we would also be able to claim that a resulting, total, internally captured, *(center of macro-mass related)*, rotating domains (effectively being spin-moments and torque elements of some internal matter-wave groups), packed inside of a certain macro mass (that is in a relative state of rest), is equal to zero as a resulting vector. However, it is not equal to zero locally (or microscopically), when a limited number of elementary torque domains is considered. This also means that a total internal torque (of a non-spinning particle) should be composed, at least, of two mutually opposed and equal torque vectors (because external resulting torque equals zero). Reasoning that way, it is more logical to consider $\tau_{int.} \cdot 2\pi = \frac{1}{2}mc^2$. **This way, we additionally support the concept that real sources of gravitation related forces are elementary, internal torque and spinning elements, dynamically packed and mutually synchronized and coupled with associated electromagnetic charges and moments, being inside an entity we identify as a compact rest mass, of course, composed of atoms, what also means that atoms are sources of Gravitation** (see how similar mass concept is already introduced in 2.2. Generalized Coulomb-Newton Force Laws).

Let us try to conceptualize the total, compact and neutral, *macro rest mass* (of certain externally non-spinning, electrically and magnetically neutral macro-particle) $\mathbf{m}$ as a specific, condensed (or equivalent) energy state that is resulting from mutual interactions of number of micro mass domains (or atoms) being internal orbiting, rotating, and spinning states (mutually coupled by surrounding electromagnetic forces). These vector values are, in-average, mutually canceling or neutralizing the macro-particle resulting orbital, spin or angular moment, and its resulting angular velocity (when observing the same particle from its external space; -from certain local laboratory reference frame). In other words, externally (in a Laboratory system), the same macro-particle is in a state of rest (standstill) and not rotating. The same macro particle internally behaves like having continuum (or an ensemble) of micro-rotating, orbiting, and spinning, mutually coupled, synchronized, and mutually neutralizing, energy carrying states, effectively creating *macroscopic vector sum of all internal orbital and spinning moments (naturally equal to zero)*. The same mass is externally considered as a source of gravitational force, described by Newton Law of Gravitation, but (as promoted in this book), only internal spinning

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and orbital moments, and associated electric and magnetic moments, dipoles and charges are real sources of gravitational attraction (see Fig. 2.2).

We could also say that for a standstill particle resulting center of a mass and angular velocity of such mass should be equal to zero, meaning that it would always have at least two mutually opposed vector components in any direction, regardless from which side we see it. Applying derivations on such center of mass angular velocity, we will be able to come to an expression where effective Orbital and Spinning Moments (meaning, torque or angular force components) will be explicitly considered (see (2.11) below). What is fundamental here is that we consider (or still hypothetically exercise and assume) that specific stable, macro particle can be presented as a unified composite state, or superposition of (***mutually synchronized***) micro spinning and rotating elements or _energy-moments-mass states_. Such internal spinning states are motional energy states, being conveniently united and packed, this way creating a total rest mass of the macro-particle in question, as follows,

$$\begin{Bmatrix} mc^2 = \sum_{[i\in(1,N)]} \frac{1}{2} L_i \omega_i \cong \frac{N}{2}\overline{L}\overline{\omega} = \pi N \overline{L} \cdot \overline{f} = H \cdot \overline{f} \\ (N, \overline{L}, \pi, H = \text{constants}) \end{Bmatrix} \Rightarrow \begin{Bmatrix} mc^2 = H \cdot \overline{f} \\ c^2 \cdot dm = H \cdot d\overline{f} \\ c^2 \cdot \Delta m = H \cdot \Delta \overline{f} \end{Bmatrix} \Rightarrow \begin{Bmatrix} mc^2 = 4\pi|\tau| = H \cdot \overline{f} \\ \overline{f} = \text{internal, mean} \\ \text{spinning frequency} \end{Bmatrix}$$

$$\vec{\omega} = \vec{\omega}^+ + \vec{\omega}^- = \frac{\sum_{(i)} \mathbf{J}_i \vec{\omega}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \vec{L}_i}{\sum_{(i)} \mathbf{J}_i} = 0 \Leftrightarrow \frac{1}{\mathbf{J}} \iiint_{[V]} d\vec{L} = 0$$

$$\dot{\vec{\omega}} = \dot{\vec{\omega}}^+ + \dot{\vec{\omega}}^- = \frac{\sum_{(i)} \mathbf{J}_i \dot{\vec{\omega}}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \dot{\vec{L}}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \vec{\tau}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\vec{\tau}^+}{\sum_{(i)} \mathbf{J}_i} + \frac{\vec{\tau}^-}{\sum_{(i)} \mathbf{J}_i} = 0 \qquad (2.11)$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}, \ \dot{\vec{\omega}} \sum_{(i)} \mathbf{J}_i = \dot{\vec{\omega}}\, \mathbf{J} = \vec{\tau}^+ + \vec{\tau}^- = 0 \Rightarrow mc^2 = 2|\tau^+| \cdot 2\pi = 2|\tau^-| \cdot 2\pi = 2|\tau| \cdot 2\pi \Leftrightarrow$$

$$m = \frac{4\pi|\tau|}{c^2} = \frac{1}{c^2} \iiint_{[V]} \vec{\omega} d\vec{L}, \ |\tau| = \frac{1}{4\pi} \iiint_{[V]} \vec{\omega} d\vec{L} = \frac{mc^2}{4\pi}.$$

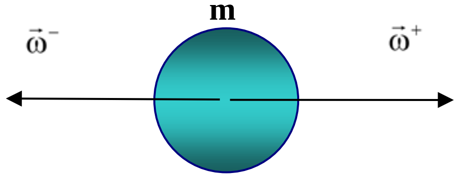

Fig. 2.2. Rest mass with internally compensated spinning and orbital moments

The main idea here has been that an absolute, resulting value of internally captured spinning moments, producing a torque (τ), presents at the same time a kind of internal energy content of the particle (when multiplied by 2 x 2π radians). It is almost evident that any neutral macro-particle should have at least two of such (mutually opposed) torque or spinning moment components in any direction so that such values are mutually canceled as vectors. Thus, we can redefine the particle total mass using mentioned rotation or spinning related concepts. The ideas presented here could also be the starting point for developing new concepts regarding Gravitation, where Newtonian attraction between masses would be explicable by the attraction between electric and magnetic fields created by an interaction of orbital and spin moments (which are internal mass or atoms constituents) like already initiated in (2.2). In Chapter 10. of this book, we can find the most complete explanation of the same situation regarding relations between masses, internal electromagnetically polarized entities, and important linear and angular moments (see (10.1.4) - (10.1.7)). Developing the concept regarding linear and rotational motions coupling, gradually appears that de Broglie matter waves, and angular forces (2.8) should be mutually related, what will be more elaborated later (see more in chapter 4.1). Obviously, (2.9) presents a simple development procedure to get the conceptual picture about the unity of linear and angular motional force components (at least being dimensionally correct). ***It is also becoming clear that Newton force formulation should be generalized to account rotation and spinning related forces and associated (and coupled) electric and magnetic polarizations.*** Here we could also speculate with

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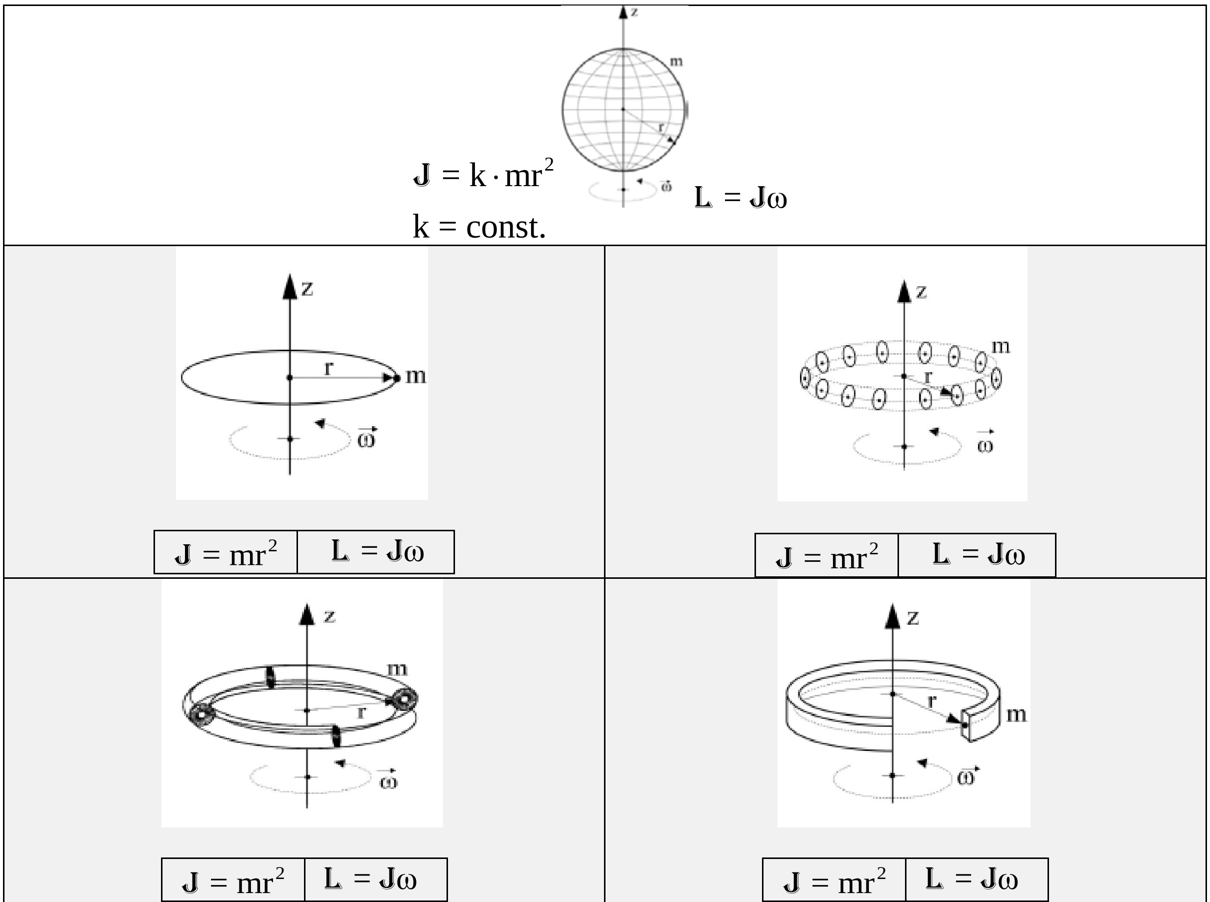
an existence of a background (or hidden) rotation of certain ether-like fluid, or Tesla's radiant energy vortices, meaning that mass in question is standstill and certain relative, rotating motion is acting around it.

Regarding gravitationally attracting masses, we could summarize the following conceptual picture. Every single, neutral (non-charged) macro-mass that has non-zero rest mass (usually meaning being composed of atoms and molecules with embedded positive and negative electric charges) should be presentable as an agglomerate of small, elementary micro-rotating or spinning mass formations. Such formations should be like uniformly and omnidirectionally distributed angular, spinning, and magnetic moments, which are in most of cases mutually cancelling as vectors (in absence of other macro masses and external forces). If two or many of such neutral masses are placed in certain spatial vicinity, mentioned micro, angular, spinning, and magnetic moments would start reorienting or polarizing on a way that an attractive (or "magneto-gravitational") force between all of them will be created. As we know, this is mathematically described as Newton-Coulomb force laws (effectively being like a force between two magnets or two electric charges; -see additional information here: *https://en.wikipedia.org/wiki/Gravitoelectromagnetism and in [71]*). We identify this force as gravitation. Of course, any magnetic force or field is on certain way and always followed by its complementary electric field since all matter in our Universe is in different states of mutually relative motions. Mentioned micro spinning masses, and associated magnetic field, and electromagnetic moments, could have number of mutually isomorphic forms (regarding spinning and orbiting energy states), as presented on Fig. 2.2.1. Such spinning, rotating or orbiting patterns can be frequently found in modelling of micro world of atoms, as well as in the macro cosmos of planetary systems and galaxies. Structure of electrons and protons could also be very well modeled as standing waves on toroidal shapes (see much more in [16,17,18, 19, 20, 22, and 120]).



(from: http://hyperphysics.phy-a str.gsu.edu/hbase/mi.html
https://en.wikipedia.org/wiki/Angular_momentum#Spin,_orbital,_and_total_angular_momentum
https://en.wikipedia.org/wiki/List_of_moments_of_inertia)

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Moment of inertia is defined with respect to a specific rotation axis. The moment of inertia of a point mass with respect to an axis is defined as the product of the mass times the distance from the axis squared. The moment of inertia of any extended object is built up from that basic definition. The general form of the moment of inertia involves an integral.

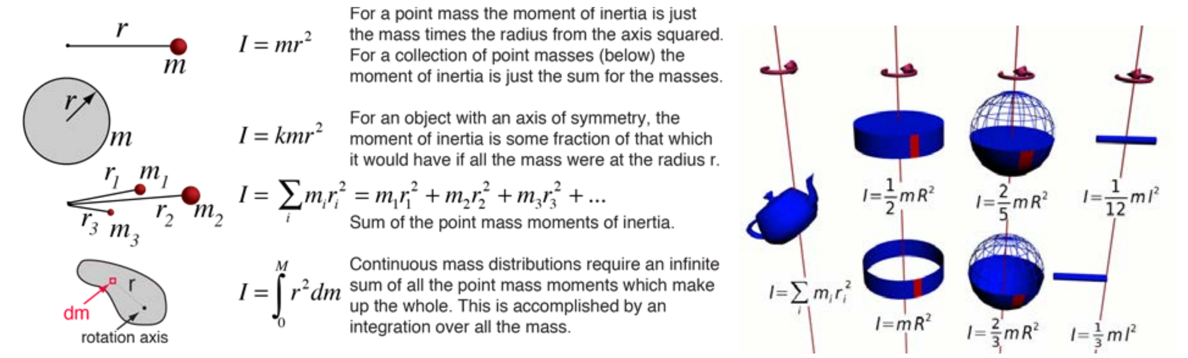

Fig. 2.2.1 Mutually isomorphic masses configurations regarding spinning

There are many levels or layers of superposition and isomorphy-related mapping applicable in explaining mass or matter structure. Generally valid mathematics shows that Fourier and Analytic Signal concepts are proving that any time-domain function or signal can be decomposed on the number of simple-harmonic (sinusoidal) components (see 4.0.2., 4.0.3., from the Chapter 4.0). If we like to address the structure of certain compact-mass or particle, we could on a similar way say that a mass is composed of a big number of internal electro-mechanical spinning, oscillatory and resonant elements (as combinations of different elementary spring-mass, and capacitance-inductance systems) that are on some way mutually superimposed, coupled, synchronized, and united. Mass is also composed of atoms and molecules, and again, we are saying here that atoms are presenting synchronized superposition of internal spinning states (as roughly elaborated in (2.11)). Such initial mass concept or atomic field structure is externally spreading between all masses, and this is creating effects of gravitational attraction.

What is interesting and innovative here is to understand the process when part of distributed, and internally compensated angular moments, including internal spinning moments, is being transformed, packed, sunk, or "injected" into certain rest mass. This way we could test, enrich, and support the framework introduced by generalized force-energy expressions all over this chapter, and especially starting from (2.3) to (2.4.3) and from (2.5) to (2.9).

Experiments (of Japanese research workers) are already confirming the change of spinning-mass weight in the field of gravitation of our planet. "Spinning mass that is spinning in the same direction and the same plane as that of the Earth weigh little more than when at rest on the Earth's surface, and when spinning in the opposite direction weigh a little less" (cited information taken from [36]).

Let us analyze familiar situation regarding spinning bullet in a rectilinear motion. It is known that a bullet spinning will significantly stabilize and strengthen the rectilinear motion of the bullet (thanks to gyroscopic effect). On some way, in this case, spinning and linear motion are mutually associated, coupled and being complementary motions, like in de Broglie matter waves hypothesis. Mechanical spinning of the bullet should on certain direct way superimpose to a corresponding de Broglie, matter-waves wavelength, and frequency.

A) To make relevant conclusions we can go step by step, starting from the situation when a bullet is performing only rectilinear motion without self-spinning, what corresponds to,

$$E_{tot} = \gamma m_0 c^2 = E_0 + E_k = m_0 c^2 + \frac{pv}{1+\sqrt{1-v^2/c^2}}, \; p = \gamma m_0 v \quad (2.11\text{-}1)$$

B) If bullet start spinning (with an energy amount E_s), we will have,

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$$E_{tot} = \gamma\left(m_0 + \frac{E_s}{c^2}\right)c^2 = \left(E_0 + E_s\right) + E_k^* = \left(m_0 + \frac{E_s}{c^2}\right)c^2 + \frac{pv + \mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}} \Rightarrow$$
$$(\gamma-1)m_0c^2 + E_s = \frac{pv + \mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}},\ (\gamma-1)m_0c^2 = \frac{pv}{1+\sqrt{1-v^2/c^2}},\ E_s = \frac{\mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}}. \quad (2.11\text{-}2)$$

C) Now, let us imagine that initially we do not have an object that has its standstill rest mass ($m_0 = 0$, v << C), or that on some way the same bullet is composed only of *matter wave packets* as photons or other motional energy states. Such superposition of anyway spinning photons is hypothetically and eventually creating the resulting, total rest mass of the same bullet $m_0 > 0$. This would produce the following energy balance,

$$E_s \rightarrow m_0c^2 = \frac{\mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}} \cong \frac{1}{2}\mathbf{L}_s\omega_s \Rightarrow \mathbf{L}_s\omega_s = 2m_0c^2 = \sum_{(i)}\mathbf{L}_{si}\omega_{si} \quad (2.11\text{-}3)$$

The idea here is that a rest mas can be created as a superposition of *spinning matter wave states* or photons.

The familiar situation regarding new understanding of gravitation in relation to spinning, orbital motions, globalized macro-cosmic-rotation, linear and angular moments and associated electromagnetic moments, fields and currents is even much more profoundly embedded (than shown in this book) in new theories, research projects and programs regarding gravitation. Development of such innovative concepts will gradually produce significant impact on everything we presently consider as solid stepstones and theories of our Physics, such as: Big-Bang, Black holes, Dark mass and energy, Fundamental forces of nature, Matter and antimatter coexistence, Relativity and Quantum theory… (read citations below).

Citation from: https://einstein.stanford.edu/SPACETIME/spacetime4.html
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The Many Faces of Spin

Many of nature's deepest mysteries come in threes. Why does space have three spatial dimensions (ones that we can see, anyway)? Why are there three fundamental dimensions in physics (mass M, length L and time T)? Why three fundamental constants in nature (Newton's gravitational constant G, the speed of light c and Planck's constant h)? Why three generations of fundamental particles in the standard model (e.g. the up/down, charm/strange and top/bottom quarks)? Why do black holes have only three properties -mass, charge, and spin? Nobody knows the answers to these questions, nor how or whether they may be connected. But some have sought for clues in the last-named of these properties: spin.

………………….

Nobel laureate C.N. Yang wrote in a letter to NASA Administrator James M. Beggs in 1983 that general relativity, "though profoundly beautiful, is likely to be amended ... whatever [the] new geometrical symmetry will be, it is likely to entangle with spin and rotation, which are related to a deep geometrical concept called torsion ... The proposed Stanford experiment [Gravity Probe B] is especially interesting since it focuses on the spin. I would not be surprised at all if it gives a result in disagreement with Einstein's theory."

……………………………………..

Gravito-Electromagnetism

…….When gravitational fields are weak and velocities are low compared to c, then this decomposition takes on a particularly compelling physical interpretation: if we call the scalar component a "gravito-electric potential" and the vector one a "gravito-magnetic potential", then these quantities are found to obey almost exactly the same laws as their counterparts in ordinary electromagnetism! (Although little-known nowadays, the idea of parallels between gravity and electromagnetism is not a new one and goes back to Michael Faraday's experiments with "gravitational induction" beginning in 1849.) One can construct a "gravito-electric field" g and a "gravito-magnetic field H from the divergence and curl of the scalar and vector potentials, and these fields turn out to obey equations that are identical to Maxwell's equations and the Lorentz force law of ordinary electrodynamics (modulo a sign here and a factor of two there; these can be chalked up to the fact that gravity is associated with a spin-2 field rather than the spin-1 field of electromagnetism). The "field equations" of gravito-electromagnetism turn out to be of great value in interpreting the predictions of the full theory of general relativity for spinning test bodies in the field of a massive spinning body such as the earth — just as Maxwell's equations govern the behavior of electric dipoles in an external magnetic field. From symmetry considerations we can infer that the earth's gravito-electric field must be radial, and its gravito-magnetic one dipolar, as shown in the diagrams below:

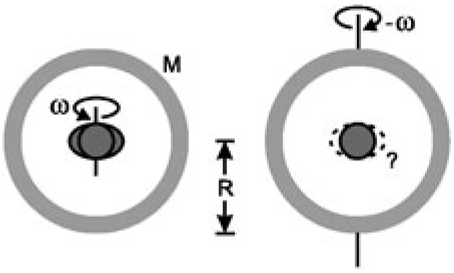
………………………..

Frame-dragging Effect

Frame-dragging in realistic experimental situations is not nearly that strong and the utmost ingenuity has to be exercised to detect it at all. Analysed in terms of the gravito-electromagnetic analogy, the effect arises due to the spin-spin interaction between the gyroscope and rotating central mass and is perfectly analogous to the interaction of a magnetic dipole ***μ*** *with a magnetic field* ***B*** *(the basis of nuclear Magnetic Resonance Imaging or MRI). Just as a torque* ***μ×B*** *acts in the magnetic case, so a gyroscope with spin* ***s*** *experiences a torque proportional to* ***s×H*** *in the gravitational case. For Gravity Probe B, in polar orbit 642 km above the earth, this torque causes the gyroscope spin axes to precess in the east-west direction by a mere 39 milliarcsec/yr — an angle so tiny that it is equivalent to the average angular width of the dwarf planet Pluto as seen from earth.*

…………………………

The calculations show that general-relativistic frame-dragging goes over to "perfect dragging" when the dimensions of the large mass (its size and density) become cosmological. In this limit, the distribution of matter in the universe appears sufficient to define the inertial reference frame of observers within it. For a particularly clear and simple explanation of how and why this happens, see The Unity of the Universe (1959) by Dennis Sciama. Had Mach lived 10 years longer, he could have predicted the existence of the extragalactic universe based on observations that the stars in the Milky Way rotate around a common center!



Would the earth still bulge, if it were standing still and the universe were rotating around it?

To put the cosmological significance of frame-dragging in concrete terms, imagine that the earth were standing still and that the rest of the universe were rotating around it: would its equator still bulge? Newton would have said "No". According to standard textbook physics the equatorial bulge is due to the rotation of the earth with respect to absolute space. On the basis of Lense and Thirring's results, however, Einstein would have had to answer "Yes"! In this respect general relativity is indeed more relativistic than its predecessors: it does not matter whether we choose to regard the earth as rotating and the heavens fixed, or the other way around: the two situations are now dynamically, as well as kinematically equivalent. ♣]

Another strong (and still hypothetical) assumption regarding rotation, spinning and explanation of Gravitation is related to Wave-Particle Duality of matter. We know that matter or mass constituents are atoms, electrons, protons, and neutrons, and all of them have spinning and magnetic moments. For all of such and familiar micro-world elementary entities or building blocks of masses, we know that de Broglie matter waves concept and wave-particle duality are the facts, experimentally and theoretically very well proven and supported (see much more in Chapters 4.0, 4.1, and 10.). Briefly summarizing, elementary entities in motion (such as atoms, electrons, protons, neutrons…) are on some way kinematically equivalent to corresponding moving wave-packets, where dominant (or mean) wavelength is, $\lambda = h/p = h/m^*v = h/\gamma mv$. In other words, conceptually we could imagine a moving particle (which naturally has internal spinning and magnetic moments) followed, externally encircled, or mathematically replaced by certain spinning, helicoidal (or spiral) waving form (being a wave-group or wave packet), which has its group and phase velocities v and u. Both, kinetic and/or wave energy of a moving micro-particle and its equivalent wave group are mutually equal (see why and how this works in Chapters 4.0, 4.1, and 10.). In other words, briefly summarizing, a particle in linear motion (having linear momentum p) is creating specific *spinning wave-group* having certain spinning or angular momentum $\mathbf{L}_s$ and spinning angular velocity ω_s . Naturally, it should also be valid (the opposite situation) that mechanically spinning particle (which was initially in the state of rest, with zero linear momentum, p=0) will obtain certain linear or axial thrust *during accelerated spinning phase* (or motion characterized with non-zero linear momentum p >0), which is colinear with the spinning moment $\mathbf{L}_s$. ***In other words, meaning of such (still hypothetical) assumptions is that with a mass in "accelerated-spinning" we can create a thrust or force acting against Gravitation (or an opposite case, depending on spinning moment direction, is that created linear thrust will increase existing gravitational force). Practically, in some spinning situations (combined with associated effects of magnetic field), torque and linear force related to the same object are mutually dependent and transformable, like in cases of photons and gyroscopes*** *(see T.4.0. Photon – Particle Analogies from Chapter 4.1, and "10.02 Meaning of natural forces", in Chapter 10.). Now we can summarize both of just elaborated, analogical situations as,*

$$\left\{\begin{array}{l} \left[\begin{array}{l} \begin{pmatrix} \text{Linear particle motion} \\ \lambda = h/p = h/m^*v = h/\gamma mv \end{pmatrix} \Leftrightarrow \begin{pmatrix} p = m^*v \\ \tilde{E} = hf \end{pmatrix} \underline{\text{Creates matter-wave spinning } (\mathbf{L}_s = J^*\omega_s)}, \\ u = \lambda f = \tilde{E}/p,\ \tilde{E} = hf = E_k = \dfrac{m^*v}{1+\sqrt{1-v^2/c^2}},\ v = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda} = \dfrac{dE}{dp} = \dfrac{dE_k}{dp} = \dfrac{d\tilde{E}}{dp} \end{array}\right] \\ \Leftrightarrow \\ \left[\begin{array}{l} \text{Mechanically spinning particle } (\mathbf{L}_s = J^*\omega_s) \ \underline{\text{can create linear particle thrust } (p = m^*v)}, \\ E_s = \dfrac{\mathbf{L}_s\omega_s}{1+\sqrt{1-v^2/c^2}} = E_k = \dfrac{pv}{1+\sqrt{1-v^2/c^2}} = \tilde{E} = hf \Rightarrow [\mathbf{L}_s\omega_s = pv] \end{array}\right] \end{array}\right\} \quad (2.11\text{-}4)$$

All what is valid for elementary and micro particles is also applicable to macro astronomic masses and systems if such systems are intrinsically periodical or repetitive (such as solar systems), except that new macro-world Planck constant H would be different compared to micro-world Planck's analog constant h (see more, later in this chapter under "2.3.3. Macro-Cosmological Matter-Waves and Gravitation").

Since spinning and magnetic moment are causally related, we can safely say that both mass spinning and mass magnetic properties should have the significant influence on Gravitation. Also, we should know that existence of magnetic properties is impossible without significant backing related to electric and spinning matter properties and vice versa (see much more in Chapters 3., 4.0, 4.1, and 10.).

[♣ COMMENTS & FREE-THINKING CORNER (only a raw material for later editing)**:**

In the next several brainstorming examples (see below A, B, C until F), we will briefly hypothesize and exercise the ideas about moments and masses couplings. We will try to suggest what could or would-be consequences if we creatively combine available options; -of course this should be tolerated with a high level of intellectual flexibility (see also equations in chapter 5, from (5.4.1) until (5.4.10)).

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- A -

Let us first imagine that kind of elementary matter-wave, or energy state, which coincidently has coupled elements of linear and rotational motions, presents an essential building block of everything that is creating a mass. For instance, let us start with a matter-wave state Ψ (specific wave-packet function) that is in linear motion, combined with elements of spinning (ω_s), which is effectively presenting quantity of linear motion with an angular (or spin) mechanical momentum, being relatively stable (non-dispersive) time-space formation. The idea here is to show that many such elementary states, which have forms of atomized energy domains (like wave packets), are effectively, after certain kind of superposition and integration, creating an entity we externally identify as a stable particle that will have its rest mass. We could start making such concept mathematically operational by introducing the elementary matter-wave function Ψ on the following way (see also analogies from the first chapter and equations starting with (4.9.0) from the chapter 4.3, addressing wave functions):

$$\left[\begin{array}{c}\left\{\begin{array}{l}E_k = E_{k-lienear} + E_{k-spinning} = \frac{1}{2}mv^2 + \frac{1}{2}J\omega_s^2 = \\ = \frac{1}{2}pv + \frac{1}{2}L_s\omega_s = \frac{p^2}{2m} + \frac{L_s^2}{2J} = \frac{1}{2}(m+\Delta m)v^2 = \\ = \frac{1}{2}m^*v^2 = \frac{1}{2}p^*v,\ \Delta m = J\left(\frac{\omega_s}{v}\right)^2 \\ p = mv,\ L_s = J\omega_s, p^* = (m+\Delta m)v = m^*v\end{array}\right\} \wedge \left\{\begin{array}{c}m\\v\\p\\vdp\end{array}\right\}\left(\begin{array}{c}\Leftrightarrow\\ \text{analog to}\end{array}\right)\left\{\begin{array}{c}J\\ \omega\\ L\\ \omega dL\end{array}\right\} \Rightarrow \\ \Rightarrow dE_k = \underline{vdp + \omega_s dL_s} = \Psi^2 dt = vdp^*(= \omega dL^*)\end{array}\right], \quad (2.11\text{-}5)$$

$\omega_s = 2\pi f_s$ (=) spinning frequency (around own particle axis, linked to mass center)

$\omega = 2\pi f$ (=) rotation around some other (externally placed) point

or,

$$\left[\begin{array}{c}\left\{\begin{array}{l}E = \sqrt{E_0^2 + p^2c^2} = E_0 + E_k = \gamma mc^2, \\ E_0 = mc^2 = \text{const.} \\ p = \gamma mv,\ \gamma = (1 - v^2/c^2)^{-0.5}\end{array}\right\} \text{and} \left\{\begin{array}{c}m\\v\\p\\vdp\end{array}\right\} \Leftrightarrow (\text{analog to}) \left\{\begin{array}{c}J\\ \omega\\ L\\ \omega dL\end{array}\right\} \Rightarrow \\ \Rightarrow dE = dE_k = c^2 d(\gamma m) = \underline{vdp + \omega_s dL_s} = \Psi^2 dt\end{array}\right] \Rightarrow$$

$$\Psi^2 = \frac{dE}{dt} = \underline{v\frac{dp}{dt} + \omega_s\frac{dL_s}{dt}} = \frac{dx}{dt}\frac{dp}{dt} + \frac{d\alpha}{dt}\frac{dL_s}{dt} = \underline{vF + \omega_s\tau} = \text{Power} = [W] \quad (2.5.1)$$

The idea behind (2.5.1) is to start introducing the concept that any particle motion (regarding its total energy content, and its internal building blocks) is composed of mutually coupled linear motion components **vdp**, and spinning motion components $\omega_s dL_s$. Both mentioned motions are intrinsically involved in creating a total particle mass, where spinning elements are creating and contributing its rest mass.

Of course, we should not forget that every linear particle motion (without measurable self-spinning) could also be presented as a rotational motion around some externally placed point (center of rotation) where involved curvature radius (radius of rotation) could be arbitrary long: $dE = dE_k = c^2 d(\gamma m) = vdp = \omega dL$. Also, simple circular motion of a particle (without self-spinning) which has mass **m** can be equally (or dually) treated as linear particle motion (on a circular path), or rotational particle motion around its center of rotation, as for example, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}J\omega^2 = \frac{1}{2}pv = \frac{1}{2}L\omega$, $dE_k = vdp = \omega dL$, $v = \omega r, J = mr^2$, $p = mv, L = J\omega$, $\frac{L}{p} = \frac{v}{\omega} = \sqrt{\frac{J}{m}} = r$.

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- B -

Motional or kinetic particle energy could be treated as having any (velocity dependent) positive value, if this energy is measured “externally”, in the space where a particle is in motion. If we attempt to solve the relativistic equation that is connecting all energy-momentum aspects of a single particle, we will find that one of the solutions for kinetic energy could be the negative energy amount that corresponds to the particle rest-mass energy, as follows,

$$\left\{\begin{array}{l} E^2 = E_0^2 + p^2c^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2 , \\ E_0 = mc^2 , \ E = \gamma mc^2 , \ E_k = E - E_0 = (\gamma - 1)mc^2 , \ \gamma = (1 - v^2/c^2)^{-1/2} \end{array}\right\} \Rightarrow$$
$$E_k^2 + 2E_0E_k - p^2c^2 = 0 \Rightarrow E_k = -E_0 \pm \sqrt{E_0^2 + p^2c^2} = -E_0 \pm E \Rightarrow$$
$$E_k = \left\{\begin{array}{l} +E_k \\ -E_0 \end{array}\right\} = \left\{\begin{array}{l} (\gamma - 1)mc^2 \\ -mc^2 \end{array}\right\} (=) \qquad (2.5.1\text{-}1)$$
$$(=) \left\{\begin{array}{l} \text{motional particle energy in "external space"} \ = \ (\gamma - 1)mc^2 \\ \text{motional particle energy internaly captured by rest mass} \ = -mc^2 \end{array}\right\} .$$

Such result, (2.5.1-1), could look illogical, and could be neglected or considered as unrealistic. The internal particle content (that is creating its rest mass) is also composed of motional (and spinning) energy components, well packed, self-stabilized and internally self-closed. Consequently, we could consider another imaginative and conceptual approach, which is stating that negative motional energy belongs to ordinary motional energy that is "frozen", packed or captured by a particle rest mass,

$$dE = dE_k = c^2 d(\gamma m) = vdp + \omega_s dL_s \Rightarrow \int_{[\omega]} \omega_s dL_s = -E_0 = -mc^2 . \qquad (2.5.1\text{-}2)$$

The process of stable rest-mass creation should just be a part of an integration (or superposition) process, based on solving equations (2.5.1) and (2.5.1-2). The energy component ωdL should stay internally blocked or captured and "frozen" by the particle structure, in most of the cases not being visible as anything that rotates externally, basically creating constant and stable particle rest-mass that mathematically disappears in the process of creating differential equations, because derivations of constants are zeros. Saying the same differently, here is tried to show that rest mass represents an accumulator, or reservoir of concentrated, self-stabilized and well-integrated, internal “spinning energy elements” (understanding that in this case the best terminology and complete conceptualization is still evolving or missing, but this should not affect the attempt to express such idea). When rest mass starts moving and interacting with other objects "externally" (for instance making a linear motion or participating in some events of scattering and impacts with other particles, or quasi-particles), internally packed “*rotational energy, spinning, and vortex content*” is somehow externally “unfolding”. Such unfolding process is becoming directly involved in de Broglie, matter waves creation, starting to interact with near objects (being indirectly measurable by its consequences such as Compton Effect, Photoelectric Effect, Diffraction of elementary particles, etc.; -For more, see also the chapter 4.0 and 4.1 of this book). The most direct externally detectable signs documenting the existence of such hidden (internally packed) rotation-related behaviors should be spin and orbital moment attributes of all elementary particles (but we also know that planets, moons, asteroids, and other astronomic objects are spinning).

- C -

To be bottom-line simple and clear, let us consider that a particle with mass **m** is moving in its laboratory system with a velocity **v**, performing externally apparent rectilinear motion (with no measurable elements of spinning: the middle column of T.2.3). Even here, we will attempt to prove that such mass in motion still presents the united-state of rotational and rectilinear motions, where some aspects of internal spinning are hidden (or packed) inside of the rest mass structure and externally not visible, but energy-vise present in the total particle energy. If the same moving particle were also spinning, this time having an externally measurable rotation, its total motional energy would get one more member, as given in T.2.3 (last, right column).

T.2.3 The motional particle with non-zero rest mass (in a Lab. System)	The particle which is only in rectilinear motion	Particle in rectilinear motion combined with spinning
Total particle energy $dE_t = dE = dE_k = vdp$	$E_t = E_0 + E_k =$ $= mc^2 + \int_{[v]} vdp$ $E_0 = E(L_0) = mc^2$ $E_k = (\gamma - 1)mc^2 = \int_{[v]} vdp$ $E_t^2 = E_0^2 + p^2c^2$ $\left\{ v << c \Rightarrow E_k = \frac{1}{2}mv^2 \right\}$	$E_t = E_0 + E_k =$ $= mc^2 + \int_{[v]} vdp + \int_{[\omega]} \omega dL$ $E_0 = E(L_0) = mc^2$ $E_k = \int_{[v]} vdp + \int_{[\omega]} \omega dL$ $E_t^2 = (E_0 + \int_{[\omega]} \omega dL)^2 + p^2c^2$ $\left\{ v << c,\ \omega = low \Rightarrow E_k = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 \right\}$
Linear momentum	$p = \gamma mv$	$p = \gamma mv$
Orbital momentum	$\omega = 0,\ L = L_0 + J\omega = L_0$ $L_0 = const.$	$\omega \neq 0,\ \vec{L} = \vec{L}_0 + J\vec{\omega}$

Of course, relations in T.2.3 should agree with linear and orbital momentum conservation laws, and with the law of total energy conservation, regardless of what kind of transformations moving particle is passing. Before being proven (and under which conceptual framework), there is still a high level of speculative and hypothetical meaning of energy transformation from the rotation (or spinning) to a rest mass content, as introduced in T.2.3. This could be equivalent to an imaginative picture that **rest-mass state presents kind of energy vortex-sink** (regarding rotational-motion energy-states: $E_t^2 = (E_0^*)^2 + p^2c^2 = (E_0 + \int_{[\omega]} \omega dL)^2 + p^2c^2 = \gamma mc^2$). In chapter 4.1, we will elaborate similar idea a bit more, by conceptualizing any particle or energy-state motion as a case of two-body, or two-states interaction. There, the first (initial) energy-state is effectively interacting with its overall vicinity (or with the rest of the universe), giving a chance (at least theoretically, or conceptually), to imagine the motion of certain equivalent center-of-mass (which corresponds to such "two-body" situation). Later, by introducing an effective center-of-mass reference system, we would be able to find that an energy-part of such "binary system" presents rotating reduced-mass around its effective center of mass. An interesting consequence of such modeling is that any linear (or curvilinear) motion is just a case of specific rotational movement, where relevant radius $\mathbf{r}$ could be sufficiently large that we would not notice elements of rotation. In other words, there is no purely linear motion in our Universe. In such cases as an appropriate differential energy-balance of such curvilinear motion, we will have as primary energy domains: $dE_t = dE = dE_k = vdp = \omega dL = c^2 d(\gamma m)$, $v = \omega r$, $\vec{r} \times d\vec{p} = d\vec{L}$.

- D -

Let us continue brainstorming and hypothesizing in the frames of the same idea, which is stating that linear and rotational motions are mutually complementary and united. If we start from the relativistic expression for a total moving particle energy, $\mathbf{E_t^2 = E_0^2 + p^2c^2}$, we would be able to notice that such energy has one static or constant energy member $\mathbf{E_0}$, and another dynamic or motional energy member $\mathbf{pc}$. Because of here "postulated intrinsic unity" of linear and rotational motions, we could merely exercise that,

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[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{pc = \text{Dynamic or motional energy part} = E_{\text{linear - motion}} + E_{\text{rotational - motion}} = E_{lm} + E_{rm}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} E_t^2 = E_0^2 + p^2c^2 = (\text{Static or constant energy part})^2 + (\text{Dynamic or motional energy part})^2 = \\ = E^2 = (\gamma mc^2)^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2, \\ E_0 = \text{Static or constant energy part} = mc^2, E_k = \text{Motional energy part} = (\gamma - 1)mc^2, \\ (pc)^2 = (E_{lm} + E_{rm})^2 = 2E_0E_k + E_k^2. \end{array}\right\}$$

$$\Rightarrow E_{rm} = pc - E_{lm} = pc - \frac{pv}{1+\sqrt{1-(\frac{v}{c})^2}} = pc(1 - \frac{\frac{v}{c}}{1+\sqrt{1-(\frac{v}{c})^2}}) = \quad (2.5.1\text{-}3)$$
$$= pc(1-\frac{u}{c}),\ u = \frac{v}{1+\sqrt{1-(\frac{v}{c})^2}}) .$$

What we are getting as a result here, (2.5.1-3), implies merely that *if rectilinear and rotational motions are mutually coupled, the energy of rotating (or associated spinning) motion should be limited and analytically dependent on parameters of its linear-motion couple (somewhat similar as in cases of resonant oscillatory circuits and electromagnetic waves).*

The more promising and more general approach regarding searching for the unity of linear and rotational motions should be to stay (as much as possible) in the framework of the well-proven aspects of Relativity theory. We will first assume that motional or kinetic energy (of a particle) would have linear motion, ($E_{\text{k-linear}} = E_{kl}$*) and spinning (*$E_{\text{k-rotation}} = E_{kr} = E_{\text{spinning}} = E_s$*) components,* $E_k = E_{\text{k-linear}} + E_{\text{k-rotation}} = E_{kl} + E_{kr}$ *. Different indexing for energy members (like, k-linear, kl, k-rotation, kr, spinning) is still present only for making more natural visual, analog, and intuitive connections with similar (often identical) expressions and elaborations in other chapters, since this book is being created (updated, modified) over a very long-time span, and one day such discrepancies and anomalies need to be rectified. Until then, the author is considering as the more essential to transfer the relevant meaning of important innovative messages (related to gravitation) to open-minded, creative-thinking and intellectually flexible readers.*

Square of a 4-vector (in the Minkowski space) of the relevant moment $\overline{P}_4 = \overline{P}\left[p = \gamma mv,\ \frac{E}{c} = \gamma mc\right]$ should be invariant regarding referential system changes, and by continuing similar exercising as before, we will have,

$$\left\{\begin{array}{l} \overline{P}_4^2 = \overline{P}^2(\vec{p}\ ,\ \frac{E}{c}) = \vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{E'^2}{c^2} = \vec{p}''^2 - \frac{E''^2}{c^2} = \text{invariant}, \\ E = E_0 + E_k = E_0 + E_{\text{k-linear}} + E_{\text{k-spinning}} = E_0 + E_{kl} + E_s \end{array}\right\} \Rightarrow \quad (2.5.1\text{-}4)$$
$$\Rightarrow \vec{p}^2 - \frac{[E_0 + E_{kl} + E_s]^2}{c^2} = \vec{p}'^2 - \frac{[E'_0 + E'_{kl} + E'_s]^2}{c^2} = \text{invariant}.$$

From (2.5.1-4) we can elaborate the following possibilities regarding particle states in a particular system of reference (kind of the center of a mass system), where the particle would have a specific state of rest,

$$\vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{[E'_0 + E'_{kl} + E'_s]^2}{c^2} = \begin{cases} -\frac{1}{c^2}(mc^2)^2 = -m^2c^2 \quad \text{if } \vec{p}' = \vec{0},\ E'_s = 0 \\ (\textit{no linear motion, no spinning}), \\ ------------------- \\ -\frac{1}{c^2}(mc^2 + E'_s)^2 \quad \text{if } \vec{p}' = \vec{0},\ E'_s \neq 0 \\ (\textit{no linear motion, only spinning}). \end{cases} \tag{2.5.1-5}$$

From (2.5.1-5) it is almost evident that spinning energy states could effectively be considered entering rest mass or rest energy states ($M = m + \frac{E'_s}{c^2}$) since we can generalize (2.5.1-5) as,

$$\begin{aligned} &\vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{[E'_0 + E'_{kl} + E'_s]^2}{c^2} = -\frac{1}{c^2}(Mc^2)^2 = -M^2c^2, \\ &E = E_0 + E_{kl} + E_s = \gamma Mc^2 = \gamma(m + \frac{E_s}{c^2})c^2,\ E_0 = Mc^2, M = m + \frac{E_s}{c^2}, E_s = E'_s, \\ &E_{kl} = (\gamma - 1)Mc^2 = (\gamma - 1)(m + \frac{E_s}{c^2})c^2,\ p = \gamma Mv = \gamma(m + \frac{E_s}{c^2})v. \end{aligned} \tag{2.5.1-6}$$

See later, as an extension of the same idea, (2.11.4), (2.11.5), (2.11.13-5) - (2.11.13-5), as well as similar elaborations in chapter 4.1.

Significant in (2.5.1-4) – (2.5.1-6) is that we are implicitly introducing "parameterized" state of rest where certain rest mass could have zero linear velocity ($v = 0, E_{kl} = 0, p = 0$) and make only spinning (around an axis passing its center of gravity: $E_{kr} = E_s \neq 0$) or being really and fully in a state of rest without any (externally measurable) linear motion and spinning ($v = 0, \vec{p} = \vec{0}, E_s = 0$). The self-spinning particle energy is practically entering its rest mass ($M = m + \frac{E_s}{c^2}$), and this way we can neglect (or account) such intrinsic or internal particle rotation and consider (externally) that particle is effectively performing only the linear motion. Of course, other "*parameterized rest-mass*" situations are also imaginable (having more energy members). The main idea or intention, here, is to show that certain mass effectively presents well-integrated, or packed spinning states, and to relate such concept to the field of gravitation.

Let us now continue making new brainstorming excursions. The expression (2.5.1-6) is effectively tracking momentum-energy states of a specific particle, which looks like being only in a linear motion (since spinning is "attached" to the rest mass state). We could also say that every linear motion is just a case of specific curvilinear motion, where a radius of rotation $\mathbf{r}$ is arbitrarily large (and where tangential particle velocity is $v = \omega r$). In the case of idealized rectilinear motion, its radius of rotation is equal to infinity ($r \to \infty$), and in all other cases, radius of rotation would be limited ($0 \leq r < \infty$). This way, we could exercise introducing an equivalent rotation into relevant 4-vectors of Relativity Theory, based on a simple analogical formulation, as for instance,

$$\left\{\overline{P}_4^2 = \overline{P}^2(\vec{p}\,,\,\frac{E}{c}) = \vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{E'^2}{c^2} = \vec{p}''^2 - \frac{E''^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2} = -M^2c^2\,,\right\} \Rightarrow$$

$$\Rightarrow \left\{\overline{\mathbf{L}}_4^2 = \overline{\mathbf{L}}^2(\vec{\mathbf{L}}\,,\,\frac{E}{\omega_c}) = \vec{\mathbf{L}}^2 - \frac{E^2}{\omega_c^2} = \vec{\mathbf{L}}'^2 - \frac{E'^2}{\omega_c^2} = \vec{\mathbf{L}}''^2 - \frac{E''^2}{\omega_c^2} = \text{invariant} = -\frac{E_0^2}{\omega_c^2} = -\mathbf{J}^2\omega_c^2\right\},$$

$$E^2 = E_0^2 + \vec{p}^2c^2 = E_0^2 + \vec{\mathbf{L}}^2\omega_c^2,\; E = \gamma Mc^2 = \frac{c^2}{rv}\mathbf{L},\; M = m + \frac{E_s}{c^2},$$

$$E_0 = mc^2 + E_s = Mc^2 = \mathbf{J}\omega_c^2 = \frac{c^2}{r\gamma v}\mathbf{L} = E - E_k\,,$$

$$\vec{p} = \gamma M\vec{v},\; \vec{\mathbf{L}} = \vec{L} + \vec{S} = \gamma\mathbf{J}\vec{\omega} = \vec{r}\times\vec{p}\,,$$

$$\vec{p}^2c^2 - (E_k + E_0)^2 = -E_0^2 \Leftrightarrow \vec{p}^2c^2 = 2E_kE_0 = 2E_k(mc^2 + E_s) \Rightarrow$$

$$E_k = E - E_0 = (\gamma - 1)Mc^2 = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\vec{p}^2c^2}{2(mc^2 + E_s)} =$$

$$= (\gamma - 1)\mathbf{J}\omega_c^2 = \frac{\mathbf{L}\omega}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\left|\vec{L}+\vec{S}\right|\omega}{1+\sqrt{1-\frac{v^2}{c^2}}}, \qquad (2.5.1\text{-}7)$$

$$v = \omega r\,,\; pv = \mathbf{L}\omega\,,\; pc = \mathbf{L}\omega_c\,,\; \frac{c}{\omega_c} = \frac{v}{\omega} = \sqrt{\frac{\mathbf{J}}{M}}\,,\; \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{\omega^2}{\omega_c^2}}},$$

$$dE = dE_k = vdp = \omega d\mathbf{L}\,,\; pdv = \mathbf{L}d\omega\,,\; vdp + pdv = \omega d\mathbf{L} + \mathbf{L}d\omega\,.$$

It is evident that here we are testing the concept that internal, effective spinning is an intrinsic property of a rest mass (even in cases when externally we do not see or detect spinning). Of course, other "*parameterized" total-energy* situations are also imaginable (as having additional energy members).

- E -

If we accept that internal elementary particle structure is a somewhat complicated and multiple, self-closed and rotating wave formation, we cannot claim that (at the same time) there is also another finite and initial rest mass inside of it. In other words, what we see and measure as a particle rest mass, externally, should be the product of specific internal wave-energy packing (making impression externally that this is a stable and solid particle). Consequently, none of the variables in differential relations in (2.5.1) can be treated as constant,

$$d(\gamma m) = md\gamma + \gamma dm,\; d(\gamma mv) = mvd\gamma + \gamma vdm + \gamma mdv = \frac{1}{c^2}dE\,, \qquad (2.5.2)$$

However, after integration, we could have the constant rest mass as a product.

- F -

Another message that could be extracted starting from (2.5.1) and later, is that resulting force acting on any mass in motion (not only between elementary particles) should have one linear, $F = F_{linear}$, and one angular, $\tau = F_{angular}$ or spinning component. Moreover, the **original Newton force definition, as the time derivation of linear momentum,** $F = \frac{dp}{dt}$**, should be (analogically) generalized to have both, angular,** $\tau = \frac{d\mathbf{L}}{dt}$ **and linear force component/s**, as for instance,

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[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\frac{dE}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = vF + \omega\tau = \frac{1}{dt}\cdot\left[dx\cdot\frac{dp}{dt}\right] + \frac{1}{dt}\cdot\left[d\alpha\cdot\frac{dL}{dt}\right] =$$
$$= \frac{1}{dt}\cdot\left[\begin{pmatrix}\text{Energy realized}\\ \text{by linear}\\ \text{force}\\ \text{component}\end{pmatrix} = vFdt\right] + \frac{1}{dt}\cdot\left[\begin{pmatrix}\text{Energy realized}\\ \text{by angular}\\ \text{force}\\ \text{component}\end{pmatrix} = \omega\tau dt\right] = c^2\frac{d(\gamma m)}{dt} = \Psi^2 \quad (2.5.3)$$

Probably that here underlined forces concept (which belongs to classical mechanics) should be better elaborated, but the principal message is already given: **Rotation and linear motion are always united, and their force components (F and τ) are mutually interacting respecting rules of Vector-Algebra** (of course after correctly applying necessary dimensional arrangements)**.**

Let us, for a moment, just forget all questions regarding specific fields and forces, which are involved in realizing certain motion, and start from the general platform, if an object (particle, wave, etc.) has a specific energy and certain effective mass. The most common to all the matter motions would be that any of them should have one linear and one angular force component. First, linear force component acting along its axial path, (displacement Δx), and second that is sweeping an angle on its circular path (angle segment $\Delta\alpha$), like indicated in (2.5.3), including certain external, static, or relative rest energy level $\mathbf{E_0}$:

$$E_{tot.} = F_{linear}\cdot\Delta x \;+\; F_{angular}\cdot\Delta\alpha \;+\; E_0 = E_{k-linear} \;+\; E_{spinning} \;+\; E_0 = E_t = \gamma Mc^2 \quad (2.6)$$

Let us now (mathematically) rearrange the expression for the linear force definition (regarding the force acting on a moving particle in linear motion, based on Newton force definition: $\mathbf{F} = \mathbf{F_{linear}} = \mathbf{dp/dt}$), replacing mass by its relativistic energy equivalent, on the similar way as it was already realized in developing force expressions (2.4) - (2.4-3).

$$F = \frac{dp}{dt} = \frac{d(\gamma mv)}{dt} = \frac{d(\frac{E_t}{c^2}v)}{dt} = \frac{d(\frac{E_0+E_k}{c^2}v)}{dt} = \underline{\frac{E_t}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_t}{dt}} =$$
$$= \frac{E_0}{c^2}\frac{dv}{dt} + \frac{E_k}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_0}{dt} + \frac{v}{c^2}\frac{dE_k}{dt} = \underline{\frac{E_0}{c^2}\frac{dv}{dt} + \frac{E_k}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_k}{dt}} = F_{linear}(=)\left[\frac{kg\times m}{s^2}\right], \quad (2.7)$$
$$E_k = E_{k\text{-linear}} = (\gamma - 1)mc^2.$$

By analogy with (2.7), we could hypothetically "reinvent" the expression for the missing angular (or rotational) force definition, known as torque,

$$\tau = \frac{dL}{dt} = \frac{E_t}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_t}{dt} = \underline{\frac{E_0}{\omega_c^2}\frac{d\omega}{dt} + \frac{E_s}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_s}{dt}} = F_{angular}(=)\left[\frac{kg\cdot m^2}{s^2}\right], \quad (2.8)$$
$$E_k(=)E_{k-spinning} = E_s,\; \omega_c(=)[c/r](=)[1/s].$$

Since the total energy, (2.6), and an entire resulting force, should have both, linear and rotational (or spinning) elements (here mentioned as angular force elements), we can merely combine (2.7) and (2.8),

$$\left.\begin{array}{l} E_{tot.} = E_t = \iiint\limits_{[x,\alpha]}(vdp + \omega dL) = F_{linear}\cdot\Delta x \;+\; F_{angular}\cdot\Delta\alpha \;+\; E_0 = \\ = (\frac{E_0}{c^2}\frac{dv}{dt} + \frac{E_k}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{dE_k}{dt})_{linear}\cdot\Delta x + \\ +(\frac{E_0}{\omega_c^2}\frac{d\omega}{dt} + \frac{E_k}{\omega_c^2}\frac{d\omega}{dt} + \frac{\omega}{\omega_c^2}\frac{dE_k}{dt})_{angular}\cdot\Delta\alpha \;+\; E_0 = \sqrt{E_0^2 + p^2c^2} \\ E_k = E_{k-linear} + E_{k-angular} = F_{linear}\cdot\Delta x \;+\; F_{angular}\cdot\Delta\alpha = \int\limits_{[\Delta t]}\Psi^2(t)dt. \end{array}\right] \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \overline{E}_t = E_0 \pm I\cdot pc = E_t \cdot e^{\pm I\theta} = \sqrt{E_0^2 + p^2c^2} \cdot e^{\pm I\cdot \mathrm{arctg}\frac{pc}{E_0}} = \gamma Mc^2 \cdot e^{\pm I\theta} = \gamma \overline{M}c^2, \\ \overline{M} = M\cdot e^{\pm I\theta} = M\cdot\cos\theta \pm I\cdot M\cdot \sin\theta = M_r \pm I\cdot M_i,\ I^2 = -1, \\ \gamma Mc^2 = \sqrt{E_0^2 + p^2c^2} \Leftrightarrow (\gamma M)^2 = (\frac{E_0}{c^2})^2 + (\frac{p}{c})^2, \gamma = 1/\sqrt{1-(\frac{v}{c})^2}, \\ E_0 = mc^2 + E_s = Mc^2, \vec{p} = \gamma M\vec{v},\ \overline{P} = \gamma\overline{M}v = p\cdot e^{\pm I\theta},\ E_s = E_{k-spinning}, \\ \theta = \mathrm{arctg}\frac{pc}{E_0} = \mathrm{arctg}(\gamma\frac{v}{c}) = \mathrm{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}}, \\ (\Rightarrow \overline{\Psi}^2 = \frac{\partial \overline{E}_t}{\partial t} = \pm Ic\frac{\partial p}{\partial t}). \end{bmatrix} \quad (2.9)$$

Apparently (if we consider (2.7) - (2.9) as relevant), linear momentum p should present some kind of mixed entity that is combining elements of linear motion and some sort of associated rotation (or spinning), as effectively introduced by L. de Broglie, matter waves concept (see later (4.3-0); $\vec{p} = \frac{\omega}{v}\vec{L} = \gamma m\vec{v}, \vec{L} = \frac{v}{\omega}\vec{p} = \frac{v}{\omega}\gamma m\vec{v}$ in chapter 4.1. In chapter 10 of this book, we can find the most complete explanation of the same situation regarding relations between important linear and angular moments (see (10.1.4) - (10.1.7)).).

The next (intuitively and analogically comprehensive) situation would be that when group and phase velocity of a specific energy-moments qualified state in motion, are not mutually equal, we would experience an extended complexity of gravitational and inertial mass (related to **P**article-**W**ave duality). This will implicate necessity for reformulation of Newton laws.

Gravitation, mass spinning and energy

Newton law express gravitational attraction between two masses,

$$F_{12} = -F_{21} = G\frac{m_1 m_2}{r^2} \Leftrightarrow F_{12} + F_{21} = 0 \ .$$

If we imagine that two isolated masses that are mutually attracting with their own gravitational force would experience virtual and infinitesimal displacement along the line which is connecting their centers, an effective consequence will be that mutual mass exchange could happen (such as $dm_1 = -dm_2$), but the total mass of such system is conserved ($m_1 + m_2 = const.$), as follows,

$$F_{12}dr + F_{21}dr = 0 \Leftrightarrow c^2 dm_1 + c^2 dm_2 = 0 \Leftrightarrow dm_1 + dm_2 = 0, dm_1 = -dm_2 \Rightarrow m_1 + m_2 = const.$$

Obviously, gravitational attraction is presenting, or implicitly complying with a mass exchange between mutually attracting masses ($dm_1 = -dm_2$). Here we are considering only attraction between purely gravitational masses (but something similar should be extendable to all kind of energy states).

Let us explore motional and inertial matter states where relevant group and phase velocity are mutually equal. Of course, group and phase velocity (v, u) should be measured related to a coordinate system linked to the inertial state in question,

$$\left\{\begin{matrix} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, \\ v = u \end{matrix}\right\} \Rightarrow \lambda\frac{du}{d\lambda} = 0 \Rightarrow u = \lambda f = const. = C.$$

Based on Relativity theory assumptions we know that group and phase velocity could be mutually equal only in two cases:

$$\{v = u\} \Rightarrow \begin{cases} v = u = 0 \\ v = u = C \text{ (= universal constant equal to speed of light)} \end{cases}.$$

For both limiting cases, we could say that group and phase velocity are not only mutually equal but also constant (meaning that there no acceleration is involved). With the help of some imagination (and little bit of a creative brainstorming guesswork), we could ask ourselves if such two cases ($v = u = C \vee 0$) are in some ways (under certain conditions) describing the same motional state. Such "motional states" (if realistic) could be something like freely propagating photons, or mass in a state of rest (as a "frozen matter-waves condensate"). Anyway, we have arguments to say that motional states in our universe are states of relative motions (including states of rest), where all related velocities are relative to a certain system of reference. Only speed of light (based mostly on somewhat questionable assumptions, within Relativity theory) has a specific privileged or exceptional character, which, in here introduced guesswork, could be applicable to a certain state of rest ($(v = u = 0) \Leftrightarrow (v = u = C)$).

In other cases, when group (or phase) speed is very small, we will have

$$\left\{ v <<< C \Rightarrow v \cong 2u \cong u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} \right\} \Rightarrow v = -\lambda \frac{dv}{d\lambda} \Rightarrow \lambda v = 2\lambda u = 2\lambda^2 f = \lambda_0 v_0 = const.$$

Inertial states are generally related to uniform motions, where there is no force involved (force equals zero). *Here, we should be very much careful with the understanding of universal constant C. Such constant exist, has velocity dimensions, and specific definite value, but there are measurements and mathematical explorations showing situations when the speed of light is not constant (when compared to constant C), as follows,*

$$\left\{ \begin{array}{l} F = \frac{dp}{dt} = \frac{d(\gamma mv)}{dt} = \frac{d(m^*v)}{dt} = m^*\frac{dv}{dt} + v\frac{dm^*}{dt} = 0 \\ m^* = \gamma m,\ \gamma = (1 - v^2/c^2)^{1/2} = \frac{m^*}{m} \end{array} \right\} \Rightarrow \left\{ dp = m^*dv + vdm^* = 0 \Leftrightarrow \frac{dm^*}{m^*} = -\frac{dv}{v} \right\} \Rightarrow$$

$$\Rightarrow \left| \frac{m^*}{m_0} \frac{v}{v_0} \right| = 1,\ m^*v = m_0 v_0 = p = \gamma mv = \text{ Const. },\ (m_0, v_0 = \text{constants})$$

Obviously, for uniform, inertial, and linear motions, linear momentum is conserved and constant ($p = \gamma mv = m^*v = m_0 v_0$), but this is also giving the impression that only group-speed of such motion is uniform and constant. Also, we know that one of the universally valid conservation laws is linear momentum conservation (which is on some way directly interfering with the definition of linear, uniform, and inertial motions). Here, to give a more complete explanation of linear inertial motions, we are exploring different relations between group and phase velocity, because group and phase velocity are belonging to a more selective and more prosperous concept which can help in establishing bridges between worlds of particles and waves (since Newton laws are not sensitive to such conceptualization).

We also know that many uniform and inertial motions in our universe are rotational or orbital (such as planetary motions, motions associated to atoms internal structure…). By analogy (with linear uniform motions), in cases of uniform angular, rotational, or orbital motions, orbital momentum should also be conserved ($\vec{L} = \vec{r} \times \vec{p} = \gamma m \cdot \vec{r} \times \vec{v} = m^* \cdot \vec{r} \times \vec{v} = m_0 \cdot \vec{r} \times \vec{v}_0$) and constant, meaning that both of universally valid *moments conservation laws* are mutually related. More advanced concepts applicable to stable orbital motions are implicating that qualifications like *states of "uniform and inertial"* will be extended and upgraded by addressing associated phenomenology of *stationary, standing waves and quantized, periodical motional states* (what will be shown later).

………………………

Consequently, it would not be surprising (see literature under [23] – [26]) if one day we conclude that natural forces and matter charges, we are presently considering as mutually distinctive, or very much different and specific, have deeper origins and differences only within electromagnetic charges, and their "packing formats". This is also valid for interactions linked to conservation laws related to coupled, orbital, spin, and linear moments ..... ♣]

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER…** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

2.3.1. Extended Understanding of Inertial States

Following the same patterns of thinking, here we are arriving closer to the possibility and necessity to **reformulate the <u>universal law and concepts of inertia</u> regarding free, natural, uniform, stable, stationary, and relative motions (considering as a generally valid case, the existence of specific intrinsic coupling of linear and rotational (or orbital) motions, including spinning, characterized by $\left\{[m, J], [p, \dot{p}], \left[L, \dot{L}\right]\right\}$). Of course, linear, and angular motions are also coupled with associated electromagnetic reality. Mentioned specific coupling of linear and circular motions is deeply related to the Particle-Wave Duality and Matter-Waves concepts elaborated in this book (equally valid for micro and macro world motions). Thoughts of inertial movements presented in this book agree with innovative inertia concepts elaborated in [36], by Anthony D. Osborne, and N. Vivian Pope, but this should be extended or united with elements of Matter-Waves and Particle-Wave duality concepts.**

New understanding of inertial motions could be initially formulated as: "All naturally free and inertial, stable and stationary, uniform motions of objects, particles and energy states in our universe have intrinsic elements of mutually coupled linear and rotational, or orbital motions (see (2.5.1-7)), tending to keep unchanged the combination of their initial (linear and angular) moments-related states, unless acted upon by an unbalanced (external) force.

For changing an inertial-motion state of an object, at least one of external (unbalanced) linear ($F = \frac{dp}{dt} = \dot{p} = F_{linear}$ ***) or orbital (or torque*** $\tau = \frac{dL}{dt} = \dot{L} = F_{angular}$ ***) force components should be involved."***

Saying the same differently we should consider that every stable, non-dispersive, motional mass formation (particle or mass-energy equivalent E/c²), in relation to certain system of reference, being in some of its inertial states, should present certain stationary and stable (coordinates invariant) orbital state of the following mutually correlated attributes: $\left\{[m, J], [p, \dot{p}], \left[L, \dot{L}\right]\right\}$***, naturally following the path of least resistance, or least action. Specifically coupled linear and orbital elements of specific inertial motion*** $\left\{[m, J], [p, \dot{p}], \left[L, \dot{L}\right]\right\}$ ***have a natural tendency to host or populate self-closed standing matter-waves. Here, we should not forget that the mentioned mechanical attributes are also intrinsically coupled with associated electromagnetic properties or charges.*** *See later* (2.9.4) as the step forward in expressing here extended universal law of Inertia.

Let us try to clarify additionally what an inertial state is. <u>*Since inertial motions are presenting a tendency to maintain the same state of orbital motion (ideally endlessly), we could say that this is a kind of uniform, steady, stationary, often resonant, or standing matter-waves state, which is conserving, or keeping stable, specific and*</u>

important parameters, such as its linear and/or orbital moment ($p = const. \Rightarrow F = dp/dt = 0$, *and/or* $L = const. \Rightarrow \tau = dL/dt = 0$). When an object, body or energy-state has such steady motional conditions, we could consider it as an inertial state. Of course, we know that many stable motional states also belong to uniform rotational or spinning motions, and we should extend the meaning of Newtonian inertial states to such steady rotational motions, and to their combinations, as for instance, like orbital and spinning motions of planets in stable solar systems and spinning of subatomic micro particles. ***In fact, (since every linear motion could be presented as an asymptotic case of certain orbital motion with radius of rotation increasing to infinity) linear and angular momentum of certain motion are not separately or independently conserved values (as formulated in Classical Mechanics). Consequently, every motion of certain entity has at the same time both linear and angular moments (being mutually coupled and convertible, very much analogical to couplings between an electric and magnetic field in cases of electromagnetic waves, or electromagnetic oscillators and resonant circuits)***. Later in this chapter, we will demonstrate that inertial planetary, orbital, and periodical motions are really hosting standing matter waves (see 2.3.3. Macro-Cosmological Matter-Waves and Gravitation).

In the first chapter of this book (**1.1 Inertia, Inertial Systems, and Inertial Motions**) we already introduced the concept of inertia and inertial systems, based on (mass – capacitor) analogies, as summarized in T.1.8. It started to be clear that such idea is only locally applicable "in-average" to relative (or dominant) stability within specific periodical motion, where synchronized and standing matter-waves are involved. In fact, every stable and uniform relative motion, or inertial-state motion is in direct relation with periodic movements and associated matter-waves phenomenology. When mentioned matter-waves are becoming uniform, stationary, stable, periodical, and like self-closed standing waves, we can define or describe such states as being inertial states. This is now becoming a more complicated situation than simple and isolated inertial states of linear motions, as initially specified by I. Newton.

Since any linear (or rectilinear) motion is only an idealization and boundary case of certain rotating or orbital motion (with sufficiently large radius of rotation $\mathbf{r}$, where actual velocity in question is $v = \omega_r r$), we could say that any linear inertial motion, and inertial system, are mostly belonging to a specific uniform (or stabilized) rotational movement. We can also conclude from Kepler laws that there is a natural tendency of moving masses towards creating circular or elliptic, closed orbits, when Newton-Coulomb attractive forces are balanced by a repulsive centrifugal force (since in opposite case, we would have only a global tendency towards masses agglomerations). Another natural property of microcosmic entities is to have and maintain stable angular and spin moments. Uniform, stabilized, steady, stationary, and essentially periodical, rotational motions are also presentable as standing-waves resonant structures, or known as possible hosting zones for standing-waves formations. ***In the world of Physics, we are considering as universally valid and coincidently applicable laws of conservation of linear and orbital moments (including total energy conservation).*** Let us follow the first and most traditional description of an inertial state, when its linear moment $\mathbf{p}$ is constant (without specifying the state of its orbital moment L), and combine it with de Broglie, matter waves conceptualization. Also, we will have in mind that linear motion is just a particular case of specific rotating motion with sufficiently large radius **r**. This will produce the following chain of conclusions,

$$\underline{p = \gamma mv = const., \Rightarrow \lambda = \frac{h}{p} = \frac{2\pi r}{n} = Const., 2\pi r = n\lambda = \overline{const.},}\ r = \frac{\overline{const.}}{2\pi} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} (\overline{P}_4)^2 = (p, \frac{E}{c})^2 = inv. \Leftrightarrow p^2 - (\frac{E}{c})^2 = -\left(\frac{mc^2}{c}\right)^2, E = mc^2 + E_k = mc^2 + \tilde{E} = \gamma mc^2 \\ \tilde{E} = hf = E_k = \frac{\gamma mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\gamma J\omega_r^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{L\omega_r}{1+\sqrt{1-\frac{v^2}{c^2}}} = \\ = \frac{2\pi L f_r}{1+\sqrt{1-\frac{(2\pi f_r r)^2}{c^2}}} = \frac{Lc^2}{2\pi r^2}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)\frac{1}{f_r} = \frac{Lc^2}{vr}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right), \\ \gamma mv^2 = \gamma J\omega_r^2 = pv = L\omega_r, v = \omega_r r, \omega_r = 2\pi f_r, u = \lambda f = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{array} \right\} \Leftrightarrow$$

$$\Leftrightarrow \underline{p = \gamma mv = \frac{h}{\lambda} = \frac{nh}{2\pi r} = \frac{\gamma Jv}{r^2} = \frac{L}{r} = const. \Leftrightarrow (r = \underline{const.}, L = CONST.)} \quad (2.9.1)$$

The fact that linear moment **p** *is constant, is producing that at the same time, relevant orbital moment* L *(of the same inertial motion: see equations* (4.3-0) - a,b,c,d,e,f,g,h,i,j,k... in chapter 4.1*) is also constant.* *<u>This is valid only under the assumption that such uniform motion is ideally circular and hosting specific stable, standing-waves formation (</u>*$\underline{2\pi r = n\lambda = n\frac{h}{p} = \overline{const.}}$*<u>).</u>* For any other closed or elliptic orbit, we can analogically apply Wilson-Sommerfeld-Bohr integrals and keep the same meaning of standing waves formations, leading to the constant orbital moment L .

$$\left\{ \oint_{C_n} p_r dr = n_r h \Rightarrow \oint_{C_n} L d\alpha = n_\alpha h \right\} \Leftrightarrow L = n_\alpha \frac{h}{2\pi} = Constant, \quad (2.9.2)$$
$$0 \le \alpha \le 2\pi, \ n_r = 1, 2, 3..., \ n_\alpha = 1, 2, 3..., n \ .$$

Now we have a much better picture what inertial states are. ***To satisfy coincidently both linear and angular moments conservation (what we are anyway considering as universally valid), it is essentially necessary to have a standing matter-waves structure, and this is equivalent to an inertial state*** *(here related to closed, periodical, circular motions).*

Later, we will show that the framework of (2.9.1) and (2.9.2) is equally applicable to microcosmic and astronomical (orbital) motions, considering that instead of micro-world Planck constant **h**, we should apply new "*Planck gravitational constant*" $H >>> h$ (see later "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"). Similar elaboration regarding new constants which are analog to Planck constant (but almost generally applicable to any self-closed macro motion which creates standing waves) can be found in the chapter 4.1, around equations (4.3-0), (4.3-0)-a,b,c,d,e,f,g,h,i,j,k... See also illustrations on Fig.4.1.1 and Fig.4.1.1a.

Here we are not facing something very new, and too original. It is the fact that everything in our universe, from galaxies to elementary particles is in states or relative motions to its environment, either rotating or spinning, or manifesting like being effectively related to certain kind of rotation or having linear and angular motional elements (of course, also naturally connected to coupled oscillatory motions). Here mentioned, natural and mutually coupled linear and rotational motional elements are like the sort of coupling between electric and magnetic fields when an electromagnetic wave is being created. The familiar type of coupling is also described in chapter 4.0 with relations between an original wave function and its Analytic Signal, Hilbert-transform couple. Later, we will discover that in cases of seemingly linear particle motions (where rotation is not externally detectable), there is still a presence of coupling between rotation and linear motion, where rotating energy elements are captured by the internal rest mass structure (see more complete explanation in chapter 10, equations (10.1.4) - (10.1.7)).

Initially, Newton First Law, or Law of Inertia is only saying that **an object at rest tends to stay at rest, and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force**. Such formulation is primarily addressing steady rectilinear or other linear, inertial motions of masses. We will see that, to extend and upgrade our understanding about Gravitation, we need to understand and accept that orbital, circular, rotational, and spinning motions can also be inertial motions (because of natural and direct coupling between linear and rotational motions). We could even say that all movements in our universe are cases of rotational or orbital movements, where a relevant radius of rotation could be arbitrary long. This way, we will transform Newton First Law (of linear inertial motions) towards the law of Inertia addressing only rotational and orbital movements (as the most general case), where rectilinear or linear motions are only specific (extreme and asymptotic) cases of orbital motions. What is underlined in this book is that masses in motions should be surrounded or guided with a (Hilbert) couple of gravitation-related fields (manifesting as coupled linear and rotational motional states), which are in a way analog to the concept of coupling between electric and magnetic fields known in Electromagnetic theory (see much more about electromagnetic and mechanical effects coupling in Chapter 3.). Such extended Newton Law of Inertia should also be a familiar form to similar phenomenology known in electromagnetism under the Faraday and Lenz's laws. Of course, this is becoming evident when proper analogical comparisons and conclusions are implemented, like in the first chapter; -see "T.1.8 Generic Symmetries and Analogies of the Laws of Physics", and in equations under (4.2), in chapter 4.1. For instance, Faraday's law of electromagnetic induction states that $u = -\frac{d\Phi}{dt}$, where u is the electromotive force (emf) in volts, and Φ is the magnetic flux in webers. Further, Lenz's law gives the direction of the induced emf, thus: The emf induced in an electric circuit always acts in such a direction that the current it drives around the circuit opposes the change in magnetic flux that produces the emf. In other words, electric current or flow of electrical charges is like a mass flow, by their common tendency to resist initial motion changes (and something similar applies to inertial spinning motions). Inertia could also be understandable in the conceptual framework of electromagnetic dipoles polarization, which could be the real source of gravitation (see (2.4-8) - (2.4-10)), because most of the particles and other masses in our universe are composed of atoms, and atom constituents are electrons, protons, and neutrons. Of course, the neutron is effectively presenting certain "exotic coupling" between a proton and an electron. Since the mass difference between a proton and an electron is enormous (1836 times), every non-linear, transient, and accelerated motion of some mass will result in specific (internal) electric dipoles polarization. Presence of electric dipoles will be linked to the applicability of Coulomb force, Faraday and Lenz's laws, giving another background for understanding and conceptualizing Inertia and gravitation).

If we consider that coordinate-systems invariance of the square of a 4-vector energy-momentum (in Minkowski space) is something that should be respected, we will be able to give much more general framework regarding forces related to linear and rotational motions, as for example,

$$\overline{P}_4^2 = (\vec{p}, \frac{E}{c})^2 = \text{inv.} \Rightarrow \left[\int_{[\Delta t]} \frac{d}{dt}(\vec{p}, \frac{E}{c})dt\right]^2 = \text{inv.} \Rightarrow \left[\int_{[\Delta t]} \frac{d\vec{p}}{dt}dt, \frac{1}{c}\int_{[\Delta t]} \frac{dE}{dt}dt\right]^2 = \text{inv.} \Rightarrow$$

$$\Rightarrow \left[(\int_{[\Delta t]} \mathbf{F}dt, \frac{1}{c}\int_{[\Delta t]} \frac{vdp}{dt}dt)\right]^2 = \text{inv.} \Rightarrow (\int_{[\Delta t]} \mathbf{F}dt, \frac{1}{c}\int_{[\Delta t]} v\mathbf{F}dt)^2 = \text{inv.} \Rightarrow (\int_{[\Delta t]} \mathbf{F}dt, \frac{1}{c}\int_{[\Delta t]} \mathbf{F}dr)^2 = \text{inv.} \quad (2.9.3)$$

Now we could develop different scenarios regarding forces involved in linear motions, where one of them is,

$$\left(\int_{[\Delta t]} \mathbf{F}dt\right)^2_{(1)} - \left(\frac{1}{c}\int_{[\Delta t]} \mathbf{F}dr\right)^2_{(1)} = -\left(\frac{1}{c}\int_{[\Delta t]} \mathbf{F}dr\right)^2_{(2)} . \quad (2.9.4)$$

If we consider any linear motion as relatively good approximation for specific rotational or orbital movement (with the corresponding radius of rotation $\mathbf{r}$), we could transform 4-vector of linear momentum into a 4-vector of belonging orbital momentum and get invariance expression that is combining orbital (or torque $\vec{\tau} = \frac{d\mathbf{L}}{dt}$, $\vec{\mathbf{L}} = \sum_{(i)}(\vec{L}_i + \vec{S}_i)$, $\vec{S}_i =$ spin) and linear forces ($\mathbf{F} = \frac{dp}{dt}$), as for instance,

$$\left\{(\vec{p}, \frac{E}{c})^2 = \text{inv.} \Rightarrow p^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c})^2 \Leftrightarrow p^2c^2 - E^2 = -E_0^2 \Leftrightarrow p^2c^2 - (\gamma mc^2)^2 = -(mc^2)^2\right\} \Rightarrow$$

$$\Rightarrow \left[\frac{\vec{r}}{r} \times (\vec{p}, \frac{E}{c})\right]^2 = \left[\frac{1}{r}(\vec{\mathbf{L}}, \frac{E}{c}\vec{r})\right]^2 = \text{inv.}, \Leftrightarrow (\frac{\vec{\mathbf{L}}}{r}, \frac{E}{c})^2 = \text{inv.} \Rightarrow$$

$$\Rightarrow \left[\int_{[\Delta t]} \frac{d}{dt}(\frac{\vec{\mathbf{L}}}{r}, \frac{E}{c})dt\right]^2 = \text{inv.} \Rightarrow \left[\int_{[\Delta t]} \left(\frac{d}{dt}(\frac{\vec{\mathbf{L}}}{r}), \frac{1}{c}\frac{dE}{dt}\right)dt\right]^2 = \text{inv.} \Rightarrow$$

$$\Rightarrow\left[\int_{[\Delta t]}\left(\frac{r\frac{d\vec{L}}{dt}-\vec{L}\frac{dr}{dt}}{r^2}\ ,\ \frac{v}{c}\frac{dp}{dt}\right)dt\right]^2=\text{inv.}\Rightarrow\left\{\int_{[\Delta t]}\left[\left(\frac{1}{r}\frac{d\vec{L}}{dt}-\frac{v}{r^2}\vec{L}\right),\ \frac{v}{c}\vec{F}\right]dt\right\}^2=\text{inv.}\Rightarrow$$

$$\Rightarrow\left\{\int_{[\Delta t]}\left[\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right),\ \frac{v}{c}\vec{F}\right]dt\right\}^2=\text{inv.}\Leftrightarrow\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\ ,\ \int_{[\Delta t]}\frac{v}{c}\vec{F}dt\right]^2=\text{inv.}\Leftrightarrow \tag{2.9.5}$$

$$\Leftrightarrow\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\ ,\ \frac{1}{c}\int_{[\Delta t]}\vec{F}dr\right]^2=\text{inv.}\Leftrightarrow\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\right]^2-\left[\frac{1}{c}\int_{[\Delta t]}\vec{F}dr\right]^2=-(\frac{E_0}{c})^2$$

There would be many exciting options resulting from future creative and intellectually flexible development of (2.9.4) and (2.9.5) because (2.9.5) is combining different force and orbital momentum components and expressing or supporting *here extended Law of Inertia*. For instance, an (extended) inertial state should conserve its linear and orbital moments as shown in (2.9.1). This will transform (2.9.4) and (2.9.5) into another controversial and challenging (to think about) result,

$$\left(\vec{p}=\underline{\text{const.}},\vec{L}=\overline{\text{const.}}\right)\Rightarrow\left(\vec{F}=0,\vec{\tau}=0\right)\Rightarrow$$

$$\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\ ,\ \frac{1}{c}\int_{[\Delta t]}\vec{F}dr\right]^2=\text{inv.}\Leftrightarrow\left\{\begin{array}{l}\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\right]^2_{(1)}-\left[\frac{1}{c}\int_{[\Delta t]}\vec{F}dr\right]^2_{(1)}=-(\frac{E_0}{c})^2\\ \left(\int_{[\Delta t]}\vec{F}dt\right)^2_{(1)}-\left(\frac{1}{c}\int_{[\Delta t]}\vec{F}dr\right)^2_{(1)}=-\left(\frac{1}{c}\int_{[\Delta t]}\vec{F}dr\right)^2_{(2)}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\left(-\vec{L}\int_{[\Delta t]}\frac{vdt}{r^2}\right)^2=-(\frac{E_0}{c})^2\Leftrightarrow\left(\int_{[\Delta t]}\frac{vdt}{r^2}\right)^2=\left(\int_{[\Delta t]}\frac{dr}{r^2}\right)^2=-(\frac{E_0}{c\vec{L}})^2,\\ \left(\int_{[\Delta t]}\vec{F}dt\right)^2=\left[\int_{[\Delta t]}\left(\frac{1}{r}\vec{\tau}-\frac{v}{r^2}\vec{L}\right)dt\right]^2,-\left(\frac{1}{c}\int_{[\Delta t]}\vec{F}dr\right)^2=-(\frac{E_0}{c})^2\end{array}\right\}. \tag{2.9.6}$$

2.3.2. Rotation and stable rest-mass creation

Let us imagine that certain particle (like a thin disk, or thin walls toroidal object) is rotating around the fixed-point **O** (see the picture Fig.2.4 below; -case A), with a radius of rotation r. The same particle is then passing through transformation, becoming only the spinning of a compact and small particle ($r \to 0$), and now performing only rotation around its own axis around the same fixed-point **O** (see the picture Fig.2.4 below, case B). All energy and momentum conservation laws should be satisfied between all phases, or motional-states transformation between A and B. Eventually, (let us imagine), the same particle (state B) is finally transformed into an equivalent "standstill" mass (see the picture Fig.2.4 below, case C), of course appropriately interacting with its environment. Here we are trying to speculate how motional energy of any kind (especially rotating and spinning) could be "transformed or packed" into a standstill rest mass. To describe such (still hypothetical) process, we will imaginatively speculate with an analogical formulation of energy forms between linear and rotational motion (see T.2.4 and T.2.5), presently without entering an analysis if, when, how, and why such analogical comparisons are realistic, applicable, possible, or valid. ***In fact, here we (hypothetically) assume that linear and angular moments of certain inertial motion (or a steady state) are not always independently conserved (as formulated in Classical Mechanics), being mutually coupled and convertible (as electric and magnetic fields are always coupled in cases of electromagnetic phenomenology).***

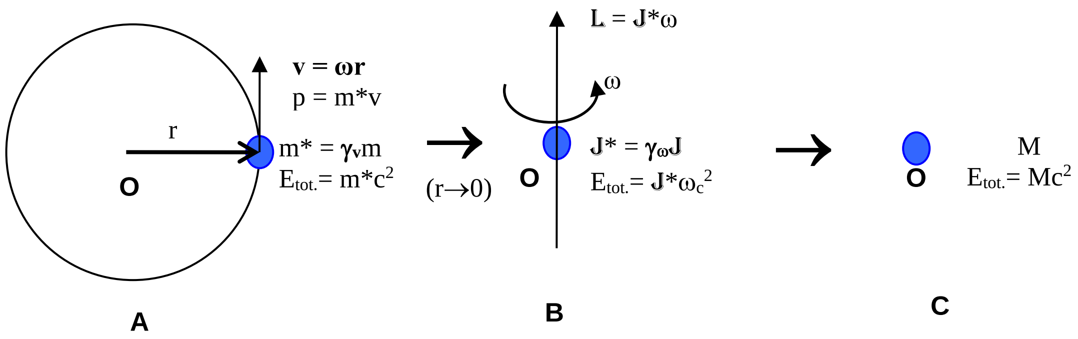

Fig.2.4. Hypothetical evolution of a rotating mass towards standstill mass

Case A: Particle **m** is only rotating around some externally fixed point (not spinning around its own axis).
Case B: Particle **m** is only spinning around its own axis and not making any other motion.
Case C: Particle **m** "energy-wise" transformed into a standstill or rest mass **M** (where revolving and spinning energy is also included).

In all cases, here-involved particles could also have a form of a thin disk, or thin walls, hollow toroidal object.

T.2.4. Motional or kinetic energy expressions formulated based on analogies

Case → Value ↓	A Linear motion	B Spinning	C Standstill
Linear speed: v	$v = v_A$	$\leq c$	0
Angular speed: ω	$\omega = \omega_A$	$\omega = \omega_B$	0
Initial Rest Mass: m	$m = m_A$	m_B	m_C
Motional mass: m*	$m_A^* = m^* = \gamma_v m_A$	$m_B^* = m^*$	0
Total mass: M	$M = m_A^*$	$M = m_B^*$	$M = m_A^* = m_B^* = m^*$
Linear Moment: p	$p = p_A = mv, p^* = p_A^* = m^* v$	Not applicable, (p = 0)	Not applicable, (p = 0)
Static Moment of Inertia: I	J_A	J_B	J_C
"Motional" Moment of Inertia: I*	J_A^*	J_B^*	0
Angular moment: L	$L_A = J_A^* \cdot \omega_A = L$	$L_B = J_B^* \cdot \omega_B = L_A = L$	= 0, internally $= L$, externally
Total Energy: $E_{tot.}$	$E_{tot.A} = m_A^* \cdot c^2 = J_A^* \cdot \omega_{cA}^2 = \sqrt{(mc^2)^2 + p^2c^2}$	$E_{tot.B} = m_B^* \cdot c^2 = J_B^* \cdot \omega_{cB}^2$	$E_{tot.A} = E_{tot.B} = E_{tot.C} = M \cdot c^2$
Motional Energy: E_m, E_k	$E_m = E_k = (m_A^* - m) \cdot c^2 = (\gamma_v - 1)mc^2 = (J_A^* - J_A) \cdot \omega_{cA}^2 = (\gamma_\omega - 1) J_A \omega_{cA}^2 = \frac{pv}{1 + \sqrt{1 - v^2/c^2}}$	$E_m = (m_B^* - m) \cdot c^2 = (\gamma_v - 1)mc^2 = (J_B^* - J_B) \cdot \omega_{cB}^2 = (\gamma_\omega - 1) J_B \omega_{cB}^2 = \frac{L\omega}{1 + \sqrt{1 - \omega^2/\omega_c^2}}$	= 0 , externally $= M \cdot c^2$, internally
Rest Energy: E_0	$m_A \cdot c^2 = J_A \omega_{cA}^2$	$m_B \cdot c^2 = J_B \omega_{cB}^2$	$M \cdot c^2$
Lorentz factor for linear motion: γ_v	$\gamma_v = (1 - v^2/c^2)^{-0.5} = \gamma$	Not applicable	Not applicable
Analogically formulated "Lorentz factor" for the rotational motion: γ_ω	$\gamma_\omega = (1 - \omega_A^2/\omega_{cA}^2)^{-0.5}$	$\gamma_\omega = (1 - \omega_B^2/\omega_{cB}^2)^{-0.5}$	Not applicable
	$\gamma_v = \gamma_\omega = \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \omega^2/\omega_c^2}}$ $v = \omega r \Leftrightarrow c = \omega_c r, \frac{\omega}{\omega_c} = \frac{v}{c}, \omega_c = c\sqrt{\frac{m}{J}} = c\sqrt{\frac{m^*}{J^*}} = \omega \frac{c}{v}.$		

Let us additionally elaborate the same (hypothetical) situation from Fig.2.4 regarding total motional energy transformation towards a rest mass (with internally captured and packed, rotational, motional energy of the process: A → B → C), by extending kinetic energy expressions from the table T.2.4 into equivalent energy expressions as given in T.2.5. In fact, here we are searching how to transform an orbiting mass (with a non-zero radius of rotation) into an ***only spinning***, or ***only-standstill*** mass (when orbiting radius reduces to zero). The more realistic example from Physics, addressing such situations, could be to analyze differences between orbiting electrons (on stationary orbits), and spinning electrons (see chapters 8 and 10).

T.2.5. Motional or kinetic energy expressions formulated based on analogies

	Linear motion: Case A	Spinning: Case B
Kinetic Energy	$E_k = (m^* - m)\cdot c^2 = (\gamma_v - 1)mc^2 =$ $= \frac{\gamma_v mv^2}{1+1/\gamma_v} = \frac{\gamma_v (mr^2)\omega^2}{1+1/\gamma_v} = \frac{\alpha \cdot \mathbf{J} \cdot \omega^2}{1+1/\gamma_v}$ $= \frac{\mathbf{J}^* \cdot \omega^2}{1+1/\gamma_v} = \frac{p \cdot v}{1+1/\gamma_v} = \frac{\gamma_v (mr^2\omega)\omega}{1+1/\gamma_v}$ $= \frac{\mathbf{L} \cdot \omega}{1+1/\gamma_v} = \frac{\mathbf{L} \cdot \omega}{1+\sqrt{1-v^2/c^2}},$ $dE_k = vdp = \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}$ $\mathbf{J}^* = \gamma_v (mr^2) = \alpha \cdot \mathbf{J} = m^* r^2 =$ $= \frac{p \cdot v}{\omega^2} = \frac{p \cdot r}{\omega} = \frac{\mathbf{L}}{\omega},$ $m^* = \gamma_v m,\ v = \omega r,\ p = \gamma_v mv = m^* v,$ $\mathbf{L} = \mathbf{J}^* \omega = \alpha \cdot \mathbf{J} \cdot \omega = p \cdot r, \alpha > 0,$ $\alpha = \frac{\gamma_v mv^2}{\mathbf{J} \cdot \omega^2} = \frac{p \cdot v}{\mathbf{J} \cdot \omega^2} = \frac{\mathbf{J}^*}{\mathbf{J}} = \frac{p \cdot r}{\mathbf{J} \cdot \omega} = \frac{\mathbf{L}}{\mathbf{J} \cdot \omega}$	$E_k = \frac{\alpha \cdot \mathbf{J} \cdot \omega^2}{1+1/\gamma_v} = \frac{\mathbf{J}^* \cdot \omega^2}{1+1/\gamma_v} =$ $= \frac{\mathbf{L} \cdot \omega}{1+1/\gamma_v} = \frac{\mathbf{L} \cdot \omega}{1+\sqrt{1-v^2/c^2}}$ $dE_k = \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}$
	Case C: Here, the externally visible rotation should somehow be transformed into “internally captured” rotation, $r \to 0$, and mass is becoming standstill.	
	$\text{Lim}(\frac{\mathbf{J}^* \cdot \omega^2}{1+1/\gamma_v})_{r \to 0} = E_{tot.} = E_k = \frac{\mathbf{J}^* \cdot \omega^2}{2} = \frac{\mathbf{L} \cdot \omega}{2} = m^* c^2 = Mc^2,$ $(r \to 0) \Rightarrow v = \omega r \to 0, \gamma_v \to 1.$	

In all other situations of combined motions where certain mass is performing linear motion and spinning around its own axis (for instance, connecting case A and case B), motional energy will be

$$dE_k = vdp + \omega d\mathbf{L} = c^2 dm^* = dE_{tot.}. \tag{2.11.1}$$

In case if “rotating particle” is a photon, its motional energy will be, based on data from T.2.5:

$$\tilde{E} = E_k = \text{Lim}(\frac{p \cdot v}{1+1/\gamma_v})_{v \to c} = \text{Lim}(\frac{\mathbf{L} \cdot \omega}{1+1/\gamma_v})_{v \to c} = p \cdot c = \mathbf{L} \cdot \omega \tag{2.11.2}$$

Apparently, if we would like to get Planck’s expression for photon energy $\tilde{E} = hf$, from (2.11.2), $E_k = \mathbf{L} \cdot \omega$, the following conditions should be satisfied:

$$\begin{aligned} & E_k = \tilde{E} = p \cdot c = \mathbf{L} \cdot \omega = hf \\ & \omega = 2\pi f,\ \mathbf{L} = h/2\pi,\ p = hf/c = mc,\ m = hf/c^2 \\ & pr = (hf/c)r = \mathbf{L} = h/2\pi \Rightarrow \\ & \Rightarrow r = c/2\pi f,\ 2\pi r = c/f = \lambda = h/p = h/mc. \end{aligned} \tag{2.11.3}$$

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We already know (from analyses of Compton and Photoelectric Effects) that photon energy is equal to $\tilde{E} = hf$. From Maxwell electromagnetic theory we know that coupled electric and magnetic field vectors (of a photon) are rotating (or spinning) along its path of propagation. Consequently, our picture about ***the photon, as an energy-packet, can get some additional conceptual grounds related to modeling it with a kind of equivalent spinning disk, ring, or torus*** *as in (2.11.3)*. Such chain of conclusions is indeed over-simplified, but still good to make some indicative findings based on analogies. From large experimental and theoretical knowledge base accumulated in Quantum Theory, we also know that other elementary energy quanta in the world of microphysics, which are not necessarily photons (and maybe not forms of electromagnetic energy), also have energies that can be expressed by $\tilde{E} = hf$. By analogy, a kind of intrinsic rotation is also involved there as in case of photons. Furthermore, experimentally is known how sufficiently high-energy photon can be fully transformed into an electron-positron couple, or how contact of an electron and positron (or some other particle and its anti-particle) is annihilating initial participants and producing, for instance, two high-energy photons. A bit later (see: (2.11.9-1) - (2.11.9-4)), it will be shown how an electron can be analogically modeled as a rotating ring, torus, or disk, and in the chapter 4.1 (*see T.4.3 and (4.5-1) - (4.5-4)*), similar ideas about explaining matter waves will also be elaborated.

The intention here is to exercise the idea that elementary "seeds and grains" of all macro-objects, or everything that has a rest mass, are particularly packed and coupled, spinning, and orbiting, self-closed and standing matter-waves, of some simpler (sinusoidal) matter waveforms. Eventually, (most probably) we would find only photons as real, most-elementary matter constituents or waveforms (in a broad background of everything else created based on electromagnetic energy structural formatting; - see also T.4.0, chapter 4.1). Specific energy packing of different matter waves, causally related to rotation and spinning, is creating particles that are self-standing, and have non-zero rest masses, and here is the meaning of the concept that every mass should be an energy-packing format. Presently or also historically, we consider mass as being the source or direct cause of gravitation. We are gradually starting to get familiar with a concept that internal mass content are rotating matter waves (captured as atoms and other elementary particles in different forms of stationary and standing waves with spin properties, including associated electromagnetic attributes). In addition, we know that all masses in our universe are in mutually relative motions (having linear and angular moments). All of that is pointing to the conclusion that Newton law of gravitation should be upgraded with some dynamic, velocities and orbital moments dependent members, and that real sources of gravity are linked to certain "vortex-radiant-energy waves", and mater-waves fields between motional masses, radiating from atoms. In Chapter 10. of this book, we can find the most complete explanation of the same situation regarding still hypothetical background velocity parameters, and Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7)).

Relativistic mass (as defined in Relativity theory) is a mass that is velocity-dependent, and it is effectively increasing when its velocity is increasing. We could ask here, what is effectively increasing. Can we see, measure, touch or visualize such effects of mass increase. The short answer is that an effective matter-wave energy equal to a mass motion (or to its kinetic) energy is creating and increasing mentioned (velocity dependent) matter-wave mass $\tilde{m}$. This way, we are introducing the notion of the matter-wave mass $\tilde{m}$, and we can present that this motional matter-wave mass is something what is effectively coming from a particle kinetic or wave energy, as for example,

$$m = m_0 + \tilde{m} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_0 \Rightarrow \tilde{m} = m - m_0 = m_0 \frac{1-\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = m\left(1-\sqrt{1-\frac{v^2}{c^2}}\right),$$

$$E_k = (m-m_0)c^2 = m_0 c^2 \frac{1-\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} = \tilde{m}c^2 = \tilde{E} = pu = mvu = mv\lambda f = hf \Rightarrow \quad (2.11.3\text{-}1)$$

$$\Rightarrow \lambda = \frac{h}{mv} = \frac{h}{p}, \ u = \lambda f \ (=) \text{phase velocity}, \ p = mv = (m_0 + \tilde{m})v = m_0 v + \tilde{m}v.$$

What will really materialize or happen regarding this <u>*matter-wave mass* $\tilde{m}$</u> depends on number of factors, such as influence of other interaction participants, and on reaching necessary energy-level thresholds, while satisfying involved conservation laws and boundary conditions (related to a moving mass in question). For instance, we expect to generate new particles, photons, electrons etc. Here, ***with (2.11.3-1), we already have all significant elements for the foundation of de Broglie, matter-waves hypothesis (that should obviously be applicable not only to a microworld Physics).***

The process of "fusion between the rest mass and its matter-wave, spinning or rotating energy-mass equivalent" ($E_{spinning}/c^2$) can be conceptualized even simpler. Let us imagine that certain standstill (rest) mass m is passing the process (of its energy transformation) from the level **A**, through the level **B**, and ending with the level **C**, where relevant energy levels are defined by the following tables (T.2.6.1. and T.2.6.2.):

T.2.6.1. A process of rotational energy "injection" into a rest mass

	A	B	C
Levels/States of mass transformation	<u>Initial Standstill, Rest Mass</u>	<u>Only Spinning Mass</u> Mass from **A** starts spinning	Final, new, <u>Equivalent Rest Mass</u> after the spinning mass is being transformed into a standstill mass
Rest Mass	$m = m_0$	$m = m_0$	$M = m + E_{spinning}/c^2$
Total Mass	$m = m_0$	$M = m + E_{spinning}/c^2$	$M = m + E_{spinning}/c^2$
Rest Energy	mc^2	mc^2	Mc^2
Total Energy	mc^2	Mc^2	Mc^2
Motional Energy	0	$E_{speening} = E_s$	0

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Energy evolution should be analogically presentable as stated in T.2.6.2.

T.2.6.2. Energy Analogies between Linear and Spinning motions			
	Only linear motion	Only spinning	Combined
Motional or kinetic energy	$E_k = \frac{1}{2}mv^2 = \frac{1}{2}pv$	$E_s = \frac{1}{2}J\omega_s^2 = \frac{1}{2}L\omega_s$	$E_k + E_s = \frac{1}{2}mv^2 + \frac{1}{2}J\omega^2 = = \frac{1}{2}(m+\Delta m)v^2 = \frac{1}{2}Mv^2$
Motional or kinetic energy (relativistic, analogical formulation)	$E_k = (m - m_0)c^2 = = (\gamma_L - 1)m_0c^2$ $\gamma_L = (1 - v^2/c^2)^{-1/2}$ $m = \gamma_L m_0$	$E_s = (J - J_0)\omega_c^2 = = (\gamma_s - 1)J_0\omega_c^2 = c^2\Delta m \approx \frac{1}{2}J\omega_s^2 = \frac{1}{2}L\omega_s$ (for $\omega_s << \omega_c$), $\gamma_s = (1 - \omega_s^2/\omega_c^2)^{-1/2}$ (Analogically formulated)	$(m - m_0)c^2 + (J - J_0)\omega_c^2 = = (\gamma_L - 1)m_0c^2 + (\gamma_s - 1)J_0\omega_c^2 = = E_k + E_s = = [(m+\Delta m) - m_0]c^2 = = (\gamma_L - 1)(m_0 + \Delta m)c^2 = = (M - m_o)c^2$
Total rest energy	$m_0c^2 = (\sum_{(i)} m_{0i})c^2$	$J_0\omega_c^2 == (\sum_{(i)} J_{0i})\omega_c^2$ (Analogically formulated)	$(m_0 + \Delta m)c^2 = (J_0 + \Delta J)\omega_c^2$ (Analogically formulated)
Total motional energy	$\gamma_L m_0 c^2$	$\gamma_s J_0 \omega_c^2$	$\gamma_L(m_0 + \Delta m)c^2 = \gamma_s(J_0 + \Delta J)\omega_c^2$
	$dE = dE_k = vdp$	$dE = dE_k = \omega_s dL$	$dE = dE_k = vdp + \omega_s dL$

Of course, here (in T.2.6.1 and T.2.6.2.) we are not saying how, when, and under which circumstances, a “rest mass carrier” would capture its spinning energy. What is implicitly stated here (regardless of terminology that is still conditional) is that *under certain conditions spinning energy can be transformed, caught or packed in the form of a stable rest mass, and that every rest mass is an equivalent “packing” form of a certain amount of “frozen rotating or spinning energy”*.

The generalized case of every particle in motion (which has non-zero rest mass m), based on situations from (2.5.1-4) - (2.5.1-6), T.2.4, T.2.5 and T.2.6, is that total particle energy should always have a combination of static (rest), rotating, and liner (rectilinear or curvilinear) motion energy members, as for instance,

$$E_{tot} = (m + E_{spinning}/c^2)c^2 + \frac{pv}{1+\sqrt{1-v^2/c^2}} = Mc^2 + E_{k-lin} = \sqrt{(Mc^2)^2 + p^2c^2} = \gamma Mc^2 = E,$$
$$M = m + E_{spinning}/c^2, \ p = \gamma Mv, \ E_{k-lin} = E_k = \frac{pv}{1+\sqrt{1-v^2/c^2}}, \quad (2.11.4)$$
$$dE = dE_{tot} = dE_k = vdp = \omega dL.$$

Even the rest mass m, from (2.11.4), is also presenting a “frozen rotating energy state” which was created or stabilized in some earlier phase of initial particle creation (what could implicate a preexistence of succession of such states). The same ideas will be

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more elaborated later in this chapter, regarding innovative modeling of an electron (see: (2.11.9-1) - (2.11.9-4)), also around equations (2.11.13-5) - (2.11.13-5), and in chapter 4.1 (see T.4.3 and (4.5-1) - (4.5-4)). Imaginative, structural "in-depth" particle modeling, which is also considering particle past, during its formation, is presented in Chapter 6., (Different possibilities for mathematical foundations of multidimensional universe; -See comments around equations from (6.18) to (6.23)).

From (2.11.4), we can also extract the roots of particle-wave duality, *if we accept that matter-wave energy (at least in cases of micro-world of atoms and its constituents) is equal to a relevant particle kinetic or motional energy*, $\tilde{E} = E_k = E_{tot} - Mc^2 = hf$, *and that it can also be analogically presented as the wave-packet or photon energy in (2.11.3)*, as for instance,

$$\begin{aligned}
&p = \frac{1}{c}\sqrt{E_{tot}^2 - (Mc^2)^2} = \frac{1+\sqrt{1-v^2/c^2}}{v}(E_{tot} - Mc^2),\\
&\frac{v}{1+\sqrt{1-v^2/c^2}} = c\sqrt{\frac{E_{tot} - Mc^2}{E_{tot} + Mc^2}} = u = \lambda f = v_{phase},\\
&\lambda = \frac{c}{f}\sqrt{\frac{E_{tot} - Mc^2}{E_{tot} + Mc^2}} = \frac{\left[\frac{(E_{tot} - Mc^2)}{f}\right]}{p} = \frac{\tilde{E}/f}{p} = \begin{cases} \frac{h}{p} \text{ (for micro world)} \\ \frac{H}{p} \text{ (for macro world)} \end{cases}, (H,h) = \text{constants}\\
&\tilde{E} = E_k = E_{tot} - Mc^2 = E_{tot} - mc^2 - E_{spinning} = \begin{cases} hf \text{ (for micro world)} \\ Hf \text{ (for macro world)} \end{cases}, H >>> h .
\end{aligned} \tag{2.11.5}$$

Nothing in (2.11.5) is strictly saying that h should only be the Planck constant ($h \approx 6{,}6260755\times10^{-34}$ J.s), when we deal with macro-world objects. Later, (in “2.3.2. Macro-Cosmological Matter-Waves and Gravitation”), we will find that matter-waves relations, as in (2.11.5), are equally applicable to gravitational orbital motions of planetary systems, except that Planck constant **h** will be replaced with another constant H with similar meaning $\Rightarrow \lambda = \frac{H}{p}, E_k = \tilde{E} = Hf , H >>> h$. It can be demonstrated that such kind of quantization and matter waves concept ($\lambda = H/p, \tilde{E} = Hf$) is even applicable in cases of “Vortex shedding flowmeter” or oscillatory flowmeter (based on detecting the vibrations of the downstream vortices caused by the barrier placed in a moving stream). The vibrating frequency of vortex shedding can be related to the velocity of the liquid flow; see more in chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i…

The philosophical and conceptual consequence of results from (2.11.5) is that particle-wave duality and matter waves’ nature is related to the internal spinning energy content of a particle in motion. For instance, photon modeling as a rotating disk, ring, or torus (2.11.2) - (2.11.3), and similar innovative modeling of electron structure (2.11.9-1) - (2.11.9-4) are indicative examples for such concepts. Consequently, matter waves should be an “*external unfolding manifestation*” of already existing “*internally-folded*” and rotating matter waves. This will be elaborated later in the same chapter, and much more in the chapters 4.1, 4.2 and 4.3. In other words, masses in motion are creating a space-time web or matrix, capturing, and sensing each other by creating rotational mater-waves motions (supported by Kepler, Newton, and Coulomb laws), and at the same time manifesting forces, and fields or Gravitation. Also, rotating solar system (a spinning sun with many planets rotating and spinning around) could be effectively conceptualized as a (big) single spinning mass in the process of agglomeration (or mass integration), thanks to gravitation as a natural "vortex-sink-like" tendency of mutually approaching masses. *In other words, rotating and spinning energy effectively belongs (or migrates) to a relevant rest mass (or it is eventually creating a rest mass).* Here is good to mention that the total mass of all planets in our solar system (or in any other solar system) is presenting only a few percent of the mass of our Sun (just to visualize how ***small particles*** are all planets compared to a local Sun). Planets in a certain planetary system are mutually interacting, rotating, and spinning around their sun, having specific planetary orbits, this way following or respecting the structure of gravitational and other (mutually coupled) electromagnetic fields involved in such situations, as well as respecting basic energy-momentum conservation laws. We are still considering gravitation as the central force between two masses, acting on the line connecting their centers (Newton law), and such simplified concept will be significantly upgraded, eventually (see later in the same chapter an extension of the same concept around equations (2.11.10) – (2.11.20); - "2.3.2. Macro-Cosmological Matter-Waves and Gravitation").

The micro-world of sufficiently isolated atoms, subatomic particles, and photons is dominantly respecting Planck-Einstein-de-Broglie's, energy-wavelength formulations, $\tilde{E} = hf, \lambda = h/p, \quad u = \lambda f = \omega/k, v = d\omega/dk$. *The idea here is to show that for macro-objects, like planets, also exists specific and characteristic wavelength, analog to de Broglie matter-waves wavelength, but no more proportional to Planck's constant. The problem concerning de Broglie type of wavelength for macro-objects is that such wavelength (*$\lambda = h/p$*) is extremely small and meaningless, if we are using Planck's constant* h. *Since macro-objects in motion should also create associated matter-waves, like any micro-world object, the macro-wavelength in question should be much bigger. It will be shown later (by analyzing planetary motions) that presently known micro-world Planck constant* h *should be replaced by another analogous (much bigger) macro-world constant* H *(see (2.11.10) – (2.11.20)), especially valid in cases of self-closed, standing matter waves (such as solar systems are). Other macro matter-waves respecting (analogical) concept are waves on a water surface formed by some moving object (a boat), where the water surface is visualizing matter waves of a moving object. Similar matter waves understanding could also be associated to a pendulum motion, if we observe the pendulum from another inertial reference system that is in relative movement to the pendulum system of reference.*

To show more directly that rotation (or spinning) is an essential (ontological) source responsible for particles creation, we can again start from the relativistic particle expression that is connecting particle's total energy $E = E_{tot} = \gamma mc^2$, its linear-motion momentum $p = \gamma mv = \gamma_v mv$, and its rest mass $m = E_0 / c^2$,

$$\left[E_{tot}^2 = E_0^2 + p^2c^2\right]/c^2 \Leftrightarrow \left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2. \qquad (2.11.6)$$

If the particle is at the same time performing kind of circular (rotational) motion around a specified fixed point (without spinning around its own axis), where r_0 is the distance between that fixed point and moving particle, we can consider that the particle also has certain orbital momentum $\vec{L} = \vec{r}_0 \times \vec{p}$, $L = r_0 \cdot p$, $dE = \omega \cdot dL = v \cdot dp = dE_k = dE_{tot}$. Now, we can multiply both sides of (2.11.6) with the radius of rotation r_0 and get kind of relativistic relation where relevant orbital moments are involved,

$$\begin{aligned} \left(\frac{E_{tot}}{c}\right)^2 &= \left(\frac{E_0}{c}\right)^2 + p^2 \\ \left(r_0 \cdot \frac{E_{tot}}{c}\right)^2 &= \left(r_0 \cdot \frac{E_0}{c}\right)^2 + (r_0 \cdot p)^2 \\ L_{tot}^2 &= L_0^2 + L^2 \end{aligned} \tag{2.11.7}$$

From the analogical point of view, to facilitate comparison between (2.11.6) and (2.11.7), it is reasonable to create (2.11.8), for exposing analogies of linear and orbital moments, such as,

$$\left\{ \begin{array}{l} \left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2 \\ p_{tot}^2 = p_0^2 + p^2 \\ p_{tot} = \frac{E_{tot}}{c} = \gamma_v mc \\ p_0 = \frac{E_0}{c} = mc \\ p = \gamma_v mv \\ \\ \gamma_v = (1 - v^2/c^2)^{-0.5} \\ 0 \le v \le c \end{array} \right\} \underset{\rightleftarrows}{\text{analog or equivalent to}} \left\{ \begin{array}{l} \left(\frac{L_{tot}}{r_0}\right)^2 = \left(\frac{L_0}{r_0}\right)^2 + \left(\frac{L}{r_0}\right)^2 \\ L_{tot}^2 = L_0^2 + L^2 \\ L_{tot} = r_0 \cdot p_{tot} = r_0 \cdot \frac{E_{tot}}{c} = r_0 \cdot \gamma_v mc = \gamma_\omega J \omega_c \\ L_0 = r_0 \cdot p_0 = r_0 \cdot \frac{E_0}{c} = r_0 \cdot mc = J \omega_c \\ L = r_0 \cdot p = r_0 \cdot \gamma_v mv = \gamma_\omega J \omega \\ \\ \gamma_\omega = (1 - \omega^2/\omega_c^2)^{-0.5} \\ 0 \le \omega \le \omega_c \end{array} \right\}. \tag{2.11.8}$$

Concluding based on (2.11.8), (2.5.1-6), (2.5.1-7), T.2.4, T.2.5, and T.2.6 is becoming evident that the rest mass $m = E_0 / c^2$ should have an origin causally related to certain spatially localized and stabilized spinning of its internal constituents (or to L, L_0, J, ω, ω_c) because,

$$\left\{\left(\frac{E_{tot}}{c}\right)^2 = \left(\frac{E_0}{c}\right)^2 + p^2\right\} \equiv \left\{\left(\frac{L_{tot}}{r_0}\right)^2 = \left(\frac{L_0}{r_0}\right)^2 + \left(\frac{L}{r_0}\right)^2\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \frac{E_{tot}}{c} = \frac{L_{tot}}{r_0} = p_{tot} = \gamma_v mc = \frac{\gamma_\omega J \omega_c}{r_0}, \\ \frac{E_0}{c} = \frac{L_0}{r_0} = p_0 = mc = \frac{J\omega_c}{r_0}, \\ p = \frac{L}{r_0} = \gamma_v mv = \frac{\gamma_\omega J\omega}{r_0}, \\ dE = \omega \cdot dL = v \cdot dp = dE_k = dE_{tot} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} m = \frac{L_{tot}}{r_0\gamma_v c} = \frac{L}{r_0\gamma_v v} = \frac{\gamma_\omega J\omega_c}{r_0\gamma_v c} = \frac{\gamma_\omega J\omega}{r_0\gamma_v v} = \frac{L_0}{r_0 c} = \frac{J\omega_c}{r_0 c} = \\ = \frac{E_{tot}}{\gamma_v c^2} = \frac{p_{tot}}{\gamma_v c} = \frac{p}{\gamma_v v} = \frac{p_0}{c} = \frac{E_0}{c^2} = \frac{1}{c^2}\iiint_{[\omega]} \vec{\omega} d\vec{L} = \frac{J\omega_c^2}{c^2}, \\ \frac{\omega}{\omega_c} = \frac{v}{c} = \frac{L}{L_{tot}} = \frac{cp}{E_{tot}} = \frac{p}{\gamma_v p_0} \end{array}\right\}. \quad (2.11.9)$$

In addition to such conceptualization, we can say that all kind of motions in our universe are curvilinear, or combinations of linear and rotational (see later T.4.3 and (4.5-1) - (4.5-4) in chapter 4.1). If this were not the case, we would not have stable particles with non-zero rest masses. Also, based on (2.11.1) to (2.11.5) and (2.11.7), we know that internal particle structure (its rest mass) has natural elements of rotation, which are directly coupled to all kind of externally rotating motions (including oscillatory and resonant states). Such kind of coupling of internal and external aspects of rotation should be part of explanation of Gravitation.

The unity, or complementary nature of linear and rotational motions (including universal law of Inertia), should be naturally integrated into laws of energy and moments conservation, as well as being the part of 4-vector of Energy-Momentum in the Minkowski space (as shown in (2.9.5), and later). Let us summarize different (mutually analogical and somewhat speculative) situations when a specific particle has different kinds of motional components, as for example,

Particle is performing only rectilinear motion:

$$(p, \frac{E}{c})^2 = inv., p = \gamma mv, E = \gamma mc^2, E_0 = mc^2 \Rightarrow$$

$$p^2c^2 - E^2 = -E_0^2,\ E = E(p),\ \frac{\partial}{\partial p}\left(p^2c^2 - E^2 = -E_0^2\right) = 0 \Leftrightarrow \quad (2.9.5\text{-}1)$$

$$2c^2p - 2E\frac{\partial E}{\partial p} = 0 \Rightarrow \frac{\partial E}{\partial p} = c^2\frac{p}{E} = c^2\frac{\gamma mv}{mc^2} = v,\ dE = vdp$$

Particle is performing only circular motion:

Depending how we define a dominant inertial system here, we could have at least two different situations, for example,

a)
$$\left\{\begin{pmatrix} p=\gamma mv, E=\gamma mc^2, v=\omega r, \vec{v}=\vec{\omega}\times\vec{r}, \vec{p}\times\vec{r}=\vec{L}, \\ \frac{1}{2}mv^2=\frac{1}{2}J\omega^2 \Leftrightarrow mv^2=J\omega^2=pv=L\omega, p=\frac{L}{r}, \\ v=\omega r \Rightarrow c=\omega_c r,\ \omega_c=\frac{c}{r} \end{pmatrix}, (p,\frac{E}{c})^2=inv.\right\} \Rightarrow \tag{2.9.5-2}$$

$$p^2c^2-E^2=-E_0^2 \Leftrightarrow (\frac{L}{r})^2c^2-E^2=-E_0^2 \Leftrightarrow (\frac{\vec{L}}{r},\frac{E}{c})^2=inv.$$

$$E=E(L)\Rightarrow \frac{\partial}{\partial L}\left[(\frac{L}{r})^2c^2-E^2=-E_0^2\right]=0 \Leftrightarrow 2\frac{c^2}{r^2}L-2E\frac{\partial E}{\partial L}=0\Rightarrow$$

$$\frac{\partial E}{\partial L}=\frac{c^2}{r^2}\cdot\frac{L}{E}=\frac{c^2}{r^2}\cdot\frac{J\omega}{J\omega_c^2}=(\frac{c}{\omega_c r})^2\omega=\omega,\ dE=\omega dL\ .$$

Another imaginable situation for making relation between two inertial systems addressing the same circular motion is,

b)
$$\left[(\vec{p},\frac{E}{c})^2=inv.\Rightarrow p^2-(\frac{E}{c})^2=-(\frac{E_0}{c})^2 \Leftrightarrow p^2c^2-E^2=-E_0^2 \Leftrightarrow p^2c^2-(\gamma mc^2)^2=-(mc^2)^2\right]\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix} \left[\begin{matrix}\left(\frac{\vec{r}}{r}\times(\vec{p},\frac{E}{c})\right)^2=\left(\frac{1}{r}(\vec{L},\frac{E}{c}\vec{r})\right)^2=inv., \\ \gamma=\gamma_1,\ \gamma_2=1,\ \vec{L}=\overline{const.}\end{matrix}\right]\Rightarrow\frac{1}{r_1^2}\left[\vec{L}_1^2-(\frac{E}{c})^2r_1^2\right]=\frac{1}{r_2^2}\left[\vec{L}_2^2-(\frac{E_0}{c})^2r_2^2\right]\Rightarrow \\ \Rightarrow L=L_1=L_2=\frac{r_1r_2}{c}\sqrt{\frac{E^2-E_0^2}{r_2^2-r_1^2}}=mcr_1r_2\sqrt{\frac{\gamma_1^2-1}{r_2^2-r_1^2}}=const.,\Rightarrow r_1r_2\sqrt{\frac{\gamma_1^2-1}{r_2^2-r_1^2}}=const.\end{matrix}\right\} \tag{2.9.5-3}$$

$$L=\frac{r_1r_2}{c}\sqrt{\frac{E^2-E_0^2}{r_2^2-r_1^2}}, E^2=E_0^2+\frac{L^2(r_2^2-r_1^2)c^2}{(r_1r_2)^2}, 2E\frac{\partial E}{\partial L}=2L\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2},$$

$$\frac{\partial E}{\partial L}=\frac{L}{E}\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2}=\frac{L}{E}\frac{c^2}{r_1^2}-\frac{L}{E}\frac{c^2}{r_2^2}=(\omega_1-\omega_2)=\omega,\ E=\frac{L}{\omega}\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2}=J\frac{(r_2^2-r_1^2)c^2}{(r_1r_2)^2}=\gamma mc^2\Rightarrow$$

$$J=\gamma m\frac{(r_1r_2)^2}{(r_2^2-r_1^2)}.$$

Particle is performing only spinning:

If a particle is in a kind of state of rest (by the fact of not being in linear motion), but spinning (around certain of its axes), it is evident that the total particle energy should have two components, one of them mc^2 and another equal to the energy of spinning $E_s \neq 0$,

$$\left\{\begin{matrix}\left(p=0, E=E_0+E_s=mc^2+E_s, E_s\neq 0, v=0,\ \vec{L}=\vec{S}\right), \\ (p,\frac{E}{c})^2=inv.\Leftrightarrow(0,\frac{E}{c})^2=inv.\Leftrightarrow(0,\frac{E_0+E_s}{c})^2=inv.\end{matrix}\right\}\Rightarrow$$

$$\frac{E^2}{c^2}=\frac{E_0^2+2E_0E_s+E_0^2}{c^2}\Leftrightarrow E^2=E_0^2+2E_0E_s+E_0^2 \tag{2.9.5-4}$$

$$E=E(S)\Rightarrow\frac{\partial}{\partial S}\left[E^2=E_0^2+2E_0E_s+E_s^2\right]\Leftrightarrow 2E\frac{\partial E}{\partial S}=2E_0\frac{\partial E_s}{\partial S}+2E_s\frac{\partial E_s}{\partial S}\Rightarrow$$

$$\frac{\partial E}{\partial S}=\frac{E_0}{E}\frac{\partial E_s}{\partial S}+\frac{E_s}{E}\frac{\partial E_s}{\partial S}=\frac{1}{E}(E_0+E_s)\frac{\partial E_s}{\partial S}=\frac{\partial E_s}{\partial S}=\omega_s, dE=dE_s=\omega_s dS$$

A particle is performing circular motion and spinning:

$$\left[\begin{pmatrix} p=\gamma mv, E=\gamma mc^2, v=\omega r, \vec{v}=\vec{\omega}\times\vec{r}, \vec{p}\times\vec{r}=\vec{L}, \\ \frac{1}{2}mv^2=\frac{1}{2}J\omega^2 \Leftrightarrow mv^2=J\omega^2=pv=L\omega, p=\frac{L}{r}, \\ v=\omega r \Rightarrow c=\omega_c r,\ \omega_c=\frac{c}{r}, \\ (\vec{p},\frac{E}{c})^2=inv. \Rightarrow p^2-(\frac{E}{c})^2=-(\frac{E_0}{c})^2 \Leftrightarrow \\ p^2c^2-E^2=-E_0^2 \Leftrightarrow p^2c^2-(\gamma mc^2)^2=-(mc^2)^2 \end{pmatrix}, \begin{pmatrix} \vec{L}\to\vec{L}+\vec{S}=\overrightarrow{const.},\ E\to E+E_s \\ \left(\frac{\vec{r}}{r}\times(\vec{p},\frac{E}{c})\right)^2=\left(\frac{1}{r}(\vec{L},\frac{E}{c}\vec{r})\right)^2=inv. \end{pmatrix}\right] \Rightarrow$$

$$\Rightarrow \frac{1}{r^2}\left[(\vec{L})^2-(\frac{E}{c})^2r^2\right]=\frac{1}{r^2}\left[-(\frac{E_0+E_s}{c})^2r^2\right] \Rightarrow \frac{(\vec{L})^2}{r^2}-(\frac{E}{c})^2=-(\frac{E_0+E_s}{c})^2,$$

$$E^2=\frac{(\vec{L})^2}{r^2}c^2+(E_0+E_s)^2, E=E(L) \Rightarrow \frac{\partial}{\partial L}\left[(\frac{L}{r})^2c^2-E^2=-(E_0+E_s)^2\right] \Leftrightarrow \tag{2.9.5-5}$$

$$2\frac{c^2}{r^2}L-2E\frac{\partial E}{\partial L}=-2E_s\frac{\partial E_s}{\partial L} \Rightarrow 2E\frac{\partial E}{\partial L}=2L\frac{c^2}{r^2}+2E_s\frac{\partial E_s}{\partial L} \Leftrightarrow$$

$$\frac{\partial E}{\partial L}=\frac{L}{E}\cdot\frac{c^2}{r^2}+\frac{E_s}{E}\frac{\partial E_s}{\partial L}=\omega+\frac{E_s}{E}\omega_s .$$

Matter Waves and Particle-Wave Duality Concept:

Let us take into consideration only the energy-momentum state of certain motional mass that is in linear and inertial motion. We will be able to find (at least) two additional, mutually equivalent ways to present the same mass as performing rotation and spinning. The common for all such equivalent motional states (of the same mass) would be equivalency of their kinetic energy between linear and rotational states of the same motional energy. Of course, all other conservation laws should be satisfied. It is not difficult to explain such (mathematical) strategy. Intuitively clear is that every case of linear mass motion is only a case of an orbital rotation around certain center (if we consider that radius of such rotation can be arbitrarily long). In frames of such concept, all masses-motions are cases of certain rotational or curvilinear movements.

The kinetic energy of the motional particle (in inertial, linear motion) is presentable in the following way,

$$E_k=(\gamma-1)mc^2=\frac{pv}{1+\sqrt{1-v^2/c^2}}$$

$$\begin{bmatrix} \overline{P}_4=(p,\frac{E}{c}) \Rightarrow p^2-\frac{E^2}{c^2}=-m^2c^2 \\ E^2=m^2c^4+p^2c^2=E_0^2+p^2c^2 \\ p=m\gamma v, \gamma=(1-v^2/c^2)^{-0.5} \\ dE_k=vdp=mc^2d\gamma=dE_{tot}=d\tilde{E} \\ E=E_{tot}=\gamma mc^2,\ E_k=(\gamma-1)mc^2 \end{bmatrix} \tag{2.9.5-6}$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

If we now consider the same mass may rotate around the specific center (with a large radius of rotation $\vec{R}$, having angular revolving frequency $\omega_c = 2\pi f_c$), we can present such motion as,

$$E_k = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = (\gamma-1)mc^2, \vec{v} = \vec{\omega}_c \times \vec{R},\ L_c\omega_c = pv\,. \tag{2.9.5-7}$$

The same mass on the same linear path could also make helicoidal spinning (around its propagation path), with spinning angular frequency $\omega_s = 2\pi f_s$ and spinning moment L_s. This is another type of rotation (we could say a mathematical equivalent to the same motion), which will have the same kinetic or spinning energy (as before). Mass in question will effectively perform helix motion when observed from the common center of rotation.

$$\begin{aligned} &E_k = \frac{L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = (\gamma-1)mc^2, \\ &\vec{v} = \vec{\omega}_c \times \vec{R},\ L_s\omega_s = L_c\omega_c = pv,\ (\vec{L}_s, \vec{\omega}_s, \vec{p}, \vec{v} \text{ mutually colinear}) \end{aligned} \tag{2.9.5-8}$$

It looks that the spinning (which is creating a helix line around the path of mass propagation) is only an artificial mathematical example to show how motional energy of such spinning could be equal to the kinetic energy of linear and rotational motion initially introduced. However, we could also try to give another meaning to such spinning. For instance, to consider it as being a specific de Broglie matter waves generator. Matter waves generated by such helicoidal motion will have the wavelength λ_s, frequency f_s, group velocity v, and phase velocity u, as follows,

$$\begin{aligned} &\lambda = \lambda_s = \frac{H}{p}, k = \frac{2\pi}{\lambda_s} = \frac{2\pi}{H}p,\ H = \text{Const.} \\ &u = \lambda_s f_s = \frac{\lambda_s\omega_s}{2\pi} = \frac{L_s\omega_s / p}{1+\sqrt{1-v^2/c^2}} = \frac{v}{1+\sqrt{1-v^2/c^2}} \\ &E_k = \frac{L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = pu = \tilde{E} = Hf_s \\ &\left\{\begin{aligned} &dE_k = vdp = \omega_s dL_s = mc^2 d\gamma = dE_{tot} = d\tilde{E} = pdu + udp \\ &v = u - \lambda\frac{du}{d\lambda} \end{aligned}\right\} \Rightarrow \\ &\Rightarrow \frac{du}{v-u} = \frac{dp}{p} \Rightarrow \frac{d\lambda}{\lambda} = -\frac{dp}{p} \Rightarrow \\ &\ln\left|\frac{\lambda}{\lambda_0}\right| = -\ln\left|\frac{p}{p_0}\right| \Leftrightarrow \left|\frac{\lambda p}{\lambda_0 p_0}\right| = 1 \Leftrightarrow \lambda p = \lambda_0 p_0 = \text{Const.} = H \\ &\vec{v} = \vec{\omega}_c \times \vec{R},\ L_s\omega_s = L_c\omega_c = pv,\ (\vec{L}_s, \vec{\omega}_s, \vec{p}, \vec{v} \text{ mutually colinear}). \end{aligned} \tag{2.9.5-9}$$

Now (in (2.9.5-9)) we have a perfect match of mutually non-contradicting parameters of matter waves integrated with motional mass properties. Imaginatively introduced helix matter wave could also be interpreted as an Analytic Signal wave function couple, where $\Psi(t)$ is the original, linear-motion and power-related function, and $\hat{\Psi}(t)$ is its phase-shifted Hilbert couple, both creating $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\ \Psi\ (t)$. Now, from the relevant Analytic Signal model we will be able to determine matter waves frequency, phase, wavelength, and other results as presented in (2.9.5-9). See much more about Analytic Signals in the chapters 4.0, 4.1 and 10. This is giving an improved conceptual picture of de Broglie matter waves compared to the contemporary view of matter waves (see much more in chapter 4.1 around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,I,k...). Since Planck constant **h** and its macro-cosmos equivalent **H** are mutually much different ($H >>> h$), number of questions related to unity and harmony between Quantum and Relativistic world theories will be initiated. What is common for micro and macro world constants **h** and **H** is that systems, objects, or entities where such constants are naturally significant are different, self-closed, standing matter-waves structures.

[♣ COMMENTS & FREE-THINKING CORNER:

As an illustration of the rest mass origins in rotation, let us exercise (or hypothesize) by considering an electron (in its state of rest) as being space-limited or localized, self-stabilized and internally captured electromagnetic wave that is creating a standing-waves structure on a closed circular line or on a toroidal form (the same also applicable to a proton, positron etc.). When following the same idea, it is becoming evident that the amount of electromagnetic energy (or specific group of photons), which is on that way creating an electron did not have any initial rest mass. Only after the act of standing-wave creation, we can associate the rest mass to an electron. The origin of electron rest mass is related to the fact that electromagnetic standing waves are being created on a self-closed circular path, while the internal content of such rest mass presents the matter wave or motional energy. Here, at the same time, we are also establishing a specific association between an electron and state of electromagnetic energy that is rotating on a closed circular line. Such internally rotating electromagnetic waves formation, or kind of group of photons (or rotating distributed mass), should have certain wave energy amount equal to $\tilde{E}_c = hf_c$, or maybe to $\tilde{E}_c = n \cdot hf_c,\ n = 1, 2, 3...$. Consequently, since we know (experimentally) the amount of electron rest mass, we could say that its total (rest) energy is, $E_e = m_e c^2 = \tilde{E}_c = hf_c \Rightarrow f_c = \frac{m_e c^2}{h}$ or maybe $f_c = \frac{m_e c^2}{n \cdot h},\ n = 1, 2, 3, ...$, where f_c is relevant, mean, standing-electromagnetic-waves frequency. By analogy, we could now consider that electron in some ways presents an equivalent rotating and distributed mass, which has a form of a spinning ring, disk, or torus. Now we will take another strategy, based on analogies, and address the rotation of such distributed mass as, $dE_e = c^2 dm_e = v_e dp_e = \omega_e d\mathbf{L}_e$, where ω_e is the effective angular speed of such rotating and distributed mass. For the sake of generality, we will assume that $\omega_e = 2\pi f_e \neq 2\pi f_c$. Since standstill electron should present stable, fixed, stationary structure, it is clear that $\omega_e = 2\pi f_e$ should be specific constant angular frequency (or sort of equivalent, mechanical, rotating, angular speed), and that such spinning ring, disk, or torus should have steady angular momentum. Let us now make the integration,

$$\begin{aligned} & dE_e = c^2 dm_e = \omega_e d\mathbf{L}_e = hdf_c \Rightarrow \int dE_e = \int c^2 dm_e = \int \omega_e d\mathbf{L}_e = \int hdf_c \Rightarrow \\ & E_e = m_e c^2 = \omega_e \mathbf{L}_e = \omega_e^2 \mathbf{J}_e = hf_c \Rightarrow \mathbf{J}_e = \frac{m_e c^2}{\omega_e^2} = \frac{hf_c}{\omega_e^2}. \end{aligned} \tag{2.11.9-1}$$

If we limit our elaborations only to spinning rings or toroidal structures (see (2.11.3)), where dimensional conditions (relations between relevant diameters) are enabling to assume that moment of inertia is $\mathbf{J}_e = m_e r_e^2$, we will be able to find the effective radius of an electron as,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\mathbf{J}_e = \frac{m_e c^2}{\omega_e^2} = \frac{hf_c}{\omega_e^2} = m_e r_e^2 = m_e\left(\frac{v_e}{\omega_e}\right)^2, \; r_e = \frac{v_e}{\omega_e}, v_e = c \Rightarrow r_e = \frac{c}{\omega_e} = \frac{1}{\omega_e}\sqrt{\frac{hf_c}{m_e}}. \tag{2.11.9-2}$$

Of course, the solution that is more general would be $\mathbf{J}_e = \frac{n \cdot hf_c}{\omega_e^2}, \; r_e = \frac{1}{\omega_e}\sqrt{\frac{n \cdot hf_c}{m_e}}.$

Here we are underlining, as a general case, the difference between the angular speed of equivalent mass spinning and its internal, comparable electromagnetic energy angular frequency $\omega_e = 2\pi f_e \neq 2\pi f_c$. The reason is that once we are treating electron almost mechanically, as an equivalent spinning ring or spinning torus, and in parallel, we are developing the idea that internal content of such spinning object effectively presents electromagnetic standing waves (see later, in chapter 4.1, more about the spinning nature of matter waves; -equations (4.3-1) – (4.3-3)).

In the very beginning of this exercise, we assumed that electron rest mass is created by self-closing of an electromagnetic standing-waves structure, on a circular line, and this concept is giving the opportunity to describe elements of such standing waves as,

$$\begin{bmatrix} 2\pi r_e = k \cdot \lambda_c \Leftrightarrow \lambda_c = \frac{2\pi r_e}{k}, \; k = 1,2,3... \\ m_e c^2 = hf_c \Leftrightarrow f_c = \frac{m_e c^2}{h} = \frac{\mathbf{J}\omega_e^2}{h} = \frac{m_e r_e^2 \omega_e^2}{h} \end{bmatrix} \Rightarrow \lambda_c f_c = u_c = c = \frac{2\pi r_e}{k} \cdot \frac{m_e c^2}{h} \Rightarrow$$

$$k \cdot h \cdot c = 2\pi r_e m_e c^2 \Rightarrow r_e = \frac{k}{2\pi} \cdot \frac{h}{m_e c} = \frac{c}{\omega_e} \Leftrightarrow \omega_e = \frac{m_e c^2}{k \cdot \frac{h}{2\pi}} = 2\pi f_e = \frac{1}{r_e}\sqrt{\frac{hf_c}{m_e}}, \text{ or} \tag{2.11.9-3}$$

$$\omega_e = \frac{m_e c^2}{k \cdot \frac{h}{2\pi}} = 2\pi f_e = \frac{1}{r_e}\sqrt{\frac{n \cdot hf_c}{m_e}}, \; k, n = 1,2,3...$$

Let us now analyze the same situation, seeing the particle (electron, for instance) from its "*external space*" (now we do not care what is happening inside such particle, and we only see that this is a compact, stable particle, which has the rest mass m). If the same elementary particle is in a process of motion (relative to a Laboratory system: see (2.11.5)), de Broglie matter wave, which is associated to such particle (like a moving electron, positron, proton…), can be specified as,

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m v} = \frac{h}{mc} \cdot \frac{c}{\gamma v} = \lambda_c \cdot \left(\frac{c}{\gamma v}\right), \; \lambda_c = \frac{h}{mc} = \text{const.}, \gamma = 1/\sqrt{1 - \frac{v^2}{c^2}},$$

$$f = \frac{1}{\Delta t} = \frac{E_k}{h} = \frac{mc^2}{h}(\gamma - 1) = f_c \cdot (\gamma - 1) = \frac{1}{\Delta t_c} \cdot (\gamma - 1), \; f_c = \frac{mc^2}{h} = \text{Const.} \Rightarrow \Delta t = \frac{\Delta t_c}{(\gamma - 1)},$$

$$u = \lambda f = \lambda_c f_c \cdot \frac{\frac{\gamma v}{c}}{1+\gamma} = \frac{\gamma}{1+\gamma} \cdot v = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{v}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_k}{p}, \;\; \lambda_c f_c = u_c = c, \tag{2.11.9-4}$$

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{2u}{1 + \frac{uv}{c^2}} = (1 + \frac{1}{\gamma})u \;,$$

$$\Rightarrow 0 \leq u < 2u \leq \sqrt{uv} \leq v \leq c.$$

By comparing here obtained results with similar results available in other literature, we will certainly notice differences. The significance of this example is related to how internal electron structure is conceptualized and how it is connected to "*external*" understanding of matter waves (or how internal matter waves are related and connected to familiar, external matter waves). We are also paving the way of understanding how particles with stable rest masses are being created, and how field of Gravitation is appearing externally. The vision of the particle-wave dualism of all matter in our universe, underlined here, is that "*external*" matter waves are an extension, or unfolding of the internal wave structure of matter (in the process of motion), in addition to the complexity of 2-body or n-body interactions. If we now start analyzing two-body problem (or binary interactions), considering internal and external wave nature of moving bodies, as well as their rest and velocity-dependent masses, we will be able to address number of known particle-wave manifestations on a much more tangible and conceptually clearer way than we find presently in physics. This will certainly initiate the process of brainstorming contributions that will gradually upgrade and optimize the same (here elaborated) concept, eventually leading to a much more complete modeling of elementary particles and atoms, and to the conclusion that electron cannot be only a point particle with "magic" attributes (valid also for other elementary particles). See similar and in many aspects more profound modeling of elementary particles in literature from [16] to [20]; Authors: Bergman, David L., and Lucas, Jr., Charles W, http://www.commonsensescience.org/papers.html.

By the way to mention that authors Bergman, David L., and Lucas, Jr., Charles W, besides ingenious and original modeling of elementary particles, are, in the same package, softly and sporadically (for some reason) promoting their religious convictions. The author of this book is not recommending such religious background attachments to be considered as a here-relevant references (because there is no need and scientifically defendable place for such inclusions). ♣**]**

2.3.3. Macro-Cosmological Matter-Waves and Gravitation

Gravitation as addressed in General Relativity theory is still not too far from the old Newton and Classical Mechanics results. For what we can measure, verify, and use within our planet and our solar system, Newton and Kepler's framework of gravitation seems still particularly good. Here is also convenient to mention the existence of much earlier foundations of Kepler laws, established in a period of flourishing of Arab science and culture, much before Kepler translated and revitalized such old concepts. Nevertheless, for a larger scale of cosmic laboratories (like an open cosmic space with galaxies), astronomic observations are indicating that we need to have certain updated and redesigned theory of gravitation. A. Einstein with his General Relativity theory applicability did not go too far from Newton theory predictions (both experiencing the same weak sides). He introduced kind of mathematically demanding and almost inoperative, spatial-temporal, or modified geometry-related interpretation of gravitation (saying that a space is curved around mass and energy formations, and that we need to use curved-space geometry to describe motions within such spatial-temporal deformations caused by presence of masses, because curved spatial-temporal deformations have acceleration-related motional components, and Gravitation equals to uniform accelerated motions). In other words, masses are anyway formations of agglomerated, stored, and stabilized energy, or being formations of atoms. We know from Classical Mechanics forces as gradients of local, spatial energy-mass or potential energy concentrations (see more in Chapter 10. under "10.02 Meaning of natural forces", and in this chapter, under "2.2.1. WHAT THE GRAVITATION REALLY IS"). This way Einstein replaced universal force definition with an equivalent or isomorphic concept saying that curved, spatial-temporal geometry around masses is deformed (being not Euclidean), and naturally producing components of accelerating motions or Gravity-related forces. Geometrically guided planetary and satellites motions, where our mechanical and cosmic engineering is presently involved, are still mutually comparable, or identical to Newton theory results (except applied mathematics is much more complex in Einstein's case). When Einstein's General Relativity theory is applied within our solar system, everything works perfectly well, as in Newton theory case, but predictions of General Relativity related to vast cosmic spaces with number of spiral galaxies are in some cases doubtful, incorrect, and requesting to consider existence of some still unknown, virtual, missing, and undetectable dark masses and dark energy zones… This is valid both for predictions based on Newton's and Relativity theory.

The mainstream of contemporary scientific authorities, dealing with Gravitation and modern Physics, are showing a tendency to artificially unite, extend, forge and fit present Relativity and Quantum theories (by introducing missing, virtual, and artificial items, like dark energy and dark masses). An army of well obeying disciples of such teachings are presently searching for mentioned "*dark entities*", getting number of official mainstream recognitions and funding for such research. This looks like prematurely taking contemporary Relativity and Quantum theories too seriously, almost as perfectly-well established, final, and solid step-stones or facts (or like hesitating to modify something what is falsely considered as already being well and forever constructed).

Consequently, by respecting such convictions, everything else in Physics, like surrounding scientific theories and concepts, should be (almost ideologically and dogmatically) subordinated to mentioned and postulated, by consensus created, somewhat artificial grounds (of Relativity and Quantum theory), and if something in such theories is missing or presenting problems, we will simply and conveniently invent whatever missing, only as a name, and postulate it as something that should exist (like dark matter and dark energy, tunnel-effects, different fundamental, natural forces, "stochastic miracles", and what (in most of cases) are only some imaginative names and labels placed on black-boxes, without real meaning).

Mathematical modeling and processing based on mechanical grounds of Newton Gravitation and Relativity theory, works well, but only in laboratories within our planet, or within any stable solar system (at least in our part of cosmic space). When applying mentioned theories to much larger astronomic distances of our universe, or to extremely small subatomic environment, we will realize that such theories are not complete, not sufficiently and mutually compatible, not universally applicable, and that we really need certain new and better theory of Gravitation (instead postulating, by consensus accepting, and/or inventing virtual, missing dark matter and energy labels, as presently practiced). Consequently, we need to have an appropriately modified and updated Relativity and Quantum theory (including better, more convenient, and larger mathematical environment for dealing with such problematic; -on some way this being comparable with the evolution of Mathematical Analysis from Real Numbers towards Complex Numbers, and later towards Hypercomplex numbers and Analytic Signal functions).

Classical mechanics, Newton theory of Gravitation, and Relativity theory are presently defined and operating well within a smooth, continuous, deterministic, spatial-temporal environment. Contemporary Quantum theory is conceptualized on a way that relevant matter domains' motions and energy-states are statistically and probabilistically modeled (as averaged values, functions, and distributions), being combined with some generalized and not mathematically clear, multi-parameter discretization and quantization of everything what there presents energy-moments states, motions, interactions..., (or being too much fuzzy). Such artificial, theoretical, or postulated mathematical practices and platforms (as known in present Relativity and Quantum theory) cannot be easily and naturally implemented and united without certain additional, conceptual, theoretical, and innovative redesign work. It will also be necessary to apply more general, unifying mathematical concepts (than presently practiced), like "Analytic Signal", joint time-frequency analysis based on using Hilbert Transform (established by Denis Gabor; -see [57] Michael Feldman), and "Kotelnikov-Shannon, Whittaker-Nyquist Sampling and Signals Recovery Theory" (see more in chapters 4.0 and 10).

One of the interesting trends in a new understanding of Gravitation is related to the successful conceptualization, modeling and quantizing of planetary systems, orbits, and motions, analogically to N. Bohr atom modeling (what will be specifically addressed in this chapter, and it is also elaborated in Chapter 8.). Familiar (to N. Bohr atom structure) modeling is also producing correct and verifiable results, based on relevant astronomic observations and measurements in different planetary or solar systems (see more in the same chapter around (2.11.13) - (2.11.13-5) - (2.11.14) including T.2.8.). Even Micro-World, Wave-Particle Duality and Matter-Waves concepts (that are already well implemented and integrated into atoms and micro-world events modeling) can be analogically extended to macro systems like planetary systems, ***but this time without big or any need to use stochastic and probabilistic modeling***. *In this book, the concept of particle-wave duality and (de Broglie) matter waves is being extended and enriched with ideas about complementarity of linear and rotational (or spinning) motions, becoming analogically and universally applicable to both, micro, and macro world of physics, or to subatomic and astronomic spaces (analog to complementarity of electric and magnetic fields; -see much more about wave-particle duality in Chapters 4.0, 4.1. and 10., and about electromagnetic and mechanical analogies, symmetry, couplings, and interactions in Chapter 3.).*

New understanding of Gravitation

Briefly summarizing, gravitation (as promoted in this book; -see "2.2.1. WHAT THE GRAVITATION REALLY IS", and Chapter 8. about extended planetary-atom modeling) is the phenomenology ***linked to spatially complex, stationary, and standing, cosmic matter waves***, manifesting within structurally and multi-dimensionally resonating universe, where orbital, linear and spinning inertial motions, are complementarily and structurally united. Formations of masses in such stabilized, structurally-resonating (and rotating) universe are occupying nodal zones of highest energy-mass densities and highest oscillatory accelerations, where oscillating amplitudes are minimal (like in half-wave resonators in High Power Ultrasonics technology, and like in cases of acoustic or ultrasonic levitation, where we can easily notice the presence of ***attractive forces*** acting towards mentioned nodal zones; -see more in [150] and [151]). Of course, such spatially resonating, stationary and standing-waves of orbiting and spinning structures are taking forms of atomic, solar, stellar, and galactic systems, respecting laws of energy-momentum conservations, while involved linear and rotational (or spinning) motions are specifically united and mutually complementary, like in cases of electromagnetic fields, mass-spring and/or inductance-capacitance oscillatory circuits. Matter waves and Particle-Wave Duality manifestations, including associated and strongly coupled mechanical and electromagnetic complexity (as elaborated in this book in Chapter 3., and on a similar way as presented in [71], from Dr. Jovan Djuric, "Magnetism as Manifestation of Gravitation"), are enabling "energy-momentum communications" within such dynamic and self-stabilizing, standing-matter-waves, and resonant spatial formations. What the principal macro or global "external, devine source" of mentioned vibrations is, and how such universal and overwhelming, structural (micro and macro world), standing-waves oscillations, spinning and rotations are being initially created, synchronized and maintained within our Universe, are still unanswered questions (see familiar elaborations in *[99] from Konstantin Meyl).*

Set of updated and generalized second order differential Wave Equations (evolving from Classical Wave Equation and redesigned Schrödinger equation), formulated using the Analytic Signal, Complex and Hypercomplex functions, based on Hilbert transformation, are presently the best mathematical framework to address structural oscillations within our micro and macro-Universe (as exercised in this book; -see Chapter 4.3). If we imaginatively extend the same framework and concepts, we will realize that Nikola Tesla's ideas about Dynamic Gravity theory are familiar to here-summarized thoughts, and compatible or complementing with Rudjer Boskovic's universal Natural Force descriptions (see more in [6], [97], [98], [99] and [117]).

We could significantly simplify the understanding of Gravitation as follows: Since we know that content of gravitational masses are atoms, and atoms have number of internal constituents effectively performing orbital and spinning motions, this way creating magnetic moments, the most probable source of gravitational attraction should be such (mutually interacting) internal magnetic moments, presenting some spatially distributed, and specifically polarized elementary magnets inside involved masses. There is always a certain number of not self-compensated internal "magnets" inside macro masses, and thanks to existence of global (and accelerated) macro-motions within our Universe, mentioned internal magnetic elements are mutually polarizing (or orienting) on a way that forces between neighbours' masses are respecting attractive-forces, Coulomb or Newton laws (also being applicable on the same way for forces between magnets). Also, effectively spinning, and rotating electrons and protons have very much different masses, and associated electric dipoles polarization will be facilitated, supporting attractive Coulomb forces. Such accelerated, by motion-induced polarizations and standing waves formations, and associated effects of attractive forces, our contemporary physics, "unintentionally" recognized as Gravitation, and as being based only on some magic masses' attraction (what is not correct). Masses' motions (including oscillations) are additionally producing matter-waves that will be part of surrounding, stationary and standing matter-waves and fields, that dominantly should have an electromagnetic nature, but we are still incorrectly conceptualizing such mixed and dynamic, electromagnetic effects as being only an independent and self-standing force of Gravitation, based only on mutual and inexplicable, static masses attraction. In the first chapter of this book, about analogies in Physics, it is demonstrated that only static and electromagnetically neutral mases cannot be sources of Gravitation. But, analogically concluding, oscillating electric charges, properly oriented dipoles and magnetic moments or fluxes, including associated linear and angular mechanical moments, forces and torques (emanating from involved atoms and agglomerated masses) should be the real sources of gravitation (see more in Chapters 3. and 8.). Consequently, most of contemporary theories based on old (and still practiced) concepts about Relativity, Gravitation and other natural forces should significantly evolve, or be completely replaced with better and new concepts.

In the case of micro-universe of atoms and elementary particles, de Broglie matter waves are manageable using the following relations (see more in chapter 4.1 and Chapter 10., concerning PWDC):

Wave or motional energy (=) $\tilde{E} = hf = E_k$,
Matter-waves wavelength (=) $\lambda = h/p$,
Phase velocity (=) $u = \lambda f = \omega/k = E_k/p$,
Group or particle velocity (=) $v = d\omega/dk = dE_k/dp$.

Let us now try to construct or exercise what could be the *macro-universe equivalent* to de Broglie matter-waves concept. The idea here is to show that solar systems, planets, satellites and similar macro-objects (in orbital and spinning motions) are also analogically respecting certain periodicity and "standing macro matter-waves packing rules", like de Broglie matter waves in a micro-universe (analogically like in N. Bohr atom model; -see more in Chapter 8.), but instead of Planck constant **h**, new macro-world constant $\mathbf{H} >>> \mathbf{h}$ is becoming relevant in a similar way as $\mathbf{h}$ is in a micro world of Physics. See an indicative introduction to such concept given by equations (2.11.5) - (2.11.9), (2.11.9-1) - (2.11.9-4) and (2.9.5-1) - (2.9.5-5).

The best for exercising mentioned brainstorming is to start from the Kepler's third law (of planetary orbital motions), which is also applicable to all satellite and lunar, inertial motions around a specific planet or big mass. Let us temporarily focus our attention only on idealized circular motions, where the radius of rotation is R, to be able to use simpler mathematical expressions (approximating diffrent orbital planetary motions, as being circular, where $\mathbf{R}$ is a planet semi-major orbital radius). Kepler's third law is showing that the period T of a planet (or satellite) with mass $\mathbf{m}$, orbiting around a big mass $M >>> m$ (or its sun), is given by (2.11.10),

$$T^2 = \left(\frac{4\pi^2}{G(M+m)}\right)R^3 \cong \left(\frac{4\pi^2}{GM}\right)R^3 \quad . \tag{2.11.10}$$

We will later also need to extract from Newton-Kepler theory the expression for a maximal orbital or escape velocity v_e, and escape kinetic energy E_e (when planet, rocket or satellite would escape its stationary, circular orbit), which can be found as,

$$\left(E_e = \frac{1}{2}mv_e^2 = \frac{GmM}{R}\right) \Leftrightarrow v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2 R = 2GM = \text{constant.} \cdot \tag{2.11.11}$$

Escape velocity v_e is a "*flexibility-parameter of boundary orbital-stability limit*" of all motions within and around specific planet or sun (where relevant planet or sun could be approximated as a local center of mass for such mutually related motions). Later, we will see that similar relation "$v^2R = v_n^2R_n = \text{constant}$" is also valid for all planets of certain solar system. For instance, for all planets of our solar system, we always have the same constant, $v^2R = v_n^2R_n = \mathbf{1.3256E+20}$ (see this result later in the same chapter inside T.2.3.3). For other planetary and satellite systems, such constants will be different. We will then see that mentioned relations are consequences of periodicity and standing, macro-matter-waves structures within stable planetary and satellite systems.

Kepler laws are also showing the intrinsic <u>tendency of (mutually approaching) motional masses, planets, and satellites, to eventually stabilize in some form of elliptic, rotational, orbital and inertial motions around certain big mass (local star or sun), which is at the same time very close to a local center of mass (concerning relevant planetary or solar system).</u> For having an additional and supporting background, see introductory elaborations in this chapter, around equations (2.4-11) – (2.4-17)), where we can find that for the stability of a specific orbital motion (planetary system), the main request is that its total orbital and spinning momentum is conserved (meaning constant). If this (about orbital motions) were not the global tendency of mutually approaching masses, our universe would collapse in the process of permanent masses attraction and agglomeration. We also know that the conservation of orbital and spin moments is equally valid and important on a micro-world scale (analogically as we find in N. Bohr atom model). The dominant tendency of macro masses, also valid for micro and elementary particles, is to create stable (standing matter-waves), periodic and orbital motions, based on balancing between involved attractive forces with repulsive centrifugal forces (see new trends in modeling atoms and elementary particles in literature references from [16] to [22], Bergman, Lucas, Kanarev and others). Natural, non-forced uniform and stable, orbital planetary motions are at the same time inertial (uniform, continuous, self-closed and self-standing), periodical motions, which are coincidently conserving their linear and orbital moments, and potentially hosting standing matter waves formations, as shown in (2.9.1). Also, consequences of stable self-closed standing matter wave's orbital formations (that are causally related to periodical motions) are various energy, spin and orbital momentum relations and quantizing situations, being very much analogical to N. Bohr atom model (see more in Chapters 8., 10., and in literature references under **[63]**).

Citation from [63], under ***25) - Spin - orbit coupling in gravitational systems:***
We employ in this work the analogy existing between electromagnetism and gravitation [1]. We extend this analogy to include all phenomena occurring at atomic level and assume that they also do occur at the gravitation level and are governed by analogous rules (equations). The spin-orbit interaction that exists in hydrogen atom, due to the magnetic field if we introduce the concept of gravitomagnetic field that is analogous to the ordinary magnetic field. We have seen that the spin-orbit interaction is the same interaction that Einstein attributed to the curvature of the space [2]. And since all planets do have spin, the spin-orbit interaction is intrinsically prevailing in all star-planet systems. Bear in mind that some atoms can have zero total spin angular momentum. Note that the spin of planets remained a kinematical quantity in Newton and Kepler formulation of planetary motion. But we will show here that the spin is a dynamical quantity without which the planets would not remain stable in their orbits. Moreover, without spin there is no orbital motion. How much a planet should spin will depend on how much it is needed to conform with the orbital one. Equating the gravitational energy of a star-planet system to the spin-orbit interaction yields a formula that relates the primary star-planet system parameters to each other. Moreover, we found that such a system exits only if the spin and orbital angular momenta are proportional to the planet mass to the star mass ratio. This condition represents a dynamical balance between the two angular momenta. We call the resulting equation the Kepler's fourth law which represents the missing equation (law) to determine a star-planet system completely.
…………
As in hydrogen atom, which is analogous to the solar system, there is an interaction between the internal magnetic field arising from the electron orbital momentum, and its spin angular momentum. This is normally known as the spin-orbit interaction. The gravitomagnetic field is analogous to the magnetic field arises from the motion of the electron around the nucleus.

Let us now attempt to show that (like in case of de Broglie matter waves applied on Bohr's hydrogen, or planetary atom model) a circular planetary (or satellite) orbit, or its perimeter, is susceptible to host some gravitational (also electromagnetic and inertial), orbital, standing matter-wave. *Such macro matter-wave should have an orbital frequency* $f_o = f_{on}$, *wavelength* $\lambda_o = \frac{2\pi R}{n} = \lambda_{on}$, $n = \text{Integer}$, *group or orbital speed v (equal to planet semi-major orbital radius velocity), and associated phase speed* $u = \lambda_o f_o = u_n$. *Integer n should serve as a principal quantum number, being mainly related to the number of days in one year of certain planet orbiting its sun. The same quantum number could be related to presence of satellites, moons, and to involved angular and spinning moments, since relevant (and associated) standing-waves structure will be affected by all involved items (what introduces additional quantum numbers for arranging structural and spatial, standing waves packing and synchronization). Familiar matter-waves related conceptualization and results are* *initially addressed in the chapters 4.1, 10, and in this chapter among equations (2.9.1), (2.9.2), (2.11.5), (2.11.14)-h, in tables T.2.3.3-a, T.2.3.3-1, and such conceptualization is shown widely applicable, both in a micro and macro world of our Universe. One of good examples showing coupling of linear and spinning motions are cases of vortex-shedding flow-meter, presented around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,I, from the Chapter 4.1. Many ideas showing or constructing rich and well-operating (astronomic observations verifiable) analogy between planetary systems and N. Bohr atom model, can be found in [63], Arbab I. Arbab, [64], Marçal de Oliveira Neto, and in [67] Johan Hansson.*

In Chapter 8. of this book, under "8.3. Structure of the Field of Subatomic Forces" (see equations from (8.64) until (8.74)), we can find proposals how to conceptualize spatial standing-waves-structured forces, emanating from internal atom field structure, where both orbital and radial quantizing rules are applicable and mutually synchronized. It will be challenging to apply similar modeling to stable planetary systems. One of the consequences of such modeling could be that gravitational forces between agglomerated atoms and masses are the result of Lorentz-forces and Coulomb attractions between half-wave resonating (and rotating) mass-dipoles (or better to say attractions between electromagnetically polarized, charged and mutually oscillating dipoles).

Effectively, here we attempt to present motional energy of an orbiting planet as an equivalent macro matter-wave packet or wave group (which is the concept often and successfully applied in the micro-world physics). Specific planetary rotation around certain sun (see below (2.11.12)) has a period $T = T_m$ (or this is its one planetary year duration) and frequency of such (mechanical) rotation is $f_m = 1/T = 1/T_m$. f_m is not necessarily the frequency f_o of the associated, orbital, standing and macro matter-wave. For understanding the difference between mechanical (mass or particle) revolving frequency f_m and orbital macro matter-wave frequency f_o, we will first assume (and prove later) that $f_m \neq f_o$. Since the framework of this exercise implicitly accepts that relevant planetary or satellite (orbital) velocities are much lower compared to the light speed ($v << c$), we could safely say that any planetary or group velocity (its orbital velocity) should be two times higher than its phase velocity, $v = 2u = 2\lambda_o f_o$. See better explanation why and when $v = 2u$ in Chapters 4.0, 4.1, and 10., with equations (4.0.78) – (4.0.81) ... In other words, if the analogy with de Broglie matter-waves hypothesis also applies to planetary orbital motions, then the kinetic energy of specific planet should be equal to its equivalent matter-wave energy (or its matter-wave packet), $E_k = \tilde{E} = \frac{1}{2}mv^2 = Hf_0$, where H is a kind of gravitational, Planck's-analog, constant (all of that being very much analogical to matter waves and PWDC, as presented in Chapter 10.; -see "10.00 DEEPER MEANING OF PWDC").

Now we can find mentioned orbital frequency, wavelength, group, and phase speed of such (hypothetical), planetary standing matter-wave as (2.11.12),

$$\left\{\begin{array}{l} 2\pi r = n\lambda_o,\ T = \frac{1}{f_m} = \frac{2\pi}{\omega_m} = T_m,\ v = \frac{2\pi R}{T} = 2\pi R f_m = \omega_m R \cong 2u,\ n = \text{Integer} \\ u = \lambda_o f_o = u_n \cong \frac{1}{2} v = \frac{\pi R}{T} = \pi R f_m,\ T^2 = \left(\frac{4\pi^2}{GM}\right) R^3 = \frac{1}{f_m^2}, v_e = \sqrt{\frac{2GM}{R}}, \\ v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = v_n, u = \frac{v}{1+\sqrt{1-v^2/c^2}} = u_n \end{array}\right\} \Rightarrow$$

$$\lambda_o = \frac{2\pi R}{n} = \lambda_{on},\ f_o = \frac{u}{\lambda_o} = f_{on} = n\frac{u}{2\pi R} = n\frac{f_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi R^{3/2}}, f_m = \frac{2f_o}{n} = \frac{1}{T} = \frac{\sqrt{GM}}{2\pi R^{3/2}}, \tag{2.11.12}$$

$$u = \frac{v}{2} = \frac{1}{2}\sqrt{\frac{GM}{R}} = \frac{1}{2\sqrt{2}} v_e,\ v = 2u = \sqrt{\frac{GM}{R}} = \frac{1}{\sqrt{2}} v_e << c,\ m << M, \forall n = \text{Integer}.$$

Based on the group or planet's orbital speed, $v = \frac{2\pi R}{T} = 2u = 2\lambda_0 f_0 = \left(2\frac{H}{p} f_0\right)$ from (2.11.12), the wave energy or kinetic energy and gravitational Planck constant H, of an orbiting planet, which has mass $m << M$, and naturally keeps its angular momentum $\mathbf{L}$ = constant, can be mutually supporting and connected as:

$$\tilde{E} = E_k = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = \frac{1}{4}mv_e^2 = \frac{GmM}{2R} = \frac{E_e}{2} = \frac{1}{2}\cdot\left(\frac{GmM}{R^2}\right)\cdot R = \frac{1}{2}\cdot F_{m-M}\cdot R =$$

$$= \frac{m}{2}\left(\frac{2\pi R}{T}\right)^2 = \frac{8m\pi^2 R^2}{n^2} f_o^2 = 2m(\pi R f_m)^2 = (2\pi m R^2 f_m)\cdot(\pi f_m) = \mathbf{L}\pi f_m = (\frac{2\pi}{n}\mathbf{L})\cdot f_o = Hf_o, \tag{2.11.13}$$

$$\left\{\begin{array}{l} \mathbf{L} = n\frac{H}{2\pi} = 2\pi m R^2 f_m,\ p = mv = \frac{\tilde{E}}{u} = \frac{Hf_o}{u} = \frac{H}{\lambda_o} = n\frac{H}{2\pi R} = m\sqrt{\frac{GM}{R}},\ G = 6.67\cdot 10^{-11} Nm^2kg^{-2}, \\ F_{m-M} = F_g = \frac{GmM}{R^2} = \frac{mv^2}{R},\ \lambda_o = \frac{H}{p} = \frac{2\pi r}{n},\ f_o = \frac{nf_m}{2} = \frac{n}{2T} = \frac{n\sqrt{GM}}{4\pi R^{3/2}}, H = \frac{2\pi}{n}\mathbf{L} = \text{const.} \end{array}\right\}.$$

$$\Rightarrow \left\{\begin{array}{l} E_k = \tilde{E} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(2\frac{H}{p}f_0\right)^2 = 2\frac{H^2}{mv^2}f_0^2 = Hf_0 \\ 2\pi R = n\lambda_0 = n\frac{H}{mv},\ H = \text{Const.}\ n = 1, 2, 3, \ldots \end{array}\right\} \Rightarrow H = \frac{2\pi m v R}{n} = \frac{2\pi m_i v_i R_i}{n_i} \Rightarrow$$

$$\Rightarrow \frac{m_i v_i R_i}{n_i} = \frac{m_j v_j R_j}{n_j} = \frac{H}{2\pi}.$$

If we formulate an Analytic Signal, power-related wave, or Phasor function, that represents matter-wave of a specific orbiting planet (since we know its kinetic or wave energy), the same results, (2.11.12) and (2.11.13), should be associable to such complex matter-wave (see more about Analytic Signal in the chapters 4.0, 4.1 and 10.).

[♣ COMMENTS & FREE-THINKING CORNER:

To get more tangible feeling what should be measurable effects of mentioned "standing gravitational waves" (or gravitational field phenomena that has phase velocity $u = \lambda_o f_o \cong \frac{1}{2} v = \frac{\pi R}{T} = \pi R f_m$ *), we could analyze tidal waves on our planet Earth in relation to the Earth spinning and Moon rotation around the Earth, as well as Earth-Moon rotation around our local Sun. This way we should be able to establish predictable and measurable* *correlations (which should comply with (2.11.12) and (2.11.13)). Of course, it will also be necessary to consider proper values for* $u, \lambda_o, f_o, v, R, T, f_m$*, valid for Moon's rotation around Earth.*

We could also consider the orbital, matter wave frequency f_o *as the* ***time-frequency-train reference*** *for measuring our real-time flow. We are effectively using such time reference on different ways, since it has remarkably high stability, like quartz crystal or atomic clock oscillators, and it is the most significant for measuring our time flow (what will become more evident later; -see (2.11.14)-g)).*

Let us address ***gravitational field intensity and potential****. The* ***gravitational intensity or gravitational field*** $I = E_g$ *is traditionally defined as the gravitational force experienced by a unit mass* m *when placed in the gravitational field of another mass. From Newton's gravitational force we will get,*

$F_{m-M} = F_g = G\frac{mM}{R^2}$*, we can get,* $I = \frac{F_g}{m} = G\frac{M}{R^2}$.

In this book (see the first chapter about Analogies), we know, based on analyzed electromechanical analogies, that ***mass itself should not be the real and only source of gravitation. Only an oscillatory mass, which has linear and orbital moments, is (analogically and still hypothetically) better fitted to present gravitational charge*** *(see later (2.11.13-6) - (2.11.13-8), and more complete explanations in chapter 10, equations (10.1.4) - (10.1.7)). Effectively similar or familiar conclusions (what real sources of gravity are) can be drawn from the number of works of Dr. Jovan Djuric, [71]. Consequently, Newton gravitational force should have specific hidden or intrinsic, linear, and angular velocity parameters, embedded inside gravitational constant* G *(here, this unknown linear velocity parameter is* $v_0 = \text{const.}$*).*

We could now formulate another equivalent expression for Newton gravitational force, as being dependent of the product of relevant moments of attracting masses,

$$F_{m-M} = F_g = \frac{GmM}{R^2} = \frac{G}{v_0^2}\frac{(mv_0)\cdot(Mv_0)}{R^2} = \frac{G}{v_0^2}\frac{(p_m)\cdot(p_M)}{R^2} = K_G\frac{p_m \cdot p_M}{R^2},\ K_G = \frac{G}{v_0^2} = \text{const.}.$$

Based on such modified formulation of Newton force, and newly introduced gravitational charges p_m *and* p_M*, we can redefine an adjusted* ***gravitational intensity (or gravitational field*** $I^* = E^*_g$*) as,*

$$I^* = \frac{F_g}{p_m} = \frac{K_G p_M}{R^2} = \frac{G}{v_0^2}\frac{p_M}{R^2} = \frac{G}{v_0^2}\frac{Mv_0}{R^2} = \frac{G}{v_0}\frac{M}{R^2} = \frac{I}{v_0}.$$

We see, after we compare the ordinary (traditional) definition of gravitational intensity, and modified gravitational intensity, that qualitatively nothing significant changed (since $v_0 = \text{const.}$*), because we are the part of the same global rotation (or motion), but we also see that kind of hidden, background velocity parameter has its influence on modified Gravitational Intensity.*

Consequently, if we respect analogical predictions (from the first chapter about analogies in Physics), we need to admit (still hypothetically) that real sources of gravitation are relevant *and mutually coupled, linear and angular momenta. In such cases, Gravitational field intensity will be given by expressions (2.11.13-6) - (2.11.13-8).*

*The **gravitational potential** V at a certain point in the gravitational field is traditionally defined as the work done in taking a unit mass **m** from that point to infinity against the force of relevant gravitational attraction. From such definition, we have,*

$$V(R) = \frac{\text{Energy}}{\text{unit mass}} = -\frac{1}{m}\int_r^\infty F dR = -\frac{1}{m}\int_r^\infty \frac{GMm}{R^2} dR = -\int_r^\infty \frac{GM}{R^2} dR = -\int_r^\infty I\, dR = -\frac{GM}{R} (=) \left[\frac{m^2}{s^2}\right].$$

*If we now consider that real charges (or sources) of gravitation are masses with linear and/or orbital moments, as in (2.11.13-6) - (2.11.13-8), we can introduce **modified gravitational potential** as,*

$$V^*(R) = \frac{\text{Energy}}{\text{unit moment}} = -\frac{1}{mv_0}\int_r^\infty F dR = -\frac{1}{mv_0}\int_r^\infty \frac{GMm}{R^2} dR = -\frac{1}{v_0}\int_r^\infty \frac{GM}{R^2} dR = -\int_r^\infty I^* dR = -\frac{G}{v_0}\frac{M}{R} (=) \left[\frac{m}{s}\right],$$

Practically, here we are recognizing that every (relatively small) mass in a gravitation field or on an orbit around certain planet (or a big mass), including the situation when such smaller mass is touching a big mass, still has certain linear, orbital, or rotating speed (or intrinsically associated linear moment), because big mass (or planet) is orbiting about its sun.

$$V^*(R) = \frac{E_{km}}{mv} = -G\frac{M}{2Rv} = -\frac{v}{2} = -\frac{R}{2}\omega_m = -\frac{G}{v_0}\frac{M}{R}, v_0 = 2\frac{G}{v}\frac{M}{R} = 2G\frac{M}{R^2\omega_m} = 2v$$

$$\left\{\begin{array}{l} E_{km} = \frac{R}{2}\left|F_g\right| = G\frac{mM}{2R} = G\frac{m_r m_c}{2R} = \frac{mv^2}{2} = \frac{J_m\omega_m^2}{2} \\ \left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R} = \frac{J_m\omega_m^2}{R} = \frac{2E_{km}}{R} \end{array}\right\},$$

alternatively, if we consider that only angular moments are dominantly relevant, we will get,

$$V^*(R) = \frac{E_{km}}{J_m\omega_m} = \frac{1}{2}\omega_m , \left(E_{km} = \frac{J_m\omega_m^2}{2}\right)..$$

We can again see, when we compare ordinary (traditional) definition of gravitational potential, and modified gravitational potential, that nothing significantly changes, and that a relevant angular velocity (or angular moment) should be the most significant for hypothetically and analogically innovated expressions of gravitational field intensity and potential (see later the same resume in (2.11.13-6) - (2.11.13-8)).

*Now we can redefine **gravitational potential energy**. The work obtained in bringing a body from infinity to a point in the gravitational field is called the gravitational potential energy of the body at that point. Potential energy $U = E_p$ is usually presented mathematically as,*

$$U = E_p = -F_g \cdot R = G\frac{mM}{R} = K_G\frac{p_m \cdot p_M}{R}.$$

As we can see, the traditional definition of gravitational potential energy is identical to the modified gravitational potential energy (and it is evident that introducing the new meaning of gravitational charges should not be a problem). The gravitational potential energy at infinity is assumed (to be) zero.

Here we also see that we similarly redefine gravitational intensity and potential as we do with electric charges and fields. This could be an intuitive or indicative argument in relation to relevant electromechanical analogies, in a direction that gravitation could be a specific hidden manifestation of electromagnetic forces, since rotating and spinning motions are producing electric charges separation (or electric dipoles) and spinning matter states are related to elementary magnets that are getting properly arranged during mentioned rotation.

*As the support to (2.11.13), it is convenient to mention that total mechanical energy (without rest-mass energy), $E_m = E_k + E_p$, for an object with mass **m**, in a closed circular orbit with radius **R**, in a central gravitational field around a body with mass **M**, such as a planet orbiting about local sun, is equal to the sum of its kinetic energy $E_k = \frac{1}{2}mv^2$, and its potential, or "positional" energy $E_p = -G\frac{mM}{R}$. The gravitational potential energy is defined as a negative value, equal to the kinetic energy that the object would gain by falling from an infinite distance to its current position. At considerable distances from the Sun, the object would have zero potential energy (since it would not have picked up any speed, by falling). Objects close to the Sun have considerable (although negative) potential energies, corresponding to the speed they would gain by dropping a long way.*

$$E_m = E_k + E_p = \frac{1}{2}mv^2 - \left(G\frac{mM}{R^2}\right)\cdot R,\ F_c = F_g = m\frac{v^2}{R} = G\frac{mM}{R^2} \Rightarrow 2E_k = mv^2 = G\frac{mM}{R} \Rightarrow$$

$$\Rightarrow E_m = E_p + E_k = \frac{1}{2}G\frac{mM}{R} - G\frac{mM}{R} = -G\frac{mM}{2R} = \frac{1}{2}E_p\,,\ E_k = G\frac{mM}{2R} = -E_m = -\frac{1}{2}E_p\ .$$

Remarkably similar, or better to say identical result for E_m also holds for an elliptical orbit, after we generalize it by replacing the radius of orbit R by the relevant orbital semi-major axis, as usually applied for Newton's derivation of Kepler's third law. Now, such total mechanical energy is constant and has similar forms for circular and elliptical orbits ($E_m = -G\frac{mM}{2R} = \frac{1}{2}E_p$, where R is semi-major axis). In ideal circular orbits, since there, speed v is constant, kinetic, and potential energies are constant. In elliptical orbits, the kinetic and potential energy is not constant, but somewhat variable on the way that one is large when the other is small and vice versa. For elliptic orbits, where $0 < e < 1$ is the eccentricity of the elliptical orbit, the following equations can be derived:

T.2.7.

Energy type	**R = R$_{min.}$**	**R = R$_{max.}$**
Potential, E_p	$2E_m/(1-e)$	$2E_m/(1+e)$
Kinetic, E_k	$-E_m(1+e)/(1-e)$	$-E_m(1-e)/(1+e)$

*We see that when orbit eccentricity $e = 0$, all latest results for elliptical orbits again correspond to a circular orbit result. The larger the eccentricity, **e**, the larger is the variation of the potential and kinetic energies during each period of the orbital motion.*

----

*What is the meaning of the gravitation force and associated matter waves energy between two masses **m** and **M** can be briefly explained based on simplified two-body problem analysis? Here, we will use the same symbolic and meanings for associated parameters, as in (2.11.10) – (2.11.13). If we have two isolated, static (or standstill) masses, **m**, and **M**, in the same inertial, reference frame, where a distance between them is equal to R, we can say that the total energy, $E_{tot.}$, of such system and Newton force of gravitation F_g, between them, are,*

$$E_{tot.} = mc^2 + Mc^2 = (m+M)c^2 = m_c c^2,\ F_g = -G\frac{mM}{R^2}, m_c = m + M \cdot \quad (2.11.13\text{-}1)$$

*If such masses are in the same reference frame and have specific mutually relative motion (where **m** has velocity $\vec{v}_1$ and **M** has velocity $\vec{v}_2$), the total energy of such two-body system and force of gravitation between them are,*

$$\begin{aligned} &E_{tot.} = mc^2 + \frac{1}{2}mv_1^2 + Mc^2 + \frac{1}{2}Mv_2^2 = \\ &= (m+M)c^2 + \frac{1}{2}(m+M)v_c^2 + \frac{1}{2}m_r v_r^2 = m_c c^2 + \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2, \\ &F_g = -G\frac{mM}{R^2} = -G\frac{m_r m_c}{R^2},\ m_c = m + M,\ m_r = \frac{mM}{m+M},\ \vec{v}_r = \vec{v}_1 - \vec{v}_2, \vec{v}_c = \frac{m\vec{v}_1 + M\vec{v}_2}{m+M}. \end{aligned} \quad (2.11.13\text{-}2)$$

If, also, each of masses is self-spinning (has its spin moment), the same situation with the total energy (like elaborations around equations (2.5.1-4) - (2.5.1-7) from the same chapter) and force of gravitation will be,

$$\left\{ \begin{aligned} &E_{tot.} = mc^2 + \frac{1}{2}mv_1^2 + \frac{1}{2}J_{s1}\omega_{s1}^2 + Mc^2 + \frac{1}{2}Mv_2^2 + \frac{1}{2}J_{s2}\omega_{s2}^2 = \\ &= (m + \Delta m_s)c^2 + \frac{1}{2}(m + \Delta m_s)v_1^2 + (M + \Delta M_s)c^2 + \frac{1}{2}(M + \Delta M_s)v_2^2, \\ &\Delta m_s c^2 = \frac{1}{2}J_{s1}\omega_{s1}^2,\ \Delta M_s c^2 = \frac{1}{2}J_{s2}\omega_{s2}^2 \end{aligned} \right\} \Rightarrow \quad (2.11.13\text{-}3)$$

$$\Rightarrow \left\{ \begin{aligned} &E_{tot.} = m^*c^2 + \frac{1}{2}m^*v_1^2 + M^*c^2 + \frac{1}{2}M^*v_2^2 = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}m_r^*v_r^2 \\ &m^* = m + \Delta m_s,\ M^* = M + \Delta M_s, \\ &F_g = -G\frac{m^*M^*}{R^2} = -G\frac{m_r^*m_c^*}{R^2}, m_c^* = m^* + M^*,\ m_r^* = \frac{m^*M^*}{m^*+M^*},\ \vec{v}_r = \vec{v}_1 - \vec{v}_2, \vec{v}_c = \frac{m^*\vec{v}_1 + M^*\vec{v}_2}{m^*+M^*} \end{aligned} \right\}.$$

Within one or the other option (for masses without, or with self-spinning), we are coming to the possibility to express reduced-mass kinetic energy $\frac{1}{2}m_r^*v_r^2 = E_r$ as specific orbital, rotation energy $\frac{1}{2}J_r^*\omega_r^2$ of relative mass m_r^* about its center of mass m_c^*, and this energy corresponds to the associated matter wave energy $\tilde{E}$ of relative mas in its (would or could be) orbital-like motion (as in (2.11.13)),

$$E_{tot.} = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}m_r^*v_r^2 = m_c^*c^2 + \frac{1}{2}m_c^*v_c^2 + \frac{1}{2}J_r^*\omega_r^2,\ E_r = \frac{1}{2}m_r^*v_r^2 = \frac{1}{2}J_r^*\omega_r^2 = \tilde{E}. \quad (2.11.13\text{-}4)$$

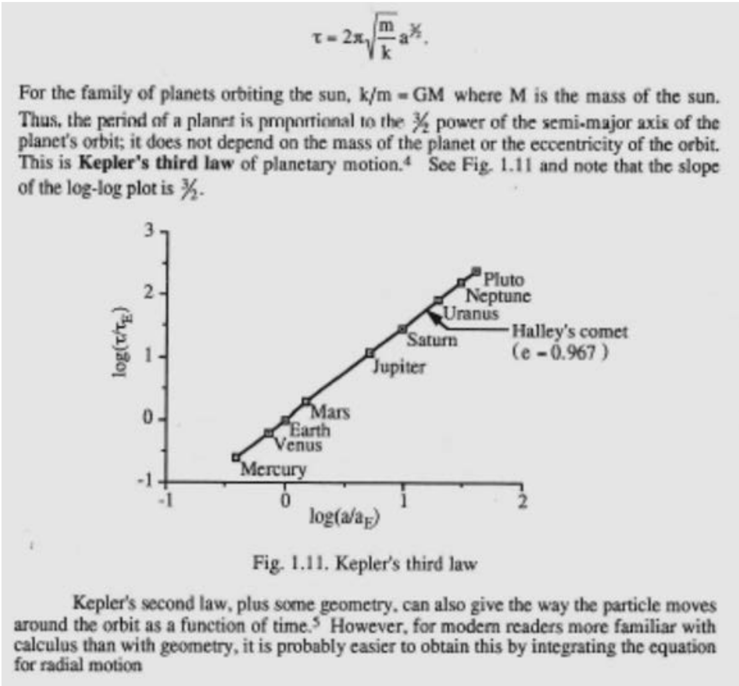
*Now, we will introduce the decisive, essential, questionable, and innovative assumption regarding gravitation. **If we assume that rotating (orbital-like) motional energy $\frac{1}{2}J_r^*\omega_r^2$, of the relative mass $m_r^* = \frac{m^*M^*}{m^*+M^*} = \mu$, about $m_c^* = m^* + M^*$, is equal to one half of the work (or energy) of Newton gravitational force $F_g \cdot R$ (between masses m^* and M^*, along the distance R, necessary to unite masses m^* and M^*, as already seen in (2.11.13)), we will be able to develop, or reformulate the same (already known) form of the Kepler Third Law** (2.11.10), as follows,*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\begin{bmatrix} 2\tilde{E} = 2E_r = 2\cdot\frac{1}{2}J_r^*\omega_r^2 = 2\cdot\frac{1}{2}m_r^*v_r^2 = F_g\cdot R = G\frac{m^*M^*}{R^2}\cdot R = G\frac{m_r^*m_c^*}{R} = -E_p = -2E_m, \\ E_m = E_p + E_k = -G\frac{m^*M^*}{2R} = \frac{1}{2}E_p = -E_r\,,\ E_k = G\frac{m^*M^*}{2R} = -E_m = -\frac{1}{2}E_p = E_r \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} 2\cdot\frac{1}{2}v_r^2 = G\frac{m_c^*}{R} = 2\cdot\frac{1}{2}(\omega_r R)^2 = 2\cdot\frac{1}{2}(2\pi f_r R)^2 = 4\pi^2 f_r^2 R^2 = \frac{4\pi^2R^2}{T_r^2}, \\ v_r = \omega_r R = 2\pi f_r R = \frac{2\pi R}{T_r} = \frac{2\pi R}{T},\ f_r = f_m = \frac{1}{T} = \frac{1}{T_r} \end{bmatrix} \Rightarrow \quad (2.11.13\text{-}5)$$

$$\Rightarrow T_r^2 = \frac{4\pi^2}{Gm_c^*}\cdot R^3 \Leftrightarrow T^2 = \frac{4\pi^2}{G(M+m)}\cdot R^3 \cong \frac{4\pi^2}{GM}R^3 .$$



$$\tau = 2\pi\sqrt{\frac{m}{k}}\,a^{3/2}.$$

For the family of planets orbiting the sun, k/m = GM where M is the mass of the sun. Thus, the period of a planet is proportional to the 3/2 power of the semi-major axis of the planet's orbit; it does not depend on the mass of the planet or the eccentricity of the orbit. This is **Kepler's third law** of planetary motion.[4] See Fig. 1.11 and note that the slope of the log-log plot is 3/2.

Fig. 1.11. Kepler's third law

Kepler's second law, plus some geometry, can also give the way the particle moves around the orbit as a function of time.[5] However, for modern readers more familiar with calculus than with geometry, it is probably easier to obtain this by integrating the equation for radial motion

The picture was taken from Lagrangian and Hamiltonian Mechanics, M. G. Calkin
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With (2.11.13-5), (2.4-13), (2.4-5.1) and with many familiar elaborations in Chapter 4.1, we are supporting and defending the concept of the real existence of planetary, macro matter waves, as introduced in (2.11.12) and (2.11.13). ***We also see that natural inertial, meaning orbital motions and self-closed standing matter-waves structures are appearing coincidently.*** *Different quantizing (or integer dependent) formulas are also the consequence of standing waves formations. Later, we will collect more arguments in a direction that gravitation-related matter waves (and gravitation) are most probably the consequences and effects of fundamentally electromagnetic, electromechanical, electro and magnetostrictive background.*

From widely elaborated electromechanical analogies, (see chapter 1), and from common definitions for the electric (and magnetic) field, we can analogically speculate about a new meaning for the field of gravitation, where real sources of gravity will be

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[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

involved angular and linear moments (and associated electromagnetic charges and fluxes), instead of masses. *For instance, the gravitational force between masses m and M, where m is orbiting about M, can be formulated as,*

$$\begin{aligned}
&\left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R} = \frac{J_m\omega_m^2}{R} = \frac{2E_{km}}{R}, \left(v = v_m = \omega_m R\right),\\
&v_m = \sqrt{G\frac{M}{R}} = \omega_m R,\ \omega_m = \sqrt{G\frac{M}{R^3}},\\
&v_r = \sqrt{G\frac{m_c}{R}} = \omega_{mr} R,\ \omega_{mr} = \sqrt{G\frac{m_c}{R^3}}, \frac{v_m}{v_r} = \frac{\omega_m}{\omega_{mr}}.
\end{aligned} \tag{2.11.13-6}$$

Using the analogical field-intensity definition, where a field is developed from its force divided by its source-charge, and if the relevant gravitational charge is either linear or angular momentum, (not a mass, like in traditional Newtonian gravitation), we will have,

$$E_g = \left\{\begin{matrix} \frac{F_g}{mv} \\ or \\ \frac{F_g}{J_m\omega_m}\end{matrix}\right\} = \left\{\begin{matrix} \frac{mv^2}{mvR} \\ or \\ \frac{J_m\omega_m^2}{J_m\omega_m R}\end{matrix}\right\} = \left\{\begin{matrix} \frac{v}{R} = \omega_m \\ or \\ \frac{1}{R}\omega_m\end{matrix}\right\}. \tag{2.11.13-7}$$

*In both cases of (*2.11.13-7*), here modified field of gravitation E_g is directly proportional to the proper angular (or orbital) velocity ω_m (as we see in (2.11.13-7)).*

*The standard and traditional definition of the gravitational field (where only a mass is the source of gravitation) is much different from (*2.11.13-7), for example,

$$E_g = \frac{F_g}{m} = G\frac{M}{R^2} = \frac{v^2}{R} = \omega_m^2 R. \tag{2.11.13-8}$$

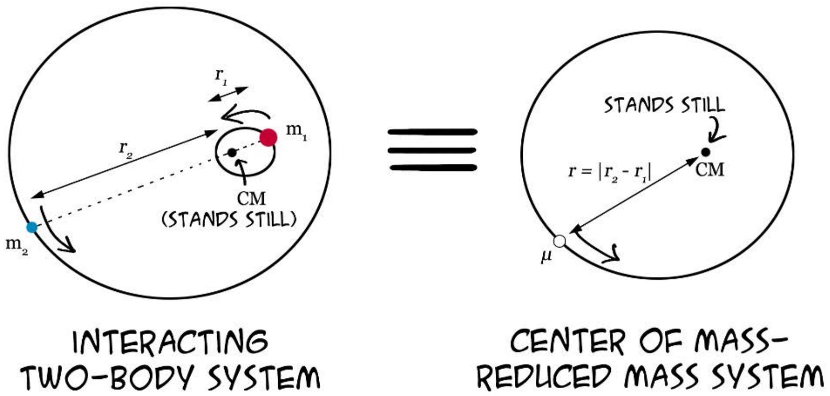
Consequently, here we are on some intuitive and brainstorming (or hypothetical) way generating conclusions that the most relevant gravitation-related sources should be angular, orbital and spinning moments and relevant derivatives as linear forces and torques as angular forces (see more in chapter 4.1). This could be additionally supported if we try to specify what is common, for both micro-world of atoms and subatomic entities, and the macro-world of planetary systems and galaxies. The typical common items are micro and macro systems, and events with rotations, spinning states, and orbital motions, all of them characterized by angular moments and dimensionally or directly proportional to certain relevant angular velocity, very much similar as we see in the new definition of the gravitational field (2.11.13-7). In Chapter 1. of this book (about analogies), we also find that relevant sources of gravitation (based on analogical conclusions) are not masses, but linear and orbital moments, or electric charges and magnetic moments and fluxes (related to rotation and spinning). This is similar or at least sufficiently common theoretical platform about the dominant place of rotation and spinning in our universe, as we can find in publications under [36], Anthony D. Osborne & N. Vivian Pope.

Practically and briefly summarizing, we could say that:

1° All-natural motions in our universe are curved, and
2° All stable, stationary, periodical, and inertial motions should be orbital motions.
3° Also, our Universe is, globally, intrinsically, and holistically rotating (oscillating and resonating) on all micro and macro structural levels.

Consequently, only angular, or orbital and spinning moments should be the most relevant regarding phenomenology we understand and describe under gravitation. Since spinning, rotation and orbiting are very often coupled with associated magnetic fields and moments, here is the place for understanding electromagnetic background nature of gravity.

Until here, we mostly analyzed somewhat static and simple-motion situations between two (electromagnetically neutral, and internally balanced) masses, but similar conclusions can be drawn from the general two-body problem. For instance, the same energy balance and gravitational attraction between two bodies, as given with (2.11.13-2) and (2.11.13-6), is also applicable in cases of motional masses, when we transform such two-body system to an equivalent orbiting motion of the reduced mass around its central mass. ***The more appropriate conceptualization here will be to treat the two-body problem as an impact situation and to search for evolving angular and rotating elements in such mutually interacting motions.*** *In other words, every two-body motional system is generating elements of angular, or somewhat circular (spiral and orbital) accelerated movements, what we can see when we play with different, mutually related reference frames, as presented with (2.11.13-9), and on the picture, below.*



Taken from: *https://quantumredpill.files.wordpress.com/2013/01/two-body-cm-systems.png*

Mentioned two or multibody systems with evolving elements of orbiting and circular motions are naturally creating matter-waves. If involved masses are also electromagnetically charged, and have spinning moments, mentioned revolving, circular, and resulting spinning motions will be intensified. In addition, if electromagnetically neutral or non-charged masses are getting closer, because of specified self-generated and evolving elements of angular and rotational movements, we can naturally expect to get internal electromagnetic dipoles polarizations (inside of masses, because masses are composed of molecules, atoms, electrons, protons, and neutrons, all of them are on some way oscillating, rotating and spinning). This way, certain kind of electric and magnetic dipoles-related internal currents and magnetic fields will be created (since masses m and M are also performing linear and circular motions, depending on the point of view, what is influencing and stimulating spontaneous, but organized electromagnetic dipoles polarization). Mentioned internal electromagnetic dipoles-related currents and fluxes are effectively creating a spatial situation like parallel wires with electric currents passing in the same direction, this way magnetically attracting each other, what we detect as gravitation. At the same time, because of periodical, spatial and temporal motions *of involved masses and because of associated elements of angular, rotational, spinning and helical movements, matter waves will be naturally created.*

Two-body energy balance and involved gravitational force with elements of linear and circular motions can be summarized as,

$$E_{tot} \cong mc^2 + \frac{1}{2}mv_m^2 + Mc^2 + \frac{1}{2}Mv_M^2 = m_c c^2 + \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2 =$$

$$= \boxed{m_c c^2 + \frac{1}{2}J_m\omega_m^2 + \frac{1}{2}J_M\omega_M^2 = m_c c^2 + \frac{1}{2}J_c\omega_c^2 + \frac{1}{2}J_r\omega_r^2},$$

$$\left|F_g\right| = G\frac{mM}{R^2} = G\frac{m_r m_c}{R^2} = \frac{mv^2}{R}\left(= \frac{m_r v_r^2}{R}\right) = \frac{2E_{km}}{R} = \boxed{\frac{J_m\omega_m^2}{R}},$$

$$\left(\begin{array}{l} E_{km} = \frac{1}{2}mv_m^2 = \frac{1}{2}J_m\omega_m^2 = \frac{R\left|F_g\right|}{2} = G\frac{mM}{2R} = G\frac{m_r m_c}{2R},\ v = v_m = \omega_m \cdot R \cong 2u, \\ \Psi_m^2(t)dt = d\tilde{E}_m = dE_{km} = v_m dp_m = d\left(\frac{1}{2}mv_m^2\right) = mv_m dv_m = p_m dv_m = \\ = \omega_m d\left(J_m\omega_m\right) = \left(J_m\omega_m\right)d\omega_m,\ \Delta\Psi_m(x,t) - \frac{1}{u_m^2}\frac{\partial^2\Psi_m(x,t)}{\partial t^2} = 0,\ u_m = \lambda_m f_m = v_m/2 \end{array}\right), \quad (2.11.13\text{-}9)$$

$$\omega_m = 2\pi f_m,\ n\lambda_m = 2\pi R,\ \vec{v}_c = \frac{m\vec{v}_m + M\vec{v}_M}{m+M} = K\frac{J_m\vec{\omega}_m + J_M\vec{\omega}_M}{J_m + J_M} = K\vec{\omega}_c,\ K = Const.$$

See much more in chapter 4.1, where similar problems of helical matter waves are additionally elaborated. In addition, if mutually approaching and interacting particles or masses already have spinning and electromagnetic moments and charges, and if specific spontaneous electromagnetic dipoles polarization is produced, beside gravitational forces and effects, we will need to account presence of Coulomb force interactions, indicating that all mentioned forces and fields essentially have an electromagnetic origin or background.

Now is a right place to mention an analogy between here-introduced concepts (of couplings and equivalency between linear and angular motions) as already presented in this chapter and later in chapter 4.1 (on illustrations on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, and within equations under (4.3) and later).

Citation took from the Internet: *http://www.school-for-champions.com/science/gravitation_orbit_center_of_mass_derivation.htm#.V6SSvKL4i6E*

Derivation of Circular Orbits Around Center of Mass

by Ron Kurtus (revised 14 May 2011)

Circular orbits of two objects around the center of mass (CM) between them require tangential velocities that equalize the gravitational attraction between the objects.

Tangential velocities tend to keep the objects traveling in a straight line, according to the Law of Inertia. If gravitation cases an inward deviation from straight-line travel, the result is an outward centrifugal force. By setting the gravitational force equal to the centrifugal forces, you can derive the required tangential velocities for circular orbits.

The orbit equations can be in simplified forms when the masses of the two objects are the same and when the mass of one object is much greater than that of the other.

Questions you may have included:

- *What are the factors involved in the derivation?*
- *What are the equations for the velocities of the objects?*
- *What happens when one object is much larger than the other is?*

This lesson will answer those questions. Useful tool: Units Conversion

Factors in determining orbital velocities

The linear tangential velocities required for two objects to be in circular orbits around the center of mass (CM) between them is found by comparing their gravitational force of attraction with the outward centrifugal force for each object.

***Note**: A linear tangential velocity is a straight-line velocity is perpendicular to the axis between the two objects. It is tangent to the curved path and is different from rotational speed.*

(See Center of Mass and Tangential Gravitational Motion for more information.)

Assume no initial radial velocities

When two objects in space are traveling toward the general vicinity of each other, they both have radial and tangential velocities concerning the center of mass (CM) between them. However, to simplify the derivation for circular orbits, we will only look at the case where there are no inward or outward radial velocities and be concerned about the tangential velocities.

This is like the case of Newton's cannonball going into orbit or sending a satellite into orbit around the Earth.

(See Gravity and Newton's Cannon for more information.)

Since there is no radial motion, a separation between the objects remains constant, which is a requirement for circular orbits.

The gravitational force of attraction

The gravitational force of attraction between two objects is:

$$F = GMm/R^2$$

where

- ***F** is the force of attraction between two objects in newtons (N)*
- ***G** is the Universal Gravitational Constant = $6.674 \cdot 10^{-17}$ N-km^2/kg^2*

- ***M*** *and* ***m*** *are the masses of the two objects in kilograms (kg)*
- ***R*** *is the separation in kilometers (km) between the objects, as measured from their centers of mass*

Note*: Since force is usually stated in newtons, but motion between astronomical bodies is usually stated in km/s, an adjusted value for* ***G*** *is used, with* $N\text{-}km^2/kg^2$ *as the unit instead of* $N\text{-}m^2/kg^2$. ***G*** *is also sometimes stated as* $6.674 \cdot 10^{-20}\ km^3/kg\text{-}s^2$.

(See Universal Gravitation Equation for more information.)

Separation of objects

As the objects orbit the CM, their total separation, ***R****, remains constant. The individual separations between the objects and CM are also constant and determined by* ***R*** *and their masses:*

$$R = R_M + R_m$$

where

- R_M *is the separation between the center of object* ***M*** *and the CM in km*
- R_m *is the separation between the center of object* ***m*** *and the CM in km*

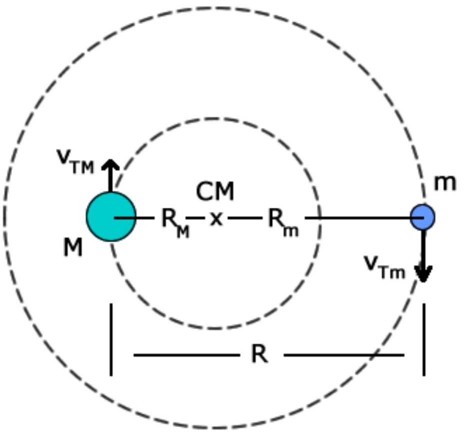
The values of R_M *and* R_m *are according to the equations:*

$$R_M = mR/(M + m)$$

$$R_m = MR/(M + m)$$

(See Center of Mass Definitions for more information.)

The factors involved can be seen in the illustration below:



Factors in objects orbiting CM

Note*: Although the Earth orbits the Sun in a counterclockwise direction, we usually indicate motion in a clockwise direction.*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

(See Direction Convention for Gravitational Motion for more information.)

Centrifugal force

The centrifugal inertial force on each object relates to its circle of travel:

$F_M = Mv_{TM}^2/R_M$,

$F_m = mv_{Tm}^2/R_m$,

where,

- F_M *is the centrifugal inertial force on mass* ***M***
- v_{TM} *is the tangential velocity of mass* ***M***
- F_m *is the centrifugal inertial force on mass* ***m***
- v_{Tm} *is the tangential velocity of mass* ***m***

Note*: Centrifugal force is caused by inertia and is not considered a "true" force. It is sometimes called a pseudo- or virtual force.*

Substituting $R_M = mR/(M + m)$ *and* $R_m = MR/(M + m)$ *in the above equations gives you:*

$F_M = Mv_{TM}^2(M + m)/mR$

$F_m = mv_{Tm}^2(M + m)/MR$

Solve for individual velocities

Since the centrifugal force equals the gravitational force for a circular orbit, you can solve for the velocity.

The object with mass m

In the case of the object with mass ***m****:*

$F_m = F$

Substitute equations:

$mv_{Tm}^2(M + m)/MR = GMm/R^2$

Multiply both sides by ***MR*** *and divide by* ***m****:*

$v_{Tm}^2(M + m) = GM^2/R$

Divide both sides by ***(M + m)****:*

$v_{Tm}^2 = GM^2/R(M + m)$

Take the square root:

$v_{Tm} = \pm\sqrt{[GM^2/R(M + m)]}$

This means the velocity can be in either direction for a circular orbit. Since direction is not relevant here:

$v_{Tm} = \sqrt{[GM^2/R(M + m)]}$ *km/s*

The object with mass M

Likewise, for the object of mass ***M****:*

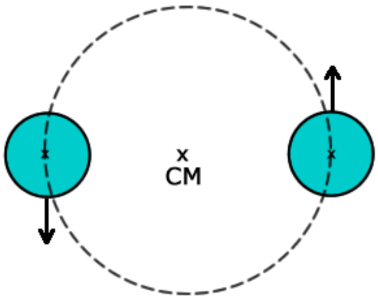
$v_{TM} = \sqrt{[Gm^2/R(M + m)]}$ *km/s*

Sizes of objects

The equations for the tangential orbital velocities can be simplified when both objects are the same size, as well as when one object has a much greater mass than the other does.

Objects have the same mass

There are situations in space where two stars have close to the same mass and orbit the CM between them. Astronomers call them double stars.



Double stars follow the same orbit around CM

If the objects are the same mass, then ***M = m*** *and the velocity equation for each becomes:*

$v_{TM} = \sqrt{[Gm^2/R(m + m)]}$ *km/s*

The equation reduces to,

$v_{TM} = \sqrt{[Gm/2R]}$ *km/s*

Since both objects or stars have the same orbital velocity and the same separation from the CM, they follow the same orbit around the CM.

One object much more massive than other

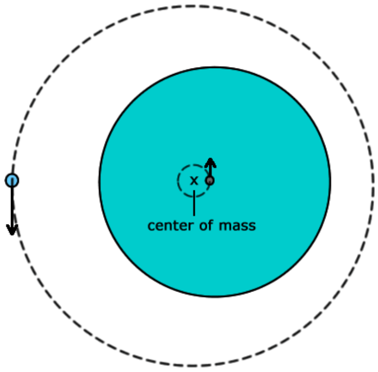
Another situation often seen in space is, when one object is much larger than the other is. In this case, the CM between them is almost at the more massive object's geometric center. This results in simplifying the equation for orbital velocity. The small object then seems to orbit the more massive object.

For example, the CM between a satellite orbiting the Earth is near the geometric center of the Earth. Likewise, the CM between the Earth and the Sun is near the center of the Sun.

*Suppose **M >> m** (**M** is much greater than **m**). Then:*

$$M + m \approx M$$

where ≈ means "approximately equal to".



Orbits, when one object is much larger than other, is

*Substitute **M + m ≈ M** into the equation for the velocity of the smaller object:*

$$v_{Tm} = \sqrt{[GM^2/R(M + m)]}$$

$$v_{Tm} = \sqrt{(GM^2/RM)}$$

Reducing the equation results in:

$$v_{Tm} = \sqrt{(GM/R)} \text{ km/s}$$

This is the same as the standard equation for the orbital velocity of one object around another.

(See Orbital Motion Relative to Other Object for more information.)

Summary

When two objects are moving at the correct tangential velocities, they will go in circular orbits around their CM. The velocity equations are determined by setting the gravitational force equal to the outward centrifugal forces caused by their tangential velocities.

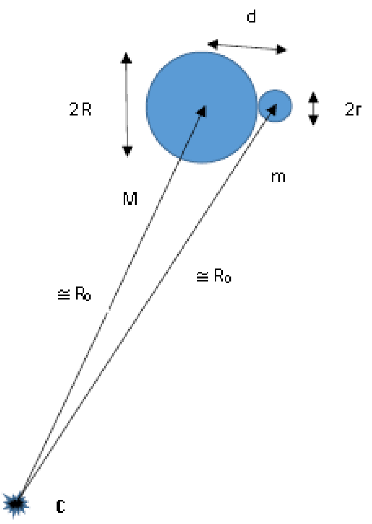
The velocity equations are:

$$v_{Tm} = \sqrt{[GM^2/R(M + m)]} \text{ km/s}$$

$$v_{TM} = \sqrt{[Gm^2/R(M + m)]} \text{ km/s}$$

When the mass of each object is the same, the velocity equation is simplified. The same is true when the mass of one object is much greater than that of the other.

To illustrate (or approximate on a highly speculative way) the meaning of global, universal, cosmic, or holistic rotation (which could be hidden or invisible for us, but we suppose that it effectively exists and produces gravitational force), let us imagine that small mass m is sitting on a big mass M, being attracted by mutual gravitational force. For instance, M could be certain planet and m will be a small spherical object, where the following approximations are applicable: M >>> m, R >>> r, d <<< R_0. Here, d is the distance between centers of M and m, and R_0 is the distance from certain distant and common, dominant, or significant center C of global or holistic (universal) rotation (see the picture below). We do not know where exactly the center of universal cosmic rotation is to place our reference system there, but mathematically we could operate with such imaginative (and still hypothetical) center of rotation.



Both masses m and M, are effectively rotating synchronously (or coincidently) around their global and universal cosmic center of rotation, here marked with C. Both masses also have the same angular or revolving frequency around center C. Such (on some way) hidden, and mathematically effective rotation could also be specific complicated angular motion locally presentable as rotation (since center C should not always be center of mass of the relevant local, planetary, or solar system). Between masses, M and m, we can safely say that it exists an attractive gravitational force, as follows,

$$\left|F_g\right| = G\frac{mM}{d^2} = \frac{mv_0^2}{R_0} = \frac{2E_{km}}{R_0} = \frac{J_m\omega_m^2}{R_0} = \frac{J_m\omega_0^2}{R_0} = m\,\omega_0^2\,R_0,\ \omega_M = \omega_m = \omega_0 = \frac{v_0}{R_0}.$$

If we now assume that gravitational force between m and M is the consequence of global, universal rotation (about center C), where relevant orbital moments are dominant factors (instead of involved masses), we will have,

$$G\frac{mM}{d^2} = G_1\frac{J_m\omega_m \cdot J_M\omega_M}{d^2} = G_1\frac{J_m J_M\omega_0^2}{d^2} = G_1\frac{mR_0^2 \cdot MR_0^2}{d^2} = G_1\frac{mMR_0^4}{d^2}$$
$$= \frac{mv_0^2}{R_0} = \frac{J_m\omega_m^2}{R_0} = \frac{J_m\omega_0^2}{R_0} = m\,\omega_0^2\,R_0,\ \omega_M = \omega_m = \omega_0 = \frac{v_0}{R_0} \Rightarrow$$
$$\Rightarrow R_0 = \frac{GM}{d^2\omega_0^2}, G_1 = \frac{G}{R_0^4}...$$

We could also speculate that universal cosmic rotation (probably combined with linear, spinning, and helical motion) is the primary cause of internal electrostatic and magnetic polarizations of involved masses, this way giving grounds to explain gravitational attraction as an electromagnetic dipoles' attraction. This example is just a brainstorming draft of a future, more elaborated modeling. ♣]

The magnitude of the angular momentum $\mathbb{L}$ from (2.11.13), of a periodically orbiting planet or satellite, its relevant, orbital mean-radius $\mathbf{R}$ (or semi-major axis), and associated characteristic speeds are quantized. Such orbital quantization is based on a planet-associated resonant and standing matter-waves (having a particular group and phase velocity, like in any periodical wave motion), respecting (in average) simple geometrical fittings (such as $2\pi R = n\lambda_o$), and can be summarized as results shown in (2.11.14), below.

Similar concepts and results are presented in [43], M. Pitkänen; [38], [39], F. Florentin Smarandache and Vic Christianto; [40], D. Da Rocha and Laurent Nottale; [64], Marçal de Oliveira Neto; and in **[125],** Markus J. Aschwanden; (see in (2.11.14)).

Here is a place to underline that nobody of mentioned authors is interpreting such results in a direct and robust relation to group and phase velocity of associated matter-waves groups, or wave-packets obtained by superposition of significant number of mutually agglomerated harmonic, elementary waves, like in the micro-world physics (what is an innovative contribution in this book).

$$\left[\mathbb{L} = pR = mvR = mR^2\omega_m = mR^2\frac{2\pi}{T} = 2\pi mR^2 f_m = m\sqrt{GMR} = n\frac{H}{2\pi} = n\hbar_{gr.} = const,\ m \cong \mu = \frac{mM}{m+M}\right] \Rightarrow$$

$$\Rightarrow \begin{cases} \left[\begin{array}{l} R = R_n = n^2\dfrac{H^2}{4\pi^2 GMm^2} = n^2\dfrac{GM}{v_0^2} = \dfrac{GM}{v_n^2} = n\dfrac{\lambda_o}{2\pi} = n\dfrac{\hbar_{gr.}}{mv},\ \lambda_o = \dfrac{H}{mv} = \dfrac{H}{p} = \dfrac{2\pi R}{n} = \lambda_{on}, \\ T = \dfrac{2\pi R}{v} = \dfrac{1}{f_m} = \dfrac{nT_0}{2} = \dfrac{n}{2f_o} = \dfrac{(nH)^3}{4\pi^2 G^2 M^2 m^3} = T_m,\ v = u - \lambda_o\dfrac{du}{d\lambda_o} = -\lambda_0^2\dfrac{df_0}{d\lambda_0} \cong 2u = 2u_n, \\ f_o = n\dfrac{f_m}{2} = n\dfrac{1}{2T} = n\dfrac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on},\ T_o = \dfrac{1}{f_o} = \dfrac{2T}{n} = \dfrac{4\pi R}{nv},\ E_k = \tilde{E} = \tilde{E}_n = Hf_o = n\dfrac{Hf_m}{2}, \\ v = v_n = \dfrac{v_0}{n} = \dfrac{2\pi}{nH}GMm = 2u = \dfrac{v_e}{\sqrt{2}} = \sqrt{\dfrac{GM}{R_n}},\ \ u = u_n = \lambda_o f_o = \dfrac{1}{2}\sqrt{\dfrac{GM}{R_n}} = \dfrac{v_0}{2n} \cong \dfrac{v}{2}, \\ v_0 = nv_n = \dfrac{2\pi}{H}GMm = \dfrac{GMm}{\hbar_{gr.}} = 2un = \dfrac{nv_e}{\sqrt{2}} = n\sqrt{\dfrac{GM}{R_n}},\ \ n, n_i = \text{Integers}],\ v << c. \end{array}\right] \Rightarrow \\ \Rightarrow \left[\begin{array}{l} \begin{Bmatrix} \text{For two planets on orbits 1 and 2: } H = const. \Rightarrow \\ \dfrac{R_1}{R_2} = \dfrac{n_1^2}{n_2^2}\dfrac{m_2^2}{m_1^2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1 m_2}{n_2 m_1}\right)^3 = \left(\dfrac{R_1}{R_2}\right)^{3/2} = \dfrac{v_2}{v_1}\cdot\dfrac{R_1}{R_2} = \dfrac{T_{1m}}{T_{2m}} = \dfrac{n_1}{n_2}\cdot\dfrac{f_{o2}}{f_{o1}} = \dfrac{n_1}{n_2}\cdot\dfrac{T_{o1}}{T_{o2}} \\ \dfrac{v_1}{v_2} = \dfrac{u_1}{u_2} = \dfrac{\lambda_{o1}f_{o1}}{\lambda_{o2}f_{o2}} = \dfrac{n_2}{n_1}\cdot\dfrac{m_1}{m_2} = \sqrt{\dfrac{R_2}{R_1}} \Leftrightarrow \underline{R_i v_i^2 = R_j v_j^2},\ \ \dfrac{\lambda_{o1}}{\lambda_{o2}} = \dfrac{n_2}{n_1}\cdot\dfrac{R_1}{R_2} \end{Bmatrix} \\ ------------------ \\ \begin{Bmatrix} \text{For the same planet passing between two orbits: } H = const. \Rightarrow \\ (m_1 \cong m_2), \dfrac{R_1}{R_2} = \left(\dfrac{n_1}{n_2}\right)^2, \dfrac{T_1}{T_2} = \left(\dfrac{n_1}{n_2}\right)^3 \end{Bmatrix} \end{array}\right] \end{cases} \quad (2.11.14)$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Effectively, sitting on results and assumptions from (2.11.14), Titus – Bede's law (related to quantization of planetary orbits) is significantly rectified and optimized by Markus J. Aschwanden; **[125]** - "Self-organizing systems in planetary physics: Harmonic resonances of planet and moon orbits". Mentioned reference, **[125]**, is strongly supporting and reinforcing here elaborated model of planetary, standing matter-waves, and additionally giving more of legitimacy to the corrected and upgraded Titus-Bode law.

[♣ COMMENTS & FREE-THINKING CORNER:

The same or equivalent, quantizing-like results, and conclusions (as in (2.11.14)) can be formulated almost directly, analogically, and much faster if we consider that in certain solar system, the Sun analogically presents a proton, and planets are like electrons orbiting around. Quantizing is applicable if the system can be approximately treated as a *2-body problem*. If we exploit the mathematical identity between the electrostatic Coulomb force in the hydrogen atom, and Newton's "static" gravitational force, and systematically substitute $\frac{Ze^2}{4\pi\varepsilon_0} \to GmM$ (or $Ze^2 \leftrightarrow mM$, $\frac{1}{4\pi\varepsilon_0} \leftrightarrow G$) in all relevant results known from hydrogen atom analyzes, where M is mass of the sun, m is mass of certain planet, H is macrocosmic planetary constant analog to Planck constant h, ($H >>> h$) and G is gravitational constant. See much more of such background in [63] Arbab I. Arbab, and in [67], including other familiar publications from Johan Hansson [77], Newtonian Quantum Gravity. "*Gravito-static versus electrostatic analogy*" should not be only a mathematical curiosity, coincidence, and academic discussion option, after we consider as realistic the possibility that solar system elements are mutually electrically (and magnetically) polarized like mutually attracting electric (and magnetic) dipoles and multi-poles (since there are electromagnetic fields and forces around them). Such electromagnetic polarization option is already presented in this chapter (see ***2.2. Generalized Coulomb-Newton Force Laws***; -equations from (2.3) until (2.4-10)). In addition, the Chapter 8. of this book (Bohr Model) develops and presents most of analogical, quantized results (see results from (8.23) to (8.33)), as found in (2.11.14), where mutual correspondence and full analogy of such results can be established by applying "*Gravito-static versus electrostatic analogy*". Of course, here analogy means more than mathematical similarity or identity, since in the case of gravitation within planetary systems relevant results are also correct, verified by astronomic measurements, and other theoretical and experimental observations. Consequently, here we deal with accurate empirical, natural, and scientific facts and the main consequence should be that gravitation and associated electromagnetic complexity are coincidently present and mutually coupled, at least in cases of solar or planetary systems (see also elaborations around equations (2.11.20) - (2.11.22) and Fig.2.6.).

Let us directly apply analogical substitutions of quantized expressions relevant for orbiting electrons, and their associated standing matter waves (as shown in [77], Johan Hansson, Newtonian Quantum Gravity, in results from the Chapter 8., Bohr model…, and in (2.4-8)), to results (8.4), and (8.26) - (8.30). This way, we will create analogical and quantized, standing-waves expressions as summarized in T.2.8., comparable to results from (2.11.14), valid and correct for orbiting planets (including planetary macro matter waves),

T.2.8. N. Bohr hydrogen atom and planetary system analogies

We will apply the following analogies and formal replacements:

$$\left\{\frac{1}{4\pi\varepsilon_0}\frac{q\cdot Q}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{(ze)\cdot(Ze)}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{zZ\cdot e^2}{r^2}\Leftrightarrow G\frac{m\cdot M}{r^2}\right\}\Rightarrow$$

$$\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2},\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}\right]\Leftrightarrow\left[G\frac{m^2}{r^2},G\frac{M^2}{r^2}\right], q=ze\Leftrightarrow m\sqrt{4\pi\varepsilon_0 G},\ Q\Leftrightarrow Ze=M\sqrt{4\pi\varepsilon_0 G},$$

$$zZe^2\Leftrightarrow mM4\pi\varepsilon_0 G,\ e^2\Leftrightarrow\frac{mM}{zZ}4\pi\varepsilon_0 G,\ e\sqrt{zZ}\Leftrightarrow\sqrt{mM}\sqrt{4\pi\varepsilon_0 G},\frac{1}{4\pi\varepsilon_0}\leftrightarrow G,\ h\leftrightarrow H,\quad\Rightarrow$$

$$\left[\frac{e}{\sqrt{4\pi\varepsilon_0 G}}\sqrt{\frac{zZ}{m+M}}\Leftrightarrow\sqrt{\frac{mM}{m+M}}=\sqrt{\mu}\right]\Rightarrow\frac{e}{\sqrt{4\pi\varepsilon_0}}\sqrt{\frac{zZ}{m+M}}\Leftrightarrow\sqrt{\frac{mM}{m+M}}\cdot\sqrt{G}=\sqrt{\mu}\cdot\sqrt{G}$$

$$\frac{ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow m\cdot\sqrt{G},\ \frac{Ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow M\cdot\sqrt{G},\ \mu=\frac{mM}{m+M}$$

$$\Rightarrow\left[\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}\Leftrightarrow G\frac{m\cdot M}{r^2}\right]\Rightarrow\left[\frac{Ze^2}{4\pi\varepsilon_0}=\frac{e}{\sqrt{4\pi\varepsilon_0}}\cdot\frac{Ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow m\sqrt{G}\cdot M\sqrt{G}\right]\Rightarrow$$

$$\left[\frac{e}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow m\sqrt{G},\ \frac{Ze}{\sqrt{4\pi\varepsilon_0}}\Leftrightarrow M\sqrt{G}\right]\Rightarrow Z\Leftrightarrow\frac{M}{m};\ \frac{1}{4\pi\varepsilon_0}\leftrightarrow G,\ h\leftrightarrow H,\ \frac{Ze^2}{4\pi\varepsilon_0}\Leftrightarrow Gm\cdot M$$

Effectively, here we assume that every mass or agglomerated atoms have the corresponding amount of polarizable electric dipoles or electric charges in a way that Coulomb and Newton's laws are mutually equivalent or replaceable (see (2.4-4.1) - (2.4-4.3) developed earlier in this second Chapter). This way, masses attraction can be treated as adequately oriented electric dipoles attraction (or by analogically extending the same conceptualization to associated magnetic dipoles attraction, since in such dynamically stable and self-closed systems electric and magnetic performances are mutually balanced like in capacitance-inductance or mass-spring resonant circuits). Here we also assume the existence of some omnipresent, holistic angular cosmic motion (including oscillatory and resonant states), being the ultimate cause or source of mentioned intrinsic and volumetric masses, electromagnetic polarization. In case if our Universe has more dimensions than 4 [more than (x, y, z, t)], we could imagine that mentioned holistic motion would be recognizable from, for us still not detectible, higher dimensional spaces. In some distant parts of our universe (for instance concerning spiral galactic formations...), such universal angular motion, and associated electromagnetic polarization could produce stronger Coulomb or Newtonian attractions (than in our part of Cosmos), and we maybe wrongly associate improvable "dark mass and dark energy" mystifications to such very much clear, natural, and explicable phenomenology (see [121], Raymond HV Gallucci).

Analogically crated relations between N. Bohr atom model and planetary systems are, as follows:

the phase velocity of an electron wave	the phase velocity of a planetary wave
$u_s\approx\frac{1}{2}v=\frac{Ze^2}{4nh\varepsilon_0}=\lambda_s f_s,\ n=1,2,3...$ (8.26)	$u_n\approx\frac{1}{2}v=\frac{\pi GmM}{nH}=\lambda_n f_n, n=1,2,3,...$ $(u_n=\lambda_{on}f_{on}=\frac{\mathbf{1}}{\mathbf{2}}\sqrt{\frac{GM}{R_n}}=\frac{v_0}{2n}\cong\frac{v}{2},$ (2.11.14))

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

the group velocity of an electron wave	the group velocity of a planetary wave
$v_s \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0}$, (8.27)	$v_n \approx 2u_n = \frac{2\pi GmM}{nH}$, ($v_n = \frac{v_0}{n} = 2u = \frac{v_e}{\sqrt{2}} = \sqrt{\frac{GM}{R_n}} = \frac{2\pi}{nH}GmM$, (2.11.14))

a frequency of an orbital electron wave	a frequency of an orbital planetary wave
$f_s \approx \frac{mZ^2e^4}{8n^2h^3\varepsilon_0{}^2}$ (8.28)	$f_n \approx \frac{2\pi^2G^2m^3M^2}{n^2H^3}$, ($f_n = f_{on} = n\frac{f_m}{2} = n\frac{1}{2T_n} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = \frac{\tilde{E}_n}{H}$, (2.11.14))

a wavelength of an orbital electron wave	a wavelength of an orbital planetary wave
$\lambda_s = \frac{h}{p} \approx \frac{2nh^2\varepsilon_0}{mZe^2}$ (8.29)	$\lambda_n = \frac{H}{p} \approx \frac{nH^2}{Gm^2M}$, ($\lambda_n = \lambda_{on} = \frac{H}{mv} = \frac{H}{p} = \frac{2\pi R}{n}$, (2.11.14))

the energy of a stationary electron wave	the energy of a stationary planetary wave
From N. Bohr atom model: $\varepsilon_s = \varepsilon_n = hf_s = \frac{1}{2}\mu v^2 = \frac{\mu Z^2e^4}{8n^2h^2\varepsilon_0^2}$ (8.30)	$\varepsilon_n = \tilde{E}_n = Hf_n \approx \frac{1}{2}\mu v^2 = \frac{2\pi^2G^2m^3M^2}{n^2H^2}$, ($E_k = \tilde{E}_n = Hf_{on} = n\frac{Hf_m}{2}$, (2.11.14))
--	--
From the solutions of the Schrödinger equation in spherical coordinates: $\varepsilon_s = \varepsilon_n = \frac{(\frac{h}{2\pi})^2}{2\mu a_0^2 n^2}$,	Analogically formulated, hypothetical: $\varepsilon_n = \frac{(\frac{H}{2\pi})^2}{2\mu a_{0g}^2 n^2}$, (hypothetical; -analogically created)
$\left\{ \begin{array}{l} a_0 = \frac{(\frac{h}{2\pi})^2}{\mu e^2} = \text{Bohr radius}, \\ \frac{2\mu e^2}{(\frac{h}{2\pi})^2\delta} = \frac{2j+k+1}{2} = j+\ell+1 = n, \\ \ell(\ell+1) = \frac{k^2-1}{4}, \left(\frac{\delta}{2}\right)^2 = -\frac{2\mu\varepsilon_s}{(\frac{h}{2\pi})^2}, k = 2\ell+1, \\ \delta = \frac{2\mu e^2}{(\frac{h}{2\pi})^2 n} = \frac{2}{a_0 n}, \quad (k, j, \ell, n) = \text{integers}. \end{array} \right\}$	$\left\{ \begin{array}{l} a_{0g} = \frac{(\frac{H}{2\pi})^2}{G\mu^2 M} = \text{Gravitational, Bohr radius}, \\ \frac{2\mu^2 M}{(\frac{H}{2\pi})^2\delta_g} = \frac{2j+k+1}{2} = j+\ell+1 = n, \\ \ell(\ell+1) = \frac{k^2-1}{4}, \left(\frac{\delta_g}{2}\right)^2 = -\frac{2\mu\varepsilon_s}{(\frac{H}{2\pi})^2}, k = 2\ell+1, \\ \delta_g = \frac{2\mu^2 M}{(\frac{H}{2\pi})^2 n} = \frac{2}{a_{0g} n}, (k, j, \ell, n) = \text{integers}. \end{array} \right\}$

a radius of an electron orbit		a radius of a planetary orbit
$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi \mu e^2 Z}$ (8.4) -------------------------------- $\langle r_{n,\ell} \rangle = \frac{a_0}{2}\left[3n^2 - \ell(\ell+1)\right]$, $\langle r^2_{n,\ell} \rangle = \frac{a_0^2 n^2}{2}\left[5n^2 + 1 - 3\ell(\ell+1)\right]$, $\left\langle \frac{1}{r_{n,\ell}} \right\rangle = \frac{1}{a_0 n^2}, \left\langle \frac{1}{r^2_{n,\ell}} \right\rangle = \frac{1}{a_0^2 n^3 (\ell + 1/2)}$, $n = k + \ell = 1, 2, 3...$ $\ell = 0, 1, 2, ..., n-1$ $m = -\ell, -\ell+1, ..., \ell-1, \ell$		$R_n = \frac{n^2 H^2}{4\pi^2 G m^2 M}$, $(R_n = n^2 \frac{GM}{v_0^2} = \frac{GM}{v_n^2} = n \frac{\lambda_{on}}{2\pi} = \frac{n^2 H^2}{4\pi^2 G m^2 M}$, (2.11.14)) ----------------------------- (hypothetical; -analogically created) $\langle R_{n,\ell} \rangle = \frac{a_{0g}}{2}\left[3n^2 - \ell(\ell+1)\right]$, $\langle R^2_{n,\ell} \rangle = \frac{a_{0g}^2 n^2}{2}\left[5n^2 + 1 - 3\ell(\ell+1)\right]$, $\left\langle \frac{1}{R_{n,\ell}} \right\rangle = \frac{1}{a_{0g} n^2}, \left\langle \frac{1}{R^2_{n,\ell}} \right\rangle = \frac{1}{a_{0g}^2 n^3 (\ell + 1/2)}$, $n = k + \ell = 1, 2, 3...$ $\ell = 0, 1, 2, ..., n-1$ $m = -\ell, -\ell+1, ..., \ell-1, \ell$

Also, for the specific planet, we can analogically determine gravitational fine-structure constant,

Fine-structure constant	Gravitational fine-structure constant
$\alpha = \frac{e^2}{4\pi\varepsilon_0 (\frac{h}{2\pi}) c} \cong \frac{1}{137}$	$\alpha_g = \frac{G m^{*2}}{(\frac{H}{2\pi}) c}$

Of course, in T.2.8., reduced masses $\mu = \frac{m_1 m_2}{m_1 + m_2}$ are different in case of Bohr atom model and for planets of a specific solar system (because different masses m_1 and m_2 are involved).

We can see that almost all (easily verifiable) analogically formulated results in T.2.8. are correct, since we already have such results from different astronomic observations, publications, and analyses. Here, we have striking analogical and quantitative, direct proportionality between involved masses and (analogically) involved electric charges. How could this be possible, when we know that gravitational attraction also exists between electromagnetically neutral or internally, spatially, and statistically, electromagnetically compensated masses (when macroscopically measurable magnetic and electric fields do not exist)? One of the logical and straightforward explanations is that involved masses, meaning involved agglomerations of atoms are (for some reason) only slightly, electrically, and magnetically, internally polarized (almost beyond our experimental recognition), and creating (in average) uniformly organized very weak electric and magnetic dipoles. Such dipoles, when spatially correctly organized, are exercising Coulomb types of attractions (since the same form of Coulomb law is equally applicable to electric charges and magnets, and at the same time this is analogically equivalent to Newton gravity force law, where only involved masses are considered). We assume here that quantitatively certain mass is directly proportional to the number of its internal constituents, atoms, or the number of involved electrostatic dipoles and/or elementary magnets belonging to atom constituents. Since we also know (from the first chapter of this book) that direct analogies between electric and magnetic charges (or fluxes) are not masses, but linear and spinning or orbital moments, to complete the analogical picture and understanding (concerning Coulomb and Newton laws), here we only miss certain (relatively constant, intrinsic or background) linear and/or angular speed of our Universe. This, about global background velocity, could be specific global, omnipresent, holistic, cosmic motion (where linear and angular movements, rotation and spinning are combined). Mentioned rotation, and associated centrifugal force are influencing weak (spatially properly polarized, organized, or aligned) electromagnetic dipoles (thanks to an enormous mass difference between electrons and *protons), this way producing Coulomb type of electromagnetic forces, and we analogically (but essentially mistakenly) describe such effects as Newtonian interaction between involved masses. As we know, electrons and protons, and almost all subatomic entities have spinning moments and magnetic properties. In cases of totally macro-neutral or macro-compensated masses, involved internal electromagnetic states will cancel mutually and spatially. However, if specific global, background rotation (of our Universe) anyway exist, small electrostatic dipoles polarization will appear on surfaces of involved masses and involved internal masses domains will (in average) experience some spinning or elementary-magnets alignments. Both, electric dipoles, and internal magnetic moments (or magnetic dipoles) are respecting Coulomb law forces, and in some cases, we mistakenly consider such effects as a manifestation of gravitation. In examples of significant rotating galactic masses, mentioned electromagnetic dipoles are much stronger, favorably and dominantly polarized, producing stronger Coulomb or (looks-like) gravitational attraction, and we wrongly consider this as an argument that some associated, additional dark, invisible mass (matter or energy) should exist in order to defend observed and inexplicable masses attraction, and to prolong the theoretical and conceptual validity of Newton and A. Einstein gravity-related theories. In other words, all of that serves to stop or prevent searching for new theory, which is better explaining what the Gravitation really is. Effectively, only oscillating masses, atoms and externally extended atomic force fields are real sources of Gravitation (instead of static masses), and all of such structures are mutually coupled and synchronized, since in cosmic systems we find applicability of such atoms modeling (see much more in Chapter 8.).*

Planetary systems, as mechanical and periodical-motions systems are on some way creating or respecting stationary and standing waves structure, like mechanical resonators (as already demonstrated in this chapter with equations from (2.11.10) until (2.11.13). Also, planetary systems behave as being very much analogically predictable (and presentable) with N. Bohr's atom model, or like electromagnetic resonators, where similar stationary waves also exist, and where electrostatic forces are dominant. Consequently, we should expect the existence of substantial and direct electromechanical couplings (between associated electric and magnetic dipoles, or multipoles) within such dualistic resonant systems (on some way like the piezoelectric effect). Such mechanical and electromagnetic resonant planetary systems are anyway united. Every stable planetary system should behave as a single, internally, and externally, synchronized resonant system, having specific and mutually harmonized, structural, radial, and orbital (standing waves) resonant frequencies, when observed only as a mechanical, or just as an electromagnetic resonator (also being analogical to manifestations of acoustic levitation; -see more in [150] and [151]). Every mechanical or electromagnetic perturbation of such coupled resonators will synchronously produce similar electromagnetic or mechanical disturbance (like in piezoelectric devices).

This electromechanical coupling should explain the essential (or ontological) electromagnetic nature of Gravitation since within stationary or standing matter waves we will have nodal zones with effects of attractive forces, which are creating masses-agglomerations (like in Newtonian attractions). To satisfy unity of radial, axial, orbital and transversal, resonant behaviors of structurally resonant planetary systems, electromagnetic waves should have both transversal and longitudinal components, meaning that Maxwell equations should be conveniently upgraded to support longitudinal waves. One attempt or proposal for such upgrading is initiated in the third chapter of this book (see familiar relations (3.7-1) and (3.7-2)). Nikola Tesla measured mentioned structural and stationary planetary, resonant waves, both as mechanical and electromagnetic waves, and this way, most probably, formulated his ideas about new Dynamic Theory of Gravitation (but unfortunately never published or finalized it). See literature references under [97], [98], [99] and [117].

Ling Jun Wang; -Citation (see [122]), … "presented a theory of unification of gravitational force and the electromagnetic force based on the generalization of Newton's law of gravitation to include a dynamic term inferred from the Lorentz force of electromagnetic interaction. The inclusion of this dynamic term alone in the gravitational force is enough to develop the entire dynamic theory of gravitation parallel to that of electrodynamics".

Familiar ideas about the extension of the Lorentz force are elaborated in the third chapter of this book. If we connect electric and magnetic (dipoles and multipoles) polarizations, and global (holistic) motions and rotations within our Universe, with Lorentz force effects, the picture of unity between gravitation and electromagnetism will be much clearer and more indicative.

The bottom-line simplified explanation about masses coupling is related to the fact that masses are composed of atoms. Atoms internally have number of discrete, stationary, and standing waves energy states, meaning resonant states, or we could also say physical resonators. Resonators with mutually overlapping spectral characteristics are being naturally synchronized (in zones where they have the same resonant frequencies). This way, compact and united macro mass starts behaving like a big, united atom with number of internal, discrete (atomic and molecular) energy states. Since all macro masses are such kind of complex resonant states, it is natural to expect that mentioned (mutually overlapping) resonant states from any of two separate masses will again mutually synchronize and on some way energetically communicate (by creating standing electromagnetic waves between them), producing the effects of Gravitation (see more in Chapter 8.). Familiar innovative concept about Gravitation, where mases of planets are in states of permanent mutual electromagnetic energy exchanges and coupling, being at the same time transmitters and receivers of electromagnetic energy, and where relevant solar system structure is creating standing waves fields between the sun and planets, including many of additional imaginative and challenging excursions towards other domains of modern Physics, can be found in [144], Poole, G. (2018) Cosmic Wireless Power Transfer System and the Equation for Everything.

Citation from [144] "**Abstract**: By representing the Earth as a rotating spherical antenna several historic and scientific breakthroughs are achieved. Visualizing the Sun as a transmitter and the planets as receivers the solar system can be represented as a long wave radio system operating at Tremendously Low Frequency (TLF). Results again confirm that the "near-field" is Tesla's "dynamic gravity", better known to engineers as dynamic braking or to physicists as centripetal acceleration, or simply (g). …
A new law of cosmic efficiency is also proposed that equates vibratory force and pressure with volume acceleration of the solar system. Lorentz force is broken down into centripetal and gravitational waves. …
Spherical antenna patterns for planets are presented and flux transfer frequency is calculated using distance to planets as wavelengths. The galactic grid operates at a Schumann Resonance of 7.83 Hz, …
The Sun and the planets are tuned to transmit and receive electrical power like resonating Tesla coils".

As we know, original N. Bohr's Planetary Atom Model is upgraded, and successfully exploited, by applying Schrödinger's equation, and ideas of particle-wave duality. Consequently, we should be able, because of the validity of mentioned "Gravito-static versus electrostatic analogies" (based on $\frac{Ze^2}{4\pi\varepsilon_0} \rightarrow GmM$), to analogically apply relevant wave functions and Schrödinger equation to familiar planetary, and other astronomic situations (with periodic and circular, orbital, and inertial motions). Elaborations and analyzes of planetary systems, starting from (2.11.12) until (2.11.14), are anyway clearly indicating that planetary systems, presented as macro-cosmological matter-waves, behave very much analogically as known in microphysics. Consequently, here we have enough grounds to apply Schrödinger equation (see such attempts, later, around equations (2.11.20) - (2.11.22), and Fig.2.6.). The Analogy in question is not at all establishing different, non-doubtful, definitive grounds that *probabilistic methodology* of quantum theory is relevant here (opposite to what certain authors are implicitly forging as the fact). Schrödinger equation is applicable here mostly because stable, self-closed orbits, hosting periodical and standing waves, create necessary conditions to formulate and apply such equation (and there is nothing to connect it with probabilities and statistics). Much more striking, challenging, and significant here is the fact that gravitation and electromagnetic field are on some way (more than only analogically) connected, and that sources of gravity are most probably of electromagnetic nature (see much more of similar ideas in [72], Dr. László Körtvélyessy. The Electric Universe).

Citation taken from **[63]**, under **24)**: Arbab Ibrahim Arbab. The Generalized Newton's Law of Gravitation versus the General Theory of Relativity. **Journal of Modern Physics.**

"We have shown that the gravitomagnetism and the general theory of relativity are two theories of the same phenomenon. This entitles us to accept the analogy existing between electromagnetism and gravity fully. Hence, electromagnetism and gravity are unified phenomena. The precession of the perihelion of planets and binary pulsars may be interpreted as a spin-orbit interaction of gravitating objects. The spin of a planet is directly proportional to its orbital angular momentum and mass, weighted by the Sun's mass. Alternatively, the spin is directly proportional to the square of the orbiting planet's mass and inversely proportional to its velocity".

Since in T.2.8., we have the analogical expression for gravitational, fine structure constant $\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c}$, and since ordinary (atomic and electromagnetic) fine structure constant is known as extraordinarily stable, α = 7.2973525698(24)x10^{-3}, α^{-1}=137.035999074(44), we could search for (analogic) conditions when gravitational fine structure constant will be equal to atomic, or electromagnetic fine structure constant, meaning,

$$\alpha_g = \frac{Gm^{*2}}{(\frac{H}{2\pi})c} \cong \alpha = 7.2973525698(24) \cdot 10^{-3} \cdot \tag{2.11.14-1}$$

From (2.11.14-1) we can determine the hypothetical (analogically founded) value for macro-cosmological or gravitational Planck constant H, as,

$$H = \frac{2\pi G}{\alpha c} m^{*2} \cong 137.035999074 \frac{2\pi G}{c} m^{*2} \,. \tag{2.11.14-2}$$

On a similar way, as we are converging gravitational fine-structure constant to the atomic or electromagnetic fine structure constant $\alpha_g \to \alpha$, $m^* \to m^*_{minimal}$, gravitational or macro cosmological Planck constant H, from (2.11.14-2), would be at the same time (analogically) converted into microworld Planck constant h, meaning $H \to h$. The minimal value of the mass $m^* = m^*_{minimal}$, which has specific gravitational mass, when is still meaningful or possible to detect and measure effects of gravitation in the Newtonian framework, could be one among masses of the proton, neutron, or electron, but here we will find that this is not the case.

$$\begin{aligned} &\left(H \to h = \frac{2\pi G}{\alpha c} (m^*_{minimal})^2 \cong 137.035999074 \frac{2\pi G}{c} (m^*_{minimal})^2 \right) \Rightarrow \\ &m^*_{minimal} \cong \sqrt{\frac{hc}{137.035999074 \cdot 2\pi G}} = 1.44308 \cdot 10^{-19} \ [kg] \end{aligned} \,. \tag{2.11.14-3}$$

We can calculate from (2.11.14-3) that the minimal mass, which has specific gravitational meaning (under here introduced analogical framework) $m^* \cong 1.44308 \cdot 10^{-19}$ [kg], is $8.62766 \cdot 10^7$ times bigger than the mass of the proton, or $8.61592 \cdot 10^7$ times more significant than the mass of the neutron, and $1.58417 \cdot 10^{11}$ times bigger than the mass of an electron.

Anyway, it is experimentally known that the validity of Newton gravitational force law (between two masses) is testable and provable until the lower distance limits of 300 micrometers (approximately). One of the conclusions here could be that gravitation has a meaning only for a relatively large group of electromagnetically polarizable atoms (for instance, for a minimum of $8.62766 \cdot 10^7$ *hydrogen atoms), above certain threshold mass amount (*$m^* \cong 1.44308 \cdot 10^{-19}$ [kg]*), and for distances between two masses higher than 300 micrometers. All of that is indicating that gravitation could be a manifestation of electromagnetic forces between masses with specific electric dipoles polarization (as speculated at the beginning of this chapter, around equations from (2.4-7) to (2.4-10)). If electromagnetic forces and charges are essential sources of gravitation, consequently, what we expect to detect as gravitational waves should be some very low-frequency electromagnetic waves. In cases of stable solar or planetary systems, we should be able to find such standing and stationary, macro electromagnetic field structures between planets and a local sun. It is still too early to draw definite conclusions, but at least, here we got specific indicative numbers (regarding validity of gravitation), under certain sufficiently well-defined and challenging conditions.*

In T.2.8. we explored formal analogies based on a comparison between the Bohr planetary atom model and a real planetary system such as,

$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M, e \Leftrightarrow m \cong \frac{mM}{m+M} = \mu, Z \Leftrightarrow \frac{M}{m}, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM, Ze^2 \to mM \cong \mu M\right\}.$$
(2.11.14-4)

From the first chapter of this book (see T.1.2 until T.1.8) we know that when respecting Mobility system of electromechanical analogies, electric charges are analog to linear and orbital moments, meaning that Newton law of Gravitation should present specific force-field between linear and orbital moments, like already exercised and summarized in T.2.2, T.2.2-2 and (2.4-5.1). If we consider that in the Newton law of gravitational force, instead of mutually attracting masses, we should have attraction of corresponding linear and orbital moments (which are on some way implicitly present, but still hidden, and somewhat hypothetical), we can reformulate mentioned initial analogies (2.11.14-4) on the following way,

$$(G = gv_c^2 = Const) \Rightarrow \tag{2.11.14-5}$$
$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow g\frac{(mv_c)(Mv_c)}{r^2} = g\frac{p_c \cdot P_c}{r^2}, Ze \Leftrightarrow Mv_c = P_c, e \Leftrightarrow mv_c = p_c \cong \frac{mM}{m+M}v_c = \mu v_c, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow g, h \Leftrightarrow H\right\} \Rightarrow$$
$$\Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to g\cdot(p_c \cdot P_c) \Rightarrow Ze^2 \Leftrightarrow p_c \cdot P_c = mMv_c^2 \cong \mu Mv_c^2\right\}$$

Practically, in (2.11.14-5) we see a way to extract (or expose) missing or hidden velocities of interacting masses from Newton gravitational constant, $G = gv_c^2 = Const.$, where speeds of both masses **m** and **M** are the same and equal to $v_c \cong const.$. This way, instead of static masses product, mM we created the outcome of associated linear moments $p_c \cdot P_c = (mv_c)(Mv_c)$, satisfying analogy that electric charge corresponds to linear momentum (like in T.1.8), and consequently transforming Newton law of gravitation to be the force between involved mechanical (linear and/or orbital) moments. In other words, both masses, **m** and **M,** are globally moving concerning specific reference system with a certain velocity, and making certain, linear and/or oscillatory motion (like oscillating dipoles). It is also evident that such a speculative and intuitive situation (regarding hidden, or background velocity parameters) should be better elaborated and explained. ***Of course, later we also need to find a way to involve angular and spin moments of interacting masses in a Newton law, but what is important here is to show that Newton force of gravitation could evolve towards richer conceptualization***. In chapter 10 of this book, we can find the complete explanation of the same situation regarding unknown or background velocity parameters and Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7)).

Analogies between the Bohr atom model and planetary systems are very much striking and indicative (see T.2.8. and Chapter 8.). Since an extended atom modeling is also related to magnetic, orbital and spin moments, we can (analogically and hypothetically) expect that planetary systems should operate within a similar environment of involved orbital moments, spins and surrounding electric and magnetic fields, and standing matter waves (because of stable periodical motions). Similar explorations and modeling are exercised all over this book, and in number of publications from [36], Anthony D. Osborne, & N. Vivian Pope, [63], Arbab I. Arbab, and in [71], from Jovan Djuric; "Magnetism as Manifestation of Gravitation". If we have necessary technical and observational means, we should be able to visualize relevant electromagnetic matter-waves, and other masses-distribution-related standing waves structure of planetary and galactic systems. Another option that is hypothetically radiating from T.2.8. is that this is not only a system of analogies between electromechanical, gravitational, and electromagnetic entities, but much more something like describing the same (anyway united) phenomenology using combined mechanical and electromagnetic concepts. In other words, here we may have mutually analog and equivalent, mechanically, electromagnetically, and electromechanically coupled entities of the same force that is in physics (by chance and in different historical periods) independently conceptualized either as Newtonian mechanics and gravitation, and as electromagnetism related Coulomb force. Here, we are on the way to propose possible unification of electromechanical phenomenology, as already exercised around equations and expressions (2.4-7) - (2.4-10), earlier in this chapter. Different motions of masses are on some way creating internal, spatially (or volumetrically) distributed electromagnetic entities, dipoles, moments and charges, and this way we get a chance to

describe planetary or mechanical motions on different ways; either dominantly mechanically or using electromagnetic conceptualization. ♣]

It can also be roughly (numerically) verified that quantum number n, which appears in $2\pi R = n\lambda_o$, (2.11.12) - (2.11.14), is taking the same order of magnitude as number of days in a year of relevant planet, indicating where we should search for the meaning of planetary, standing matter waves quantizing (see (2.11.14)-g, h).

Analog to vortex shedding phenomenology, known in fluid motions as “Karman Vortex Street”, we could say that certain kind of “Planetary Karman Street” is on some way following planets and astronomic size objects, like a helix oscillatory tail. As shown in the chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i such vortex shedding is in agreement with matter waves quantization $\lambda = H/p, \tilde{E} = Hf$, or on a similar way equivalent to (2.11.12), (2.11.13) and (2.11.14). ***For instance, the frequency of vortex shedding is directly proportional to relevant fluid or particle velocity, meaning that in (2.11.14) we may have*** $f_o = f_{on} = const \cdot v = C(n) \cdot v$***, what is explaining that*** v_0 ***for the specific planetary system could be constant, as follows,***

$$f_o = const \cdot v = C(n) \cdot v = C(n) \cdot v_n \Rightarrow$$

$$\left\{ \begin{array}{l} R_n = n^2 \frac{GM}{v_0^2} = \frac{GM}{v_n^2} = n\frac{\lambda_{on}}{2\pi} = \frac{n^2 H^2}{4\pi^2 G m^2 M},\ v_n = \frac{v_0}{n} = 2u = \frac{v_e}{\sqrt{2}} = \sqrt{\frac{GM}{R_n}} = \frac{2\pi}{nH} GmM \\ R_i v_i^2 = R_j v_j^2 \Rightarrow R_n v_n^2 = R_n \frac{v_0^2}{n^2} = GM \Leftrightarrow R_n v_n^2 = Rv^2 = constant, \end{array} \right\} \Rightarrow \quad (2.11.14)\text{-a}$$

$$v_0 = \frac{nv_e}{\sqrt{2}} = nv_n = \frac{2\pi}{H} GMm = \frac{GMm}{\hbar_{gr.}} = 2un = n\sqrt{\frac{GM}{R_n}},\ n = 1, 2, 3...$$

$$f_o = n\frac{f_m}{2} = n\frac{1}{2T} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = \frac{n}{4\pi R_n}\sqrt{\frac{GM}{R_n}} = \left(\frac{n}{4\pi R_n}\right) v_n = \left(\frac{v_0^2}{4\pi n GM}\right) v_n = \left(\frac{1}{4\pi R_n}\right) v_0 = C(n) \cdot v_n.$$

[♣ COMMENTS & FREE-THINKING CORNER:

There is increasing evidence from astronomical measurements (spectral, Doppler redshifts of the electromagnetic radiation passing about galactic centers; - [37] Tifft, [40] Nottale, [41] Rubćić, A., & J. Rubćić, [43] M. Pitkänen) that v_0 *(appearing in (2.11.14) and (2.11.14)-a) is a characteristic velocity parameter applicable for many planetary systems (like other universal or fundamental constants known in Physics) having the value* $v_0 = n\sqrt{\frac{GM}{R_n}} = 144.7 \pm 0.7\ km/s \cdot$ ***Nottale is showing in [40] that such fundamental velocity constant is observed from the planetary scales to the extragalactic scales (see the diagram below). His theoretical predictions, based on “Scale Relativity Theory” agree very well with the observed values of the actual planetary orbital parameters, including those of the asteroid belts.*** *Mentioned observations are supporting the legitimacy of all other quantized parameters (from (2.11.14)) like orbital radius, phase, and group velocity* R_n, u_n, v_n*, etc.*

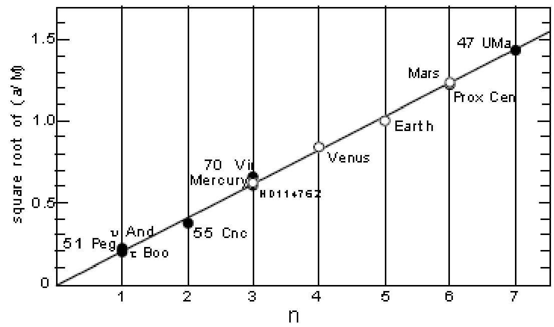


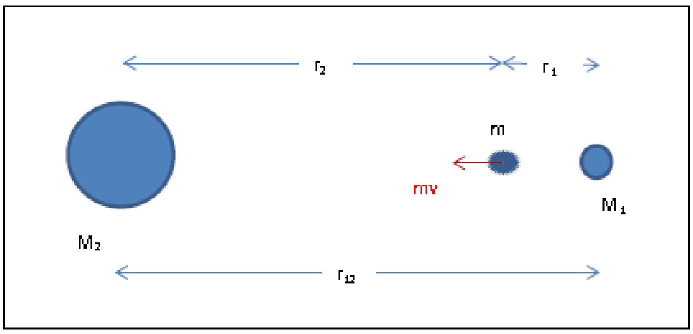
Figure 2. Square root of the ratio (a/M) (a in AU and M in $M_{\odot}$), where a is the semi-major axis of a planet and M the mass of its star, versus n integer, for inner solar system planets and extra-solar companions. The line corresponds to the law $\sqrt{a/GM} = n/w_0$, with $w_0 = 144$ km/s.

This picture is taken from [40]; -the "Letter to the Editor; Scale-relativity and quantization of extra-solar planetary systems. L. Nottale", DAEC, CNRS et Université Paris VII, Observatoire de Paris-Meudon, F-92195 Meudon Cedex, France, Received 9 May 1996 / Accepted 5 September 1996.

Some gravitational, or inertial standing waves field structure, which also has specific electromagnetic nature (whatever that means), really exist around and behind planets in stationary orbital motions. See more of supporting remarks later, related to measured red shifts, around equations (2.11.15) to (2.11.19), and [63], Arbab I. Arbab, [67], Johan Hansson and [68], Charles W. Lucas, Jr. … All of that is giving chances that some "Planetary Karman Street" should exist behind every planet in orbital motion. Since certain electromagnetic nature is intrinsically incorporated (see [63]) into such planetary and gravitational formations (and quantization of planetary systems is the fact), the imprints and traces of "Planetary Karman Streets" should exist and be measurable as electromagnetic and Doppler-shifts spectral signatures (as Tifft measured), and very probably, as charged particles currents and plasma-related manifestations.

Let us observe two astronomic objects with masses M_1 and M_2. One of them $M_1 << M_2$ could be small planet or satellite, and the other M_2 could be a bigger planet or local sun, or we could have two independent and "self-standing" cosmic masses M_1 and M_2. The distance between M_1 and M_2 is $r_{12} = r_1 + r_2$, as presented in the following picture.

Let us now imagine that specific small mass $m <<< M_1$ is projected (like a gun bullet) from M_1 towards M_2. We could assume that initial mass, speed, and linear moment of the bullet-mass m are constant and known ($(r_1 = \text{minimum}) \Rightarrow m = m_0 = \text{const. } v = v_0 = \text{Const., } p = p_0 = m_0 v_0$). We could also consider that M_1 and M_2 are relatively stable and static masses.



There is also an attractive field of gravitation between all involved masses m, M_1 and M_2, making that bullet mass m will have an increasing speed and linear moment. This time, we will neglect the possible presence and influence of electromagnetic fields and forces. To be more general, we could imagine that all of the involved masses should have certain linear and angular moments (what will be correct in cases of planetary systems), but for analyzing this example, we will assume that masses M_1 and M_2

are sufficiently static (or approximately standstill) and stable. The objective here will be to find evolving effective mass, velocity, energy, and momentum of a small gun bullet m.

The first step in such analyzes is to apply energy and momentum conservation laws (this time neglecting possible involvement of angular moments and electromagnetic fields and forces),

$$\begin{aligned}&\vec{P}_{total} = \vec{p}_m + \vec{p}_1 + \vec{p}_2 = const.\\&\vec{p}_m = m\vec{v}, \vec{p}_1 = M_1\vec{v}_1, \vec{p}_2 = M_2\vec{v}_2 \Leftrightarrow (\vec{p}_m = \gamma m_0\vec{v}, \vec{p}_1 = \gamma_1 M_{10}\vec{v}_1, \vec{p}_2 = \gamma_2 M_{20}\vec{v}_2)\\&(\vec{p}_{m0} = m_0\vec{v}_0) \Rightarrow \vec{p}_{10} = M_{10}\vec{v}_{10}, \vec{p}_{20} = M_{20}\vec{v}_{20},\\&E_{total} = E_m + E_1 + E_2 = (m_0c^2 + E_{km}) + (M_{01}c^2 + E_{kM1}) + (M_{02}c^2 + E_{kM2}) = const.\\&(\vec{P}_{total}, \frac{E_{total}}{c}) = invariant. \Rightarrow \vec{P}^2_{total} - \frac{E^2_{total}}{c^2} = -\frac{E^2_0}{c^2},\ E_0 = m_0c^2 + M_{01}c^2 + M_{02}c^2,\\&E_{km} = (\gamma - 1)m_0c^2 \cong \frac{1}{2}mv^2, E_{kM1} = (\gamma_1 - 1)M_{01}c^2 \cong \frac{1}{2}M_1v_1^2, E_{kM2} = (\gamma_2 - 1)M_{02}c^2 \cong \frac{1}{2}M_2v_2^2\end{aligned} \qquad (2.11.14)\text{-a-1}$$

It is evident that bullet mass m, its velocity, and momentum will be dependent on speeds and moments states of M_1 and M_2. The small mass m is in the field of attractive gravitational forces of masses M_1 and M_2 (acting in mutually opposite directions and being distance-dependent, based on Newton law), meaning that all the involved masses will have evolving and mutually dependent moments.

It is also possible to present motional mass m as a matter-wave packet or photon, where we could start exploiting associated group and phase speed, wave energy, matter-wave wavelength, and matter-wave frequency. For instance, in cases of microparticles like electrons, protons, positrons, etc. it will be, $\tilde{E}_m = hf = E_{km} = (\gamma - 1)m_0c^2 \cong \frac{1}{2}mv^2,\ p = \gamma m_0 v$ *and* $v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{dp},\ u = \lambda f = \frac{\tilde{E}}{p}$. *We will find that useful (and analogical) matter-wave characteristics of the bullet mass m, or an equivalent photon (like wavelength and frequency) will also evolve, being distance, velocity, and all initial masses dependent (basically getting certain observer-dependent Doppler, red and blue frequency shifts). Here is a place to underline that Planck's constant* h *is applicable only for cases involving microparticles and photons. Typical examples where we can verify the existence of such analogical particle-wave parallelism situations are innovative analyzes of Compton, and photoelectric effects, including the continuous spectrum of x-rays (or photons), caused by impacts of electrons accelerated in an electrical field between two electrodes (see such analyzes in chapter 4.2). For other macroparticles, certain new, and analogical* H *constant will be more appropriate instead of Planck constant* h *(especially in cases when self-closed, matter-waves structures are being involved or created).*

The next step can be to imagine that the kinetic energy of a small bullet-mass m (as in analogical cases of elementary microparticles and photons) will be replaced or cinematically represented by an equivalent matter-wave or photon energy $\tilde{E}_m$ *(see equations under (4.2) and T.4.0 from the chapter 4.1),*

$$\begin{bmatrix}(0 \le 2u \le \sqrt{uv} \le v \le c, m = \gamma m_0) \Rightarrow E_{km} = \tilde{E}_m = hf = (m - m_0)c^2 = \frac{pv}{1 + 1/\gamma}, \gamma = 1/\sqrt{1 - \frac{v^2}{c^2}},\\(0 \le v \cong 2u << c) \Rightarrow E_{km} = \tilde{E}_m = hf \cong \frac{1}{2}mv^2 = \frac{1}{2}pv\end{bmatrix}, \qquad (2.11.14)\text{-a-2}$$

$$\begin{bmatrix}(v \cong u \cong c) \Rightarrow \left(\frac{\tilde{E}_m}{p} \cong \frac{d\tilde{E}_m}{dp}\right) \Leftrightarrow \frac{dp}{p} \cong \frac{d\tilde{E}_m}{\tilde{E}_m} \Rightarrow \tilde{E}_m = hf \cong \frac{E_0}{p_0}p \cong cp, \frac{E_0}{p_0} = \frac{E_a}{p_a} = \frac{E_b}{p_b} = ... = \frac{E_m}{p}\\(E_0 = m_0c^2, p_0 = m_0 c) = constants \Rightarrow v = \frac{d\tilde{E}_m}{dp} \cong \frac{E_0}{p_0} \cong c,\ u = \lambda f = \frac{\tilde{E}_m}{p} \cong c\end{bmatrix}.$$

From results in (2.11.14)-a-2 we see that relevant matter wave model (in cases of microparticles and photons) could have its initial particle-like part (like a non-zero rest mass, $(E_0, p_0) = constants$), and waving-tail part concerning its phase velocity. For macro masses or macroparticles, we should be able to construct similar mathematical modeling with new H constant (like already exercised for planetary or solar systems, in this chapter). Of course, this mathematically challenging situation could be much better elaborated, and we should not forget that until here we neglected angular moments and associated electromagnetic complexity.

Interactions between Photons and Gravitation

If we detect and analyze photons emitted from very distant sources (as stars or galaxies), we will be able to make any judgments about evolving photons' parameters related to gravitational influences of masses, which are on the way between photons source and photons receiver. Light waves coming to our astronomic observatories are carrying imprints, modulations, or signatures of planetary and galactic systems that are between a distant source of light and our observatory. Tifft, [37], performed number of spectral analyzes of light from remote sources, and found that in such cases we often have certain quantized or discrete frequency shifts (or "redshifts"), which are most probably gravitational (or electromagnetic) imprints of astronomic objects that are on the way of involved light waves. Of course, we should not exclude the possibility to detect "blue shifts" within the same framework. Here, we are directly faced with the very probable existence of quantized gravitational and electromagnetic structures, and macro-cosmological matter waves (as exercised in this book), interfering, and interacting with photons propagating around. If Gravitational intensity, related velocities, and orbital diameters of planetary systems (along with light ways propagation) are naturally quantized, this will produce that received light waves from such distant sources will also be on certain similar way quantized (towards red or blue Doppler shifts).

*Effectively, here we assume that it should exist certain gravitational force-interaction between photons and big gravitational masses around. **Initial conditions**, relations, assumptions, and necessary mathematical relations (see chapter 4.1) applicable to a photon propagating from a very distant source (towards its observer or receiver) are:*

$$\left[\tilde{E} = hf = \tilde{m}c^2, V = V(r) = V_0 = \mathbf{0}\right] \Leftrightarrow \left[u = v = u_0 = v_0 = c\right] \Rightarrow \tilde{E}_0 = hf_0 = \tilde{m}_0 c^2 \,(=)\, \text{initial photon},$$

$$V = V(r) = \frac{GM}{r} = \text{gravitational potential} \,(=) \left[\left(\frac{m}{s}\right)^2, \text{velocity squared}\right], G = \text{gravitational const.}$$

$$\tilde{E}_0 = \text{photon energy on its distant source where } V = V_0 = \text{Lim}\left(V\right)_{r\to\infty} = \mathbf{0},\ V = V(r) = \frac{GM}{r},$$

Index "0" means that certain value relates to its distant radiation source, where $V = V_0 = \mathbf{0}$,

Photon source or emitter (in this case) has negligible mass and zero gravitational intensity.

$$\tilde{E} = hf = \tilde{m}c^2 = \text{wave energy of a photon},$$

$$\tilde{m} = \text{mass-equivalent of a photon} = \tilde{E}/c^2,$$

$$d\tilde{E} = hdf = c^2 d\tilde{m} = vd\tilde{p} = Fdr = -V\tilde{m}\frac{dr}{r},$$

$$F = \text{Force acting on a photon} = \frac{d\tilde{p}}{dt} = -G\frac{M\tilde{m}}{r^2} = -V\frac{\tilde{m}}{r},$$

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{d\tilde{p}} = \text{group speed},$$

$$u = \lambda f = \frac{\tilde{E}}{\tilde{p}} = \text{phase speed},$$

$$\mathbf{0} \le \mathbf{2}u \le \sqrt{uv} \le v \le c,$$

$$\lambda = \frac{h}{\tilde{p}} = \text{photon wavelength}$$

$$F = \frac{d\tilde{E}}{dr} = h\frac{df}{dr} = c^2\frac{d\tilde{m}}{dr} = v\frac{d\tilde{p}}{dr} = -G\frac{M\tilde{m}}{r^2}$$

*Possible accurate approximations (regarding propagating photon) that can be quickly developed from just stated **initial conditions** are:*

$$\tilde{m} = \tilde{m}_0 \cdot e^{\frac{V}{c^2}} \cong \tilde{m}_0 \cdot (1 + \frac{V}{c^2}) \cong \frac{\tilde{m}_0}{\sqrt{1 - \frac{2V}{c^2}}} \geq \tilde{m}_0, \; V << c^2$$

$$f = f_0 \cdot e^{-\frac{V}{c^2}} \cong f_0 \cdot (1 - \frac{V}{c^2}) \cong \frac{f_0}{\sqrt{1 + \frac{2V}{c^2}}} \leq f_0,$$

$$\lambda = \frac{h}{\tilde{p}} = \frac{\lambda_0}{u_0} \frac{\tilde{E}}{\tilde{p}} \cdot e^{\frac{V}{c^2}} = \lambda_0 \frac{\tilde{E}}{c\tilde{p}} \cdot e^{\frac{V}{c^2}} \cong \lambda_0 \frac{\tilde{E}}{c\tilde{p}} \cdot (1 + \frac{V}{c^2}) \cong \frac{\lambda_0 \frac{\tilde{E}}{c\tilde{p}}}{\sqrt{1 - \frac{2V}{c^2}}} \geq \lambda_0 \frac{\tilde{E}}{c\tilde{p}},$$

$$\lambda_0 = \frac{h}{\tilde{p}_0} = \frac{c}{f} \cdot e^{-\frac{V}{c^2}} \cong \frac{c}{f} \cdot (1 - \frac{V}{c^2}) \cong \frac{\frac{c}{f}}{\sqrt{1 + \frac{2V}{c^2}}} \leq \frac{c}{f}$$

$$\tilde{E} = hf_0 \cdot e^{-\frac{V}{c^2}} = \tilde{E}_0 \cdot e^{-\frac{V}{c^2}} = hf = \tilde{m}c^2 \cong \tilde{E}_0 \cdot (1 - \frac{V}{c^2}) \cong \frac{\tilde{E}_0}{\sqrt{1 + \frac{2V}{c^2}}} \geq \tilde{E}_0 = hf_0,$$

$$u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = \lambda f_0 \cdot e^{-\frac{V}{c^2}} = \frac{\lambda}{\lambda_0} u_0 \cdot e^{-\frac{V}{c^2}} = c\frac{\lambda}{\lambda_0} \cdot e^{-\frac{V}{c^2}} \cong c\frac{\lambda}{\lambda_0} \cdot (1 - \frac{V}{c^2}) \cong \frac{c\frac{\lambda}{\lambda_0}}{\sqrt{1 + \frac{2V}{c^2}}} \geq c\frac{\lambda}{\lambda_0},$$

$$u_0 = \lambda_0 f_0 = \frac{\tilde{E}_0}{\tilde{p}_0} = c = v_0,$$

$$\left\{\begin{array}{l} \tilde{p} = \tilde{p}_0 + \frac{\tilde{m}_0 V}{v_0} = \tilde{p}_0 + \frac{\tilde{m}_0 V}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} + \frac{\tilde{m}_0 V}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} + \frac{hf_0 V}{c^3} = \tilde{m}c \cdot e^{\frac{V}{c^2}} + \tilde{m}_0 c\frac{V}{c^2} \\ \tilde{p} = \tilde{m}_0 c \cdot e^{2\frac{V}{c^2}} + \tilde{m}_0 c\frac{V}{c^2} = \tilde{m}_0 c \cdot \left(e^{2\frac{V}{c^2}} + \frac{V}{c^2}\right) \cong \tilde{p}_0 \cdot \left(1 + 3\frac{V}{c^2}\right) \cong \frac{\tilde{p}_0}{1 + \frac{V}{c^2}} \cong \tilde{p}_0\sqrt{1 - 2\frac{V}{c^2}}, \\ \tilde{p}_0 = \tilde{m}_0 c = \frac{hf_0}{c} = \frac{hf}{c} \cdot e^{\frac{V}{c^2}} \cong \frac{hf}{c}\left(1 + 3\frac{V}{c^2}\right) = \tilde{m}c \cdot \left(1 + 3\frac{V}{c^2}\right) \cong \frac{\tilde{m}c}{1 + \frac{V}{c^2}} \cong \tilde{m}c\sqrt{1 - 2\frac{V}{c^2}}, \\ \tilde{m} = \tilde{m}_0 \cdot e^{\frac{V}{c^2}} \cong \tilde{m}_0 \cdot (1 + \frac{V}{c^2}) \cong \frac{\tilde{m}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cong \frac{\tilde{m}_0}{1 + 3\frac{V}{c^2}}. \end{array}\right\} \Rightarrow$$

$$\Rightarrow \tilde{p} = \frac{h}{\lambda} \cong \tilde{m}c \cdot \left(1 + 3\frac{V}{c^2}\right)^2 = \tilde{p}_0 \cdot \left(1 + 3\frac{V}{c^2}\right) \cong \tilde{m}c \cdot \left(1 - 2\frac{V}{c^2}\right) \cong \tilde{p}_0 \cdot (1 + \frac{V}{c^2}) \cdot \left(1 + 3\frac{V}{c^2}\right)^2 \cong$$

$$\cong \frac{\tilde{p}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cdot \left(1 + 3\frac{V}{c^2}\right)^2 \cong \tilde{m}c\sqrt{1 - 2\frac{V}{c^2}} \cong \frac{\tilde{m}c}{1 + \frac{V}{c^2}} \cong \frac{\tilde{p}_0}{\sqrt{1 - 2\frac{V}{c^2}}} \cong \tilde{m}c.$$

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{d\tilde{p}} = \frac{\tilde{E}_0}{c^2}\frac{dV}{d\tilde{p}}\cdot e^{-\frac{V}{c^2}} = \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot e^{-\frac{V}{c^2}} \cong \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot(\mathbf{1}-\frac{V}{c^2}),$$

$$v_0 = \tilde{m}_0\left(\frac{dV}{d\tilde{p}}\right)_{V\to 0} \Rightarrow v_0 d\tilde{p} = \tilde{m}_0 dV \Rightarrow v_0(\tilde{p}-\tilde{p}_0) = \tilde{m}_0 V \Rightarrow v_0 = \frac{\tilde{m}_0 V}{\tilde{p}-\tilde{p}_0} = c,$$

$$v = \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot e^{-\frac{V}{c^2}} = c\cdot e^{-\frac{V}{c^2}} \cong \tilde{m}_0\frac{dV}{d\tilde{p}}\cdot(\mathbf{1}-\frac{V}{c^2}) = c(\mathbf{1}-\frac{V}{c^2}) \cong \frac{c}{\sqrt{\mathbf{1}+\mathbf{2}\frac{V}{c^2}}},$$

*Several of here obtained results are similar or identical to published results about **Gravitational Redshift and Gravitational Effects on Light Propagation** concerning General relativity Theory. What is significant here is that we confirm that big cosmic, gravitational systems are an integral part of universal **P**article-**W**ave **D**uality **C**oncept (also confirmable on different ways). ....♣]*

As the direct support to quantizing concepts, assumptions and results found in (2.11.14), and (2.11.14)-a, ($v_n^2 R_n = GM = \underline{const.}$), we can verify (based on very long time known, and many times published measurements) that product between Semi-major Axis of planet revolution **R**, and square of a mean Semi-major orbital (or group) velocity **v**, for each of members of certain stable planetary, or satellite system, is a constant number, $v^2R = v_n^2 R_n = \text{constant}$ (see the table T.2.3.3, below).

T.2.3.3

Planets	m, Planet mass [kg]	R, Semi-major Axis of revolution around the Sun (mean radius of rotation) [m]	Mean Semi-major Orbital (or group) velocity, v (=) [m/s]	v^2R [m³/s²]	m/v [kg s/m]	nH = C₁(m/v)
Mercury	3.3022.E+23	**5.80E+10**	**4.7828E+04**	**1.3256E+20**	6.9043238270469.E+18	5.7506047172213.E+39
Venus	4.8690.E+24	**1.08E+11**	**3.5017E+04**	**1.3256E+20**	1.3904674872205.E+20	1.1581190412636.E+41
Earth	5.9742.E+24	**1.50E+11**	**2.9771E+04**	**1.3257E+20**	2.0067179469954.E+20	1.6715195460789.E+41
Mars	6.4191.E+23	**2.28E+11**	**2.4121E+04**	**1.3256E+20**	2.6612080759504.E+19	2.2165176631950.E+40
Jupiter	1.8988.E+27	**7.78E+11**	**1.3052E+04**	**1.3256E+20**	1.4547961998161.E+23	1.2116983645068.E+44
Saturn	5.6850.E+26	**1.43E+12**	**9.6383E+03**	**1.3256E+20**	5.8983430687984.E+22	4.9127243050721.E+43
Uranus	8.6625.E+25	**2.87E+12**	**6.7951E+03**	**1.3253E+20**	1.2748156760018.E+22	1.0615524611320.E+43
Neptune	1.0278.E+26	**4.50E+12**	**5.4276E+03**	**1.3256E+20**	1.8936546539907.E+22	1.5772231515805.E+43
Pluto	1.5000.E+22	**5.91E+12**	**4.7365E+03**	**1.3257E+20**	3.1668953868891.E+18	2.6379031231061.E+39
AVERAGE	2.9650.E+26	**1.78087E+12**	**1.9599E+04**	**1.3256E+20**	2.6280461756991.E+22	2.1888788009444.E+43

The relation $v^2R = v_n^2 R_n = \text{constant}$ is originally discovered only mathematically, by finding strong numerical relationships between involved factors (based on measured data), but here is theoretically and conceptually founded (as the consequence of standing matter waves formations), getting much higher significance and generalized weight. It can be additionally confirmed on many similar examples and looks as generally applicable to all stable solar and satellite systems, and it is substantially related to the satellite escape velocity (2.11.11). There is a big chance that such relation could already be considered as the law of contemporary Physics if it were properly understood and respected before the establishment of Kepler and Newton laws (at least, it is not inferior compared to Kepler and Newton laws). It is possible to show that Newton gravitational force between two masses (one of them, $\mathbf{m} = \mathbf{m_1}$, rotating on a stable circular orbit around bigger mass $\mathbf{M} = \mathbf{m_2}$) can be postulated, invented, or analogically formulated from mentioned relation $v^2R = v_n^2R_n = \text{constant}$. Based on analogies from the first chapter, summarized in T.1.8., and Coulomb-Newton force laws, as given in T.2.2, T.2.2-2, (2.1), (2.2), (2.4-5.1), (2.11.14-5), we can see that what should be analog to Coulomb electrostatic force between two electric charges q_1 and q_2 is similar relationship between two linear moments p_1 and p_2,

$$\left\{\begin{array}{l} F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{R^2}\left(\begin{array}{c}\leftrightarrow\\ \text{analogical}\end{array}\right)F_g = \frac{1}{4\pi g_p}\cdot\frac{p_1p_2}{R^2} = \frac{1}{4\pi g_p}\cdot\frac{m_1v_1\cdot m_2v_2}{R^2} \\ q_{1,2}\leftrightarrow p_{1,2} = m_{1,2}v_{1,2}\end{array}\right\}\Rightarrow$$

$$F_g = \frac{v_1v_2}{4\pi g_p}\cdot\frac{m_1m_2}{R^2} = G\frac{m_1m_2}{R^2} = G\frac{mM}{R^2},\ \left(G = \frac{v_1v_2}{4\pi g_p} = \text{const.}\leftrightarrow\frac{1}{4\pi\varepsilon_0}\right)\Leftrightarrow$$

$$\Leftrightarrow\left\{\begin{array}{l} v^2R = \text{constant}\Leftrightarrow v^2 = \frac{\text{constant}}{R}\Leftrightarrow mv^2 = m\cdot\frac{\text{constant}}{R}\Rightarrow \\ F_c = \frac{mv^2}{R} = m\cdot\frac{\text{constant}}{R^2}\left(\begin{array}{c}\leftrightarrow\\ \text{analogical}\end{array}\right)F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{R^2}\Rightarrow \\ F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{R^2}\left(\begin{array}{c}\leftrightarrow\\ \text{analogical}\end{array}\right)F_g = \frac{1}{4\pi g_p}\cdot\frac{p_1p_2}{R^2} = G\frac{mM}{R^2} = F_c(R) \\ G = \frac{v_1v_2}{4\pi g_p} = \frac{v_c^2}{4\pi g_p} = \frac{c^2}{4\pi g} = \text{const.}\Rightarrow v_1v_2 = v_c^2 = \text{ constant}\end{array}\right\}. \qquad (2.11.14)\text{-b}$$

The way expression for Newton gravitational force developed here (in (2.11.14)-b) is implicating that anyway, in any case of the gravitational attraction of masses, we should have elements of stationary rotations (on stable and inertial-motion orbits with standing matter wave structures) to be able to apply such force law. If in some instances we do not see such elements of stable orbiting (between attracting masses), this is most probably because we are the part of specific complex or more substantial scale rotation (concerning a larger or more general reference frame). In other words, the gravitational force is not, and should not only be a central force $F_c(R)$ between static masses. Dynamic parameters like linear and/or angular moments should also be on some essential way involved here, in the broader reference frame (as it is very well supported in [36]).

We still do not have solid arguments to be undoubtedly and generally sure in $(v^2R = \text{constant}) = GM$, but this looks very convincing, based on astronomic observations (see T.2.3.3), and as such is intuitively (and analogically) invented or postulated by Newton (see [61], Mark McCutcheon).

From published literature is known that Gravitational force and Coulomb force are two familiar examples with $F_c(R)$ being proportional to $1/R^2$. Both, neutral and electrically charged masses in such force field with negative $F_c(R)$ (presenting an attractive force) obey Kepler's laws of planetary motion.

-Also, the force-field of a spatial harmonic oscillator is central, with $F_c(R)$ proportional to **R,** and negative.

-Bertrand's theorem formulates more significant support to Kepler-Newton Laws, when saying, $F_c(R) = -k/R^2$, and $F(R) = -kR$, are the only possible central force fields with stable and closed orbits.

We could explore other consequences of "$v^2R = v_n^2R_n = \text{constant}$" concerning orbital quantization, to estimate numerical value of gravitational Planck constant H as follows (see T.2.3.3, (2.11.13) and (2.11.14)),

$$\begin{gathered} v^2R = v_n^2R_n = \text{constant} \Leftrightarrow 2\pi R v^2 = 2\pi \cdot \text{constant} = C_1 = 2\pi GM \Rightarrow \\ \Rightarrow \begin{Bmatrix} 2\pi R = n\lambda_0 \\ \lambda_0 = H/p \\ n\lambda_0 v^2 = C_1 \end{Bmatrix} \Rightarrow \begin{Bmatrix} H = \frac{2\pi}{n}\cdot(v^2R)\cdot(\frac{m}{v}) = \frac{C_1}{n}(\frac{m}{v}) = \frac{2\pi m\sqrt{GMR}}{n} \\ nH = C_1(\frac{m}{v}) = 2\pi GM(\frac{m}{v}) = 2\pi m\sqrt{GMR}, n = \text{Integer} \end{Bmatrix}, \\ \Rightarrow \begin{Bmatrix} H = \frac{C_1}{n}(\frac{m}{v}) = \text{Const.} \\ \tilde{E} = E_k = \frac{mv^2}{2} = Hf_0 \end{Bmatrix} \Rightarrow \frac{m}{nv} = \text{const.} \Leftrightarrow \frac{m_i}{m_j} = \frac{n_i}{n_j}\cdot\frac{v_i}{v_j}, \frac{m_iv_i^2}{m_jv_j^2} = \frac{n_iv_i^3}{n_jv_j^3} = \frac{f_{0i}}{f_{0j}}. \end{gathered} \tag{2.11.14)-c}$$

From data in T.2.3.3 it is possible to find that an "average gravitational Planck constant" **H** applicable in case of our planetary system could be somewhere inside the following estimations:

$$\begin{Bmatrix} H = \frac{2\pi m\sqrt{GMR}}{n}, n = \text{Integer}, \\ \langle H \rangle \in \left(\frac{9.33\cdot E{+}42}{n}, \frac{2.1888788009444\cdot E{+}43}{n}\right) \end{Bmatrix} \Rightarrow \left(2\pi m\sqrt{GMR} \leq 2.1888788009444\cdot E{+}43 \Rightarrow R \leq \frac{0.121362271}{m^2GM}\cdot E{+}86\right). \tag{2.11.14)-d}$$

It is almost obvious from (2.11.14)-a,b,c,d that in $v^2R = v_n^2R_n = \text{constant} = GM$ something could be wrong with **G**, since estimated **H** is too far from being the universal constant, meaning that Newton law of gravitation should have certain weak sides. What remains is that we could creatively exploit relations:

$$\left(\frac{m}{nv} = \text{const.}\right) \Leftrightarrow \left(\frac{m_i}{m_j} = \frac{n_i}{n_j}\cdot\frac{v_i}{v_j}\right) \Leftrightarrow (H = \text{Const.}) \Rightarrow \left(\frac{m_iv_i^2}{m_jv_j^2} = \frac{n_iv_i^3}{n_jv_j^3} = \frac{f_{0i}}{f_{0j}}\right), \tag{2.11.14)-d1}$$

and draw new conclusions and consequences regarding relations between gravitation, **H**-constant, and planetary masses.

We can also exploit $H = \frac{2\pi mvR}{n} = \frac{2\pi m_iv_iR_i}{n_i}$ from (2.11.13), and calculate the set of possible (or approximate) values for the gravitational Planck-like constant H, as,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

T.2.3.3-a

Planets	m, Planet Mass,	R, Semi-major Axis of revolution around the Sun (mean radius of rotation)	v, Mean Semi-major Orbital (or group) velocity	n, number of days in one planetary year	$\frac{m_i v_i R_i}{n_i}$	H (=) Gravitational Planck constant
	[kg]	[m]	v (=) [m/s]	[1]	[H/2p] (=) [kg m^2/s]	[H] (=) [kg m^2/s]
Mercury	3.3022E+23	5.8000E+10	4.7828E+04	1.5000E+00	6.1069E+38	3.8351E+39
Venus	4.8690E+24	1.0800E+11	3.5017E+04	9.2500E-01	1.9907E+40	1.2501E+41
Earth	5.9742E+24	1.5000E+11	2.9771E+04	3.6600E+02	7.2893E+37	4.5777E+38
Mars	6.4191E+23	2.2800E+11	2.4121E+04	6.7000E+02	5.2690E+36	3.3089E+37
Jupiter	1.8988E+27	7.7800E+11	1.3052E+04	1.0500E+04	1.8363E+39	1.1532E+40
Saturn	5.6850E+26	1.4300E+12	9.6383E+03	2.4200E+04	3.2378E+38	2.0333E+39
Uranus	8.6625E+25	2.8700E+12	6.7951E+03	4.2700E+04	3.9563E+37	2.4846E+38
Neptune	1.0278E+26	4.5000E+12	5.4276E+03	8.9700E+04	2.7986E+37	1.7575E+38
Pluto	1.5000E+22	5.9100E+12	4.7365E+03			
Average	*2.9650E+26*	*1.7813E+12*	*1.9599E+04*	*2.1017E+04*	*2.8529E+39*	*1.7916E+40*

Obviously that n, as the principal quantum number (from T.2.3.3-a, temporarily specified as the number of days during one planetary year), because of existence of moons and satellites, should be combined or composed from different quantum numbers in relation to planets' orbital and spinning moments, what analogically also exist in the N. Bohr atom model (see T.2.8. N. Bohr hydrogen atom and planetary system analogies).

Another aspect of $v^2R = v_n^2R_n = \text{constant}$ is that this is also the way to determine the planetary (or satellite) escape speed, based on (2.11.11). For instance, for planets of our Solar system, we have $v^2R \cong 1.3256E+20$, (see T.2.3.3), and similar escape speed for every particular planet can be found as $v_{en} \cong 1.41 \cdot v_n$ (meaning that specific planet can be removed from its stable orbit if its orbital velocity will be suddenly increased 1.41 times):

$$\left(E_e = \frac{1}{2}mv_e^2 = \frac{GmM}{R}\right) \Leftrightarrow v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2R = 1.3256E+20 = 2GM \Rightarrow$$
$$\Rightarrow v_{en} = \sqrt{\frac{2GM}{R_n}} = \sqrt{\frac{1.3256E+20}{R_n}} = v_n\sqrt{2} = \frac{v_0}{n}\sqrt{2} = u_n 2\sqrt{2},$$
$$\frac{v_{en}}{v_n} = \frac{1}{v_n}\sqrt{\frac{2GM}{R_n}} = \frac{1}{v_n}\sqrt{\frac{1.3256E+20}{R_n}} = \frac{v_0}{nv_n}\sqrt{2} = \frac{u_n 2\sqrt{2}}{v_n} = \sqrt{2},\ v_{en} \cong 1.41 \cdot v_n \ . \quad (2.11.14)\text{-d2}$$

[♣ COMMENTS & FREE-THINKING CORNER:

***If matter-wave Earth's wavelength is related only to the period of one Earth day**, such Earth wavelength will be:*

$$\lambda_{1-day} = vT_{(1-day)} = \frac{H}{p} = \frac{H}{mv} = 2.56617E+9\ [m] \Rightarrow H = mv^2T_{(1-day)} \cong 4.56E+38,$$

$T_{(1\text{-}day)}$ = 8.62E+04 [s],

v = 29771 [m/s] (=) Earth Mean Semi-major Orbital (or group) velocity,

m = 5.9742 E+24 [kg] (=) Earth mass,

R = 1.4959826 x 10^{11} m (=) Earth Mean, or Semi-major Orbital radius.

Since one Earth year has 365.26 Earth days, it should also have 365.26 single wavelengths, and we can easily verify that, $365.26 \cdot \lambda_{1-day} \cong 2\pi R \cong 9.399E+11\ [m]$, meaning that n = 365.26, and associated, relevant macrocosmic Planck constant (in this case) could be $H \cong 4.56E+38$. If we

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

check on a similar way, the same situation regarding other planets in our solar system, we will get indicative and encouraging, not extremely large divergences, or dispersion of results related to orbital perimeters, relevant matter wavelengths, and this way calculated macrocosmic Planck constant. Naturally, after certain process-refinement, we could generalize such concepts and results (as elaborated in the Appendix, under Chapter 10). See later T.2.3.3-1, where the same idea is applied to all planets of our solar system. ♣]

It is almost evident that the more complete picture about quantization in stable planetary systems should also take care about additional angular and spinning quantum numbers (of involved planets, moons, asteroids, meteorites, and satellites). In [64], Marçal de Oliveira Neto, we can find (effectively based on (2.11.14)) very precisely and convincingly presented, fitted, and calculated, quantizing results, applied to our planetary system.

The integer "**n**", or some kind of quantum number (appearing in all expressions from (2.11.12) until (2.11.14)-a,b,c,d) could be an arbitrarily high number, and this is presenting a difficulty regarding understanding and using precise and meaningful quantization of planetary systems. It will be much easier if we could say, for instance considering our Solar system, that Mercury is the first and closest planet orbiting our Sun, and it should be characterized as the orbit number 1 (one). The same way, Venus is on the second planetary orbit around the Sun, and it should be characterized as the orbit number 2. Earth and Mars will have orbits 3 and 4, etc. Such orbital numbers can be considered as principal, significant quantum numbers. Here, we will use symbol "i" for mentioned numbers (**i = 1, 2, 3, …**). Obviously, such orbital numbers are not at all equal to integer "**n**" appearing in (2.11.12) - (2.11.14)-d. We can try to present the integer **n** in relation to orbital quantum number i, as $n = iN$, where **N** is a certain constant number (also integer), being the same (and valid) for all planets of certain planetary system. ***We will consider that every planetary system has its own characteristic number N, and its own, unique constant H, while*** $v_n^2 R_n = \text{constant}$ ***.*** Now, relations developed under (2.11.14) will evolve to,

$$\Rightarrow \left[\begin{array}{c} \left\{ \begin{array}{l} \text{For two planets on orbits 1 and 2:} \\ H = \text{const.} \Rightarrow \frac{R_1}{R_2} = \frac{n_1^2}{n_2^2}\frac{m_2^2}{m_1^2} = \left(\frac{n_1 m_2}{n_2 m_1}\right)^2, \frac{T_1}{T_2} = \left(\frac{n_1 m_2}{n_2 m_1}\right)^3 \end{array} \right\} \\ ------------------- \\ \left\{ \begin{array}{l} \text{For the same planet passing between two orbits:} \\ (m_1 \cong m_2), H = \text{const.} \Rightarrow \frac{R_1}{R_2} = \left(\frac{n_1}{n_2}\right)^2, \frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3 \end{array} \right\} \end{array} \right] \Rightarrow$$

$$\Rightarrow \left[\left\{ \begin{array}{l} \text{For two planets on orbits i and j: } H = \text{const.} \Rightarrow \\ \frac{R_i}{R_j} = \left(\frac{n_i m_j}{n_j m_i}\right)^2 = \left(\frac{i \cdot N \cdot m_j}{j \cdot N \cdot m_i}\right)^2 = \left(\frac{i \cdot m_j}{j \cdot m_i}\right)^2, \\ \frac{T_i}{T_j} = \left(\frac{n_i m_j}{n_j m_i}\right)^3 = \left(\frac{i \cdot N \cdot m_j}{j \cdot N \cdot m_i}\right)^3 = \left(\frac{i \cdot m_j}{j \cdot m_i}\right)^3 \\ i, j \in [1,2,3,...],\ n_i = i \cdot N, n_j = j \cdot N, n_i = \frac{i}{i-1} n_{i-1} \end{array} \right\}, \left\{ \begin{array}{l} \text{For the same planet passing between two orbits:} \\ (m_i \cong m_j), H = \text{const.} \Rightarrow \\ \frac{R_i}{R_j} = \left(\frac{n_i}{n_j}\right)^2 = \left(\frac{i}{j}\right)^2, \frac{T_i}{T_j} = \left(\frac{n_1}{n_2}\right)^3 = \left(\frac{i}{j}\right)^3 \\ i, j \in [1,2,3,...],\ n_i = i \cdot N, n_j = j \cdot N, n_i = \frac{i}{i-1} n_{i-1} \end{array} \right\} \right]. \quad \text{(2.11.14)-e}$$

Relations (2.11.14)-e are also identical, analogical, or equivalent to links found in [64], Marçal de Oliveira Neto. Here is the chance to exploit and extend results presented in [64]. As we can see in [64], mentioned relations are creatively and "ad hock" fitted (by Marçal de Oliveira Neto), to satisfy relevant astronomic observations, to the following forms,

$$\left(\frac{R_i}{R_j}=\left(\frac{i}{j}\right)^2, \frac{T_i}{T_j}=\left(\frac{i}{j}\right)^3\right) \Leftrightarrow \left(R_i = R_j\cdot\left(\frac{i}{j}\right)^2, T_i = T_j\cdot\left(\frac{i}{j}\right)^3\right) \Rightarrow$$

$$\Rightarrow \text{(as in [64], Marçal de Oliveira Neto)} \Rightarrow \tag{2.11.14)-f}$$

$$\Rightarrow R_{i+1} = R_i\cdot\left(\frac{i+1}{i}\right)^2, T_{i+1} = T_i\cdot\left(\frac{i+1}{i}\right)^3 \Rightarrow$$

$$\Rightarrow R_{i,j} = R_{j,j}\cdot\left(\frac{i^2+j^2}{j^2+j^2}\right), T_{i,j} = T_{j,j}\cdot\left(\frac{i^2+j^2}{j^2+j^2}\right)^{3/2}$$

$$i, j \in [1,2,3,...],\ n_i = i\cdot N, n_j = j\cdot N, n_1 = 1\cdot N, n_2 = 2\cdot N, n_3 = 3\cdot N, ...$$

Citation from [64]:

*"The application of Eq. (2.11.14)-f can be illustrated by considering the mean planetary radii and orbital periods, in astronomical units, within the solar system. The semi-major axis, which is the same as the mean planetary distance to the Sun, is expressed in terms of the mean distance from the Earth to the Sun (designated as one astronomic unit or AU). In astronomical units, the orbital period of the Earth (one year) defines the unit of time. Mercury is the closest planet to the Sun (hence its orbit corresponds to n = 1), and its observed mean radius (R_1) and orbital period (T_1) are 0.387 AU and 0.241 years, respectively. Based on these values, the other mean planetary radii and orbital periods can be calculated from Eq. (2.11.14)-f by setting **content i** in the range between 1 and 13 (Table 1).*

Regarding planetary orbits where i = 2 and i = 3, the sum of the squares of i , and a second ad-hoc integer j, taking values from 0 to i, must be considered using expressions analogous to those of Eq. (2.11.14)-f. This procedure may be illustrated by reference to the series of orbits with i = 2. Starting from the state i = 2, j = 2, which is associated with the orbit of Mars, the state i = 2, j = 1 corresponds to Earth's orbit and i = 2, j = 0 corresponds to that of Venus. The respective mean planetary radii and orbital periods are given by the following calculations:

$$R_{2,1} = R_{2,2}\left(\frac{2^2+1^2}{2^2+2^2}\right) = 0.9677,\ R_{2,0} = R_{2,2}\left(\frac{2^2+0^2}{2^2+2^2}\right) = 0.7742,$$

$$T_{2,1} = T_{2,2}\left(\frac{5}{8}\right)^{3/2} = 0.953,\ T_{2,0} = T_{2,2}\left(\frac{4}{8}\right)^{3/2} = 0.682.$$

in which the values R2,2 = 1.548 and T2,2 = 1.928 correspond to the parameters for Mars. It is worth noting that the splitting of states associated with i equal to 2 or 3 is analogous to the spectral series derived from modern atomic theory. Moreover, this splitting of states occurs in the region corresponding to the distance of Jupiter from the Sun (i = 4) and may be linked with the unusual characteristic of this planet that, together with its many rings and satellites, almost constitutes a mini-solar system in its own right.

Regarding the terrestrial and the gas giant planets, as well as the dwarf planets Pluto, Makemake and Eris, the theoretical mean radii and orbital periods predicted by this model are in reasonable agreement with the observed values (Johnston's Archive; Space and Astronomy, 2010).

Furthermore, there is a significant agreement between the theoretical and observed results (Table 1) regarding the positions of some asteroids found in the solar system. The model predicts, for example, the orbits of the inner (i = 3; j = 0; HIL) and outer (i = 3; j = 1; HOL) limits of the Hungaria asteroids at mean observed radii between 1.780 and 2.000 AU. The asteroids Vesta (i = 3; j = 2) and Camilla (i = 3; j = 3) are correctly located in the inner (2.361 AU) and outer (3.477 AU) rings of the main asteroid belt, which lies between the orbits of Mars and Jupiter and contains approximately 2000 objects orbiting the Sun. The asteroid Chiron, a Centaur object, is positioned between the orbits of Saturn and Uranus at an observed mean radius of 13.698 AU. Moreover, the calculated mean radius of 24.768 AU is associated with the recently discovered asteroids, also Centaur bodies, named Nessus and Hylonome, whose mean distances are 24.617 and 25.031 AU from the Sun, respectively. Additionally, this model predicts the orbit of trans-Neptunian objects in the region of space where the Plutoids are found, including that of the asteroid 1999 DE9 (i = 12) at an observed mean radius of 55.455 AU, a value that accords very well with the theoretical result of 55.728 AU (Johnston's Archive; Space and Astronomy, 2010)."

Table 1 from [64]

Orbital position		Planet / Asteroid	R (=) Mean radius (AU),		T (=) Orbital period (years)	
i	j		Calculated	Observed	Calculated	Observed
1	1	Mercury	0.387	0.387	0.241	0.241
2	0	Venus	0.774	0.723	0.682	0.615
2	1	Earth	0.968	1.000	0.953	1.000
2	2	Mars	1.548	1.523	1.928	1.881
3	0	HIL	1.742	1.780	2.300	2.375
3	1	HOL	1.935	2.000	2.694	2.828
3	2	Vesta	2.515	2.361	3.994	3.630
3	3	Camilla	3.483	3.478	6.507	6.487
4		Jupiter	6.192	5.203	15.424	11.864
5		Saturn	9.675	9.537	30.125	29.433
6		Chiron	13.932	13.698	52.056	50.760
7		Uranus	18.963	19.191	82.663	83.530
8		Nessus	24.768	24.617	123.392	122.420
9		Neptune	31.347	30.069	175.689	163.786
10		Pluto	38.700	39.808	241.000	251.160
11		Makemake	46.827	45.346	320.771	309.880
12		1999 DE9	55.728	55.455	416.448	412.960
13		Eris	65.403	68.049	529.477	558.070

At least, with (2.11.14)-e and (2.11.14)-f, and results from [64], Table 1, we have specific, indicative and intuitive, justification of the relation that connects newly introduced, orbital quantum number $i = 1, 2, 3, ...$, and initial quantum number **n**, as $n_i = i \cdot N$, where **n** is figuring in T.2.3.3, and in equations (2.11.12) - (2.11.14)-a,b,c,d. Here, **N** is specific constant (and integer), valid for all planets of its solar system. This way, we also have an insight regarding a deeper understanding of quantization in stable planetary and asteroid systems, with the overwhelming analogy with Bohr planetary atom model (see also [63], Arbab I. Arbab, and [67] Johan Hansson). Since now we have orbital quantum numbers (taken from [64]) associated to planets of our Solar system, $i \in [1, 10], n_i = i \cdot N$, it will be possible to make additional numerical speculations about gravitational, macrocosmic, Planck-like constant **H**, by introducing specific values of $i \in [1, 10]$ into last, right column of T.2.3.3.

Since constant N can be almost arbitrary big integer (and **n** is like number of days in a year for relevant planet; -see T.2.3.3-1), we can also conclude from (2.11.14), that for specific stable planetary system, exist the common, sufficiently high-frequency time-train (or frequency carrier), which is universally applicable for time-flow counting, for all planets belonging to the same solar system, as follows.

$$\left\{\begin{array}{l} f_o = n\frac{f_m}{2} = n\frac{1}{2T} = n\frac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on}\,,\ T_o = \frac{1}{f_o} = \frac{2T}{n} = \frac{4\pi R}{nv} \\ H = \frac{2\pi m\sqrt{GMR}}{n},\quad n = i\cdot N,\ i = 1,2,3...\ N = \text{integer} \end{array}\right\} \Rightarrow$$

$$f_o = \frac{1}{T_o} = i\cdot N\cdot\frac{f_m}{2} = i\cdot N\cdot\frac{1}{2T} = i\cdot N\cdot\frac{\sqrt{GM}}{4\pi R^{3/2}} = f_{on}\ (=),\ f_m = \frac{2f_0}{n},$$

$$T_o = \frac{1}{f_o} = \frac{2T}{i\cdot N} = \frac{4\pi R}{i\cdot N\cdot v}\,,\ T = T_m = T_y = \frac{1}{f_m} = \frac{2\pi R}{v} = \frac{nT_0}{2} = \frac{n}{2f_o} = \frac{(nH)^3}{4\pi^2G^2M^2m^3}(=)\,\text{one year}\,,$$

$$\tilde{E} = Hf_o = \tilde{E}_n = n\frac{Hf_m}{2} = E_k = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = G\frac{mM}{2R}. \qquad (2.11.14)\text{-g}$$

This looks like establishing a precise mathematical way for understanding planetary systems synchronization, discretization, gearing, and digitalization, enriching our understanding of stability and integration of planets inside their solar systems (still without any need for using probability and statistics as in modern quantum theory).

As an example, let us creatively apply (2.11.14)-a,b,c, (2.11.14)-g, and standing matter waves concept from Chapter 10, to (all planets of) our Solar system, as found in T.2.3.3-1, which is created (as the spreadsheet, MS Excel table, with 22 columns), using known astronomic data and observations (mostly from very recent NASA publications). We will just start from the obvious fact that the number of days in a year N_{dy} (for every planet), multiplied with one-day time-duration $T_{(1-day)} = \frac{T_y}{N_{dy}} = \frac{T_y}{n} = \frac{T}{n} = \frac{T_0}{2} = \frac{1}{2f_0}$ is equal to the whole year time-duration $T_y = T = T_m$ (of a relevant planet). Consequently, number of days in a year $N_{dy} = \frac{T_y}{T_{(1-day)}} = n = i\cdot N$ (for every planet), multiplied with one planetary matter-wave wavelength $\lambda_{(1-day)}$ (found for every planet as, $\lambda = \lambda_{(\text{one day})} = \frac{H}{p} = V\cdot T_{(\text{one day})}$) is equal to the orbital circumference of a relevant planet $2\pi R = n\lambda = N_{dy}\lambda$, calculated using the following relations,

$$\left\{\begin{array}{l} 2\pi r = n\lambda_0 = N_{dy}\lambda_{(1-day)}\,,\lambda_0 = H/p,\ v_n^2R_n = v^2R\ = GM = \underline{const.},\ v \cong 2u = 2\lambda_{(1-day)}f_{(1-day)} \\ H = \frac{2\pi mvR}{n} = \frac{2\pi m\sqrt{GMR}}{n} = \frac{2\pi mGM}{nv} = \lambda_0 p = \lambda_{(1-day)}mv,\ f_0 = f_{(1-day)} = \frac{1}{2T_{(1-day)}} \\ \lambda = \lambda_0 = \lambda_{(1-day)} = vT_{(1-day)} = 2uT_{(1-day)} = \frac{H}{p} = \frac{H}{mv} \Rightarrow H = mv^2T_{(1-day)}\,, N_{dy} = \frac{T_y}{T_{(1-day)}} = n \\ T_{(1-day)} = \frac{\lambda_{(1-day)}}{v} = \frac{\lambda_{(1-day)}}{2u}, u = \lambda_{(1-day)}f_{(1-day)} = \frac{\lambda_{(1-day)}}{2T_{(1-day)}}, v_0 = nv_n = nv = N_{dy}v = n\sqrt{\frac{GM}{R_n}} \end{array}\right\} \Rightarrow$$

$$\Rightarrow H = \frac{2\pi mvR}{n} = \frac{2\pi m\sqrt{GMR}}{N_{dy}} = \frac{2\pi mGM}{nv} = mv^2T_{(1-day)} = \frac{mv^2T_y}{N_{dy}} = \qquad (2.11.14)\text{-h}$$

$$= 2E_kT_{(1-day)} = \frac{E_k}{f_{(1-day)}} = \lambda p = \lambda mv.$$

Relations (2.11.14)-h are almost in the full agreement with the helically spinning matter waves concept (associated with moving masses), as elaborated mainly in chapter 4.1. The table T.2.3.3-1 is created using relations from (2.11.14)-h, by applying relevant planetary data (taken from NASA publications). There, we can see that specific data initially known only from astronomic measurements (and from other observations) are getting completely verifiable, calculable, and confirmable from the here-established conceptual framework of planetary standing matter-waves (see Appendix, Chapter 10, where standing matter-waves concept is additionally summarized). We can also see that only planet Saturn is still an exception (for an order of magnitude) related to predictions from (2.11.14)-h and results from the table T.2.3.3-1 (see columns 19 and 22). If we would like to make Saturn behaving as other planets in relation to T.2.3.3-1, its mean orbital radius should be about 10 times larger, compared to what we presently know regarding Saturn (but the final answer related to the planet Saturn will be more complicated than such simple solution).

T.2.3.3-1 Gravitational Planck Constant & Standing Matter Waves of our Solar System (see columns from 1 to 22)

1	2	3	4	5	6	7	8	9	10
Planets	**Mean Radius of rotation / Semi-major orbital radius around the Sun, R (=) [m]**	**Orbit Circumference = $2\pi R$ (=) [m]**	**Planet mass, m (=) [kg]**	**Sun mass, M (=) kg**	**G**	**π**	**Average, Orbital (group) velocity, v (=) [m/s]**	**Average, Orbital phase velocity, $u = \lambda_{(1\text{-day})} f_{(1\text{-day})} = v/2$ (=) [m/s]**	**Linear, orbital moment, p = mv (=) [kg·m /s]**
Mercury	5.79E+10	3.60E+11	3.30E+23	1.99E+30	6.67E-11	3.14	4.74E+04	2.37E+04	1.56E+28
Venus	1.08E+11	6.80E+11	4.87E+24	1.99E+30	6.67E-11	3.14	3.50E+04	1.75E+04	1.70E+29
Earth	1.50E+11	9.40E+11	5.97E+24	1.99E+30	6.67E-11	3.14	2.98E+04	1.49E+04	1.78E+29
Mars	2.28E+11	1.43E+12	6.42E+23	1.99E+30	6.67E-11	3.14	2.41E+04	1.20E+04	1.54E+28
Jupiter	7.78E+11	4.89E+12	1.90E+27	1.99E+30	6.67E-11	3.14	1.31E+04	6.53E+03	2.48E+31
Saturn	1.43E+12	8.96E+12	5.68E+26	1.99E+30	6.67E-11	3.14	9.64E+04	4.82E+04	5.48E+31
Uranus	2.87E+12	1.80E+13	8.68E+25	1.99E+30	6.67E-11	3.14	6.80E+03	3.40E+03	5.90E+29
	4.50E+12	2.83E+13	1.02E+26	1.99E+30	6.67E-11	3.14	5.43E+03	2.72E+03	5.57E+29
AVERAGE	**1.24E+12**	**7.80E+12**	**3.33E+26**	**1.99E+30**	**6.67E-11**	**3.14**	**3.22E+04**	**1.61E+04**	**1.01E+31**

11	12	13	14	15	16	17
Sidereal Orbit period/Period of full rotation around the Sun/Length of Year, T_y (=) [Earth days]	**Sidereal Orbit period/Period of full rotation around the Sun/Length of Year, T_y (=) [s]**	**Sidereal Rotation Period / One self-revolution period / Length of Day (=) Rotation period, $T_{(1\text{-day})}$ (=) [Earth days]**	**Sidereal Rotation Period / One self-revolution period / Length of Day (=) Rotation period, $T_{(1\text{-day})}$ (=) [s]**	**$\lambda_{(1\text{-day})} = vT_{(1\text{-day})} = \lambda = H/mv$ (=) [m]**	**$H = mv^2 T_{(1\text{-day})} = 2 E_k \cdot T_{(1\text{-day})} = E_k / f_{(1\text{-day})}$ (=) [kg·m²/s]**	**Sidereal Number of days in a year (=) [Columns 11,12,13, 14] (=) $T_y / T_{(1\text{-day})} = N_{dy}$**
87.969	7.58E+06	58.646	5.05E+06	2.39E+11	3.74E+39	1.50E+00
224.700	1.94E+07	243.018	2.09E+07	7.33E+11	1.25E+41	9.25E-01
365.260	3.15E+07	0.997	8.59E+04	2.56E+09	4.55E+38	3.66E+02
686.980	5.92E+07	1.026	8.84E+04	2.13E+09	3.29E+37	6.70E+02
4332.820	3.73E+08	0.414	3.56E+04	4.65E+08	1.15E+40	1.05E+04
10755.700	9.27E+08	0.444	3.83E+04	3.69E+09	2.02E+41	2.42E+04
30687.150	2.64E+09	0.718	6.19E+04	4.21E+08	2.48E+38	4.27E+04
60190.030	5.19E+09	0.671	5.78E+04	3.14E+08	1.75E+38	8.97E+04
1.34E+04	**1.16E+09**	**3.82E+01**	**3.29E+06**	**1.40E+11**	**2.02E+40**	**2.10E+04**

18	19	20	21	22
$C = N_{dy}\lambda_{(1\text{-}day)} = N_{dy}\, vT_{(1\text{-}day)}$ (=) (Column-17) * (Column-15) (=) Orbit Circumference (=) [m]	**Orbit Circumference/Orbit Circumference (=) (Column-3)/(Column-18) (=) 2πR/C**	**Orbital, Kinetic energy, $E_k = mv^2/2 = H\cdot f_{(1\text{-}day)}$ (=) [kg·m²/s²]**	$H = \frac{2\pi m\sqrt{GMr}}{N_{dy}}$ **(=) [kg·m²/s]**	**H-constant (Column-21)/ (H-constant Column-16) (=) H/H**
3.59E+11	1.002825	3.70E+32	3.83E+39	1.024627
6.78E+11	1.002785	2.98E+33	1.25E+41	1.002949
9.37E+11	1.002751	2.65E+33	4.57E+38	1.003012
1.43E+12	1.002693	1.86E+32	3.31E+37	1.007272
4.87E+12	1.002842	1.62E+35	1.16E+40	1.003635
8.93E+13	0.100281	2.64E+36	2.03E+39	0.010042
1.80E+13	1.002928	2.01E+33	2.49E+38	1.003490
2.82E+13	1.002624	1.51E+33	1.75E+38	1.002312
7.78E+12	**1.002779**	**2.45E+34**	**2.02E+40**	**1.006757**

In the table T.2.3.3-1, column 17, and earlier in T.2.3.3-a, we can find that calculated number of days in a year, $n = N_{dy}$ (in relation to H constant) is not an integer (as under ideal and mathematically preferable conditions should be), since here we are operating with mean or average values of related orbital parameters (and still neglecting involved spin characteristics). This is also linked to the reference platform from where our astronomic measurements are valid, and to the fact that solar or planetary systems are dynamically stable, space and time-evolving motions.

It is evident that in T.2.3.3-1, we are getting significant results (see Columns 19 and 22) by implicitly because all planets of our solar system (including the Sun) have orbital and spinning moments. In column 19, we find that values of planetary orbits-circumferences, calculated on two different ways (compared to known astronomic measurements, as in column 3, and to standing macro matter-waves concept, as in column 18) are producing almost identical values. In column 22, we can also find that macrocosmic **H** constant values, calculated on two different ways (one based on known astronomic data, and the other based on standing macro matter-waves concept), are mutually almost identical. This way, we are building the legitimacy of standing macro matter-waves concept in relation to planetary systems. Such a situation should be much better exploited to enrich our understanding of periodicity and standing matter waves quantization within stable solar systems. For instance, planet Earth's Moon is (helically) rotating around planet Earth, and its mean orbit circumference is 2.41E+09 m. In the column 15 of T.2.3.3-1, we can find that "1-day" Earth wavelength, $\lambda_{(1\text{-}day)} = vT_{(1\text{-}day)} = \lambda = H/mv$ is 2.56E+09 m, not very much different from 2.41E+09 m, meaning that Earth's Moon should be on some way captured or channeled by helical macro-matter-wave field associated to planet Earth's orbital motion. ***Since calculated H constants (columns 16 and 21) are still too much mutually different, this is indicating that additional, new quantizing, or new standing waves parameters should be considered, meaning that presented modeling is still oversimplified. The most promising strategy here would be to consider specific electromagnetic background involved in the structuring of planetary systems.***

[♣ COMMENTS & FREE-THINKING CORNER:

We can see that solar or planetary systems are respecting (or fully complying to) standing waves, spatial arrangements. This is on some more complicated way also valid for galaxies. Standing waves in question are radial and angular or circular formations of waves, meaning that for every solar system,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

all its planets and local sun are properly participating, being part of well-integrated and mutually synchronized structure of spatial, macrocosmic standing waves. Temporarily, we could say that mentioned standing waves are waves of gravitational field, but, the most probable nature of such waves is within an electromagnetic phenomenology, naturally coupled with acoustical or mechanical oscillations.

The nature of all standing waves is that in relevant nodal spots or zones, there are only attractive, agglomerating forces, acting towards nodal spots. Such effects of attraction can easily be demonstrated (and measured) when experimenting with ultrasonic, half-wavelength resonators (that are producing standing waves), and in cases of acoustic or ultrasonic levitation effects realized within standing acoustic waves in air, or in other fluids (see more in [150] and [151]). In such cases, we always see that masses (or particles) are agglomerating and achieving stable, standstill positions within nodal spots of standing waves. Such analogy should be also valid for planets in orbital motions. The hypothetical assumption here is that every planet or mass presents on some way agglomerating spatial-temporal and standing-waves nodal-formation (when it is defendable to use such conceptualization). Acoustic standing waves levitation effects should be extendable, or naturally coupled and synchronized to similar, intrinsically associated, standing electromagnetic waves (since masses are composed of electromagnetically polarizable atoms). See similar elaborations in [99] from Konstantin Meyl.

Existence of standing waves is requesting to have certain (external) source of vibrations. It should exist, in a surrounding cosmic background (of our universe), something what is producing resonant oscillations, and driving such complex 3-dimensional or multidimensional, resonating universe. For macro systems, resonant frequencies could be extremely low (even below 1 Hz). Of course, we know that standing cosmic matter-waves exist concerning solar systems, since we can verify this mathematically and by comparison with known astronomic measurements (as already elaborated in this chapter).

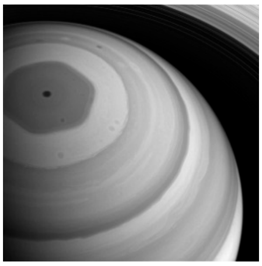
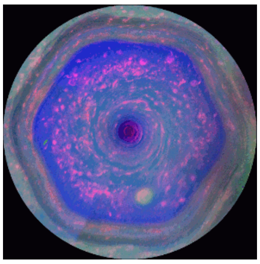
Every time when we have standing waves, with matter or masses fitting into such formations (like in cases of planetary systems), we also have associated electromagnetic dipoles (or multipoles) polarizations, organized within the same structure of standing waves, since masses are composed of atoms, and atoms internally have electric charges, with spinning and magnetic properties. Electric charges inside electrically (and magnetically) neutral atoms can be polarized creating spatially oriented electric dipoles (and multipoles) because of effects of accelerated motions, since electrons have almost 2000 times smaller mass compared to protons. Internal spinning within atoms (of electrons, protons, neutrons...) is creating number of small magnets, and such magnets (or magnetic moments) will be aligned or organized within the same structure of externally structured, macroscopic standing waves. At the same time, we will have macrocosmic or macroscopic 3D formations of *gravitation and big masses related* standing matter-waves, and similar (coincidently time-space synchronized) standing waves structure with electric and magnetic dipoles. It is obvious that for creating standing waves we also need to have certain material medium or fluid.

Let us now hypothetically assume that an ideal vacuum is anyway filled with certain fine fluidic medium (having some small particles), which is behaving as an ideal gas, and we will call such medium an ether. Since mentioned ether anyway has measurable electromagnetic constants (magnetic and electric permeability or susceptibility constants), that means that such ether should present certain material medium, and it can carry electromagnetic oscillations, fields, and forces (since it has some of electromagnetic properties, and it can be electromagnetically polarized). Of course, ether is a weak carrier-medium for magnetic and electric fields and waves. Since photons and electromagnetic waves are propagating in fluids, open space, and in an ideal vacuum, this means that such ether has certain exotic material nature (being even something what we are still not able to conceptualize, or something what belongs to multidimensional universes). Gravitation should also be a field structure acting within the spatial-temporal matrix of mentioned ether fluid. This is the reason why gravitation is not extraordinarily strong force when compared with electromagnetic forces known in our electromagnetic and engineering practices.

Professor, Dr. Jovan Djuric, [71], proved that every small, non-magnetic mass of different metals, inorganic and organic matter like a wood, is able to self-orient (or self-align) in the direction of local, dominant geomagnetic lines, meaning to align with an existing magnetic field of its local planet. Such effects are very weak, slow evolving and difficult to be noticed, but Prof. Djuric found a way to make successful experimental presentations of such effects, and presented relevant mathematical modeling, which can be additionally developed and optimized. That means all organic and inorganic mases (not only masses with ferromagnetic properties) are being slightly influenced (or physically oriented) by external magnetic field, effectively creating what we still conceptualize as forces of gravitation. In a macrocosmic environment (as certain planetary or solar system is), we will always find standing waves formations, and inside such standing matter-waves we should be able to detect presence of (synchronized) magnetic and electric fields, also structured as the same spatial, macrocosmic standing waves formations (being analog to acoustic levitation effects). Here we are closing the loop of explanations, knowing that nodal spots of all standing waves are manifesting as centers of attractive forces, and by assuming that such attractive forces are creating gravitation (see more in [99] from Konstantin Meyl). This way conceptualizing, we are approaching Nikola Tesla [97], and Rudjer Boskovic [6] ideas about dynamic gravitation and universal natural forces. N. Tesla speculated about certain (standing waves structured) streaming, or some fine matter flow of "radiant energy" between mutually attracting masses (associating on A. Einstein accelerated elevator), this way explaining effects of gravitation, what we could compare with an ether streaming around and between attracting masses. Rudjer Boskovic, also gave his contribution to gravitation, qualitatively describing the shape of certain universal natural force that should act between and inside all masses, or other corpuscular matter structures that externally manifests as Newton force of gravitation. Of course, this is noticeably short, and an oversimplified conceptual explanation related to understanding gravitation, and it can be combined with ether-flow effects between masses, since all standing waves also have circulation of mutually transforming (and oscillating) kinetic and potential energy amounts... ♣]

Saturn rings should present a perfect (observational) case of specific *standing-waves-like* mass density distribution, where we could search for "signatures" and effects of associated orbital standing waves (or gravity related matter waves). Since Saturn also has a strong magnetic field, and its rings are rotating (becoming somewhat electrically charged and polarized), we could conceptualize specific electromagnetic explanation of the structure of Saturn rings, analogical to N. Bohr model. We could also speculate that involved gravitational nature and attracting force effects (about Saturn and its rings) are direct consequences of a primarily electromagnetic phenomenology, since isolated and static magnetic and electric field components cannot exist as mutually separated in dynamic (motional) situations. This way we will imaginatively enter the space of Nikola Tesla Dynamic Gravity speculations.

Citation from https://solarsystem.nasa.gov/news/531/saturns-famous-hexagon-may-tower-above-the-clouds/ "A new long-term study using data from NASA's Cassini spacecraft has revealed a surprising feature emerging at Saturn's northern pole as it nears summertime: warming, a high-altitude vortex with a hexagonal shape, akin to the famous hexagon seen deeper down in Saturn's clouds.
The finding, published Sept. 3 in Nature Communications, is intriguing because it suggests that the lower-altitude hexagon may influence what happens above and that it could be a towering structure hundreds of miles in height.


"The edges of this newly-found vortex appear to be hexagonal, precisely matching a famous and bizarre hexagonal cloud pattern we see deeper down in Saturn's atmosphere," said Leigh Fletcher of the University of Leicester, lead author of the new study.
Saturn's cloud levels host most of the planet's weather, including the pre-existing north polar hexagon. This feature was discovered by NASA's Voyager spacecraft in the 1980s and has been studied for decades; a long-lasting wave potentially tied to Saturn's rotation, it is a type of phenomenon also seen on Earth, as in the Polar Jet Stream.

For more on the new study, visit the European Space Agency's story here: http://sci.esa.int/cassini-huygens/60589-saturn-s-famous-hexagon-may-tower-above-the-clouds/"

[♣ Anyway, Newton-Kepler foundations of gravitation can be presently understood mostly as the best intuitive guess about planetary orbits fitting, based on observations, while several of structural and theoretical miss-concepts and autocorrecting steps are approximately and creatively implemented, producing still sufficiently useful mathematical model. This will have significant impact on our future and improved understanding of Gravitation, orbital motions, and micro-world modeling of motions within atoms. See the following citation from [127].

Non-Conservativeness of Natural Orbital Systems

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The Newtonian mechanic and contemporary physics model the non-circular orbital systems on all scales as essentially conservative, closed path zero-work systems and circumvent the obvious contradictions (rotor-free 'field' of 'force', in spite of its inverse proportionality to squared time-varying distance) by exploiting both energy and momentum conservation, along specific initial conditions, to be arriving at technically more or less satisfactory solutions, but leaving many of unexplained puzzles. In sharp difference to it, in recently developed thermo-gravitational oscillator approach movement of a body in planetary orbital systems is modeled in such a way that it results as consequence of two counteracting mechanisms represented by respective central forces, that is gravitational and anti-gravitational accelerations, in that the actual orbital trajectory comes out through direct application of the Least Action Principle taken as minimization of work (to be) done or, equivalently, a closed-path integral of increments (or time-rate of change) of kinetic energy. Based on the insights gained, a critique of the conventional methodology and practices reveals shortcomings that can be the cause of the numerous difficulties the modern physics has been facing: anomalies (as gravitational and Pioneer 10/11), three or more bodies problem, postulations in modern cosmology of dark matter and dark energy, the quite problematic foundation of quantum mechanics, etc. Furthermore, for their overcoming, indispensability of the Aether as an energy-substrate for all physical phenomena is gaining a very strong support, and based on recent developments in Aetherodynamics the Descartes' Vortex Physics may become largely reaffirmed in the near future.

1. Introduction

Following the Newton's fitting of elliptical planetary orbits to the single central force inversely proportional to the square of its distance to the Sun, all natural systems
- from atomic to galactic scales - have been treated as non-conservative (work over closed loop in the field of potential force equaling to zero). The exclusive reliance on gravitation as the only central force does not allow for the formally exact prediction of the planet's trajectories in accordance with the Kepler's First law [1], and furthermore orbit fitting to an elliptical shape is contingent on the initial conditions [2]. The basic shortcoming of Newton's theory of orbital motion is the presumed absence of the tangential acceleration component, quite contrary to well established observational results, which is deduced either from the 'naive' interpretation of the Kepler's Third law, which actually is related to the aver- age values of the orbital radius and elapsed time, or from the improper interpretation of Kepler's Second law as angular momentum, its presumed constancy implying only the circular motion.

For theoretical foundations and practical calculations, the factual time-dependence of the force (thus non-zero rotor field) is neglected, and one proceeds from the constancy of the sum of kinetic and potential energies, on one side, and the constancy of the angular momentum, on the other, although in actuality neither of the two is the case.

Only recently, within explorations of biological molecular systems, as well as in certain domains of particle physics, the need starts arising for looking at such systems as non-conservative, the so-called "open systems", which within the classical formalisms turn out to become the "non-integrable" orbital systems (inability to be reduced to "circular coordinates" by even applying the time-varying transformations of the coordinate systems). This has led to modifications and specializations of the formalisms of the classical axiomatic mechanics having been developed by Euler, Lagrange, Hamilton, Noether and others for essentially conservative systems to be applicable to the non-conservative ones. However, a critical analysis of the matters suggests that all the natural orbital systems are open, that is non-conservative (including the planetary, atomic and galactic ones), and that neither the energy nor the (angular) impulse is constant over the time, so that the very basic foundations turn out to be erroneous.

………………..

Another resurfacing of the work not intended for publication is Feynman's scrutinizing and attempting to over- come the noticed week point in Newton's geometrical fit- ting of elliptical orbits to the central force inversely proportional to the squared distance is the above first cited [1], where Feynman had attempted to correct the inconsistency of Newton's geometrical fitting of the elliptic path to the squared distance inverse central force. It is deplorable indeed, that Feynman did not persevere and was not able to apply his favorite Least Action Principle to that problem, instead of stepping into the further support the otherwise unsoundingly set-up quantum mechanics by calculation of the (notably non-zero!?) works on all possible paths of an electron and assigning their reciprocal values to the probabilities, and further going into quite controversial development of the "Quantum Gravity".

…………………

2. Critique of the conventional approach in solving the Kepler's/Newton's problems

When it comes to determining the intrinsic feature of an orbital system, that is whether is it conservative or non-conservative, by all means of prime importance is the topic of a system energy balancing — evaluation of difference between the de-facto performed work and the (knowingly) available applied energy (re)sources.

……………………….

If the former exceeds the latter, or if the traditionally conceived and established law of sum of kinetic and potential energy conservation does not 'hold', we must be missing the awareness of the true nature mechanisms and the availability of the unaccounted for 'environmental' effective energy input(s).

As the historically firstly considered, the Sun's planetary orbital system should indeed be the right one for these considerations, in particular that the established theory and its further developments have detrimentally affected all other physics' and in general science do- mains — form the atom- to galactic-levels, and from chemistry to biology. In direct relation to the orbital energy balancing stands the concept of energy conservation with the related work over a closed path being equaled to zero, as intrinsic feature of the so-called potential fields (the 'central' force vector field having form of gradient of a scalar potential field). …………..♣]

As an example, how to consider mutually coupled (and mutually interacting) orbital and spinning moments (of solar systems) as vectors, it is sufficiently illustrative to see familiar conceptualization in chapter 4.1, presented by the table "T.4.2.1, Analogies Between **n**-Body Coupled Inertial Motions in a Laboratory System". Such approach should result in the more precise numerical estimation of macrocosmic Planck constant **H**, since here, for every planet, we have different **H** constant because we are neglecting orbital and spinning moments as mutually coupled vectors (see (2.11.14)-h and T.2.3.3-1, columns 16 and 21). All orbital and spinning moments of specific solar system are so well mutually integrated and coupled, that effective, planetary moments should be established somewhat similar as in two-body problem, where we will create central and reduced moment of inertia, as well as center of inertia angular velocity, and relative angular and spinning speeds for every planet. This will be like reduced and center of mass terms in the two-body problem, but now using terms of rotational and spinning motions.

For instance, the individual solar or planetary system can be characterized by the following set of parameters:

$$\begin{aligned}
&m_i (=) \text{mass of certain planet, } i \in [1, 2, 3, ...] \\
&M_s (=) \text{mass of the local sun} \\
&\vec{v}_i (=) \text{ planet orbital velocity } (=) \ \vec{\omega}_i R_i \\
&\vec{v}_s (=) \text{sun orbital velocity (relative to local galaxy center)} \\
&\vec{\omega}_i (=) \text{planet angular velocity} \\
&R_i (=) \text{planet mean orbital radius} \\
&I_i (=) \text{planet moment of inertia } (\cong) \ m_i R_i^2 \\
&I_{sun} (=) \text{sun moment of inertia} \\
&\vec{L}_i (=) I_i \vec{\omega}_i \cong m_i R_i^2 \vec{\omega}_i = m_i R_i \vec{v}_i \ (=) \text{planet orbital moment} \\
&\vec{L}_{si} (=) \text{planet spin moment } (=) \ I_s \vec{\omega}_s \\
&\vec{L}_{sun} (=) \text{total angular moment of local sun } (=) \ I_{sun} \vec{\omega}_{sun}
\end{aligned}$$

Since solar systems are (sufficiently and longtime) stable, we can consider that some of the orbital and spin moments of all planets, and the local sun is conserved (or constant), and this way we will be able to determine the value of local macrocosmic Planck constant **H**, as,

$$\begin{aligned}
&\vec{L}_{total} = \vec{L}_{sun} + \sum_{(i)} (\vec{L}_i + \vec{L}_{si}) = I_c \vec{\omega}_c = \overrightarrow{const.} \Rightarrow \left|\vec{L}_{total}\right| = \frac{H}{2\pi} \Rightarrow \\
&\Rightarrow \left|\vec{\omega}_c\right| = \frac{\left|\vec{L}_{total}\right|}{I_c} = \frac{H}{2\pi I_c} = 2\pi f_c = \frac{2\pi}{T_c} \Leftrightarrow H = 4\pi^2 I_c f_c = \frac{4\pi^2 I_c}{T_c}.
\end{aligned} \tag{2.11.14)-i}$$

Apparently, in a larger picture, if we attempt to determine unique value of macrocosmic **H** constant, we should not neglect the contribution of all (involved) orbital and spin moments, as well as participation of associated, mutually coupled electromagnetic and other fields and forces within solar systems (as roughly conceptualized in (2.11.14)-i). If such **H** is a stable and constant value, we could speculate around "entanglement ideas" that all mutually coupled orbital and spin moments within the specific stable solar system are communicating with enormously high speed (or instantaneously).

With (2.11.14)-i we are effectively presenting (or replacing) a whole solar or planetary system with a single, central, spinning solar mass M_c, which also has its center of mass velocity $\vec{v}_c$ (relative to local galaxy center), as follows,

$$M_c = M_s + \sum_{(i)} m_i \ , \ \vec{v}_c = \frac{M_s \vec{v}_s + \sum_{(i)} m_i \vec{v}_i}{M_c}. \quad (2.11.14)\text{-j}$$

If we consider our local galaxy center as a new reference frame, our solar system M_c will make orbital motion around galaxy center (having orbital velocity $\vec{v}_c$), and planets will create progressive helical movements around mass M_c and direction of $\vec{v}_c$ (as in (2.11.14)-j). Orbital motions of planets about the local sun are elliptical or close to circular (in the reference system linked to the local sun), but this is because we neglect that complete solar system is also orbiting about its local galaxy center (observed from the reference system linked to a galaxy center in question). This way, we are again coming to clear helical motions concepts that are in any case associated with linear movements, like elaborated in chapter 4.1 (valid both for micro and macro physics world). The much simpler example for visualizing such helical motion is rotating (or orbiting) movement of the specific moon about its local planet (from the reference system linked to the local sun or local planetary system center of mass). Here is also the more in-depth background or nature of matter waves and particle-wave duality (and not at all in a probability or possibility of "*chances that something could happen*"). Modern micro particles accelerators and colliders are also generating streams of energy-momentum products that are respecting similar unity of linear and spinning motions, particle-waves duality, and helical motions framework.

*On specific planetary system, we can attempt to create and apply the most common, universally valid, space-time measuring referential frame linked to the local galaxy center, on the following way. Mass M_c from (2.11.14)-j will be placed in the center of the (newly created), orthogonal axes (X-Y) plane, and Z-axis will be perpendicular to such (X-Y) plane and collinear (or coaxial) with the center of mass velocity $\vec{v}_c$. Total spinning moment of mass M_c will have its value $\vec{L}_{total}$ relative to such (X, Y, Z) coordinate system (using similar mathematics as in (2.11.14)-i). At the same time, we can place another **(x, y, z)** reference system (linked to the center of a local solar system) where M_c is again in the center of orthogonal axes **(x-y)** plane, and perpendicular **z**-axis has the direction of $\vec{L}_{total}$. Of course, **(X-Y)** and **(x-y)** planes in general case are not overlapping, except having common (0,0) and (0, 0, 0) center, and $\vec{L}_{total}$ will be different in (X-Y-Z) and (x-y-z) coordinates. We will always be able to use coordinates rotation and make connections between orthogonal **(x, y, z)** and **(X, Y, Z)** reference frames. The proposed concept is effectively describing an equivalent (and big, remarkably high mechanical quality factor) gyroscope replacing the complete solar system and could be more complicated than here simplified example. Such equivalent gyroscope is spinning, making precession around different axes, and at the same time, creating a large-scale rotational motion or orbiting in the **(X, Y, Z)** coordinate system (linked to galaxy center). Such solar-system gyroscope has its center of mass velocity $\vec{v}_c$, its mass M_c, and its linear, orbital, and spinning moments, including electromagnetic moments and associated electromagnetic properties and charges.*

*Now, we can introduce the hybrid, four-dimensional, space-time coordinate system **(x, y, z, t)** where the time flow or time direction (or time axis) will be linked to the center of mass velocity $\vec{v}_c$ (from (2.11.14)-j), similar as in Minkowski space of Relativity theory, using the planetary coordinates basis $(x, y, z, Iv_c t)$. Here, **"I"** presents Hypercomplex imaginary unit ($I^2 = -1$), composed from three more elementary imaginary units, as introduced in chapter 4.0. This way, we will make the most natural space-time frame for describing and visualizing helical planetary motions in relation to local galaxy center (see the picture below, taken from [57], showing different aspects of an Analytic Signal of specific attenuated oscillatory process), or for visualizing helical motions of moons around certain planet.*

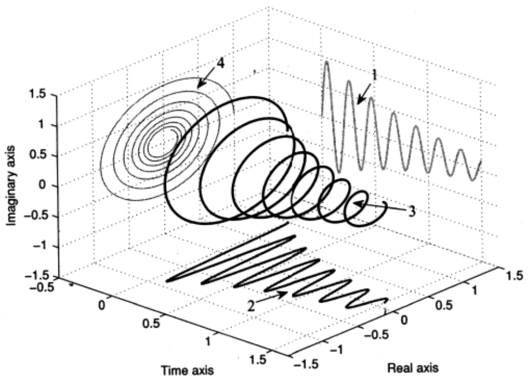
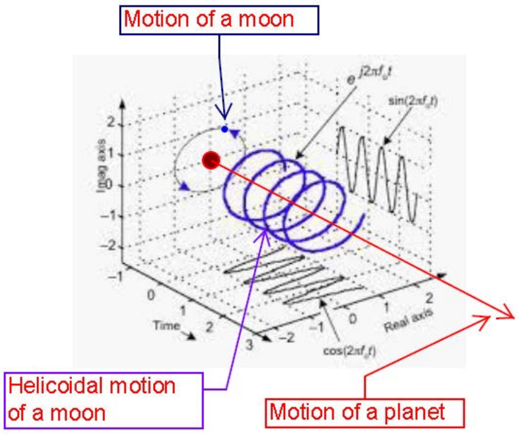


Figure 2.3 The HT projection (1), the real signal (2), the analytic signal (3), and the phasor in complex plain (4) (Feldman, ©2011 by Elsevier)



The ultimate evolution of such conceptualization is to arrive to unite and extended Minkowski, 4-vectors Hypercomplex space (x, y, z, Ict) *and Hypercomplex Analytic Signal functions (based on Hilbert transform, HT), to characterize different motions and associated energy-momentum relations (as elaborated in the chapters 4.0., 4.3. and 10.). If we intellectually, creatively, and philosophically extrapolate this situation, we will understand that similar conceptualization applies to the whole micro and macro universe.*

What is relatively new and original here (around equations (2.11.12) - (2.11.14)-a,b,c,...) is the introduction of a mutually related group and phase, orbital, planetary velocities (compatible to universally valid wave-motions concepts, including compatibility with Lorentz transformations and Relativity theory 4-vectors of energy-momentum relations). Such foundations implicitly introduce and defend the idea of a wave-group, or wave-packet about orbital, planetary matter-waves (very much analogical to Quantum theory wave packets and wave functions). This directly opens a way to analogical implementation of wave functions and wave equations concepts in modeling planetary systems (as Schrödinger and Heisenberg did in Quantum theory). Later, we could enrich and extend the same modeling by applying ideas like Bohr-Sommerfeld's quantization conditions (see Appendix, Chapter 10; - "PARTICLES AND SELF-CLOSED STANDING MATTER WAVES"). Creations of N. Bohr and his followers about planetary atom model (in the early steps of old quantum mechanics) are much more natural and applicable to here elaborated matter-waves of planetary systems, than to the atom model. It is also clear that certain direct and substantial analogy and the connection between such micro and macro world matter-waves conceptualization really exist (see [63], [64], [67] and [68]).

Using equations and relations from (2.11.14)-a,b,c..., we can predict and verify surprisingly exact quantization of celestial orbits in the specific solar system (see such extremely well-documented analyses in [38], [39] and [64]). This additionally confirms, or at least supports, the validity of planetary standing waves field structure (as elaborated in this book). Since planets' orbiting periods are exceptionally long, involved frequencies are small and almost meaningless. Of course, this is only a matter of our perception, scaling and measurement reference systems. Since the force of Gravitation is too weak, compared to electric and magnetic forces, and since Newton and Coulomb force laws are mathematically identical, this implicates that Gravitation could be indirect (still hidden behind our intellectual horizons) electromagnetic forces manifestation. The same idea has been suggested earlier in this book, based on small electric and magnetic dipoles (or multi-poles) polarizations, equivalent to relevant field charges displacements, resulting in certain unbalanced electromagnetic distributions (because masses of electrons and protons are enormously different, and always locally and globally rotating, spinning, and creating electromagnetic fields). This is giving chances to electromagnetic forces to work and show effects that are still classified as Gravitation (see (2.4-6) - (2.4-10)). Effects of planetary system quantization, which are becoming verifiable by proper conceptualization, observations, measurements, and calculations, that are mainly imitating quantization in electromagnetic fields, like in atoms, are indicatively showing that planetary systems should have much more of electromagnetic nature than presently considered (see [67]). The apparent consequence of quantization in planetary systems is that magnetic fields, electric currents, and charges should be much more involved in maintaining the dynamic and stationary structure and stability of planetary systems, and this should be valid for all matter in our universe on micro and macro scale.

What matters here is that the same mathematics concepts are applicable and working on micro and macro world scale (of course, not taking it literally, and without intellectual flexibility), both for planetary systems with masses and gravitational forces, and within Coulomb and other electromagnetic interactions between electrically charged microparticles (inside atoms). After establishing such grounds and analogical platforms, it will be imaginable to apply the framework of Schrödinger's, wave quantum mechanics, backward to orbiting planetary systems (see such mathematical modeling later; -equations (2.11.20 – (2.11.23)). There is the significant difference between Schrödinger's wave mechanics, and quantum mechanics of micro-world (as presently established, within Copenhagen interpretation), and its analogical application on planetary systems, as promoted here. We will find out that intrinsically probabilistic, and ontologically stochastic wave function, and associated mathematical concepts and practices, are not at all necessary, natural, and best choice to address all micro and macro-world matter waves (except when mathematical conditions for such modeling are met, and when we do not have a better opportunity).

Anyway, impressive, seducing and, in its own work-frames, operative mathematics of Quantum Theory will be still applicable and complementary tool to any other modeling of matter waves (at least for an "in-average" addressing of wave motions). It is incredible how the group of creative people (founders of Quantum Theory) invested such enormous efforts and wonderful imagination, creating an isomorphic, non-realistic, fantastic, and artificial mathematical structure (as a shadow or projection of real world around us), that is producing beneficial, and practically good results. Even more amazing is how they convinced or almost ideologically influenced a countless number of followers to admire such creation as the final one and the best made (and fire of such foundations is still burning). We too often find in literature, publications, and interviews (about microphysics), endlessly repeating statements, sounding like Buddhist mantra, that there is no one theory in humans' history such successful and useful as contemporary Quantum Theory (of course, including unmistakable Quantum Electrodynamics). Even doubts about it are forbidden (at least being unprofessional and unacceptable), and should be eradicated or punishable, as effectively (of course, not very explicitly, like here) interpreted by some of the ultraorthodox warriors of present days Quantum Theory. Others (some of them brilliant minds) who privately, most probably have doubts (in modern Quantum Theory), and see the same situation *somewhat differently, are anyway staying on the temporary stable grounds of not going explicitly against officially established mainstream.*

How good predictions in (2.11.14)-a,b,c… are, and could be, is related to the facts that (in contemporary mechanics) we are still approximating, neglecting or omitting certain elements of a real situation (concerning stable planetary systems) in the following aspects:

a) **We consider that all relevant planetary masses are tiny and homogenous, isotropic solid balls, compared to their common central mass or sun, and that the sun is in the state of rest (without rotation in their common center of mass). The reality is that any planetary system, including its sun, is effectively rotating around its common center of mass (and the center of the sun is not overlapping with the common center of mass). Also, some of the planets are close to solid balls, some of them still have a liquid core, and almost all of them are non-homogenous and have different forms of anisotropy and specific moments of inertia. Much more correct is to say that any Solar System moves through space, orbiting the center of its galaxy, and the planets trace out spirally (or helically) looking paths in space (what is noticeable, for instance, if we place the reference coordinate system in the center of galaxy).**

b) **On some way, we (implicitly) approximate that all planets and sun are rotating in the same (almost) flat plane, what is not the general case (not valid for our or any other solar system), meaning that all orbital and spinning moments of participants should be adequately considered (as vectors) when calculating macrocosmic H constant.**

c) **Also, we neglect interplanetary, orbital, and associated electromagnetic interactions, considering that allied forces and fields between every planet and its sun are dominant.**

d) **Moreover, we are still not enough precisely considering planets spinning concerning the total angular moment conservation.**

e) **We also do not consider too seriously the consequences of the orbital motion of solar systems concerning its local galactic center (what is effectively, in a larger scale, producing helical planetary motions). Since all solid bodies, like planets and satellites, are electromagnetically behaving like conductive metal masses, mentioned helical pats could be on a proper way considered as electric wires (or conductors), as Nikola Tesla proposed long time ago. Consequently, gravitational forces between such moving masses could be on some way presented as electromagnetic forces between wires with electric currents (since imaginative helical wires or planets-paths are anyway experiencing influences of surrounding electromagnetic fields and fluxes).**

f) **If quantization of planetary orbits (based on complex standing waves arrangements) has specific real meaning, as it looks to be, we should introduce additional (angular) quantum numbers (concerning orbital and spin moments).**

Let us go back to planetary systems, where our reference frame is linked to the common center of inertia (or center of mass) of such system (not considering motions relative to galactic center…). It will become evident that expression for planetary macro-wave (motional or kinetic) energy, $E_k = Hf_o$ from (2.11.13), is directly analog to Planck's wave-quantum energy of a photon, $\tilde{E} = hf$, as well as macro equivalent for a wavelength $\lambda = \frac{H}{p}$ is also analog to a micro-world de Broglie matter wavelength $\lambda = \frac{h}{p}$, where new "macro-world Planck-like constant" H is,

$$
\begin{aligned}
&H = H(m,R,n) = \frac{2\pi}{n} L = \frac{2\pi}{n}\frac{GMm}{v} = \frac{2\pi m\sqrt{GMR_n}}{n} = \frac{2\pi GMm}{v_0} = \\
&= \frac{2\pi R_n vm}{n} = \frac{\tilde{E}}{f_o} = \tilde{E}T = const. >> h,\ (n \in [1,2,3...], v << c,\ m << M) \qquad (2.11.15)\\
&\Rightarrow \left(\hbar = \frac{h}{2\pi}\right) \text{analog to} \left(\hbar_{gr.} = \frac{H}{2\pi} = \frac{L}{n} = \frac{m\sqrt{GMR_n}}{n} = \frac{GMm}{v_0}\right).
\end{aligned}
$$

The difference between Planck's constant h and analog constant of planetary macro waves H is that h is already known as universally valid constant (for world of atoms elementary particles and photons), and H could be different for every planetary and satellite system…. Of course, for specific planetary system, and for a sufficiently high integer $n = n_{max.}$ in (2.11.15), we should be able to find when H will be equal to h, as for instance,

$$
\begin{aligned}
&H = h = \frac{2\pi m\sqrt{GMR}}{n_{max.}} = \frac{2\pi GMm}{v_0} = 6.626\ 0693(11)\cdot 10^{-34}\, J\cdot s,\\
&n_{max.} = v_0\sqrt{\frac{R}{GM}} = 0.948252278\cdot 10^{34}\cdot m\sqrt{GMR} = 0.77443828\cdot 10^{29}\, m\sqrt{MR},\\
&(v_0 = 0.77443828\cdot 10^{29}\sqrt{G}\cdot m\cdot M),
\end{aligned}
$$

but such high quantum numbers are obviously unrealistic for characterizing macrocosmic objects and planetary systems.

In fact, for the specific planetary system (where each of planets concerning a common Sun could be approximated as a Binary System), we should be able to find H that will be the same constant for each planet. For instance (see (2.11.14) and (2.11.14-20)), if the ratio between any two of H constants (2.11.16), applied for planets (with circular orbits) from the same solar system, should be equal to one, then we can say that H is at least locally applicable constant, as for instance,

$$H = H_1 = H_2 \Rightarrow \frac{H_1}{H_2} = \frac{H(m_1, R_1, n_1)}{H(m_2, R_2, n_2)} = \frac{n_2}{n_1}\frac{m_1}{m_2}\sqrt{\frac{R_1}{R_2}} = \frac{n_2}{n_1}\frac{m_1}{m_2}\frac{v_2}{v_1} = 1,\ (n_1, n_2) \in [1, 2, 3...),$$

$$\Rightarrow \left\{n_2 \cdot m_1 \cdot \sqrt{R_1} = n_1 \cdot m_2 \cdot \sqrt{R_2} \Leftrightarrow n_2 \cdot m_1 \cdot v_2 = n_1 \cdot m_2 \cdot v_1\right\} \Rightarrow$$

$$\frac{L_2}{L_1} = \frac{m_2}{m_1}\frac{v_1}{v_2} = \frac{n_2}{n_1},\ \frac{L_2}{n_2}\frac{n_1}{L_1} = \frac{n_1}{n_2}\frac{m_2}{m_1}\frac{v_1}{v_2} = 1 \ \Rightarrow \tag{2.11.16}$$

$$\frac{H}{2\pi} = \frac{L_1}{n_1} = \frac{L_2}{n_2} \Rightarrow \frac{\left|\vec{L}_1 + \vec{L}_2\right|}{n_1 + n_2} = \frac{\left|\vec{L}_i\right|}{n_i} = \frac{GMm}{v_0} \Rightarrow \frac{H}{2\pi} = \frac{\sum_{(i)}\left|\vec{L}_i\right|}{\sum_{(i)} n_i} = \frac{GMm}{v_0}.$$

The limitations of **H** constant expressions related to (2.11.15) and (2.11.16) are that we still approximate all orbits with circles (which are in the same plane) and we do not consider any planetary “self-spinning” momentum. Another limitation involved here is that orbital and spin moments conservation is entirely valid only if we bring in consideration (as a vector) a resulting total moment (including spin moments) of all planets, moons, and satellites of a solar system in question (see [36], Anthony D. Osborne, & N. Vivian Pope).

Consequently, after implementing more elaborated analyses, we should be able to find more general and more precise expressions for **H** (see (2.11.14)-i,j). Present comments regarding planetary-world **H** constant are still indicative brainstorming directions serving to establish the grounds for defending the utility of such constants. Another exciting situation here is how to explain that micro-world **h**-constant (or Planck constant) is unique and universally valid for all atomic and subatomic entities (or we just consider it as universally valid). Do we have (in our Universe) a succession of **H**-constants, starting from certain big **H**-numbers (for galactic formations) which are gradually descending towards smaller numbers with unique and constant **h**-value at the opposite subatomic end, could be a question to answer? *One common fact is almost apparent:* ***h*** *or* ***H*** *constants are products of stable, periodical, circular, or closed domain (standing waves) motions where orbital moments are conserved (meaning constant).*

[♣ COMMENTS & FREE-THINKING CORNER (still in preparation and brainstorming phase)**:**

Here we are combining dynamics of orbital motions with certain kind of stable space packing expressed by the necessity of standing waves formation, which is in close relation to proper angular (and spin) moments conservation, also applicable to inclinations of planetary orbits. In addition, since certain wave-like spatial-periodicity and stable packing in periodical planetary motions exist, it could be presentable as integer multiple "n_α" of angular segments $\alpha = \frac{2\pi}{n_\alpha}$, capturing the angle of a full circle that is in average equal $n_\alpha \alpha = 2\pi = n\lambda_o / R,\ n_\alpha = 1, 2, 3...$ (of course, indicative for an idealized and oversimplified situation, just to give a direction of thinking about angular quantizing),

$$\begin{aligned} &L = n\frac{H}{2\pi} = \frac{n}{n_\alpha}\cdot\frac{H}{\alpha} = \frac{H}{\lambda_o}R = \frac{2\pi\sqrt{GM}}{n\lambda_o}\cdot mR^{3/2} = m\sqrt{GMR}, \\ &\alpha = \frac{2\pi}{n_\alpha} = \frac{n}{n_\alpha}\cdot\frac{H}{L} = \frac{n}{n_\alpha}\cdot\frac{\lambda_o}{r} = \frac{2\pi\sqrt{GM}}{n_\alpha L}\cdot mR^{1/2},\ (n, n_\alpha) \in [1, 2, 3, ...). \end{aligned} \tag{2.11.17}$$

For instance, spherical coordinate system (which should naturally be most applicable here) has one radial coordinate and two angular, and we should use minimum three different quantum numbers for describing such spatial standing waves packing. The more general approach regarding inclinations of planetary orbits, instead (2.11.17) should be an angular or spatial quantizing of relevant orbital matter waves, like in A. Sommerfeld quantization and semi-classical quantization of angular momentum (see [40], D. Da Roacha and L. Nottale). See later more of supporting background under "Wavelength analogies in different frameworks", T.4.2, as well as extended matter-waves conceptualization with equations 4.3-1, 4.3-2, 4.3-3 and Fig.4.1.2, Fig.4.1.3 and Fig.4.1.4, all from the chapter 4.1.

The ideas, modeling and documented astronomic observations about solar systems quantization of orbital radius and relevant velocities (like results in (2.11.14)) are already known from the publications of William Tifft, Rubćić, A., & J. Rubćić, V. Christianto, Nottale, L. and their followers (see literature under [37], [38], [39], [40], [41], and [42]). The possibility, suggested by the observation of velocity quantization (72 km/s, Tifft, [37]) in the redshifts of galaxies, that wave-particle duality with a much larger value of Planck's constant may apply at galactic distances is also examined. For instance, in (2.11.14), we have the phase velocity found as, $u = u_n = \frac{1}{2}\sqrt{\frac{GM}{R}} = \frac{v_0}{2n} \cong \frac{v}{2}$. The galactic (phase) velocity redshifts measured by Tifft are often found to be around 72 km/s (see [37]), and the best-known estimate for specific (galactic) velocity v_0 is, $v_0 = 144.7 \pm 0.7\left[\frac{km}{s}\right] = 2\,x\,72.35 \pm 0.35\left[\frac{km}{s}\right]$ (see (2.11.14) and (2.11.14)-a). Of course, for higher values of quantum numbers $n = 2, 3$, we should be able to detect other galactic (phase-velocity related) redshifts such as, $u = \frac{v_0}{2n} \cong \frac{144}{2n} = \frac{72}{n}\left[\frac{km}{s}\right],\ n = 1, 2, 3, ... \Rightarrow\ u \in (\frac{72}{2} = 36, \frac{72}{3} = 24, \frac{72}{4} = 18...)\left[km/s\right]$. It is also found that orbital velocities (see (2.11.14)) of planets and satellites belonging to our Solar System, $v_n = \sqrt{\frac{GM}{R_n}}$ multiplied by **n** (**n = 1, 2, 3…**) are equal to the multiple of a fundamental velocity, which is close to $24\left[km/s\right]$. Also, increments of the intrinsic galactic redshifts are found to be $\cong 24\left[km/s\right]$ (see [40] Nottale; [41] Rubćić, A., & J. Rubćić; [43] M. Pitkänen), very much like the predictable situation regarding quantized orbital, planetary and satellite velocities from (2.11.14). Surprisingly, in here mentioned literature regarding redshifts, nobody related such case to phase velocity on the way as conceptualized here (related to orbital, macro-cosmological matter waves). Since measured spectral redshifts are really affected by orbital phase velocity $u = u_n$, it is almost evident that here hypothesized standing-waves field structure should exist. ♣**]**

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

The same problematic (related to redshifts and, why not to "blueshifts") we can conceptualize concerning matter-wave duality in the following way:

-An observer on our planet (or on a satellite orbiting our planet) is analyzing spectral content of a light coming from a specific distant galaxy.
-Distant galaxy is composed of many stars, solar systems, asteroids, meteorites, etc.
-Majority of such galaxy entities are solid bodies in mutually relative motions (and in a motion in relation to the distant observer), and many of them have specific magnetic field and emitting light or photons (including direct and secondary emissions of light).
-Since mentioned galaxy entities are motional bodies, we could associate matter-waves fields and wave properties to such motions (as we did all over this book).
-Let us consider that specific and dominant mass **M** of the galaxy in question has certain center-of-mass or group velocity **v**. At the same time, such group velocity is presentable as,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda},\ \lambda = \frac{H}{Mv}, u = \lambda f\ .$$

-If the mass **M** is emitting photons, and if our observer detects and analyze spectral signature of such photons, we should naturally have an interference and superposition between light waves or photons and matter waves of motional galactic mass **M**.
-Any light waves should also be presentable as having active group and phase velocity,

$$v_p = u_p - \lambda_p \frac{du_p}{d\lambda_p} = -\lambda_p^2 \frac{df_p}{d\lambda_p}, u_p = \lambda_p f_p\ .$$

-In such situation, group, and phase speeds of the resulting light emission, being detected by the distant observer, are naturally modulated by matter-waves of the motional galactic body **M**. Such modulation should produce red and blue (Doppler), spectral-shifts, and our distant observer should be able to detect such frequency alterations. Let us consider (for mathematical simplicity, to avoid using vectors) only extreme cases when we would have just additions or only subtractions of corresponding (group and phase) velocities. Resulting, effective (modulated) light received by the distant observer will have new, modified group and phase velocity (v^* and u^*),

$$v^* = v \pm v_p - \lambda \frac{du}{d\lambda} \mp \lambda_p \frac{du_p}{d\lambda_p} = -\lambda^2 \frac{df}{d\lambda} \mp \lambda_p^2 \frac{df_p}{d\lambda_p} = u^* - \lambda^* \frac{du^*}{d\lambda^*} = -\lambda^{*2} \frac{df^*}{d\lambda^*} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} v^* = v \pm v_p, u^* = u \pm u_p, \\ \lambda^* \frac{du^*}{d\lambda^*} = \lambda \frac{du}{d\lambda} \pm \lambda_p \frac{du_p}{d\lambda_p}, \\ \lambda^{*2} \frac{df^*}{d\lambda^*} = \lambda^2 \frac{df}{d\lambda} \pm \lambda_p^2 \frac{df_p}{d\lambda_p}, \\ u = \lambda f, u_p = \lambda_p f_p, u^* = \lambda^* f^* \end{Bmatrix}.$$

-Also, we could exercise Relativity Theory concepts stating that whatever we do with photons or light waves, resulting group speed (of modulated photons) will always stay constant, equal to the speed of light, $v^* = c = \text{constant}$. This will directly support the facts that our observer should detect mass-velocity related spectral (or frequency) shifts, and this is what William Tifft measured. Of course, here-established concept should be additionally developed and much better elaborated (for instance, concerning additions of velocities), but qualitative conclusions regarding the origin of red and blue shifts are already evident. A similar chain of thinking and findings could be applied to the Michelson-Morley experiment, possibly indicating why Earth motion is not influencing the speed of light inside of such old experimental framework, and how to organize more appropriate experiment, where immediate and entanglement coupling and synchronizations between light beams will be significantly eliminated. *Another reason for confusion produced by old Michelson-Morley experiments is maybe related to the situation that light beams or photons are oscillating only transversally and streaming or flow of ether should be kind of laminar or linear motion.*

All of that (about measured redshifts) is also, implicitly suggesting that electromagnetic dipoles polarizations between rotating (and spinning) astronomic objects could also be involved here, creating associated electromagnetic field structure (as speculated earlier in this chapter; -see equations from (2.4-6) to (2.4-10)). W. Tifft type of redshift can analogically be related to (still hypothetical) "**Planetary Vortex Shedding**" phenomenology, known in fluid motions as "Karman Vortex Street". If certain kind of "Planetary Karman Street" is on some way following planets and astronomic size objects (like an oscillatory, or helix tail), the light coming from distant sources, and passing such spatial zones of electromagnetic "Planetary Karman Streets" will be velocity-modulated, causing measured **redshifts**, and probably in some cases "**blue-shifts**". See more about vortex shedding in the chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,i. Measured frequency shifts of electromagnetic waves coming from distant sources and passing planetary systems (probably applicable to other cosmic rays, x-rays, maybe neutrinos…), are showing certain quantized and predictable, velocity-dependent repetition nature. Such quantization can be numerically related to quantizing of a planetary group and phase velocities (like in (2.11.12) - (2.11.14)), what is pawing an open way towards confirmation of innovative concepts regarding gravitation, as promoted in this book. Of course, here we should not neglect other types of signal modulations, like amplitude and phase modulations of light waves passing complex (*electromagnetic and gravitational*) structure of "Karman Streets" or helicoidally shaped field-tails associated to planetary orbital motions. Why we do not easily see such (spinning), helix field tails, is most probably related to incredibly low frequencies of such phenomena (calculated based on our SI time unit). Such slow spinning effects have much more chances to be influential (or detectable) on planetary orbits when a planet has a relatively big mass, high orbital speed and relatively small perimeter, like in case of planet Mercury (precession of the perihelion). We also know that rotational and spinning motions are often coupled with associated magnetic fields, and we know that gyromagnetic ratios are maintaining constant values both in a micro and in the macrocosmic world, making the same situation more complex, but also more defendable and self-sustaining. Many planets, moons, and other astronomic systems have relatively stable magnetic fields, meaning that rotation and/or spinning (or helix motion) should be somewhere in the background (regardless how low or high is associated spinning frequency). We already have all elements and facts supporting here conceptualized, an extended theory of gravitation;

-we only need to learn how to recognize, understand and use such facts. *In chapter 10 of this book, we can find the most complete explanation of the familiar situation regarding unknown or background velocity parameters and Newtonian attraction between important linear and angular moments (see (10.1.4) - (10.1.7))*.

An extraordinary publication from Charles W. Lucas, Jr. (see [54]), "The Symmetry and Beauty of the Universe" is explaining spiraling quantized orbits of planets and moons about the planets, linked to the universally present chiral symmetry based on new, universal electrodynamics force law (which is also addressing the updated force of gravitation). This symmetry can be observed on all size scales from the smallest elementary particle to the structure of the universe. Such spiraling phenomenology (in macro astronomical systems) should be in direct relation to planetary orbits quantizing, Tifft redshifts, and "Planetary Vortex Shedding" phenomenology (describing the same reality).

Anyway, our macro-universe is known to behave like a big and very precise astronomic clock, where periodical motions are its intrinsic property. It will be just a matter of finding or fitting proper integers $(n, n_{\alpha}) \in [1, 2, 3, ...)$ into above given (or similar) macro matter-waves relations, to support here presented concept. Of course, the situation analyzed here is presently addressing only purely circular planetary orbits (for having mathematical simplicity and faster introduction), and in later analyses, we would need to consider elliptic and other self-closed planetary orbits (and, most probably, we will generate additional quantum numbers or integers like n, n_{α}). Quantizing of planetary orbital motions presented here is realized using extremely simple, geometrical concepts analog to N. Bohr atom model, such as $n\lambda = 2\pi r_n, \lambda = H/p = H/mv_n$. The natural development of such quantized model of planetary systems will be in some ways like the evolution of Bohr's planetary atom model towards Sommerfeld's atom model (related to the period before the wave and probabilistic quantum mechanics and Schrödinger equations started to be dominant theoretical approach). For micro-world, we are merely implementing or associate universal quantization by default, since this is well-known practice tested in many cases (Planck, L. de Broglie, Einstein, Sommerfeld…). We should not forget that analogical planetary, orbital quantizing is also valid because of global periodical motions, and macro-universe conservation of important orbital and spinning moments (see (2.9.1) and (2.9.1)).

Also, as a significant theoretical background and support to the innovative concept of Macro-Cosmological stability and gravitation (presented here) the following reference should be considered: [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

[♣ COMMENTS & FREE-THINKING CORNER:

There is another, slowly emerging support (as well as empirical confirmations coming from satellites technology), to unity and coupling of linear, circular, and spinning motions, related to Liapunov (or Lyapunov) stability concept applied on spinning satellites. The following resume is partially taken from the Internet (Wikipedia):

*"A **spin-stabilized spacecraft** is a satellite which has the motion of one axis held (relatively) fixed by spinning the spacecraft around that axis, using the gyroscopic effect.*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

The attitude of a satellite or any rigid body is its orientation in space. If such a body initially has a fixed orientation relative to inertial space, it will start to rotate, because it will always be subject to small torques. The most natural form of attitude stabilization is to give the rigid body an initial spin around an axis of minimum or maximum moment of inertia. The body will then have a stable rotation in inertial space. Rotation about the axis of minimum moment of inertia is at an energy maximum for a given angular momentum, whereas rotation about the axis of maximum moment of inertia is at a minimum energy level for a given angular momentum. In the presence of energy loss, as is the case in satellite dynamics, the spin axis will always drift towards the axis of maximum moment of inertia. For short-term stabilization, for example, during satellite insertion, it is also possible to spin-stabilize the satellite about the axis of minimum moment of inertia. However, for long-term stabilization of a spacecraft, spin stabilization about its axis of maximum moment of inertia must be used".

Surprisingly or by coincidence, we can find supporting background to an appearance of helical matter waves associated with bullets motion in modern guns. Rifling is the process of making helical grooves in the barrel of a gun or firearm, which imparts a spin to a projectile around its long axis. This spin serves to "gyroscopically stabilize" the projectile, improving its aerodynamic stability and accuracy. Bullet stability depends primarily on gyroscopic forces, the spin around the longitudinal axis of the bullet imparted by the twist of the rifling. Once the spinning ball is pointed in the direction the shooter wants, it tends to travel in a straight line until outside forces such as gravity, wind and impact with the target influence it. Without spin, the bullet would tumble in flight. Modern rifles are only capable of such fantastic accuracy because the ball is stable in flight (thanks to the gyroscopic effect). Even spherical projectiles must have a spin to achieve any sort of acceptable accuracy.

Let us elaborate bullet-spinning connection with matter-waves associated with the same bullet. As we know, all linear motions (inertial and non-inertial) are relative motions. Depending on observer's reference frame we can conclude that body is in relative movement to something else, and often different observers can differently describe which object is moving and which one is static (because such motions are mutually relative).

We could say something similar for rotational and spinning motions. Spinning and rotation in a specific mechanical system is also related to something, meaning being a relative angular motion from observer's point of view, concerning specific axis, to a fixed or moving frame of reference, etc. To say what is rotating and what is in a stable state, concerning certain angular motion, also belongs to specific relative movements, meaning one motional state has described another state.

Now we can go back to a spinning bullet in linear motion. Mentioned spinning is relative to the longitudinal axis of the ball, meaning its axis is stable (static, rigid) and a bullet is spinning around. If observer's reference frame is fixed to the longitudinal axis and revolving around the same axis, such observer will notice that bullet is not rotating, (just to say what here means relative spinning motion). Since bullet's accuracy and path-stability is enormously increased (thanks to gyroscopic effects) consequently, guns-related technology found a way to give spinning to bullets. We could analogically reverse the cause and effect in the same situation, and say that particle in inertial, stationary, and stable motion should have helical, spinning matter wave around its longitudinal axis of propagation, because such matter wave and moving particle are in mutually relative spinning motion (either one or the other is spinning).

As we know, most of the astronomical objects that can be considered as satellites, or as behaving in a similar way (like orbiting planets, moons, asteroids…) are also naturally spinning. All of that is either complementary or in a direct agreement with here elaborated ideas about "Macro Cosmological Matter Waves", supporting the idea that ***there is a natural tendency of all (micro and macro) objects in linear motions to be on some way coupled to spinning***. Something similar is additionally conceptualized in Chapter 4.1, around equations (4.3-0) and (4.3-0)-a,b,c,d,e,f,g,h,I,j,k... In addition, (in this book) we are also considering rest masses as energy "condensed, frozen or stabilized spinning motions" (as already exercised in this chapter: "2.3.2. Rotation and stable rest-mass creation").

Familiar situations (concerning couplings between linear motions and associated spinning) we could also find by analyzing laser beams (or beams of photons), and beams of electrons in different interactions with matter states. In addition, we could say that electrons are just specific energy-momentum formatting and packing states of high-energy photons (since a high-energy photon can produce electron-positron couple, and electron and positron are mutually annihilating and producing photons). As we know, both, photons and electrons have spin attributes, and both are presenting perfect examples of dualistic Wave-Particle objects (as widely elaborated in this book; -just to mention some of them as, Compton Effect, Photoelectric Effect, interference effects, scatterings, and diffractions situations…).

Since gravitational force is anyway too weak in comparison to other forces (like electromagnetic and nuclear), if our laboratory for such measurements is only on our planet, this will not be enough and correct approach. We need to consider the macro-Universe or Space around us (with many of galaxies and other objects), as a much more relevant laboratory for observing and learning about gravitation, and for testing new theories related to gravitation. We already know that photons or light waves (high-frequency electromagnetic radiation) are interacting with gravitation of big masses, also interacting with electric and magnetic fields, as well as with matter of any kind. It is logical to consider that light should also interact with other (still hypothetical) aspects of gravitation. If motional masses have associated field complement to gravitation (for instance something like spinning helix tail, or matter wave, or Karman Street, Vortex Shedding entity, like waves behind the boat crossing large and quiet water surface), light beams passing through such *Vortex Shedding Zone* should interact with it. The result of spectral measurements will be that received light is on some specific way modified, or modulated; -for instance frequency, phase and amplitude modulated (including Doppler Shifts), as measured by W. Tifft. Now we can consider our Cosmos as a big laboratory for testing new theories about gravitation. For instance, first, we can search for remote, stable, clear, strong, and distant source of light with known (measurable) spectrum, and we qualify it in a situation when we know (see) that there is an open and empty space between the light source and our Observatory. Such spectral measurements we can consider as our reference measurements. In the second step, we need to wait until the spatial position of the Observatory is changed and make similar spectral (time and frequency domain) measurements when the same light will pass through certain galactic or planetary zones and compare new measurements with reference measurements. The differences should be related to specific signal (or spectral) modifications and modulations caused by certain motional, gravitational, electromagnetic, and other matter states present in the concerned "*Vortex Shedding"* spatial zones (since light will interact with such matter and fields states). Of course, if we replace distant light source with something else (with cosmic rays of another kind) we will again be able to apply a similar concept regarding spectral signatures comparison. Another (more controllable) approach will be to have two satellites in stationary orbits around our planet (preferably on mutually opposite orbital ends) and to send laser beams between satellites on a way that reference beams will pass empty spatial areas between them. This way we will establish (measure) reference spectral signatures. In a second step, we will target "Planetary Karman Streets" of other planets (from the same solar system), passing there the same laser beams and registering new spectral signatures. By simple comparisons with reference situations, we should be able to find if "Planetary Karman Streets" exist, and if and how laser beams are modulated by such toroidal and spiraling spatial zones (of planetary and satellite motions; - see [54]). The nature of "Planetary Karman Streets" and Tifft redshifts should be causally linked to a complex structure of associated spiraling electromagnetic and gravitation-related fields (*most probably having very low frequencies for our SI* unit of time).

The signal analysis that will be applied in such cases should show specific correlations with motional and field states that are present in galaxies and planetary systems under investigation. If we are lucky, good mathematicians with fresh concepts and modeling about new gravitation-related phenomenology, and if we are well equipped with measurements and signal processing tools, we will be able to prove new theories about gravitation.

In [52], (Rainer W. Kühne: Gauge Theory of Gravity Requires Massive Torsion Field), we can find another supporting theory which is promoting necessity of linear, torsion and spinning motions coupling on the following way (just the abstract): "One of the greatest unsolved issues of the physics of this century is to find a quantum field theory of gravity. According to a vast amount of literature, unification of quantum field theory and gravitation requires a gauge theory of gravity, ***which includes torsion and an associated spin field****. Various models including either massive or massless torsion fields have been suggested".*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER…** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

For stable and planar solar systems, we already know quantization rules (2.11.14) applicable for an orbit radius and relevant planetary (or tangential) velocity. Most stable solar systems are not necessarily planar, and we should consider the existence of similar quantization (or spatial orbits packing) regarding angular orbits positions (or orbit inclination towards specific reference orbital plane).

To generalize the same concept (already elaborated with equations from (2.11.13) to (2.11.17)) for any closed planetary orbit, we can apply Wilson-Bohr-Sommerfeld action integrals (used in supporting N. Bohr's Planetary Atom Model). Wilson-Bohr-Sommerfeld action integrals (see [9]), related to any periodical motion on a self-closed stationary orbit C_n, applied over one period of the movement, present the kind of general quantifying rule (for all self-closed standing waves, which are energy carrying structures, having constant angular momentum). Sommerfeld (see chapter 5; equations (5.4.1)) extended Bohr atom model to cover elliptic (and circular) electron (or planetary) orbits, where the semi-major axis is **"a"** and semi-minor axis is **"b"**. We can (just to initiate brainstorming in that direction) analogically (also still hypothetically, and highly speculatively) apply the same strategy on a planet which has mass **m** and which is rotating around its sun, which has mass **M** >> **m**, on the following way,

$$\left.\begin{array}{l} \left\{\oint_{C_n} \mathbb{L} d\alpha = n_\alpha H,\ \mathbb{L} = \text{Constant},\ 0 \le \alpha \le 2\pi\right\} \Rightarrow \mathbb{L} = n_\alpha \frac{H}{2\pi},\ n_\alpha = 1,2,3...,n \\ \left\{\oint_{C_n} p_r dr = n_r H \right\} \Rightarrow \mathbb{L}(\frac{a}{b} - 1) = n_r \frac{H}{2\pi},\ n_r = 0,1,2,3... \end{array}\right\} \Rightarrow \qquad (2.11.18)$$

$$\Rightarrow a = \frac{m+M}{(mM)^2} \frac{(\frac{H}{2\pi})^2}{G} n^2 \cong \frac{(\frac{H}{2\pi})^2}{GMm^2} n^2 = a_0 n^2,\ b = a\frac{n_\alpha}{n} = a_0 n_\alpha n,\ n \equiv n_\alpha + n_r = 1,2,3,4...$$

In addition to (2.11.18), for a certain stable (planar) planetary system with several planets (or even for our universe) it should also be valid that its total angular momentum is constant (including spinning moments of planets, moons, and asteroids),

$$\left\{\begin{array}{l} \left[\begin{array}{l} \vec{\omega}_c = \frac{\sum_{(i)} J_i \vec{\omega}_i}{\sum_{(i)} J_i} = \frac{\sum_{(i)} \vec{\mathbb{L}}_i}{\sum_{(i)} J_i} \\ \oint_{C_n} \mathbb{L} d\alpha = n_\alpha H,\ \oint_{C_n} p dr = n_r H \\ n_\alpha, n_r \in [1,2,3,...] \end{array}\right] \Rightarrow \left[\begin{array}{l} \vec{\mathbb{L}} = \sum_{(i)} \vec{\mathbb{L}}_i = \sum_{(i)} J_i \vec{\omega}_i = \vec{\omega}_c \sum_{(i)} J_i = \text{const.}(= n\frac{H}{2\pi}) \\ \sum_{(i)} \oint_{C_n} \mathbb{L}_i d\alpha = \sum_{(i)} n_i H_i = nH = \text{const.} \end{array}\right] \\ \& \\ \left[(\vec{p}, \frac{E}{c})^2 = \text{inv.} \Rightarrow p^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c})^2 \Leftrightarrow p^2c^2 - E^2 = -E_0^2 \Leftrightarrow p^2c^2 - (\gamma mc^2)^2 = -(mc^2)^2\right] \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \left[\begin{array}{l} \left(\frac{\vec{r}}{r} \times (\vec{p}, \frac{E}{c})\right)^2 = \left(\frac{1}{r}(\vec{\mathbb{L}}, \frac{E}{c}\vec{r})\right)^2 = \text{inv.}, \\ \gamma = \gamma_1,\ \gamma_2 = 1,\ \vec{\mathbb{L}} = \overrightarrow{\text{const.}} \end{array}\right] \Rightarrow \frac{1}{r_1^2}\left[\vec{\mathbb{L}}_1^2 - (\frac{E}{c})^2 r_1^2\right] = \frac{1}{r_2^2}\left[\vec{\mathbb{L}}_2^2 - (\frac{E_0}{c})^2 r_2^2\right] \Rightarrow \\ \Rightarrow \mathbb{L} = \mathbb{L}_1 = \mathbb{L}_2 = \frac{r_1 r_2}{c}\sqrt{\frac{E^2 - E_0^2}{r_2^2 - r_1^2}} = mcr_1 r_2 \sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.}, \Rightarrow r_1 r_2 \sqrt{\frac{\gamma_1^2 - 1}{r_2^2 - r_1^2}} = \text{const.} \end{array}\right\} \qquad (2.11.19)$$

Of course, if we have combinations of orbital (**L**) and spin moments (**S**), we will need to replace $\mathbb{L}$ with **L**+**S**. As we can see in [36], Anthony D. Osborne, & N. Vivian Pope effectively and analogically (could be unintentional, too) made a significant extension of Sommerfeld concept to the macro-world of planets, stars, and galaxies, and it is evident that this way the new chapter of Cosmology and Astronomy is being initiated. ♣]

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Elements of specific stable space-time structure with periodical motions (planetary systems, for instance) are mutually coupled by fields and forces integrating them into a stable macro system, and essential associated condition or consequence regarding such stability is the creation of standing waves of involved fields. Positions and paths of planets (inside such periodical-motions systems) are defined by stable or stationary energy-momentum conditions of the system in question, which are related to system minimal energy dissipation, or maximal mechanical quality factor conditions (for instance, found by solving relevant Euler-Lagrange-Hamilton equations). To give an idea how we could evolve this quantum-like conceptualization of Gravitation it would be beneficial to see the Appendix (at the end of this book) that is innovatively treating "**Bohr's model of hydrogen atom and particle-wave dualism".** Of course, some other time, ideas paved with (2.11.10) - (2.11.21) should be better elaborated, extended, and verified, *but significant and innovative brainstorming breakthrough is already made.* What we should, conceptually and imaginatively, visualize and upgrade here is that we are no more dealing only with time-stable and spatially isolated linear (and circular) planetary orbits and discrete planetary masses. Masses of planets in orbital motions are embedded in certain energy-momentum, time and spatial standing-waves distributions that are presenting material, mass-energy extensions, links, and bridges between all elements of such planetary systems. What we see as planetary masses and orbits (described by Kepler and Newton laws) are only space-time localized effective centers (or channels) of such energy-momentum agglomerations that are (in the broader space-time frame) structured as standing waves.

Based on the planetary macro-waves conceptualization which is presented from (2.11.12) until (2.11.19) we can also create kind of Schrödinger equation valid for such standing-waves situations of quantized mass (or energy-momentum) distributions (primarily related to planetary systems). Here we should consider relevant (equivalent) mass in its extended meaning as relativistic, velocity-dependent, spatially distributed and coupled with surrounding energy-momentum states and fields (familiar to conceptualization given in "**2.2. Generalized Coulomb-Newton Force Laws"**, equations (2.3) - (2.4-3)). This time, the important wave function Ψ is causally related to a planet or satellite motional energy on a self-closed path or orbit, or to a relevant radius of orbiting (like having spatial standing waves on a self-closed and oscillating circular string). Geometrically and analogically, this is modeling based on formulating closed (three dimensional or multi-dimensional) spatial structures where all relevant and mutually coupled motional elements with certain periodicity are becoming stationary and stable. The first association related to any standing waves formation is that this should also be a kind of resonance. Moreover, such periodical structures or states have an integer number of specific elementary wavelengths or an integer number of other relevant elementary domains, and such states can be qualified as quantized states. Really, there is nothing more significant to implement or profit from Quantum Theory here.

For creating a better idea about such "standing waves *packing",* it is useful to see equations under (2.9.5-9), *Chapter 4.1, T.4.2., Wavelength analogies in different frameworks,* and *Chapter 5, T.5.3., Analogical Parallelism between Different Aspects of Matter Waves*. If we insist on creating some clear, preliminary, and conceptual visualization of planetary orbital motions with associated gravitational matter-waves structure (as spatially distributed energy-momentum states, enclosed in ***toroidal*** forms), this could be intuitively linked to the illustration on Fig. 2.6. and strongly related to future creative modeling and innovative solutions resulting from equations (2.11.20) to (2.11.23).

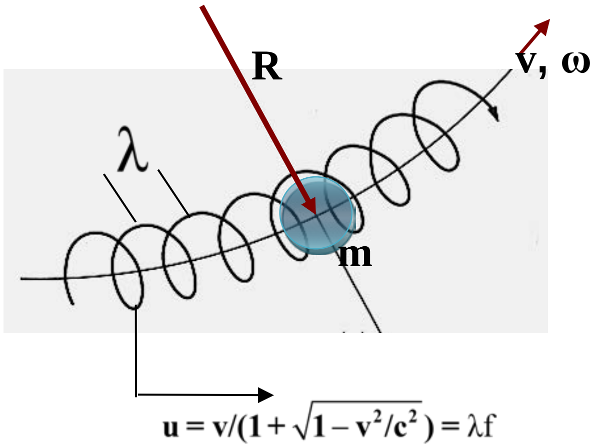

Fig.2.6. Gravitational matter waves and orbital planetary motion

Natural way of modeling such geometrical forms (standing waves) is causally related to Analytic Signal concept (see chapter 4.0. and [57]) and Schrödinger equation, which has its conceptual, historical, and analogical origins in generalization of d'Alembert, Classical Wave Equation, which is related to standing waves oscillations of an ordinary (self-closed) string. In addition, the same, Classical wave-equation (in specific analogical form of d'Alembert equation) has been known in different fields of Classical Mechanics, Fluid Mechanics, Acoustics and Maxwell Electromagnetic Theory long before being "renamed, modified and analogically applied" to waves phenomenology in micro-Physics by Schrödinger and other Quantum Theory founders. Another striking analogy (showing that a mass in motion should have some kind of associated helix-spinning field or oscillating matter wave tail) is related to fluid flow vortices and vortex flow-meters, as speculated in ***chapter 4.0 and 4.1; -see equations (4.3-0), and (4.3-0)-a,b,c,d,e,f,g,h,l,j,k...***

This concept of associated helicoidally spinning field (around a path of linear motion) is causally related to the Analytic Signal modeling (as presented in chapter 4.0). Let us consider a specific linear movement of a particle, or an equivalent wave group (in the same state of linear motion). If we present this motion (meaning its power or force function, or relevant field function) with specific wave function $\Psi(t)$***, using the Analytic Signal model, we can create another, associated wave function*** $\hat{\Psi}(t)$***, where a couple of such functions (*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$***) are generating the Complex Analytic Signal*** $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\ \Psi\ (t)$ ***(see much more in chapter 4.0). Now, based on such Analytic Signal modeling, we can determine de Broglie or matter-wave frequency, wavelength, amplitude, and phase functions. The healthy, and fundamental mathematics, well connected to the stable and naturally stable body of Physics (without artificial and postulated theoretical concepts), is***

always producing good and realistic mathematical predictions, meaning that both $\Psi(t)$ and $\hat{\Psi}(t)$ must be realistic, measurable wave functions of something that exists in our Physics and Universe. One of the examples for such coupled wave functions is the electromagnetic field that is combining electric and magnetic field functions in a similar way as realized in the Analytic Signal model. Here, as a case causally related to gravitation, we have the situation that any linear and spinning or helix wave motion should be on the same way coupled (creating an Analytic Signal).

[♣ COMMENTS & FREE-THINKING CORNER:

Anyway, in many cases, we can conclude that linear and helix or spinning and rotating motions (of masses) are mutually complementary and united (concerning matter-waves, or PWDM elaborated in this book). Such concepts could be, (imaginatively, creatively, and analogically) extrapolated from atoms to planetary systems and galactic formations. On some way, our universe is globally rotating and spinning, following helix-like paths of associated matter-waves. What we see as red or blue, Doppler shifts (of electromagnetic radiation) coming from a remote deep space, could be effects of such globally present, macro-rotating effects. As we know, the tangential velocity of the certain rotating mass, v_t is equal to the product of relevant orbital velocity, ω, and relevant radius R, $v_t = \omega R$. Hubble's law is maybe saying something similar, such as, $v = (v_t) = H_0 R$, where H_0 is Hubble constant, which could be certain metagalaxy, orbital (or helicoidally spinning associated matter-waves), tangential velocity. ♣]

Citation took from the Internet; -Wikipedia, the free encyclopedia:

*"**Hubble's law** or **Lemaître's law** is the name for the astronomical observation in physical cosmology that: (1) all objects observed in deep space (intergalactic space) are found to have a Doppler shift observable relative velocity to Earth, and to each other; and (2) that this Doppler-shift-measured velocity, of various galaxies receding from the Earth, is proportional to their distance from the Earth and all other interstellar bodies. In effect, the space-time volume of the observable universe is expanding, and Hubble's law is the direct physical observation of this process.[1] It is considered the first observational basis for the expanding space paradigm and today serves as one of the pieces of evidence most often cited in support of the Big Bang model. Although widely attributed to Edwin Hubble, the law was first derived from the General Relativity equations by Georges Lemaître in a 1927 article where he proposed that the Universe is expanding and suggested an estimated value of the rate of expansion, now called the **Hubble constant**.[2][3][4][5][6] Two years later Edwin Hubble confirmed the existence of that law and determined a more accurate value for the constant that now bears his name.[7] The recession velocity of the objects was inferred from their redshifts, many measured earlier by Vesto Slipher (1917) and related to velocity by him.[8]*

*The law is often expressed by the equation $v = H_0 D$, with H_0 the constant of proportionality (the **Hubble constant**) between the "proper distance" D to a galaxy (which can change over time, unlike the comoving distance) and its velocity v (i.e. the derivative of proper length with respect to cosmological time coordinate; see Uses of the appropriate distance for some discussion of the subtleties of this definition of 'velocity'). The SI unit of H_0 is s^{-1}, but it is most frequently quoted in (km/s)/Mpc, thus giving the speed in km/s of a galaxy 1 megaparsec (3.09×10^{19} km) away. The reciprocal of H_0 is the Hubble time.*

As of 3rd Oct 2012, the Hubble constant, as measured by NASA's Spitzer Telescope and reported in Science Daily, is 74.3 ± 2.1 (km/s)/Mpc"

The formulation of the Schrödinger equation is well-known and, in this book, additionally elaborated and generalized (later, in chapter 4.3). Anyway, from different publications we already have definite confirmation that Schrödinger equation is well applicable to solar systems quantizing (see [63], Arbab I. Arbab, and [67], Johan Hansson), since results of Schrödinger equations related to N. Bohr hydrogen atom are directly generating all results of planetary orbit parameters quantizing, as in (2.11.14), when we apply analogical replacement $\boxed{\frac{Ze^2}{4\pi\varepsilon_0} \rightarrow GmM}$. Since orbiting planets are respecting certain periodicity and "*macro matter-waves packing rules*", like standing, de Broglie matter-waves in a micro-universe (see explanations around equations (2.9.5-9), (2.11.5) - (2.11.9), (2.11.9-1) - (2.11.9-4), (2.9.5-1) - (2.9.5-5) and (2.11.12) - (2.11.14)), we are in the position to apply relevant, generalized Schrödinger-like equations, as equations (4.10), to planetary orbital motions. Of course, it is essential to address accurately all relevant parameters and analogical replacements.

[♣ COMMENTS & FREE-THINKING CORNER:

Let us first consider the specific chain of thinking and logical conclusions with several unavoidable facts and step-stones of modern Physics, such as:

1. *Schrödinger equation is something that works well in the world of microphysics (and modern Quantum Theory), where it is unavoidable and producing correct results. Exceptionally brilliant and the early triumph of Schrödinger equation (in spherical coordinates) was experienced with exploring N. Bohr, hydrogen atom model, when all experimentally known results, as well as results of old quantum and atom theory, have been calculated based on Schrödinger equation results. We could only argue about historical Schrödinger equation foundations and development, which has been like tricky patchwork, with mathematical trial and errors experiments, until Schrödinger constructed or intuitively fitted the right and useful, present form of his equation. However, since it is working well, and producing useful results, we forgot how it was created or postulated, without enough systematic and logical arguments.*

2. *In this book, (see chapter 4.3) it is anyway shown that there is another mathematical approach and modeling, which starts from Classical, universally valid wave equation and produces almost the same, but a logically consistent, more generally correct, more logical and intuitively clear family of equations compatible to Schrödinger equation. This is realized based on Analytic Signal and Hilbert Transform modeling, without using Probability and Statistics décor, and without almost arbitrary and complicated postulations.*

3. *Anyway, old, or innovated Schrödinger equations are working on the same or similar way and producing perfect (spectral) results related to N. Bohr, hydrogen atom model (just to start with). Consequently, we should conclude that something exciting and significant for Physics and our Universe should be linked to mentioned Schrödinger equations family.*

4. *Number of authors, including the author of this book, theoretically concluded, experimentally explained, and supported by astronomic measurements and observations, that there is the striking analogy between results of N. Bohr hydrogen atom model (regarding quantized electron energy, velocity, radius…), and similar results applicable to planetary or solar systems (see in this chapter equations (2.11.12) – (2.11.14) and table T.2.8.). Nobody presently claims that such analogy, and mutually comparable, and by measurements verifiable results are entirely correct, but what we can verify is very much indicative. Naturally, we need to admit that there is undoubtedly familiar, and monumentally simple, intrinsic, ontological, and experimental, unifying background (of micro and macrocosmic entities) here.*

5. *Since Schrödinger equation has its extremely significant place regarding N. Bohr, hydrogen atom model, and such atom model have striking analogies with planetary systems (with similar periodical motions), the logical conclusion is that we should be able to explain quantizing within*

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stable solar systems by exploring relevant (customized) Schrödinger equations. Of course, somewhat innovated Schrödinger equations (based on Analytic Signal Wave-function) should eventually be formulated in spherical coordinates (as in case of hydrogen atom model) to better address Gravitation. Results of gravitational, planetary system Schrödinger equation will enrich our understanding of Gravitation, far better compared to Newton and Relativity Theory concepts. ♣]

It is essential to underline that Schrödinger-like, analogically formulated wave equation, applicable to gravitational fields, and macro-mechanical motions within planetary systems has almost nothing to do with stochastic and probability concepts applied in contemporary Quantum Theory.

Let us briefly specify logical, a mathematical chain of initial and final forms and conclusions in the process of analog formulation of such wave equation. We can start by complying with generalized Schrödinger equation (4.10) from the Chapter 4.3, which will address deterministic (certainly non-stochastic) planetary and satellites orbital motions, including associated (deterministic and dimensional) wave functions in a field of gravitation, as for instance,

$$\left\{\begin{array}{l}
\left[\begin{array}{l}
\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} + \mathbf{E_0} - U_p)\overline{\Psi} = \mathbf{0}, \\
\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi} - \mathbf{U_p}\,\overline{\Psi} = -(\tilde{E} + \mathbf{E_0})\,\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} - \mathbf{U_p}\,\overline{\Psi} = \frac{\hbar^2}{\tilde{E} + \mathbf{E_0} - U_p}\cdot\frac{\partial^2\,\overline{\Psi}}{\partial t^2} - \mathbf{U_p}\,\overline{\Psi} = \\
= \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi} - \mathbf{U_p}\,\overline{\Psi}, \quad (\frac{\mathbf{E_{total}} - \mathbf{U_p}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\,\overline{\Psi}}{\partial t^2} = \mathbf{0}\,, \frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0}, \\
\nabla = \mathbf{i}\frac{\partial}{\partial \mathbf{x}} + \mathbf{j}\frac{\partial}{\partial \mathbf{y}} + \mathbf{k}\frac{\partial}{\partial \mathbf{z}} = \mathbf{grad},\ \Delta = \nabla^2 = \frac{\partial^2}{\partial \mathbf{x}^2} + \frac{\partial^2}{\partial \mathbf{y}^2} + \frac{\partial^2}{\partial \mathbf{z}^2}\,,\ \overline{\Psi} = \Psi + \mathbf{j}\hat{\Psi},\ \mathbf{j}^2 = -\mathbf{1}
\end{array}\right] (\mathbf{4.10-1}), \text{chapter 4.3} \\
\& \\
\left[\begin{array}{l}
\mathbf{v} = \omega_m\mathbf{R} \cong \mathbf{2u} = 2\lambda_0\mathbf{f_0} = \frac{\mathbf{2\pi r}}{\mathbf{T}} = 2\pi\mathbf{Rf_m} = \sqrt{\frac{\mathbf{GM}}{\mathbf{R}}} << \mathbf{c},\ (\mathbf{R} = \mathbf{R_0}\frac{\mathbf{1+e}}{\mathbf{1+e\cdot\cos\theta}}), \\
\omega_m = 2\pi\mathbf{f_m} = \frac{\mathbf{4\pi f_0}}{\mathbf{n}} = \frac{2\pi}{\mathbf{T}} = \frac{\sqrt{\mathbf{GM}}}{\mathbf{R}^{3/2}} = \frac{\mathbf{v}}{\mathbf{R}},\ \mathbf{p} = \mathbf{mv} = \frac{\mathbf{H}}{\lambda_0} = \frac{\mathbf{nH}}{2\pi\mathbf{R}},\ \mathbf{n} = \mathbf{1,2,3,...} \\
\mathbf{L} = \mathbf{n}\frac{\mathbf{H}}{2\pi} = \mathbf{pR} = \mathbf{mvR} = \mathbf{mR}^2\omega_m = \mathbf{mR}^2\frac{2\pi}{\mathbf{T}} = 2\pi\mathbf{mR}^2\mathbf{f_m} = \sqrt{\mathbf{GMR}}\cdot\mathbf{m}, \\
\tilde{E} = \mathbf{E_k} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{mv}^2 = \mathbf{Hf_0} = \frac{\mathbf{GMm}}{\mathbf{2R}},\ \mathbf{U_p} = -\frac{\mathbf{GMm}}{\mathbf{R}},\ \mathbf{E_k} - \mathbf{U_p} = \frac{\mathbf{3}}{\mathbf{2}}\frac{\mathbf{GMm}}{\mathbf{R}}
\end{array}\right] \begin{array}{l}(\mathbf{2.11.13})\\(\mathbf{2.11.14}),\\(\mathbf{2.11.15})\end{array} \\
\& \\
\left[\begin{array}{l}
\mathbf{h} \leftrightarrow \mathbf{H} = \frac{2\pi\sqrt{\mathbf{GMR}}}{\mathbf{n}}\cdot\mathbf{m},\ \hbar = \frac{\mathbf{h}}{2\pi} \leftrightarrow \hbar_{\mathbf{gr.}} = \frac{\mathbf{H}}{2\pi} = \frac{\sqrt{\mathbf{GMR}}}{\mathbf{n}}\cdot\mathbf{m} = \frac{\mathbf{GMm}}{\mathbf{v_0}}, \\
\mathbf{m} \leftrightarrow \frac{\mathbf{mM}}{\mathbf{m+M}} = \mu \cong \mathbf{m} << \mathbf{M},\ \frac{\mathbf{Ze}^2}{4\pi\varepsilon_0} \to \mathbf{GmM}.
\end{array}\right]
\end{array}\right\} \Rightarrow$$

$$\left[\begin{array}{l} \frac{(\frac{H}{2\pi})^2}{2m}\Delta\overline{\Psi} + (E_k + E_0 - U_p)\overline{\Psi} = 0,\ U_p = -\frac{GMm}{R},\ E_k = \frac{GMm}{2R},\ E_0 = mc^2, \\ \\ \frac{(\frac{H}{2\pi})^2}{2m}\Delta\overline{\Psi} - U_p\overline{\Psi} = -(E_k + E_0)\overline{\Psi} = -j\hbar\frac{\partial\overline{\Psi}}{\partial t} - U_p\overline{\Psi} = \frac{\hbar^2}{E_k + E_0 - U_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} - U_p\overline{\Psi} = \\ = j\hbar u\nabla\overline{\Psi} - U_p\overline{\Psi},\ (\frac{E_{total} - U_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,\ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0,\ u = v/2. \end{array}\right] \quad (2.11.20)$$

The solutions of (2.11.20) will show that planetary and satellite orbits (here populated by a wave function $\overline{\Psi}$) are closed circular or elliptic lines, but only approximately. Certain harmonic, exceptionally low frequency standing waves, having helical paths, or amplitude-modulation of relevant radius, with toroidal and helix, field-envelope should be measurable, when planets and satellites' orbits are very precisely monitored (because of rotation, spinning and mutual interactions among participants).

The closest, extraordinary, and unique publications about existence and grounds of such (helix and toroidal) planetary motions are coming from Lucas Jr. Charles when he is explaining Chiral Symmetry of Spiraling Planetary Orbits (on Surface of a Toroid) about the Sun (see [54]). Unfortunately, publications and ideas of Mr. Lucas are not enough addressed in the mainstream of officially supported science, probably because he is too original and sometimes gravitating around arbitrary, religiously flavored environments, this way maybe creating some minor doubts regarding his scientific objectivity and ideological neutrality. Anyway, regardless of personal ideological preferences of Mr. Lucas, his conceptualization of spiraling planetary orbits on a surface of the toroid is amazingly seducing and significant contribution to understanding Gravitation (and familiar to ideas and concepts elaborated in this book).

It is evident that the future development of here-introduced macro-cosmological matter-waves concept will significantly enrich our understanding of Gravitation.

To get generally valid and entirely natural solutions $\overline{\Psi} = \overline{\Psi}(\mathbf{R},\theta,\varphi)$ of (2.11.20), involved operators ($\nabla, \nabla^2 = \Delta$) should be applied in spherical, polar coordinates ($\mathbf{R},\theta,\varphi$). In addition, elliptic planetary orbits should be considered (of course, after we upgrade all equations from (2.11.12) until (2.11.18), which are valid for ideal circular orbits, into new and equivalent expressions applicable for elliptic planetary orbits, and consequently, all of that will modify differential equations found in (2.11.20)). The same gravitational, Schrödinger equation, (2.11.20), in its natural, spherical coordinates will eventually look like the equation applicable on N. Bohr, the hydrogen atom model,

$$\frac{(\frac{H}{2\pi})^2}{2m}\left[\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\cdot\overline{\Psi} + U_p(R) = E_k\cdot\overline{\Psi}, \quad (2.11.21)$$
$$\overline{\Psi} = \overline{\Psi}(R,\theta,\phi) = X(R)\cdot Y(\theta,\phi),$$

where $\mathbf{X(R)}$ is expressible in terms of associated Laguerre functions, and $\mathbf{Y(\theta,\phi)}$ are the spherical harmonic functions.

To find solutions of (2.11.21) will not be easy, but to rely on analogies between a specific planetary system and N. Bohr, hydrogen atom model, and directly make results conversions, like in T.2.8., will be much more comfortable, since the validity of analogical replacements, $\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M, e \Leftrightarrow m \cong \frac{mM}{m+M} = \mu, Z \Leftrightarrow \frac{M}{m}, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM\right\}$ is shown as working very well. For instance, to get an (analogical) idea about possible spatial shapes of gravitational, matter-wave function from (2.11.21), we can see the picture given below, which is addressing hydrogen wave function (taken from Quantum Theory, standard literature):

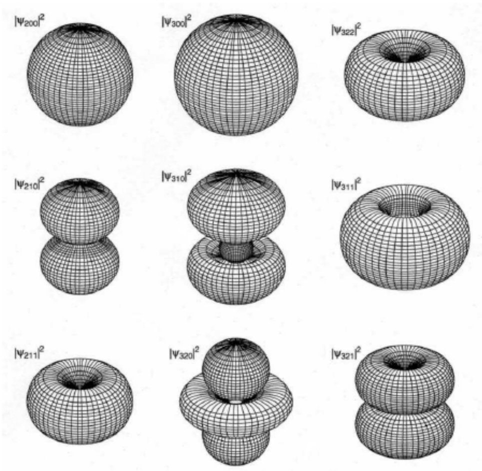

Fig.2.6. Surfaces of the constant $\left|\Psi\right|^2$ for the first few hydrogen wavefunctions

In fact, more correct explanation of the ideas found in (2.4) - (2.11.21) could be complicated compared to here-presented, but for the purpose of introducing new concepts about Wave and Quantum Gravitation, Particle-Wave Duality, force-field charges, and unification between linear and rotational elements of every motion, hire initiated, conceptual platform is already sufficiently clear. To understand the broader meaning of wave functions it is useful to see the chapter "4.3 Wave Function and Generalized Schrödinger Equation"; -equations: (4.33-1), (4.41-1) to (4.45), T.4.2 and T.4.3, as well as matter-waves conceptualization around equations (4.3) and (4.3-1) in the chapter 4.1.

Also, based on Parseval's identity (that is universally valid, and connecting time and frequency domains of any wave function), we can establish very realistic and deterministic meaning of the matter-waves, wave function. For more information, see chapter 4.0, and equations (4.0.4).

The wave function $\overline{\Psi} = \overline{\Psi}(\mathbf{R}, \theta, \varphi, \mathbf{t})$, instead of being certain probability, any oscillating amplitude, or specific displacement, or harmonically modulated orbital radius (like in (2.11.20) and (2.11.21)), could analogically get an extended meaning of spatially distributed, matter-wave power (see (2.11.22)). In such situations, (instead of Probability and Statistics concepts and postulations), number of mutually linked, universally valid conditions and relations would be naturally satisfied, as for instance,

$$\Psi^2 = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} = \omega_m \frac{d\mathbf{L}}{dt} = vF = \omega_m \tau = \text{Power} = P,$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{array}{l} E_k = \tilde{E} = \frac{1}{2}mv^2 = mvu = pu = 2mu^2 = \frac{1}{4}mv_e^2 = \frac{GmM}{2R} = \frac{1}{2}\cdot\left(\frac{GmM}{R^2}\right)\cdot R = \frac{1}{2}\cdot F_{m-M}\cdot R = \\ = \frac{m}{2}\left(\frac{2\pi R}{T}\right)^2 = \frac{8m\pi^2R^2}{n^2}f_o^2 = 2m(\pi R f_m)^2 = (2\pi m R^2 f_m)\cdot(\pi f_m) = L\pi f_m = (\frac{2L\pi}{n})\cdot f_o = Hf_o = \\ = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \\ = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\overline{U}(\omega)\right|^2 d\omega = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\right]^2 d\omega = \int_0^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \\ = \int_{-\infty}^{+\infty}P(t)dt\ (=)\ [J] \end{array}\right\},$$

$$\left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = H\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}}, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{Hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p},\ f_s = k/2\pi, \lambda = \frac{H}{p}, \\ \Rightarrow 0 \le 2u \le \sqrt{uv} \le v \le c \end{array}\right\}, \tag{2.11.22}$$

because, regarding orbital planetary motions we have harmonic and periodical wave-functions (and motions), which create stable, self-closed, spatial standing-waves, and resonant-like field states. The nature of fields and forces involved here is related to mechanical motions and gravitation. Such movements are also mixed with associated electromagnetic fields (at least because mutually identical mathematical forms of Newton and Coulomb's force laws are applicable). We should not exclude the possibility that Gravitation has its primary and essential origin in Electromagnetism (see such electromagnetic forces conceptualizations around equations from (2.4-6) to (2.4-10), in the same chapter; -see also [54]). Deterministic, *square of the power-related macrocosmic wave-function* from (2.11.21) and (2.11.22) is a product between two of relevant, mutually linked, dynamic and motional-energy related values, such as relevant current and voltage, or force and velocity, etc. (see (4.0.82) from the chapter 4.0, with more of supporting background), as follows,

$$\Psi^2(t,R) = P(t,R) = \frac{d\tilde{E}}{dt}(=)\text{Active Power}\ (=)$$

$$(=)\left\{\begin{array}{lll} i(t)\cdot u(t) & (=) & [\text{Current}\cdot\text{Voltage}],\text{ or} \\ f(t)\cdot v(t) & (=) & [\text{Force}\cdot\text{Velocity}],\text{ or} \\ \tau(t)\cdot\omega(t) & (=) & [\text{Orb.}-\text{moment}\cdot\text{Angular velocity}],\text{or} \\ (\vec{E}\,x\,\vec{H})\cdot\vec{S} & (=) & [\overrightarrow{\text{Pointyng Vector}}]\cdot\overrightarrow{\text{Surface}} \\ ----- & (=) & --------- \\ s_1(t)\cdot s_2(t) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{array}\right\}. \tag{4.0.82}$$

It is also important to mention that specific, new, unbounded, creative, and intellectually flexible approach should still be implemented in a future fruitful merging between the wave-function environment from (2.11.22), (4.0.82), and wave equations (2.11.21) to formulate new wave and quantum gravity theory. What we have here, so far, are just early brainstorming and first intuitive steps.

The opinion of the author of this book about gravitational waves is that if such waves exist, we will find that this is certain cosmic, electrostrictive and/or magnetostrictive oscillatory and waving effect (like in cases of piezoelectric and magnetostrictive transducers), including associated electromagnetic waves, fields, and forces, manifesting on particles distributions and motions (analogical to waves in fluids). What effectively exist (and what is measurable) in our astronomic environment are different electromagnetic waves, photons, streams of electrically charged and neutral particles, and surrounding electric and magnetic fields. Also, in our cosmic environment, we can find different mass-energy-momentum flow and streaming situations of different particles, waves, and fluids. Mentioned, primarily electromagnetic effects (being as original sources of vibrations) are producing secondary, temporally, and spatially evolving electromechanical effects, by contemporary physics specified as gravitational force and waves. The principal sources of gravitation are not static masses, but rather motional and oscillating masses with linear and orbital moments, and all masses are specific packing formats of specifically structured and polarized electromagnetic entities (like electrons, protons, neutrons, positrons, photons… and their combinations). The fact is that our Universe is already united and stable, regardless of we (or our Physics) miss the proper global unification theory of all-natural forces and fields. Since our Universe is firm and united, consequently, the same natural force should oversee the micro and macro world of physics. The best and most logical candidate for such universal force is electromagnetic fields' related phenomenology (see more in Chapter 3.). Structural or spatial stability and organizing of our Universe is described by R. Boskovic universal natural force, [6], and such force, by its nature, should be single and unique (meaning cannot be gravitational plus electromagnetic plus weak and strong nuclear force).

2.3.3-5 Uncertainty and Entanglement in Gravitation

Until present, certain defendable legitimacy regarding planetary and gravitational wave functions and wave equations has been established. Consequently, whenever we have wave functions, we can analyze associated couples of mutually conjugate, space, time, and spectral domains. Uncertainty Relations, which are generally applicable to such situations (in mathematics), are mutually relating durations of mutually conjugate, original, and spectral domains, of involved wave functions. To better understand such, generally valid uncertainty relations, it is recommendable to read chapter 5. of this book (around relations (5.5)),

$$\begin{aligned}&\mathrm{TF} > \frac{1}{2},\ \mathrm{T}\Omega > \pi,\ \Omega = 2\pi\mathrm{F}\\&\mathrm{T} - \text{absolute time duration of the } \Psi \text{ function}\\&\mathrm{F} - \text{absolute frequency duration of the } \Psi \text{ function.}\end{aligned} \tag{5.5}$$

Obviously, in cases of planetary systems, Planck constant **h** has not its natural place there and new and analogous **H** >> **h** constant is becoming much more relevant. The number of uncertainty relations can be now formulated for gravitational and planetary wave-functions, on a similar way as practiced in Quantum Theory concerning constant **H** (for covering much more extensive background about wave functions and Uncertainty Relations; -see chapters 4.0 and 5). As shown in chapter 5, we will conclude that typically mechanical and motional entities, as practiced in present interpretations of gravitation, should be enriched with naturally associated electromagnetic set of relevant parameters (see equations from (5.2) until (5 .4.1)), if we like to have more complete picture about Gravitation. ***What makes Uncertainty Relations, equally applicable (on the same way), both in micro and macro world of Physics, is the fact that simple, geometric, or spatial dimensions, durations and size of certain rest-mass are different and smaller when compared with mass-energy-momentum matter-wave packet associated to the same mas (including all associated electromagnetic items). Within Uncertainty Relations, we should naturally operate with matter-waves durations, both in relevant original and spectral domains.*** ***High Power Mechanical, ultrasonic or acoustical energy, moments, forces, oscillations and vibrations, or audio signals and music, can be created and transferred by applying different signal-modulating techniques on laser beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); - See relations from Chapter 10. under (10.2-2.4) and literature references from [133] until [139].***

Another, still fantastic, imaginative, and hypothetical vision, or prediction, concerning planetary wave functions, will be a necessity of "Gravitational Entanglement" (see more about entanglement in Chapter 4.3, equations (4.10-12) and in Chapter 10). If we search what could or should be such entanglement in Astronomy and Gravitation, we will come to concepts of globally coupled orbital and spinning moments of planetary and galactic formations, where all such rotating and spinning states (belonging to members of specific planetary system) are mutually communicating, balancing, and compensating every perturbation by infinite speed. Such orbital moments communication should be present locally or internally (inside of specific planetary system), and globally or externally concerning the larger astronomical environment. See literature under [36], Anthony D. Osborne, & N. Vivian Pope, where something similar or equivalent to Gravitational Entanglement is also promoted.

[♣ COMMENTS & FREE-THINKING CORNER:

2.3.3-6 Rudjer Boskovic and Nikola Tesla's theory of Gravitation

The most exciting and most profound brainstorming conceptualization of Gravitation, based on elaborations presented in this book, is to understand and develop it in a familiar framework already established by Rudjer Boskovic [6], as the "**Universal Natural Force**", including intuitive concepts of similar "**Dynamic Theory of Gravity**", as sporadically commented by Nikola Tesla, [97]. We can imaginatively and creatively (with much of positive intellectual freedom) see that forces and fields discussed by R. Boskovic and N. Tesla are already present within forces keeping atoms stable (like in the N. Bohr's atom model). We can also see from the results presented in this book (*see chapter "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"; -equations from (2.11.10) until (2.11.22), and "T.2.8. N. Bohr hydrogen atom and planetary system analogies"*) that solar or planetary systems have big level of analogy with N. Bohr's atom model (and vice versa). An innovative extension of N. Bohr's atom model (see "8. BOHR's MODEL OF HYDROGEN ATOM AND PARTICLE-WAVE DUALISM") is elaborating that such structures (like hydrogen atom, or analogically conceptualized planetary systems; - see equations (8.64) until (8.74), and "8.3. Structure of the Field of Subatomic Forces") should be surrounded with a complex force-field $\mathbf{F}(r, \theta, \phi, t)$ that could be modeled as Rudjer Boskovic Universal Natural Force and be also conceptually compatible to Nikola Tesla's Dynamic Force of Gravity, since N. Tesla anyway had R. Boskovic as the primary source of his ideas about gravitation. Here is convenient to rethink about extended foundations of gravitation in "2.2.1. WHAT THE GRAVITATION REALLY IS", at the beginning of this chapter, where is speculated that all atoms and masses (in our Universe) are

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continuously and synchronously communicating electromagnetically, mechanically and electromechanically.

*We also know that electric charge and magnetic flux are naturally bipolar entities (able to create dipole structures). Something similar should be valid for linear and angular moments, what is already known and related to action and reaction forces, electromagnetic induction, and different inertial effects. In other words, since gravitational force is known only as an attractive force, to satisfy mentioned bipolarity, some real mass-energy-momentum flow should exist as a reaction-force complement to gravitation, what N. Tesla conceptualized as "**radiant energy**" and mass flow from all masses towards other masses, [97].*

We could speculate that, on some way, electric charges, magnetic fluxes, linear and angular moments are always mutually coupled, complementarily integrated, or packed, presenting the most important source of natural fields and forces. This is already kind of General Field Unification Platform, which is in harmony with Rudjer Boskovic's "Universal Natural Force", [6], and Nikola Tesla's "Dynamic Force of Gravity", [97].

*Citation from PowerPedia, on Internet; -"Tesla's Dynamic Theory of Gravity: The **Dynamic Theory of Gravity** of Nikola Tesla explains the relation between gravitation and electromagnetic force as a unified field theory (a model over matter, the aether, and energy). It is a unified field theory to unify all the fundamental forces (such as the force between all masses) and particle responses into a single theoretical framework".*

Also, all fields and wave phenomena (presently known in Physics) should be considered as a natural evolution of elementary, microparticles and fluids' states towards diversity of macro momentum-energy or mass states, being a kind of "communicating, coupling and gluing medium" in a space between particles. Of course, waves are always oscillations of a certain medium, or fluidic and elastic matter states (where energy can fluctuate between its kinetic and potential forms). Consequently, in an absolute vacuum state, where electromagnetic waves, neutrinos and various cosmic radiation are propagating, it should exist some fluidic matter (still not well conceptualized in contemporary physics), as N. Tesla stated many times, [97]. In the same context, particles could be considered as specifically condensed (or solidified) energy states of self-sustaining, internally folded forms of rotating matter waves (where the process of such stabilizing is causally related to internal standing-waves formations). See much more of similar ideas in [117], Jean de Climont.

R. Boskovic's and N. Tesla universal and dynamic force $\mathbf{F}(r, \theta, \phi, t)$, to comply with Bohr's planetary atom model, and to analogical solar systems modeling (as elaborated earlier in this chapter), should have, at least two force components $\mathbf{F_1}(r, \theta, \phi, t) + \mathbf{F_2}(r, \theta, \phi, t)$, (one being potential, and the other solenoidal vector field, as already exercised in the chapter 8.), as for instance:

$$\mathbf{F}(r, \theta, \phi, t) = \mathbf{F_1}(r, \theta, \phi, t) + \mathbf{F_2}(r, \theta, \phi, t),\ \nabla\times\mathbf{F} \neq 0,\ \nabla\mathbf{F} \neq 0 \Rightarrow$$
$$\Rightarrow \begin{cases} \nabla\times\mathbf{F}_1 = 0, \nabla\mathbf{F}_1 \neq 0 \\ \nabla\times\mathbf{F}_2 \neq 0, \nabla\mathbf{F}_2 = 0 \end{cases} . \quad (2.11.23)$$

Inside an atom, mentioned forces and fields have electromagnetic nature. Since we already know that striking analogy between atoms and planetary systems exist, we could also make model of the field of gravitation on a similar way by exploring the possibility that gravitational force is an analogical extension of interatomic forces (2.11.23), within the framework of the Extended Bohr's atom model (as presented in chapter 8). This way, we will model gravitational forces as a composition (or superposition) of one solenoidal and one potential vector field (as in (2.11.23)), but on the way as R. Boskovic suggested, and N. Tesla commented (see more in the chapter 8.).

Gravitational attraction is implicitly suggesting that there is certain energy fluctuation and gravitational potential (which has certain associated speed) between mutually attracting masses (see supporting elaborations around equations (2.4-11) – (2.4-17)). Presence of such energy fluctuations and communications (related to standing waves phenomenology) would also influence or alternate the meaning of particle velocity (in a field of gravitation that has solenoidal and potential vector components).

In the chapter 8. of this book (8.3. Structure of the Field of Subatomic Forces), we can find familiar elaborations about Gravitation as matter-waves exchange effects between atoms and Universe. "The condition of the balance of the potential energy of all attractive and all repulsive forces (see (8.74)) within an atom may be added on by a hypothesis about the existence of permanent communication, by an interchange of electromagnetic quanta between stationary states of a nucleus and electron-waves-shell of an atom. This is happening synchronously and coincidently, in both directions, so that the internal field of an atom always captures such interchange, i.e., could not be noticed in the external atom space, if an atom is neutral, self-standing, not connected to other atoms, and non-excited. Here we could profit from analogically extension of the same conceptualization, hypothetically saying that mentioned (bidirectional) electromagnetic, quantized exchanges between atom nucleus and electrons' shell are explicable by the force of gravitation, penetrating almost endlessly towards infinity of an outer, external atom space (outside of atoms, towards other atoms and cosmic formations). In other words, all atoms are on such way connected within our Universe, continuously radiating, and receiving electromagnetic waves.

Similarly, Nikola Tesla, [97], conceptualized existence of specific "radiant energy", or radiant fluid flow from all atoms towards the universe (and vice versa). Outside of neutral atoms and other masses, we have a dominant presence of forces we qualify as gravitational attraction. Every mechanical force should be a time derivative of certain momentum, F = dp/dt. Since action and reaction forces are always mutually compensating and synchronously present, we could imagine a continuous, steady flow of certain "radiant" fluid, where microelements of such fluid have linear moments, p = mv (this way to be able to conceptualize existence of gravitational force in comparison with A. Einstein elevator).

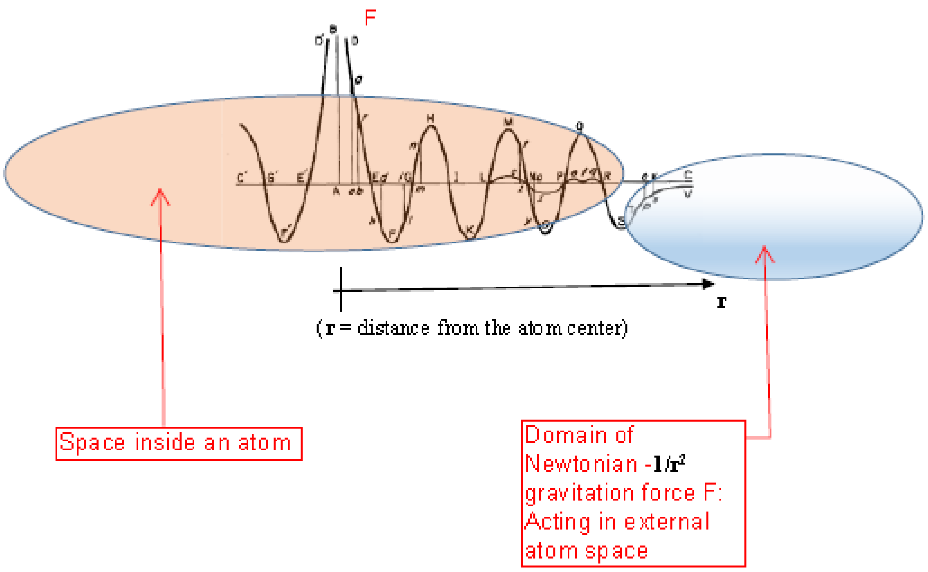
We also know that planetary or solar systems are analogically structured as atoms, respecting standing matter-waves resonant packing and couplings; -See "2.3.3. Macro-Cosmological Matter-Waves and Gravitation, 2.8. N. Bohr hydrogen atom and planetary system analogies".

External reaction forces in question (outside of atoms and masses) belong to attractive effects manifesting as Gravitation (being essentially and primarily of electromagnetic nature, since atoms are anyway communicating internally and externally by exchanging photons, including what belongs to cosmic radiation). Practically, all atoms, other particles, more significant astronomic objects, and our Universe are mutually communicating bi-directionally (or omni-directionally) by radiating electromagnetic energy, and by receiving an echo of "Electromagnetic-Gravitation" related forces.

The roots of such interpretation of an atomic or macro mass alternating force-field, which looks like standing waves resonant structure (of course, created after few derivations or integrations), are present in the works from Rudjer Boskovic, [6], about universal natural force ([6], Principles of the Natural Philosophy), as well as in certain papers published in «Herald of Serbian Royal Academy of Science» between 1924 and 1940 (J. Goldberg 1924; V. Žardecki, 1940). Nikola Tesla's, [97], Dynamic theory of Gravitation, is also close to Rudjer Boskovic's unified natural force (see the picture below), and to a here-elaborated concept about extended Bohr's atom model, summarized with (8.74) in chapter 8. Another source of familiar ideas we can find in [73], Reginald T. Cahill, Dynamical 3-Space; -Emergent Gravity." R. Boskovic is practically explaining where repulsive (anti-gravitation) force elements are, since our generalized experience is that natural forces and charges should have positive and negative, or attractive and repulsive nature. Such effects of attractive and repulsive forces, analog to gravitation, we can produce and easily detect with ultrasonic resonators (where nodal zones are zones of only attractive forces, and anti-nodal zones are manifesting only repulsion).

$$\oiiint_{\left\{ r\in[0,\infty],\ \theta\in[0,2\pi],\ \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \right\}} \mathbf{F}d\mathbf{R} = \mathbf{0} \quad , \qquad (8.74)$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.



Rudjer Boskovic's Universal Natural Force Function

Nikola Tesla, [97], made several patents (and presented affirmative and successful experiments) showing that his "radiant fluid" or steady radiant mass flow exists, and has electromagnetic nature (being able to carry positive and negative electric charges and mechanical moments). Consequently, we can draw conclusions that all atoms and masses in our universe are mutually communicating, being familiar to Rudjer Boskovic Universal Natural Force, [6], as mutually coupled and tuned resonators, thanks to surrounding and coupling fluidic medium named as an ether (that is useful until we find another more convenient name, and better conceptualization). Properties of such ether (based on Nikola Tesla concepts) have both electromagnetic and mechanical nature (having extremely fine and small carriers of mechanical and electromagnetic moments and charges, as well as having dielectric and magnetic character).

2.4. How to unite Gravitation, Rotation, and Electromagnetism?

It should already be evident that all possible forces and fields in our Universe are anyway united, regardless of we know how to formulate the Unified Field Theory. In given (philosophical) frontiers we can mention the best starting points for creating a new (analogous and hypothetical) field structure of rectilinear and rotational motions, which would (conveniently) follow Faraday-Maxwell electromagnetic field definition. For instance, Lorentz and Laplace forces are the explicit connection between the rectilinear motion of electrical charge (current) and magnetic field and can be transformed to analog forms of a specific interaction between participants being in rectilinear and rotational movement (using concepts of analogies already elaborated here). The Ampere-Maxwell's, Biot-Savart's, and Faraday's induction laws can serve to complete the previous situation mathematically, for a more precise description of "linear-rotational" fields (just by transforming mentioned laws into corresponding analog expressions). See larger background about electromagnetic and mechanical complexity and essential origins of Gravitation in Chapter 3.

Many attempts are already known in modern science regarding the formulation of the Maxwell-like theory of gravitation and explaining the origins of inertia. The traditional formulation of gravitation field takes mass as a (primary) source of gravity. However, in this book it is demonstrated that more essential (and dominant) sources of gravity-related phenomenology should be found in different interactions between moving objects, such as, between their linear and/or angular (or spinning) moments, which are coupled with certain electric and magnetic moments or dipoles, and between their motional, and state of rest energies. (See also Chapter 4.1 of this book for more supporting elements regarding associated de Broglie, matter waves). Revitalizing and updating Wilhelm Weber's force law (to cover electromagnetic and gravity related interactions, with combined linear and rotational motion elements) would be a healthy platform towards establishing a new Maxwell-like theory of gravitation (see literature under [28] and [29]).

Most probably that many force/fields manifestations and components of constant, or accelerated movements, (such as Coriolis, centrifugal, centripetal, gyroscope-effect, pendulum oscillations, inertial and similar forces, Gravitomagnetic induction from General Relativity Theory, etc.) could conveniently be incorporated, interpreted, and mutually united with here proposed concepts about gravitation. It is conceptually already clear what the author of this book is suggesting related to links between rotation, linear motion, and electromagnetism (see chapter 10; -equations (10.1.4) - (10.1.7)). See also equations (4.18), (4.22) - (4.29), (5.15) and (5.16).

After establishing a new platform for understanding the complementary nature of "linear-rotational" fields and motions (see chapters 4.0 to 4.3 of this book), we shall have an open way for creating a full set of "Gravity-Rotation" field equations, making them initially analog with Maxwell equations of an electromagnetic field. Later, we could modify and upgrade such equations up to the most meaningful and useful forms that will correspond to the reality of different natural fields and forces (see the development of equations (4.22) - (4.29)). Later (in Chapter 3), it will be shown that Maxwell Theory should also be slightly upgraded to become compatible for unification with the upgraded theory of Gravitation (see also literature [23] – [26]).

Anyway, to present a significant and new insight regarding gravitation, we would need to introduce unique and original concepts that do not show only redundant and analogical variations of already known field theories. Let us initiate one of such thoughts, as follows.

2.5. New Platforms for Understanding Gravitation

Gravitation can also be conceptualized by making analogies with mechanical or acoustical resonators**. Let us imagine that our macro-universe or cosmos effectively presents a kind of fluid-like substance with a different particle or mass agglomerations submersed (or hanging) in such substance. Mentioned particle agglomerations (in the frames of this conceptualization) would be different cosmic objects, planets, stars, galaxies, dust, atoms, plasma states etc. Let us now imagine that such composite cosmic fluid is being mechanically vibrated by certain constant frequency (from an external, presently unknown source of mechanical vibrations). In case of performing a real experiment (just to visualize the concept and make relevant analogies), in a vessel filled with liquid that is mixed with solid particles, by vibrating such vessel we will notice creation of three-dimensional standing-waves structure, where submerged (and suspended) particles would make higher mass density, or mass agglomerations in nodal areas of standing waves. Such effects are known as acoustic and/or ultrasonic effects of levitation (see more in [150] and [151]). Nodal areas, in this case, are zones where oscillating velocities are minimal (or zero), and oscillating forces and mass-density are maximal. If we intentionally introduce a small test particle somewhere in a vicinity of any of such nodal areas with high mass density (while vessel filled with liquid and other particles is resonating), we will notice that the test particle will be attracted by the closest nodal zone (or closest particle). Of course, here we are temporarily excluding cases of involvements of possible electromagnetic forces to make the situation quite simple in its first brainstorming steps. Similar attractive force (in the vicinity of a nodal zone) can be observed in the case of resonant, standing wave oscillations of half-wavelength solid resonators or multiple half-wavelength resonators, known in ultrasonic technology). If external vibrations that are driving mentioned resonators are suddenly switched-off, the attractive forces towards nodal areas will disappear. Now we could conceptualize our universe as an equivalent (or analogical) mechanical fluid-like system that is permanently in a state of **exceptionally low frequency** resonant and standing waves oscillations (see time-frequency relations (5.14-1) in Chapter 5, Uncertainty). Such standing-waves oscillations are forcing all astronomical objects to take only certain stable nodal positions (or orbits) of the easiest agglomerating areas, which are kind of its space-matrix texture (apart from other linear and rotational motions, involved). Placed around such astronomical objects (planets, stars, galaxies…), every test mass would experience only an attractive force (similar like in cases of gravitation). Later, the same initial concept can be upgraded by considering linear and rotational motions (of submerged particles, or astronomical objects) that are again forced to comply with agglomeration rules around global standing-waves nodal areas, complying with the framework of Euler-Lagrange-Hamilton mechanics (see similar concepts in [99] from Konstantin Meyl). **Understanding of mass, as conceptualized here, is indirectly considering that any mass is a storage or modus of matter-waves energy packing or agglomerating (and at the same time kind of "frozen, rotating energy states").** The problem here could be the fact that we know that between astronomical objects in our universe there is significant "empty-space of vacuum states", and our* *imaginative fluid-substance (which should mechanically resonate) would have problems regarding performing only mechanical vibrations, but such problems will be eliminated since acoustic, mechanical, electromechanical, and electromagnetic vibrations are anyway mutually coupled. Again, we would need to understand the specific nature of such fluid-like substance that is a carrier of externally introduced mechanical vibrations on some new innovative way. Contemporary physics made efforts to show that ether-type fluids are not something that could be experimentally confirmed. In mechanics and acoustics, we already know that vacuum cannot be a carrier of mechanical vibrations, and for supporting here introduced concept of gravitation, we need to have kind of mechanical resonant and standing waves states of our universe. Most probably that specific electromagnetic, magnetostrictive and/or electrostrictive coupling nature should also be involved here (in a framework of coupled oscillators) to realize penetration of mechanical vibrations through vacuum and empty-space states (and certainly vacuum in our universe is not at all an empty space). Anyway, the situation regarding explaining gravitation, as initiated here, could be richer and different compared to present Newton, Kepler, and Einstein framework, since none of them is explaining why gravitation is only manifesting as an attractive force. At the same time, we know that standing-waves mechanical resonators are easily showing the existence of such (only) attractive forces in their nodal areas (creating acoustic levitation), and by analogy, we could make hypothetical predictions regarding what behind the force of gravitation should be (see more in [150] and [151]). The remaining question to answer here would be, where and what the source of mentioned vibrations is, or what is the source of cosmic standing waves?* ***Since all constituents of our universe are mutually connected and interacting, as well as in permanent relative motions and we know that matter-waves are associated with mass motions, this should be an important element of the answer regarding origin of mentioned intrinsic vibrations and their standing waves (in the context of understanding the nature of gravitation). *Einstein's General Relativity Theory is already explaining gravitation from specific space and fields' geometry-related modifications and deformations, taking this as a fact, not speculating (as here) that specific spatial-matrix texture could be a consequence of complex resonating, standing waves formations. Since everything that exists in our Universe is anyway mutually cross-linked, coupled, and united (in some cases most probably without our full knowledge about such unity), any new theory about Gravitation should consider electromagnetic and other forces coupled with gravitation. Of course, the ordinary Newton (static) gravitation force is for many orders of magnitude weaker than all other forces (electromagnetic, nuclear…), compared on the same scale, making that we usually neglect interactions between gravitation and other fields. If the concept proposed here has enough practical and theoretical grounds, this would be a breakthrough in the novel and better understanding of gravitation (being also applicable to other forces like electromagnetic ones). Another contemporary fields-unification theory (which is going much deeper and wider in conceptualizing a multi-dimensional space with its elementary and vibrating building blocks that are taking forms of strings and membranes) that could be in some ways familiar to here introduced concepts is the Superstrings or M-theory (which is still evolving and searching for its best foundations). We should not forget that any new concept of gravitation should be simple, elegant, and well-integrated into remaining chapters of physics that are already working well, and some attempts in creating such modeling will be made later.*

Let us review original abstracts from different publications, showing new, emerging aspects of still evolving, and future theory of gravitation:

A) **Dynamical 3-Space. Emergent Gravity**; Reginald T. Cahill, School of Chemical and Physical Sciences, Flinders University, Adelaide 5001, Australia, E-mail: Reg.Cahill@inders.edu.au . Invited contribution to: **Should the Laws of Gravitation be reconsidered**? Héctor A. Munera, ed. (Montreal: Apeiron 2011), **[73].**

The laws of gravitation devised by Newton, and by Hilbert and Einstein, have failed many experimental and observational tests, namely the borehole g anomaly, at rotation curves for spiral galaxies, supermassive black hole mass spectrum, uniformly expanding universe, cosmic filaments, laboratory G measurements, galactic EM bending, precocious galaxy formation, … The response has been the introduction of the new epicycles: "dark matter", "dark energy", and others. To understand gravity, we must restart with the experimental discoveries by Galileo, and following a heuristic argument, we are led to a uniquely determined theory of a dynamical 3-space. That 3-space exists has been missing from the beginning of physics, although it was first directly detected by Michelson and Morley in 1887. Uniquely generalizing the quantum theory to include this dynamical 3-space, we deduce the response of quantum matter and show that it results in a new account of gravity and explains the above anomalies and others. The dynamical theory for this 3-space involves G, which determines the dissipation rate of space by matter, and α, which experiments, and observation reveal to be the fine structure constant. For the first time, we have a comprehensive account of space and matter and their interaction - gravity.

B) **The Nature of Space and Gravitation; Jacob Schaff.** Instituto de Fsica, Universidade Federal do Rio Grande do Sul (UFRGS), Porto Alegre, Brazil. Email: schaf@if.ufrgs.br. Received May 12, 2012; revised June 8, 2012; accepted July 1, 2012. doi: 10.4236/jmp.2012.38097. Published Online August 2012 (http://www.SciRP.org/journal/jmp), **[74].**

Many recent highly precise and unmistakable observational facts achieved thanks to the tightly synchronized clocks of the GPS, provide consistent evidence that the gravitational fields are created by velocity fields of real space itself, a vigorous and very stable quantum fluid like spatial medium, the same space that rules the propagation of light and the inertial motion of matter. It is shown that motion of this real space in the ordinary, three dimensions around the Earth, round the Sun and round the galactic centers throughout the universe, according to velocity fields strictly consistent with the local main astronomical motions, correctly induces the gravitational dynamics observed within these gravitational fields. In this space-dynamics, the celestial bodies, all firmly rest with respect to the real space, which, forth-rightly leads to the observed null results of the Michelson light anisotropy experiments, as well as to the absence of effects of the solar and galactic gravitational fields, on the rate of clocks moving with Earth, as recently discovered with the help of the GPS clocks. This space dynamic exempts us from explaining the circular orbital motions of the planets around the Sun; likewise, the rotation of Earth exempted people from disclosing the diurnal transit of the heavens in the days of Copernicus and Galileo because it is space itself that so moves. This space dynamic also eliminates the need for dark matter and dark energy to explain the galactic gravitational dynamics and the accelerated expansion of the universe, respectively. It also straightforwardly accounts regarding well-known and genuine physical effects for all the other observed effects, caused by the gravitational fields on the velocity of light and the rate of clocks, including all the new effects recently discovered with the help of the GPS. It moreover simulates the non-Euclidean metric underlying Einstein's space-time curvature. This space dynamics is the crucial innovation in the current world conception that definitively resolves all at once the troubles afflicting the current theories of space and gravitation.

C) **Deriving gravitation from electromagnetism**. Can. J. Phys. 70, 330- 340 (1992). A. K. T. ASSIS[1]. *Department of Cosmic Rays and Chronology, Institute of Physics, State University of Campinas,* C. *P. 6165, 13081 Campinas, Sao Paulo, Brazil.* Received November I, 1991. Can. J. Phys. 70.330 (1992), **[75].**

We present a generalized Weber force law for electromagnetism including terms of fourth and higher order in v/c. We show that these additional terms yield an attractive force between two neural dipoles in which the negative charges oscillate around the positions of equilibrium. This attractive force can be interpreted as the usual Newtonian gravitational force as it is of the correct order of magnitude, is along the line joining the dipoles, follows Newton's action and reaction law, and falls off as the inverse square of the distance.

D) The Electrodynamic Origin of the Force of Gravity, Part 1; ($F = Gm_1m_2/r^2$). **Charles W. Lucas, Jr.** 29045 Livingston Drive, Mechanicsville, MD 20659-3271, bill@commonsensescience.org, **[76].**

The force of gravity is shown to be a small average residual force due to the fourth order terms in v/c of the derived universal electrodynamic contact force between vibrating neutral electric dipoles consisting of atomic electrons vibrating concerning protons in the nucleus of atoms. The derived gravitational force has the familiar radial term plus a new non-radial term. From the radial term, the gravitational mass can be defined in terms of electrodynamic parameters. The non-radial term causes the orbits of the planets about the sun to spiral about a circular orbit giving the appearance of an elliptical orbit tilted concerning the equatorial plane of the sun and the quantization of the orbits as roughly described by Bode's law. The vibrational mechanism that causes the gravitational force is shown to decay over time, giving rise to numerous phenomena, including the expansion of the planets (including the earth) and moons in our solar system, the cosmic background radiation, Hubble's redshifts versus distance (due primarily to gravitational redshifting), Tifft's quantized redshifts (Bode's law on a universal scale), Tifft's measured rapid decay of the magnitude of redshifts over time, the Tulley-Fisher relationship for luminosity of spiral galaxies, the unexpected high velocities of the outer stars of spiral galaxies, and Roscoe's observed quantization of the luminosity and size (Bode's law) of 900 spiral galaxies. Arguments are given that this derived law of gravity is superior to Newton's Universal Law of Gravitation ($F = Gm_1m_2/r^2$) and Einstein's General Relativity Theory ($G_{\mu\nu} = -8\pi G/c^2 \, T_{\mu\nu}$).

The next citation (or copy) from ***[83], David L. Bergman, editor: Selected Correspondence on Common Sense Science #1.*** ***November 2013, Volume 16, number 4. Common Sense Science, P.O. Box 767306, Roswell, GA 30076-7306, E-mail: bergmandavid@comcast.net***

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Gravity Dilemma. I seem to remember that gravity cannot be shielded. An internet search agrees. Yet I believe electromagnetism can be shielded. If so, how can gravity be of electrodynamic origin?

Russel Moe,
Wildwood, Florida

Reply by Dave Bergman. More than once I have asked myself the same question. Eventually I resolved the dilemma in this way: First, recall that matter is composed of elementary particles—mostly electrons and protons.

Second, every electron and every proton has self-generated electromagnetic fields surrounding the charged particle and spreading outward into an increasing volume of space with field intensities that decrease in accordance with the inverse square law that applies also to gravity.

Third, these electromagnetic fields have an *oscillating component* and a *non-oscillating component*, corresponding respectively to radiation of energy (including **light** energy) and **gravity**.

Fourth, actual measurements show that shielding of an oscillating electromagnetic field ranges from *no shielding* to *partial shielding* to *high shielding* in correspondence with the **wavelength** of the oscillation [D. G. Fink, Editor-in-Chief, **Standard Handbook for Electrical Engineers, Tenth Edition,** McGraw-Hill Book Company, p. 29-23, fig. 29-40 (1957).]

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Reply by Dr. Lucas. Good question. The answer is that the experiments of William J. Hooper as described in his book **New Horizons in Electric, Magnetic and Gravitational Field Theory** identify that all three types of electric and magnetic fields have different empirical properties. For instance in Chapter 1 Hooper lists 14 empirical properties of E fields. In Table 1 he gives what these properties are for electrostatic E fields, E fields dependent on dA/dt, and E fields dependent on motion $V \times B$. In particular he notes in property 6 that the motionally caused E fields cannot be shielded.

There is an interesting story regarding this E field that cannot be shielded. When Hooper discovered this effect, he invented and patented a speedometer for airplanes. When a plane was flying with respect to the surface of the earth, Hooper's speedometer measured the $V \times B$ term due to the velocity of the plane with respect to magnetic field lines of the earth being crossed.

He arranged a demonstration with the military. They flew Hooper's speedometer and compared it readings with respect to other methods. The military testers confirmed that Hooper's speedometer worked very well. However, when Hooper applied for a government contract to supply the military with these speedometers, his application was rejected by the scientific reviewers, because they said that the metal hull of the military aircraft would shield the effect. According to Maxwell's theory of electrodynamics, all three forms of the electric field have the same identical properties. The scientific reviewers ignored the fact that the military testers had tested Hooper's speedometer inside a metal-hulled military airplane, and it worked inside that shielded environment.

In our derivation of an improved and more general version of the electrodynamic force law, we treat each of the three types or sources of electric fields separately as distinctly different types of E fields.

It is the term proportional to $R(R \times V)$ in the universal electrodynamic force law that gives rise to the force of gravity. It is equivalent to Hooper's motional E field.

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specifically to the helical charged particles. Thus, inertial mass was not fundamental, but a specific, calculable property of the unit charged particles due to their electro-magnetic structure.

Your approach differs in considering a net neutral, but finite size, structured combination of particles, interacting with all the other such dipole particles in the universe. Thus, the amount of inertia demonstrated by a particle's resistance to motion is not even a specific amount, calculable from the charge, speed of light c and physical dimensions, but depends on the amount and distribution of all the other particles in the universe.

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There is a hierarchy of interactions in electrodynamics in order of decreasing strength as follows:

1. Charge to charge – Coulomb force
2. Charge to neutral electric dipole – inertial force
3. Neutral electric dipole to neutral electric dipole – gravitational force
4. Charge to neutral electric quadruple – dust or plasma aggregation & rotation
5. Neutral electric dipole to neutral electric quadruple – dust aggregation & rotation
6. Neutral electric quadruple to neutral electric quadruple – dust aggregation & rotation etc.
7. The charge to vibrating neutral electric dipole force is second order in the hierarchy.

The sentence "In the next section the interaction force that gives rise to the force of gravity will be considered." should have been "On the grand scale the interaction force that gives rise to the force of gravity is defined in terms of inertial mass as…."

> "Thus the interaction force that Einstein referred to above that gives rise to the force of inertia of any specific dipole pair is due to the vibration of that pair, and all the other vibrating, neutral, electric dipoles in the rest of the universe."

would perhaps be better stated

> "Thus the interaction force that Einstein referred to above that gives rise to the force of inertia of any specific dipole pair is due to the interaction of that dipole with all the other charges in the universe."

In a similar manner one could say that

> "The interaction force that gives rise to the force of gravity on any specific electric dipole pair is due to the interaction of that electric dipole with all the other electric dipoles in the universe."

The notion of mass is somewhat poorly defined in science. In the Standard Model of elementary particles and Einstein's theory of relativity, mass is an inherent property of a point-elementary-particle. Thus the specifics of the interactions within the structure of the particle are not taken into account. However, if one measures the *mass* of an atom, it is not the sum of the masses of the electrons, protons, and neutrons, but something less indicating that if the mass of a particle depends on internal structure, it changes within the environment of the atom. If one measures the total *charge* of an atom, it is exactly the sum of the charges of the electrons, protons, and neutrons in the atom. *Thus mass is not an inherent fixed quantity like charge!*

From the definition of the force of inertia $F = ma$, mass m is a measure of resistance to motion in some environment. In fluid dynamics the resistance to motion of a particle depends on the density of the fluid around it. Thus mass depends more on the environment than on fixed inherent local properties of the particle of mass. The total resistance to motion is not inherent to the particle, but depends on its environment.

In the work of Barnes and Bergman there is a feedback effect due to Lenz's Law on a charged elementary particle causing it to have a mass of inertia. From Mach's Principle we can see that there are two parts to this mass. The first is the local asymmetry term, and the second is the contribution on the grand scale from the rest of the universe. The first term will depend intimately on the internal structure of the particle. The second term will depend heavily on the symmetry of the universe.

There can be multiple types of contributions to inertial mass. In my work so far I have concentrated on the hierarchy of electrodynamic interactions, i.e.

1. charge to charge - Coulomb force
2. charge to electric dipole - force of inertia

3. electric dipole to electric dipole - force of gravity, etc.

Besides these supposedly primary contributions, there can be secondary and tertiary contributions just as in the case of the atom where there are fine structure and hyper-fine structure contributions. My electrodynamic force derivation of the force of inertia is able to explain the origin of the cosmic microwave background radiation and its spectrum, the unusual gyroscope experiments of Eric Laithwaite in defiance of Newton's force of inertia, the constant high velocity of the outer arms of spiral galaxies in defiance of General Relativity requiring the invention of dark matter, and the general expansion of the universe in defiance of General Relativity requiring the invention of dark energy.

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Reply by Charles Wm. (Bill) Lucas, Jr. See the URLs below for the "politically correct view" that Russell Humphreys relies upon.

http://en.wikipedia.org/wiki/Hafele%E2%80%93Keating_experiment

http://en.wikipedia.org/wiki/Time_dilation_of_moving_particles

http://en.wikipedia.org/wiki/Pound%E2%80%93Rebka_experiment

For time dilation of moving particles, Special and General Relativity theories are point-particle theories. No real elementary particles such as protons, neutron, muons, etc. are point-particles. All have both finite size and internal structure. Since in my work these elementary particles consist of multiple charge current loops, the elementary particles experience an electromagnetic feedback effect when they move that compresses the particle and increases its binding energy and effective mass. The increased binding energy causes the half-life of the muon to increase with velocity. Special Relativity theory mathematically predicts the same result, but for the wrong reasons due to its use of many idealizations such as the point-particle idealization and that space is homogeneous and isotropic that does not correspond to reality.

The gravitational redshift—a tenet extrapolated from Einstein's theory of general relativity—claims that clock rates change with gravitational potential, as a result of space-time being bent by objects of large mass. Ed Dowdye has shown that for starlight passing near the sun there is no bending of the path of the light due to general relativity theory. The only bending of starlight that occurs is when the light passes through the electrical plasma rim of the sun due to electromagnetic effects. At larger distances the predicted bendings of light due to general relativity theory are *not* observed.

With regard to the Pound-Rebka experiment at Harvard in 1959 to detect the redshift and blue-shift in light moving in a gravitational field due to clocks running at different rates at different places in a gravitational field, note that the Mossbauer effect was used. In 1958 Mossbauer had reported that all the atoms in a solid lattice absorb the recoil energy when a single atom in the lattice emits a gamma ray. The test is based on the following principle: When an electron in an atom transits from an excited state to a ground state, it emits a photon with a specific frequency and energy. When an electron in an atom of the same species in its ground state encounters a photon with that same frequency and energy, it will absorb that photon and transit to the excited state. If the photon's frequency and energy is different by even a little, the atom cannot absorb it (this is the basis of quantum theory). When the photon travels through a gravitational field, its frequency and therefore its energy will change due to the gravitational redshift. As a result, the receiving atom cannot absorb it. But if the emitting atom moves with just the right speed relative to the receiving atom the resulting Doppler shift cancels out the gravitational shift and the receiving atom can absorb the photon.

http://en.wikipedia.org/wiki/Doppler_shift

The "right" relative speed of the atoms is therefore a measure of the gravitational shift. The frequency of the photon "falling" towards the bottom of the tower is blue-shifted. Pound and Rebka countered the gravitational blue-shift by moving the emitter away from the receiver, thus generating a relativistic Doppler redshift. The energy associated with gravitational redshift over a distance of 22.5 meters is very small. The fractional change in energy is given by $\partial E/E = gh/c^2 = 2.5 \times 10^{-15}$. Therefore short wavelength high energy photons are required to detect such minute differences.

http://en.wikipedia.org/wiki/Electromagnetic_spectrum

When it transitions to its base state, the 14 keV gamma rays emitted by iron-57 proved to be sufficient for this experiment.

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The Doppler shift required to compensate for this recoil effect would be much larger (about 5 orders of magnitude) than the Doppler shift required to offset the gravitational redshift. But in 1958, Mössbauer reported that all atoms in a solid lattice absorb the recoil energy when a single atom in the lattice emits a gamma ray.

http://en.wikipedia.org/wiki/Rudolf_M%C3%B6%C3%9Fbauer

http://en.wikipedia.org/wiki/Lattice_model_(physics)

http://en.wikipedia.org/wiki/M%C3%B6ssbauer_effect

<http://en.wikipedia.org/wiki/Pound%E2%80%93Rebka_experiment#>

Therefore the emitting atom will move very little. However, the notion of a photon being emitted or absorbed on a single electron of an atom is not supported by the Mossbauer effect. The wave nature of light is supported by the Mossbauer effect where the crystal lattice acts as an antenna for emission and absorption of light.

In Dr. Lucas's electrodynamic theory of gravity, vibrating neutral electric dipoles are the source of the gravitational force. The movement of the vibrating neutral electric dipoles changes the strength of the gravitational field to match the red and blue shifts. Thus there is no role for Einstein's general relativity theory. This result is also consistent with the work of Ed Dowdye showing that there is no gravitational bending of starlight due to general relativity. Thus the so-called gravitational redshift of light supports only the electrodynamic theory of gravity, *not* General Relativity theory.

Charles W. Lucas, Jr.
Mechanicsville, Maryland

2.6. A short resume of possibilities for direct experimental and theoretical verification of innovated theory of Gravitation concerning Particle-Wave Duality and Matter Waves:

1. Vortex flow meter, Karman Vortex Streets, vortices-frequency, and Strouhal-Reynolds number, in linear and robust relationship to fluid flow velocity, can be explained using here-elaborated coupling between linear and spinning motions, including associated matter-waves and particle-wave duality concepts. A similar concept can be analogically extended to motions of planets within planetary systems. Modern engineering is using vortex flowmeters since an exceptionally long time, without real and fundamental explanation and insight why fluid flow velocity is directly and linearly proportional to vortices frequency (see Chapter 4.1, where vortex flow meter equation is developed and explained as the consequence of liner and spinning motions coupling).

2. *The work of matter-waves and associated gravitation related forces we can find in analyzing a* ***spin-stabilized satellite.*** *This is a satellite, which has the motion of one axis held (relatively) fixed by spinning the spacecraft around that axis, using the gyroscopic effect. The attitude of a satellite or any rigid body is its orientation in space. If such a body initially has a fixed orientation relative to inertial space, it will start to rotate, because it will always be subject to small torques. The most natural form of attitude stabilization is to give the rigid body an initial spin around an axis of minimum or maximum moment of inertia, meaning in the same direction where helical matter waves tend to be naturally created. The body will then have a stable rotation in inertial space. Rotation about the axis of minimum moment of inertia is at an energy maximum for a given angular momentum, whereas rotation about the axis of maximum moment of inertia is at a minimum energy level for a given angular momentum. In the presence of energy loss, as is the case in satellite dynamics, the spin axis will always drift towards the axis of maximum moment of inertia. For short-term stabilization, for example, during satellite insertion, it is also possible to spin-stabilize the satellite about the axis of minimum moment of inertia. However, for long-term stabilization of a spacecraft, spin stabilization about its axis of maximum moment of inertia must be used (read about Lyapunov stability concept). Here is the place to explain, harmonize and generalize mentioned items with* ***PWD****, matter waves conceptualization, and convenient mathematics (as vastly elaborated in this book).*

We can also find supporting background to helical matter waves associated with modern-guns bullets motion. Rifling is the process of making helical grooves in the barrel of a gun or firearm, which imparts a spin to a projectile around its long axis. This spin serves to stabilize a projectile gyroscopically, improving its aerodynamic stability and accuracy. Bullet stability depends primarily on gyroscopic forces, the spin around the longitudinal axis of the bullet imparted by the twist of the rifling. Once the spinning bullet is pointed in the direction the shooter wants, it tends to travel in a straight line, until it is influenced by outside forces, such as gravity, wind, and impact with the target. Without spin, the bullet would tumble in flight. Modern rifles are only capable of such fantastic accuracy because the bullet is stable in flight (thanks to the gyroscopic effect). Even spherical projectiles must have a spin to achieve any acceptable accuracy. We could analogically reverse the cause and effect in the same situation (of spinning bullets propagation), and say that particle in inertial, stationary, and stable motion should have helical, spinning matter wave around its long axis of propagation, because such matter wave and moving particle are in mutually relative spinning motion (either one or the other is spinning). Such factual situation (of enormously increased bullets accuracy and stability) should be verifiable by mathematical relations that are used in matter waves' conceptualization.

3. In addition, since linear and helix or spinning and rotating motions (of masses) are mutually complementary and united (in relation to matter-waves, or **PWDC** elaborated in this book, Chapter 4.1), we could imaginatively, creatively, and analogically extrapolate such concepts from atoms to planetary systems and galactic formations and try to differently address Hubble's law. On some way, our universe is globally rotating and spinning, following helix-like paths of associated matter waves. What we observe as red or blue, Doppler shifts (of electromagnetic radiation) coming from remote, deep space, could be consequences of such globally present, macro-rotating effects of distant masses. As we know, the tangential velocity of the certain rotating mass, $\mathbf{v_t}$ is equal to the product of relevant orbital velocity ω, and relevant radius $\mathbf{R}$, $v_t = \omega R$. Hubble's law is maybe saying something similar, such as, $v = (v_t) = H_0 R$, where H_0 is Hubble constant, which could be certain metagalaxy, orbital (or helicoidally spinning associated matter waves) velocity. If an expansion of our Universe is on some ways partially mistaken and masked by, or related to such macro-rotation, this will open a window into amazing new Cosmology research areas.

4. Another similar phenomenon is related to helical liquid funneling (spiral spinning) when liquid is in a vessel that has an open hole, or a sink at the bottom. Outgoing liquid flow speed should be directly proportional to the spiral, funneling frequency of the liquid in the same vessel (this is the proposal for an experimental verification). The explanation should take into consideration that every linear motion is intrinsically linked to spinning in the same direction of movement (as elaborated in this book).

5. Macro matter-waves related situation (that is analogical to micro-world, de Broglie matter-waves) are also waves on a quiet water surface created by some moving object (a boat), where the water surface is visualizing matter-waves associated to a moving object. An average wavelength of such surface water-waves should be roughly equal to $\lambda = H/p = H/mv$ (or inversely proportional to the motional object speed, where (m, H) = constants). Of course, here we should be able to show the applicability of other matter-waves relations elaborated in this book, such as:
$E_k = \tilde{E} = \frac{mv^2}{2} = \frac{pv}{2} = Hf, \quad \Rightarrow u = \lambda f = \frac{v}{2}$. For instance, we can easily measure if the phase speed of surface water-waves, u, behind moving particle (or boat), is two times smaller than particle speed v.

6. Turbulences (including vortex turbulences) are also direct manifestations, or imprints, streaming and reverberations of matter-waves associated with masses motions in and around fluid environments. Fluids (in relative motion to the specific particle) should present sensitive sensor bodies (or spatial antennas and displays) for detecting and visualizing matter waves, vortices, spinning, and turbulences, including the possibility to detect astronomic macro matter-waves. Of course, cosmic macro matter waves could be detected when observing low-frequency acoustic fields' complexity inside vast lakes and ocean spaces. Present conceptualization related to Fluid dynamics and Navier-Stokes equations should also be enriched and optimized by considering here-elaborated matter-waves manifestations, as helically rotating fields' perturbations around and behind motional particles (see chapter 4.1), and the same should apply to enriching matter-waves concepts with elements of Fluid dynamics and Navier-Stokes equations. Also, we could extend familiar analogical associations to orbital planetary motions in solar systems (see Chapter 2. Gravitation; 2.3.3. Macro-Cosmological Matter-Waves and Gravitation).

7. Many publications are showing (and experimentally documenting) weight reduction when certain spinning discs, spinning magnets, and gyroscopes, combined with other rotating and oscillating motions are specifically coupled (see much more in [36]). Such, seemingly anti-gravity effects are in fact consequences of natural, linear and rotational motion couplings (including associated electromagnetic dipoles separation and polarization), when involved spinning discs and oscillatory systems are interacting with balanced natural motions, producing unbalanced effects of weight reduction (as long as we maintain such movements). Here, we should not forget that besides Newton linear motion forces, we also have coupled effects of rotational or torque forces. A certain specific combination of implemented torque components (by spinning disks or magnets) can be the forces acting against gravitation.

8. Universally valid and known effects of diffraction of light rays, particle beams, fluid beams, jets, and similar phenomena, could also be explicable if we consider that certain repulsive force (in relation to associated matter-waves and spinning) is developing between effective mass (or matter-waves) packets of “parallel flow elements”. Such repulsive force should be causally related to the resonant half-wavelength of involved matter-waves, being the consequence of unity and couplings between linear and spinning motions. The force law that is addressing diffraction (or beams repulsion) should respect Newton-Coulomb $1/r^2$ force law (see [3]). If centers of active mass packets in described parallel motion are mutually separated by one half-wavelength, the repulsive force (or measured diffraction angle) should be maximal.

9. Theories related to “standing-waves” quantizing of planetary systems (as one elaborated in this book, in chapter 2.) that are directly analog to quantizing in Bohr atom model, are practically showing the triumph of here-elaborated matter waves and linear-rotational motions coupling concepts. In such modeling it is possible to integrate electromagnetic effects, like in early atom models (see [63] and [67]), showing existence of more complete unity between mechanical motions, gravitation, and electromagnetic fields, since such rich quantization in planetary systems (like in atoms) cannot exist without the dominant presence of electromagnetic forces and fields. In such quantized systems of synchronized and periodic (planetary) motions with standing-waves structure, it is possible to associate masses presence only to stable stationary orbits that are equivalent to spatial nodal zones (where orbital acceleration and density are maximal, and oscillating amplitudes minimal, like in resonant mechanical systems and standing waves related acoustic and/or ultrasonic levitation). See more of familiar concepts in [150], and [151] and in [99] from Konstantin Meyl.

10. Our sun and a countless number of stars can be regarded as a variety of blackbody objects (concerning blackbody radiation). Planck’s blackbody radiation formula is mathematically fitted to experimentally measured situations, but the real, essential explanation of what is happening inside of a black body cavity is still missing. We can try to estimate what happens inside a black body cavity where we have complex, random motion of hot gas particles, random light emissions, absorptions, photons, and electrically charged particles collisions and scattering (including participation of particles with magnetic moments). We only know from Planck’s formula the resulting (fitted and averaged) spectral distribution of outgoing light emission, in the case when we make a small hole on the surface of a black body, and let photons be radiated and measured in the external, free space of a black body. This external light radiation is characterized by free photons where each photon has the same phase and group velocity $v = u = c = \text{constant}$. This is not the case inside the black body cavity, since there are many mechanical and fields interactions between photons, gas particles, matter waves, and cavity walls, and there we have broad distributions of group and phase velocities of different energy-momentum entities,

$0 \le 2u \le \sqrt{uv} \le v \le c$. A significant number of wave packets (de Broglie matter wave groups with mutually united or coupled mechanical and electromagnetic properties, with linear and spinning motion components), inside a blackbody cavity, permanently interact (among themselves, as well as with the cavity and gas particles), and we cannot consider them being freely propagating wave groups, or stable and synchronized standing matter-waves formations. It is logical (as the starting point in an analysis of such case) to imagine that mean particle or group velocity of such wave groups is (in average) directly proportional to the blackbody temperature, and when gas temperature (inside a black body radiator) is relatively low, then we should dominantly have motions with non-relativistic particle velocities ($v << c \Rightarrow v \cong 2u$). When a temperature is sufficiently (or remarkably) high, we should dominantly have the case of relativistic particle motions with high speeds ($v \approx c \Leftrightarrow v \approx u \approx c$). There is a big difference between free wave groups, like free photons in open space, and mutually interacting (de Broglie) matter-waves (inside of a limited space of a black body cavity). On the contrary, in most analyzes of similar situations in modern Quantum Mechanics, we do not find that such differentiation is explicitly underlined and adequately treated (mostly we see that de Broglie matter waves are treated similarly to free photons or to other free wave groups, or as virtual and artificial probability waves). Also, in mathematical development of Planck's blackbody radiation law, we can essentially find specific particularly suitable (oversimplified, mainly poor, and unrealistic) modeling and curve-fitting situations, where phase velocities of a black body photons are always treated as the velocity of free (externally radiated) photons, or as $u = \lambda f = v = c = \text{Constant}$. This book is offering new elements to understand and develop blackbody radiation formula on a more natural way (see such elaborations in chapter 4.1 and chapter 9).

11. *Very much neglected, or still not well conceptualized approach in addressing mechanical systems and mechanical-circuits, is something that is very much known and practiced in electric-circuits and electromagnetic theory. Every electrical circuit, to be completely described and understood, should be treated as a closed circuit, or as a network with electric components, that has its front-end (input generator or electricity source) and its last-end or load. Also, all internal circuits (as network elements) should be treated and analyzed as closed current-flow circuits, both as being, either DC, or AC, or mixed currents circuits. Analogically valid is that all mechanical systems or mechanical and acoustical circuits should also have front and last ends (meaning sources and loads). Currents in mechanical systems (or circuits) are forces and angular moments, presenting temporal flow functions of linear and angular momenta. In the contemporary Physics (mechanics, acoustics, vibrations theory...) we are still too often presenting and analyzing open-ends circuits, either without front or last-ends or without both. Doing this way, we are not able to understand the real and complete nature of the specific mechanical or planetary (or astronomic) system, and its connections with other mechanical systems, and we do not see that such systems could also be connected on number of ways, realizing energy-momentum exchanges with different matter states and matter waves. Mechanical currents (meaning forces and angular moments) can have both DC and AC nature (using the analogy with electric currents; -see much more about electromechanical similarities in the first chapter of this book). Also, hypothetical, and innovative proposals presented in this book are indicating that electric currents and effective mechanical orbital moments (within the specific motion of particles and masses) could often be mutually coupled and synchronized, followed by magnetic field effects, what should produce results of gravitational attraction. Only such conceptualization, analysis, and understanding of all mechanical systems could bring the proper understanding of what is happening in our Universe. If carefully and*

imaginatively analyzed, concepts of Nikola Tesla about electromagnetic phenomenology of radiative energy and "Dynamic Gravity Theory" are creatively illustrating similar, closed electromechanical circuits, matter-waves connections, and interactions between all masses in our Universe. See more in the literature under [97] until [101], in the first chapter about Analogies, in chapter 4.1, around Fig. 4.1.6, and in chapters 8. and 9.

Now is also the right place to mention connections between here-introduced concepts of closed current circuits, couplings, analogies, and equivalency between linear and angular motions, and associated electric currents and magnetic field effects. Such links and couplings are conceptually presented on illustrations in chapter 4.1, on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, with equations under (4.3), and later.

Also, we could mention simple experiments about closed circuits of energy-momentum flow from prof. Eric Laithwaite, where he demonstrated unusual and extraordinary couplings between linear and rotational motions of spinning gyroscopes, and similar magnetic field effects (see more in [102]).

The existence of realistic matter wave's connections between masses is also elaborated in this chapter, concerning planetary macro matter waves, around equations starting from (2.11.14) until (2.11.14)-h.

2.7. **Dark Matter, Dark Energy, Hidden Invisible Mass, and similar imaginative and virtual entities** could be qualitatively, mathematically, and creatively explained by the fact that mass, moments, and energy involved in "4-vectors of Energy-Momentum" invariant expressions from Relativity Theory (Minkowski space formalism) are presentable as complex or hyper-complex mathematical, Analytic Signal functions, having Real, Imaginary and Apparent parts. See more in chapter 10 of this book (10.1 **Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors concerning matter-waves and particle-wave duality**). Such background could support macrocosmic effects of resonant synchronization, entanglements, and the existence of Dark Matter & Energy, presenting mentioned "dark" items as imaginary or apparent mass components, since a mass of our Universe is "Mass in permanent motion", where linear and angular motions are specifically united, respecting **P**article **W**ave **D**uality **C**oncepts, as widely elaborated in this book.

2.8. We anyway need to start from Newton and Einstein's visions or interpretations of Gravity, since this already works well mathematically, and works well in our spatial engineering. Where we have dilemmas, problems and questions is how to understand or explain the real sources of universal gravitation. Both of existing theories addressing Gravitation are only and superficially assuming, stating, and describing what is happening regarding attraction of masses, and using or offering practical mathematical ways and formulas to calculate elements of masses motions in the field of Gravitation (of course, respecting Classical Mechanics and conservation laws of Physics). There are still number of unanswered questions related to essential explanations of origins of Gravitation, such as:

1. Why Gravitation cannot be shielded, or how it could be shielded, reflected, attenuated, and/or differently manipulated?
2. Why present theories of gravitation deal only with attractive forces between masses? Do we have examples of repulsive gravitation related forces (what we could consider as antigravitation)?
3. Why Newton and Coulomb laws have the same mathematical form, but mass and electric charge are not mutually analogue (since electric charge is analogue to mechanical moments)?
4. What is the real background or explanation regarding Black or Dark matter, masses, and energies in relation to gravitation?

Presently, we can address such questions mostly conceptually, by offering innovative and challenging visions and answers about origins of Gravitation, and deeper explanations how Gravitation should work (in its background). It is still too early to offer ready-made antigravity devices and fully developed and mathematically well supported, new theory. Something like that could be developed and optimized, based on dedicated, serious scientific and engineering activities.

Anyway, new and more advanced theory of Gravitation will not annulate or neglect existing Newton and Einstein theories, but it should open number of new engineering options, and give much better explanations about origins of Gravitation. In this book, mentioned questions and problems are frequently addressed, with number of challenging examples and proposals, but still much better resume is waiting to be made. Briefly, we could summarize the following conclusions,

1. Is shielding and manipulating Gravitation possible**.**

a) As presently known and tested, when applying certain closed cage, envelope, metallic, or solid shield, around any mass, we do not find its weight reduction. Gravitation is on some way penetrating applied shielding. This is implicating the concept that sources of Gravitation are extended atomic fields, producing an electromagnetic dipoles polarization that is becoming part of applied shielding, being conveniently aligned on the way that applied shield is effectively transparent for such electromagnetic polarization formations. This is further implicating that around involved masses we should have specific (and omnidirectional) background vortex or rotation of an invisible and very fine fluid. This resembles the concepts of Nikola Tesla and Rudjer Boskovic regarding Gravitation and Radiant energy, this way creating standing-waves formations between mutually attracting and conveniently polarized masses (respecting Coulomb force, which effectively, or electromagnetically penetrates applied shielding).

b) There are also indications (maybe still under suspicion and highly hypothetical) that gravitation shielding, weight reduction, and gravitation field-reflecting effects could be produced using shielding objects that have specific, spatial resonant structures. Russian professor Viktor Stepanovich Grebennikov, [149], described number of effects familiar to gravitation, levitation, and temporal-spatial matter-waves phenomenology of structurally periodic geometric structures created by insects, or as natural beehive cells, dry honeycomb structures, or objects artificially assembled on a similar way, creating weight reduction.

2. Why we know only about attractive forces between masses.

   This is result of our incomplete conceptualization of Gravitation. Presently we find only attractive forces of Gravitation because our concept and modelling of gravitation is still restrictive (meaning Newton and Einstein concepts are incomplete and restrictive). Stable macro masses are agglomerations of smaller masses or involved atoms, imaginable as being created (or placed) in nodal zones of certain structurally resonating system with standing matter waves between involved masses. In such nodal zones we can naturally detect only attractive, or agglomerating forces, but standing waves structures also have anti-nodal zones, where we can detect only repulsive forces. This again means that Nikola Tesla and Rudjer Boskovic qualitatively formulated very good concepts about Gravitation, and that our Universe is structurally resonating.

3. Why Newton and Coulomb laws have the same mathematical form.

   Because all such forces initially or essentially originate from electric and magnetic charges and polarizations, where we naturally apply Coulomb type of interactions, and Newton force between masses is mathematically identical or analogue to Coulomb law. We can conceptualize and present masses as being directly proportional to involved (internal) electric and magnetic dipoles and charges, and this way, when we apply Newton law, we effectively apply Coulomb law.

4. What Black or Dark matter, masses, and energies in relation to gravitation are.

   Mass and energy are not what we find only within solid boundaries of certain "mass-energy-moments" formation. Spatially extended mass-energy formations should be differently conceptualized and characterized. Different mass-energy aspects are part of dynamic, structurally resonating Universe. Also, remote spiral galaxies (where we find that Newton and Einstein gravitation are not working well) should involve different contributions of free-standing and interacting, electric and magnetic charges.

[♣ COMMENTS & FREE-THINKING CORNER (still in preparation and brainstorming phase)**:**

2.3.3-1 Binary Systems, Kepler and Newton Laws and Matter Waves Hosting

The essential fact in the background of (2.11.10) - (2.11.14) is that gravitation is the central force. Its direction is always along a radius, either towards or away from a point, we are using as an origin or force center. The magnitude of such central force depends solely upon the distance from its origin, **r**. We can present such forces as, $F_{m-M} = \frac{GmM}{r^2} = F(r),\ \vec{F}(r) = F(r)\cdot\frac{\vec{r}}{r}$. Central forces are interesting because we find them very often in physics. The gravitational and electrostatic forces are central forces (as well as forces between permanent magnets). Much of classical mechanics or physics can be placed in the framework of elaborated applications of Newton Laws. Let us start with the Second Newton Law, $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$, and express the associated torque and angular momentum as, $\vec{\tau} = \vec{r}\times\vec{F} = \vec{r}\times m\vec{a}$, $\vec{L} = \vec{r}\times\vec{p}$. Since torque is the time derivative of angular momentum, let us find the torque for central forces (where $\vec{F}(r)$ is parallel with $\vec{r}$) as, $\left(\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \vec{r}\times m\vec{a} = \vec{r}\times\vec{F} = 0\right) \Rightarrow \vec{L} = \text{const.}$.

Consequently, an orbital planetary motion has constant angular momentum because gravitational force is the central force. A little bit later we will see that this is the real origin of quantization in physics (equally applicable to Coulomb Law related analogical situations), as well as to micro-world of atoms and elementary particles where building blocks have constant angular moments or spin characteristics.

Let us now analyze the simplified case of gravitational attraction between two masses $m_1 = m$ and $m_2 = M$ from the point of view of Binary Systems relations in their center of mass coordinate system. The total separation between the centers of the two masses is $\vec{r} = \vec{r}_1 + \vec{r}_2$. We may define the center of a mass point placed between two objects through the equations,

$$m_1 r_1 = m_2 r_2,\ r = r_1 + r_2,\ r_1 = \frac{m_2}{m_1 + m_2} r,\ r_2 = \frac{m_1}{m_1 + m_2} r. \tag{2.11.14-1}$$

From gravitational attraction between m_1 and m_2 nothing will change if we imagine that m_1 and m_2 may be in a uniform rotational motion around their common center of mass since masses will at the same time experience mutually repulsive balancing centrifugal force (possible spinning is not considered). Let us imagine that m_1 and m_2 are rotating (around their common center of mass) with certain angular speed $\omega = \frac{2\pi}{T}$, what can be described with another set of equations,

$$\begin{gathered}\omega = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v}{r},\ \frac{v_1}{v_2} = \frac{r_1}{r_2},\ p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r = m_r v_r,\ \vec{p}_1 + \vec{p}_2 = 0, \\ \vec{v}_i = \frac{d\vec{r}_i}{dt},\ \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_r = \vec{v}_1 + \vec{v}_2,\ v_1 v_2 = \omega^2 r_1 r_2,\ v_i = \omega r_i,\ \frac{v_1 v_2}{v^2} = \frac{r_1 r_2}{r^2}.\end{gathered} \tag{2.11.14-2}$$

Here v_1 and v_2 are tangential velocities of m_1 and m_2. In cases of such circular, rotational motions, every mass is experiencing certain centrifugal (mutually opposed) force with a tendency to separate them, for example,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{dp}{dt} = \frac{dp_r}{dt} = \frac{m_r v_r^2}{r} = \frac{p_r v_r}{r} \Leftrightarrow m_1 v_1 = m_2 v_2 (= p = m_r v_r = p_r) \Rightarrow$$
$$\left\{ \begin{array}{c} \Rightarrow \text{after integration} \Rightarrow \\ v_r = \frac{dr}{dt} = \frac{p_0}{m_r} \cdot \frac{r}{r_0},\ p_r = m_r v_r = p_0 \cdot \frac{r}{r_0}, F_r = F_c = \frac{dp_r}{dt} = \frac{p_0}{r_0} v_r = p_0 \omega \cdot \frac{r}{r_0},\ [p_0, r_0] = \text{constants} \end{array} \right\}. \quad (2.11.14\text{-}3)$$

If the distance between two masses m_1 and m_2 is remaining unchanged (stable orbital motions), mutually opposed (or repulsive) centrifugal forces should be balanced with similar (central) attractive force between them, which is Newton force of gravitation F_g. Conceptualizing given case of a stable Binary System this way, we are developing and formulating Kepler's third law, as follows.

$$\left\{ \begin{array}{l} F_g = G\frac{m_1 m_2}{r^2} = F_c = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = m_1 v_1 \omega = m_2 v_2 \omega = m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{m_1 m_2}{m_1 + m_2} v_r \omega = \frac{m_1 m_2}{m_1 + m_2} r\omega^2 = m_r r\omega^2 = \frac{m_r v_r^2}{r} \\ m_r = \frac{m_1 m_2}{m_1 + m_2},\ v_r = \omega r = \frac{dr}{dt},\ v_1 = \frac{m_2}{m_1 + m_2} v_r = \omega r_1,\ v_2 = \frac{m_1}{m_1 + m_2} v_r = \omega r_2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \omega = \sqrt{G\frac{m_1 + m_2}{r^3}} = \frac{2\pi}{T}, \Leftrightarrow \left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{G(m_1 + m_2)}. \quad (2.11.14\text{-}4)$$

Another conclusion radiating from here is that natural tendency of masses (regarding stable Binary Systems, or multi-mass systems) is to create uniform or stationary rotational motions (around their common center of mass), this way balancing attractive Newton force with associated centrifugal force. If such rotation is not a visible case, at least mathematically and by respecting relevant conservation laws every Binary System could be equally presentable as a case of mutually coupled rotating bodies (including rotating disks, toroids…). The coupling force in question (for instance in cases of electromagnetically neutral bodies) is the gravitation.

We could also say that boundary or asymptotic tendency (or just mathematically equivalent state in the same center of mass coordinates) of Binary Systems is that initial masses m_1 and m_2 can be effectively replaced by one bigger central mass which is equal $m_c = m_1 + m_2$ and placed in their common center of mass position (being there in a state of rest). In addition to such central mass m_c, there is another, (mathematically generated) reduced mass $m_r = \frac{m_1 m_2}{m_1 + m_2}$, which is rotating around the central mass m_c. Such reduced mass m_r will have the total kinetic energy and orbital moment of masses m_1 and m_2. The distance between m_r and m_c (or relevant circle radius) is again the same as before $r = r_1 + r_2,\ r_1 = \frac{m_2}{m_1 + m_2} r,\ r_2 = \frac{m_1}{m_1 + m_2} r,\ m_1 r_1 = m_2 r_2 \Rightarrow p = m_1 v_1 = m_2 v_2$. Angular (mechanical rotating) velocity ω of the new Binary System m_r and m_c will stay the same as found previously for Binary System of masses m_1 and m_2 ($\omega = v_1 / r_1 = v_2 / r_2 = v_r / r = 2\pi f_m$). The attractive gravitational force between m_1 and m_2 will be the same as the attractive force between m_r and m_c, for instance:

$$F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = \frac{m_r v_r^2}{r} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{m_1 v_1^2 + m_2 v_2^2}{r_1 + r_2} = m_r r\omega^2 = m_1 r_1 \omega^2 = m_2 r_2 \omega^2. \quad (2.11.14\text{-}5)$$

We can also find involved orbital moments of rotating masses m_1, m_2 and m_r, taking into account that the total orbital moment of a Binary System is conserved (constant).

$$\left\{\begin{bmatrix} L_1 = p_1 r_1 = m_1 v_1 r_1 = m_1 r_1^2 \omega = \frac{m_1 v_1^2}{\omega} = J_1 \omega, \\ L_2 = p_2 r_2 = m_2 v_2 r_2 = m_2 r_2^2 \omega = \frac{m_2 v_2^2}{\omega} = J_2 \omega, \end{bmatrix} \& \begin{bmatrix} \frac{L_1}{L_2} = \frac{J_1}{J_2} = \frac{r_1}{r_2} = \frac{v_1}{v_2}, \ p_1 = p_2 = p = p_r \\ L_r = p_r r = m_r v_r r = m_r r^2 \omega = \frac{m_r v_r^2}{\omega} = J_r \omega \end{bmatrix}\right\} \Rightarrow$$

$$\vec{L}_i = \vec{r}_i \times \vec{p}_i \Rightarrow L_i = m_i v_i r_i = m_i r_i^2 \omega = \frac{m_i v_i^2}{\omega} = J_i \omega = \frac{2}{\omega} E_{ki},$$

$$L = L_1 + L_2 = (J_1 + J_2)\omega = \frac{2}{\omega}\left(\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}\right) = \frac{2}{\omega}(E_{k1} + E_{k2}) = \frac{2}{\omega} E_{orbital} = \frac{m_r v_r^2}{\omega} = \frac{J_r \omega^2}{\omega} =$$

$$= J_r \omega = L_r = (J_1 + J_2)\sqrt{G \frac{m_1 + m_2}{r^3}} = const., \ J_r = J_1 + J_2, \ v_r = v_1 + v_2 = \omega r, \tag{2.11.14-6}$$

$$E_{ki} = \frac{1}{2} m_i v_i^2 = \frac{1}{2} p_i v_i = \frac{1}{2} m_r v_r^2 \frac{r_i}{r} = \frac{1}{2} J_i \omega^2 = \frac{1}{2} L_i \omega, \ E_{kr} = E_{ki} \frac{v_r}{v_i} = E_{k1} + E_{k2},$$

$$\frac{E_{k1}}{v_1} = \frac{E_{k2}}{v_2} = \frac{E_{kr}}{v_r} = \frac{E_{ki}}{v_i} = \frac{m_i v_i}{2} = \frac{m_r v_r}{2} = \frac{1}{2} p, v_i = \omega r_i .$$

Now we will be able to show that for (isolated) Binary Systems that are conserving total orbital moment, specific orbital (or kinetic, or motional) energy is in some way quantized, or given by similar expression like Planck's energy of a photon (except that new Planck-like H-constant will be much bigger compared to Planck constant of micro-world).

$$E_{orbital} = E_{k1} + E_{k2} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} J_1 \omega^2 + \frac{1}{2} J_2 \omega^2 = \frac{1}{2}(J_1 + J_2)\omega^2 = \frac{1}{2}(L_1 + L_2)\omega =$$

$$= \frac{1}{2} m_r v_r^2 \frac{r_1}{r} + \frac{1}{2} m_r v_r^2 \frac{r_2}{r} = \frac{1}{2} m_r v_r^2 \left(\frac{r_1}{r} + \frac{r_2}{r}\right) = \frac{1}{2} m_r v_r^2 = \frac{1}{2} J_r \omega^2 = E_r = \frac{1}{2}\left[(J_1 + J_2)\omega\right] \cdot \omega = \tag{2.11.14-7}$$

$$= \frac{1}{2}\left[const\right] \cdot \omega = Const \cdot \omega = H \cdot f = H \cdot f_0, \ \omega = 2\pi f_m = \frac{2\pi}{T} = \frac{H}{Const} \cdot f_0, \ f = f_0 \neq f_m.$$

The next significant remark here (relevant for Binary Systems) is that ***to experience an attractive gravitational force, rotating bodies should rotate in the same direction (both having mutually collinear angular speed and angular moments vectors).*** If the rotation is not externally (or macroscopically) detectable, it should be in some ways internally (intrinsically) present in Binary Systems relations. Simply, gravitation without rotation cannot be explained. We can also find expressions for such inherently associated angular velocity and angular momentum of Binary Systems as,

$$F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = F_c = \frac{m_r v_r^2}{r} = \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = m_1 v_1 r_1 \frac{v_1}{r_1^2} = m_2 v_2 r_2 \frac{v_2}{r_2^2} = J_1 \omega \frac{v_1}{r_1^2} = J_2 \omega \frac{v_2}{r_2^2} = L_1 \frac{v_1}{r_1^2} = L_2 \frac{v_2}{r_2^2} = J_r \omega \frac{v_r}{r^2} = L_r \frac{v_r}{r^2},$$

$$F_g r = G\frac{m_1 m_2}{r^2} r = G\frac{m_1 m_2}{r} = G\frac{m_r m_c}{r} = F_c r = \frac{m_r v_r^2}{r} r = m_r v_r^2 = m_r r^2 \omega^2 = J_r \omega^2 = L_r \omega = 2E_r = 2E_{orbital} = 2Hf,$$

$$\left(m_1 r_1 = m_2 r_2, \ m_1 v_1 = m_2 v_2 = m_r v_r = p\right) \Rightarrow F_t = \frac{dp}{dt} = F_\tau, \ \left(\vec{\tau} = \vec{r} \times \vec{F}_t = \frac{d\vec{L}}{dt}, \ \vec{F}_\tau = \frac{\vec{\tau}}{r} = \frac{\vec{r} \times \vec{F}_t}{r} = \frac{1}{r}\frac{d\vec{L}}{dt}\right),$$

$$L_r v_r = G m_1 m_2 = \frac{L_1 L_2}{L_1 + L_2} v_r \Leftrightarrow G = \frac{L_1 L_2}{L_1 + L_2} \frac{v_r}{m_1 m_2} = L_r \frac{\omega r}{m_1 m_2} = \frac{J_1 J_2}{J_1 + J_2} \frac{\omega^2 r}{m_1 m_2} = J_r \frac{\omega^2 r}{m_1 m_2},$$

$$J_r = \frac{J_1 J_2}{J_1 + J_2} = J_1 + J_2 = \sqrt{J_1 \cdot J_2} = \frac{m_1 m_2}{m_1 + m_2} r^2 = m_r r^2, \tag{2.11.14-8}$$

$$L_r = J_r \omega = L = \frac{L_1 L_2}{L_1 + L_2} = L_1 + L_2 = \sqrt{L_1 \cdot L_2} = \sqrt{G\frac{m_1 m_2 J_r}{r}} \Rightarrow \omega = \sqrt{G\frac{m_1 m_2}{J_r r}} = \sqrt{G\frac{m_r m_c}{J_r r}} .$$

Another conclusion to draw is that gravitational constant **G** is the measure of here elaborated intrinsic rotation or angular (mechanical revolving) speed of Binary Systems, leading to another alternative form of Kepler's third law as,

$$\frac{1}{\omega^2} = \frac{1}{(2\pi f_m)^2} = \left(\frac{T}{2\pi}\right)^2 = \frac{J_r r}{G m_1 m_2} = \frac{r^3}{G(m_1 + m_2)}, \quad (2.11.14\text{-}9)$$

f_m (=) Mechanical (planet or satellite) revolving or orbiting frequency.

Shall we have a repulsive gravitational force in cases when masses in Binary Systems are not rotating in the same direction (when important angular moments' vectors are mutually opposed or maybe not collinear) is one of the logical questions to ask here? Let us exercise what could be the answer on a similar question if mentioned masses are also self-spinning (having finite spin moments $\vec{L}_{s1}$ and $\vec{L}_{s2}$, $\vec{L}_i \rightarrow (\vec{L}_i + \vec{L}_{si})$), and how such spinning moments would influence the attractive force/s between them?

$$F_g = F_c = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = \left(\frac{\pi G}{c^4}\right)\frac{m_1 c^2 m_2 c^2}{\pi r^2} = \left(\frac{G}{c^4}\right)\frac{m_r c^2 m_c c^2}{r^2} = \left(\frac{\pi G}{c^4}\right)\frac{E_{t1} E_{t2}}{\pi r^2} = \left(\frac{\pi G}{c^4}\right)\frac{E_{tr} E_{tc}}{\pi r^2},$$

$$\left.\begin{cases} (v_i << c) \Rightarrow E_{ti} = m_i c^2 = \gamma_i m_i c^2 \Rightarrow E_{ti} \cong m_i c^2 + \frac{1}{2} m_i v_i^2 = m_i c^2 + \frac{1}{2} J_i \omega^2 = m_i c^2 + \frac{1}{2}\vec{L}_i \vec{\omega} \\ \left(\vec{L}_i \rightarrow (\vec{L}_i + \vec{L}_{si}), \vec{L}_i = J_i \vec{\omega}, \vec{L}_{si} = J_i \vec{\omega}_{si}\right) \Rightarrow E_{ti} \cong m_i c^2 + \frac{1}{2}(\vec{L}_i + \vec{L}_{si})\vec{\omega} \end{cases}\right\} \Rightarrow \quad (2.11.14\text{-}10)$$

$$F_g \cong \left(\frac{G}{c^4}\right)\frac{\left[m_1 c^2 + \frac{1}{2}(\vec{L}_1 + \vec{L}_{s1})\vec{\omega}\right]\left[m_2 c^2 + \frac{1}{2}(\vec{L}_2 + \vec{L}_{s2})\vec{\omega}\right]}{r^2} = \left(\frac{G}{c^4}\right)\frac{\left[m_r c^2 + \frac{1}{2}(\vec{L}_r + \vec{L}_{sr})\vec{\omega}\right]\left[m_c c^2 + \frac{1}{2}(\vec{L}_c + \vec{L}_{sc})\vec{\omega}\right]}{r^2}.$$

If there is a stable ground in here hypothesized exercise about gravitational force, the presence of spin and orbital moments (of participants) could increase or decrease the total gravitational force between two bodies in a Binary System (depending on relative mutual positions of important orbital and spin moments). Most probably, such contributive spin-related members are too small compared to other involved energy-related members (in cases of planetary or solar systems), and it has been not easy to notice such possibility for addressing modifications of the old Newton Law. ***Here we should enrich the same situation by paying more attention to matter-waves nature of such binary interactions by additional elaborations around equations (2.4-11) to (2.4-17) from the same chapter.***

Apparently, in the absence of repulsive centrifugal forces, planets (or orbits) of specific Solar System would collapse and unite masses with their Sun if there are no orbital rotations. Since the repulsive (centrifugal) gravitational force (as formulated here) is something exclusively related to rotation, most probably that the hidden nature of Gravitation itself is on a similarly effective way intrinsically and inherently also associated with specific (equivalent) rotation inside of matter substance of gravitational masses.

Another step in exercising and hypothesizing the same situation (regarding decoding essence of Gravitation in Binary Systems relations) is to notice connections between different aspects of (involved) energy components and work of "matter vortices" characterized by orbital and spin moments which should have certain torque. There is a tiny imaginative step from here to start thinking how to conceptualize rest masses as some "frozen or self-stabilized matter vortices' states" (since dimensionally torque is measured by the same units as energy).

Until here we did not address any of relativistic aspects of motional masses, since by the nature of astronomic, gravitational Binary Systems (or planetary systems), we can consider that in majority of relevant cases relevant orbital velocities are much smaller compared to the speed of light, and in such cases it is clearly valid,

$$(v_{1,2} << c) \Rightarrow F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} \Leftrightarrow m_1 m_2 = m_r m_c, m_r = \frac{m_1 m_2}{m_1 + m_2}, m_c = m_1 + m_2. \quad (2.11.14\text{-}11)$$

Let us now imagine that some of Binary Systems (not necessarily of exclusively gravitational nature) could be orbital speed sensitive and let us analyze the consequences (again about the important center of mass).

$$\left\{ m_i \to \gamma_i m_i = \frac{m_i}{\sqrt{1-\frac{v_i^2}{c^2}}} = m_i^* \right\} \Rightarrow \left\{ \begin{array}{l} \left(m_r = \frac{m_1 m_2}{m_1+m_2} \right) \to \frac{\gamma_1 m_1 \gamma_2 m_2}{\gamma_1 m_1 + \gamma_2 m_2} = \frac{m_1^* m_2^*}{m_1^* + m_2^*} = m_r^* = \gamma_r m_r \\ (m_c = m_1 + m_2) \to \gamma_1 m_1 + \gamma_2 m_2 = m_1^* + m_2^* = m_c^* \\ (m_1 m_2 = m_r m_c) \to \gamma_1 m_1 \gamma_2 m_2 = m_1^* m_2^* \end{array} \right\} \Rightarrow \quad (2.11.14\text{-}12)$$

$$\Rightarrow \left\{ \begin{array}{l} F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2} = G\frac{m_1^* m_2^*}{r^2} = G\frac{m_r^* m_c^*}{r^2} \\ \gamma_r = \frac{\gamma_1 \gamma_2}{\gamma_1 \frac{m_1}{m_1+m_2} + \gamma_2 \frac{m_2}{m_1+m_2}} \end{array} \right\}.$$

To make a simple validity test, of here elaborated relativistic masses relations, it would be very indicative and almost sufficient to present the case when one of masses is enormously more significant compared to other,

$$(m_1 = m << m_2 = M) \Rightarrow \left\{ \begin{array}{l} m_r = \frac{m_1 m_2}{m_1+m_2} \cong m = m_1 \Rightarrow \gamma_r = \frac{\gamma_1 \gamma_2}{\gamma_1 \frac{m_1}{m_1+m_2} + \gamma_2 \frac{m_2}{m_1+m_2}} \cong \gamma_1 \\ E_{k1} + E_{k2} = E_{kr} = E_{ki}\frac{v_r}{v_i} = (\gamma_r - 1) m_r c^2 \cong (\gamma_1 - 1) m_1 c^2 = E_{k1} \end{array} \right\} \quad (2.11.14\text{-}13)$$

what already looks like the correct result (see also equations (2.4-11) - (2.4-18)).

Many possible consequences are starting from here. For instance, any stable planetary system (with one big solar mass M_s) and number of orbiting planets with masses $\{m_i\}_{i=1}^n$ can be *decomposed and analyzed as an ensemble of simple Binary Systems* with masses m_i orbiting around $(M_c - m_i)$, for example,

$$m_i \cdot (M_c - m_i) = m_{r-i} \cdot M_c,\ M_c = M_s + \sum_{i=1}^{n} m_i,\ m_{r-i} = \frac{m_i \cdot [M_c - m_i]}{M_c},\ \forall i \in (1, n), \quad (2.11.14\text{-}14)$$

where m_i and M_s are only an approximation of a Binary System masses when $M_c \cong M_s >> \sum_{i=1}^{n} m_i$

.

Let us extrapolate two-body problem analysis to an equivalent n-body situation. For instance, imagine that n of astronomic objects (like planets, including one massive star) are mutually approaching, entering specific n-body interaction, and becoming a stable planetary or solar system (this time we will analyze such situation without considering impacts). Kinetic energy balance in such case will be:

$$\left\{ \begin{array}{l} 2-\text{body situation} \\ M_c = m_1 + m_2,\ m_r = \frac{m_1 \cdot m_2}{m_1 + m_2} = \frac{m_1 \cdot m_2}{M_c} \\ E_{k1} + E_{k2} = E_{kc} + E_{kr} \Leftrightarrow \\ \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} M_c v_c^2 + \frac{1}{2} m_r v_r^2 \end{array} \right\} \Rightarrow \text{by analogy}$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{array}{l} \text{n}-\text{body situation} \\ M_c = \sum_{(i)} m_i \ \text{(including solar mass)} \\ \sum_{(i)} E_{ki} = E_{kc} + \sum_{(i)} E_{kr-i} \Leftrightarrow \\ \frac{1}{2}\sum_{(i)} m_i v_i^2 = \frac{1}{2} M_c v_c^2 + \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 = \frac{1}{2} M_c v_c^2 + \frac{1}{2} M_r v_r^2 \end{array}\right\} \Rightarrow \quad (2.11.14\text{-}14\text{-a})$$

$$\sum_{(i)} E_{ri} = \sum_{(i)} E_{ki} - E_{kc} = \frac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 .$$

From (2.11.14-14-a) we could easily establish the following, natural understanding of planetary systems with many planets orbiting around one big central mass. We can say that each planet has its reduced mass that is exactly equal to its ordinary mass $m_{r-i} = m_i$ (not modified) and that only relative velocities of masses are modified,

$$\begin{array}{l} \frac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 \Rightarrow \\ \sum_{(i)} m_{r-i} v_{r-i}^2 = \sum_{(i)} m_i v_{r-i}^2 = \sum_{(i)} m_i (v_i^2 - v_c^2) = M_r v_r^2 \Rightarrow \\ m_{r-i} = m_i \,,\, v_{r-i}^2 = v_i^2 - v_c^2 \Leftrightarrow v_i^2 = v_c^2 + v_{r-i}^2 \end{array} \quad (2.11.14\text{-}14\text{-b})$$

From (2.11.14-14-b), helix rotation of planets (observed from specific Laboratory System) is a natural conclusion (since $v_i^2 = v_c^2 + v_{r-i}^2$), but in the important center of mass system, we will only have orbiting (or rotation) of planets around the central mass.

We could on a similar way to exercise the situation of *an ensemble of Binary Systems* with masses m_i and $(M_c - m_i)$, for example,

$$\begin{array}{l} m_{r-i} = \frac{m_i \cdot [M_c - m_i]}{M_c} \\ \frac{1}{2}\sum_{(i)} m_i (v_i^2 - v_c^2) = \frac{1}{2}\sum_{(i)} m_{r-i} v_{r-i}^2 = \frac{1}{2}\sum_{(i)} \frac{m_i \cdot [M_c - m_i]}{M_c} v_{r-i}^2 = \frac{1}{2} M_r v_r^2 \Rightarrow v_i^2 - v_c^2 = \frac{M_c - m_i}{M_c} v_{r-i}^2 \end{array} \quad (2.11.14\text{-}14\text{-c})$$

Of course, similar elaborations can additionally be extended to other n-body problems. In the familiar mainstream of thinking, we can imagine that initial participants of n-body interaction have orbital and spinning moments and implement laws of linear and orbital moments' conservation to establish a much more powerful analyzing framework that will consider mutual interactions of many-body systems. In cases when, participants also have free and/or dipole types of electromagnetic charges, the same situation is becoming richer for similar analyses. This will give us a chance to explore other non-Newtonian gravitation-related interactions between masses with spin and orbital moment's attributes, and electromagnetic charges (not to forget matter waves spinning associated with motions of masses).

2.3.3-2 Quantizing and Matter Waves Hosting

Circular orbits of stable Binary Systems (including most of stable solar or planetary systems), as conceptualized here, are presenting a uniform, stationary, periodical, and inertial motions. For inertial motions, we have seen in (2.9.1) and (2.9.2) that coincident validity and applicability of relevant linear and orbital momentum conservation is causally linked to standing matter waves formations. Consequently, stable Binary and Planetary Systems' Orbits as inertial motions, besides hosting orbiting masses could also host certain mutually synchronized standing matter waves formations, where synchronizing (or waves packing, or quantizing) criteria concerning the relevant center of mass coordinates system should be,

$$\begin{bmatrix} 2\pi r_i = n_i\lambda_i,\ \mathbf{L}_i = p_i r_i = m_i v_i r_i = m_i r_i^2\omega = \dfrac{m_i v_i^2}{\omega} = \mathbf{J}_i\omega = \dfrac{H}{\lambda_i} r_i,\ \lambda_i = \dfrac{2\pi r_i}{n_i} = \dfrac{H}{p_i}, H = \text{const.}, \\ \dfrac{\mathbf{L}_1}{\mathbf{L}_2} = \dfrac{\mathbf{J}_1}{\mathbf{J}_2} = \dfrac{r_1}{r_2} = \dfrac{v_1}{v_2} = \dfrac{n_1}{n_2} = \dfrac{E_{k1}}{E_{k2}},\ \dfrac{n_1}{r_1} = \dfrac{n_2}{r_2} = \dfrac{n_r}{r} = \dfrac{n_i}{r_i} = \dfrac{2\pi}{\lambda_i},\ n_i \in [1, 2, 3, ...], \\ \omega = \dfrac{v_1}{r_1} = \dfrac{v_2}{r_2} = \dfrac{v_i}{r_i} = \dfrac{v_1 + v_2}{r_1 + r_2} = \dfrac{v}{r},\ p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r,\ v_1 v_2 = \omega^2 r_1 r_2 \end{bmatrix} \Rightarrow$$

$$\frac{H}{2\pi} = \mathbf{L}_1\frac{\lambda_1}{2\pi r_1} = \mathbf{L}_2\frac{\lambda_2}{2\pi r_2} = \mathbf{L}_r\frac{\lambda}{2\pi r} = (\mathbf{L}_1 + \mathbf{L}_2)\frac{\lambda_1 + \lambda_2}{2\pi(r_1 + r_2)} =$$
$$= \frac{\mathbf{L}_1}{n_1} = \frac{\mathbf{L}_2}{n_2} = \frac{\mathbf{L}_1 + \mathbf{L}_2}{n_1 + n_2} = \frac{\mathbf{L}_r}{n_r} = \frac{\mathbf{L}_i}{n_i} = \frac{\mathbf{J}_i}{n_i}\omega = \frac{2}{n_i\omega}\frac{\mathbf{J}_i\omega^2}{2} = \frac{2}{n_i\omega}E_{ki} = \hbar_{gr.},$$
$$(v_i << c) \Rightarrow E_{ki} = \frac{n_i\omega}{2}\frac{H}{2\pi} = \frac{n_i v_i}{2r_i}\frac{H}{2\pi} \cong \frac{n_i 2u_i}{2r_i}\frac{H}{2\pi} = \frac{n_i u_i}{r_i}\frac{H}{2\pi} = H\frac{n_i\lambda_i f_i}{2\pi r_i} = Hf_i, \tag{2.11.14-15}$$
$$\omega r_i = v_i \cong 2u_i = 2\lambda_i f_i,\ \omega = \frac{2\pi}{T} = 2\pi f_m \cong 2\lambda_i\frac{f_i}{r_i} = 2\lambda\frac{f}{r} = \frac{4\pi}{n}f,\ f_m \cong \frac{\lambda}{\pi r}f = \frac{2}{n}f = \frac{2}{n_i}f_i,\ f = f_0,$$
$$E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{1}{2}F_g r = \frac{1}{2}G\frac{m_1 m_2}{r^2}r = \frac{1}{2}G\frac{m_1 m_2}{r} = \frac{1}{2}G\frac{m_r m_c}{r} = \frac{1}{2}F_c r = \frac{1}{2}\frac{m_r v_r^2}{r}r =$$
$$= \frac{1}{2}m_r v_r^2 = \frac{1}{2}m_r r^2\omega^2 = \frac{1}{2}\mathbf{J}_r\omega^2 = \frac{1}{2}\mathbf{L}_r\omega = E_r = \frac{H}{4\pi}(n_1 + n_2)\omega = \frac{H}{2}(n_1 + n_2)f_m = \frac{H}{2}nf_m,\ n = n_1 + n_2 = n_r.$$

We could again attempt to characterize and quantify unity of orbital moments of specific stable Solar System ($\mathbf{L}_s, n_s$) with many orbiting planets ($\mathbf{L}_i, n_i$), considering the Sun as enormously more significant mass compared to any of related planets, $m_s >> m_i$) on a similar way, for instance,

$$\left[\frac{H}{2\pi} = \frac{\mathbf{L}_1}{n_1} = \frac{\mathbf{L}_2}{n_2} = \frac{\mathbf{L}_1 + \mathbf{L}_2}{n_1 + n_2} = \frac{\mathbf{L}_r}{n_r} = \frac{\mathbf{L}_i}{n_i}\right] \Rightarrow \frac{H}{2\pi} = \frac{\mathbf{L}_s}{n_s} = \frac{\mathbf{L}_i}{n_i} = \frac{\sum_{(i)}\mathbf{L}_i}{\sum_{(i)} n_i} = \frac{\mathbf{L}_s + \sum_{(i)}\mathbf{L}_i}{n_s + \sum_{(i)} n_i} = \hbar_{gr.}, \tag{2.11.14-16}$$

where $\mathbf{L}_s$ and n_s are characteristic parameters of the common Sun, $\mathbf{L}_i$ and n_i are related to each planet. This way the same Solar System can be decomposed on many simple binary systems (where each planet and the sun are presenting one elementary Binary System). Of course, such strategy should be additionally elaborated and united with masses decomposition criteria from (2.11.14-14).

There is still certain confusion and ambiguity in physics literature regarding relations between mechanical revolving (or orbital, rotating) frequency $f_m = \omega / 2\pi$ and associated, specific orbital, matter wave frequency $f = \omega_0 / 2\pi = f_0$, and resolutions of such discrepancies are being explained by *postulating correspondence principles* (what is not a real and very scientific explanation). The background of mentioned discrepancies is closely related to the nature of wave motions, to particle-wave duality and to specific relations between a group and phase velocity of the matter wave packet (which represents an energy-momentum wave model of a moving particle). For instance, the relation between group and phase velocity (where group velocity is at the same time real, measurable particle velocity) can be found as (see chapters 4.0 and 4.1),

$$v = v_g = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda},\ u = \lambda f,\ u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \begin{bmatrix} (v << c) \Rightarrow \omega r_i = v_i \cong 2u_i = 2\lambda_i f_i \\ (v \approx c) \Rightarrow \omega r_i = v_i \cong u_i = \lambda_i f_i \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \omega = \frac{2\pi}{T} = 2\pi f_m = \frac{v_i}{r_i} \cong \frac{2u_i}{r_i} \cong 2\lambda_i\frac{f_i}{r_i} = 2\lambda\frac{f}{r} = \frac{4\pi}{n}f,\ f_m \cong \frac{\lambda}{\pi r}f = \frac{2}{n}f = 2\frac{f_i}{n_i}, f_i = \frac{n_i}{n}f, (v << c) \\ \omega = \frac{2\pi}{T} = 2\pi f_m = \omega_m = \frac{v_i}{r_i} \cong \frac{u_i}{r_i} \cong \lambda_i\frac{f_i}{r_i} = \lambda\frac{f}{r} = \frac{2\pi}{n}f,\ f_m \cong \frac{\lambda}{2\pi r}f = \frac{1}{n}f = \frac{f_i}{n_i}, f_i = \frac{n_i}{n}f, (v \approx c) \end{bmatrix} \Rightarrow$$

$$\Rightarrow E_{ki} = \frac{n_i\omega}{2}\frac{H}{2\pi} = \frac{n_i v_i}{2r_i}\frac{H}{2\pi} = \frac{2n_i u_i}{2r_i}\frac{H}{2\pi} = H\frac{n_i\lambda_i f_i}{2\pi r_i} = Hf_i \Rightarrow \quad (2.11.14\text{-}17)$$

$$\Rightarrow E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf,\ f_1 + f_2 = f,\ n_1 + n_2 = n,\ n_i\lambda_i = 2\pi r_i,$$

where,

$$\begin{bmatrix} (v << c) \Rightarrow v \cong 2u = 2\lambda f = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} \Leftrightarrow u \cong -\lambda\frac{du}{d\lambda} \Leftrightarrow 2\frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln\left|\frac{\lambda}{\lambda_0}\right|^2\left|\frac{f}{f_0}\right| = 0 \Rightarrow \left|\frac{\lambda}{\lambda_0}\right|^2\left|\frac{f}{f_0}\right| = 1 \\ \Leftrightarrow \lambda^2 f = \lambda_0^2 f_0, u\lambda = u_0\lambda_0,\ \lambda = \lambda_0\sqrt{\frac{f_0}{f}} = \frac{H}{p},\ u = u_0\frac{\lambda_0}{\lambda}, (f_0, \lambda_0) = const., p = \frac{Hf}{c}\sqrt{\frac{f_0}{f}} = \frac{nHf_m}{2c}\sqrt{\frac{f_0}{f}} \end{bmatrix}, \quad (2.11.14\text{-}18)$$

$$\begin{bmatrix} (v \approx c) \Rightarrow v \cong u = \lambda f = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} \Leftrightarrow \frac{du}{d\lambda} = \frac{fd\lambda + \lambda df}{d\lambda} \cong 0 \Leftrightarrow \frac{d\lambda}{\lambda} = -\frac{df}{f} \Rightarrow \ln\left|\frac{\lambda}{\lambda_0}\right|\left|\frac{f}{f_0}\right| = 0 \Rightarrow \\ \left|\frac{\lambda}{\lambda_0}\right|\left|\frac{f}{f_0}\right| = 1 \Leftrightarrow u = \lambda f = \lambda_0 f_0 = u_0 = c = const., (f_0, \lambda_0) = const.,\ \lambda = \lambda_0\frac{f_0}{f} = \frac{c}{f} = \frac{H}{p}, p = \frac{Hf}{c} = \frac{nHf_m}{c} \end{bmatrix}.$$

Until here we analyzed a Binary System composed of two rotating bodies (m_1 and m_2) around their common center of mass (where the total kinetic energy of both rotating participants is $E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = E_r$). Alternatively, we can present the same situation as another (artificial and equivalent) Binary System where one of involved masses $m_c = m_1 + m_2$ is much bigger than other ($m_1 = m << m_2 = M \cong m_c$) and staying at rest in their common center of mass (having zero orbital, kinetic energy), and second (much smaller) mass $m_r = \frac{m_1 m_2}{m_1 + m_2} \cong m << m_c \cong M$ is rotating around much bigger mass m_c, having again the same total orbital energy as before ($E_{orbital} = E_{k1} + E_{k2} = E_{k1} = Hf_1 = Hf = Hf_0$).

If we imagine that the last phase of a Binary System evolution is its collapse towards the creation of a single spinning mass $m_c = m_1 + m_2$ (where mechanical spinning of m_c is characterized by ω_c), we will have (still in the center of the mass coordinate system),

$$E_{orbital} = E_{k1} + E_{k2} = \frac{1}{2}m_r v_r^2 = H(f_1 + f_2) = Hf = E_r = \frac{1}{2}J_r\omega^2 = \frac{1}{2}(J_1 + J_2)\omega^2 = \frac{1}{2}J_c\omega_c^2 \Rightarrow$$

$$(J_1 + J_2)\omega^2 = J_c\omega_c^2,\ \omega_c = 2\pi f_c = \omega\sqrt{\frac{J_1 + J_2}{J_c}} = \frac{2\pi}{T}\sqrt{\frac{J_1 + J_2}{J_c}} = 2\pi f_m\sqrt{\frac{J_1 + J_2}{J_c}} = \begin{Bmatrix} \frac{4\pi f_0}{n}\sqrt{\frac{J_1 + J_2}{J_c}}, v_r << c \\ \frac{2\pi f_0}{n}\sqrt{\frac{J_1 + J_2}{J_c}}, v_r \approx c \end{Bmatrix}. \quad (2.11.14\text{-}19)$$

Binary Systems (as conceptualized here) are planar motional systems, meaning that involved circular motions are in the same fixed plane, and this is the reason why quantizing or synchronizing, or standing waves packing criteria is related only to one orbital quantum number. Here, we should not forget that involved mechanical rotations and spinning have much different angular velocities ω_m, ω_c, compared to associated (surrounding) matter waves angular velocities $\omega_0 = 2\pi f_0 = 2\pi f$. Of course, all of that is an idealization or approximation, since more real are multi-body systems, like planetary or solar systems (including micro-world and subatomic systems), where orbital single-plane circular motions are becoming multi-planar elliptic-paths motions (having quantized inclinations for relevant planetary orbits). Consequently, new quantizing or waves synchronizing rules are getting additional angular quantum numbers, like in semi-classical quantization of angular momentum (see [40], D. Da Roacha and L. Nottale). In mentioned Multi-component Systems (including Binary Systems), very appropriate quantizing and generalizing approach will be to apply, *creatively and with intellectual flexibility*, Wilson-Bohr-Sommerfeld Action Integrals, combined with familiar theoretical concepts published by Anthony D. Osborne, & N. Vivian Pope (see [36]). Ironically, the early days Classical Quantum Physics related to N. Bohr Hydrogen Atom Model is much more a Quantum approach to macrocosmic, real planetary orbital motions, than anything that explains or conceptualize a real nature of hydrogen atom. Here (in relation with Binary Solar Systems) we are still not specifying what kind of matter waves we are talking about, but a solid candidate (besides others related to inertial effects, rotation, and gravitation) that cannot be excluded are electromagnetic fields and waves.

Quantization in Physics is merely a consequence of the existence of stable Binary and Multi-component Systems and energy-momentum communications between them (but we should not forget that other, transient, and non-stable systems have a place in our universe). This is also the area where modern-day Quantum Theory started being complex and fuzzy, since for managing such situations (in the absence of real, clear, natural, and obvious conceptualization), it was necessary to establish new, primarily mathematically operating theories and postulates, which have been deductively generating "second-hand", luckily useful results.

2.3.3-3 Standing-Waves Resonators and Gravitation

Another aspect of imaginable, stable standing-waves field structures in relation to gravitation is the fact that every two masses (of specific Binary System, including static masses that are mutually touching) can be presented as a kind of half-wave ($\lambda / 2$) resonator, or a gravitation-dipole, where the distance between two of such masses is equal to $r = \lambda / 2 = c_{gr} / 2f_{gr}$. Here c_{gr} is the radial (central) gravitational-waves velocity acting along the distance r connecting centers of masses in question and f_{gr} is the relevant, resonant frequency of the associated standing wave (see (2.11.14-15) and (2.11.14-16)). This can mathematically be described as,

$$\left\{\begin{array}{l} r = \frac{\lambda}{2} = \frac{c_{gr}}{2f_{gr}} = r_1 + r_2 \,,\; r_1 = \frac{m_2}{m_1 + m_2} r \,,\; r_2 = \frac{m_1}{m_1 + m_2} r ,\; m_1 r_1 = m_2 r_2 \,, \\ E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{1}{2} F_g r = \frac{1}{2} G \frac{m_1 m_2}{r^2} r = \frac{1}{2} G \frac{m_1 m_2}{r} = \frac{1}{2} G \frac{m_r m_c}{r} = \\ = \frac{1}{2} F_c r = \frac{1}{2} \frac{m_r v_r^2}{r} r = \frac{1}{2} m_r v_r^2 = \frac{1}{2} m_r r^2 \omega^2 = \frac{1}{2} \mathbb{J}_r \omega^2 = \frac{1}{2} \mathbb{L}_r \omega = E_r = \frac{H}{4\pi} (n_1 + n_2) \omega = \\ = \frac{H}{2} (n_1 + n_2) f_m = \frac{H}{2} n f_m ,\; n = n_1 + n_2 = n_r ,\; n f_m = 2f \,, \mathbb{L}_1 \rightarrow (\mathbb{L}_1 + L_{s1}) ,\; \mathbb{L}_2 \rightarrow (\mathbb{L}_2 + L_{s2}) ,\; n \rightarrow (n + n_s) \\ \frac{H}{2\pi} = \frac{\mathbb{L}_1}{n_1} = \frac{\mathbb{L}_2}{n_2} = \frac{\mathbb{L}_1 + \mathbb{L}_2}{n_1 + n_2} = \frac{\mathbb{L}_r}{n_r} = \frac{\mathbb{L}_i}{n_i} = \frac{\mathbb{L}_1 + \mathbb{L}_2}{n} = \frac{(\mathbb{L}_1 + L_{s1}) + (\mathbb{L}_2 + L_{s2})}{n + n_s} \end{array}\right\} \Rightarrow$$

$$E_{orbital} = E_{k1} + E_{k2} = H(f_1 + f_2) = Hf = \frac{H}{2}(n_1 + n_2)f_m = \frac{H}{2}nf_m = \frac{1}{2}F_g r = \frac{G}{c_{gr}} m_1 m_2 f_{gr} = \frac{G}{c_{gr}} m_r m_c f_{gr} \Rightarrow$$

$$\Rightarrow H = H \cdot \frac{nf_m}{2f} = \frac{G}{c_{gr}}\frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{f} = 2\frac{G}{c_{gr}}\frac{m_1 m_2 f_{gr}}{nf_m} = 2\frac{G}{c_{gr}}\frac{m_r m_c f_{gr}}{nf_m} =$$

$$= 2\frac{F_g}{c_{gr}}\frac{f_{gr}}{nf_m} r^2 = \frac{F_g}{c_{gr}}\frac{f_{gr}}{f} r^2 = 2\pi\frac{L_1 + L_2}{n} = \text{constant},\ \frac{nf_m}{2f} = \frac{G}{c_{gr}H}\frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}H}\frac{m_r m_c f_{gr}}{f} = 1 \Rightarrow$$

$$\Rightarrow F_g = \frac{\pi c_{gr} nf_m}{nf_{gr}}\frac{(L_1 + L_2)}{r^2} = \frac{2\pi c_{gr} f}{nf_{gr}}\frac{(L_1 + L_2)}{r^2} = \frac{4\pi f}{n}\frac{(L_1 + L_2)}{r} = 2\pi f_m \frac{(L_1 + L_2)}{r} =$$

$$= \omega_m \frac{(L_1 + L_2)}{r} = v\frac{(L_1 + L_2)}{r^2} = G\frac{m_1 m_2}{r^2} = G\frac{m_r m_c}{r^2},$$

$$\omega_m = \omega = \frac{2\pi}{T} = 2\pi f_m = \frac{v}{r} = \frac{v_1}{r_1} = \frac{v_2}{r_2} = \frac{v_r}{r},\ \frac{v_1}{v_2} = \frac{r_1}{r_2},\ v(L_1 + L_2) = G m_1 m_2,$$

$$p_1 = m_1 v_1 = m_2 v_2 = p_2 = p = p_r = m_r v_r,\ \vec{p}_1 + \vec{p}_2 = \vec{0},\ \vec{r} = \vec{r}_1 + \vec{r}_2,$$

$$\vec{v}_i = \frac{d\vec{r}_i}{dt},\ \vec{v} = \frac{d\vec{r}}{dt} = \vec{v}_r = \vec{v}_1 + \vec{v}_2,\ v_1 v_2 = \omega^2 r_1 r_2,\ v_i = \omega r_i,\ \frac{v_1 v_2}{v^2} = \frac{r_1 r_2}{r^2}. \quad (2.11.14\text{-}20)$$

There are many challenging (still hypothetical) options regarding understanding the Gravitation starting from results found in (2.11.14-20). One of such possibilities, *offering the replacement for Newton Law* ($F_g = v\frac{(L_1 + L_2)}{r^2} = G\frac{m_1 m_2}{r^2}$)*, is that gravitational force is directly dependent on the total resulting vector of angular and spin moments of Binary System participants.* Such angular moments ($\vec{L} = \vec{L}_1 + \vec{L}_2$) are externally visible (and measurable), and some of their components could be states related to spinning, or to another kind of hidden rotation of belonging subatomic entities (see also (2.2), (2.4-5), (2.5) and (2.11)). What is significant here is that all three vectors $\vec{r}, \vec{v}, \vec{L}$ are mutually orthogonal, meaning that relevant vectors' product will produce a vector of gravitational force $\vec{F}_g$ collinear with $\vec{r}$. Consequently, now we can be sure that the origin of gravitation is in an interaction between angular, orbital and/or spin moments of mutually attracting masses.

Half-wave resonator, as an intuitive concept for explaining gravitational attraction between two pulsating or oscillating masses (elaborated in (2.11.14-20)) can also be approximated and modeled as the situation when specific springs mutually connect two masses in question (Binary System). Such springs (obviously having non-linear spring coefficients k_1 and k_2), are effectively realizing Newton gravitational force, between masses in question and can be supported by the following (at least dimensionally correct and still hypothetical) relations,

$$\left\{\begin{array}{l} F_g = k_1 r_1 = k_2 r_2 = G\frac{m_1 m_2}{r^2},\ m_1 r_1 = m_2 r_2 = m_r r,\ m_r = \frac{m_1 m_2}{m_1 + m_2}, \\ f_{gr} = \frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}} = \frac{1}{2\pi}\sqrt{\frac{k_2}{m_2}}\ (=)[Hz], \\ r_1 = \frac{m_2}{m_1 + m_2} r = \frac{\lambda_1}{4},\ r_2 = \frac{m_1}{m_1 + m_2} r = \frac{\lambda_2}{4}, \lambda_i = \frac{c_{gr-i}}{f_{gr}} = \frac{H}{p_{gr-i}}, \\ r = r_1 + r_2 = \frac{\lambda_1}{4} + \frac{\lambda_2}{4} = \frac{\lambda}{2} = \frac{c_{gr1}}{4f_{gr}} + \frac{c_{gr2}}{4f_{gr}} = \frac{c_{gr1} + c_{gr2}}{4f_{gr}} = \frac{c_{gr}}{2f_{gr}}, \\ F_g r = G\frac{m_1 m_2}{r} = G\frac{m_r m_c}{r} = k_1 r_1^2 + k_2 r_2^2 = 2Hf = nHf_m = \frac{2Gm_1 m_2}{c_{gr}} f_{gr}. \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{k_1}{k_2} = \frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{c_{gr2}}{c_{gr1}} = \frac{\lambda_2}{\lambda_1} \\ c_{gr-i} = \lambda_i f_{gr} = 4r_i f_{gr} = \frac{2r_i}{\pi}\sqrt{\frac{k_i}{m_i}} \end{array}\right\}$$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\Rightarrow \begin{cases} \lambda_1 = \frac{4m_2}{m_1+m_2} r = \frac{c_{gr-1}}{f_{gr}} = \frac{H}{p_{gr-1}}, \ \lambda_2 = \frac{4m_1}{m_1+m_2} r = \frac{c_{gr-2}}{f_{gr}} = \frac{H}{p_{gr-1}}, \\ H = \lambda_1 p_{gr-1} = \lambda_2 p_{gr-2} = \frac{4m_2}{m_1+m_2} p_{gr-1} r = \frac{4m_1}{m_1+m_2} p_{gr-2} r = \frac{c_{gr-1}}{f_{gr}} p_{gr-1} = \frac{c_{gr-2}}{f_{gr}} p_{gr-2} = \\ = H \cdot \frac{nf_m}{2f} = \frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{f} = \frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{f} = 2\frac{G}{c_{gr}} \frac{m_1 m_2 f_{gr}}{nf_m} = 2\frac{G}{c_{gr}} \frac{m_r m_c f_{gr}}{nf_m} = \\ = 2\frac{F_g}{c_{gr}} \frac{f_{gr}}{nf_m} r^2 = \frac{F_g}{c_{gr}} \frac{f_{gr}}{f} r^2 = 2\pi \frac{L_1 + L_2}{n} = \text{constant} \end{cases}$$

$$\Rightarrow \begin{cases} p_{gr-1} = m_1 \cdot \frac{G(m_1+m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = m_1 \cdot \frac{G(m_1+m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = m_1 \cdot v_1^*, \\ p_{gr-2} = m_2 \cdot \frac{G(m_1+m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = m_2 \cdot \frac{G(m_1+m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = m_2 \cdot v_2^*, \\ v_1^* = v_2^* = \frac{G(m_1+m_2)}{4c_{gr} r} \frac{f_{gr}}{f} = \frac{G(m_1+m_2)}{2c_{gr} r} \frac{f_{gr}}{nf_m} = v^* \end{cases} \quad (2.11.14\text{-}21)$$

What is interesting in (2.11.14-21) is that Binary Systems relations are conclusively showing that two masses, mutually exercising the Newton force of gravitation (as a Binary System), can be analyzed in a certain approximate way as two weakly coupled mass-spring oscillators (linked to their common center of mass), having the same resonant frequency on both sides. To achieve a global forces balance (like in cases of stable planetary systems, where attractive gravitational force is balanced by repulsive centrifugal force), attractive forces of such non-linear springs (between masses) should be compensated by equal repulsive forces of other two springs (connected in line with two masses in question in the mutually opposing directions). This way we can represent gravitational attraction between each of masses and the rest of the universe. This way (see Fig.2.5), we will be able to analyze (almost) independently, each of two mass-spring systems as an equivalent, macro $\lambda/4$ resonator, as already practiced in (2.11.14-21).

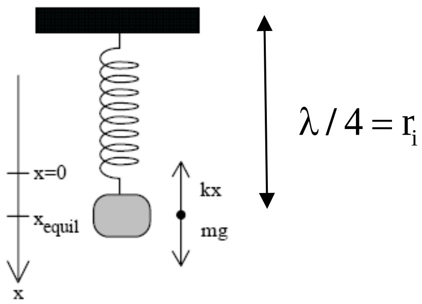


Fig.2.5. Simple Mass-Spring oscillator

The mass-spring oscillations (where mass m_i is oscillating with a certain amplitude Δr_i, Fig.2.5) can be mathematically presented by simple harmonic function $x = (\Delta r_i)\cos(\omega t + \varphi)$. In reality, we could imagine that (valid for both masses) distance r_i between a mass m_i and common center of both masses is pulsating (or harmonically oscillating) between two values: $r_i + \Delta r_i$ and $r_i - \Delta r_i$. This will (after applying few of mathematical steps valid for mass-spring systems, and applicable to particle-wave duality situations) extend the relation of proportionality between relevant elements of a Binary System in question (found in (2.11.14-21)) to,

$$\frac{k_1}{k_2}=\frac{m_1}{m_2}=\frac{r_2}{r_1}=\frac{c_{gr2}}{c_{gr1}}=\frac{\lambda_2}{\lambda_1}=\frac{(\Delta r_2)^2}{(\Delta r_1)^2}, \left(r=r_1+r_2=\frac{\lambda_1}{4}+\frac{\lambda_2}{4}=\frac{c_{gr1}}{4f_{gr}}+\frac{c_{gr2}}{4f_{gr}}=\frac{\langle c_{gr}\rangle}{2f_{gr}}=\frac{c_{gr}}{2f_{gr}}\right). \quad (2.11.14\text{-}22)$$

If such (standing waves and resonant) oscillations exist between two astronomic objects, we should be able to detect them in some way. For instance, if one of masses is our Sun and the other of masses is our planet Earth, the light beam coming from the Sun and detected on the Earth (by certain prism) should be *wavelength-modulated* producing that every specific color should have its bandwidth, directly proportional to the oscillatory speed amplitude $\omega\Delta r_i <<< c$ (like kind of Doppler effect). Such bandwidths can be measured (for many specific colors) on the Equator and somewhere far from Equator (as well as from some satellite observatory), and we should notice the differences between corresponding bandwidths. Since here we are talking about modulated and standing waves motions (between two masses), we can apply generally-valid relations between group and phase velocity, where: group velocity (of a relevant gravitational wave) is $v = c_{gr} = v_{gr}$, the phase velocity is $u = u_{gr}$, modulating planetary oscillating speed is $\Delta v = \omega\Delta r_i <<< c$, and mean group and phase velocities are $\overline{v} = \overline{c}_{gr} = \overline{v}_{gr}$, $\overline{u} = \overline{u}_{gr}$.

This would give us an idea of how to establish relations between relevant frequency and wavelengths bandwidths, as follows,

$$\left\{\begin{array}{l} v=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=u+p\frac{du}{dp}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=H\frac{df}{dp}=\frac{df}{df_s}=\frac{2u}{1+\frac{uv}{c^2}}, \\ u=\lambda f=\frac{\omega}{k}=\frac{\tilde{E}}{p}=\frac{Hf}{p}=\frac{f}{f_s}=\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}=\frac{E_k}{p},\ f_s=k/2\pi, \\ \Rightarrow 0\le 2u\le\sqrt{uv}\le v\le c \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} d\tilde{E}=Hdf=mc^2d\gamma,\quad \frac{df}{f}=(\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \\ (v\pm\Delta v)=(u\pm\Delta u)-(\lambda\mp\Delta\lambda)\frac{d(u\pm\Delta u)}{d(\lambda\mp\Delta\lambda)} \end{array}\right\}\Rightarrow\left\{\begin{array}{l} \frac{\Delta f}{\overline{f}}=(\frac{\Delta v}{\overline{v}})\cdot\frac{1+\sqrt{1-\frac{\overline{v}^2}{c^2}}}{1-\frac{\overline{v}^2}{c^2}}\cong\frac{2\Delta v}{\overline{v}}=\frac{\Delta v}{\overline{u}}, \\ \frac{du}{\Delta u}=\frac{d\lambda}{\Delta\lambda}=2\frac{du}{\Delta v}=\frac{dv}{\Delta v}, \\ \overline{v}\cong 2\overline{u}=2\overline{\lambda}\cdot\overline{f},\ \Delta v\cong 2\Delta u, \\ 2\frac{\Delta u}{\overline{v}}=\frac{\Delta v}{\overline{v}}=\frac{\Delta v}{2\overline{u}}=\frac{\Delta f}{2\overline{f}}=\frac{\Delta v}{2\overline{\lambda}\cdot\overline{f}}, \overline{\lambda}\cdot\Delta f=\Delta v \end{array}\right\}. \quad (2.11.14\text{-}22)$$

If we continue developing similar ideas about standing waves communications between masses, we should be able to explain “redshifts and blue-shifts” of the light spectra from deep and remote cosmic areas, captured by astronomic observatories on our planet.

No doubts that here we are faced with an oversimplified and accelerated mathematical and brainstorming conceptualization which is mostly useful as the first step towards familiarization with gravitational standing waves as an explanation of the nature of attractive gravitational force. Taking and proving-valid such approach will have consequences on a better understanding of origins of Gravitation.

2.3.3-4 Central Forces, Newton and Coulomb Laws

Next challenging question here is why central forces, like those that Newton and Coulomb's laws are describing, are inversely dependent from the square of the relevant distance, $F(r)=\frac{C}{r^2}, C=\text{const.}$? We can indirectly explain such situation ($F(r)=\frac{C}{r^2}$) by analyzing force components involved in orbital motions under a central force. Since in cases of central forces, relevant orbital momentum is constant $\frac{d\vec{L}}{dt}=0 \Rightarrow \vec{L}=\vec{r}\times\vec{p}=\overrightarrow{\text{const.}}$, we can conclude that vector $\vec{L}$ is perpendicular to the plane defined by the vector $\vec{r}$ and the momentum $\vec{p}$. The fact that $\vec{L}$ remains constant is saying that relevant plane ($\vec{r}$, $\vec{p}$) also remains constant (or stable), and that every orbital motion (on such plane) under central force is a stable, planar, and two-dimensional motion (which can naturally host standing waves structures without big need to give probabilistic or stochastic meaning to any of such waves). This is very much like astronomic observations documenting that many solar systems are planar, facilitating involved mathematical processing, for example,

$$L=L(r,\theta)=mr^2\frac{d\theta}{dt}=\text{const.},\ \vec{F}(r,\Theta)=\vec{F}(r)+\vec{F}(\Theta)=\vec{F}(r)=\overrightarrow{(m\frac{d^2r}{dt^2})}=m\vec{a}_r+m\vec{a}_\theta=\overrightarrow{\left[m\frac{d^2r}{dt^2}-mr(\frac{d\theta}{dt})^2\right]},$$

$$a_r=\frac{d^2r}{dt^2}-r(\frac{d\theta}{dt})^2,\ a_\theta=r\frac{d^2\theta}{dt^2}+2\frac{dr}{dt}\cdot\frac{d\theta}{dt},\ mr\frac{d\theta}{dt}+2m\frac{dr}{dt}\frac{d\theta}{dt}=0\Rightarrow\frac{d\theta}{dt}=\frac{L}{mr^2}=\frac{L}{m}\rho^2,\ \rho=\frac{1}{r},$$

$$\frac{d^2\rho}{d\theta^2}+\rho=-\frac{m}{L^2}\cdot\frac{1}{\rho^2}F(\frac{1}{\rho}),\ F(\frac{1}{\rho})=F(r)=\frac{C}{r^2}=C\cdot\rho^2\Rightarrow\frac{d^2\rho}{d\theta^2}+\rho=-\frac{mC}{L^2}\Rightarrow\rho=A\cos(\theta-\theta_0)-\frac{mC}{L^2},$$

$$r=\frac{1}{A\cos(\theta-\theta_0)-\frac{mC}{L^2}}.$$

The last equation is describing conic curves $r=r(\Theta)$, such as ellipse, parabola, and hyperbola, depending on constants $A, \Theta_0, m, C, \vec{L}$. If we chose the reference coordinates where $\Theta_0=0$, we will get for a planetary and satellite orbits $r=\frac{1}{A\cos\theta-\frac{mC}{L^2}}$ that is a conic section, which can also be transformed to $r=r_0\frac{1+e}{1+e\cdot\cos\theta}$, where **e** presents eccentricity of the orbit. For **e** < **1**, the orbit is an ellipse, for **e** = **1**, the orbit is a parabola, for **e** > **1** the orbit is a hyperbola, and for **e** = **0** the orbit is a circle (covering all cases of planetary, asteroids and satellite orbits). ------------------------------ ♣]

3. POSSIBILITIES FOR GENERALIZATION OF FARADAY-MAXWELL THEORY

The author's initial vision in this chapter is that present days Maxwell-Faraday **E**lectromagnetic **T**heory (**ET**) presents the example of an elegant, very pragmatic, natural, and sufficiently stable field-theory that should be fully integrated with **R**elativity **T**heory, much more than presently known, and practiced, inside the boundaries where **R**elativity **T**heory and **E**lectromagnetic **T**heory are operational without doubts. If the **R**elativity **T**heory (**RT**) is in some of its fundamental aspects still not natural or completed, we should refer to Maxwell-Faraday's theory as an ontologically dominant, conceptually richer, better, and more pragmatic field theory, to serve as a model and underlaying, source-theory for modifying, updating, and creating a new and better **R**elativity **T**heory. The author's guiding idea in this chapter is also to suggest possible optimizations of contemporary Maxwell theory and to address new merging domains between Maxwell Electromagnetic Theory and Relativity Theory. It is already clear from the new experimental evidence regarding electrons, electric currents, and magnetic fields, combined with rotational motions, that contemporary concepts about electrons, magnetic field and Maxwell equations should be updated, what mainstream leaders of electromagnetic theory are still not ready to admit or even analyze (see more about such revelations in [63], [68], [88], [89], [91] and [117]). This is implicating that contemporary **Q**uantum and **R**elativistic **E**lectrodynamics, **QRE**, regardless how well experimentally and theoretically proven, consistent, and mathematically well fitted to look as a completed and fully correct theory, is (most probably) still not presenting the final, the best, and the most natural merging and upgrading concept between **EM** and **RT**. In addition, presently neither Electromagnetic nor Relativity theory themselves are in their final self-standing frames that are ready for higher-level unification (see serious and much harder, but similar statements in **[113]**, from Hristopoulos, Demetris; "**Beyond Electromagnetic Theory"**). In other words, well-operating mathematical modeling of present **QRE** is successfully constructed (fitted and hybridized) by complying with known and relevant experimental facts, conservation laws, and by merging with already well-operating segments of quantum theory, electromagnetic theory, and relativity theory, but simplicity, mutual compatibility, fully natural and logical unity, and conceptual elegance are still missing there (theory is still bulky and very complex to be in its optimal, self-standing, self-explanatory, natural frames). Just to give an example about somewhat familiar situation: The Ptolemy geocentric planetary-motion theory was also mathematically working sufficiently well (after applying relevant mathematical optimizations based on mutually synchronized, periodical and stable nature of planetary motions), during exceptionally long time, but conceptually it has been completely wrong, and relevant "*society and religion-related power-establishment*" did not like to admit this during almost 1000 years because of keeping alive some of their profitable, ideological and dogmatic postulations.

We are already familiar with the terminology often used in the contemporary Physics, such as relativistic, non-relativistic, classical-mechanical, quantum-mechanical, quantum-electrodynamic, probabilistic or statistic… points of view. When we think a bit more seriously and logically, it is becoming clear that such segmentation and divisions in physics are on some way products of our separated, independent, and sporadic advances in different fields, insufficient multidisciplinary and cross-platforms knowledge, incomplete or partially wrong mathematical modeling, and missing general and unifying interdisciplinary concepts regarding different Physics chapters.

Attributes like relativistic, non-relativistic, classical-mechanical, quantum-mechanical, and similar ones, one day should be replaced by certain new and united theory that is equally well addressing all of them on a similar and mutually consistent way (using the same mathematics). Consequently, it is also clear that some of the theories, or all of them, previously mentioned, could be either partially wrong, or incomplete, or only particularly valid within some limited frontiers, or in some cases being unnatural, and that a new and unifying or generally valid theory is still missing here (see [36], IMMEDIATE DISTANT ACTION AND CORRELATION IN MODERN PHYSICS: THE BALANCED UNIVERSE). The weakest side of Maxwell theory or equations is that such equations are still not formulated to be valid (or invariant) in all inertial systems being in mutual relative motions, meaning that when we use Maxwell equations, we need to know that results are applicable or valid only for the laboratory coordinate system where we perform observations and calculations. One day, mentioned non-invariance should be corrected, and here we will try to progress towards such an objective. We also should be on some theoretical way, intellectually and perceptually ready to accept that properly updated Maxwell's theory would completely generate a new and better Relativity theory and explain or create much better foundations of Gravitation. Renewed and updated (future) Relativity theory will not be very much linked to A. Einstein axiomatic and postulated assumptions and constructions. Minkowski 4-vectors energy-momentum formalism is still the best modeling associated to Relativity theory, which can be additionally clarified, valorized and generalized, being united with mathematical practices regarding Hypercomplex Analytic Signals and Phasors concept (like used in electric circuit theory), and all of that would also materialize as a very advanced and universally applicable mathematical framework for addressing wave-particle duality, wave motions, different field theories, Mechanics and Gravitation (see more in Chapter 10).

Currently, it is also the author's position that Maxwell-Faraday electromagnetic theory still presents practically very convenient model for analogical constructing of other field theories in physics. In this book we start with analogies and symmetries, as guiding and new ideas-generating platform. Later, we anyway need to implement additional mathematical modeling and upgrading to formulate better Electromagnetic and Gravitation-related theory that will comply with known experimental facts and general conservation laws of physics (see **[34]**, **[71]**, **[99]** and **[152]**), where it is suggested how Maxwell equations could be better united and upgraded).

*Here, we shall underline and spark certain aspects of possible and still hypothetical extensions of Maxwell-Faraday theory that will support the main ideas of this book, such as an intrinsic connection and complementary nature between rectilinear-motion and spinning, what leads to better understanding of Gravitation. This is in a direct relation, analogy, or parallelism with mutual couplings and complementarity of electric and magnetic fields. Everything related to fields, forces, and matter-waves functions, can be mathematically modeled using Complex Analytic signals, and Phasors (D. Gabor model based on Hilbert transform; -see **[7]** and **[57]**). Involved, real and imaginary Analytic signal functions, should naturally represent mutually coupled electric and magnetic fields (see more about Analytic Signals in Chapter 4.0). Practically, coupling and temporal-spatial form of electric and magnetic field vectors, that are creating photons and electromagnetic waves, behaves like having* *two wave components of certain "Analytic Signal wave function" (where one part is real, and the other is an imaginary wavefunction component, being mutually phase-shifted). Only such complex waveform (or Analytic Signal function) can correctly represent electromagnetic waves, and this way it will be possible to define (or present) electromagnetic-wave phase-function, amplitude function, group and phase velocity, signal power, different moments, and energy of electromagnetic waves (including voltages, currents, and photonic wave groups) on certain rich, logical, and natural way. Poynting vector concept (for representing an electromagnetic wave) is a kind of ad-hock, intuitive, hybridized creation, and a natural assumption in the same direction as proposed here (but an innovative Analytic Signal modeling of such new electromagnetic field vector, or updated Poynting vector, should be a much better approach; -see more in Chapter 10.). Similar mathematical modeling should be applicable to all other matter waves, and natural forces, including Gravitation since Gravitation has its primary sources in electromagnetism. See familiar elaborations in [34] Dipl.-Ing. Andrija S. Radović.* ***High Power Mechanical, ultrasonic or acoustic energy, moments, forces, oscillations, and vibrations, also audio signals and music, can be created and transferred by applying different signal-modulating techniques on laser beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); - See relations from Chapter 10. under (10.2-2.4), and literature references from [133] until [139].***

Within next steps, an innovative contribution in establishing a more complete system of electromechanical analogies, and much better Maxwell equations, than already known in Physics, will be formulated. See also the first chapter of this book, as the starting platform, regarding analogical comparisons and conceptualizations in Physics, as summarized in "T.1.8 Generic Symmetries and Analogies of the Laws of Physics".

3.1. Modification of Maxwell Equations

Non-relativistic expressions of Lorentz, electric and magnetic forces (in homogeneous and isotropic, mutually orthogonal coordinates of Euclid-Galilean-Descartes space), with relative motions of participants, and combined with involved magnetic and electric fields, makes possible to establish fully mutually symmetrical or analogical forms of electric and magnetic fields (**E** and **H**), and relevant inductions (**D** and **B**), as being functions of involved mechanical-motion velocity **v**, (see **[4]**, **[34], [35], [46]**, **[47], [49]**, **[99]**),

$$\begin{aligned}
&\vec{E}(v) = \vec{E}_0 \pm \vec{v}\times\vec{B}_0 = \vec{E}_{stat.} \pm \vec{E}_{dyn.}\,,\ \vec{E}_{stat.} = \vec{E}_0\,,\ \vec{E}_{dyn.} = \vec{v}\times\vec{B}_0\,,\\
&\vec{H}(v) = \vec{H}_0 \mp \vec{v}\times\vec{D}_0 = \vec{H}_{stat.} \mp \vec{H}_{dyn.},\ \vec{H}_{stat.} = \vec{H}_0\,,\ \vec{H}_{dyn.} = \vec{v}\times\vec{D}_0\,,\\
&\left\langle \vec{B}_0 = \mu\vec{H}_0\,,\ \vec{D}_0 = \varepsilon\,\vec{E}_0,\ \vec{B}(v) = \mu\,\vec{H}(v),\ \vec{D}(v) = \varepsilon\,\vec{E}(v)\right\rangle,
\end{aligned} \quad (3.1)$$

Here, index "**0**" indicates that certain field exists in a static case when **v** = 0 (meaning in a state of relative rest, where applied indexing means: $\langle v = 0\rangle \Rightarrow \langle E(v) = E(0) = E_0 = E_{stat.},\ H(v) = H(0) = H_0 = H_{stat.}\rangle$). The field members indexed by "**dyn.** (=) dynamic", are representing fields created by motion of interaction participants (being velocity dependent). Lorentz forces, laws of electromagnetic induction, and "action-equal-to-reaction" forces could serve here for possible extension of Electromagnetism towards a better understanding and explanation of Gravitation (since electromagnetically neutral masses in relative motions are internally or structurally creating electromagnetic dipoles and multipoles, and such kind of coupled charges in different relative motions (related to external particle motions) are also producing electric and magnetic fields, forces, voltages and currents, what is eventually producing mechanical motions, and mechanical or gravitation related forces). Mentioned ideas (based on Lorentz magnetic and electric forces) could explain the origins of Gravitation, and its connections with Maxwell equations, as elaborated in **[34]** and **[152]** and in the second chapter of this book. We also know that electrically and magnetically charged micro and macro masses are producing masses attraction and repulsion effects, what are familiar effects to Gravitation, except we still do not master repulsion or anti-gravitation effects with electromagnetically neutral masses.

Mutually complementary structure (and full mathematical symmetry and analogy) between equations of electric and magnetic fields could be initially expressed (at this time still as a brainstorming and somewhat hypothetically) by transforming (or extending) the present explicit system of almost independent equations (3.1) into new, implicit, mutually linked and dependent system of equations (3.2). Electric and magnetic fields (related to the same process) are anyway and always, naturally, fully united and coupled (see later (3.4)), and should be fully mutually dependent and symmetrical, as follows,

$$\left.\begin{aligned}\vec{E}(\vec{v}) &= \vec{E}_0 \pm \vec{v}\times\vec{B}\left[\vec{E}(\vec{v}),\vec{B}_0\right] = \vec{E}\left[\vec{H}(\vec{v}),\vec{E}_0\right] = \frac{1}{\varepsilon}\vec{D}\left[\vec{H}(\vec{v}),\vec{D}_0\right] = \vec{E}_{stat.} \pm \vec{E}_{dyn.}\,,\\ \vec{H}(\vec{v}) &= \vec{H}_0 \mp \vec{v}\times\vec{D}\left[\vec{H}(\vec{v}),\vec{D}_0\right] = \vec{H}\left[\vec{E}(\vec{v}),\vec{H}_0\right] = \frac{1}{\mu}\vec{B}\left[\vec{E}(\vec{v}),\vec{B}_0\right] = \vec{H}_{stat.} \mp \vec{H}_{dyn.}\end{aligned}\right\} \Rightarrow \qquad (3.2)$$

$$(\text{Lorentz's force law}) \Rightarrow F = q_{el.}\left\{\vec{E}_0 \pm \vec{v}\times\vec{B}\left[\vec{E}(\vec{v}),\vec{B}_0\right]\right\} + q_{mag.}\left\{\vec{H}_0 \mp \vec{v}\times\vec{D}\left[\vec{H}(\vec{v}),\vec{D}_0\right]\right\}$$

Now, we could analyze different solutions of (3.2), by introducing various initial and boundary conditions regarding the nature of involved motions and propagation-media, and by exploiting relations between electric field and electric induction, and magnetic field and magnetic induction (always satisfying the well-known natural conservation laws, and laws and rules known from Faraday-Maxwell theory). The goal is to create an updated (more structurally symmetric, and *inertial system invariant*), the united and generalized system of Maxwell equations of perpetually self-interacting, self-regenerating, mutually coupled electromagnetic fields (in their integral and local forms), starting from implicit relations as given in (3.2). *Since we did not precisely say what kind of implicit functions and relations are involved in (3.2), there will be a lot of (adjusting and fitting) freedom to finalize correct mathematical modeling, later (especially if we correctly present electric and magnetic fields as components of certain Analytic signal).* It is almost obvious that we can thus achieve a much higher (mathematical) level of symmetry between intrinsically mutually interacting and mutually coupled electric and magnetic fields, and later be able to analyze some new (still not sufficiently explained, or still hypothetical) experiments, and other challenging situations in the same domain, which are anyway related to Gravitation. *There is a big chance that every couple of mutually* *dependent electric and magnetic field components or waveforms behaves like real and imaginary parts of an Analytic signal (being mutually phase-shifted for 90° and mutually transformable using Hilbert transform*, but in some cases (relevant electric and magnetic fields, currents and voltages) will have different phase differences, depending on involved electromagnetic impedances and loads configuration), and this kind of mathematical modeling will significantly enrich Electromagnetic theory and open a window towards better understanding of Gravitation (see about electromechanical analogies in the first chapter of this book).

Generally applicable and valid in real material media (not only in a homogeneous, linear, and isotropic or empty space, free of other forces and other interactions), electric and magnetic induction vectors from (3.2) should have the following symmetrical, analogical, and mutually dependent forms such as:

$$\begin{aligned}
&\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (\vec{E} + \vec{E}_{int.}),\ \vec{E}_{int.} = \frac{\vec{P}}{\varepsilon_0},\ (or\ \vec{D} = \|\varepsilon\|\ \vec{E}\ ),\\
&\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \vec{H}_{int.}),\ \vec{H}_{int} = \vec{M},\ (\text{or } \vec{B} = \|\mu\|\ \vec{H}),\\
&\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right] = \|\varepsilon\| \cdot \vec{E}(\vec{v}) = \|\varepsilon\| \cdot \vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \vec{D}_{int} + \vec{D} = \vec{D}_{stat.} \pm \vec{D}_{dyn.}\\
&\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] = \|\mu\| \cdot \vec{H}(\vec{v}) = \|\varepsilon\| \cdot \vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \vec{B}_{int} + \vec{B} = \vec{B}_{stat.} \mp \vec{B}_{dyn.}\\
&\vec{E}_{int.} = \frac{1}{\|\varepsilon\|} \vec{D}_{int} = \text{internal-polarization electric field inside of media} = \vec{E}_{stat.}, (v = 0)\\
&\vec{H}_{int.} = \frac{1}{\|\mu\|} \vec{B}_{int} = \text{internal-polarization magnetic field inside of media} = \vec{H}_{stat.}, (v = 0)
\end{aligned} \tag{3.3}$$

$$\|\varepsilon\| = \text{dielectric constant as a tensor,}$$
$$\|\mu\| = \text{magnetic permeability as a tensor.}$$

The main mathematical and intuitive strategy in (3.2) and (3.3) is to keep ***mutual analogy and symmetry*** in all mathematical expressions of mutually coupled electric and magnetic fields (related to the same situation), and always formulate them as a system of mutually dependent equations (and later to solve such equations). How such unifying concept would be made mathematically practical and operational is another question, but at least with (3.2) and (3.3) we are making first steps or foundations, explicitly formulating what the most important framework for new electromagnetic and gravitation related theory could be. In **[4]**, **[34], [46]**, **[47]** and **[99]** we can also find original modeling concepts, regarding Maxwell equations, that can support, and complement here introduced and internally or structurally symmetrical electromagnetic field concept. How else we could modify equations of magnetic and electric field to be more symmetrical, mutually coupled, and mutually analog, is already known and well presented in the available literature about such problematic. See, for instance, the book: "The Electromagnetic Field, from Albert Shadowitz, Dover Publications, 1988. If creating analogical expressions and formal mathematical symmetry within electromagnetic field and Maxwell equations has been the main objective here, this would be a simple mathematical exercise, but here, *this is only the first necessary step, or an initial, starting platform proposal to explain Gravitation (since we reasonably assume that electric and magnetic fields are always mutually coupled, symmetrical and mutually interactive, acting also* *mechanically on involved masses, what should generate effects of Gravitation). In other words, Electromagnetism is underlaying Mechanics and Gravitation (see familiar elaborations in [152]).*

We already know that any kind of electrical charge would create an electric field (or electric induction) in its vicinity, and we know that electrical charges in any kind of visible or hidden motion including oscillations (relative to something), would create effects of electric currents, which are immediately creating magnetic field around such, effectively and electrically conductive zones. In addition, particle-wave duality framework (see relevant foundations in chapters 2., 4.1., and 10. of this book) is explaining that every ***linear motional energy state***, has certain associated spinning or helical, by matter-waves created field around the path of its motion. *Analogically, following the same concept (of matter waves and particle-wave duality), any moving electron should create helical (or spiral) magnetic field structure around its path of propagation. The distance between two spiraling, successive elements (of the magnetic field solenoid) should correspond to de Broglie matter wavelength* $\lambda_e = h/m_e v_e$*, meaning that an electron is presentable as a certain electromagnetic or photonic matter-wave packet, which has its group and phase velocity, frequency, energy, and momentum (including associated magnetic field spinning or helicity). In solar systems, planets are self-spinning and rotating around their suns, complete solar systems are rotating around galactic centers, and galaxies are also rotating, meaning that complex helicoidal or solenoidal motions and associated, similar, helical, electromagnetic, and spinning fields configurations, couplings and interactions are omni-present in our Universe… After analyzing Compton, Photoelectric and electrons' diffraction and scattering effects (including x-rays creation), we already know that* $\lambda_e = h/m_e v_e$ *is an essential (and exact) property of a moving electron or its matter wave-packet, because based on such wavelength we are calculating and exactly predicting experimental results of mentioned interactions, helix-spinning effects, impacts and scatterings (like in Compton and Photoelectric effects), heavily using particles mechanics and Minkowski 4-vectors framework.*

We can try to understand the nature of (motional) electrically charged states *by analyzing the creation of an electron-positron pair from the energy-momentum content of a sufficiently energetic photon, and the annihilation of an electron-positron pair that produces two photons. These are very indicative experimental situations explicitly showing that internal content of an electron (or positron) mass or energy is certain specific form of localized standing-waves of an* ***"electromagnetic, photons-energy packing".*** *Of course, such entities are naturally respecting the conservation of total system energy and relevant orbital and linear moments. Something similar (at least by analogy and structural symmetry) should also be valid for protons (and anti-protons), and since neutron effectively presents certain mutually coupled combination of one electron and a proton (regardless of involved quarks), we could conclude that quanta of electromagnetic energy (or photons) in different* ***"packing formats"*** *(most probably) create overall mass and atoms diversity in our universe. More of familiar supporting arguments regarding standing-waves formatted matter structure can be found in Chapters 4.1. and 10.*

Richard F. Gauthier [53] publishes the attempt to present electron and positron as a specific superposition of photons; - "a photon is modeled along an open 45-degree helical trajectory. A spatial model of an electron is composed of a charged point-like quantum circulating at an extremely high frequency in a closed, double-looped helical trajectory whose helical pitch is one Compton wavelength h / mc. The two possible helicities

of the electron model correspond to the electron and the positron. With these models, an electron is like a closed circulating photon. The electron's inertia is proposed to be related to the electron model's circulating internal Compton momentum mc".

Logically concluding, Maxwell-Faraday equations and other relevant formulas concerning interactions between electric and magnetic fields, can and should be made fully mutually integrated, coupled, inertial systems invariant, symmetrical, fully mutually dependent and convertible, and mutually analogical, regarding corresponding electric and magnetic fields relevant items (like elaborated in **[34]**, **[71]**, **[89], [99]** and **[152])**. Here we still have a lot of creative space to better integrate, unite and formulate ***Maxwell equations, Ampere's Law, Faraday's Law, Lorentz force, Gauss's Law of Electrostatics and Magnetostatics, and Lenz's Law***. Since the desired level of mathematical unity and integration is still not the case in our contemporary Maxwell or Electromagnetic theory, here is the source of challenging ideas and an intellectual motivation to start with reformulation and upgrading of present electromagnetic theory (towards much higher internal symmetry of upgraded Maxwell equations, and towards creating better foundations of particle-wave duality). This approach will at the same time explain origins or essential background of Gravitation.

Anyway, there is already significant and striking symmetry and analogy between Maxwell equations and equations related to fluid-flow mechanics, and only thanks to such very tangible comparisons and analogies (between electromagnetic and fluid-flow phenomenology) creators of Electromagnetic theory succeeded to make something meaningful and practical like contemporary electromagnetic theory is (resulting in Maxwell equations). See the following citation from Tsutomu Kambe [123]:

"Fluid mechanics is a field theory of Newtonian mechanics of Galilean symmetry, concerned with fluid flows represented by the velocity field such as v(x, t) in space-time. A fluid is a medium of continuous mass. Its mechanics is formulated by extending discrete system of point masses. Associated with two symmetries (translation and space-rotation), there are two-gauge fields: E ≡ (v · ∇) v and H ≡ ∇ × v, which do not exist in the system of discrete masses. One can show that those are analogous to the electric field and magnetic field in the electromagnetism, and fluid Maxwell equations can be formulated for E and H. Sound waves within the fluid is analogous to the electromagnetic waves in the sense that phase speeds of both waves are independent of wave lengths, i.e., non-dispersive".

Contrary to what really happened (regarding creation of Maxwell theory), based on analogical, indicative, and creative conceptualization in relation to already existing fluids dynamics, the real nature and content of fluid mechanics theory should also be explicable as the consequence of certain well-established, internally, or structurally symmetric electromagnetic theory foundations (as initiated with (3,1), (3.2) and (3.3)). *This implicates existence of certain exceptionally fine and spatially penetrating fluidic medium within and around all mater states including an absolute vacuum state. Or differently saying, Electromagnetism is underlaying Mechanics and Gravitation.* Historically, evolution of this (Maxwell theory) situation was much different, since fluids phenomenology was intuitively, visually, theoretically, and experimentally already known, tangible and more accessible for an indicative and analogical conceptualization of electromagnetic field. This is additionally reinforcing the strategy (being promoted in this book) showing how to reorganize, and update Physics based on indicative analogical comparisons, conceptualizations, and conclusions. ***Since we still have some of conceptually, insufficiently clarified***

theoretical and experimental situations in relation to electromagnetism and gravitation, and knowing (or just accepting) that fluid-dynamics theory is an analogical ideas-generating source of Maxwell-equations, we should maximally use or explore such analogies (in both directions) and this way produce new theoretical insights or proposals for additional completing of Maxwell and Fluid dynamics equations, this way also contributing to better understanding of Gravitation. *Of course, a relevant fluid which is the carrier of electromagnetic (and most probably other) matter waves could be an ether, with the meaning as Nikola Tesla speculated and experimentally supported it. Anyway, the ether concept is still officially excluded and being like forbidden in a modern Physics, starting from the creation of A. Einstein Relativity theory and Michelson Morley experiments.*

Another objective is much more important in this book, and it could be revealed in formulating the following question: What are the direct and indirect consequences of such (fully symmetrical) electromagnetic compatibility and united fields reality (between an electric and magnetic field, mutually coupled as in (3.3)), when addressing phenomenology in the world of Gravitation, Mechanics of electromagnetically neutral masses like atoms, and interactions of other micro and macro particles and masses? The answer to such question in this book is still an evolving process guided by noticing and following certain level of *analogies between linear motions and electric field phenomena*, and *analogies between mechanical rotation of masses, magnetic field, and rotation of electric charges, which is in fact effectively (of course mathematically) replaceable by closed loops of electric currents and voltages*. Since electric and magnetic fields are anyway mutually coupled and complementing, it is intuitively and implicitly clear that something similar could exist between linear motions and rotations of electromagnetically neutral particles, leading us towards better, essential, and ontological explanation of matter-waves and Gravitation. There are number of publications under [63], **Arbab I. Arbab**, addressing possible and provable relations between electromagnetic and Lorentz forces and gravitation, familiar to updated Maxwell equations from this chapter, as we find in (3.2). To completely structure and prove something like that, the best and obvious strategy would be, first to establish a total mathematical symmetry, and an analogy between all involved field components of magnetic and electric fields (just to have a simpler, more intuitive, and more indicative starting platform). Then, by analogy, we can apply the same strategy to linear and rotational motions of (electromagnetically neutral) masses and find similar elements of fields complementarities between them. This time we would have analogies between mechanical and mutually coupled gravity-related fields where one field component belongs to linear motion (analog to electric field), and the other, presently still hypothetical component, belongs to certain kind of rotation (analog to magnetic field). Briefly summarizing, the intention here is to formulate all possible analogies, couplings, and symmetries between **magnetic field and rotation**, and later between the **electric field and linear motion**, as presented in the first chapter of this book within **T.1.8 (Generic Symmetries and Analogies of the Laws of Physics).** Furthermore, based on already known relations between electric and magnetic fields, the goal here is to formulate analogically possible “coupling relations” between linear motion and rotation of masses. Of course, eventually, we would need to make a new and well operating modeling or higher-level symmetry, with complementary-relations between associated electromagnetic, linear, and rotational motions, and involved electromagnetic fields, including Gravitation (what also presents an essential background of Particle-wave Duality, as promoted in this book).

We already know that such reality and analogy exist being experimentally testable, since ***High Power Mechanical, ultrasonic or acoustical energy, moments, forces, oscillations and vibrations, or audio signals and music, can also be created and transferred by applying different signal-modulating techniques on laser beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); -See literature references from [133] until [139]. In addition, we can analogically create and propagate mechanical or ultrasonic vibrations in solid wires (of any shape and length) on a similar way as electromagnetic waves and photons are propagating (as transversal waves, or being perpendicular on a direction of propagation), and such energy transfer will be like electrical currents in metal wires (see [140] European Patent Application, related to MMM technology).***

The next important objective (or project) could be to find what kind of mechanics and gravity-related entities (presently formulated independently, and out of the frames of Maxwell-Faraday electromagnetic theory) could be "analogically equivalent" or symmetrical to/with (3.2) and (3.3), to extend the concept of present Maxwell electromagnetic field equations to other mechanics and gravitation related fields and concepts known in Physics. This should not be too much surprising and hypothetical objective since whatever we know as a relatively stable (electrically neutral) rest-mass-matter is composed of atoms, and atoms are composed of internally moving and oscillating electrons, protons, and neutrons (which are structurally in a natural relation to self-closed standing waves loops, and circulations or exchanges of photons inside and outside such standing-waves stabilized structures). This is also the explanation why gravitation is a meaningless and non-existent phenomenology for very small particles and small distances (below 100 micrometers). Mentioned matter constituents (including neutrons) are directly or indirectly presenting coupled motional electrical charges, and associated *magnetic moments carriers*, cross-linked with their linear and orbital mechanical moments, *having mutually coupled electric and magnetic fields and very much stable gyro-magnetic ratios*; -see number of familiar ides in [63] and [152]).

The remaining important objective of the here-elaborated strategy is to show that mutually and implicitly dependent system of electromagnetic field equations (3.2) is naturally generating field vector solutions that on some way agree or resemble with Lorentz transformations. *This should be realized without using "a little bit artificially created and directly implementing prefabricated or prescribed" old Lorentz transformations, as presently practiced in Relativity theory*. **This will generate an innovated Special Relativity theory, intrinsically incorporated in the structure of such updated Electromagnetic theory. In other words, Renewed or updated Relativity theory would become a simple mathematical consequence of analogies, symmetries, complementarities, and interactions between mutually coupled electric and magnetic fields in relation to involved moving states. This is going to eliminate the need to use Lorentz transformations almost as formal prescriptions, how we presently practice).** This way, since we can also establish parallelism between electromagnetic and "gravity-mechanics" related fields, we should be able to extend similar (new Relativity-theory related) conclusions to motions of electromagnetically neutral masses (again showing that Relativity theory should have its roots and origins only in a properly updated Maxwell-Faraday electromagnetic theory, correctly merged with relevant conservation laws). *Contemporary Special Relativity Theory is still temporarily serving as a useful* *complement and missing theoretical substance of present Electromagnetic theory. When new, updated electromagnetic theory will be well established, present days of importance of the old and original Special Relativity Theory will largely disappear as being somewhat artificial, obsolete, redundant, not self-standing and incomplete.*

On a similar way, gravitation could be understood as being the product of matter-waves communications via electromagnetic-forces and "*matter-energy-moments-exchanges*", as (mostly electromagnetic) fields interactions between motional masses, and in line with Rudjer Boskovic and Nikola Tesla's concepts about gravitation (see [6] and [97]). In addition, it should be obvious that Electromagnetic theory (regarding relevant fields and forces) could also be theoretically or geometrically conceptualized on a similar way as in A. Einstein General Relativity theory (regarding Gravitation), as spatial-temporal deformations around involved masses and energy states with electromagnetic field charges, what effectively presents kind of trivial and always valid statement and possibility).

We should not forget that electromagnetic photons really have detectable and accountable mechanical moments, effective mass, and energy. We could imagine that the whole Universe also and essentially has an electromagnetic nature and structure (meaning that photons are naturally interacting with atoms and masses that are also created of specifically stabilized, packed, or formatted electromagnetic fields or photons; -see more about photons in Chapter 4.1). This is making fully manageable and mathematically predictable descriptions of scatterings, impacts and other mechanical interactions between photons and much more ordinary and orthodox masses and particles, as well as with electrically charged particles (what we still need to conceptualize better, as a precondition for creating an innovative conceptualization of Gravitation).

[♣ Electric and magnetic fields and relevant properties of certain matter state are always (in any natural conditions, everywhere in our Universe), mutually coupled, and motion parameters dependent. We often analyze seemingly sufficiently isolated magnetic or electric fields, currents, and charges (or electromagnetic dipoles formations), only within specific, limited scope of our laboratory and local inertial systems, where we can detect only one side or part of certain surrounding, global electromagnetic and mechanical phenomenology. This way, we see, either electric or magnetic part of certain reality, which is in a larger referential frame, anyway and always composed of mutually coupled electric and magnetic dipoles, fields and charges (which are directly coupled with associated linear and angular mechanical motions, internally and externally). Natural forces are manifesting as direct, indirect, or derived effects of electric and magnetic moments, dipoles and multipoles oscillations, attractions, and repulsions, being strongly dependent on motional states of interaction participants (as extended Maxwell equations with symmetric Lorentz forces are indicating; -see (3.2) and later). Electric and magnetic polarizations are always present between macro matter states or masses, because of holistic, omnipresent, mutually coupled, linear, angular, and oscillatory motions of all masses and atoms in our Universe. Spatial orientations of specific electromagnetic vectors and involved standing waves oscillations are influencing all forces (between masses, atoms, and elementary particles) acting within our universe. Universal binding or surrounding medium (where all masses and matter formations are submerged) could be certain, still hypothetical, fluidic state of matter (like an ideal gas), which we can

call an ether. Such ether has, at least, electric, and magnetic permeability or susceptibility, which are real, measurable, and stable constants or properties of our Universe (applicable even in an absolute or idealized vacuum). Mentioned ether fluid, besides being the carrier of electromagnetic waves and perturbations, should also have other, masses and fluids related properties and manifestations. ♣]

3.2. Generalized view about Currents, Voltages, and Charges

Currents and voltages (or potential differences) are presenting essential engineering and phenomenological, technical items or phenomena in our electromagnetic theory and engineering practices. Let us introduce, and establish, unified and generalized (mutually symmetrical and analog) definitions of electric and magnetic currents $i_{electr.}$, $i_{mag.}$, voltages $u_{electr.}$, $u_{mag.}$, and charges $q_{electr.}, q_{mag.}$, starting from electric and magnetic induction from (3.3), as follows:

$$\left.\begin{array}{l} i_{electr.} = u_{mag.} = -\dfrac{d\Phi_{electr.}}{dt} = i_{el.stat.} + i_{el.dyn.} = i_{electr.}(x,y,z,t) \ , \\ \Phi_{electr.} = \iint\limits_{S} \vec{D}d\vec{S} = q_{electr.} = q_{el.stat.} + q_{el.dyn.} = \Phi_{electr.}(x,y,z,t) \\ q_{elect.} = \iiint\limits_{\Omega} \rho_{el.}\, dV,\ i_{electr.} = \iint\limits_{\Sigma} \mathbf{J}_{electr.} \cdot d\mathbf{S} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} divD = \nabla D = \rho_{elect.} \\ \nabla \times D = \varepsilon(J_{mag.} - \dfrac{\partial B}{\partial t}) \end{array}\right\}$$

$$\left.\begin{array}{l} i_{mag.} = u_{electr.} = -\dfrac{d\Phi_{mag.}}{dt} = i_{mag.stat.} + i_{mag.dyn.} = i_{mag.}(x,y,z,t) \\ \Phi_{mag.} = \iint\limits_{S} \vec{B}d\vec{S} = q_{mag.} = q_{mag.stat.} + q_{mag.dyn.} = \Phi_{mag.}(x,y,z,t) \\ q_{mag.} = \iiint\limits_{\Omega} \rho_{mag.}\, dV,\ i_{mag.} = \iint\limits_{\Sigma} \mathbf{J}_{mag.} \cdot d\mathbf{S} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} divB = \nabla B = \rho_{mag.} \\ \nabla \times B = \mu(J_{elect.} + \dfrac{\partial D}{\partial t}) \end{array}\right\} \quad (3.4)$$

Within formulations (3.4) there is not an attempt to introduce and defend the existence of a simple, self-standing, natural magnetic charge (or magnetic monopole), since it is known that something like that cannot exist in ordinary, static, stable, or stationary and natural situations. Of course, magnetic monopoles could be artificially assembled (but not on a simple way). There are specific spatial configurations of magnets (assembled artificially) exposing to external space only one magnetic pole, while the opposite magnetic pole still exists, but it is invisible and hidden inside mentioned spatial monopole structure. Magnetic charges (to avoid term monopoles) could also, effectively, mathematically, and temporarily exist (as certain magnetic flux entities) in transient, action-reaction, "force-current-voltage-velocity" situations, when matter waves combined with mechanical motions and other interactions of involved participants (such as electromagnetic field, currents, and voltages) are being created, this way becoming complementary parts of an intrinsic electromagnetic symmetry and analogy.

The power $P(t)$, energy $\tilde{E}$, and equivalent matter-wave mass $\tilde{m}$, of certain electromagnetic field or electromagnetic wave group, in relation to (3.4), can be expressed as:

$$\begin{array}{l} P(t) = i_{electr.} \cdot u_{electr.} = u_{mag.} \cdot i_{mag.} = \dfrac{d\Phi_{electr.}}{dt} \cdot \dfrac{d\Phi_{mag.}}{dt} = \oiint\limits_{S} \vec{\Gamma} \cdot d\vec{S} = \Psi^2(t), \\ \text{where: } \vec{\Gamma} = \vec{\Gamma}(\vec{v}) = \vec{E}(\vec{v}) \times \vec{H}(\vec{v}) = \text{Poynting's vector}, \\ \Psi^2(t) = (\text{square of electromagnetic wave function}) = P(t) = \text{power}, \\ \tilde{E} = \int P(t)dt = \int \Psi^2(t)dt = \text{energy} \Rightarrow \tilde{m} = \dfrac{\tilde{E}}{c^2} = \text{effective mass}. \end{array} \quad (3.5)$$

In both (3.4) and (3.5), $\Phi_{electr.}$ and $\Phi_{mag.}$ present electrical and magnetic field fluxes (see important mathematical and analogical elaborations about power and energy of matter waves in Chapter 4.0, around (4.0.82) - (4.0.104) and later in Chapter 10.).

To temporarily avoid all questions and possible dilemmas about the real nature of different forms of electric and magnetic charges, currents, and voltages, we can start from equations (3.4) and (3.5), and replace all corresponding electric and magnetic inductions, or field members, using the expressions from equations (3.2) and (3.3). This way, going backwards (deductively), we can determine (or reinvent) all basic electromagnetic charges. In addition, we shall be able to generalize and unify the meanings of relevant static and dynamic, or transient electromagnetic parameters (such as charges, currents, voltages..., see T.5.2, (5.15) and (5.16)).

For instance, based on our present knowledge about electric charges, we consider this parameter as certain static, fixed numerical value, being number of Coulombs (or electrons). Based on analogies from the first Chapter of this book, we know or have strong indications that electric charge is not at all a static parameter. It really belongs to dynamic, motional states (like mechanical moments are), and obviously this challenging situation should be additionally clarified. ***Electric charges, when considered as being active, dynamic variables (especially on elevated potentials), should be kind of electromagnetic waves or mass emitters, being able to create kind of a antenna emitting, jet-propulsion, or thrust-force (as Nikola Tesla concluded in his experimental research)****. In addition, an attractive Coulomb force between very small-mass electric charges -q and +q, being on a mutual distance equal r, is* $\mathbf{F=-(1/4\pi\varepsilon_0)\cdot q^2/r^2}$*. The same attractive force will appear between only one electric charge q (which has mass m) and certain electromagnetically neutral mass m, since opposite charge would be electrostatically induced as an electric-dipole polarization effect within neutral mass m. As we know, for a similar case, when we have two mutually equal, electromagnetically neutral masses m and m, Newton gravitational force between them will be* $\mathbf{F=-G\cdot m^2/r^2}$*, meaning that Newton force is mathematically identical to Coulomb electrostatic force between +q and -q, indicating that Gravitation and Electromagnetism could have essentially the same electromagnetic nature. Consequently, we could reasonably assume that because of some natural reason, such as global, multiaxial, helical, and spinning motions of planets, solar systems, and galaxies (in cases of gravitational attraction; -see more in Chapter 2. around equations (2.4-4.1) - (2.4-4.3),) every mass is mathematically presentable as certain constant multiplied with one electron charge e, as follows,*

$$\begin{bmatrix} \mathbf{F=-(1/4\pi\varepsilon_0)\cdot q^2/r^2=-G\cdot m^2/r^2} \\ \mathbf{q_1=q_2=q,\ m_1=m_2=m,\ (1/4\pi\varepsilon_0)\cdot q^2=G\cdot m^2} \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{m_{1,2}=q_{1,2}/\sqrt{4\pi\varepsilon_0 G}} \\ \mathbf{q_{1,2}=N_{1,2}q_e=N_{1,2}e} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \mathbf{F=-G\cdot m_1m_2/r^2=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}=-G\cdot\left(q_1/\sqrt{4\pi\varepsilon_0 G}\right)\cdot\left(q_2/\sqrt{4\pi\varepsilon_0 G}\right)/r^2=} \\ \mathbf{=-\left(N_1e/\sqrt{4\pi\varepsilon_0 G}\right)\cdot\left(N_2e/\sqrt{4\pi\varepsilon_0 G}\right)/r^2=-N_1N_2e^2/4\pi\varepsilon_0 r^2,} \\ \mathbf{(q_1=q_2=q=Ne),\ (m_1=m_2=m)\Leftrightarrow(N_1=N_2=N=const)} \end{bmatrix} \Rightarrow \tag{3.5-a}$$

$$\Rightarrow \mathbf{F=-G\cdot m^2/r^2=-N^2e^2/4\pi\varepsilon_0 r^2=-\frac{q^2}{4\pi\varepsilon_0 r^2}\Rightarrow m^2=N^2e^2/4\pi\varepsilon_0 G\Rightarrow \boxed{m=\frac{N}{\sqrt{4\pi\varepsilon_0 G}}e}}.$$

If we now go back to the striking analogy between N. Bohr atom model and planetary or solar systems, as presented in Chapters 2. and 8. (*see "T.2.8. N. Bohr hydrogen atom and* *planetary system analogies"), based on (3.5-a) and (2.4-4.1) - (2.4-4.3)*, we will see why such analogy works well. This is implicitly indicating that all masses are dominantly composed of different combinations of electric charges, and that electromagnetic energy and photons-packing is anyway creating all elementary particles, atoms, matter, and masses around us.

……………………..

By following the usual framework of Maxwell-Faraday Electromagnetic theory, expressing all Maxwell equations in their integral and local forms, using symmetrical field forms from (3.2) and (3.3), we should be able to update, reconstruct and innovate our definition and approach to electromagnetic and gravitational charges, this way getting more universal, more united, practical, and operational definitions (of natural fields charges). This would make Maxwell theory more generally applicable, and more internally symmetric, as proposed in this book.

The biggest obstacles in reconstructing and optimizing contemporary Maxwell electromagnetic theory are related to three major deficiencies of Maxwell's equations, as only partially summarized in the following citation (see [35] from Thomas E. Phipps, Jr.):

• They are non-invariant at the lowest (first) order of approximation under inertial (Galilean) transformations.
• They are under-parameterized, in the sense that source motions are described, but not sink motions. Such an implicit promotion of the sink to a preferred motional status flouts any conceivable form of the relativity principle. (This is why invariance fails.)
• They misrepresent Faraday's observations of induction through the false implication that $\frac{\partial}{\partial t}$ can treat that for which $\frac{d}{dt}$ is mathematically necessary: *viz.,* description of the *emf* generated by *changing the shape* of a conducting circuit penetrated by magnetic flux.

The rest of analyzes and number of proposals found in the book of Thomas E. Phipps, [35], regarding Maxwell Theory and its upgrading and renovation, are so well elaborated and supported that it is better to read them from the original source, [35].

[♣ COMMENTS & FREE-THINKING CORNER

By William J. Hooper, there are three types of electric and magnetic fields still not distinctively and explicitly captured and explained by Maxwell electromagnetic theory.

Citation from: Selected Correspondence on Common Sense Science #1, David L. Bergman, editor, Common Sense Science, P.O. Box 767306, Roswell, GA 30076-7306, E-mail: bergmandavid@comcast.net):

"The experiments of William J. Hooper as described in his book ***New Horizons in Electric, Magnetic and Gravitational Field Theory*** *identify that all three types of electric and magnetic fields have different empirical properties. For instance, in Chapter 1 Hooper lists 14 empirical properties of E fields. In Table 1 he gives what these properties are for electrostatic E fields, E fields dependent on dA/dt, and E fields dependent on motion $V\times B$. In particular, he notes in property 6 that the motionally caused E fields cannot be shielded".*

Consequently, we can expect big reorganization and updating work within the present electromagnetic theory (see [152]).

Matter is anyway intrinsically united (locally and non-locally) being in coincidental (mutually synchronized) existence of all its aspects, forms, charges, fields and their multilevel and multidimensional interactions, and there is no doubt that whatever exists in certain (well-known and proven-valid) domain of physics should have analogous or closely related manifestations in some other domain... See John Stewart Bell's Interconnectedness theorem known in Quantum Mechanics established since 1964 (CERN, Geneva, Switzerland). For instance, parallelism and analogy *between Maxwell's Electromagnetic theory and Gravitation should exist at a much deeper level than presently known (because basic matter or mass constituents are atoms, composed of electrons, protons, and neutrons, and a free neutron is naturally separating on an electron and proton, meaning that all of them and ourselves are the part of the same electromagnetic world). Also, the Ampere-Maxwell's, Biot-Savart's, Lenz and Faraday's induction laws and electromagnetic Lorentz forces could be very inspiring concepts, from the point of view of analogous thinking, for richer description of "linear & rotational" fields, or motions (for instance, by conveniently transforming mentioned electromagnetic laws into corresponding analogous expressions for rectilinear and rotational motions ...).*

It looks that an extension of Maxwell's Electromagnetic theory (including different strategy regarding new Relativity theory) is also going to be made by revitalizing and upgrading of Wilhelm Weber's force law, which presents the natural unification of fundamental laws of classical electrodynamics, such as: Gauss's laws, Coulomb's law, Ampere's generalized law, Faraday's law, and Lentz's law. See literature [28] - [29].

If we consider that electric and magnetic (vector) fields should always be coupled, and structured as mutually orthogonal (especially in cases of electromagnetic waves propagation), we can try to present the composite vector of such electromagnetic field (or force) as,

$$\vec{\mathbf{F}}_{\mathbf{em}}(r, \theta, \phi, t) = \vec{\mathbf{F}}_p(r, \theta, \phi, t) + \vec{\mathbf{F}}_s(r, \theta, \phi, t),$$
$$(\vec{E} = \vec{E}_1 + \vec{E}_2 , \vec{H} = \vec{H}_1 + \vec{H}_2 = \vec{B}/\mu , \vec{B} = \vec{B}_1 + \vec{B}_2 = \mu\vec{H}), \qquad (3.5.1)$$

from which the first is ***the potential force field****,* $\vec{\mathbf{F}}_p(r, \theta, \phi, t) = \alpha_1\vec{E}_1 + \beta_1\vec{B}_1$

$$\nabla\times\vec{E}_1 = 0 , \nabla\cdot\vec{E}_1 = \frac{\rho}{\varepsilon_o} \neq 0$$
$$\nabla\times\vec{B}_1 = 0 , \nabla\cdot\vec{B}_1 \neq 0, \qquad (3.5.2)$$

and, the second is ***the solenoidal force field****,* $\vec{\mathbf{F}}_s(r, \theta, \phi, t) = \alpha_2\vec{E}_2 + \beta_2\vec{B}_2$

$$\nabla\times\vec{B}_2 = \mu_o(J + \varepsilon_o\frac{d\vec{E}}{dt}) \neq 0 , \nabla\cdot\vec{B}_2 = 0, ,$$
$$\nabla\times\vec{E}_2 = -\frac{d\vec{B}}{dt} \neq 0 , \qquad \nabla\cdot\vec{E}_2 = 0, \qquad (3.5.3)$$

where vectors of such electric and magnetic fields are,

$$\vec{\mathbf{E}}(r, \theta, \phi, t) = \vec{E}_1 + \vec{E}_2 , \vec{\mathbf{H}}(r, \theta, \phi, t) = \vec{H}_1 + \vec{H}_2 , \vec{\mathbf{B}}_{\mathbf{1,2}} = \mu_{\mathbf{o}}\vec{H}_{1,2}$$
$$\vec{\mathbf{F}}_{\mathbf{em}}(r, \theta, \phi, t) = \vec{\mathbf{F}}_p(r, \theta, \phi, t) + \vec{\mathbf{F}}_s(r, \theta, \phi, t) = \alpha_1\vec{E}_1 + \beta_1\vec{B}_1 + \alpha_2\vec{E}_2 + \beta_2\vec{B}_2 ,$$
$$\begin{bmatrix} \nabla\times\vec{\mathbf{F}}_{\mathbf{em}} = 0 \\ \nabla\cdot\vec{\mathbf{F}}_{\mathbf{em}} \neq 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \alpha_1\nabla\times\vec{E}_1 + \beta_1\nabla\times\vec{B}_1 + \alpha_2\nabla\times\vec{E}_2 + \beta_2\nabla\times\vec{B}_2 = 0 \\ \alpha_1\nabla\cdot\vec{E}_1 + \beta_1\nabla\cdot\vec{B}_1 + \alpha_2\nabla\cdot\vec{E}_2 + \beta_2\nabla\cdot\vec{B}_2 \neq 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} -\alpha_1\frac{d\vec{B}}{dt} + \beta_1\mu_o(J + \varepsilon_o\frac{d\vec{E}}{dt}) = 0 \\ \alpha_1\frac{\rho}{\varepsilon_o} + \beta_1\nabla\cdot\vec{B}_1 \neq 0 \end{bmatrix} \qquad (3.5.4)$$
$$\begin{bmatrix} \nabla\times\vec{\mathbf{F}}_{\mathbf{em}} \neq 0 \\ \nabla\cdot\vec{\mathbf{F}}_{\mathbf{em}} = 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \alpha_1\nabla\times\vec{E}_1 + \beta_1\nabla\times\vec{B}_1 + \alpha_2\nabla\times\vec{E}_2 + \beta_2\nabla\times\vec{B}_2 \neq 0 \\ \alpha_1\nabla\cdot\vec{E}_1 + \beta_1\nabla\cdot\vec{B}_1 + \alpha_2\nabla\cdot\vec{E}_2 + \beta_2\nabla\cdot\vec{B}_2 = 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} -\alpha_2\frac{d\vec{B}}{dt} + \beta_2\mu_o(J + \varepsilon_o\frac{d\vec{E}}{dt}) \neq 0 \\ \alpha_1\frac{\rho}{\varepsilon_o} + \beta_1\nabla\cdot\vec{B}_1 = 0 \end{bmatrix}$$

In addition, we could (still intuitively and hypothetically) explore the options that solenoidal field component $\vec{\mathbf{F}}_s(r, \theta, \phi, t)$ *is equal to Hilbert transform of the potential field component* $\vec{\mathbf{F}}_p(r, \theta, \phi, t)$, *or that magnetic field* $\vec{H}(r, \theta, \phi, t)$ *is (on some specific way) product of Hilbert transform of an* *electric field* $\vec{E}(r, \theta, \phi, t)$. *Presence of constants or functions* $\alpha_{1,2}, \beta_{1,2}$ *could give us a chance and freedom to adjust and comply with the present (or future) forms of Maxwell equations.*

Later, as a continuation of similar explorations, we could create an Analytic signal, complex function (or generalized Phasor) of the electromagnetic field, composed of mutually coupled (orthogonal and phase shifted) by Hilbert-transform created components (from (3.5.4). See more about Analytic signals in Chapters 4.0., and 10. Mentioned phase-shifted electric and magnetic fields, currents and voltages will have phase differences $+\pi/2$ or $-\pi/2$ *for inductive and capacitive loads or impedances (and the same is valid for mass and spring loads in mechanics), but for other resistive and complex impedance loads we will have different phase shifts (see more about electromechanical analogies in the first chapter of this book).*

Another, somewhat familiar and amazing set of ideas and source of intellectual, creative inspiration regarding electromagnetic field equations (and complex form of electromagnetic field) can be found in the publication "DERIVATION OF MAXWELL EQUATIONS AND THEIR CORRECTIONS" from Andrija S. Radovic´, ©1996, ©2003, http://www.andrijar.com and in number of papers from Prof. Jovan Djuric (literature reference under [34] and [71]).

Further evolution of such ideas can be to conceptualize most of physics-related vector fields (including gravitation and matter waves manifestations) as being mutually complementary (in pairs) composed of Potential and Solenoidal field vectors, where both are mutually linked like real and imaginary parts of certain relevant Analytic Signal or complex Phasor (being very much analogical to electric and magnetic fields in an electromagnetic wave). See more in Chapters 4.0. and 10.

Of course, here are given only initial brainstorming proposals, which should be additionally elaborated and improved. See similar ideas in relation to Bohr's hydrogen atom model in the appendix under Chapter 8. within equations (8.69) – (8.73), and in literature references under [63]. ♣]

3.3. New, Relativistic-like Formulation of Maxwell Equations

Let us now take the case of homogeneous and isotropic media (free space and vacuum) and find the simplest solutions for electric and magnetic fields, starting from the system of mutually dependent implicit equations (3.2),

$$\begin{aligned}
&\vec{E}(\vec{v}) = \vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \frac{1}{\varepsilon_0}\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right],\ \vec{H}(\vec{v}) = \vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \frac{1}{\mu_0}\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right]\\
&\vec{E}(\vec{v}) = \vec{E}_0 \pm \vec{v}\times\vec{B}\left[\vec{E}(\vec{v}), \vec{B}_0\right] = \vec{E}_0 \pm \mu_0\,\vec{v}\times\vec{H}\left[\vec{E}(\vec{v}), \vec{H}_0\right] = \vec{E}_0 \pm \mu_0\,\vec{v}\times\vec{H}(v),\\
&\vec{H}(\vec{v}) = \vec{H}_0 \mp \vec{v}\times\vec{D}\left[\vec{H}(\vec{v}), \vec{D}_0\right] = \vec{H}_0 \mp \varepsilon_0\vec{v}\times\vec{E}\left[\vec{H}(\vec{v}), \vec{E}_0\right] = \vec{H}_0 \mp \varepsilon_0\vec{v}\times\vec{E}(v),\ \varepsilon_0\mu_0 = 1/c^2.
\end{aligned} \tag{3.6}$$

One of possible solutions of (3.6) is:

$$\begin{aligned}
&\vec{E}(\vec{v}) = \frac{\vec{E}_0 - \varepsilon_0\mu_0\vec{v}\cdot(\vec{v}\cdot\vec{E}_0)}{1-\varepsilon_0\mu_0 v^2} \mp \frac{\mu_0\vec{v}\times\vec{H}_0}{1-\varepsilon_0\mu_0 v^2} = \frac{\vec{E}_0 - \frac{\vec{v}}{c^2}\cdot(\vec{v}\cdot\vec{E}_0)}{1-\frac{v^2}{c^2}} \mp \frac{\mu_0\vec{v}\times\vec{H}_0}{1-\frac{v^2}{c^2}}\ ,\\
&\vec{H}(\vec{v}) = \frac{\vec{H}_0 - \varepsilon_0\mu_0\vec{v}\cdot(\vec{v}\cdot\vec{H}_0)}{1-\varepsilon_0\mu_0 v^2} \pm \frac{\varepsilon_0\vec{v}\times\vec{E}_0}{1-\varepsilon_0\mu_0 v^2} = \frac{\vec{H}_0 - \frac{\vec{v}}{c^2}\cdot(\vec{v}\cdot\vec{H}_0)}{1-\frac{v^2}{c^2}} \pm \frac{\varepsilon_0\vec{v}\times\vec{E}_0}{1-\frac{v^2}{c^2}}\ .
\end{aligned} \tag{3.7}$$

From (3.7) we could "experience the picture" of ***mutually dependent, orthogonal and mutually complementing***, repetitively (and perpetually) ***self-generating or***

self-regenerating electric and magnetic fields and charges, when we apply Maxwell equations to (3.7): $\nabla \mathrm{x}\, \vec{H} = \vec{J} + \varepsilon\, \partial\vec{E}/\partial t$, $\nabla \mathrm{x} \vec{E} = -\mu \partial \vec{H}/\partial t$, $\nabla \vec{\mathbf{H}} = \mathbf{0}$, $\nabla \vec{E} = \rho/\varepsilon$.

The next step would be an attempt to find much more generally valid solutions for (3.2), in some similar form like we find in (3.7) and to apply definitions from (3.4) and (3.5) to such solutions. After obtaining more general forms of electric and magnetic currents and charges, we shall be able to go back and *reestablish* updated, more-symmetrical forms of all Maxwell equations compared to the present situation. By exploiting electromechanical analogies (as in the first Chapter of this book), we could go backwards to a new understanding of natural forces, including Gravitation, field charges, and their mechanical and electromagnetic moments.

It is also interesting to notice the validity of the following relations (in connection with understanding spontaneous field vectors bending and rotation):

$$\left\{\begin{array}{l} \vec{\mathbf{v}} \cdot \vec{\mathbf{E}}(\vec{\mathbf{v}}) = \vec{\mathbf{v}} \cdot \vec{\mathbf{E}}_0 \Leftrightarrow \mathbf{v} \cdot \mathbf{E}(\mathbf{v}) \cdot \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{E}}) = \mathbf{v} \cdot \mathbf{E}_0 \cdot \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{E}}_0), \\ \vec{\mathbf{v}} \cdot \vec{\mathbf{H}}(\vec{\mathbf{v}}) = \vec{\mathbf{v}} \cdot \vec{\mathbf{H}}_0 \Leftrightarrow \mathbf{v} \cdot \mathbf{H}(\mathbf{v}) \cdot \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{H}}) = \mathbf{v} \cdot \mathbf{H}_0 \cdot \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{H}}_0) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \mathbf{E}(\mathbf{v}) / \mathbf{E}_0 = \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{E}}_0) / \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{E}}) \\ \mathbf{H}(\mathbf{v}) / \mathbf{H}_0 = \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{H}}_0) / \mathbf{cos}(\vec{\mathbf{v}}, \vec{\mathbf{H}}) \end{array}\right\},$$

$$\left\{\begin{array}{l} \vec{v} \cdot \left[\vec{E}(v) - \vec{E}_0\right] = \vec{v} \cdot \Delta\vec{E} = 0 \\ \vec{v} \cdot \left[\vec{H}(v) - \vec{H}_0\right] = \vec{v} \cdot \Delta\vec{H} = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \vec{v} = 0, \quad \text{or} \quad \Delta\vec{E} = 0, \quad \text{or} \quad \cos(\vec{v}, \Delta\vec{E}) = 0 \\ \vec{v} = 0, \quad \text{or} \quad \Delta\vec{H} = 0, \quad \text{or} \quad \cos(\vec{v}, \Delta\vec{H}) = 0 \end{array}\right\}, \qquad (3.8)$$

which are equally applicable to (3.1), (3.2), (3.6) and (3.7).

....

Another, still hypothetical idea, in connection with electric and magnetic field vectors is that ***electric and magnetic fields could be mutually related like real and imaginary components of an Analytic signal*** (based on Hilbert transform, as established by D. Gabor; -see more in Chapter 4.0. of this book). In addition, we should not forget and neglect Thomas E. Phipps, Jr. elaborations and proposals (as can be found in [35]) about Maxwell and Relativity theory weak areas, regarding missing but necessary updates (which should be taken as an extremely important warning).

For instance, Maxwell equations of electromagnetic waves have the same mathematical form of Classical, second order differential wave equation, as known in Mechanics (see (4.9-1) and (4.9-2.1) from Chapter 4.3., and (4.0.82) from Chapter 4.0.), which can be analogically assembled, or mutually connected between electric and magnetic fields (in a shape of the Complex, Analytic Signal), on the following way,

$$\Delta\mathbf{E}-\varepsilon_0\mu_0\frac{\partial^2\mathbf{E}}{\partial t^2}=\Delta\mathbf{E}-\frac{1}{u^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=0,$$
$$\Delta\mathbf{B}-\varepsilon_0\mu_0\frac{\partial^2\mathbf{B}}{\partial t^2}=\Delta\mathbf{B}-\frac{1}{u^2}\frac{\partial^2\mathbf{B}}{\partial t^2}=0,\ u=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=c=\text{const.}(=)\text{light speed}$$
$$\mathbf{E}(=)\text{electric field},\mathbf{B}(=)\text{magnetic induction},\ \ \mathbf{H}(=)\text{Hilbert transform}$$
$$\overline{\Psi}=\overline{\Psi}(\mathbf{E},\mathbf{B})=\Psi(\mathbf{E},\mathbf{B})+I\cdot\hat{\Psi}(\mathbf{E},\mathbf{B}),\hat{\Psi}(\mathbf{E},\mathbf{B})=\mathbf{H}\left[\Psi(\mathbf{E},\mathbf{B})\right],\ I^2=-1$$
$$\nabla^2\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\Delta\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\Leftrightarrow\begin{Bmatrix}\Delta\Psi-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\rightarrow\Delta\Psi-\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2}=0\\ \Delta\hat{\Psi}-\frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2}=0\rightarrow\Delta\hat{\Psi}-\frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2}=0\end{Bmatrix}$$
$$\Psi^2(t,r)=P(t,r)=\frac{d\tilde{E}}{dt}(=)\text{Active Power }(=)$$
$$(=)\begin{Bmatrix}i(t)\cdot u(t) & (=) & \left[\text{Current}\cdot\text{Voltage}\right],\text{ or}\\ f(t)\cdot v(t) & (=) & \left[\text{Force}\cdot\text{Velocity}\right],\text{ or}\\ \tau(t)\cdot\omega(t) & (=) & \left[\text{Orb.}-\text{moment}\cdot\text{Angular velocity}\right],\text{or}\\ (\vec{E}\,x\,\vec{H})\cdot\vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right]\cdot\overrightarrow{\text{Surface}}\end{Bmatrix} \quad (3.7\text{-}1)$$

We still have a freedom to define (later) what means generalized wavefunction of electromagnetic waves, $\overline{\Psi}(\mathbf{E},\mathbf{B})=\Psi(\mathbf{E},\mathbf{B})+I\cdot\hat{\Psi}(\mathbf{E},\mathbf{B}),\ \hat{\Psi}(\mathbf{E},\mathbf{B})=\mathbf{H}\left[\Psi(\mathbf{E},\mathbf{B})\right],I^2=-1$, (see, for instance, (4.0.82) in Chapter 4.0. and (4.23) from Chapter 4.3.). Anyway, what we can conclude from (3.7-1) is that light speed is always constant and dependent only on some static, fixed, or some hidden, background *matter-media parameters* (effectively from a pure vacuum, dielectric, and magnetic permeability properties ε_0, μ_0), $u=1/\sqrt{\varepsilon_0\mu_0}=c=\text{const.}$. There is nothing here (regarding light speed c) about variable and mutually dependent time and length intervals contraction in compliance with Lorentz transformations. Consequently, A. Einstein, intentionally or not intentionally, or just intuitively concluded (or only postulated) that light speed should always be the same and constant (in all mutually relative motions and inertial frames). How to understand such unusual property of our Universe or Nature is still a challenging situation. Such always constant speed of light (regardless how strange it is) should also, and at least, be formally (or mathematically) defined as any other speed in Physics, meaning,

$$c=1/\sqrt{\varepsilon_0\mu_0}=u=\text{const.}=\frac{\Delta x_c}{\Delta t_c}=\frac{k\cdot\Delta x_c}{k\cdot\Delta t_c}=\frac{p\cdot\Delta x_c}{p\cdot\Delta t_c}=...\ , \quad (3.7\text{-}2)$$

where Δx_c and Δt_c are certain minimal and mutually related, spatial, and temporal durations, or intervals, $\Delta x_c=c\cdot\Delta t_c$, which are obviously intrinsically quantized, naturally always constant, and cannot be smaller. ***Such kind of conceptualization is connecting (minimal) spatial and temporal signal (or relevant matter-wave packets) positions and durations, and this could additionally clarify and explain one aspect of relativistic coordinates transformations in relation to A. Einstein postulate about always constant light speed, implicating the***

existence or revitalization of the ether (at least regarding electromagnetic phenomenology).

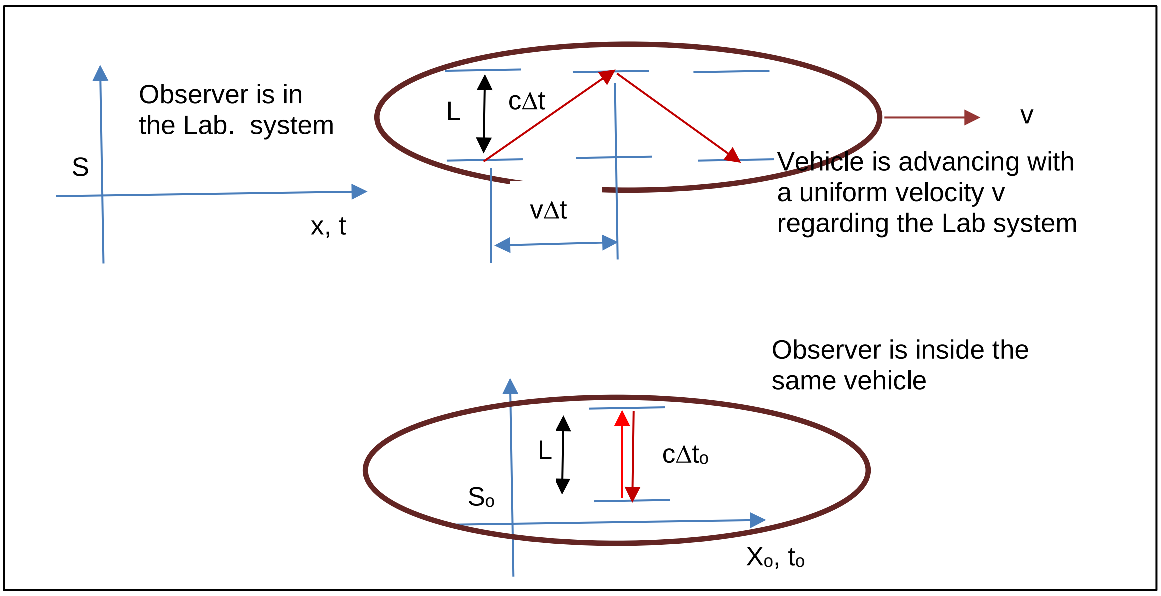
Since Classical, second order differential wave equations in Mechanics, Acoustics, Fluid Dynamics ... also have the same form/s as found in (3.7-1), where phase velocity $u = \lambda f$ could be constant (and related only to relevant material properties), and independent from spatial and temporal parameters, this will open new insights and challenging questions in relation to wave motions, Lorentz transformations, and temporal and spatial durations of motional particles and corresponding wave groups (see more about group and phase velocity in Chapters 4.0., 4.1. and 10.). Regardless of the field of application, generally valid relations between group and phase velocity v and u, of all kinds of matter waves (or wave packets) are always the same, such as,

$$u = \lambda \cdot f = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\tilde{E}}{p},\ v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda}, \tag{3.7-3}$$

meaning that there is still a work in front of us to completely understand, unite and harmonize here presented facts regarding speed of light, phase velocity, and spatial and temporal intervals relations.

[♣ COMMENTS & FREE-THINKING CORNER:

Let us imagine that inside certain (linearly moving) vehicle a light beam is shot between two mirrors (perpendicularly to the direction of linear vehicle motion), and the distance between mirrors is L. Observer within its reference frame So, linked to the vehicle, will notice or measure only the path of travel of the light-beam equal to $\mathbf{L = c \cdot \Delta t_0}$, *where* Δt_0 *is the travel-time interval or time duration in So . Another observer in the laboratory coordinate system S will detect the same light-beam travel* Δt *differently, since the system So has the velocity v in the system S. This will result in,* $\mathbf{L^2 = (c\Delta t)^2 - (v\Delta t)^2 = (c\Delta t_0)^2}$. *See all relevant details on the picture below.*



Time and space intervals of the same event in different referential frames

$$c = 1/\sqrt{\varepsilon_0\mu_0} = \text{const.} = \frac{\Delta x_c}{\Delta t_c} = \frac{k\cdot\Delta x_c}{k\cdot\Delta t_c} = \frac{p\cdot\Delta x_c}{p\cdot\Delta t_c} = \frac{\Delta x_0}{\Delta t_0} = ...$$

$$L^2 = (c\Delta t)^2 - (v\Delta t)^2 = (c\Delta t_0)^2 \Rightarrow \boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \gamma\cdot\Delta t_0} \Rightarrow$$

$$\begin{bmatrix} \Delta t_0 = k\cdot\Delta t_c = \Delta t/\gamma \\ \Delta x_0 = k\cdot\Delta x_c \\ \\ \frac{\Delta x_0}{\Delta t_0} = \frac{\Delta x_c}{\Delta t_c} = c \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta t = k\cdot\gamma\cdot\Delta t_c = \gamma\Delta t_0 \\ \boxed{\Delta x = v\cdot\Delta t = \gamma\cdot v\cdot\Delta t_0 = \gamma\cdot\frac{v}{c}\Delta x_0} \\ \\ v = \frac{\Delta x}{\Delta t} = \frac{v}{c}\cdot\frac{\Delta x_0}{\Delta t_0} = v \end{bmatrix}$$

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\gamma\Delta t_0)^2 - (\gamma\cdot v\cdot\Delta t_0)^2 = (c\Delta t_0)^2 = (\Delta x_0)^2 = L^2$$

$$(\Delta s_0)^2 = c^2(\Delta t_0)^2 - (\Delta x_0)^2 = c^2(\Delta t_0)^2 - (c\Delta t_0)^2 = 0$$

Eventually, in cases when addressing photons, (starting from (3.7-3)), we will get what we expect to have, and what we already know as correct from different points of view, as follows,

$$\begin{bmatrix} u = \lambda\cdot f = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\tilde{E}}{p} = c = \text{const.}, \quad \lambda = \frac{h}{p} \\ v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \left[\tilde{E} = cp = c\frac{h}{\lambda} = hf,\ v = \frac{d\tilde{E}}{dp} = -\lambda^2\frac{df}{d\lambda} = u = c\right] \Rightarrow c\frac{d\lambda}{\lambda^2} = -df \Rightarrow \qquad (3.7\text{-}4)$$

$$\Rightarrow \left(\frac{c}{\lambda_2} - \frac{c}{\lambda_1}\right) = (f_2 - f_1),\ \frac{\Delta x_0}{\Delta t_0} = \frac{\Delta x_c}{\Delta t_c} = \frac{\Delta x}{\Delta t} = u = v = c\ .$$

An interesting situation regarding (3.7) is that today's Maxwell theory is somewhat artificially, but still correctly formulated, or better to say successfully fitted and merged within the so-called Relativistic Electrodynamics theory. Such modified theory (as briefly exercised here) is naturally generating on some way familiar results regarding Lorentz transformations of electromagnetic field vectors. Lorentz electromagnetic field transformations known in contemporary Relativistic Electrodynamics are mathematically correct (since relevant experimental and theoretical facts are telling that Relativistic Electrodynamics is functioning very well), but apparently it is being different, compared to (3.7), because at least both are using/implementing/generating the Lorentz factor $(1-\varepsilon_0\mu_0 v^2) = (1-\frac{v^2}{c^2})$. The consequences of such situation could be that either (3.2), or some aspects of today's Relativistic Electrodynamics, or forms of Lorentz electric and magnetic forces found in (3.7) and earlier are in certain fundamental aspects incorrect and/or incomplete. In addition, we could explore a possibility that Lorentz forces and Lorentz transformations are partially miss-formulated, and maybe placed in a wrong conceptual framework, instead of being fully incorporated in Maxwell equations, as shown in [34] and [35], not forgeting that contemporary Relativity theory also has similar weak areas. An important question to ask here is how we can successfully unite or combine Maxwell and Relativity theory and create innovated, even better functioning Relativistic Electrodynamics, when both of mentioned theories have some not well composed, not completely finalized, and challenging situations, or missing spots (see much more about fundamental errors and weak spots of Relativity and Maxwell theory in [35]).

The “new” electromagnetic forces, not at all described by present Maxwell electromagnetic theory, are already known, and becoming an increasing area of scientific research (see [30]). Such new aspects of electromagnetic forces are experimentally verifiable, showing undoubtedly that present-days Maxwell equations are sooner or later going to be modified and updated. The proposal here is to establish new conceptual understanding of ***electromagnetic field as a combination of longitudinal and transversal wave motions, including natural coupling with associated linear and rotational motions*** in the mathematical framework of Analytic signals modeling. Here we could also address universal force laws formulated by Ampère and Wilhelm Weber. Ampère’s law (recently modernized on different ways, by number of authors) is increasingly receiving recognition (almost 200 years after its first formulation) regarding predicting or complying with experimental results related to forces between current elements, or between other electrically conductive channels (see literature references [28], [29], [36] and [63]). ♣]

Here is a convenient place to mention that N. Tesla also had number of extraordinary, original, and imaginative ideas, suggestions and experimental results related to electromagnetic phenomenology and Gravitation, certain of them still waiting to be fully understood and decoded (see presentations about N. Tesla ideas, experiments, concepts and results in [81] and [97]).

[♣ COMMENTS & FREE-THINKING CORNER:

N. Tesla’s heritage is still offering scientific motivation and uniting cultural ingredients among several historically, geopolitically, ideologically, and nationally different (and sometimes conflicting) flavors of our modern society. For instance, Serbs, Croats, and Americans are claiming that Tesla presents their national proud and scientific heritage. Anyway, the mainstream tendency (in relevant worldwide publications, except in Serbian ones) has been to insinuate that Tesla is everything else, and only by an insignificant and arbitrary accident, a little bit of being a Serb, or more preferably, for certain Vatican-flavored groups, he should be pure Croat. Answering such questions, Tesla clearly said that he is the Serb (from pure Serbian family with all Serbian attributes). In addition, he said that being either Serb or Croat is the same (most probably because Croatian identity started to be gradually created, and known in geographic areas under Vatican influence, dominantly and initially populated with Serbs. Serbs and Croats started to differentiate only after the Vatican or catholic ideologically politically separated from the Orthodox Christianity family and started motivating and forging available Serbs to follow Vatican-elaborated prescriptions. Of course, after several generations, many of newly labeled Croats forgot what real origins they have. Tesla, equally admired worldwide and by Serbs and Croats, had much more cosmopolitan and international nature, and all of that (regarding his national, ideological, and geographic origin) is irrelevant in relation to Tesla’s contribution to our planetary science and technology. Nevertheless, during the Second World War, some of deeply indoctrinated Croats, morally and ideologically protected and supported by Vatican and fascists, exterminated hundreds of thousands easily available Serbs, including members of Nikola Tesla family (probably, just in case, to secure being on a good side). Even present western scientific community mainstream is still glorifying Edison and Marconi as biggest inventors and contributors to planetary electromagnetic technology, forgetting that our human and technological society would stop working if Tesla’s inventions and engineering contributions would suddenly disappear (not to mention other relevant and shameful details about Edison and Marconi in relation to Tesla). It is good to remember such details to enrich and challenge our positions and feelings regarding a variety of virtual and imaginative realities applicable to the same subject. Reality and facts about Tesla are known, unique and clear (not at all pluralistic) to neutral and objective observers, and to honest, not-indoctrinated historians. Anyway, different authors and interpreters, from mentioned cultural and ideological flavors (in relation to Tesla), are giving differently colored presentations to compromise or reinforce certain temporary and slowly evolving geopolitical, ideological positions and political trends, and we know that such performances of human society are equally and largely present in ordinary life and science. At least, modern human society stopped with most brutal suppressions against too original science-related contributors and interpreters that are not mainstream compatible, as it was the case with middle-ages by Vatican organized inquisition (until less than couple hundreds of years ago), being practiced within a big part of Vatican dominated territory … Presently, we are faced with much softer, hidden, smarter and more elaborated history, science and facts modifications, manipulations, and optimizations, where Tesla is only one of instructive, convenient, and positive cases to mention and follow (of course, we should not forget the similar example of Rudjer Boskovic, who was admired by Tesla, [6]).

*Obviously, it is interesting that in (3.7) we can find characteristic Lorentz factor, usual for coordinate transformations in **S**pecial **R**elativity **T**heory (**SRT**), $\gamma^2 = 1/(1-\varepsilon_0\mu_0 v^2) = 1/(1-v^2/c^2)$, without applying or imposing anything what "smells" on **SRT**. The author of this book is still not able to offer more precise and final (or better) comments about such insights. Based only on the brainstorming and common-sense intuition we could ask ourselves if today's (Lorentz) relativistic transformations of electric and magnetic field vectors (or applied anywhere else in physics) are so much important, essential, fully correct, complete, and generally applicable to all kinds of particles, fields and wave states and motions? The remaining option is that in our contemporary physics books we still have some particularly valid (well-fitted) cases of artificially created **"relativistic"** field transformations, **"explaining"** well **"mostly already known"** experimental facts, in many cases initially discovered or implemented **without using any of present relativistic theory background**). ♣]*

Let us mention possible ideas that will test the general validity of today's Maxwell Relativistic equations. Magnetic field phenomenology is always in connection with its complementary electric field phenomenology, being mutually coupled and united in a form of electric and/or magnetic charges, fluxes, and/or currents (even when we do not consider this as an option). For instance, the magnetic field of an electromagnet can be created inside and around a ferromagnetic core, if the core is placed inside of a solenoid with circulating electrical current. Circulation of electrons in a solenoid around magnetic core presents kind of rotation (or closed electric-current loop). Any rotation also presents non-uniform accelerated motion, and **SRT** is not addressing such situations. If we now take as another example a permanent magnet, we can measure its magnetic field, without seeing a real solenoid and circulation of its electrons. We also know that piece of permanent magnet should be a case of a bit hidden rotation inside of spinning electrical (current) elements of internal magnetic domains (meaning that again some kind of hidden rotation on the atomic scale is creating a permanent magnet field). In the case of propagation of electromagnetic waves (or fields) in an open and free (of forces and interactions) space (or in a vacuum) we see neither core material nor solenoid, but again we can detect the presence of electromagnetic waves (or photons, composed of mutually coupled and rotating electric and magnetic field vectors). A kind of well-hidden rotation (or vortex phenomena) of some strange "carrier-fluid flow" should always exist in certain material form (even in a vacuum), and be responsible to create the magnetic field component, associated to some moving electric entity. However, it looks that we are still not able to detect a direct presence of such strange space-time texture (or ether). Luckily, it also looks that we do not need to master too much such kind of "carrier-fluid" for modeling electromagnetic waves, since Maxwell theory mathematically explains well the creation and propagation of electromagnetic field components without a big need to say explicitly what their "carrier-fluid" is. Explaining light or electromagnetic waves propagation could be analogically conceptualized by comparing them with alternating currents and voltages in capacitors (in dielectrics), or in complex impedances of different electric loads.

*[♣ **COMMENTS & FREE-THINKING CORNER:** Relativistic Lorentz transformations (of **SRT**) applied on (visibly or invisibly) rotating and mutually complementary and coupled states, which also perform rectilinear motion, should be (mathematically) more complex than just using simple Lorentz transformations applied only on rectilinear motions (as formally prescribed in present days **SRT**). Most probably that some of missing or limiting aspects of traditionally formulated Maxwell theory and **SRT** (such as not-counted effects of transient situations, accelerated motions and rotation, incomplete mathematical formulation of Faraday's Law of Magnetic Induction, incomplete symmetry in formulating basic Maxwell equations and/or equations of electromagnetic field…), are artificially (or mathematically) compensated by formulating **Q**uantum and **R**elativistic **E**lectrodynamics, (**QRE**). Effectively, QRE was constructed and adjusted to satisfy and explain results of (already known)*

*experimental observations. If traditional Maxwell Electrodynamics (formulated in Galilean space) was differently established, we should be able to get relativistic field transformations (as known in present days Relativistic Electrodynamics, similar to (3.7)), without any math-hybridization with **SRT**, or without using or knowing **SRT** (see number of papers indicating such options from **[23]** to **[26]** and in **[35]**). ♣]*

Certain connection between Maxwell theory and Gravitation comes from the well-known fact that quantum of electromagnetic radiation, a photon, has its equivalent, dynamic mass, hf/c^2, *momentum,* hf/c, *and spin (see analyses of Compton and Photoelectric effect, for instance). In addition, gravitation field has some influence on an equivalent photon mass. Rectilinear motion and rotation of particles (and other energy states) also make a complementary (matter-wave) couple (like rotating couple of electric and magnetic field unified in a photon formation). The creation of an electron-positron couple from high energy* γ *-photon (both naturally spinning after creation) is yet another experimental example (among others) confirming that photon (or wave energy) can be transformed into real and rotating particles. For instance, from (3.5) we can find effective or equivalent (static and dynamic) mass and momentum of electromagnetic radiation, in terms of electric and magnetic field vectors,* $\tilde{\mathbf{m}} = \tilde{\mathbf{E}}/\mathbf{c}^2 = \mathbf{m}_{\text{stat.}} + \mathbf{m}_{\text{dyn.}}$, *going back to (3.4) and (3.3), exploiting all Maxwell equations. It is already known that A. Einstein (besides many others) tried, during almost half of his life, to apply Faraday-Maxwell Theory as a guiding model for describing Gravitation and Unified Field theory. Eventually, his efforts ended without success, most probably because SRT uses some challenging and problematic assumptions and postulates, and it is not considering visible or hidden effects of rotation and other accelerated motions, as previously mentioned. Here, the similar idea is initiated (based on an analogical thinking regarding unification between Gravity and Electromagnetic field). However, before we start to realize it, first we should somehow establish the active presence and influence of the "field of rotation" that is (or should be) complementary to Gravitation (following the analogy between mutual complementary nature of electric and magnetic fields). Then, also using a bit extended and upgraded Maxwell equations as a guiding model, it should be much easier to realize above-mentioned objectives (see [34] and [35]). Here favored strategy is to try to introduce (new, more realistic, and more relevant) adjustments and modifications, both in Gravitation and Electromagnetic theory, and make them more mutually compatible for unification (specially to show that Gravitation also has its essential origins in Electromagnetic fields and forces, as suggested in [23] – [26], [63] and [152]). Here, the research area regarding fields unification is also linked to inertial and reaction forces, "fields of rotation" and de Broglie matter waves phenomenology (see (4.18), (4.19), (4.22) - (4.29), (5.15) and (5.16)), and it will be more closely analyzed in the following chapters of this book.*

[♣ COMMENTS & FREE-THINKING CORNER:

In "*P. W. Bridgman, The Logic of Modem Physics*" we can find the following statement (see in [35]):
"There is no physical phenomenon whatever by which light may be detected apart from the phenomena of the source and the sink ... Hence from the point of view of operations, it is meaningless or trivial to ascribe physical reality to light in intermediate space, and light as a thing traveling must be recognized to be a pure invention".

In other words, photons or light is measurable or detectable only directly on its source and directly on its sink or receiver, but in a space between (where photons should travel), we are not able to see photons and light waves. Of course, if we place sensors and detectors between the light source and light receiver, we will be able to detect photons. In addition, if we consider that all the energy-momentum currents, voltages, and electromagnetic motions, should belong to closed circuits, or *networks with real, physical front and last ends (meaning with sources and sinks), we can intuitively conclude or suspect that:*

a) *Information about emitted photon from its source is immediately transmitted to its receiver, regardless of distances between them, but physically received photon (or its momentum and energy) will rich its sink after a certain time because of the limited speed of photons.*

b) *Emitters and receivers (or sources and sinks) of photons are on some way, permanently (immediately and timeless) connected and synchronized by certain entanglement channels and couplings, thanks to spatial-temporal couplings and mutual domains transformability. Of course, emitters and receivers of photons are some atoms (or resonators), meaning that whole cosmos or universe was once closely and densely united, creating mutually coupled atoms, photons, and other elementary particles (effectively being presentable as some resonators), which are still in mutual entanglement relations, and effectively any distant receiver knows in advance that after a certain time it will receive specific (emitted) photon. This kind of imaginative excursions is mathematically explicable with complex Analytic signal modeling of motions, mater waves and signals existing in our Universe.*

c) *Since open (lose and free ends) circuits and networks (without sources and sinks) do not exist in our Universe, it should exist certain material media, substance, or fluid (even in a pure cosmic vacuum), connecting all entities from the world of Physics. It is another question how to model and explain such connecting channel where information is being transferred without any time delay.* ♣]

3.4. The Important Extension of Electromechanical Analogies

Since in (3.4) we introduced static and dynamic currents, charges, and fluxes (for both electric and magnetic field), the system of analogies given in T.1.3, T.1.4 and T.1.5 can be extended, separating electric and magnetic parameters, towards equivalent (dimensional) analogies, as summarized in T.3.1, T.3.2 and T.3.3:

T.3.1	[E] = [ENERGIES]= = $\int\{[U]\,[I]\}dt = \int[P]dt$	[Q] = [CHARGES] = = [C][U]	[C] = [REACTANCES] = = [Q]/[U]
Electric Field	$[C_{el.}u_{el.}^2]$ (=) $[q_{el.}u_{el.}^1]$ $[L_{el.}i_{mag.}^2]$ (=) $[\Phi_{el.}i_{mag.}^1]$	$[C_{el.}u_{el.}^1]$ (=) $[q_{el.}u_{el.}^0]$ (=) $[q_{el.}]$ (=) $[\Phi_{el}]$	$[C_{el.}u_{el.}^0]$ (=) $[q_{el.}u_{el.}^{-1}]$ (=) $[C_{el.}]$ (=) $[L_{el.}]$ (=) [C]
Magnetic Field	$[C_{mag.}u_{mag.}^2]$ (=) $[q_{mag.}u_{mag.}^1]$ $[L_{mag.}i_{el.}^2]$(=) $[\Phi_{mag.}i_{el.}]$	$[C_{mag.}u_{mag.}^1]$ (=) $[q_{mag.}u_{mag.}^0]$ (=) $[q_{mag.}]$ (=) $[\Phi_{mag}]$	$[C_{mag.}u_{mag.}^0]$ (=) $[q_{mag.}u_{mag.}^1]$ (=) $[C_{mag.}]$ (=) $[L_{mag.}]$ (=) [L]
Gravitation	$[mv^2]$ (=) $[pv^1]$	$[mv^1]$ (=) $[pv^0]$ (=) [p]	$[mv^0]$ (=) $[pv^{-1}]$ (=) [m]
Rotation	$[J\,\omega^2]$ (=) $[\mathbb{L}\,\omega^1]$	$[J\omega^1]$ (=) $[\mathbb{L}\omega^0]$ (=) $[\mathbb{L}]$	$[J\omega^0]$ (=) $[\mathbb{L}\omega^{-1}]$ (=) [J]

T.3.2	[U] = [VOLTAGES] = = d[X]/dt	[I] = [CURRENTS]= = d[Q]/dt	[Z] =[IMPEDANCES]=[U]/[I] (= [mobility] in mechanics)
Electric Field	$u_{el.} = d\Phi_{mag.}/dt = i_{mag.}$	$i_{el.} = dq_{el.}/dt = u_{mag.}$	$Z_{el} = u_{el.} / i_{el.} = i_{mag.}/ u_{mag.} = Y_{mag.}$
Magnetic Field	$u_{mag.} = d\Phi_{el.}/dt = i_{el.}$	$i_{mag.} = dq_{mag.}/dt = u_{el.}$	$Z_{mag.} = u_{mag.} / i_{mag.} = i_{el.} / u_{el.} = Y_{el.}$
Gravitation	$v = dx/dt$	$F = dp/dt$	$Z_m = v / F$
Rotation	$\omega = d\alpha/dt$	$\tau = d\mathbb{L}/dt$	$Z_R = \omega / \tau$

T.3.3	[R] = [RESISTANCES]= = $[Z]_{real}$	[X] = [DISPLACEMENTS] = = $\int[U]\,dt$	[P] = [POWER] = d[E]/dt = = [U][I]
Electric Field	$R_{el.}$	$\Phi_{mag} = L_{mag}i_{el} = q_{mag}$	$u_{el.}i_{el.}$
Magnetic Field	$R_{mag.}$	$\Phi_{el.} = L_{el.}\,i_{mag.} = q_{el.}$	$u_{el.}i_{el.}$
Gravitation	R_m	$x = S\,F$	$v\,F$
Rotation	R_R	$\alpha = S_R\tau$	$\omega\tau$

Based on extended analogies from T.3.1, T.3.2 and T.3.3 (regardless of used terminology and symbols that could be further improved), certain creatively founded future analyses can open new and/or reinforce already presented hypotheses (see **[3]**). This will lead to a more general theory of gravitation, complemented with the field effects of rotation. It is also clear that here we are only touching the bottom levels of possible unification concepts for constructing the higher level of Universal Field theory (based on analogies and symmetries as shown in **T.1.8 Generic Symmetries and Analogies of the Laws of Physics,** Chapter 1.). Here we are mostly searching for indicative and challenging ideas, facts and analogies that will create a background useful for foundations of a New Field theory *(The more pragmatic and more general field unification platform will be introduced in chapter 4. and 5. of this paper. See development of equations (4.22) - (4.29), (5.15) – (5.16)).*

The fact is that analogies could serve as a powerful unifying and predicting research tool, only if properly formulated, and if all bottom-line, elementary elements of formulated analogy charts and tables are fully mutually coherent and analogically replaceable. As we can see, until the present, such level of bottom-line coherence and symmetry (regarding established analogies) is remarkably present in Physics.

Another approach to analogies would be to start from Energy (in any of its packing formats) considering it as the primary building entity of everything that we deal with in our universe, and then express all other physics-related values (such as: Charges, Reactance, Voltages, Impedances, Resistances, Displacements and Power, found in T.3.1, T.3.2 and T.3.3) in terms of Energy. This way we could try to formulate more powerful analogies and symmetries, leading to generally acceptable, and Unified Field theory.

[♣ COMMENTS & FREE-THINKING CORNER:

From T.3.2 we can easily notice that the product between electric and magnetic impedances (of the same electric circuit) is equal to 1 ($Z_{el.} \cdot Z_{mag} = \frac{u}{i} \cdot \frac{i}{u} = 1$*). Let us extend this analogically, searching for mechanical impedance (or mobility) of a linear motion, and for the mechanical impedance of the rotation or spinning that belongs to the same motion, where the product of both would also be equal to 1 (or to some constant). We shall see that this is still not possible to satisfy as a generally valid case since* $Z_m \cdot Z_R = \frac{v}{F} \cdot \frac{\omega}{\tau} \neq \text{const.}$ *(because all the involved variables* v, F, ω, τ *are multidimensional, space-time dependent). If we take the product between relevant mechanical impedance and admittance* $Z_m \cdot Y_R = \frac{v}{F} \cdot \frac{\tau}{\omega}, Y_R = \frac{1}{Z_R} = \frac{\tau}{\omega}$*, we will see that the result could be interesting (most probably invariant to referential system changes).*

As the first example, let us intentionally take the case of a circular mass motion that can be presented as linear particle motion along the circular line or rotational motion around its center of rotation. In both cases, we should get the same mechanical impedance and all other relevant energy members should be mutually equal ($dE = v \cdot dp = \omega \cdot dL, v = \omega \cdot r$ *).*

$$\left\{\begin{array}{l} \vec{p} = m\vec{v},\ \vec{L} = \vec{r} \times \vec{p},\ \vec{F} = \frac{d\vec{p}}{dt}, \vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}, \\ dE = v \cdot dp = \omega \cdot dL, v = \omega \cdot r \end{array}\right\} \Rightarrow$$

$$\Rightarrow\begin{cases} Z_m = \frac{v}{F} = \frac{dE\cdot dt}{(dp)^2} = \frac{\omega}{\tau}\cdot(\frac{v}{\omega})^2 = \frac{dE\cdot dt}{(dL)^2}\cdot(\frac{v}{\omega})^2 = \frac{\omega}{\tau}\cdot r^2 = Z_r\cdot r^2 = \\ = \frac{\omega}{(\frac{\tau}{r^2})} = \frac{\omega}{\pi(\frac{\tau}{\pi r^2})} = \frac{\omega}{\pi(\frac{\tau}{S})} = \frac{\omega}{\frac{\pi}{S}\tau} = \frac{\omega}{\tau^*},\ S = \pi r^2, \\ \tau^* = \frac{\pi}{S}\tau = \frac{\pi}{S}\frac{dL}{dt} = \frac{dL^*}{dt},\ \vec{L}^* = \frac{\pi}{S}\vec{L} = \frac{\pi}{S}(\vec{r}\times\vec{p}) = J\vec{\omega}, \\ Z_r = \frac{\omega}{\tau} = \frac{dE\cdot dt}{(dL)^2} = \frac{v}{F}\cdot(\frac{\omega}{v})^2 = \frac{dE\cdot dt}{(dp)^2}\cdot(\frac{\omega}{v})^2 = \frac{v}{F}\cdot\frac{1}{r^2} = Z_m\cdot\frac{1}{r^2} = \\ = \frac{v}{F}\cdot\frac{\pi}{\pi r^2} = \frac{v}{F}\cdot\frac{\pi}{S} = \frac{v}{\frac{S}{\pi}\cdot F} = \frac{v}{F^*},\ Z_m\cdot Z_r = \frac{v}{F}\cdot\frac{\omega}{\tau} \neq 1, \\ F^* = \frac{S}{\pi}\cdot F = \frac{S}{\pi}\cdot\frac{dp}{dt} = \frac{dp^*}{dt},\ \vec{p}^* = \frac{S}{\pi}\vec{p} = \frac{S}{\pi}m\vec{v}. \end{cases} \quad (3.9)$$

$$\Rightarrow Z_m\cdot Y_m = \frac{v}{F}\cdot\frac{\tau}{\omega} = \frac{v}{F}\cdot\frac{F}{v}\cdot r^2 = r^2 = \frac{S}{\pi}$$

The idea here, based on analogies (where the electromagnetic field should have its analogous and hypothetical mechanical couple, which still has no name, but it should present certain specific combination of linear and spinning motions), is to search for another case of combined linear and spinning motions that could produce $Z_m\cdot Y_R = \frac{v}{F}\cdot\frac{\tau}{\omega} = \text{const.}$ *. The only presently known candidate for being a complementary field couple of any motional particle should be an associated de Broglie matter wave, and such matter wave field should present axially (and helicoidally) spinning structure around the particle. This is very much analog to a photon that can analogically be presented as an equivalent wave-packet in some ways as a moving particle, (* $m_f = hf/c^2, p_f = m_f c = hf/c, \tilde{E}_f = hf, \lambda_f = h/p_f = c/f$ *; -see analyses of Compton and Photoelectric effects). From electromagnetic theory, we also know that photon is rotating (or its mutually complementary electric and magnetic field vectors are rotating along the common propagation path). There is only a small (hypothetical) step to associate similar properties to any particle in relation to its de Broglie matter wave.*

3.4.1. The Hyperspace Communications and Light

Let us imagine hypothetically that light is the manifestation of some oscillating phenomena connected to the multidimensional world (which has more than four dimensions, or which has a different set of dimensions, comparing to our perceptible world). We could try to communicate with this (multidimensional) world just by manipulating and modulating the dynamic and transient aspects of light (or photons and electromagnetic waves) that are under our control in our 4-dimensional universe. For instance, let us imagine that we create a convenient spatially closed mirror-wall container (like a toroid), then introduce certain light source inside, and let light to reach its equilibrium state (after almost countless number of reflections, scattering and interference effects inside of such mirror-wall container). Practically, we can say that a certain amount of (electromagnetic) energy is continuously being introduced and fluctuating inside of a closed-space of a toroidal mirror-wall container. If our container is in the state of rest, the internal "light energy fluid" in its equilibrium could create certain (stable) center of gravity, or center of inertia, somewhere in internal space of the container. Incident or input light energy would be partially transformed into an output heat dissipation on the container walls. This toroidal light container is an object that we see as a fully spatially closed body in our four-dimensional universe. Contrary, the same "light-energy fluid" inside of such container is only virtually bounded by the closed mirror walls of the container, but effectively not bounded at all, if our hypothetical starting point that light belongs to a multidimensional world (which has more than four dimensions) is correct. One of the actions we could make to create meaningful information going out of our four-dimensional world is to modulate (dynamically and non-stationary) the "light energy carrier fluid" inside of mentioned toroidal container (by external mechanical, ultrasonic and/or electromagnetic-field oscillations and/or by certain complex motion, or rotation applied on the mirror-walls of the light energy container. We could also imagine producing similar effects by some other electrodynamics means such as applying frequency, phase, and amplitude modulation...), this way permanently moving, modulating and/or rotating effective center of inertia of the "light mass" captured in the same container. Such locally unbalanced modulated energy (or information) will not be *transformed only into heat energy (on container walls), and (still hypothetical) part of it should penetrate a multidimensional world since energy and momentum conservation should be valid in a multidimensional world too (of course if such hypothetical world exists). A similar communication system (apparatus) combined with convenient sensors could be used as a receiver of signals coming from the multidimensional world into our 4-dimensional world (of course all of that is still highly hypothetical and oversimplified). By the opinion of the author of this paper, here proposed experimental arrangement could be a probable connecting channel between our 4-dimensional world and higher dimensions of some more complex universe, if the initial hypothesis about multidimensional light nature is correct. Of course, given example or experiment proposal should be considered only as a big simplification of the apparatus for future communications between multidimensional worlds, and it is still highly speculative). If a light is not the convenient carrier (or messenger) for making contacts within multidimensional Universe, we could try to find another phenomenology or matter waves, which exist in our 4-dimensional world and maybe penetrates higher dimensions if such signal-carrier and higher dimensions exist. Maybe we can speculatively reproduce the above-described process of locally unbalanced, dynamic (and non-stationary) modulation (where the effective center of gravity would have an unstable position, this way carrying the information towards higher dimensions). ♣]*

4. DE BROGLIE MATTER WAVES and QUANTUM THEORY

4.0 WAVE FUNCTIONS, WAVE VELOCITIES AND UNCERTAINTY RELATIONS

The strategy of using wave packets, or wave groups, or just wave functions, which in certain essential aspects are serving to represent real moving particles, has the significant place in explaining and understanding Particle-Wave Duality and Uncertainty Relations (even more crucial than presently seen in Physics). What we can find in the literature regarding wave functions and wave velocities (group and phase velocity) concerning comparison with an “equivalent-particle velocity”, is often oversimplified, and narrow-conditions valid, or “statistically averaged”. Consequently, often everything (regarding wavefunctions and relevant velocities) looks either not enough convincing, somewhat artificial, or mostly as assembled to produce some “forged results”, without giving a non-doubtful impression about the general applicability of such models and conclusions to all possible waves-related phenomena known in Nature. This is probably the case because most of the authors dealing with mentioned items are intentionally forcing almost every domain of institutionalized Physics to be on some way (as much as possible) compliant to Orthodox Quantum (**QT**) and Relativity (**RT**) theories, practically fitting their ways of presentation and modeling to be “**QT** and **RT** mainstream correct”, without taking any chance to challenge the foundations of contemporary **QT** and **RT**.

Here, we will first attempt to rectify and generalize concepts dealing with wave-motion functions and velocities in general, and specifically concerning a particle-equivalent, wave-packet velocities by using the Analytic Signals modeling (see [57], [109], [110], and [111]). We will also see that such mathematics is equally applicable to anything that is propagating and oscillating as a wave or particles (what we can associate both to a micro and macro universe, to Mechanical or an Electromagnetic phenomenology). Such situation is not in line with a tendency of the contemporary **QT** to address and forge most of wavefunction conceptualization concerning a micro world of photons and subatomic entities to comply with Statistics and Probability modeling. It is even becoming a custom and fashionable situation that everybody who would like to be recognized and respected in a Modern Physics-Theater should start his preaching with some glorifying, with explicit and vocal statements about enormous significance and eternal validity, applicability, and success of the contemporary, Orthodox **QT**.

In this book, the first and significant step, regarding matter waves and Wave-Particle Duality, would be to establish (or accept) the most applicable, most common, rich, and universal mathematical model that would cover all (physics-related) wave and oscillatory motions in a joint, time-space and frequency domain (or to create proper, natural, and universally applicable wave functions), assuming that particles in motion can be successfully modeled as wave groups, or wave packets. Something as that could presently look like an ambitious task, but it will be shown as realistic and feasible if for involved wave functions, we use Dennis Gabor, Analytic Signals modeling, based on Hilbert Transform (see [57], Michael Feldman, [109], Poularikas A. D., including [110], and [111]).

Regarding waves and oscillations, we can say that everything what we know as such phenomena in Physics should belong to the same concept, theory, and mathematical modeling. Fourier Signal Analysis is clearly showing that all kind of spatial and temporal (or combined domains) signals and wave-shapes can be decomposed on elementary simple-

harmonic or sinusoidal waves (or vice-versa). We could safely say that Joseph Fourier is the real inventor or founder of the Wave-Particle Duality theory (and Dennis Gabor with his Analytic Signal model only significantly optimized and updated Fourier Analysis concept). Later on, Luis de Broglie discovered other connections (and unity) between particle and wave motions. In addition, all waves known in Physics and Mathematics are on the same way mutually interfering, diffracting, refracting, scattering, and superimposing (meaning, should be described with the same mathematical modeling). Any kind of particles and energy-momentum states in motions is presentable as a superposition of some simple-harmonic, or other elementary waveforms, creating matter-waves groups or packets. Here we search how to connect, replace, and explain mechanical properties of moving particles or masses with matter-wave motions of equivalent wave-groups. Probability waves and similar formulations about waves in microphysics and QT could be mathematically useful until certain "averaged" level, but also being misleading, unrealistic and illogical, abstract, and artificial concepts for explaining non-statistical motional situations where we need immediate phase and amplitude insights. Waves and oscillations always have mutually linked spatial and temporal characteristics (even when extended towards multidimensional universes). We could simplify a wave-motions analyses by focusing only on temporal and/or spatial shapes of waves ... The best-known mathematical model for presenting and analyzing all waves in Physics is the Analytic Signal concept. It is, (well and naturally) connected with de Broglie, matter-waves concept.

Let us start from an arbitrary and energy-finite waveform $\Psi(t)$, which is presented using the Analytic Signal model (first time introduced by Dennis Gabor, [57]), in the form of the product of specific amplitude and phase function; - see (4.0.1). This is the closest, ready-made, Analytic Signal, wave function model where we have generalized amplitude and phase functions separated (like in cases of simple-harmonic signals' modulating techniques), and where we can (later) prove that the amplitude function propagates with a group velocity v, and the phase function has its phase velocity u. Once more to repeat, all kind of wave functions $\Psi(t)$ of energy-finite motions known in Physics are presentable using the Analytic Signal modeling. Later, we will show more profoundly, and with more arguments, why Analytic Signal modeling is taken as the optimal and natural waveforms modeling.

Most of the contemporary (and old) analyses elaborating wave motions in Physics (and dealing with wave functions), are still insignificantly present in the scientific literature, on a way that is explicitly and dominantly using the Analytic Signal concepts for wave functions modeling. Since different interpretation-related frameworks are still in use regarding analyses of wave motions, this presents a disadvantage in mathematical physics, since essential, analogical, and multidisciplinary conclusions, unification, and generalizations are still unclear and complicated tasks. The significant differences, only superficially seen when using the Analytic Signal or Fourier Transform for presenting and analyzing different wave functions, are not so much mutually distinctive, evident and immediately clear (without investing mathematical efforts of deeper insights). This is, most probably, the reason why the scientific community in many cases continues (by intellectual rigidity and inertia) to use the old and well-established, but not too far-reaching, methods (of signal analysis) based mostly on Fourier analysis. Majority of authors in this field are repeating, and in some cases "creatively" copying in their publications, what somebody else published a long time ago (regarding wave motions and wave functions), instead of accepting much better mathematical modeling options. It is already necessary to enter a new, combined space-time-frequency universe of dynamic and instantaneous Analytic Signals and

Hilbert transform-based waveforms modeling and analysis, to catch all finesses and advantages of such signal analysis world, and to distinguish it from the ordinary Fourier analysis based, averaging methods. We could say that the modeling and applicability differences and advantages between Fourier analysis and Analytic Signal concepts are comparable to differences between the operations within Real Numbers and Complex Numbers (where the domain of Real Numbers is just a tiny area of the Complex Numbers Analysis).

Briefly stating, Analytic Signal modeling gives a chance to extract (from any arbitrary energy-finite waveform, which could be of relevance in Physics) the instantaneous or immediate (time-and-frequency-dependent) amplitude and phase signal functions, including instantaneous signal frequency and power. All of them are presentable in their temporal, spatial and frequency domains (including all mutually analogical amplitude and phase spectral functions in their frequency domains). It is also worth mentioning that starting from a wave function presented as a Complex Analytic Signal, development of Schrödinger, Klein-Gordon and many other well-known differential wave equations of Physics is a relatively easy and straightforward task without a need to implement limited, not-natural hybridizations and only temporarily convenient "ad hock patchwork" as practiced in QT.

Going directly to the most useful analytic signal forms for wavefunctions, let us consider that $\Psi(t)$ is our original, time-domain wave function, wave packet (or wave group), and $\hat{\Psi}(t)$ is its Hilbert transform, both being real-value functions,

$$\begin{aligned}
&\Psi(t)=\frac{1}{\pi}\int_0^{\infty}\left[U_c(\omega)\cos\omega t+U_s(\omega)\sin\omega t\right]d\omega=\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\cos(\omega t+\Phi(\omega))\right]d\omega=\\
&=a(t)\cos\varphi(t)=-H\left[\hat{\Psi}(t)\right],\\
&\hat{\Psi}(t)=H\left[\Psi(t)\right]=\frac{1}{\pi}\int_0^{\infty}\left[U_c(\omega)\sin\omega t+U_s(\omega)\cos\omega t\right]d\omega=\frac{1}{\pi}\int_0^{\infty}\left[A(\omega)\sin(\omega t+\Phi(\omega))\right]d\omega=\\
&=a(t)\sin\varphi(t)=\Psi(t)*\frac{1}{\pi t}=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\Psi(\tau)}{t-\tau}d\tau=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\Psi(t-\tau)}{\tau}d\tau\ ,\ H=H_t\,(=)\ \text{Hilbert transform}\ .\\
&\mathrm{Mod}\left[H\right]=1,\ \mathrm{Arg}\left[H\right]=-\pi/2\ .
\end{aligned} \tag{4.0.1}$$

where $\mathbf{a(t)}$ is the instantaneous signal amplitude or signal envelope, $\cos\varphi(t)$ is the signal carrier function, $\varphi(t)$ is the signal phase function, and $\omega(t)=d\varphi(t)/dt=2\pi f\,(t)$ is the instantaneous signal frequency (all in a time domain). Analogical functions in the signal frequency domain are $A(\omega)$, as the signal amplitude, and $\Phi(\omega)$ as the signal phase function. The Hilbert transform $\mathbf{H}$ is a kind of filter, which shifts phases of all elementary (simple harmonic) components of its input ($\Psi(t)$) by $-\pi/2$ (see the picture below). The same, time-domain function $\Psi(t)$, transformed into Complex, time-domain Analytic Signal, or complex Phasor function $\overline{\Psi}(t)$ has the following form,

$$\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\,\Psi\,(t) = \frac{1}{\pi}\int_{0}^{\infty} U(\omega)e^{j\omega t}\,d\omega = \frac{1}{\pi}\int_{0}^{\infty} A(\omega)e^{j(\omega t + \Phi(\omega))}\,d\omega$$

$$= a(t)e^{j\varphi(t)},\ \Psi(t) = \frac{1}{2}\left[\overline{\Psi}(t) + \overline{\Psi}^*(t)\right],\ \overline{\Psi}^*(t) = \Psi(t) - j\hat{\Psi}(t)\ ,\ j^2 = -1,$$

$$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)} = e^{R_e\left[\ln\overline{\Psi}(t)\right]},\ \dot{a}(t) = \frac{\Psi(t)\dot{\Psi}(t) - \hat{\Psi}(t)\dot{\hat{\Psi}}(t)}{a(t)} = a(t)\mathrm{Re}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right],$$

$$\varphi(t) = \mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)} = \mathrm{Im}\left[\ln\overline{\Psi}(t)\right] = \text{instantaneous phase}, \tag{4.0.2}$$

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = \dot{\varphi}(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \mathrm{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right] = 2\pi f(t) = \text{instantaneous angular frequency.}$$

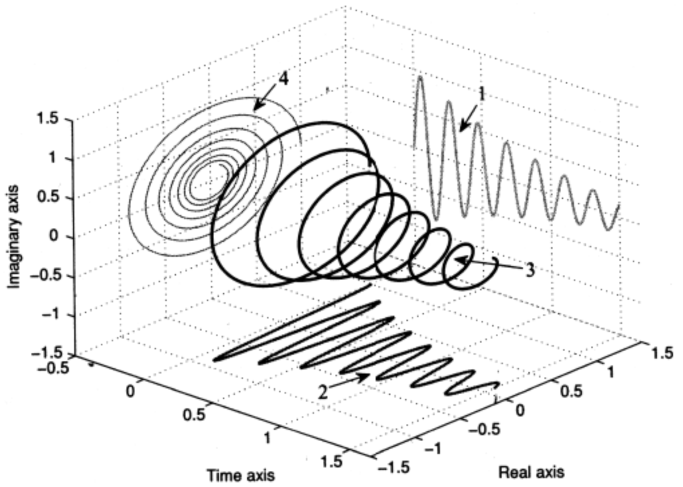


Figure 2.3 The HT projection (1), the real signal (2), the analytic signal (3), and the phasor in complex plain (4) (Feldman, ©2011 by Elsevier)

The frequency-domain, wave functions of an Analytic Signal are,

$$A(\omega) = U(\omega)e^{-j\Phi(\omega)},\ \Phi(\omega) = -\arctan\left[U_s(\omega)/U_c(\omega)\right],\ j = \sqrt{-1}$$

$$\overline{U}(\omega) = U_c(\omega) - jU_s(\omega) = \int_{-\infty}^{+\infty}\Psi(t)e^{-j\omega t}dt = A(\omega)e^{j\Phi(\omega)} = A(\omega)\cos\Phi(\omega) + jA(\omega)\sin\Phi(\omega), \tag{4.0.3}$$

$$\begin{pmatrix} U_c(\omega) = A(\omega)\cos\Phi(\omega) = -H\left[U_s(\omega)\right],\ H = H_\omega\,(=)\ \text{Hilbert transform}, \\ U_s(\omega) = H\left[U_c(\omega)\right] = A(\omega)\sin\Phi(\omega) = U_c(\omega) * \frac{1}{\pi\omega} = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{U_c(\omega)}{\omega - \Omega}d\Omega = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{U_c(\omega - \Omega)}{\Omega}d\Omega. \end{pmatrix}$$

In the analysis of waveform velocities in this book (which follows), we will use all here mentioned, real and complex time-domain and frequency-domain Analytic Signal wave functions, (4.0.1) - (4.0.3).

For completer understanding of Analytic Signal function and its advantages in comparison to all other waveform presentations and analyses, it would be useful to go to already published Analytic Signal and Hilbert Transform related literature listed under [57], [58], [59] and [79] (to focus analyses of this book only to wave functions, wave velocities and closely related topics).

*As we can see, Analytic Signal (established by Dennis Gabor, [57]) is a kind of functions-shape modeling where we are "casting" an arbitrary-shaped and energy-finite function into a "phasor molds" of sinus and cosines functions. Such (generalized amplitude and phase) functions exist in the real and complex, time, and frequency domains. This is offering significant advantages for arbitrary wave-functions analyses and characterizations, on the same way as known for simple harmonic wave functions, and we can easily make analogical comparisons and associations between amplitude, phase and frequency of a simple-harmonic, sinusoidal forms, and generalized amplitude, phase, and frequency of Analytic Signal forms of arbitrary shaped functions. The mathematical power of such modeling is being reinforced by the fact that all waveforms known in physics and nature are composed of elementary sinusoidal functions. The similar (but much more simplified) concept is already very successfully applied as complex **phasors** notation and mathematical operations concerning complex functions of (simple-harmonic) currents, voltages, and impedances in electro-technique and electronics (see more in the same chapter under "4.0.11. Generalized Wave Functions and Unified Field Theory"). **In this book, Analytic Signal will be considered as the optimal and most natural modeling framework for de Broglie or all matter waves, where real analytic signal component** $\Psi(t)$ **describes the state of motion we know, measured, and qualified in our real space-time domain. Associated, phase shifted signal** $\hat{\Psi}(t)$ **creates the imaginary component of a relevant analytic signal** $\overline{\Psi}(t)$**, but both** $\Psi(t)$ **and** $\hat{\Psi}(t)$ **should be considered as existing, natural and relevant signals (only being time and space, phase-shifted and mutually orthogonal).** Most probably that (after applying necessary mathematical processing and dimensional arrangements) we will be able to prove that wave functions of any mutually-coupled electric and magnetic field vectors (like photons or electromagnetic waves) are behaving as a united Analytic signal of an electromagnetic field couple of* $\Psi(t)$ *and* $\hat{\Psi}(t)$*, or effectively being a couple of relevant electric and magnetic field wave functions. Here, we should avoid oversimplified speculations with nonrealistic magnetic monopoles.* ***The Universe or Nature is on some realistic and detectable way always synchronously creating both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***matter wave functions, and here are the roots of matter waves, particle-wave duality and entanglement effects known in quantum physics. Since Analytic Signal concept (meaning operating with*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***functions) is mathematically well defined and being the part of a significant and stable mathematical body of signal analysis, also well and smoothly connected to mathematical waves processing in Physics, we should consider that both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***in every case of matter-waves are something that synchronously exists, and should be detectable in our Universe. The best example or candidate for supporting such a situation should be the structure of electromagnetic fields and photons. If we know, see or detect only one wave component of certain natural motion and we consider it as being*** $\Psi(t)$**, we should be sure that sooner or later, corresponding and complementary** $\hat{\Psi}(t)$ ***will be theoretically and experimentally discovered (what belongs to a deeper meaning of the matter-wave duality).***

The same situation could be particularly interesting in all kinds of transient, switching, pulsing, and waves creating situations concerning electromagnetic currents, voltages, and waves. In mentioned situations, we usually analyze, as real and most relevant signals (meaning currents, voltages, and electric and magnetic fields), conveniently formulated $\Psi(t)$ ***functions, but Nature, or relevant Physics, is immediately creating both*** $\Psi(t)$***, and*** $\hat{\Psi}(t)$ ***(see the works and publications of Rudjer Boskovic, Nikola Tesla, Eric P. Dollard, and Konstantin Meyl; -see [6], [97], [98] and [99]). Something similar or equivalent should exist as a natural coupling between linear and spinning or angular motions. In addition, we should not forget that general solution of Classical second order differential wave equations always have (at least) two wave functions (or two wave groups that can be considered being two complex Analytic Signals). Mentioned wave groups are synchronously propagating in mutually opposed directions, naturally being in mutual synchronization and entanglement relation. This is valid both for*** $\Psi(t)$ **and** $\hat{\Psi}(t)$***), and this should also be valid (or always present) in the real world of all matter-waves situations. Anyway, in some cases we cannot easily visualize or explain such waves propagations (since this sometimes looks like one wave component is propagating along positive temporal and spatial direction, and the other is advancing in the negative time and space direction, or in any other imaginable and mathematically possible combination of spatial and temporal directions;*** *- see more in the chapter 4.3-MATTER WAVES AND WAVE EQUATIONS) and in Chapter 10. Here we could* *draw the philosophic conclusion such as: "In our Universe, past, present and future are mutually balanced, coupled and present synchronously in every moment of our existence".*

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The amount of ***Entanglement* and quality of resonant states** *between two matter-wave groups, functions, or Analytic Signals, we could express with calculating Coherence factors (see more about coherence factors at the end of this chapter around definitions (4.0.83), (4.0.87) and (4.0.109)). Between two Complex Analytic signals, or wave functions,* $\overline{\Psi}_1(r,t)$ and $\overline{\Psi}_2(r,t)$ *it is possible to find the measure of their mutual spatial-temporal and spectral coherence (or non-coherence), using the following coherence factors:*

$$\overline{K}_{r,t} = \frac{\iiint_{(-\infty,+\infty)} \overline{\Psi}_1(r,t)\cdot\overline{\Psi}_2(r,t)\cdot dr\cdot dt}{\iiint_{(-\infty,+\infty)} \left|\overline{\Psi}_1(r,t)\right|\cdot\left|\overline{\Psi}_2(r,t)\right|\cdot dr\cdot dt}\ ,\ \overline{K}_{k,\omega} = \frac{\iiint_{(-\infty,+\infty)} \overline{U}_1(k,\omega)\cdot\overline{U}_2(k,\omega)\cdot dk\cdot d\omega}{\iiint_{(-\infty,+\infty)} \left|\overline{U}_1(k,\omega)\right|\cdot\left|\overline{U}_2(k,\omega)\right|\cdot dk\cdot d\omega}$$

4.0.1. Signal Energy Content

Let us first find the energy $\tilde{E}$ carried by an analytic signal waveform $\Psi(t) = a(t)\cos\varphi(t)$. Generally applicable Parseval's theorem is connecting time and frequency domains of signal's wave functions, and such expression has the meaning of signal's (or wave) (motional) energy in cases when $\Psi^2(t) = P(t) = d\tilde{E}/dt$ is modeled to present an instantaneous signal power $P(t)$, as follows,

$$\begin{aligned}
&\tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \\
&= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\overline{U}(\omega)\right|^2 d\omega = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{\infty}\left[A(\omega)\right]^2 d\omega = \int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \\
&= \int_{-\infty}^{+\infty}P(t)dt\ (=)\ [J](=)[Ws], \\
&\rho(\tilde{E})_t = \frac{d\tilde{E}}{dt} = \Psi^2(t) = P(t)\ (\Leftrightarrow)\left[\frac{a(t)}{\sqrt{2}}\right]^2 (=)\ [W]\ ,\ t\in(-\infty,+\infty)\ , \\
&\rho(\tilde{E})_\omega = \frac{d\tilde{E}}{d\omega} = \frac{\Psi^2(t)}{d\omega/dt}(\Leftrightarrow)\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 (=)\ \left[Js = Ws^2\right],\ \omega\in(0,+\infty), \\
&\frac{\tilde{E}}{\omega(t)} = \frac{a^2(t)\int_{-\infty}^{+\infty}a^2(t)dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]} = \frac{\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]\int_{-\infty}^{+\infty}\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]}(=)\left[Js = Ws^2\right], \\
&\text{if } \left\{\frac{\tilde{E}}{\omega(t)} = \frac{h}{2\pi} \Rightarrow \tilde{E} = hf\right\} \Leftrightarrow \text{elementary quantum of wave-packet energy.}
\end{aligned}$$

(4.0.4)

From the expressions for signal energy (4.0.4), it is evident that a total signal energy content is captured and propagating only by the signal amplitude (or envelope) function $a(t)$ or $A(\omega)$, both in time and frequency domain,

$$\tilde{E} = \int_{-\infty}^{+\infty} \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{0}^{\infty} \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega, \tag{4.0.5}$$

also, it is evident that signal phase functions, $\varphi(t)$ and $\Phi(\omega)$ do not directly participate in a total signal energy content (do not carry signal energy at all). This is a significant fact to notice to understand the meaning of signal velocities (and this is universally valid for all waves and signals in micro and macrophysics). We can also say that the wave *velocity of the signal amplitude (or signal envelope) is the velocity of the total signal energy propagation*. This velocity is known as a group velocity (in particle-wave duality concepts, group velocity corresponds to particle velocity, and a wave packet is the wave-equivalent of a motional particle; -such situations will be analyzed in detail later). The speed of the signal-carrier or phase function is known as a phase velocity, and it should present the signal carrier velocity or velocity of primary simple-harmonic signal elements (which are creating a wave packet). Here, the signal carrier functions in a time-domain are $\cos\varphi(t)$, or $e^{j\varphi(t)}$, and in a frequency-domain $\cos\Phi(\omega)$ or $e^{j\Phi(\omega)}$, and we can use both of them (equally and analogically), depending on preferences and mathematical advantages when operating with trigonometric or complex functions.

Experimentally provable and theoretically well-supported knowledge is that for Physics-related (analytic) signals or wave packets like photons, electrons and many other energy-quanta and elementary-particles, the following expressions, properties, and parameters are well known, applicable and mutually very closely related and coupled:

a) *Important property of Analytic Signals (between their corresponding time and frequency domains) is that all phase functions (* $\cos\varphi(t)$, $e^{j\varphi(t)}$, $\cos\Phi(\omega)$, $e^{j\Phi(\omega)}$*), regardless in which domain formulated, have the same phase velocity* $u = \lambda f$*. Also, all amplitude functions (* $a(t)$, $A(\omega)$*), regardless in which domain formulated, have the same group velocity v. Also, there is a well-known (universally applicable in physics) equation connecting group and phase velocity of wave functions (* $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$*). Analytic Signal modeling presents the best native mathematical environment for conceptualizing de Broglie matter-waves and wave-particle duality. See more in the chapters 4.1, 4.3 and 10.*

b) *M. Planck-Einstein, wave-packet energy, valid for frequency narrow-band or band-limited waves and wave groups (like photons) is,*

$$\tilde{E} = hf\ (=)\left[\text{Kinetic energy} = E_k = (\gamma - 1)mc^2\right],$$

c) *Einstein-Minkowsky's energy-momentum, 4-vector relations,*

$$\overline{P}_4 = \overline{P}(p, \frac{E}{c}),\ \overline{P}^2 = \vec{p}^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2},\ \vec{p} = \gamma m\vec{v},\ E_k = (\gamma - 1)mc^2,$$

$$E_0 = mc^2,\ E = \gamma E_0 = \gamma mc^2 = E_0 + E_k \Rightarrow \vec{p}^2 c^2 + E_0^2 = E^2$$

d) *de Broglie matter wave wavelength*

$\lambda = h/p = u/f$,

e) *In fact, all of them, a), b), c) and d), when used to describe the same momentum-energy matter-wave state in a joint spatial-temporal universe, are entirely mathematically compatible, complementary, and analytically united by the equation that is connecting relevant group and phase velocity* $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$*, being the core of Particle-Wave Duality (see more of supporting elements later, in the same chapter, around equations (4.0.73) - (4.0.76), and in Chapter 10.).*

f) *Also, we will realize that there is a kind of associated spinning, as a helix-path field around moving particle, which is defining (de Broglie) matter-wave wavelength (and all of that will also be analyzed in detail, later). The best mathematical environment to exploit all the here-mentioned signal properties (and to unite matter-waves and particle-wave duality foundations originating from L. de Broglie, M. Planck, A. Einstein, W. Heisenberg, E. Schrödinger...) is the Analytic Signal concept. De Broglie matter waves have the frequency and phase described by the corresponding Analytic Signal model, as defined in (4.0.2). From an Analytic Signal frequency and phase, we can determine matter wave wavelength,* $\lambda = h/p = u/f$*, and group and phase velocity relations* $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$ *(see more in Chapters 4.1 and 10.). Relevant Analytic Signal wave functions that are naturally describing de Broglie matter waves are power and motional energy-related functions, including corresponding field and force functions. Later, (if desired) by creating normalized, non-dimensional wave functions, we will be able to reproduce Quantum Theory approach to the same problematic. See more in the chapters 4.1, 4.3 and 10.*

Regarding particle-wave duality concept based on presenting a moving particle, which has certain mass, with an equivalent wave packet, the problem appears in understanding that group and phase velocity of the wave packet are mutually different. For non-relativistic particle motion ($v << c$), its phase velocity is close to one half of its group velocity, and at the same time the particle that should be well-represented on that way is known as a stable and time-space localized (without energy residuals and waving tails). For the time being, the best understanding of differences between a particle and its equivalent matter wave group is that <u>only the wave-packet amplitude function carries the whole wave-packet or particle (motional) energy and propagates with a group or particle velocity</u>, what is universally valid, based on (4.0.4) and (4.0.5). The other, phase-part of the wave-packet, that is somehow space-time retarded behind the particle, like its waving tail (which has a phase velocity u), is anyway not carrying the energy, and it is not presenting a big problem for particle-wave duality mathematical modeling. Development of group and phase velocity of a wave packet and relations between them will be addressed a bit later, but since this is essential for understanding the particle-wave duality, it is necessary to mention in advance the significance of the problematic we are dealing with (see the development of wave velocities starting from equations (4.0.6) until (4.0.46)). At the end of this book, we will again take a much closer look to physics-related wavefunctions structure and properties (see Chapter 10.). The Analytic Signal model is naturally representing arbitrary waveforms and giving the unique opportunity to extract instantaneous signal amplitude and phase functions, instantaneous signal frequency, and all other signal characteristics, both in time and frequency domains. The fascinating idea here is to treat all wave motions, oscillations, wave packets and signals (relevant in Physics) using the Analytic Signal modeling. Such modeling is so rich, natural, selective, and informative, that also Quantum Theory wave functions should be based on the Analytic Signals modeling. Starting from here, all other wave equations (known in Mathematical Physics) can be easily and logically developed, using clear, step-by-step, and almost elementary mathematical methods (*without introducing whatever works, such as add-in, patch-in, fit-in, and "by divine inspiration, fallen from the sky" equation members, helping rules, operators and postulates*). Something like that is shown in Chapter 4.3 (where almost all wave equations and associated operators, known in present Quantum Theory, are developed starting from an Analytic Signal wave function). Since there is a lot of available literature regarding Hilbert transform and Analytic Signal modeling, let us only summarize the most important properties and expressions valid for presenting arbitrary and energy-finite waveforms as Analytic Signals, here given in the table T.4.0.1.

T.4.0.1

The parallelism between Time and Frequency Domains	**Analytic Signal**	
	Time Domain	**Frequency Domain**
Complex Signal	$\overline{\Psi}(t) = a(t)e^{j\varphi(t)}$ $= \Psi(t) + j\hat{\Psi}(t)$ $= \frac{1}{\pi}\int_0^{\infty} U(\omega)e^{j\omega t}d\omega$ $= \frac{1}{\pi}\int_0^{\infty} \overline{U}(\omega)e^{-j\omega t}d\omega$ $= \frac{1}{\pi}\int_0^{\infty} A(\omega)e^{j(\omega t+\Phi(\omega))}d\omega$	$\overline{U}(\omega) = A(\omega)e^{j\Phi(\omega)}$ $= U_c(\omega) - jU_s(\omega)$ $= \int_{-\infty}^{+\infty} \Psi(t)e^{-j\omega t}dt$ $= \int_{-\infty}^{+\infty} \overline{\Psi}(t)e^{j\omega t}dt$ $= \int_{-\infty}^{+\infty} a(t)e^{-j(\omega t+\varphi(t))}dt$
Real and imaginary signal components	$\Psi(t) = a(t)\cos\varphi(t) =$ $= -H\left[\hat{\Psi}(t)\right],$ $\hat{\Psi}(t) = a(t)\sin\varphi(t) =$ $= H\left[\Psi(t)\right]$	$U_c(\omega) = A(\omega)\cos\Phi(\omega) =$ $= -H\left[U_s(\omega)\right],$ $U_s(\omega) = A(\omega)\sin\Phi(\omega) =$ $= H\left[U_c(\omega)\right],$
Signal Amplitude	$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)}$	$\mathbf{A(\omega) = \sqrt{U_c^2(\omega) + U_s^2(\omega)}}$
Instant Phase	$\varphi(t) = \mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}$	$\mathbf{\Phi(\omega) = \arctan\frac{U_s(\omega)}{U_c(\omega)}}$
Instant Frequency	$\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial\mathbf{t}}$	$\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\omega}$
Signal Energy	$\tilde{E} = \int_{-\infty}^{+\infty} \lvert\overline{\Psi}(t)\rvert^2 dt =$ $= \int_{-\infty}^{+\infty} \Psi^2(t)dt =$ $= \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt =$ $= \int_{-\infty}^{+\infty} \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt$ $\int_{-\infty}^{+\infty} \Psi(t)\cdot\hat{\Psi}(t)dt = 0$	$\tilde{E} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert\overline{U}(\omega)\rvert^2 d\omega =$ $= \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_c^2(\omega)d\omega =$ $= \frac{1}{2\pi}\int_{-\infty}^{+\infty} U_s^2(\omega)d\omega =$ $= \int_0^{\infty} \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega$ $\int_{-\infty}^{+\infty} U_c(\omega)\cdot U_s(\omega)d\omega = 0$

<table>
<tr><td>Central Frequency</td><td>$\omega_c = \frac{\int_{[t]} \omega(t)\cdot a^2(t)dt}{\int_{[t]} a^2(t)dt} = 2\pi f_c$</td><td>$\omega_c = \frac{\int_{[\omega]} \omega\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]} [A(\omega)]^2 d\omega} = 2\pi f_c$</td></tr>
<tr><td>“Central Time Point”</td><td>$t_c = \frac{\int_{[t]} t\cdot a^2(t)dt}{\int_{[t]} a^2(t)dt}$</td><td>$t_c = \frac{\int_{[\omega]} \tau(\omega)\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]} [A(\omega)]^2 d\omega}$</td></tr>
<tr><td>Standard Deviation</td><td>$\sigma_\omega^2 = \frac{1}{\Delta t}\int_{[t]} \left|\omega(t)-\omega_c\right|^2 dt$</td><td>$\sigma_t^2 = \frac{1}{\Delta\omega}\int_{[\omega]} \left|\tau(\omega)-t_c\right|^2 d\omega$</td></tr>
<tr><td rowspan="2">Uncertainty Relations
generally applicable to micro and macro world of Physics</td><td colspan="2">$\pi \le \sigma_\omega\cdot\sigma_t < \left|\omega(t)\cdot\tau(\omega)\right| \cong \omega_c\cdot t_c \cong \Omega\cdot T \le \frac{\pi}{2\cdot\delta t\cdot\delta f} = \frac{\pi^2}{\delta t\cdot\delta\omega}$ (!?)
$0 < \delta t\cdot\delta\omega = 2\pi\cdot\delta t\cdot\delta f < \pi < \Omega\cdot T \le \frac{\pi}{2\cdot\delta t\cdot\delta f} = \frac{\pi^2}{\delta t\cdot\delta\omega}$</td></tr>
<tr><td colspan="2">$\delta t\cdot\delta f \le 2\cdot\delta t\cdot\delta f\cdot T\cdot F \le 1/2,$
$\delta t \le \frac{1}{2F},\ \delta f \le \frac{1}{2T},\ F\cdot T > \frac{1}{2},$
$\delta t\cdot\delta f < \frac{1}{2}, \delta t\cdot\delta\omega < \pi,\ \Omega\cdot T > \pi$
$0 < \delta t\cdot\delta f < \frac{1}{2} < F\cdot T \le \frac{1}{4\cdot\delta t\cdot\delta f}.$

Shannon-Kotelnikov optimal signal-sampling relations in time and frequency domains: $\delta t =$ maximal time sampling interval, $\delta f =$ maximal frequency sampling interval $\Omega = 2\pi F$ and T are the total, or absolute signal-lengths in its frequency and time domains; ($\omega = 2\pi f, \delta\omega = 2\pi\delta f$).</td></tr>
</table>

Other forms of Uncertainty Relations (see later in this chapter for complete development of such relations):

$$0 < \delta t\cdot\delta\omega = 2\pi\cdot\delta t\cdot\delta f < \pi \le \sigma_\omega\cdot\sigma_t < \Omega\cdot T \le \frac{\pi}{2\cdot\delta t\cdot\delta f} = \frac{\pi^2}{\delta t\cdot\delta\omega}\ ,\ \left|\omega(t)\cdot\tau(\omega)\right| \cong \omega_c\cdot t_c \cong \pi$$

$$0 < \delta t\cdot\delta f < \frac{1}{2} \le \sigma_f\cdot\sigma_t < F\cdot T \le \frac{1}{4\cdot\delta t\cdot\delta f}\ ,\ \left|f(t)\cdot\tau(\omega)\right| \cong f_c\cdot t_c \cong \frac{1}{2}$$

$$0 < \delta t\cdot\delta f = \delta x\cdot\delta f_x < \frac{1}{2} \le \sigma_t\cdot\sigma_f = \sigma_x\cdot\sigma_{f\text{-}x} < F\cdot T = F_x\cdot L \le \frac{1}{4\cdot\delta t\cdot\delta f} = \frac{1}{4\cdot\delta x\cdot\delta f_x}\ ,$$

$$0 < \delta t\cdot\delta\tilde{E} = \delta x\cdot\delta p < \frac{H}{2} \le 2\pi\sigma_t\cdot\sigma_{\tilde{E}} = \sigma_x\cdot\sigma_p < \tilde{E}\cdot T = P\cdot L \le \frac{H}{4\cdot\delta t\cdot\delta f} = \frac{H}{4\cdot\delta x\cdot\delta f_x}\ ,$$

H (=) Either equal to Planck constant h for micro world,
or analog H >> h constant for planetary and astronomic systems
("H" is not presenting Hilbert transform, here).

4.0.2. Resume of Different Analytic Signal Representations

In many practical cases related to physics (non-dispersive, band-limited, finite-energy signals), Analytic Signal modeling is giving a chance to present an arbitrary-shaped signal to look as a simple-harmonic signal $a(t)\cos\varphi(t)$, which has its frequency and phase functions, as for instance,

$$\Psi(t) = a(t)\cos\varphi(t) = a(t)\cos\left[\omega_0 t + \varphi(t)_{residual}\right],\ \varphi(t) = \omega_0 t + \varphi(t)_{residual}\ ,\ \ \omega_0 = const.\ .$$

We can also imagine more complex situations regarding signal-phase functions of non-linear and dispersive signals such as,

$$\Psi(t) = a(t)\cos\varphi(t) = a(t)\cos\left[\omega_0 t + \frac{\omega_1}{T_1}t^2 + \frac{\omega_2}{T_2^2}t^3 + ...\varphi(t)_{residual}\right]$$

$$\varphi(t) = \omega_0 t + \frac{\omega_1}{T_1}t^2 + \frac{\omega_2}{T_2^2}t^3 + ...\varphi(t)_{residual}\ ,\ \ \omega_{0,1,2...} = const.\ \ T_{1,2...} = Const.\ .$$

Analytic Signals can be presented in different ways, giving the chance to reveal the internal structure of waveforms from different points of view. Here are summarized several of such possibilities (mainly as superposition or multiplication of elementary simple-harmonic signals; -see below).

$$\overline{\Psi}(t)=\Psi(t)+I\,\hat{\Psi}(t)=a_0(t)e^{I\,\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t)+I\,\sin\varphi_0(t)\right]=$$

$$= a_0(t)e^{\sum_{(k)} i_k\varphi_k(t)} = \sum_{(k)} a_k(t)e^{i_k\varphi_k(t)} = \sum_{(k)} \overline{\Psi}_k(t)\ ,$$

$$\mathbf{\overline{H}\left[\Psi(t)\right] = \overline{\Psi}(t) = \Psi(t) + I\cdot\hat{\Psi}(t) = a(t)\cdot e^{I\cdot\varphi(t)}}$$

$$\mathbf{\overline{H} = 1 + I\cdot H,\ I^2 = -1}$$

Analytic Signal presented as SUPERPOSITION	Analytic signal presented as MULTIPLICATION
$\overline{\Psi}(t) = \sum_{(k)} \overline{\Psi}_k(t) = \sum_{(k)} \Psi_k(t) + I\sum_{(k)} \hat{\Psi}_k(t)$	$\overline{\Psi}(t) = \Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t) + I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t) = $ $= \Psi_n(t)\prod_{(i=0)}^{n-1} e^{I\cdot\varphi_i(t)}$

4.0.2.1. Relations between Additive and Multiplicative Elements

$$\Psi(t)=\sum_{(k)}\Psi_k(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t)=a_0(t)\cos\varphi_0(t)=a(t)\cos\varphi(t)=-H\left[\hat{\Psi}(t)\right]$$

$$\hat{\Psi}(t)=\sum_{(k)}\hat{\Psi}_k(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)=a_0(t)\sin\varphi_0(t)=a(t)\sin\varphi(t)=H\left[\Psi(t)\right]$$

$$\overline{\Psi}_k(t)=\Psi_k(t)+i_k\hat{\Psi}_k(t)=a_k(t)e^{i_k\varphi_k(t)}\ ,\Psi_k(t)=-H\left[\hat{\Psi}_k(t)\right],\ \hat{\Psi}_k(t)=H\left[\Psi_k(t)\right]$$

$$\overline{\Psi}(t)=\Psi(t)+I\hat{\Psi}(t)=\frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)}+e^{-I\varphi_k(t)})+\frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)}-e^{-I\varphi_k(t)})\right\}$$

See more about energy related structural hierarchy of matter waves when presented as more elementary wave functions multiplication in Chapter 6., under “6.1. Hypercomplex, In-depth Analysis of the Wave Function”).

Brainstorming proposal: Let us imagine that we could present an “atom-field wave function” as the following multiplicative and additive analytic signal:

$$\overline{\Psi}(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}\cos\varphi_i(t)+I\Psi_n(t)\prod_{(i=0)}^{n-1}\sin\varphi_i(t)=\Psi_n(t)\prod_{(i=0)}^{n-1}e^{I\cdot\varphi_i(t)}=$$

$$=\Psi_{atom}(t)\cdot\left[\cos\varphi_{electrons}(t)\right]\cdot\left[\cos\varphi_{protons}(t)\right]\cdot\left[\cos\varphi_{neutrons}(t)\right]+\quad\dots$$

$$+I\Psi_{atom}(t)\cdot\left[\sin\varphi_{electrons}(t)\right]\cdot\left[\sin\varphi_{protons}(t)\right]\cdot\left[\sin\varphi_{neutrons}(t)\right]=$$

$$=A_{electrons}(t)\cdot e^{I\varphi_{electrons}}+A_{protons}(t)\cdot e^{I\varphi_{protons}}+A_{neutrons}(t)\cdot e^{I\varphi_{neutrons}}$$

This could open new opportunities to analyze interatomic and nuclear forces, and interactions between all involved participants, internally and externally.

4.0.2.2. Signal Amplitude or Envelope

$$a(t)=a_0(t)=\left|\overline{\Psi}(t)\right|=\sqrt{\Psi^2(t)+\hat{\Psi}^2(t)}=a_n(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)=\Psi_{n+1}(t)\prod_{(i=1)}^{n}\cos\varphi_i(t)$$

$$a_k(t)=\Psi_{k+1}(t)=\left|\overline{\Psi}_k\right|=\sqrt{\Psi_k^2(t)+\hat{\Psi}_k^2(t)}=a_{k+1}(t)\cos\varphi_{k+1}(t)=a_n(t)\prod_{(i=k+1)}^{n}\cos\varphi_i(t),\ k<n$$

$$a_{n-1}(t)=\Psi_n(t)=\left|\overline{\Psi}_{n-1}(t)\right|=\sqrt{\Psi_{n-1}^2(t)+\hat{\Psi}_{n-1}^2(t)}=a_n(t)\cos\varphi_n(t)=\frac{\Psi_0(t)}{\prod_{(i=0)}^{n-1}\cos\varphi_i(t)}$$

4.0.2.3. Signal Instantaneous Phase

$$\varphi_0(t)=\varphi(t)=\text{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}=\sqrt{\sum_{(k)}\varphi_k^2(t)},\ \varphi_k(t)=\text{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}$$

$$I^2=i_1^2=i_2^2=...=i_n^2=-1\ ,\ i_ji_k=0,\forall\, j\neq k\ \ \text{(hypercomplex imaginary units)}$$

$$I\varphi_0(t)=I\varphi(t)=i_1\varphi_1(t)+i_2\varphi_2(t)+...+i_n\varphi_n(t)=\sum_{k=1}^{n}i_k\varphi_k(t)$$

$$e^{i_k\varphi_k(t)}=\cos\varphi_k(t)+i_k\sin\varphi_k(t),\ \ \varphi_0^2(t)=\sum_{k=1}^{n}\varphi_k^2(t)$$

4.0.2.4. Signal Instantaneous Frequency

$$\omega_i(t) = 2\pi f_i(t) = \frac{\partial\varphi_i(t)}{\partial t},\ i = 0,1,2,...k,...n$$

4.0.2.5. More of interesting relations

$$\bar{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t),$$

$$\cos\varphi_k = \frac{1}{2}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}),\ \sin\varphi_k = \frac{1}{2i}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}),$$

$$\varphi_k(t) = \mathrm{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)},\ \varphi_0{}^2(t) = \sum_{(k)}\varphi_k{}^2(t),\ \ \omega_k(t) = \frac{\partial\varphi_k(t)}{\partial t} = 2\pi f_k(t),$$

$$a_k{}^2(t) = a_{k-1}{}^2(t) + \hat{a}_{k-1}{}^2(t) = \Psi_{k+1}{}^2(t) = \Psi_k{}^2(t) + \hat{\Psi}_k{}^2(t),$$

$$a_0{}^2(t) = \left|\bar{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)}a_k{}^2(t) + 2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t),\ \forall i,j,k \in [1,n].$$

$$\Psi(t) = \frac{a_n(t)}{2^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}),\ \hat{\Psi}(t) = \frac{a_n(t)}{(2i)^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}),$$

$$\bar{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}.$$

4.0.2.6. Hyper-complex Analytic Signal

(This is only a draft... should be better elaborated and combined with Quaternions concept)

$$\bar{\Psi}(r,t) = \bar{\Psi}(x,y,z,t) = \Psi(r,t) + I\cdot H\left[\Psi(r,t)\right] = \Psi(r,t) + I\cdot\hat{\Psi}(r,t) =$$

$$= \Psi(r,t) + i\cdot\hat{\Psi}_x(r,t) + j\cdot\hat{\Psi}_y(r,t) + k\cdot\hat{\Psi}_z(r,t) =$$

$$= \bar{\Psi}_i + \bar{\Psi}_j + \bar{\Psi}_k = \left|\bar{\Psi}(r,t)\right|\cdot e^{I\cdot\varphi(r,t)},\ \bar{\Psi}_{i,j,k} = \Psi_{i,j,k} + \begin{bmatrix} i \\ j \\ k \end{bmatrix}\cdot\hat{\Psi}_{i,j,k} = \left|\bar{\Psi}_{i,j,k}\right|\cdot e^{\begin{bmatrix} i \\ j \\ k \end{bmatrix}\cdot\varphi_{i,j,k}},$$

$$\Psi(r,t) = \Psi_{x,y,z}(r,t) = \Psi_x(r,t) + \Psi_y(r,t) + \Psi_z(r,t) = \Psi_i + \Psi_j + \Psi_k,\ \hat{\Psi}_{x,y,z}(r,t) = H\left[\Psi_{x,y,z}(r,t)\right]$$

$$\left|\bar{\Psi}(r,t)\right|^2 = \left[\Psi(r,t)\right]^2 + \left[\hat{\Psi}(r,t)\right]^2 = \Psi^2 + \hat{\Psi}^2 = \left|\bar{\Psi}\right|^2,\ \varphi(r,t) = \mathrm{arctg}\frac{\hat{\Psi}(r,t)}{\Psi(r,t)} = \varphi,$$

$$\left|\bar{\Psi}_{i,j,k}\right|^2 = \left[\Psi_{i,j,k}\right]^2 + \left[\hat{\Psi}_{i,j,k}\right]^2,\ \varphi_{i,j,k} = \mathrm{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi_{i,j,k}},\ \omega_{i,j,k} = \frac{\partial\varphi_{i,j,k}}{\partial t}$$

$$\left|\overline{\Psi}\right|^2 = \left|\overline{\Psi}_i\right|^2 + \left|\overline{\Psi}_j\right|^2 + \left|\overline{\Psi}_k\right|^2 = \Psi^2 + \hat{\Psi}^2 ,$$
$$\Psi = \left|\overline{\Psi}\right| \cdot \cos\varphi ,\ \hat{\Psi} = \left|\overline{\Psi}\right| \cdot \sin\varphi = H\left[\Psi\right],$$
$$\Psi_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right| \cdot \cos\varphi_{i,j,k} ,\ \hat{\Psi}_{i,j,k} = H\left[\Psi_{i,j,k}\right] = \left|\overline{\Psi}_{i,j,k}\right| \cdot \sin\varphi_{i,j,k} ,$$
$$I \cdot \hat{\Psi}(r,t) = i \cdot \hat{\Psi}_i + j \cdot \hat{\Psi}_j + k \cdot \hat{\Psi}_\kappa = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \hat{\Psi}(r,t),$$
$$I \cdot \varphi(r,t) = i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k = e^{I(\frac{\pi}{2}+2m\pi)} \cdot \varphi(r,t) ,\ I^2 = i^2 = j^2 = k^2 = -1 ,$$
$$\hat{\Psi} = \left|\overline{\Psi}\right| \cdot \sin\varphi = H\left[\Psi\right] = \sqrt{\hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2},$$
$$\Psi_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right| \cdot \cos\varphi_{i,j,k} ,\ \hat{\Psi}_{i,j,k} = H\left[\Psi_{i,j,k}\right] = \left|\overline{\Psi}_{i,j,k}\right| \cdot \sin\varphi_{i,j,k}$$
$$I \sin\varphi = i \cdot \sin\varphi_i + j \cdot \sin\varphi_j + k \cdot \sin\varphi_k ,$$
$$I = i \cdot \frac{\varphi_i}{\varphi} + j \cdot \frac{\varphi_j}{\varphi} + k \cdot \frac{\varphi_\kappa}{\varphi} = i \cdot \frac{\hat{\Psi}_i}{\hat{\Psi}} + j \cdot \frac{\hat{\Psi}_j}{\hat{\Psi}} + k \cdot \frac{\hat{\Psi}_\kappa}{\hat{\Psi}} = e^{I(\frac{\pi}{2}+2m\pi)} ,\ \frac{\varphi_i}{\varphi} = \frac{\hat{\Psi}_i}{\hat{\Psi}} ,\ \frac{\varphi_j}{\varphi} = \frac{\hat{\Psi}_j}{\hat{\Psi}} ,\ \frac{\varphi_k}{\varphi} = \frac{\hat{\Psi}_k}{\hat{\Psi}},$$
$$\frac{\varphi_{i,j,k}}{\varphi} = \frac{\hat{\Psi}_{i,j,k}}{\hat{\Psi}} = \frac{H\left[\Psi_{i,j,k}\right]}{H\left[\Psi\right]} = \frac{\operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi}}{\operatorname{arctg}\frac{\hat{\Psi}}{\Psi}} ,$$
$$\left[?!(\text{working on...}) \left(\begin{array}{l} \sin\varphi = \sqrt{(\sin\varphi_i)^2 + (\sin\varphi_j)^2 + (\sin\varphi_k)^2}, \\ \cos\varphi = \sqrt{(\cos\varphi_i)^2 + (\cos\varphi_j)^2 + (\cos\varphi_k)^2}, \varphi_i + \varphi_j + \varphi_k = \pi, \\ \Psi = \Psi_i + \Psi_j + \Psi_k \ ,\ \Psi^2 = \Psi_i^2 + \Psi_j^2 + \Psi_k^2 \ ,\ \Psi_i\Psi_j + \Psi_j\Psi_k + \Psi_i\Psi_k = 0 , \\ \hat{\Psi} = \hat{\Psi}_i + \hat{\Psi}_j + \hat{\Psi}_k \ ,\ \hat{\Psi}^2 = \hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2 \ ,\ \hat{\Psi}_i\hat{\Psi}_j + \hat{\Psi}_j\hat{\Psi}_k + \hat{\Psi}_i\hat{\Psi}_k = 0 , \\ \varphi = \varphi_i + \varphi_j + \varphi_k \ ,\ \varphi^2 = \varphi_i^2 + \varphi_j^2 + \varphi_k^2 \ ,\ \varphi_i\varphi_j + \varphi_j\varphi_k + \varphi_i\varphi_k = 0 . \end{array} \right) \right] \tag{6.10}$$

In chapters 4.1 and 10, we can find proposals "how to apply Hypercomplex functions or Phasors of energy-momentum vectors" (see equations (4.3-0)-p,q,r,s…). In chapter 4.3, we will exercise that all famous wave equations of Quantum Theory can be naturally, smoothly, and easily developed from Analytic and Hypercomplex representations of wave functions (without implementing or postulating certain "ad hoc patchwork"). In chapter 6.0 (around equations (6.8) - (6.13)), it will be demonstrated how we can structure multidimensional universe using Hypercomplex functions. ***What here looks familiar to Hypercomplex, or Quaternion concepts is certain kind of 3-dimensional structuring of common imaginary unit*** $I^2 = i^2 = j^2 = k^2 = -1$ ***(what could be endlessly extended, by creating new "imaginary-units' triplets", from every (lower level) imaginary unit, this way introducing new foundations of multidimensionality).***

Remarks: Hypercomplex or Quaternions *wavefunction signal phase function* is also presentable as Hypercomplex function, having its real and imaginary parts, such as,

$$\overline{\Psi}(r,t) = \overline{\Psi}(x,y,z,t) = \Psi(r,t) + I \cdot H\left[\Psi(r,t)\right] = \Psi(r,t) + I \cdot \hat{\Psi}(r,t) =$$
$$= \Psi(r,t) + i \cdot \hat{\Psi}_x(r,t) + j \cdot \hat{\Psi}_y(r,t) + k \cdot \hat{\Psi}_z(r,t) =$$

$$= \overline{\Psi}_i + \overline{\Psi}_j + \overline{\Psi}_k = \overline{\Psi}'_i \cdot \overline{\Psi}'_j \cdot \overline{\Psi}'_k = \left|\overline{\Psi}(r,t)\right| \cdot e^{\overline{\varphi}(r,t)},\ \overline{\Psi}_{i,j,k} = \Psi_{i,j,k} + \begin{bmatrix} i \\ j \\ k \end{bmatrix} \hat{\Psi}_{x,y,z},$$

$$\overline{\varphi}(r,t) = \varphi_R(r,t) + I\,\varphi_I(r,t) = \varphi_R(r,t) + i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k = \left|\overline{\varphi}(r,t)\right| e^{\overline{\varphi}(r,t)},$$

$$I \cdot \varphi_I(r,t) = i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k,\ \ I^2 = i^2 = j^2 = k^2 = -1.$$

This will give us more substantial mathematical-modeling freedom, since ***every Hypercomplex function (or number) can be presented either as a summation or as multiplication of ordinary complex functions*** (with only one imaginary unit).

4.0.3. Generalized Fourier Transform and Analytic Signal

The Analytic Signal modeling of the wave function can easily be installed in the framework of the Fourier Integral Transform, which exists based on simple-harmonic functions $\cos \omega t$, as follows,

$$\Psi(t) = a(t)\cos\varphi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})e^{j2\pi ft}df = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\overline{H}\left[\cos 2\pi ft\right]\right\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

(4.0.5-1)

$$U(\frac{\omega}{2\pi}) = A(\frac{\omega}{2\pi})e^{j\Phi(\frac{\omega}{2\pi})} = \int_{-\infty}^{\infty} \Psi(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty} \Psi(t)\left\{\overline{H}^*\left[\cos 2\pi ft\right]\right\}dt = F\left[\Psi(t)\right],\ \ \omega = 2\pi f,$$

where the meaning of symbols is:

F (=) Direct Fourier transform,

$\mathbf{F^{-1}}$ (=) Inverse Fourier transform,

$\mathbf{\overline{H} = 1 + jH}$ (=) Complex Hilbert transform, $\mathbf{j^2 = -1}$,

$\overline{H}^* = 1 - jH$ (=) Conjugate complex Hilbert transform.

$$\overline{H}\left[\cos\omega t\right] = e^{j\omega t},\quad H\left[\cos\omega t\right] = \sin\omega t,$$

$$\overline{H}^*\left[\cos\omega t\right] = e^{-j\omega t},\quad H\left[\sin\omega t\right] = -\cos\omega t,$$

$$e^{\pm j\omega t} = (1 \pm jH)\left[\cos\omega t\right],$$

$$\overline{H}\left[\Psi(t)\right] = \overline{\Psi}(t) = \Psi(t) + jH\left[\Psi(t)\right] = \Psi(t) + j\hat{\Psi}(t),$$

$$\overline{H}^*\left[\Psi(t)\right] = \overline{\Psi}^*(t) = \Psi(t) - jH\left[\Psi(t)\right] = \Psi(t) - j\hat{\Psi}(t).$$

The further generalization of the Fourier integral transformation can be realized by replacement of its simple-harmonic functions basis $\cos \omega t$ by some other, convenient signal-elements basis $\boldsymbol{\alpha(\omega, t)}$. Now, the wave function (in the generalized framework of Fourier transform) can be presented as,

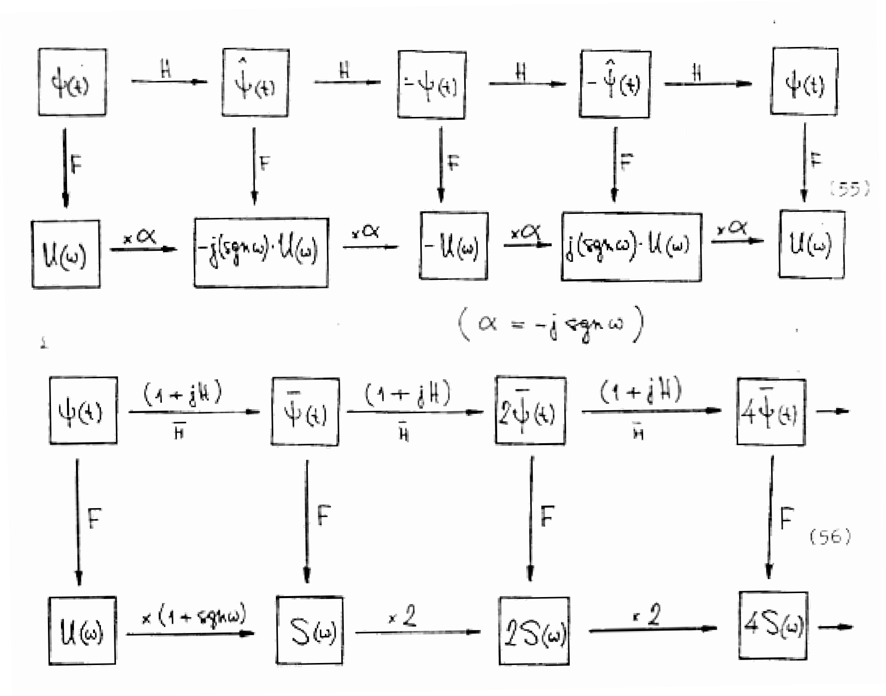
$$\Psi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\bar{H}\left[\alpha(\omega,t)\right]\right\} df = \mathbf{F}^{-1}\left[U(\frac{\omega}{2\pi})\right],$$
$$U(\frac{\omega}{2\pi}) = \int_{-\infty}^{\infty} \Psi(t)\left\{\bar{H}^{*}\left[\alpha(\omega,t)\right]\right\} dt = \mathbf{F}\left[\Psi(t)\right]. \tag{4.0.5-2}$$

For instance, we could exercise such signals quantization (analysis, decomposition, synthesis, signals reconstruction, and superposition) by using new, elementary signals $\alpha(\omega, t) = \frac{\sin\Omega t}{\Omega t}\cos(\omega t)$ or $\alpha(\omega, t) = e^{-\beta t}\cdot\frac{\sin\Omega t}{\Omega t}\cos(\omega t)$, or much better using certain of wavelets family similar to Gaussian-Gabor wave packets, or wave pulses (see later (4.0.33)-(4.0.36) and (10.2-3), being in relation to Kotelnikov-Shannon-Nyquist Theorem). Gaussian pulses and wave packets have the same *amplitude or envelope shape* in both time, and frequency domains, and such shapes are securing optimal (compacted) signal localization in its time and frequency domains. The idea here is to establish and shape elementary basis functions $\boldsymbol{\alpha(\omega, t)}$ which are the most natural and generally applicable, elementary wave groups or wave-packets (applicable to forming photons and other wave groups from the world of Physics).

Here are more relations connecting Hilbert and Fourier transformation:

$$F\left[\Psi(t)\right]\cdot(-j\cdot\operatorname{sgn}\omega) = F\left[\hat{\Psi}(t)\right] = -j\cdot\operatorname{sgn}\omega\cdot U(\omega),$$
$$F\left[\hat{\Psi}(t)\right]\cdot(-j\cdot\operatorname{sgn}\omega) = -F\left[\Psi(t)\right] = -U(\omega),$$
$$F\left[\Psi(t)\right]\cdot(1+j\cdot\operatorname{sgn}\omega) = F\left[\bar{\Psi}(t)\right] = S(\omega),$$

$$H\left[\Psi(t)\right] = \hat{\Psi}(t),\ \Psi(t) = -H\left[\hat{\Psi}(t)\right], (1+jH)\cdot\bar{\Psi}(t) = \bar{\Psi}(t),\ H^{2} = -j(Y-H),$$
$$H\left[H(\Psi(t))\right] = H^{2}\left[\Psi(t)\right] = -\Psi(t),\ H^{4}\left[\Psi(t)\right] = \Psi(t),$$
$$\bar{H}\left[\bar{\Psi}(t)\right] = 2\bar{\Psi}(t) = 2\bar{H}\left[\Psi(t)\right],\ H\left[\bar{\Psi}(t)\right] = -j\bar{\Psi}(t), \tag{4.0.5-3}$$
$$H\cdot\bar{H} = \bar{H}\cdot H = Y,\ H\cdot(\bar{H}\cdot H) = (H\cdot\bar{H})\cdot H,$$
$$Y\left[\bar{\Psi}(t)\right] = 2\bar{\Psi}(t) = H\left[\bar{\Psi}(t)\right],\ Y\left[\Psi(t)\right] = -j\bar{\Psi}(t),$$
$$H\left[t\cdot\Psi(t)\right] = t\cdot\hat{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt,\ \bar{H}\left[t\cdot\Psi(t)\right] = t\cdot\bar{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt,$$
$$H\left[(t+\tau)\cdot\Psi(t)\right] = (t+\tau)\cdot\hat{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt,\ \bar{H}\left[(t+\tau)\cdot\Psi(t)\right] = (t+\tau)\cdot\bar{\Psi}(t) + \frac{1}{\pi}\int_{-\infty}^{\infty}\Psi(t)\,dt,$$
$$H\left[\frac{d^{n}\Psi(t)}{dt}\right] = \frac{d^{n}\hat{\Psi}(t)}{dt},\ \bar{H}\left[\frac{d^{n}\Psi(t)}{dt}\right] = \frac{d^{n}\bar{\Psi}(t)}{dt}.$$


ψ(t)
H
$\hat{ψ}$(t)
-ψ(t)
-$\hat{ψ}$(t)
F
(55)
U(ω)
×α
-j(sgn ω)·U(ω)
-U(ω)
j(sgn ω)·U(ω)
(α = -j sgn ω)
(1 + jH)
$\bar{H}$
$\bar{ψ}$(t)
2$\bar{ψ}$(t)
4$\bar{ψ}$(t)
(56)
×(1 + sgn ω)
S(ω)
×2
2S(ω)
4S(ω)

(4.0.5-4)

4.0.4. Meaning of Complex Analytic Signal Functions in Physics

The practice of using complex functions in Physics and Electronics has a long time been considered mostly as a convenience to simplify complicated mathematical expressions and equations processing (especially for avoiding operations with trigonometric functions). All of that has a much broader and more profound meaning when adequately applied (what has not been quite uniformly and correctly established practice in different domains of Physics, yet). Briefly concluding, the only proper extension and generalization of real-variables wave (or function) to a corresponding complex variable function (with tangible meaning in Physics) should be formulated as an Analytic Signal function (what is generally still not the case in Physics). Even in electronics, or basic electro-technique, where operating with complex functions such as current, and voltage ***phasors,*** *are the common practice, this is not realized on the grounds of an Analytic Signal modeling, but luckily, what realized, it is compatible and not-contradictory to such approach (see more at the end of this chapter under "4.0.11. Generalized Wave Functions and Unified Field Theory").*

The fact is that significant arbitrary wave function elements (such as instantaneous amplitude, phase, frequency…) cannot be found if we do not take into account both original wave function Ψ and its Hilbert couple $\hat{\Psi}$, meaning that in reality both of such wave functions should coincidently exist (on some way), explaining the meaning of a complex Analytic Signals. In other words, *the Nature (or the world of Physics) always creates mutually coupled, mirror-states and events (which are mutually orthogonal, phase-shifted functions). In such cases, Hilbert transform, and Complex Analytic Signal model are the right and best mathematical tools to formulate mentioned coupled entities* (however, often we do not notice or realize an intrinsic, or coincidental existence of such coupled states, because in most of measurements, or observation cases we see, or detect only one of them). Here is a part of the explanation related to Quantum Theory mathematical structure, which is also (and incompletely) modeling mentioned mutually coupled, dual (or conjugate) matter-wave states on a formally different way that should be isomorphic to an Analytic signal structure (what is still not fully and correctly satisfied). *For instance, well-known, and somewhat controversial particle-wave duality theory, as elaborated in the contemporary Quantum Theory, is mathematically, verbally, experimentally, stochastically, and philosophically (but not ontologically and deterministically) explaining how in some cases objects like photons, electrons, and other elementary particles could behave like waves, and in other cases like particles. Such particle-wave duality should also be explicable concerning duality (and complementarity) of an original wave function Ψ and its Hilbert couple $\hat{\Psi}$.* We could conditionally say that if Ψ represents specific ***moving-particle*** model (or wave group), then $\hat{\Psi}$ represents its wave model (and vice versa), since both, Ψ and $\hat{\Psi}$ (as well as their frequency-domain counterparts), exist coincidently, and have the same energy content $\tilde{E}$ (of course, this should be elaborated better), as for instance,

$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty} \left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty} \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty} \left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{0}^{\infty} \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega \,. \quad (4.0.4)\ (4.0.5)$$

Ψ and $\hat{\Psi}$ have the same group and phase velocity (to be shown later), they are mutually orthogonal (phase shifted for $\pi/2$), and on the certain way "*energetically*" mutually exclusive since $\int_{-\infty}^{+\infty} \Psi(t) \cdot \hat{\Psi}(t) dt = 0$. All of that should belong to the mathematical background of the particle-wave duality and wave functions, and there is still no need to introduce *probability* wave functions when we already have such rich, exact, operational, and explicit wave functions modeling (as operating with Analytic Signals). Complex *Analytic Signal formulation of wave functions is the ultimate generalization of* ***PHASORS*** *concept (which is in cases of simple harmonic functions very successfully applied in electro-technique and electronics), and there is still a space to be much more extended in Physics, and to everything that is moving, oscillating, and waving in time and/or spatial domains (see more in Chapter 10.). Relativity theory, regarding significant imaginary and complex entities and functions is presently still not in a big harmony with here elaborated Complex Analytic Signals and Phasors, but Minkowski-Einstein 4-vectors could be treated as Complex functions and on some convenient way modified to be as Analytic Signals or Phasors.*

The celebrated physicist Paul Dirac was even more imaginative, intellectually vocal, and more convincing in conceptualizing an "ocean" of densely packed negative energy "phantom-like" electrons in a pure vacuum state of matter. This way, he "invented" anti-matter particle named positron, which without such creative imagination could also be found either as certain Ψ-function, or its Hilbert couple $\hat{\Psi}$, or anyway described inside the framework of Analytic Signal functions (and eventually, positron was experimentally discovered, much more by chance and without Dirac's, and any of here-mentioned theoretical support, but later, backwards associated to Dirac's prediction). Dirac also gave in the same scientific package his imaginative meaning of complex or imaginary functions concerning relativistic particle energy, what his faithful followers found as an exceptional and brilliant-mind contribution to physics. This way, he "predicted" the existence of other anti-matter particles, and some of them were, by chance, experimentally (and most probably independently from any Dirac's influence and prediction) discovered a few years later. This prediction has been presented in some books as a brilliant success of the science of 20th century AD, also showing that operational "Ptolemy-type theories" can still exist, and that we are in some cases, during certain period, not able to recognize them as such. Ptolemy is mentioned here concerning its geocentric teaching that is conceptually and fundamentally wrong, but practically and mathematically was operational and giving sufficiently correct results (during certain, very long historical period…). Richard Feynman later upgraded and optimized mentioned Dirac's concepts, making them very practical in the form of Feynman's diagrams. Later, similar imaginative and seducing concepts evolved towards the Zero Point, Electromagnetic Quantum Vacuum Fluctuations (ZPF). Here, we could say that *every real motion-state expressed as certain Ψ-function should have its Hilbert couple $\hat{\Psi}$, which is another and equally real, motional state (existing both mathematically and on some specific way in the real world of physics, like electric and magnetic field components in cases of electromagnetic waves and photons).* The interpretation how such coupling, imaging and phase shifting of complementary and dual states manifests in Physics is another question, but it is undoubtedly related to de Broglie matter waves, and maybe including Dirac's, Feynman's and ZPF concepts. In electronics and electro-technique, when operating with current, voltage, impedance, and power-related **phasors** (meaning with complex and analytic signal functions), we know that real values of mentioned phasors are presenting direct emitted or outgoing energy (or real and active power) flow from its source towards its load or sink. At the same time, imaginary components of the same power-related functions are presenting reactive or reflected (backward) energy flow. Similar meanings (of direct and reflected energy flow) we can analogically associate to different matter waves in other domains of physics. This way we will avoid arbitrary and not-realistic applications or interpretations of imaginary and complex functions in Physics.

It should not be too far from reality that Ψ and its Hilbert couple $\hat{\Psi}$ are, on some way, in a mutual (and instant or immediate) synchronizing, coupling, and communicating relation of Quantum entanglement (based on signal energy relations (4.04) and (4.05)). At least, we know that entanglement is an experimentally verifiable reality of specifically and coincidently created and coupled particle-wave states like photons, electrons, protons, atoms, and small atomic clouds (see more about entanglement in [56]).

[♣ **COMMENTS & FREE-THINKING CORNER:**

Of course, there is no progress and advance without creative imagination, and eventually, wrong concepts, which are on a certain level useful, innovative, and operational, will be gradually replaced or improved by better concepts. In that name we can introduce another of such (still brainstorming) concepts, as follows: *"Since atoms, as elementary bricks of matter, are composed of electrons, protons, and neutrons, and all of them manifest particle and wave properties, being presentable as wave packets or wave functions, we can associate to each of them certain wave function Ψ_i. From the properties of Analytic Signals (elaborated here), we know that all of them should also have intrinsically coupled and phase-shifted Hilbert components $\hat{\Psi}_i$". How such intrinsic nature of coupled signals behaves inside atoms (or inside planetary or solar systems) it is another question to answer? Is this kind of thinking leading to reinventing positrons, antiprotons, positive, negative, and neutrally charged particles, and maybe "dark matter" energy states? Another creative aspect of "Hilbert-type duality" could be a duality of linear and rotational motions, meaning that every particle (or wave) in linear motion should have its "motional Hilbert couple", which could be some spinning motion (and vice-versa). For instance, linear motion of an electrically charged particle has its electric field vector, but it should also have an associated, solenoidal magnetic field vector.* Is de Broglie Particle-Wave Duality describing something familiar to that?

If we accept the philosophical position that all mathematically consistent, internally coherent, self-standing theories, concepts, and models (based on tangible Physics) are potentially presenting certain reality of our Universe, which may, one day be materialized as a verifiable Physics knowledge, we could already see the roots and foundations of Duality in Physics in relation to Analytic Signal Wave Functions Modeling. Of course, such concepts, models and theories should be well integrated with the overall body of relevant Mathematics and Physics, avoiding creations that are extensively axiomatic, dogmatic, or based on many postulates and assumptions (like the case is in the present Relativity and Quantum theory).

What is interesting regarding physics-related, energy-finite-signals (found in our universe) is that such signals are not only *time-dependent functions* but in most cases *space-time dependent*. However, from our point of view (related to measurements and observations in certain space-location) we often experience signals as only time or space dependent functions. Most of the above-presented mathematical relations (starting from (4.0.1) to T.4.0.1) could (equally and reasonably) exist, having physics-related meaning if we replace a time variable $\mathbf{t}$ with corresponding space variable $\mathbf{x}$, for instance, $\mathbf{t} \rightarrow \mathbf{x}$, $\omega = \omega_t = 2\pi f \rightarrow \omega_x = 2\pi f_x$, $\Psi(t) \rightarrow \Psi(x)$. *The reason for such time-space symmetry is that whichever wave function, found as a good model for representing specific sufficiently stable moving particle, energy state or other, relatively stable (non-transient and non-dispersive) motion, should intrinsically have a structure that takes care about its space and time parameters matching and integrity. This is creating mutual harmony and unity between them (usually expressed by simple mathematical relations between relevant time and space-related parameters).* If this has not been the case in our universe, we would not have temporally and spatially stable objects and recognizable patterns of different motions and astronomic formations, which are mutually analogical, symmetrical, and respecting the same fundamental conservation laws of Physics (see (4.0.46)). It will be shown later that mentioned temporal-spatial integrity, stability and synchronization between relevant time, space and their frequency domains is also closely related to relevant group and phase velocity, and to optimal time and space, mutually dependent signal sampling intervals known in Signal Analysis in relation to Shannon-Kotelnikov-Nyquist signal sampling theorem (see more about such situations in chapter 10). Analyses of that kind will also lead to an extension, revision, and generalization of universally valid Uncertainty Relations, mistakenly considered as originating in Physics from W. Heisenberg, but merely taken from universally valid Signal analysis and associated mathematics.

The unfortunate event (regarding quantum physics) is that Dennis Gabor, the inventor of the Analytic Signal concept, came too late with his mathematical invention, after the stepstones of Quantum Theory have been "*strongly and by consensus established*" and celebrated founders of Quantum Theory were (probably) already too tired, and not ready to make (very new) Quantum Theory-house redesign. Also, it could probably be embarrassing to admit that few of the Nobel prizes were already awarded to something that maybe was not the best possible, unique, extraordinary, and brilliant creation, as sometimes vocally announced by QT founders, being still non-doubtfully maintained by many of their followers and believers. To keep established harmony, the most appropriate (and wrong) was to continue as nothing serious happened (and, most probably that not many people noticed the relevance of such new mathematical concepts, unintentionally mixing them with similar items from Fourier Signal Analysis). Also, most of the followers of the Orthodox Quantum Theory teachings are proudly asking themselves and suggesting to others: Why to change foundations or something else in present Quantum Theory? Where the problem is, considering that present-days Quantum Theory is mathematically working very well (in its own, self-defined, or postulated frames and assumptions). We should hope that this (about QT) would not become like Ptolemy's geocentric system that unfortunately (and wrongly) resisted as the unique, mainstream-accepted, and most accurate teaching (about Sun and planetary motions) for an exceedingly long time.

-- ♣]

4.0.5. Wave Packets and mathematical strategies regarding Wave Velocities

Let us first introduce general wave properties regarding most elementary, simple harmonic waves as usually modeled in physics.

Oscillations at a point (only time-dependent) can be presented as,

$$\Psi(t) = a \cdot \cos\ 2\pi ft = a \cdot \cos\omega t \tag{4.0.6}$$

Traveling, planar waves (in one-dimensional motions) are characterized by,

$$\begin{aligned} &\Psi(t,x) = a \cdot \cos 2\pi\ f(t - \frac{x}{u}) = a \cdot \cos 2\pi (ft - \frac{x}{\lambda}) = a \cdot \cos\ (\omega t - kx) = \\ &= a \cdot \cos k\,(ut - x) = \Psi(x - ut), \\ &\omega = 2\pi f\ (=)\ \text{angular frequency},\ k = \frac{2\pi}{\lambda}\ (=)\ \text{wave number}, \\ &u\ = \lambda f = \frac{\omega}{k}\ \ (=)\ \text{phase velocity}, \end{aligned} \tag{4.0.7}$$

and traveling (also planar) waves as 3-dimensional space motion, including a time dimension,

$$\Psi(t,r) = a \cdot \cos\ (\omega t - kr) = \Psi(r - ut),\ \vec{r} = \vec{r}(x, y, z)\ . \tag{4.0.8}$$

In reality (what is often being neglected), every wave that has the same wave-shape during motion (like elementary sinusoidal signals), presented in one-dimension (traveling along the **x** or **r** axis), is usually composed of two waves, traveling in mutually opposite directions (below marked as (+) and (-) directions), judging by general solutions of relevant (differential) wave equations. The complete (generalized) wave function of such wave motion, applicable also to arbitrary function shapes, is $\Psi(t,r) = \Psi^{(+)}(r - ut) + \Psi^{(-)}(r + ut)$. It can be proven that for all of particular wave functions such as $\underline{\Psi^{(+)}(r - ut)}$, or $\underline{\Psi^{(-)}(r + ut)}$, or $\underline{\Psi^{(+)}(r - ut) + \Psi^{(-)}(r + ut)}$, (when the wave propagates in linear media, without dispersion, at the same speed independent of wavelength, in both directions, and independent of amplitude) is applicable the same partial, differential Wave Equation (known also as Classical and/or d 'Alembert wave equation), $\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2} (=) \nabla^2\Psi$. What is even more significant is that solutions of such Wave Equation can be generalized with a simple, complex form as,

$$\Psi(t,r) = a \cdot e^{i(\omega t - kr)} + b \cdot e^{i(\omega t + kr)} \Leftrightarrow \begin{bmatrix} \Psi(t,r) = a \cdot \cos(\omega t - kr) + b \cdot \cos(\omega t + kr) \\ (=)\ \Psi^{(+)}(r - ut) + \Psi^{(-)}(r + ut) \end{bmatrix},$$ what

Schrödinger applied when he created his wave equation. In this book, further generalization is also practiced considering that solutions of Wave Equation are Complex, Analytic Signal functions, as in (4.0.2). We will see later that such approach can efficiently produce and explain all the essential wave equations known from

Quantum Theory, on a much simpler way compared to contemporary Quantum Theory, axiomatic and stochastic, mathematical elaborations).

Of course, the wave velocity (here phase velocity) **u** will depend on the medium properties through which the wave is propagating.

All other, more complex waveforms and wave packets are presentable as an integral or discrete superposition of elementary, simple-harmonic waves, as shown in (4.0.1), including a superposition of simple waves (4.0.8). Synchronous propagation of two, coupled wave components in mutually opposed directions should always be considered as generally present and valid reality for all wave motions (having a much deeper meaning in the world of Physics, than presently seen).

If a wave velocity of the waveform (or wave packet) is independent of wavelength, each elementary wave (and thus the wave packet) travels at the same speed (what is *valid for propagations in linear media*).

If wave velocity depends on wavelength (*valid for propagations in non-linear media*), each elementary wave travels at a different speed, compared to the wave packet speed (or group speed).

The general condition (regarding an arbitrary waveform) for extracting the waveform phase velocity, **u**, is that signal phase function would become constant. The meaning of that is that we would be able to travel parallel with signal phase, always seeing the same phase point (being linked to the same phase value: or effectively we will not see that signal carrier function $\cos\varphi(t)$ is propagating or being time dependent. Of course, the first step is to present the waveform as an Analytic Signal function). Satisfying this will mean that we are also traveling parallel to a wave in question, with waves phase velocity. Mathematically, this could be summarized as,

$$\left\{\begin{array}{l}\varphi(t) \to \varphi(t,x) = const. \\ \omega t \to \omega t\ -\ kx \\ \varphi'(t,x) = \varphi''(t,x) = const.\end{array}\right\} \Rightarrow \left\{\frac{dx}{dt}, \frac{\Delta x}{\Delta t}\right\} (= u =\ \text{phase velocity}). \qquad (4.0.9)$$

Group velocity, **v**, is the velocity of the signal amplitude or its envelope function $\mathbf{a(t)}$. Now the same situation could be visualized if we imagine that we are traveling parallel to signal amplitude and we would always see only one point of the amplitude function (for instance, signal envelope peak value), for instance,

$$\left\{\begin{array}{l}a(t) \to a(t,x) = const. \\ \omega t \to \omega t - kx \\ a'(t,x) = a''(t,x) = const.\end{array}\right\} \Rightarrow \left\{\frac{dx}{dt}, \frac{\partial x}{\partial t}\right\}\ (= v = \text{group velocity}). \qquad (4.0.10)$$

We could also create a new analytic signal form, which is equal only to the signal amplitude function, and apply similar method as in case of phase velocity (here we are treating the signal amplitude function as a new wave function, which would have its own, newly calculated phase and amplitude functions).

$$\left\{\begin{array}{l} a(t) \to a(t,x) = (\sqrt{a^2(t,x) + [H(a(t,x))]^2})\cos\left[\arctan\frac{H(a(t,x))}{a(t,x)}\right] = \\ = a_1(t,x)\cos\varphi_1(t,x);\ \ \varphi_1(t,x) = const.,\quad \omega t \to \omega t - kx \\ a_1(t,x) = \sqrt{a^2(t,x) + [H(a(t,x))]^2},\ \varphi_1(t,x) = \arctan\frac{H(a(t,x))}{a(t,x)} \end{array}\right\} \Rightarrow \tag{4.0.11}$$

$$\Rightarrow \left\{\frac{dx}{dt}, \frac{\partial x}{\partial t}, \frac{\Delta x}{\Delta t}\right\}_1 \Rightarrow (v = \text{group velocity}) .$$

Let us now compare the "**external and internal**" signal structure in a signal time-domain, (4.0.1). *Internally*, (inside of the integral) we have the infinitesimal superposition of elementary waveforms,

$$U_c(\omega)\cos\omega t + U_s(\omega)\sin\omega t = A(\omega)\cos(\omega t + \Phi(\omega)), \tag{4.0.12}$$

Externally (after integration) we have a similar analytic signal form, $\Psi(t) = a(t)\cos\varphi(t)$. In the following several steps (1°, 2°, 3° …), it will be shown how group and phase velocities can be found, and what the consequences regarding modeling elementary waveforms that are building elements of other more complex waveforms, are. Terms "**internal and external**" signal structures are conditionally and temporarily introduced in this book before we find terminology that is more appropriate.

1°

In both cases (*time-wise and frequency-wise*: (4.0.1) – (4.0.3)) there is an amplitude function ($A(\omega)$ or $\mathbf{a(t)}$) and a phase function as an argument of the cosine function ($\cos(\omega t + \Phi(\omega)) = \cos\Phi(\omega,t)$ or $\cos\varphi(t)$), and we could say that both, "**internal and external**" signal waveforms are presenting Analytic Signal forms (being created in the same way and looking mutually similar).

For the signal integrity and its existence as the lasting traveling waveform it would be necessary that group and phase velocities of "**internal and external**" wave functions are mutually equal (in other words, here we will analyze only non-dispersive traveling waves).

Since the integration of $A(\omega)\cos(\omega t + \Phi(\omega))$ is made taking into account only frequency ω (not time), it is almost evident that *the wave function* $A(\omega)\cos(\omega t + \Phi(\omega))$ *and wave function* $\mathbf{a(t)\cos\varphi(t)}$ *should have the same velocities*. Of course, all waveforms in question should be conveniently presented in time and space coordinates (briefly: $\omega t \to \omega t - kx$), what is applicable in all cases of non-dispersive traveling waves.

2°

We can also compare the "**external and internal**" signal structure, (4.0.1) – (4.0.3), in a signal frequency domain to address the group and phase wave velocity differently. This time we will take the complex signal forms to have more condensed mathematical expressions and easier comparison.

$$\begin{aligned}&\overline{U}(\omega)=A(\omega)e^{-j\Phi(\omega)}=U_c(\omega)-jU_s(\omega)=\int_{-\infty}^{+\infty}\overline{\Psi}(t)e^{j\omega t}\,dt \quad =\int_{-\infty}^{+\infty}a(t)e^{j(\omega t+\varphi(t))}\,dt\\&\overline{\Psi}(t)\;=a(t)e^{-j\varphi(t)}\quad =\Psi(t)+j\hat{\Psi}(t)\quad =\frac{1}{\pi}\int_0^{\infty}U(\omega)e^{j\omega t}d\omega\;=\frac{1}{\pi}\int_0^{\infty}A(\omega)e^{j(\omega t+\Phi(\omega))}d\omega\end{aligned} \tag{4.0.13}$$

The formal analogy is obvious: both complex signal forms, $\overline{U}(\omega)$ and $\overline{\Psi}(t)$ (one in the frequency domain, and the other in the time domain, representing the same signal or the same wave) should have the same wave velocities; - the phase functions, $\Phi(\omega)$ and $\varphi(t)$, should generate a phase velocity $\mathbf{u}$, and the amplitude functions, $A(\omega)$ and $\mathbf{a(t)}$ should generate a group velocity $\mathbf{v}$. Also, we see again that all "**internal and external**" signal functions are presenting the structure of Analytic Signal forms (of course, differently formulated). Before we start searching for mathematical expressions of wave velocities, all waveforms in question should be extended to have time and space coordinates, or to represent traveling and non-dispersive waveforms (briefly: $\omega t \rightarrow \omega t - kx$, $\omega = \omega(k)$).

The forms of group and phase wave velocities should equally (and analogically) be presentable regarding ordinary time-space $(\mathbf{t}, \mathbf{x})$ variables, and concerning spectral variables $(\omega, \mathbf{k})$, $\omega=\omega(k)=\omega_t=2\pi f_t=2\pi f$, $k=\omega_x=2\pi f_x=\frac{2\pi}{\lambda}$,

$$\left\{\begin{array}{l}\varphi(t, x)=\text{const.}\\ \omega t\rightarrow\omega t\,-\,kx\,,\,\omega=\omega(k)\\ \Phi(\omega, k)=\text{Const.}\end{array}\right\}\Rightarrow\left\{\frac{dx}{dt},\frac{\Delta x}{\Delta t},\frac{\omega}{k}\right\}(=)\,u=\text{phase velocity}, \tag{4.0.14}$$

$$\left\{\begin{array}{l}a(t,x)=\text{const.}\\ \omega t\rightarrow\omega t\,-\,kx,\,\omega=\omega(k)\\ A(\omega\,,\,k)=\text{Const.}\end{array}\right\}\Rightarrow\left\{\frac{dx}{dt},\frac{\partial x}{\partial t},\frac{d\omega}{dk}\right\}(=)\,v=\text{group velocity}\cdot \tag{4.0.15}$$

3°

Let us now unite the wave-velocities search strategies for "**external and internal**" signal structures, both in time and frequency signal domains. We will now create the simplest possible wave group that is composed only of two elementary waveforms that are mutually infinitesimally close (where closeness is measured by small differences between their space, time, and frequency variables), and find its phase and group velocity. By the nature of the mathematical formulation of all Analytic Signals, we can say that amplitude or envelope functions $\mathbf{a(t,x)}$ and $A(\omega, k)$ are placed in the lower frequency spectrum area (being slowly evolving), compared to carrier or phase functions $e^{j(\omega t - kx+\varphi(t,x))}$ and $e^{j(\omega t - kx+\Phi(\omega, k))}$ (also, by the way it is also an excellent time to say, this is also well-known property of analytic signals, not analyzed here). Consequently, we have a chance to simplify the determination of group and phase velocity, since when making a superposition of two infinitesimally close wave elements we will be able to consider that their amplitude functions remain constant (and that only signal carrier phase functions are significant variables). Practically, instead of integrating with total integral limits, we will take only two of sub-integral elementary waveforms (both in time and frequency domains), that are mutually infinitesimally close, and find what their superposition will create,

$$\left.\begin{cases} \overline{y}(t,x) = a(t,x)e^{j(\omega t - kx + \varphi(t,x))} = a(t,x)e^{j\Theta(t,x)} \\ \overline{Y}(\omega,k) = A(\omega,k)e^{j(\omega t - kx + \Phi(\omega,k))} = A(\omega,k)e^{j\Theta(\omega,k)} \\ \Theta\ (t,x) = \omega t - kx + \varphi(t,x) \\ \Theta(\omega,\ k) = \omega t - kx + \Phi(\omega,k) \\ \overline{U}(\omega,k) = A(\omega,k\ )e^{-j\Phi\ (\omega)} = \int_{-\infty}^{+\infty} a(t,x)e^{j(\omega t - kx + \varphi(t))}\, dt = \int_{-\infty}^{+\infty} \overline{y}(t,x)\cdot dt \\ \overline{\Psi}(t,x) = a(t,x)e^{-j\varphi(t)} = \frac{1}{\pi}\int_{0}^{\infty} A(\omega,k)e^{j(\omega t - kx + \Phi(\omega))}\, d\omega = \frac{1}{\pi}\int_{0}^{\infty} \overline{Y}(\omega,k)\cdot d\omega \end{cases}\right\} \Rightarrow \qquad (4.0.16)$$

$$\begin{aligned} \overline{\Psi}_{1+2} &= \frac{1}{2}\left[\vec{y}_1(t,x) + \overline{y}_2(t,x)\right] = \frac{1}{2}a(t,x)\cdot\left\{ e^{j[\Theta(t,x) - d\Theta(t,x)]} + e^{j[\Theta(t,x) + d\Theta(t,x)]} \right\} \\ \overline{U}_{1+2} &= \frac{1}{2}\left[\overline{Y}_1(\omega,\ k) + \overline{Y}_2(\omega,\ k)\right] = \frac{1}{2}A(\omega,\ k)\cdot\left\{ e^{j[\Theta(\omega,\ k) - d\Theta(\omega,\ k)]} + e^{j[\Theta(\omega,\ k) + d\Theta(\omega,\ k)]} \right\} \end{aligned} \qquad (4.0.17)$$

$$\begin{aligned} &\overline{\Psi}_{1+2} = \frac{1}{2}a(t,\ x)e^{j\Theta(t,\ x)}\cdot\left[e^{-jd\Theta(t,x)} + e^{+jd\Theta(t,x)}\right] = a(t,\ x)e^{j\Theta(t,\ x)}\cdot\cos\left[d\Theta(t,\ x)\ \right] = \\ &= a(t,\ x)e^{j\varphi(t,\ x)}\cdot\cos\left[d\Theta(t,\ x)\ \right]\cdot e^{j(\omega t - kx)} = \overline{\Psi}(t,x)\cdot\cos\left[d\Theta(t,\ x)\ \right]\cdot e^{j(\omega t - kx)} = \\ &= \overline{\Psi}(t,x)\cdot\cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\varphi(t,\ x)\right]\cdot e^{j(\omega t - kx)} \\ &--------------- \\ &\overline{U}_{1+2} = \frac{1}{2}A(\omega,\ k)e^{j\Theta(\omega,\ k)}\cdot\left[e^{-jd\Theta(\omega,\ k)} + e^{+jd\Theta(\omega,\ k)}\right] = A(\omega,\ k)e^{j\Theta(\omega,\ k)}\cdot\cos\left[d\Theta(\omega,\ k)\right] = \\ &= A(\omega,\ k)e^{j\Phi\ (\omega,\ k)}\cdot\cos\left[d\Theta(\omega,\ k)\right]\cdot e^{j(\omega t - kx)} = \overline{U}(\omega,k)\cdot\cos\left[d\Theta(\omega,\ k)\right]\cdot e^{j(\omega t - kx)} = \\ &= \overline{U}(\omega,k)\cdot\cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\Phi(\omega,\ k)\right]\cdot e^{j(\omega t - kx)} \end{aligned} \qquad (4.0.18)$$

$$\begin{aligned} &\cos\left[d\Theta(t,\ x)\ \right] = \cos\left\{d\left[\omega t - kx + \varphi(t,\ x)\right]\right\} = \\ &= \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\varphi(t,\ x)\right] - \sin\left[d(\omega t - kx)\right]\cdot\sin\left[d\varphi(t,\ x)\right] = \\ &= \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\varphi(t,\ x)\right] \end{aligned}$$

$$\begin{aligned} &\cos\left[d\Theta(\omega,\ k)\right] = \cos\left\{d\left[\omega t - kx + \Phi(\omega,\ k)\right]\right\} = \\ &= \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\Phi(\omega,\ k)\right] - \sin\left[d(\omega t - kx)\right]\cdot\sin\left[d\Phi(\omega,\ k)\right] = \\ &= \cos\left[d(\omega t - kx)\right]\cdot\cos\left[d\Phi(\omega,\ k)\right] \end{aligned}$$

General conditions to be satisfied (regarding (4.0.18)) in order to find phase velocity are $\boldsymbol{\varphi(t,x) = const.}$ and $\Phi(\omega,k) = Const.$, making that $d\varphi(t,x) = 0$, and $d\Phi\ (\omega\ ,\ k) = 0$, what simplifies the expressions of elementary waveforms in time and frequency domains,

$$\left.\begin{cases} \overline{\Psi}_{1+2} = \overline{\Psi}(t,x)\cdot\cos\left[d(\omega t - kx)\right]\cdot e^{j(\omega t - kx)} \\ ---------------- \\ \overline{U}_{1+2} = \overline{U}(\omega,k)\cdot\cos\left[d(\omega t - kx)\right]\cdot e^{j(\omega t - kx)} \end{cases}\right\} \Leftrightarrow \left\{ \frac{\overline{\Psi}_{1+2}}{\overline{U}_{1+2}} = \frac{\overline{\Psi}(t,x)}{\overline{U}(\omega,k)} \right\} \qquad (4.0.19)$$

The only common carrier frequency, phase function member in both elementary wave functions ($\overline{\Psi}_{1+2}$ and $\overline{U}_{1+2}$) is $\mathbf{e}^{\mathbf{j}(\omega t-\mathbf{kx})}$, and their velocities should be the signal phase velocity, found as usually (when the phase is constant),

$$\begin{aligned} &\omega t - kx = const \Leftrightarrow \omega t_1 - kx_1 = \omega t_2 - kx_2 \Leftrightarrow \omega(t_2 - t_1) - k(x_2 - x_1) = 0 \Leftrightarrow \\ &\Leftrightarrow \omega \Delta t - k\Delta x = 0 \Leftrightarrow \frac{\Delta x}{\Delta t} = \frac{\omega}{k} \end{aligned} \tag{4.0.20}$$

Since in this analysis we started creating the superposition of two infinitesimally close elementary waves, (4.0.16), it is evident that their phase velocity should be found as,

$$u = (\lim \frac{\Delta x}{\Delta t})_{\Delta t \to 0} = \frac{dx}{dt} = \frac{\omega}{k} = \text{phase velocity}. \tag{4.0.21}$$

From the other joint member, $\cos[d(\omega t - kx)]$, of two elementary waveforms ($\overline{\Psi}_{1+2}$ and $\overline{\mathbf{U}}_{1+2}$), we would be able to find the group velocity, again using the same constant phase argument as in the case of phase velocity,

$$\begin{aligned} &\omega t - kx = const \Leftrightarrow d(\omega t - kx) = 0 \Leftrightarrow k(\frac{\omega}{k} - \frac{dx}{dt})dt - (x - t\frac{d\omega}{dk})dk = 0 \Leftrightarrow \\ &\Leftrightarrow \left[\frac{dx}{dt} = \frac{\omega}{k} = \text{phase velocity}\right] \text{and} \left[x - t\frac{d\omega}{dk} = 0\right] \Rightarrow x = t\frac{d\omega}{dk} \Rightarrow \\ &\Rightarrow v = \frac{\partial x}{\partial t} = \frac{d\omega}{dk} = \text{group velocity} \end{aligned} \tag{4.0.22}$$

It is also possible to find the functional connection between phase and group velocity in the following form,

$$v = u + k\frac{du}{dk} = \frac{d\omega}{dk} \ . \tag{4.0.23}$$

Of course, we could immediately find the elementary sub-integral, waveforms as,

$$\begin{aligned} &\left\{\begin{array}{l} \overline{\Psi}_{1+2} = \overline{\Psi}(t,x) \cdot \cos[d(\omega t - kx)] \cdot e^{j(\omega t - kx)} = \overline{\Psi}(t,x) \cdot e^{j(\omega t - kx)} = a(t,x) \cdot e^{j\varphi(t,x)} \cdot e^{j(\omega t - kx)} \\ ---------------- \\ \overline{U}_{1+2} = \overline{U}(\omega,k) \cdot \cos[d(\omega t - kx)] \cdot e^{j(\omega t - kx)} = \overline{U}(\omega,k) \cdot e^{j(\omega t - kx)} = A(\omega,k) \cdot e^{j\Phi(\omega,k)} \cdot e^{j(\omega t - kx)} \end{array}\right\} \Leftrightarrow \\ &\Leftrightarrow \left\{\begin{array}{l} \overline{\Psi}(t,x) = \overline{\Psi}_{1+2} \cdot e^{-j(\omega t - kx)} = a(t,x) \cdot e^{j\varphi(t,x)} \\ \overline{U}(\omega,k) = \overline{U}_{1+2} \cdot e^{-j(\omega t - kx)} = A(\omega,k) \cdot e^{j\Phi(\omega,k)} \end{array}\right\} \Leftrightarrow \left\{\frac{\overline{\Psi}_{1+2}}{\overline{U}_{1+2}} = \frac{\overline{\Psi}(t,x)}{\overline{U}(\omega,k)}\right\}, \\ &\overline{U}(\omega,k) = A(\omega,k)e^{-j\Phi(\omega,k)} = \int_{-\infty}^{+\infty} \overline{y}(t,x) \cdot dt = \int_{-\infty}^{+\infty} \overline{\psi}_{1+2} dt = \int_{-\infty}^{+\infty} \overline{\Psi}(t,x) \cdot e^{j(\omega t - kx)} dt \\ &\overline{\Psi}(t,x) = a(t,x)e^{-j\varphi(t,x)} = \frac{1}{\pi}\int_0^{\infty} \overline{Y}(\omega,k) \cdot d\omega = \frac{1}{\pi}\int_0^{\infty} \overline{U}_{1+2} \cdot d\omega = \frac{1}{\pi}\int_0^{\infty} \overline{U}(\omega,k) \cdot e^{j(\omega t - kx)} \cdot d\omega \ , \end{aligned} \tag{4.0.24}$$

however, going too fast towards the result, we would not be able to find the expression for a group velocity. The group velocity here sounds a little bit strange, since the amplitude modulating function is $\cos[d(\omega t-kx)]=1$, making a signal amplitude flat. We should not forget that here we started with the superposition of two infinitesimally close elementary waveforms, what was the essential condition to treat both of their amplitude functions as virtually constant (and if this was not the case some other amplitude modulating function such as $\cos[\Delta(\omega t-kx)]$ would materialize and have a much more significant influence on the signal amplitude). We have here also the proof that the superposition of only two simple waveforms (resulting in $\overline{\Psi}_{1+2}$ and $\overline{U}_{1+2}$) is sufficiently suitable for representing the original waveform when searching for group and phase velocity of the integral waveform. This is possible since the original signal phase and amplitude are not lost in this process (both in time and frequency terms). This way we are also confirming, that "**external and internal**" signal structures, in time and frequency domain, have the same wave velocities (where the original waveform is equal to the superposition result of an infinite number of such elementary waveforms).

4°

Since we already know that sub-integral elementary waveform would generate the same wave velocities as the original waveform (here we already applied the names: internal and external waveforms), we can construct another way of finding expressions for wave velocities. Let us consider that the wave function $\Psi(t)$ is the narrow frequency band wave-packet, or wave-group, which should be a wave-model of certain moving-particle. In other words, the wave-packet $\Psi(t)$ presents the (discrete or infinitesimal) superposition of numbers of mutually similar and narrow-zone concentrated elementary waves $y(t)=A(\omega)\cos[\omega t+\Phi(\omega)]$. By introducing the assumption that the frequency band of the wave-packet $\omega\in\Omega$ is very narrow, we would be in a position to treat the signal amplitude function $A(\omega)$ as approximately constant, $A(\omega)\cong A(\omega_0)\cong const.$ in the area around central frequency point $\omega_0\in\Omega$. Under given conditions, we will be able to make the following approximations,

$$\left\{\begin{aligned}&\Psi(t)=\frac{1}{\pi}\int_0^\infty[U_c(\omega)\cos\omega t+U_s(\omega)\sin\omega t]d\omega=\\&=\frac{1}{\pi}\int_0^\infty[A(\omega)\cos(\omega t+\Phi(\omega))]d\omega=a(t)\cos\varphi(t)\end{aligned}\right\}\Rightarrow$$

$$\begin{aligned}&\Psi(t)=a(t)\cos\varphi(t)=\frac{1}{\pi}\int_0^\infty[A(\omega)\cos(\omega t+\Phi(\omega))]d\omega\cong\\&\cong\frac{A(\omega_0)}{\pi}\int_{[\Omega]}[\cos(\omega t+\Phi(\omega))]d\omega\cong\\&\cong\frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos(\omega_i t+\Phi(\omega_i))=\frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i)\\&\Theta(\omega)=\omega t+\Phi(\omega)\end{aligned}\qquad(4.0.25)$$

Now, since we already know that the superposition of only two elementary waveforms would carry the information about characteristic signal velocities, we can accelerate this process if the total wave-packet has only two elementary waveforms,

$$\begin{aligned}&\Psi(t)=a(t)\cos\varphi(t)\cong\frac{A(\omega_0)}{\pi}\int_{[\Omega]}\left[\cos(\omega t+\Phi(\omega))\right]d\omega\cong\frac{\Omega}{\pi}A(\omega_0)\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i)\Rightarrow\\&\sum_{[\omega_i\in\Omega]}\cos\Theta(\omega_i)\Leftrightarrow\cos\left[\Theta(\omega_0)-\delta\Theta\right]+\cos\left[\Theta(\omega_0)+\delta\Theta\right]=\cos\left[\Theta_0-\delta\Theta\right]+\cos\left[\Theta_0+\delta\Theta\right]=\\&=2\cos\delta\Theta\cdot\cos\Theta_0\ ,\ \frac{\Omega}{\pi}A(\omega_0)=\text{const.}\end{aligned} \tag{4.0.26}$$

Let us now extend the signal to have all other space and frequency related variables,

$$\begin{aligned}&\omega t\to\omega t-kx\,;\ \omega=\omega(k);\ \Theta(\omega)=\omega t+\Phi(\omega)\to\Theta(\omega,k)=\omega t-kx+\Phi(\omega,k)\Rightarrow\\&2\cos\delta\Theta\cdot\cos\Theta_0=2\cos\left\{\delta\left[\omega t-kx+\Phi(\omega,k)\right]\right\}\cdot\cos\left[\omega_0 t-k_0x+\Phi(\omega_0,k_0)\right]=\\&=2\cos\left[k(\frac{\omega}{k}-\frac{\delta x}{\delta t})\cdot\delta t+(x-t\frac{\delta\omega}{\delta k})\cdot\delta k+(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t})\delta k\cdot\delta t\right]\cdot\cos\left[\omega_0t-k_0x+\Phi(\omega_0,k_0)\right],\end{aligned} \tag{4.0.27}$$

$$\left\{\begin{aligned}&\delta\left[\omega t-kx+\Phi(\omega,k)\right]=(\omega\cdot\delta t+t\cdot\delta\omega)+\delta\omega\cdot\delta k-(k\cdot\delta x+x\cdot\delta k)-\delta k\cdot\delta x+\delta\Phi(\omega,k)=\\&=k(\frac{\omega}{k}-\frac{\delta x}{\delta t})\cdot\delta t+(x-t\frac{\delta\omega}{\delta k})\cdot\delta\ k+(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t})\delta k\cdot\delta t;\ \Phi(\omega,k)=\text{const.}\Leftrightarrow\delta\Phi(\omega,k)=0\end{aligned}\right\}$$

The wave function $2\cos\delta\Theta\cdot\cos\Theta_0$ has the form of an amplitude-modulated elementary wave, where $\cos\Theta_0$ presents the carrier-function and $2\cos\delta\Theta$ presents its amplitude function. Both, amplitude, and wave-carrier function have different phases, and consequently, they should have different, but mutually related velocities: where the velocity of the amplitude function will be the group-velocity = **v** and velocity of the carrier function will be the phase-velocity = **u**. Now we can find the phase velocity, as usual, (when the phase of the carrier-function is constant),

$$\omega_0t-k_0x+\Phi(\omega_0,k_0)=\text{const.}\Leftrightarrow u=\frac{dx}{dt}=\frac{\omega_0}{k_0}=\frac{\omega}{k}=\text{phase velocity}\ ,\ (\omega,\ \omega_0)\in\Omega\,, \tag{4.0.28}$$

In addition, from the constant-phase of the amplitude signal-member, we will be able to find the group velocity as,

$$\begin{aligned}&k(\frac{\omega}{k}-\frac{\delta x}{\delta t})\cdot\delta t+(x-t\frac{\delta\omega}{\delta k})\cdot\delta k+(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t})\delta k\cdot\delta t=\text{Const.}\Leftrightarrow\\&\frac{\omega}{k}=\frac{\delta x}{\delta t}=\frac{dx}{dt}=u\,,\ (x-t\frac{\delta\omega}{\delta k})\cdot\delta k+(\frac{\delta\omega}{\delta k}-\frac{\delta x}{\delta t})\delta k\cdot\delta t=\text{Const.}\Rightarrow\\&\Rightarrow\frac{\delta\omega}{\delta k}=\frac{\delta x}{\delta t}=\frac{dx}{dt}=v=\text{group velocity}\end{aligned} \tag{4.0.29}$$

5°

What we can conclude and summarize from already presented steps (from 1° to 4°) regarding waveform velocities is:

a) That the most elementary (and non-dispersive) waves, which are building elements of all other waveforms we know in physics, should have forms of specific simple harmonic functions both in time and space coordinates (and being like wavelets belonging to Gaussian pulses and Gabor wavelets. See later (4.0.35) in relation to Kotelnikov-Shannon Theorem). The meaning of energy atomizing, discretization and quantization in Physics should also be closely related to here-elaborated signal analysis and synthesis in the frameworks of Analytic Signal and Kotelnikov-Shannon theorem.

b) That there is a significant level of structural symmetry and integrity regarding constructing well-operating mathematical models of wave functions in time, space, or joint time-space domains ($\Psi(t), \Psi(x), \Psi(t, x)$), and that similar symmetry is also extending to frequency-momentum domains with spectral functions ($A(\omega), A(k), A(\omega, k)$ and $\Phi(\omega), \Phi(k), \Phi(\omega, k)$). Mentioned symmetry is particularly well-exposed if Analytic Signals modeling is applied, as for instance: Every waveform element, or most elementary signal of other more complex waveforms, "**Internally and Externally**", is carrying all important information regarding waveform velocities, both in time and frequency domains, where terms “**internally and externally**” have the same meaning as already explained in earlier steps. In other words, saying the same, all elementary signals, and other time-space and frequency dependent parameters of relevance for conceptualizing different wave motions in the world of Physics, are mutually dependent, connected, proportional and well-united, as for instance: $x = x(t)$, $\omega = \omega(k)$, $v = v(x, \omega)$...

c) That the total signal energy is carried only by the signal amplitude or envelope function either in time or frequency domain, which is propagating with a group velocity.

d) That the only sufficiently narrow, elementary, “*time-space-frequency-energy*” limited and finite, (band-limited in all domains), Gaussian-Gabor signal forms are of the most significant relevance in wave motions analyses. Such building blocks of matter (and our Universe) are synthesizing or decomposing all other more complex waveforms. It seems that the highest preferences of Nature are in communicating between all relatively stable energy-carrying states using such, Gaussian band-limited elementary waves, because such wave packets are well defined in their temporal and spectral domains. Kotelnikov signals analysis or synthesis is exploiting such wave packets.

Let us now respect here summarized facts (a, b, c and d) and construct the following, elementary and band-limited wave-packet or wave group,

$$\Psi(t, x) = a(t, x)\cos\varphi(t, x) = \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}\left[A(\omega)\cos(\omega t - kx + \Phi(\omega))\right]d\omega = \tag{4.0.30}$$

$$= \frac{1}{\pi}\int_{k_0-\Delta k}^{k_0+\Delta k}\left[A(k)\cos(\omega t - kx + \Phi(k))\right]dk \,.$$

Of course, the most general form and properties of any wavefunction that is not narrow-band limited (valid also for (4.0.30)), is already given at the beginning of this chapter with (4.0.1) – (4.0.5) and T.4.0.1. In other words, elementary wave groups or wave packets (4.0.30) could present kind of the elementary signal basis of any other wave function or signal, meaning that we will be able to use such elementary signals and synthesize or decompose much more complex and even ***non-band-limited*** signals. It will be shown later (see chapter 4.3), that starting from such analytic wave functions and wave packets, (4.0.1) – (4.0.5) and (4.0.30), we will be able to establish generalized forms of Schrödinger wave equation, as complex forms of Classical wave equations (on a different and more elementary, natural, and deterministic way compared to quantum theory practices).

Another challenging situation with the narrow-band limited signal like (4.0.30) could be to connect it with single photon energy $\tilde{E}_o = hf_o$, $h = const.$, *for example,*

$$\tilde{E}_o = \int_{-\infty}^{+\infty} \Psi^2(\omega t - kx)dt = \int_{k_o-\Delta k}^{k_o+\Delta k} \left[\frac{A(\omega t - kx)}{\sqrt{2}}\right]^2 dk = \int_{\omega_o-\Delta\omega}^{\omega_o+\Delta\omega} \left[\frac{A(\omega t - kx)}{\sqrt{\pi}}\right]^2 d\omega = Const\cdot \omega_o = (2\pi\cdot Const)\cdot f_o$$

Since here we are analyzing a band-limited and finite, frequency very narrow elementary wave-packet (4.0.30), we can express the dispersion function $\omega = \omega(k)$ in the close vicinity of $\omega_0, k_0;\ \omega_0 = \omega(k_0)$ as,

$$\begin{aligned}
&\omega(k) = \omega(k_0) + (k-k_0)(\frac{d\omega}{dk})_{(k=k_0)} + \frac{1}{2}(k-k_0)^2(\frac{d^2\omega}{dk^2})_{(k=k_0)} + ... \cong \\
&\cong \omega(k_0) + (k-k_0)(\frac{d\omega}{dk})_{(k=k_0)} \ .
\end{aligned} \tag{4.0.31}$$

Now, the elementary and band-limited wave-packet can be formulated as,

$$\begin{aligned}
&\Psi(t,x) = \frac{A(k_0)}{\pi}\int_{k_0-\Delta k}^{k_0+\Delta k} \cos\left[\omega t + (k-k_0)(\frac{d\omega}{dk})_0\cdot t - kx + \Phi(k_0)\right] dk = \\
&= \frac{2A(k_0)}{\pi}\Delta k \frac{\sin \Delta k\cdot\left[(\frac{d\omega}{dk})_0\cdot t - x + \Phi(k_0)\right]}{\Delta k\cdot\left[(\frac{d\omega}{dk})_0\cdot t - x + \Phi(k_0)\right]}\cos(\omega_0 t - k_0 x) = a(t,x)\cos\varphi(t,x)\ , \\
&\frac{2A(k_0)}{\pi}\Delta k = constant,\ \Phi(k_0) = \ CONST.,\ \cos(\omega_0 t - k_0 x) = \cos\varphi(t,x).
\end{aligned} \tag{4.0.32}$$

The constant phase of the carrier wavefunction will generate the phase velocity, and the constant phase of the amplitude function will generate the group velocity (as in all the cases presented earlier),

$$\begin{aligned}
&\omega_0 t - k_0 x = const. \Rightarrow u = \frac{dx}{dt} = \frac{\omega_0}{k_0} = phase\ velocity \\
&(\frac{d\omega}{dk})_0\cdot t - x + \Phi(k_0) = Const. \Rightarrow v = \frac{dx}{dt} = (\frac{d\omega}{dk})_0 = group\ velocity.
\end{aligned} \tag{4.0.33}$$

Almost the same example in different and mutually similar variants is very often present in most of Quantum Theory books in the chapters explaining group and phase velocity. Here, it is a little bit more convincing, more general, and more evident why, and when, conclusions based on such band-limited elementary signals are correct. We can also show that any other, more complex, or arbitrary (energy-finite) waveform can be presented as the superposition of other elementary signals. In Mathematics, modern Telecommunications Theory, and Digital Signal Processing practice, we can find many methods and formulas for discrete signal representations or signals sampling. This means that time-continuous signals, or wave functions, could be adequately represented (errorless, without residuals) if we implement sufficiently short time-increments of signal sampling, and create discrete series of such signal samples. For instance, if a continuous wave-function, $\Psi(t)$ is a frequency band-limited (and at the same time it should also be an energy-finite function), by applying Kotelnikov-Shannon sampling theorem, we can express it concerning its samples-value, $\mathbf{\Psi(n\cdot\delta t)}$, as for instance (see **[8]**),

$$\begin{aligned}&\Psi(t)=a(t)\cos\varphi(t)=\sum_{n=-\infty}^{+\infty}\Psi(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}=\\&=\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}\cos\varphi(n\cdot\delta t),\ \Psi(n\cdot\delta t)=a(n\cdot\delta t)\cos\varphi(n\cdot\delta t),\end{aligned}\tag{4.0.34}$$

Where Ω is the highest frequency in the spectrum of $\Psi(t)$, and we could consider that Ω is the total frequency duration of the signal $\Psi(t)$.

Since sampling frequency-domain of signal-amplitude, Ω_L, is always in a lower frequency range than the frequency range of its phase function $\Omega=\Omega_H$, and since the total signal energy is captured only by the signal amplitude-function, we should also be able to present the same signal as:

$$\begin{aligned}&\Psi(t)=a(t)\cos\varphi(t)=\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}\cos\varphi(n\cdot\delta t)=\\&=\left[\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)}\right]\cdot\left[\sum_{n=-\infty}^{+\infty}\cos\varphi(n\cdot\delta t)\frac{\sin\Omega_H(t-n\cdot\delta t)}{\Omega_H(t-n\cdot\delta t)}\right],\\&\overline{\Psi}(t)=\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}e^{j\varphi(n\cdot\delta t)}=a(t)e^{j\varphi(t)},\end{aligned}\tag{4.0.35}$$

$$\begin{aligned}&a(t)=\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t_L)\frac{\sin\Omega_L(t-n\cdot\delta t_L)}{\Omega_L(t-n\cdot\delta t_L)},\ \cos\varphi(t)=\sum_{n=-\infty}^{+\infty}\cos\varphi(n\cdot\delta t)\frac{\sin\Omega_H(t-n\cdot\delta t)}{\Omega_H(t-n\cdot\delta t)}\\&\delta t\le\frac{\pi}{\Omega}=\frac{1}{2F}<\delta t_L\ ,\ \Omega=\Omega_H=2\pi F=2\pi F_H,\ \delta t_L\le\frac{\pi}{\Omega_L}=\frac{1}{2F_L}\ ,\ \Omega_L=2\pi F_L<\Omega.\end{aligned}\tag{4.0.36}$$

As we can see by simple comparison, the sampled waveforms under the summation signs, given by (4.0.35) and (4.0.36), have the same form as an elementary wave-packet (4.0.32), what is giving much more weight and importance to Kotelnikov-Shannon sampling theorem (regarding modeling natural waveforms). *In fact, (4.0.35)* *and (4.0.36) are also defining kind of signals atomizing, packing, and formatting, what would become important later when we start making equivalency relations between wave-packets and moving-particles (see also (4.0.38) - (4.0.44)).*

Now, by applying analogy with already analyzed examples (regarding waveform velocities), we can express the same signal, (4.0.35), as the function of extended time-space variables as,

$$\left\{\begin{array}{l} x = x(t),\ \omega = \omega(k), \\ \omega t \rightarrow \omega t - kx,\ \omega\cdot\delta t \rightarrow \omega\cdot\delta t - k\cdot\delta x, \\ \Omega t \rightarrow \Omega t - Kx,\ \Omega\cdot\delta t \rightarrow \Omega\cdot\delta t - K\cdot\delta x \\ \Omega(t - n\cdot\delta t) \rightarrow \Omega(t - n\cdot\delta t) - Kx = K\cdot\left[(\frac{\delta\omega}{\delta k})\cdot(t - n\cdot\delta t) - x\right] \\ n\cdot\delta t \rightarrow n\cdot\delta t - \frac{K}{\Omega}n\cdot\delta x,\ \frac{\delta\omega}{\delta k} = \frac{\Omega}{K},\ \Omega = 2\pi F,\ K = 2\pi F_x \\ \delta t \leq \frac{\pi}{\Omega} = \frac{1}{2F},\quad \delta f \leq \frac{1}{2T},\ \delta x \leq \frac{1}{2K} \end{array}\right\} \Rightarrow \tag{4.0.37}$$

$$\begin{aligned} \Psi(t) &= a(t)\cdot\cos\varphi(t) = \left[\sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin\Omega_L(t - n\cdot\delta t_L)}{\Omega_L(t - n\cdot\delta t_L)}\right]\cdot\cos\varphi(t) = \\ &= \left[\sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin\Omega_L(t - n\cdot\delta t_L)}{\Omega_L(t - n\cdot\delta t_L)}\right]\cdot\cos\varphi(t) = \\ &= \left[\sum_{n=-\infty}^{+\infty} a(n\cdot\delta t_L)\frac{\sin K_L\cdot\left[(\frac{\delta\omega}{\delta k})\cdot(t - n\cdot\delta t_L) - x\right]}{K_L\cdot\left[(\frac{\delta\omega}{\delta k})\cdot(t - n\cdot\delta t_L) - x\right]}\right]\cdot\cos\varphi(t)\ , \end{aligned} \tag{4.0.38}$$

$$A(f) = A(\frac{\omega}{2\pi}) = \sum_{n=-\infty}^{+\infty} A(n\cdot\delta f_L)\frac{\sin 2\pi T_L(f - n\cdot\delta f_L)}{2\pi T_L(f - n\cdot\delta f_L)}\ ,\ \delta f_L \leq \frac{1}{2T_L}. \tag{4.0.39}$$

Let us consider that there is a sufficiently high number of samples, N and M, that would adequately represent or reconstruct the same amplitude wave-functions, (4.0.38) and (4.0.39), both in time and frequency domain (to capture the same and total signal energy amount). Kotelnikov-Shannon sampling theorem combined with Parseval's theorem is giving the option to express the signal-energy as,

$$\begin{aligned} \tilde{E} &= \int_{-\infty}^{+\infty}|\Psi(t)|^2 dt = \frac{1}{\pi}\int_0^{\infty} A^2(\omega)\ d\omega = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\cdot dt = \delta t\cdot\sum_{(n)}|\Psi(n\cdot\delta t)|^2 \\ &= \frac{1}{2}\cdot\delta t_L\cdot\sum_{(n)} a^2(n\cdot\delta t_L) = 2\cdot\delta f_L\cdot\sum_{(m)} A^2(m\cdot\delta f_L) = \frac{\delta\omega_L}{\pi}\cdot\sum_{(m)} A^2(m\cdot\delta f_L), \end{aligned}$$

$$F_L < F_H \,,\; \delta t_L > \delta t_H = \delta t \,,\; n \in [1,2,3... N],\; m \in [1,2,3... M],$$
$$T_L = N \cdot \delta t_L = \frac{1}{2F_L},\; F_L = \frac{\Omega_L}{2\pi} = M \cdot \delta f_L = \frac{1}{2T_L},\; \Omega_L T_L > \pi \,,$$
$$\frac{\delta f_L}{\delta t_L} = \frac{\delta\omega_L}{2\pi \cdot \delta t_L} = \frac{N}{M} \cdot \frac{F_L}{T_L} = \frac{1}{2\pi} \cdot \frac{N}{M} \cdot \frac{\Omega_L}{T_L} = \frac{\sum_{(n)} a^2(n \cdot \delta t_L)}{8\pi \cdot \sum_{(m)} A^2(m \cdot \delta f_L)} \,, \tag{4.0.40}$$
$$\frac{1}{2} \cdot \frac{N}{M} \cdot \frac{\pi}{T_L^2} < \frac{\delta f_L}{\delta t_L} = \frac{\delta\omega_L}{2\pi \cdot \delta t_L} = \frac{N}{M} \cdot \frac{F_L}{T_L} = \frac{1}{2\pi} \cdot \frac{N}{M} \cdot \frac{\Omega_L}{T_L} < \frac{1}{2\pi} \cdot \frac{N}{M} \cdot \frac{\Omega_L^2}{\pi}.$$

Since the *amplitude-function-frequency-interval* F_L could be for orders of magnitude lower than *carrier-function-frequency-interval* $F_H = F$, it is clear that also many samples necessary to reconstruct the signal amplitude $a(t)$ could be for an order of magnitude lower than the number of samples which is reconstructing the total wave function $\Psi(t)$. Here it is also the beginning of the explanation of how particles with non-zero rest masses could be created by specific superposition or packing of elementary wave-packets.

If we are interested in exploring the signal-energy options, we can continue developing only the signal amplitude function (as an Analytic Signal function), making the next similar step,

$$\left\{\begin{array}{l} a_0(t) = a(t) = \sum_{n=-\infty}^{+\infty} a(n \cdot \delta t_L) \frac{\sin \Omega_L (t - n \cdot \delta t_L)}{\Omega_L (t - n \cdot \delta t_L)} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_0 = \tilde{E} = \frac{1}{2} \cdot \delta t_{L0} \cdot \sum_{(n)} a^2(n \cdot \delta t_{L0}) = 2 \cdot \delta f_{L0} \cdot \sum_{(m)} A^2(m \cdot \delta f_{L0}) \\ \delta t_{L0} = \delta t_L \,,\; \delta f_{L0} = \delta f_L \end{array}\right\} \Rightarrow \tag{4.0.41}$$

$$\Rightarrow \left\{\begin{array}{l} a_1(t) = \sum_{n=-\infty}^{+\infty} a_1(n \cdot \delta t_{L1}) \frac{\sin \Omega_{L1}(t - n \cdot \delta t_{L1})}{\Omega_{L1}(t - n \cdot \delta t_{L1})}, \\ a_1(t) = \sqrt{a_0^2(t) + \{H[a_0(t)]\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_1 = \frac{1}{2} \cdot \delta t_{L1} \cdot \sum_{(n)} a_1^2(n \cdot \delta t_{L1}) = 2 \cdot \delta f_{L1} \cdot \sum_{(m)} A_1^2(m \cdot \delta f_{L1}) \\ \Leftrightarrow \tilde{E}_0 = \frac{1}{2^2} \cdot \delta t_{L0} \cdot \delta t_{L1} \cdot \sum_{(n)} a_1^2(n \cdot \delta t_{L1}) = 2^2 \cdot \delta f_{L0} \cdot \delta f_{L1} \cdot \sum_{(m)} A^2(m \cdot \delta f_{L1}) \end{array}\right\} \Rightarrow \tag{4.0.42}$$

$$\Rightarrow \left\{\begin{array}{l} a_2(t) = \sum_{n=-\infty}^{+\infty} a_2(n \cdot \delta t_{L2}) \frac{\sin \Omega_{L2}(t - n \cdot \delta t_{L2})}{\Omega_{L2}(t - n \cdot \delta t_{L2})}, \\ a_2(t) = \sqrt{a_1^2(t) + \{H[a_1(t)]\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_2 = \frac{1}{2} \cdot \delta t_{L2} \cdot \sum_{(n)} a_2^2(n \cdot \delta t_{L2}) = 2 \cdot \delta f_{L2} \cdot \sum_{(m)} A_2^2(m \cdot \delta f_{L2}) \Rightarrow \\ \Leftrightarrow \tilde{E}_0 = \frac{1}{2^3} \cdot \delta t_{L0} \cdot \delta t_{L1} \cdot \delta t_{L2} \cdot \sum_{(n)} a_2^2(n \cdot \delta t_{L2}) = 2^3 \cdot \delta f_{L0} \cdot \delta f_{L1} \cdot \delta f_{L2} \cdot \sum_{(m)} A^2(m \cdot \delta f_{L2}) \end{array}\right\} \Rightarrow \tag{4.0.43}$$

…

…

$$\Rightarrow \left\{ \begin{array}{l} a_k(t) = \sum_{n=-\infty}^{+\infty} a_k(n\cdot\delta t_{Lk}) \dfrac{\sin \Omega_{Lk}(t - n\cdot\delta t_{Lk})}{\Omega_{Lk}(t - n\cdot\delta t_{Lk})}, \\ a_k(t) = \sqrt{a_{k-1}^2(t) + \left\{H\left[a_{k-1}(t)\right]\right\}^2} \Leftrightarrow \\ \Leftrightarrow \tilde{E}_k = \dfrac{1}{2}\cdot\delta t_{Lk}\cdot\sum_{(n)} a_k^2(n\cdot\delta t_{Lk}) = 2\cdot\delta f_{Lk}\cdot\sum_{(n)} A_2^2(n\cdot\delta f_{Lk}) \Rightarrow \\ \Leftrightarrow \tilde{E}_0 = \dfrac{1}{2^{k+1}}\cdot\prod_{n=0}^{k}\delta t_{Ln}\cdot\sum_{(n)} a_k^2(n\cdot\delta t_{Lk}) = 2^{k+1}\cdot\prod_{n=0}^{k}\delta f_{Ln}\cdot\sum_{(n)} A^2(n\cdot\delta f_{Lk}) \end{array} \right\}, \quad (4.0.44)$$

Until the level "k", when we would arrive at the most representative amplitude function (since this mathematical process should converge to a specific result, because initially, we assumed that we are dealing only with energy-finite functions). This process, (4.0.40) - (4.0.44), looks motivating for a creative brainstorming very much. It could be that *here we are just unveiling new signal-atomizing and formatting techniques (being most relevant in the world of Physics), and probably coming closer to the explanation why and when Planck's and de Broglie relations regarding quantized wave-packet energy and particle wave-properties are working well.*

4.0.6. Traveling Wave Packets and Waves Dispersion

Velocities of the waveforms (4.0.41) - (4.0.44) are again equal to the already known group and phase velocity (found on the same way as before). Also, we are now able to extract another property of the group velocity, which is showing that the same proportionality is conserved between signal's time, space, and frequency-durations (of course, valid only in cases of non-dispersive waves, where signal propagation velocity is frequency independent),

$$v=\frac{\delta\omega}{\delta k}=\frac{\Omega_{Lm}}{K_{Lm}}=\frac{\Omega_{Hm}}{K_{Hm}}=\frac{\delta x}{\delta t}=\frac{dx}{dt}=\frac{d\omega}{dk}\ ,\ m=0,1,2,3,... \tag{4.0.45}$$

Here is good to mention that based on familiar considerations regarding "spatial and time couplings", as eleborated in [36], from Anthony D. Osborne and N. Vivian Pope (Light-Speed, Gravitation, and Quantum Instantaneity), we should be able to recreate and update almost complete Relativity Theory (avoiding using original A. Einstein's concepts).

For the group and phase velocities (as found in all previously shown examples), it seems essential that a waveform time-domain function $\Psi(t)$ should be fully compliant and extendable to an equivalent waveform in a time-space domain $\Psi(t,x)$. Moreover, the new *time-space waveform,* $\Psi(t,x)$*, should represent a non-dispersive traveling wave (meaning that signal propagation velocities are not frequency dependent).* As we have seen, there are straightforward rules for such waveform transformations (based on a much more general symmetry of space and time variables, which is in the background here), that can be described as:

a) *First, an arbitrary waveform* $\Psi(t,x)$ *should be presented as a superposition of elementary waves, where simple harmonic functions are involved, such as* $\cos\omega t$*,* $\sin\omega t$*,* $\frac{\sin\Omega t}{\Omega t}\cos(\omega t)$ *,* $e^{-\beta t}\cdot\frac{\sin\Omega t}{\Omega t}\cos(\omega t)$, (eventually being similar to Gaussian pulses and Gabor wavelets)*…*

b) *Time and frequency variables which are arguments of simple-harmonic functions can be extended, for instance,*

$$x=x(t),\ \omega=\omega(k),$$
$$\omega t\rightarrow\omega t-kx,\ \omega\cdot\delta t\rightarrow\omega\cdot\delta t-k\cdot\delta x,$$
$$\Omega t\rightarrow\Omega t-Kx,\ \Omega\cdot\delta t\rightarrow\Omega\cdot\delta t-K\cdot\delta x\ ,$$

alternatively, *(4.0.46)*

$$x=x(t),\ \omega=\omega(k),$$
$$kx\rightarrow kx-\omega t,\ k\cdot\delta x\rightarrow k\cdot\delta x-\omega\cdot\delta t,$$
$$Kx\rightarrow Kx-\Omega t,\ K\cdot\delta x\rightarrow K\cdot\delta x-\Omega\cdot\delta t\ .$$

In (1.46) all minus "-" signs could be replaced by plus "+" signs, changing only a direction of propagation of a waveform in question. Furthermore, in some cases, we *could combine inwards and outwards signals propagation, without influencing already established conclusions regarding signal integrity and signal velocities. Such simple rules can be well explained and supported by analyzing many elementary wave phenomena known in Physics, where only simple harmonic waveforms and oscillations are involved. Here, we are just generalizing or applying mentioned rules to any other arbitrary waveform, which presents non-dispersive traveling wave. Effectively, matter-waves and oscillations related to the world of micro-physics, electrons, atoms, and photons… are in most cases presentable as energy-finite functions, composed of simple harmonic and elementary waveforms or wavelets (applicable also in cases of macro-particles motions, which are effectively composed of similar simple harmonic waveforms).*

In fact, instead of extending $\Psi(t) \to \Psi(t,x)$*, we could start from* $\Psi(t,x) \to \Psi(t,0)$ *and go to* $\Psi(t), x = \text{const.}$ *(and then deductively extract steps mentioned above), but for the analysis presented here it was mathematically simpler to start from* $\Psi(t)$*). There are many different strategies (in available literature) regarding waves dispersion, considering many possible situations. Here, we are limiting our framework to items directly applicable to basic particle-wave phenomenology among elementary particles, micro-particles, photons, and de Broglie matter waves (such as different interactions, diffractions and interferences related to elementary-particles, to Compton and Photoelectric Effect, to particles annihilation, or creation, etc.). In all such examples, it looks that Nature is showing or respecting quite simple interaction rules between wave-packets and particles (where group velocity has its significant place).*

General case of ***Dispersion Relation*** $\omega = \omega(k)$*, valid for any wave motion, will be addressed later (see (4.0.80) in this chapter) as,*

$$\omega(k) = 2\pi f = ku = \frac{kv}{1+\sqrt{1-v^2/c^2}} = \frac{2\pi}{h}\frac{pv}{1+\sqrt{1-v^2/c^2}} =$$
$$= \frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \cdot \frac{\hbar}{m}k^2 \cong \begin{Bmatrix} \frac{\hbar}{2m}k^2, \text{for } v \cong \frac{\hbar}{m}k = 2u << c, u \cong \frac{\hbar}{2m}k = \frac{v}{2} << c \\ ck, \quad \text{for } v \approx u \approx c \end{Bmatrix}, \tag{4.0.80-1}$$

Consequences concerning (4.0.80-1) are in some ways very much challenging (regarding the relativistic and quantum-mechanical understanding of mass, momentum, and wave propagation vector), for instance,

$$m(v) = \frac{m}{\sqrt{1-v^2/c^2}} \cong \begin{Bmatrix} m, \quad \text{for } v << c \\ \frac{\hbar k}{c}\sqrt{1-v^2/c^2} = \frac{p}{c}\sqrt{1-v^2/c^2} = \frac{mv}{c} \cong m, \text{ for } v \approx c \end{Bmatrix},$$
$$p(v) = \frac{mv}{\sqrt{1-v^2/c^2}} \cong \begin{Bmatrix} mv, \quad \text{for } v << c \\ \frac{\hbar k}{c}v\sqrt{1-v^2/c^2} = \frac{pv}{c}\sqrt{1-v^2/c^2} = \frac{mv^2}{c} \cong mv \cong mc, \text{ for } v \approx c \end{Bmatrix}, \tag{4.0.80-2}$$
$$k(v) = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p = \frac{2\pi}{h}\frac{mv}{\sqrt{1-v^2/c^2}} \cong \begin{Bmatrix} \frac{2\pi}{h}mv, \quad \text{for } v << c \\ \frac{k}{c}v\sqrt{1-v^2/c^2} = \frac{2\pi}{h}\frac{pv}{c}\sqrt{1-v^2/c^2} = \frac{2\pi}{h}\frac{mv^2}{c} \cong \frac{2\pi}{h}mv \cong \frac{2\pi}{h}mc, \text{ for } v \approx c \end{Bmatrix}.$$

4.0.7. Wave Packets already exercised in Physics

From many different experimental observations (Davisson-Germer, G.P. Thomson, Photo Effect, and Compton Effect…), we are faced with the reality that all microparticles, (regardless of origin) exhibit diffraction and interference effects, and have de Broglie wavelength $\lambda = h/\tilde{p}$, where $\tilde{p} = \sqrt{2mE} = \gamma\, mv$, $h = 6.626176 \cdot 10^{-34} J \cdot s$. *Also, the same value for Planck's constant* h *is also found applicable for mathematical curve fitting of black body radiation formula, for explaining Compton and Photoelectric effects, for quantifying energy-momentum relations regarding photons, and for processing almost any other kind of sub-atomic, micro, and elementary particles. This looks like introducing some energy atomization, discretization, or quantization in the form of elementary wave packets, wave groups, or energy quanta (*as in here-elaborated signal analysis and synthesis in the framework of Analytic Signal and Kotelnikov-Shannon theorem*). This has been the reason to exercise the concept where specific wave group or wave packet could replace moving particle (of course only regarding few of mentioned aspects).*

As we will see, the wave-packet or wave-group is modeled to have its group wave-velocity that should be equal to certain equivalent-particle velocity. Also, it will be shown that the wave-packet energy should be equal to the equivalent-particle kinetic energy. For instance, an electromagnetic quant of energy, or photon, behaving as a wave-packet, is experimentally showing waves or particles properties (in different experimental situations). In Physics is widely demonstrated applicability and compatibility of Planck's wave-packet energy-expression as $\tilde{\mathbf{E}} = \mathbf{hf}$ (originally found and applied only for photons, and later extended to other microparticles), with Energy and Momentum conservation laws, as well as with de Broglie matter-waves wavelength, $\lambda = h/\tilde{p}$ (where $\tilde{m} = hf/c^2$, $\tilde{p} = hf/c$, $u = \lambda f$), without precisely and completely showing what really makes those relations correct. Mentioned mathematics was working well in explaining many experimental situations, and this has been the justification of Planck's energy expression, and de Broglie's wavelength). <u>Similar wave (or motional) energy relation $\tilde{E} = Hf$ is even applicable to planetary or solar systems, but the constant "H" is no more equal to already known Planck constant "h" (see the second chapter: 2.3.3. Macro-Cosmological Matter-Waves and Gravitation). Moreover, similar relations (with specific H constant) are also applicable to vortex flow meters, related to fluid flow vortices, as speculated in chapter 4.1; -see equations (4.3-0), (4.3-0)-a,b,c,d...</u>

The answer to a simple question how and why only one characteristic (central) frequency (multiplied by Planck's constant h, or by another H constant) can represent the motional wave-energy of a narrow-band wave-group (or what means that frequency, and what kind of wave group is behind) should be found. The first intuitive and logical starting point could be to imagine that this is just the mean frequency, $\bar{f}$, of the corresponding narrow-band, elementary matter-wave-group calculated in relation to its energy (where a wave group, or wave packet, or de Broglie matter-wave is composed of infinite number of elementary waves, covering certain (relatively small) frequency interval: $\mathbf{0} \leq \mathbf{f}_{min.} \leq \bar{\mathbf{f}} \leq \mathbf{f}_{max.} < \infty$). The energy of such wave-group (established by connecting Parseval's and Planck's energy forms) is,

$$\tilde{E} = \int_{-\infty}^{+\infty} \Psi^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\,dt = \frac{1}{\pi}\int_0^{\infty} A^2(\omega)\,d\omega = h\overline{f} = h\frac{u}{\lambda} = pu. \tag{4.0.47}$$

Now (by definition) we can find the mean frequency of such wave-packet as,

$$\overline{f} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\frac{1}{2}\int_{-\infty}^{+\infty} f(t)\cdot a^2(t)\cdot dt}{\frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\cdot dt} = \frac{\frac{1}{2}\int_{-\infty}^{+\infty} f(t)\cdot a^2(t)\cdot dt}{\tilde{E}}, \tag{4.0.48}$$

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = \dot{\varphi}(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \mathrm{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right] = 2\pi f(t).$$

Moreover, we can replace it into the wave energy expression, (4.0.47),

$$\tilde{E} = \frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega = h\overline{f} = h\frac{\frac{1}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega}{\tilde{E}} \Rightarrow \tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2 d\omega. \tag{4.0.49}$$

Using one of the most general formulas valid for all definite integrals (and applying it to (4.0.49)), we can prove that the wave energy (of a wave-group in question) should be equal to the product between Planck's constant (h or H) and mean frequency of the wave group in question, as follows:

$$\left\{\begin{array}{l} \int_a^b f(x)\cdot g(x)dx = f(c)\int_a^b g(x)dx,\ a<c<b,\ g(x)\geq 0, \\ f(x) \text{ and } g(x) - \text{continuous in } [a\leq x\leq b], \\ f(x)=f,\ \ g(x)=[A(\omega)]^2>0,\ x=\omega\in(0,\ \infty) \end{array}\right\} \Rightarrow$$

$$\tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f\cdot[A(\omega)]^2\,d\omega = \frac{h}{\pi}\cdot\overline{f}\cdot\int_0^{\infty}[A(\omega)]^2\,d\omega = hf\cdot\tilde{E} = h\overline{f}\cdot\tilde{E} \Rightarrow \tag{4.0.50}$$

$$\Rightarrow \tilde{E} = h\overline{f} = hf,\ \Delta\tilde{E} = h\cdot\Delta f,\ f = \overline{f}.$$

If Planck's energy of a photon or any other equivalent narrow-band wave-group deals with a mean frequency of that wave-group, the same should be valid for de Broglie wavelength, as well as for its phase and group velocities. Consequently, all of them could be treated as mean values describing the motion of an "effective center of inertia, or center of gravity" of that wave-group). We do not need specifically to mark them as mean-values, as it was the case with mean-frequency in (4.0.50) since we know that all of them should anyway be mean values $\overline{\mathbf{f}} = \mathbf{f},\ \overline{\boldsymbol{\lambda}} = \boldsymbol{\lambda},\ \overline{\mathbf{u}} = \mathbf{u},\ \overline{\mathbf{v}} = \mathbf{v}$), and applicable only to narrow frequency band signals (where $\lambda = h/p,\ \tilde{E} = hf,$ $v = u - \lambda du/d\lambda = dE/dp = d\omega/dk,\ u = \lambda f = \tilde{E}/p = \omega/k\ ...$). Also, for sufficiently narrow-banded signals, mean-values of a group and phase velocity should also be presentable as:

$$\bar{v}_g = \bar{v} = \frac{\frac{1}{\pi}\int_0^\infty v\cdot[A(\omega)]^2\,d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2\,d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2\,d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2\,d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2\,d\omega}{\tilde{E}} = \frac{\delta\omega}{\delta k}, \quad (4.0.51)$$

$$\bar{v}_f = \bar{u} = \frac{\frac{1}{\pi}\int_0^\infty u\cdot[A(\omega)]^2\,d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2\,d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2\,d\omega}{\frac{1}{\pi}\int_0^\infty [A(\omega)]^2\,d\omega} = \frac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2\,d\omega}{\tilde{E}} = \frac{\omega_0}{k_0}. \quad (4.0.52)$$

From the expressions for mean-group and mean-phase velocity (in cases of narrowband waveforms, when we can approximate the signal frequency-domain amplitude as a constant) we should also be able to find Planck's expression for the energy of an elementary wave-group, $E = hf = \frac{h}{2\pi}\omega$, as follows,

$$\left\{\begin{array}{l} \bar{v}_g = \bar{v} = \dfrac{\frac{1}{\pi}\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2\,d\omega}{\tilde{E}} = \dfrac{\delta\omega}{\delta k} \Rightarrow \tilde{E} = \dfrac{1}{\pi}\int_0^\infty \dfrac{\delta k}{\delta\omega}\cdot\dfrac{d\omega}{dk}\cdot[A(\omega)]^2\,d\omega \Rightarrow \\ \Rightarrow d\tilde{E} \cong \dfrac{[A(\omega)]^2}{\pi}d\omega = 2[A(\omega)]^2\,df \end{array}\right\} \vee$$

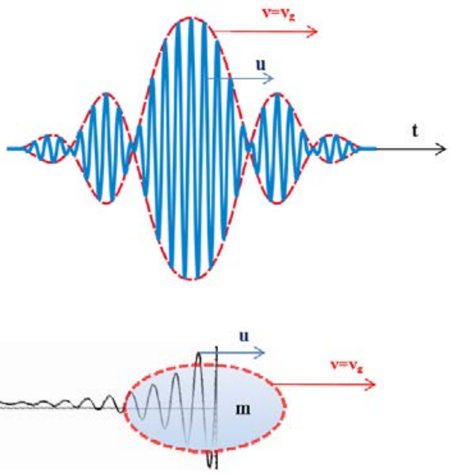
$$\vee\left\{\begin{array}{l} \bar{v}_f = \bar{u} = \dfrac{\frac{1}{\pi}\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2\,d\omega}{\tilde{E}} = \dfrac{\omega_0}{k_0} \Rightarrow \tilde{E} = \dfrac{1}{\pi}\int_0^\infty \dfrac{k_0}{\omega_0}\cdot\dfrac{\omega}{k}\cdot[A(\omega)]^2\,d\omega \\ \Rightarrow d\tilde{E} \cong \dfrac{[A(\omega)]^2}{\pi}d\omega = 2[A(\omega)]^2\,df \end{array}\right\} \Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} \delta\tilde{E} = 2[A(\omega_0)]^2\cdot\delta f \\ \tilde{E} = 2[A(\omega_0)]^2\cdot f = hf \\ A(\omega) \cong A(\omega_0) \cong Const. \\ 2[A(\omega_0)]^2 = h = \text{Planck constant} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \Delta\tilde{E} = h\cdot\Delta f\ ,\ \delta\tilde{E} = h\cdot\delta f \\ \dfrac{\Delta\tilde{E}}{\Delta f} = \dfrac{\tilde{E}}{f} = h, \dfrac{\Delta\tilde{E}}{\tilde{E}} = \dfrac{\Delta f}{f} \end{array}\right\}. \quad (4.0.53)$$

Of course, since parts of conclusions (here) are based on “narrow-band” approximations, we can only get a good feeling regarding the background conditions that are supporting Planck’s expression for an elementary quant of wave-energy. *Briefly summarizing, we can say that Planck's energy-formula should be well applicable only to narrowband elementary wave-packet forms.* What that means in all its aspects, when extended to wideband and complex, spatial and temporal waveforms, and how it is fully connected to all other moving macroparticles with non-zero rest masses remain to be found. Luckily, we already know that Planck's energy-quant (or energy-packet concept), united with de Broglie wavelength and relativistic particle-energy expressions is perfectly applicable in explaining the number of experiments (such as Compton and Photoelectric effect etc. See also Chapter 10. “PARTICLES AND SELF-CLOSED STANDING MATTER WAVES”.). *Presently, we clearly see that Nature is using its methods of waveforms sampling, analysis, and synthesis, applying them equally to waves and particles, experiencing or moving specific energy content from one packing-format to another (conveniently combining infinitesimal and discrete signal processing approach). This is already an excellent starting point for further analyses (see later, in the same chapter (4.0.73) - (4.0.76)).*

Most probably that time and frequency domain of wave functions of elementary quanta of energy (like photons and other elementary matter wave packets and particles) belong to a family of finite-energy Gaussian pulses and Gabor wavelets. Such waveforms guarantee the best possible time-frequency resolution. This way, the same wave-packet or signal can be well defined and localized both in its time and frequency domain).

Of course, formation of more complex objects or wave groups (or macroparticles), which cannot be related to narrow frequency banded cases, should be presentable as a convenient integration or superposition of relevant narrow-banded elementary wave groups, where (4.0.50) is applicable (see the oversimplified picture below to get an idea about particle and wave group equivalency). One of the highly likely and natural mathematical frameworks for such signals-integration (or superposition) can be found in applying concepts based on Analytic Signal and Kotelnikov-Shannon theorem.



Moving particle and equivalent wave-group

If we agree that an energy quantum of an electromagnetic wave-packet or photon is a narrow-band signal, with limited time and frequency durations ($\Delta t = T,\ \Delta f = F = \frac{\Delta\omega}{2\pi}$), it will be,

$$\tilde{E} = \int_{[\Delta t=T]} \Psi^2(t)\,dt = \frac{1}{2}\int_{[\Delta t=T]} a^2(t)\,dt \cong \frac{1}{2}\bar{a}^2 \cdot \Delta t =$$

$$= \frac{1}{\pi}\int_{[\Delta\omega=2\pi F]} A^2(\omega)\,d\omega \cong \frac{1}{\pi}\bar{A}^2 \cdot \Delta\omega = 2\bar{A}^2 \cdot \Delta f = h\bar{f} \quad \Rightarrow$$

$$\left\{\begin{array}{l} d\tilde{E} = \frac{1}{2}\overline{a}^2 \cdot dt = \frac{1}{\pi}\overline{A}^2 \cdot d\omega = 2\overline{A}^2 \cdot df = hdf = \frac{h}{2\pi}d\omega, \overline{a}^2 \cdot T \cong 4\overline{A}^2 \cdot F, \\ \Delta t = T,\ \Delta f = F = \frac{\Delta\omega}{2\pi},\ h = 2\overline{A}^2\frac{\Delta f}{\overline{f}} = \frac{\overline{a}^2 \cdot T}{2\overline{f}} = \text{Constant}, \end{array}\right\} \Rightarrow$$

$$\Rightarrow \Delta\tilde{E} \cong h\Delta f = \frac{h}{2\pi}\Delta\omega,\ h\overline{f} = 2\overline{A}^2 F = \frac{\overline{a}^2 \cdot T}{2} = \tilde{E}.$$

For such time and frequency limited durations signals, where $\Delta t = T$ and $\Delta f = F = \frac{\Delta\omega}{2\pi}$ are its total temporal and frequency durations, it should be valid $TF \cong \frac{1}{2}$, and we will have, $\Delta\tilde{E} \cong hF \cong \frac{h}{2T} \Rightarrow \Delta\tilde{E} \cdot \Delta t \cong \frac{h}{2}$.

From Signal and Spectral Analysis we know that the generally valid case is, $TF > \frac{1}{2}$, producing that $\Delta\tilde{E} \cdot \Delta t \geq \frac{h}{2}$. Now we could say that Planck's energy-quantum formula $\tilde{E} = hf$ is on certain way additionally described and proven valid. In other words, Nature is communicating with such finite energy portions.

Of course, here we could safely say that **h** should be a constant number (what is the well-known case). By intellectual inertia and following analogy from what we know in Quantum Theory, we know that **h** is Planck's constant, and we experience that this is only valid and applicable for atoms, photons, and micro-world of Physics. We would need additional criteria to find numerical values of such **h** constants that are applicable for different matter-waves situations, and for macro world of Physics, such as Solar Systems (since $h = 2\overline{A}^2\frac{\Delta f}{\overline{f}} = \frac{\overline{a}^2 \cdot T}{2\overline{f}}$ = Constant).

....

Citation from [79]: "Other essential and significant examples include the Gabor transform, and the wavelet transform both of which transform a signal f(t) in the time-frequency domain (t–w plane). In other words, these new transforms convey essential information about the nature and structure of a signal in the time-frequency domain simultaneously. In 1946, Dennis Gabor, a Hungarian-British physicist, an engineer and a 1971 Nobel Prize winner in physics, introduced the *windowed Fourier transform* (or the *Gabor transform)* of a signal f(t) concerning a window function g, denoted by $\tilde{f}_g(t,\omega)$ and defined by

$$G[f](t,\omega) = \tilde{f}_g(t,\omega) = \int_{-\infty}^{+\infty} f(\tau) \cdot g(\tau - t)e^{-i\omega t}d\tau = \left\langle f, \overline{g}_{t,\omega} \right\rangle, \quad (1.2.6)$$

Where f and $g \in L^2$ (R) with the inner product $\langle f, g \rangle$. "

Gabor (1900-1979) first recognized the significant weaknesses of the Fourier transform analysis of signals and realized the great importance of localized time and frequency concentrations in signal processing. All these motivated him to formulate a fundamental method of the Gabor transform for decomposition of the signal regarding elementary signals (or wave transforms). Gabor's pioneering approach has now become one of the standard models for time-frequency signal analysis. It is also important to point out that the Gabor transform $\tilde{f}_g(t,\omega)$ is referred to as the canonical coherent state representation of f in quantum mechanics. In the 1960s, the term "coherent states" was first used in quantum optics".

*[♣ **COMMENTS & FREE-THINKING CORNER:***

We could additionally test the Planck's radiation law, regarding photon energy $\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega$*. It is well proven that a photon has the wave energy equal to the product between Planck's constant* $\mathbf{h}$ *and photon's frequency* $\mathbf{f}$*. Photon is also a wave phenomenon, and it should be presentable using a certain time-domain wave function* $\Psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$*, expressed in the form of an Analytic Signal. Since the Analytic Signal presentation gives the chance to extract instantaneous signal amplitude* $\mathbf{a}(\mathbf{t})$*, phase* $\varphi(\mathbf{t})$*, and frequency* $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial \mathbf{t}} = 2\pi\mathbf{f}(\mathbf{t})$*, let us extend and test the meaning of Planck's energy when: instead of constant photon frequency* $\mathbf{f}$ *(valid for a single photon), we take its wave-group, mean wave-frequency,* $2\pi\overline{f} = \overline{\omega}$*, of the time-domain photon wave-function* $\Psi(\mathbf{t})$*. Since we already know that the photon is a "relatively concentrated wave-group", its time-variable frequency-function would also be very narrow band-limited (and could easily be replaced by photon's mean frequency value). However, just for mathematically exercising such an opportunity, we will proceed with this idea, and maybe find some additional conditions applicable to all physics related wave-groups, as follows,*

$$d\tilde{E} = \Psi^2(t)dt = \hat{\Psi}^2(t)dt = \left|\overline{\Psi}(t)\right|^2 dt = \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = [\frac{A(\omega)}{\sqrt{\pi}}]^2 d\omega =$$

$$= P(t)dt = hdf(t) = \frac{h}{2\pi} d\left[\frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)}\right], \ h = \text{const.},$$

$$\tilde{E} = \int_{[T]} \Psi^2(t)dt = \int_{[T]} \left[a(t)\cos\varphi(t)\right]^2 dt = \int_{[T]} a^2(t)dt,$$

$$P(t) = \frac{d\tilde{E}}{dt} = \Psi^2(t) \ (\Leftrightarrow) \left[\frac{a(t)}{\sqrt{2}}\right]^2 (=) \left[W\right] \ , t \in (-\infty, +\infty) \ ,$$

$$P(\omega) = \frac{d\tilde{E}}{d\omega} = \frac{\Psi^2(t)}{d\omega/dt} = \left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 (=) \left[Js = Ws^2\right], \ \omega \in (0, +\infty),$$

$$\Psi(t) = a(t)\cos\varphi(t) = -H\left[\hat{\Psi}(t)\right] \ , \ \hat{\Psi}(t) = a(t)\sin\varphi(t) = H\left[\Psi(t)\right],$$

$$a(t) = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)} \ , \ a^2(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{\overline{\omega}}, \ \varphi(t) = \arctan\frac{\hat{\Psi}(t)}{\Psi(t)},$$

$$f(t) = \frac{\omega(t)}{2\pi} = \frac{1}{2\pi}\frac{\partial\varphi(t)}{\partial t} = \frac{1}{2\pi}\dot{\varphi}(t) = \frac{1}{2\pi}\frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)} = \frac{1}{2\pi}\text{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right].$$

$$\tilde{E} = hf = \frac{h}{2\pi}\omega(t) \ (\Leftrightarrow) \ \tilde{E} = h\overline{f} = \frac{h}{2\pi}\overline{\omega},$$

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = 2\pi f(t) \ (\Leftrightarrow) \ \overline{\omega}(t) = \left\langle\frac{\partial\varphi(t)}{\partial t}\right\rangle = 2\pi\left\langle f(t)\right\rangle = 2\pi\overline{f} = \overline{\omega},$$

$$\overline{\omega} = \frac{1}{T}\int_{[T]} \omega(t)dt \ (\Leftrightarrow) \ \frac{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt}{\int_{[T]} a^2(t)\cdot dt} \ (=) \ \tilde{E}\frac{2\pi}{h} \Rightarrow$$

$$\Rightarrow \frac{\tilde{E}}{\overline{\omega}} = \frac{\left[\int_{[T]} a^2(t)\cdot dt\right]^2}{\frac{1}{T}\int_{[T]} \omega(t)\cdot a^2(t)\cdot dt} = \frac{\int_{[T]} a^2(t)\cdot dt}{\frac{1}{T}\int_{[T]} \omega(t)dt} = \frac{a^2(t)\int_{-\infty}^{+\infty} a^2(t)dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)\right]} = \frac{h}{2\pi} = \text{Const.} \quad (4.0.54)$$

More about photon wave function can be found in the Annex at the end of this book: ***WAVE FUNCTION OF THE BLACK BODY RADIATION AND A PHOTON****.*

Depending on how we calculate the mean frequency, we should be able to prove at least one of the above-given relations (see the last line). Also, we should be able to find the family of wave functions that are describing photon or any other elementary wave-packet in a time domain. In any case, we should be able to see how universal Planck's energy law could be applicable regarding the energy of arbitrary wave functions. See more about a similar problem in the following article from Dr. Juluis S. Bendat: ***THE HILBERT TRANSFORM AND APPLICATIONS TO CORRELATION MEASUREMENTS.***

What is interesting in the analysis of Dr. Bendat and others is that a group or phase velocity is directly proportional to the square root of signal frequency $\mathbf{v} = \mathbf{v}_g \approx \sqrt{\mathbf{f}}$. This agrees with Planck's elementary-quanta radiation-energy expression $\tilde{\mathbf{E}} = \mathbf{hf}$ since we already know that any motional or kinetic energy is proportional to the square of the group velocity, and it should be directly proportional to the dominant wave-packet frequency $\mathbf{E}_k = \tilde{\mathbf{E}} \approx \mathbf{v}^2 = \mathbf{v}_g^2 \approx \mathbf{f}$. It should be very natural and deterministic, mathematical explanation regarding Planck's elementary-quantum energy, and here we are approaching well to it. ♣]

4.0.8 Uncertainty Relations and Waveform Velocities

In Physics, the Uncertainty Principle is usually linked to Heisenberg's Uncertainty Relations. For real, correct, and full understanding of many relations that are linked to the Uncertainty Principles, it would be, for the time being, better to forget that Heisenberg made any invention regarding Uncertainty. In other words, we should know (or learn) that Uncertainty is not married almost exclusively with Quantum Theory and with Heisenberg. It is also the current case that Uncertainty, as presented in contemporary Physics (mostly in Quantum Mechanics), is sporadically (and unintentionally) applied as a useful supporting background for some misinterpretations, and for justifications of some conceptual, and methodological Uncertainties in Physics (author’s comment). For better foundations and explanations regarding mathematical and physics related Uncertainty relations or inequalities (what should be the same, united concept or theory), much more space will be devoted in chapter 5. of this book. Here, we will address only generally valid and surrounding grounds of Uncertainty relations that should be universally valid in signal analysis and physics.

A more general approach to Uncertainty relations (in connection with the quantum manifestations of energy-moments formats) should start from the specific finite wave function $\Psi(t)$. Here we are treating the square of the wave function as a power, (4.0.4): $\Psi^2(\mathbf{t}) = \mathbf{Power} = \mathbf{dE}/\mathbf{dt}$, but we could also treat $\Psi(t)$ as a dimensionless function (after proper normalization) without influencing the results of the analysis that follows.

It is an advantage of Analytical Signals that they cover only natural domains of real-time and frequency: $-\infty < \mathbf{t} < \infty, \mathbf{0} \leq \mathbf{f} < \infty$, opposite to the traditional Signal Analysis (Fourier Analysis), where frequency can also take negative values. In many other aspects, Analytic Signals are giving equivalent results, as in the case of Fourier Signal Analysis, including producing additional, time-frequency dependent, dynamic and spectral signal properties, what Fourier analysis is not able to deliver.

The mutual relations between time and frequency signal-domain-segments and their isolated points of a specific waveform $\Psi(t)$, are not suitable for 1:1 (one-to-one) mapping or imaging. Moreover, they can be mutually related, and precisely localized on both sides, only within certain approximating relations, given by the following Uncertainty relations (or domains-mapping restrictions),

$$0<\delta t\cdot\delta f<\frac{1}{2}\leq F\cdot T\leq\frac{1}{4\cdot\delta t\cdot\delta f},(\delta t\leq\frac{1}{2F})<<T,\ (\delta f\leq\frac{1}{2T})<<F,$$
$$0<\delta t\cdot\delta\omega=2\pi\cdot\delta t\cdot\delta f<\pi\leq\Omega\cdot T\leq\frac{\pi}{2\cdot\delta t\cdot\delta f}=\frac{\pi^2}{\delta t\cdot\delta\omega}, \tag{4.0.55}$$

where $\mathbf{T}$ is the total time duration of the signal, $F=\Omega/2\pi$ is its total frequency-duration, δt is the maximal time-sampling interval of the signal, and $\delta f=\delta\omega/2\pi$ is the maximal frequency-sampling interval (all of them compliant to Nyquist and Shannon-Kotelnikov signal sampling theorems; $t\in[\mathbf{T}]$, $0\leq\mathbf{T}<\infty$, $-\infty<\mathbf{t}<\infty$, and $\mathbf{f}\in[\mathbf{F}]$, $0\leq\mathbf{F}<\infty$, $0\leq\mathbf{f}<\infty$, $2\pi F=\Omega, 2\pi f=\omega$). Of course, only mathematically judging, here we know that absolute time and frequency signal durations (total signal lengths) cannot be precisely found in both time and frequency domains. Only in the frames of capturing the dominant part of the signal energy in both domains (for instance we request to have minimum 99 % of total signal energy in both domains), we can speak about absolute and total signal durations. Also, if a relevant wave or signal functions are presentable like Gaussian-Gabor (bell-curve shaped) signal forms (both in time and frequency domains), we will be able to apply more accurately, uncertainty relations and signal localization. The other fact known from Physics is that Nature (or our universe) is always presenting a kind of attenuating, filtering and modifying medias for all signals and waves propagation. Thus, even mathematically ideal, unlimited, or infinite spectral durations would be naturally transformed into finite signal-lengths (simply low energy and remarkably high frequency spectral components of the signal would be absorbed, dissipated, or filtered by the media where a signal propagates). Concerning such kind of background, we can use the terms of finite signal durations in all its domains. The next crucial mathematical foundation compatible to operations with limited and finite spectral or domain lengths (in time, space, and all frequency-related domains) can be found in signal analysis and synthesis (or in signal sampling rules) defined by Kotelnikov and Shannon's theorem. This will be again addressed later.

We know that Nature (or everything what is in motion, and what belongs to Physics) also respects mentioned domains mapping and imaging restrictions. In Physics, such mathematical relations are "domesticated" as Uncertainty Relations and widely applied for different explanations of measured data. Physics related literature made in certain cases negative contributions by popularizing, over-simplifying, mystifying and incompletely explaining what Uncertainty Relations should mean (instead, firstly clarifying all mathematical aspects of domains mapping and imaging restrictions, and then starting with a bottom-line, and simple physics related conceptual explanations). Since we already know that Nature (and Mathematics) respects Uncertainty Relations (maybe not exactly as we presently describe such Uncertainty in micro-Physics), we can safely say that Nature also respects our time and frequency signal modeling techniques (Fourier and Hilbert transform, Nyquist sampling rules, Analytic Signal concept, Kotelnikov-Shannon theorem, etc.). Consequently, we can say that whatever we see as a motional form in our Universe is composed of elementary simple harmonic signal-forms and can be decomposed on such elementary waveforms.

It will be shown later that even stable and virtually non-moving forms (particles with non-zero rest masses) were in their past (during creation) assembled by a convenient superposition of specific elementary waveforms.

In such motional particles-related situations, it seems that Nature has conveniently transformed all "*Signal Uncertainties*", applicable to equivalent wave-groups (recognizable by the symbol $\leq$**,** as found in (4.0.55)), into stabilized "*Waveform Certainties*" (characterized by symbol =). This is indicatively describing formation of stable elementary particles and atoms (which are also stable in states of rest, but effectively being objects where internal standing-waves structures are involved as building elements). In all other dynamic cases, where something is moving, Uncertainty relations (4.0.55) are applicable in their original mathematical forms. If we compare the mathematical Uncertainty relations (4.0.55), with similar Uncertainty relations found in Orthodox Quantum Theory, we will notice differences between them. For instance, in the contemporary Quantum Theory signal durations, are captured differently than here, taking relevant and shorter, statistical-deviations intervals. This produces different quantitative uncertainty relations. Instead of $\mathbf{F \cdot T \geq 1/2}$ in Quantum Theory, we often find $\mathbf{F \cdot T} \geq 1$, but both of such uncertainty relations would become mutually identical if the same signal-intervals were taken into consideration. The same problem sometimes gets much more complicated in contemporary Quantum Theory, when other assumptions and statistics and probability related criteria are applied.

Since energy-finite signals or waves in real physics always propagate in certain space, during a certain time, similar uncertainty relations should exist considering space-related parameters (length along the axis of propagation $\mathbf{x}$, and spatial-periodicity $\mathbf{k} = \mathbf{2\pi} \cdot \mathbf{f_x}$). Just for making analogies, the ordinary meaning of frequency in a time-domain could also be considered as a time-related-frequency $\omega = 2\pi \cdot f_t = 2\pi \cdot f$, and $\mathbf{k} = \mathbf{2\pi} \cdot \mathbf{f_x}$ would be a space-related frequency (see better explanation in Chapter 10.). By analogy with time-frequency-domain Uncertainty Relations (4.0.55), to speed up this process of explanations, we could create similar (but extended) uncertainty relations, where time is analogically replaced by corresponding *signal propagation length,* or *signal spatial duration*, $\mathbf{t \leftrightarrow x, T \leftrightarrow L}$. Moreover, the time-related frequency is replaced by space-related frequency, $\omega \leftrightarrow k, \Omega \leftrightarrow K$, where $\mathbf{L}$ is the total signal length, or total signal, spatial duration, and $\mathbf{K = 2\pi F_x}$ is the total signal spatial frequency interval. Since here we mathematically observe the same waveform $\Psi(t, x)$ in different domains, it is evident that relevant Uncertainty Relations in time and space domains should be mutually analogical, united, and coupled. Thus, such relations describe a stable proportionality between all relevant signal-domain intervals (making that such waveform would also be relatively stable, compact, progressive, and non-dispersive), and in this way, we create an extended Uncertainty Relations chain, such as,

$$0 < \delta t \cdot \delta\omega = 2\pi \cdot \delta t \cdot \delta f = \delta x \cdot \delta k = 2\pi \cdot \delta x \cdot \delta f_x < \pi \leq \Omega \cdot T = K \cdot L\,,$$
$$0 < \delta t \cdot \delta f = \delta x \cdot \frac{\delta k}{2\pi} = \delta x \cdot \delta f_x < \frac{1}{2} \Rightarrow \bar{v} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} \Leftrightarrow \left\{\frac{dx}{dt} = \frac{d\omega}{dk}\right\}. \tag{4.0.56}$$

We can also see that the average velocity $\bar{\mathbf{v}}$ associated to extended Uncertainty relations (4.0.56), at least dimensionally corresponds to the signal's $\Psi(t, x)$ group velocity (and that such group velocity is a measure of proportionality between corresponding original and spectral domains durations). Later we will see that the velocity found here really has strong grounds in Physics (meaning that *as long the signal or an equivalent particle (represented by such signal) is in waving or motion, they will have the same group and phase velocity, and relevant wave energy would correspond to the particle kinetic energy*).

Here, to speed up the conclusion making process, we merely applied analogies to create extended uncertainty relations (4.0.56), but in Physics, such relations are an experimentally known and mathematically well-supported fact. By creating δ -differences, it is evident that we here consider elementary sampling intervals that are minimal steps defined by Nyquist-Kotelnikov-Shannon signal-sampling rules. By comparing such signal-intervals with total signal-durations in time and frequency domains, we find that δ-differences present minimal signal atomizing steps. By multiplying the last form of Uncertainty Relations (4.0.56) with Planck's constant, to get an elementary Planck's energy of a wave packet, and implementing de Broglie wavelength, we can formulate the following Uncertainty Relations,

$$\left\{\begin{array}{l} 0 < h \cdot \delta t \cdot \delta f = h \cdot \delta x \cdot \frac{\delta k}{2\pi} = h \cdot \delta x \cdot \delta f_x < \frac{h}{2} < h \cdot F \cdot T = h \cdot F_x \cdot L = h \cdot \frac{K}{2\pi} \cdot L \ , \\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p \ , \ \lambda = \frac{h}{p} \ , \ h = const. \ , \ \tilde{E} = hf \end{array}\right\} \Rightarrow \quad (4.0.57)$$

$$\Rightarrow \boxed{0 < \delta t \cdot \delta \tilde{E} = \delta x \cdot \delta p < \frac{h}{2} < \tilde{E} \cdot T = P \cdot L}$$

In early-days of Quantum Theory when mathematical background of Uncertainty was dominant (and no more in the contemporary Quantum Theory), absolute or total signal durations in all domains (effectively real signal durations), $\mathbf{F}, \mathbf{T}, \mathbf{F_x}, \mathbf{L}, \mathbf{K}$ *were considered as* Δ *-intervals, where,* $F = \Delta f$, $\tilde{E} = h \cdot F = h \cdot \Delta f$, $T = \Delta t$, $L = \Delta x$, $\frac{h}{2\pi} \cdot K = \Delta p \cdot$ *Such early-days Quantum Theory (or QT) Uncertainty Relations initially are known as,* $\Delta\tilde{E} \cdot \Delta t = \Delta p \cdot \Delta x > \frac{h}{2}$, *what is entirely identical to (4.0.57),* $\tilde{\mathbf{E}} \cdot \mathbf{T} = \mathbf{P} \cdot \mathbf{L} > \frac{\mathbf{h}}{\mathbf{2}}$. *Later, in Quantum Theory literature, such absolute or total signal durations are transformed (philosophically, stochastically, and conceptually) into signal standard deviations (since the group of QT founders mutually agreed that everything in the micro world should be quantifiable and predictable only by probability functions and waves). The other (right) side of the same inequality has also been changed from* $\frac{\mathbf{h}}{\mathbf{2}}$ *to* $\mathbf{h}$ *(what is possible when considering different signal duration intervals, as statistical standard deviations). This way, renewed, probabilistic Uncertainty Relations got their latest quantum-mechanical form (becoming less generally valid),*

$$\Delta\tilde{E} \cdot \Delta t = \Delta p \cdot \Delta x \geq h \ . \quad (4.0.58)$$

In most of contemporary Quantum Theory presentations regarding Uncertainty Relations, and everywhere else in the physics literature, there is an overwhelming tendency of using (in as many situations as possible) the constant $\hbar = \frac{\mathbf{h}}{\mathbf{2\pi}}$ *instead of*

$\mathbf{h}$. *This is in some cases maybe not entirely mathematically defendable, but it is somehow "stochastically thinking" and approximately acceptable, being anyway in the same order of magnitude (and this is anyway experimentally not easy to verify). Here we will try to avoid such approach, remaining less quantum-theory-fashionable.*

In this book, we will keep as relevant to the original, mathematically most robust, and entirely defendable form of Uncertainty (or inequality) Relations, where absolute or total signal durations are explicitly involved, such as,

$$0 < \delta t \cdot \delta \tilde{E} = \delta x \cdot \delta p < \frac{h}{2} < h \cdot F \cdot T = \frac{h}{2\pi} \cdot K \cdot L = \tilde{E} \cdot T = P \cdot L \ . \qquad (4.0.59)$$

Here (in (4.0.59)), $\tilde{\mathbf{E}} = \mathbf{hF}$ *could be a total wave energy of the signal, and* $\mathbf{P} = \frac{\mathbf{h}}{2\pi}\mathbf{K}$ *is its total linear moment (analogically concluding). Of course, later, in cases when conditions for statistical conceptualization would be satisfied, the same inequality or Uncertainty Relations can easily be extended and transformed into statistical standard deviations inequality relations (as presently practiced in Quantum Theory).*

What we can see is that the Physics related formulation of Uncertainty Relations is also related to the Planck's wave-packet energy ($\tilde{\mathbf{E}} = \mathbf{hF}$). As we know from earlier wave-packet velocity analyses, such energy relation is well applicable only to "narrow-banded" waveforms. We also know that such wave-packet concept can be applied as the motional-particle model in two aspects: *when the average group velocity of the wave-packet corresponds to the particle (center of mass) velocity, and when energy of the wave-packet is equal to the kinetic energy of the equivalent-particle,* because in both cases we consider only motional energy. Also, de Broglie wavelength, easily understandable for wave-packets and photons (which do not have rest masses), fits well and very much analogically into the moving particle-equivalent, or particle matter-wave concept (where non-zero rest mass exists). Here are the essential grounds of particle-wave duality in Physics: Motional energy of any origin, belonging to any energy carrying entity, has wave properties defined by de Broglie-Planck-Einstein matter-wave mathematical formulas (see much more in Chapter 10.).

It is interesting to find the same Uncertainty Relations (4.0.56) if we would like to analyze a sudden signal duration change for an arbitrary, Δ-signal interval. To address such a question let us start again from total signal durations in its time and frequency domains,

$$T \cdot F = L \cdot F_x > \frac{1}{2},\ T \cdot \Omega = L \cdot K > \pi. \qquad (4.0.60)$$

If a particular transformation, as in (4.0.61), happens to a wave function $\Psi(t)$, changing its time and frequency lengths, $\mathbf{T}$ and $\mathbf{F}$, for the (mutually dependent or mutually coupled) amounts Δt and Δf, such signal transformation will automatically influence all other space, time and energy-related parameters of $\Psi(x, t)$ to change, producing similar Uncertainty relations, as already known. Effective physical signal spatial length $\mathbf{L}$ and total signal wave-energy $\tilde{\mathbf{E}}$ would also change, for instance:

$$\left\{\begin{array}{l} T \to T \pm \Delta t > 0, \quad F \to F \pm \Delta f > 0, \quad L \to L \pm \Delta x > 0 \quad , \quad K \to K \pm \Delta k > 0 \\ \Delta\tilde{E} = h\,\Delta f = \overline{v}\Delta p = F\Delta x \quad , \quad F = \dfrac{\Delta p}{\Delta t} = \text{force} \quad , \\ 0 < \delta t \cdot \delta f = \delta x \cdot \dfrac{\delta k}{2\pi} = \delta x \cdot \delta f_x < \dfrac{1}{2} < F \cdot T = \dfrac{1}{2\pi} \cdot K \cdot L = F_x \cdot L \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \tilde{E} \to \tilde{E} \pm \Delta\tilde{E}, \\ \overline{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\delta x}{\delta t} = \dfrac{\delta\omega}{\delta k} = \dfrac{\delta\tilde{E}}{\delta p} \end{array}\right\}. \tag{4.0.61}$$

In cases when time and frequency changes in (4.0.61) would be either positive or negative, we will have:

$$T \cdot F > \frac{1}{2} \Leftrightarrow \left\{\begin{array}{l} (T+\Delta t)\cdot(F+\Delta f) > \dfrac{1}{2} \\ (T-\Delta t)\cdot(F-\Delta f) > \dfrac{1}{2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \left[T^2-(\Delta t)^2\right]\cdot\left[F^2-(\Delta f)^2\right] > \dfrac{1}{4} \\ T\cdot F + \Delta t \cdot \Delta f > \dfrac{1}{2} \\ T\cdot\Delta f + \Delta t \cdot F > 0 \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} 1-(\dfrac{\Delta t}{T})^2-(\dfrac{\Delta f}{F})^2+(\dfrac{\Delta t}{T})^2\cdot(\dfrac{\Delta f}{F})^2 > \dfrac{1}{4} \\ 1+(\dfrac{\Delta t}{T})\cdot(\dfrac{\Delta f}{F}) > \dfrac{1}{2T\cdot F}, T\cdot F > \dfrac{1}{2} \\ (\dfrac{\Delta t}{T})+(\dfrac{\Delta f}{F}) > 0 \end{array}\right\}. \tag{4.0.62}$$

By analogy (just to shorten this process, and avoid lengthy introductions), let us replace the signal total time duration **T** with the signal total spatial length **L**, and signal total frequency duration $\mathbf{F = F_t = \Omega/2\pi}$ by signal total spatial frequency duration $\mathbf{F_x = K/2\pi}$, and unite results for time and spatial lengths cases,

$$L \cdot F_x > \frac{1}{2} \Leftrightarrow \left\{\begin{array}{l} (L+\Delta x)\cdot(F_x+\Delta f_x) > \dfrac{1}{2} \\ (L-\Delta x)\cdot(F_x-\Delta f_x) > \dfrac{1}{2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \left[L^2-(\Delta x)^2\right]\cdot\left[F_x^2-(\Delta f_x)^2\right] > \dfrac{1}{4} \\ L\cdot F_x + \Delta x \cdot \Delta f_x > \dfrac{1}{2} \\ L\cdot\Delta f_x + \Delta x \cdot F_x > 0 \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} 1-(\dfrac{\Delta x}{L})^2-(\dfrac{\Delta f_x}{F_x})^2+(\dfrac{\Delta x}{L})^2\cdot(\dfrac{\Delta f_x}{F_x})^2 > \dfrac{1}{4} \\ 1+(\dfrac{\Delta x}{L})\cdot(\dfrac{\Delta f_x}{F_x}) > \dfrac{1}{2L\cdot F_x}, L\cdot F_x > \dfrac{1}{2} \\ (\dfrac{\Delta x}{L})+(\dfrac{\Delta f_x}{F_x}) > 0 \end{array}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} 1-(\frac{\Delta t}{T})^2-(\frac{\Delta f}{F})^2+(\frac{\Delta t}{T})^2\cdot(\frac{\Delta f}{F})^2=1-(\frac{\Delta x}{L})^2-(\frac{\Delta f_x}{F_x})^2+(\frac{\Delta x}{L})^2\cdot(\frac{\Delta f_x}{F_x})^2>\frac{1}{4} \\ 1+(\frac{\Delta t}{T})\cdot(\frac{\Delta f}{F})=1+(\frac{\Delta x}{L})\cdot(\frac{\Delta f_x}{F_x})>\frac{1}{2T\cdot F}=\frac{1}{2L\cdot F_x},\ T\cdot F=L\cdot F_x>\frac{1}{2} \\ (\frac{\Delta t}{T})+(\frac{\Delta f}{F})=(\frac{\Delta x}{L})+(\frac{\Delta f_x}{F_x})>0 \end{array}\right\} \quad (4.0.63)$$

The real waveforms-related domains-uncertainties are given by the set of inequalities in (4.0.63). Now we can extend the *uncertainty relations chain* for a couple more members, which implicitly include all possible Δ -variations found in (4.0.63),

$$\begin{array}{l} 0<\delta t\cdot\delta f=\delta x\cdot\delta f_x<\frac{1}{2}<F\cdot T=F_x\cdot L\leq\frac{1}{4\cdot\delta t\cdot\delta f}=\frac{1}{4\cdot\delta x\cdot\delta f_x}, \\ 0<\delta t\cdot\delta\tilde{E}=\delta x\cdot\delta p<\frac{h}{2}<\tilde{E}\cdot T=P\cdot L\leq\frac{h}{4\cdot\delta t\cdot\delta f}=\frac{h}{4\cdot\delta x\cdot\delta f_x}. \end{array} \quad (4.0.64)$$

The signal durations (or absolute domain lengths) can be changed only for an integer number of (Nyquist) sampling intervals δt, δx, δf and δf_x. To keep the non-dispersive signal integrity and stable mutual-proportionality of selected total signal-durations in all domains, the average group-velocity found for small sampling intervals should also be equal to the average group-velocity, when much larger space, time and frequency intervals are involved,

$$\begin{array}{l} \overline{v}=\frac{\Delta x}{\Delta t}=\frac{\delta x}{\delta t}=\frac{n\cdot\delta x}{n\cdot\delta t}\ (=\frac{\delta f}{\delta f_x}=\frac{n\cdot\delta f}{n\cdot\delta f_x}=\frac{\delta\omega}{\delta k}=\frac{\Delta\omega}{\Delta k}=\frac{dx}{dt}=\frac{d\omega}{dk}), \\ \Delta x=n\cdot\delta x,\ \Delta t=n\cdot\delta t,\ n=1,2,3,... \end{array} \quad (4.0.65)$$

The most interesting here is that an average group velocity, serving as domains proportionality measure, is not found like any other velocity, and is presently related only to entire signal lengths. Here, could be a part of the answer why light or electromagnetic waves velocity in open space and vacuum, $c=1/\sqrt{\varepsilon_0\mu_0}$ *, has a significant and unique place in Physics (like in (4.0.66)). This is probably the case, because such waveforms (or photons) while propagating in an open space, are manifesting as almost continuous motional waveforms. Their space and time domains are permanently shifted for certain constant amount, expressed by Uncertainty relations, producing almost constant group wave-speed. Whatever we try to do to change the speed of light, the same proportionality would re-appear and again stabilize the photons' group velocity. Once after we succeed to destroy the group-velocity balance between different signal domains, the signal will disappear (being dispersed, absorbed, or dissipated). For instance, continuing the same brainstorming thinking, we could say that since the light speed is the highest signals speed presently known in Physics, the following limitation should be valid,*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{n \cdot \delta x}{n \cdot \delta t} = \frac{\delta f}{\delta f_x} = \frac{n \cdot \delta f}{n \cdot \delta f_x} = \frac{\delta \omega}{\delta k} = \frac{\Delta \omega}{\Delta k} = \frac{\Omega_L}{K_L} = \frac{\Omega_H}{K_H} \leq c \ , \qquad (4.0.66)$$

what could be a beneficial relationship to address the mutual coupling and proportionality limits of absolute signal durations in their space, time, and frequency domains, additionally. Even if some velocity dependent changes of characteristic signal lengths and intervals would happen, without destroying signal integrity (like in case of Lorentz transformations), the average signal group velocity (or domain lengths proportionality) would be conserved. This looks like the recognition that space and time dimensions (signal length or distance **x**, *and signal time duration* **t**) *are mutually connected or convertible by a specific constant which has velocity dimension (* $\Delta x = \bar{v} \cdot \Delta t, \ \bar{v} = const.$*). Of course, there is an analog situation for signal durations in corresponding spectral domains (*$\Delta \omega = \bar{v} \cdot \Delta k, \ \bar{v} = const.$*). It would be interesting to read books and publications from Anthony D. Osborne, & N. Vivian Pope,* ***[36],*** *where, in some ways, the familiar concept about space-time coupling and proportionality is presented (on a somewhat simpler level).*

Uncertainty Relations and similar concepts in Physics and Mathematics should be equally valid or applicable to a micro and macro universe, being completely united, as well as appropriately connected with mathematical or statistical errors' analysis, and with natural mathematical relations of finite-differences, when we describe and localize specific particle and its equivalent matter-wave group, in all relevant domains. The most important, dominant, always valid, and universally applicable, are only mathematical uncertainty relations (not Quantum Theory Uncertainty or Heisenberg relations). Mathematical Uncertainty relations are presenting inequality relations between absolute, original, and spectral (or total) signal domains durations, and nothing else. Later, mentioned mathematical relations could be conveniently applied on relevant Quantum Theory concepts, and on some probabilistic way fit Quantum Mechanical Uncertainty to comply with mathematical Uncertainty (but not vice versa). Of course, the message here is that the same uncertainty or inequality relations should connect mutually conjugate, original, and spectral domains of a specific signal or wave function, both in mathematics and in physics (and everywhere else in a micro and macro universe, when applicable). Until present, Quantum Theory realized such objective only on the platform of Statistics, and only for the world of microphysics, but this should not be the only and final option as vocally presented in the contemporary Quantum Theory publications (see more, later in this chapter, and in Chapter 5. of this book).

Citation: https://phys.org/news/2014-12-quantum-physics-complicated.html#jCp "............. An international team of researchers has proved that two peculiar features of the quantum world previously considered distinct are different manifestations of the same thing. The result is published 19 December in *Nature Communications*. Patrick Coles, Jedrzej Kaniewski, and Stephanie Wehner made the breakthrough while at the Centre for Quantum Technologies at the National University of Singapore. They found that 'wave-particle duality' is simply the quantum 'uncertainty principle' in disguise, reducing two mysteries to one.

"The connection between uncertainty and wave-particle duality comes out very naturally when you consider them as questions about what information you can gain about a system. Our result highlights the power of thinking about physics from the perspective of information," says Wehner, who is now an Associate Professor at QuTech at the Delft University of Technology in the Netherlands. ....

Wave-particle duality is the idea that a quantum object can behave like a wave, but that the wave behavior disappears if you try to locate the object. It is most directly seen in a double slit experiment, where single particles, electrons, say, are fired one by one at a screen containing two narrow slits. The particles pile up behind the slits not in two heaps as classical objects would, but in a stripy pattern like you would expect for waves interfering. At least, this is what happens until you sneak a look at which slit a particle goes through - do that, and the interference pattern vanishes.

The quantum uncertainty principle is the idea that it is impossible to know specific pairs of things about a quantum particle at once. For example, the more precisely you know the position of an atom, the less precisely you can know the speed with which it is moving. It is a limit on the fundamental knowability of nature, not a statement on measurement skill. The new work shows that how much you can learn about the wave versus the particle behavior of a system is constrained in precisely the same way.

Wave-particle duality and uncertainty have been fundamental concepts in quantum physics since the early 1900s. ....

It is possible to write equations that capture how much can be learned about pairs of properties that are affected by the uncertainty principle. Coles, Kaniewski, and Wehner are experts in the form of such equations known as 'entropic uncertainty relations', and they discovered that all the maths previously used to describe wave-particle duality could be reformulated concerning these relations. ....

In earlier papers, Wehner and collaborators found connections between the uncertainty principle and other physics, namely quantum 'non-locality' and the second law of thermodynamics”.

4.0.9. Uncertainty Relations in Quantum Theory

Understanding Uncertainty relations in physics, (presently still on mathematical level) is also related to our choice of signal duration intervals. Until here we have used (or talked about) real, absolute, or total signal interval lengths. Now we will once more extend already established Uncertainty Relations of absolute signal duration intervals, taking into consideration corresponding signal standard deviations intervals.

Since the Orthodox Quantum Mechanics mostly deals with statistical distributions and probabilities, interval lengths are represented by signal variance intervals, which are statistical or standard deviations of certain variables around their mean values. Consequently, mathematical expressions of basic Uncertainty Relations, when using variance intervals or statistical deviations, present another aspect of Uncertainty Relations (not mentioned before, but very much present in today's Quantum Mechanics literature). This should be appropriately integrated into a chain of all other, already known Uncertainty Relations. The statistical concept of variance is used to measure the signal's energy spreading in time and frequency domains. For instance, for a finite wave function (4.0.1), we can define the following variances (see **[7]**, pages: 29-37, and **[8]**, pages: 273-277):

$$(\sigma_t)^2 = \Delta^2 t = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}(t-\langle t\rangle)^2\left|\overline{\Psi}(t)\right|^2 dt = \int_{-\infty}^{+\infty} t^2 \frac{\left|\overline{\Psi}(t)\right|^2}{\tilde{E}} dt - \langle t\rangle^2 < T^2,$$
$$(\sigma_\omega)^2 = \Delta^2\omega = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}(\omega-\langle\omega\rangle)^2\left|A(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{+\infty}\omega^2\frac{\left|A(\omega)\right|^2}{\tilde{E}}d\omega - \langle\omega\rangle^2 < (2\pi F)^2, \qquad (4.0.67)$$
$$\omega = 2\pi f,\ \sigma_\omega = 2\pi\sigma_f,\ \tilde{E} = \left\|\overline{\Psi}(t)\right\|^2 = \int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\int_{0}^{+\infty}\left|A(\omega)\right|^2 d\omega,$$

Where meantime and mean frequency should be found as:

$$\langle t\rangle = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} t\left|\overline{\Psi}(t)\right|^2 dt,\ \langle\omega\rangle = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}\omega\left|A(\omega)\right|^2 d\omega = 2\pi\langle f\rangle = 2\pi\overline{f}\ . \qquad (4.0.68)$$

If two functions, $\Psi(t)$ and $A(\omega)$, form a Fourier-integral pair, then they cannot both be of short duration. The scaling theorem supports this,

$$\Psi(at) \leftrightarrow \frac{1}{|a|}A(\frac{\omega}{a}), \qquad (4.0.69)$$

*where **"a"** is a real constant. The above assertion, (4.0.69), also known as the **Uncertainty Principle**, can be given various interpretations, depending on the meaning of the term "duration".*

Using the time and frequency variances, (4.0.67), as the significant signal duration intervals, found for a finite wave function $\Psi(t)$, it is possible to prove the validity of the following Uncertainty Principle (see **[7]** and **[8]**):

If $\sqrt{t}\Psi(t) \to 0$ for $|t| \to \infty)$,

then $2\pi TF > TF > \sigma_t \sigma_\omega = 2\pi\sigma_t\sigma_f = \sqrt{(\Delta^2 t)(\Delta^2\omega)} = 2\pi\sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{2}$. (4.0.70)

In the variance relations (4.0.70) we consider as apparent that absolute (or total) time and frequency durations, **T** and **F,** can never be shorter than time and frequency variances, $\boldsymbol{\sigma_t}$ and $\boldsymbol{\sigma_f}$ (and usually, they should be much larger than σ_t and σ_f). It is also clear that statistical, (4.0.70), and Quantum Mechanic's aspect of Uncertainty should be fully integrated with absolute interval values Uncertainty Relations, (4.0.64), for instance,

$$0 < \delta t \cdot \delta f = \delta x \cdot \delta f_x < \frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L \leq \frac{1}{4 \cdot \delta t \cdot \delta f} = \frac{1}{4 \cdot \delta x \cdot \delta f_x},$$

$$0 < \delta t \cdot \delta\tilde{E} = \delta x \cdot \delta p < \frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L \leq \frac{h}{4 \cdot \delta t \cdot \delta f} = \frac{h}{4 \cdot \delta x \cdot \delta f_x},$$

alternatively, in cases when normal, Gauss distributions are applicable it should also be valid,

$$\frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < 36 \cdot \sigma_t \cdot \sigma_f = 36 \cdot \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L,$$

$$\frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < 36 \cdot 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = 36 \cdot \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L. \quad (4.0.71)$$

To maintain the same signal domains proportionality (regarding non-dispersive wave packets and stable particles), the average group velocity (as concluded before for absolute interval lengths, (4.0.65) - (4.0.66)) should also depend (in the same way) on all relevant signal lengths expressed as standard deviations,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} = \frac{\Delta\omega}{\Delta k} = \frac{\Omega_L}{K_L} = \frac{\Omega_H}{K_H} = \frac{dx}{dt} = \frac{d\omega}{dk} \cong \frac{\sigma_x}{\sigma_t} = \frac{\sigma_\omega}{\sigma_k} = \frac{\sigma_f}{\sigma_{f-x}}. \quad (4.0.72)$$

If an overall signal domains proportionality has not been maintained stable, we would have a mess or disorder in particle-waves duality situations, and stable particles would not be a part of our universe (see (4.0.73) and (4.0.74)).

Here, as the supporting fact to (4.0.72), is a very convenient place to mention again the well-known and extraordinary expression for electromagnetic waves or photons speed, which is given only concerning static materials and space properties (or as a function of electric and magnetic permeability). Such relation is showing that dynamic time domain parameters of certain wave-packet (here photon or electromagnetic waves) are perfectly united and synchronized with its space and spectral domain parameters,

$$c = \frac{1}{\sqrt{\mu\varepsilon}}. \quad (4.0.72\text{-}1)$$

Generalizing the same concept about space-time unity and compactness of finite moving entities (as matter-waves and particles), we become familiar with the idea that even stable and solid particles (without visible waving properties) are presenting specially packed stationary states of relevant (at least) 4-dimensional signals, or wave functions (see more in Chapter 10). Maximal velocity c can be considered here as a natural boundary factor (or space-time proportionality factor), which secures relevant space-time signal compactness, integrity, and stability. *See familiar ides in **[36]**, Anthony D. Osborne, & N. Vivian Pope.*

4.0.10. Wave Group Velocities, Moving Particles and Uncertainty Relations

Let us additionally explore the same idea (about sudden signal duration changes) using the moving particle Energy-Momentum 4-vector from the Minkowski-space of Relativity Theory, by (mathematically) introducing mutually coupled variations of a total system energy and total momentum, applying "discrete, central differentiations" method (based on symmetrical, central differences),

$$\overline{P}_4 = \overline{P}\left[\vec{p} = \gamma m\vec{v}, \frac{E}{c} = \gamma mc\right], \overline{P}^2 = \vec{p}^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, E_0 = mc^2, E = \gamma E_0 \Rightarrow \tag{4.0.73}$$

$$\begin{aligned}
&\vec{p}^2c^2 + E_0^2 = E^2, (p \to p \pm \Delta p) \Leftrightarrow (E \to E \pm \Delta E) \Rightarrow \\
&\left\{\begin{matrix}(p+\Delta p)^2c^2 + E_0^2 = (E+\Delta E)^2 \\ (p-\Delta p)^2c^2 + E_0^2 = (E-\Delta E)^2 \\ \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} = \frac{\Delta\omega}{\Delta k}\end{matrix}\right\} \Leftrightarrow \left\{c^2 \cdot p\,\Delta p = E\,\Delta E \Leftrightarrow \frac{\Delta E}{\Delta p} = c^2\frac{p}{E} = \overline{v}\right\} \Rightarrow \\
&\Rightarrow \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\Delta E}{\Delta p} = c^2\frac{p}{E} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\delta\omega}{\delta k} \le c.
\end{aligned} \tag{4.0.74}$$

What we can see from (4.0.74) is that sudden energy-momentum changes in a specific motion (moving particle, here) are causally related to its average center-of-mass velocity, which is at the same time equal to the system average group velocity, as shown in (4.0.72). ***Here, we could make an intuitive conclusion that Minkowski 4-vectors and Analytic Signal concept are mutually related.*** *See more in Chapter 10. This now gives the new and more tangible meaning of Uncertainty Relations, which should be well analyzed, before we make other conclusions.*

*Before, in (4.0.72), we also found that the signal group velocity should be dimensional and quantitative, signal domains proportionality factor, and now we can additionally reconfirm (with (4.0.74)) that this is indeed the case (see also **[36]**).*

The idea here is to show that so-called *Uncertainty Relations are causally related to a velocity of matter-waves propagation. The meaning of that is that at the same time when specific motional object or signal would experience a sudden change of its energy-momentum related parameters, matter waves are automatically created. This produces results captured by Uncertainty Relations (indirectly saying that there is not any real Uncertainty in its old and traditional meaning, and apparently, here we are not dealing with probabilities and standard deviations.* Also, we can express the average group velocity $\overline{\mathbf{v}}$ associated with the transformations (4.0.74) as:

$$\begin{aligned}
&\left\{\begin{matrix} u = \frac{\omega}{k}, \omega = ku \\ (k \to k \pm \Delta k/2) \Leftrightarrow (u \to u \pm \Delta u/2)\end{matrix}\right\} \Rightarrow \Delta\omega = (k + \frac{1}{2}\Delta k)(u + \frac{1}{2}\Delta u) - (k - \frac{1}{2}\Delta k)(u - \frac{1}{2}\Delta u) = \\
&= k\Delta u + u\Delta k \Leftrightarrow \overline{v} = \frac{\Delta\omega}{\Delta k} = u + k\frac{\Delta u}{\Delta k} = \left(\frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\Delta E}{\Delta p} = c^2\frac{p}{E} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\delta\omega}{\delta k}\right) \Leftrightarrow \\
&\Leftrightarrow \left\{v = u + k\frac{du}{dk} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = \frac{dx}{dt} = \text{immediate group velocity}\right\}.
\end{aligned} \tag{4.0.75}$$

Such group velocity (obtained using central, symmetrical, finite differences) is fully analog to its differential form where infinitesimal signal changes are involved. By merging average group velocity with Uncertainty Relations, we again see that they are mutually compatible,

$$\left\{\begin{array}{l} \overline{v} = u + k\frac{\Delta u}{\Delta k} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta x}{\Delta t} = \text{average group velocity} \\ \textit{and} \\ \left|\Delta x\ \Delta p\right| = \left|\Delta t\ \Delta\tilde{E}\right| = h\left|\Delta t\ \Delta f\right| > h/2,\ \Delta\tilde{E} = h\Delta f\ , \\ 0 < \delta t\cdot\delta f = \delta\ x\cdot\delta f_x < \frac{1}{2} \le F\cdot T = F_x\cdot L \le \frac{1}{4\cdot\delta t\cdot\delta f} = \frac{1}{4\cdot\delta x\cdot\delta f_x} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = u + k\frac{\Delta u}{\Delta k} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} = \frac{dx}{dt} = \frac{d\omega}{dk}. \qquad (4.0.76)$$

An interesting fact regarding average group velocity "$\overline{\mathbf{v}}$" in (4.0.75) and (4.0.76), which could pass unnoticed, is that the full analogy between "Δ-form" for an average group velocity "$\overline{\mathbf{v}}$" and differential or infinitesimal "d-form" for immediate group velocity "v" is not made as an approximation, automatically by formal and simple (mutual) replacing of infinitesimal difference "d" with central, discrete delta-difference "Δ". The development of the average group velocity (as given here) is applying finite <u>symmetrical central differences</u> on basic definitions of group and phase velocity and this way we arrive to entirely correct results without the real need for infinitesimal assumptions like $\Delta \to d\,(\to 0)$. Such methodology must be entirely correct (judging by results), showing that there is a deterministic (as well as analogical) connection between the physics of continuum, and physics of discrete or finite steps (like signals sampling, analysis, and synthesis, based on Nyquist-Kotelnikov-Shannon theorems and concepts). It can be shown that in many equations relevant for mathematical physics it is possible to replace infinitesimal, differential analyses methods with operations based on using central differences. To support better such statements, it would be necessary to devote a time to learn about the properties of central, symmetrical differences. ***We could also say that laws of physics, including equations, and concepts where we have high level of mutual similarity, parallelism, and the same consequences or results when operating with finite Δ -intervals, or with infinitesimal d-intervals, are much stronger, universal and naturally stable, than if infinitesimal d-intervals are not being mathematically (or formally) identical or proportional to similar Δ -relations.***

*[♣ **COMMENTS & FREE-THINKING CORNER:** Here is the place to address another important aspect of United symmetries and analogies between different (mutually coupled) conservation laws of physics, originating from the concept of Minkowski-space 4-vectors used in Relativity Theory. For instance (4.0.73) presents some space-time unification of energy and momentum conservation laws, making that the following expression is always invariant regarding different reference frames which are mutually in a uniform relative motion,*

$$\vec{p}_1^2 - \frac{E_1^2}{c^2} = \vec{p}_2^2 - \frac{E_2^2}{c^2} = ... = \vec{p}_n^2 - \frac{E_n^2}{c^2} = \text{invariant}$$

Also, if the relativistic space-time interval (as presently formulated in Relativity Theory) is on the same way reference-frame invariant, it will be:

$$(\Delta r)_1^2 - c^2(\Delta t)_1^2 = (\Delta r)_2^2 - c^2(\Delta t)_2^2 = ... = (\Delta r)_n^2 - c^2(\Delta t)_n^2 = \text{invariant}$$

$$-\left[1 - \frac{1}{c^2}\frac{(\Delta r)_1^2}{(\Delta t)_1^2}\right]c^2(\Delta t)_1^2 = -\left[1 - \frac{v_1^2}{c^2}\right]c^2(\Delta t)_1^2 = ... = -\left[1 - \frac{v_n^2}{c^2}\right]c^2(\Delta t)_n^2 = \text{invariant}.$$

What we can see from here presented invariant expressions, is that only couples which are relevant for (average) group velocity, are involved in such invariant expressions and that analogically we could formulate new invariant expressions such as,

$$(\Delta\omega)_1^2 - c^2(\Delta k)_1^2 = (\Delta\omega)_2^2 - c^2(\Delta k)_2^2 = ... = (\Delta\omega)_n^2 - c^2(\Delta k)_n^2 = \text{invariant},$$

$$-\left[1 - \frac{v_1^2}{c^2}\right]c^2(\Delta k)_1^2 = ... = -\left[1 - \frac{v_n^2}{c^2}\right]c^2(\Delta k)_n^2 = \text{invariant},$$

$$(\Delta\theta)_1^2 - \omega_c^2(\Delta t)_1^2 = (\Delta\theta)_2^2 - \omega_c^2(\Delta t)_2^2 = ... = (\Delta\theta)_n^2 - \omega_c^2(\Delta t)_n^2 = \text{invariant}, \ \omega = d\theta/dt$$

$$\mathbf{L}_1^2 - \frac{E_1^2}{\omega_c^2} = \mathbf{L}_2^2 - \frac{E_2^2}{\omega_c^2} = ... = \mathbf{L}_n^2 - \frac{E_n^2}{\omega_c^2} = \text{invariant}, \ \omega_c = \text{Const.},$$

where only absolute signal durations are involved.

On the same way, analogically, it should be possible to formulate many new invariant relations (of course still hypothetically valid until being proven from a more general platform). ♣]

Now is also possible to prove the validity of the following expressions for group and phase velocity,

$$\left\{\begin{array}{l} v = \frac{dE}{dp} = \frac{dE_k}{dp} = \frac{d}{dp}\left[\gamma mc^2\right] = \frac{d}{dp}\left[(\gamma-1)mc^2\right] = \frac{d}{dp}\left[\frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}}\right], \\ u = \frac{E_k}{p} = \lambda \cdot f, \quad v = u + p\frac{du}{dp}, \quad p = \gamma mv, \quad \lambda = \frac{h}{p}, \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \end{array}\right\} \Rightarrow \tag{4.0.77}$$

$$\Rightarrow \left\{\tilde{E} \Leftrightarrow E_k\right\} \Rightarrow \left\{\begin{array}{l} u = \dfrac{v}{1+\sqrt{1-\dfrac{v^2}{c^2}}} = \lambda \cdot f = \dfrac{\tilde{E}}{p} \leq c \ , \\ v = \dfrac{d\omega}{dk} = \dfrac{d\omega/dv}{dk/dv} \ , \\ k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}\gamma mv \ , \quad \omega = 2\pi f = \dfrac{E_k}{h} \\ E_k = (\gamma - 1)mc^2 \end{array}\right\} , \tag{4.0.78}$$

Here is directly proven (contrary to the position of contemporary Quantum Theory) that *<u>wave packet energy</u> corresponds only to motional or kinetic particle energy and that group velocity corresponds to the particle velocity (or that the motional energy is propagating by group velocity). Quantum Theory considers that a wave-packet, which represents a specific particle, relates to a total particle energy, including its rest mass, what is not correct.* The phase velocity found here is only relevant for elementary, simple harmonic wave components propagation and it has the same limits as group velocity, $\mathbf{0 \leq 2u \leq \sqrt{uv} \leq v \leq c}$.

The other useful relations between motional energy, group and phase velocity are,

$$\left\{\begin{array}{l} \left\{E_k \Leftrightarrow \tilde{E} = pu, \ pv = E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right] = \dfrac{\gamma^2-1}{\gamma}mc^2\right\}, p = \gamma mv, \\ \left\{(\dfrac{u}{v}) = \dfrac{\tilde{E}}{pv} = \dfrac{pu}{E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right]} = \dfrac{1}{1+\sqrt{1-(\frac{v}{c})^2}} = \dfrac{\gamma}{\gamma+1}\right\}, \\ \left\{\lambda = \dfrac{h}{p}, k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}p, \omega = 2\pi f, \dfrac{d\lambda}{\lambda} = -\dfrac{dp}{p} = -\dfrac{dk}{k} = -\dfrac{df}{f}, u = \dfrac{\omega}{k}, v = \dfrac{d\omega}{dk}\right\} \end{array}\right\} \Rightarrow \tag{4.0.79}$$

$$\Rightarrow \left\{\begin{array}{l} v = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda} = u + p\dfrac{du}{dp} = \dfrac{d\omega}{dk} = \dfrac{d\tilde{E}}{dp} = h\dfrac{df}{dp} = \dfrac{df}{df_s} = \dfrac{2u}{1+\frac{uv}{c^2}} , \\ u = \lambda f = \dfrac{\omega}{k} = \dfrac{\tilde{E}}{p} = \dfrac{hf}{p} = \dfrac{f}{f_s} = \dfrac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{E_k}{p} \quad \Rightarrow \\ \Rightarrow \quad 0 \leq 2u \leq \sqrt{uv} \leq v \leq c \ , \\ d\tilde{E} = hdf = mc^2 d\gamma, \quad \dfrac{df}{f} = (\dfrac{dv}{v}) \cdot \dfrac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \Rightarrow \dfrac{\Delta f}{\overline{f}} = (\dfrac{\Delta v}{\overline{v}}) \cdot \dfrac{1+\sqrt{1-\frac{\overline{v}^2}{c^2}}}{1-\frac{\overline{v}^2}{c^2}} \\ \omega = 2\pi f = ku = \dfrac{kv}{1+\sqrt{1-v^2/c^2}} = \dfrac{2\pi}{h}\dfrac{pv}{1+\sqrt{1-v^2/c^2}} = \dfrac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \cdot \dfrac{\hbar}{m}k^2, k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}p \end{array}\right\} \tag{4.0.80}$$

$$\frac{df}{f} = \frac{d\tilde{E}}{\tilde{E}} = -(\frac{v}{u}) \cdot \frac{d\lambda}{\lambda} = (\frac{dv}{v})(1+1/\gamma)\gamma^2 = -(1+\frac{1}{\gamma}) \cdot \frac{d\lambda}{\lambda} \cong \left\{\begin{array}{l} 2\dfrac{dv}{v} = -2\dfrac{d\lambda}{\lambda} \ , \quad \text{for} \quad v << c \\ -\dfrac{d\lambda}{\lambda} \ , \quad \text{for} \quad v \approx c \end{array}\right\}$$

$$\omega(k)=\frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\cdot\frac{\hbar}{m}k^2\cong\begin{Bmatrix}\frac{\hbar}{2m}k^2, \text{for } v<<c\\ ck, \quad \text{for } v\approx c\end{Bmatrix}\Rightarrow$$

$$m(v)=\frac{m}{\sqrt{1-v^2/c^2}}\cong\begin{Bmatrix}m, \quad \text{for } v<<c\\ \frac{\hbar k}{c}\sqrt{1-v^2/c^2}=\frac{p}{c}\sqrt{1-v^2/c^2}=\frac{mv}{c}\cong m, \text{ for } v\approx c\end{Bmatrix},$$

$$p(v)=\frac{mv}{\sqrt{1-v^2/c^2}}\cong\begin{Bmatrix}mv, \quad \text{for } v<<c\\ \frac{\hbar k}{c}v\sqrt{1-v^2/c^2}=\frac{pv}{c}\sqrt{1-v^2/c^2}=\frac{mv^2}{c}\cong mv\cong mc, \text{ for } v\approx c\end{Bmatrix},$$

$$k(v)=\frac{2\pi}{\lambda}=\frac{2\pi}{h}p=\frac{2\pi}{h}\frac{mv}{\sqrt{1-v^2/c^2}}\cong\begin{Bmatrix}\frac{2\pi}{h}mv, \quad \text{for } v<<c\\ \frac{k}{c}v\sqrt{1-v^2/c^2}=\frac{2\pi}{h}\frac{pv}{c}\sqrt{1-v^2/c^2}=\frac{2\pi}{h}\frac{mv^2}{c}\cong\frac{2\pi}{h}mv\cong\frac{2\pi}{h}mc, \text{ for } v\approx c\end{Bmatrix}.$$

$$\Rightarrow\begin{Bmatrix}\frac{\tilde{E}}{mc^2}=\frac{hf}{mc^2}=\frac{1}{\sqrt{1-(\frac{v}{c})^2}}-1=\gamma-1=\frac{\tilde{E}}{E_0},\ E_0=mc^2=const.,\\ \frac{\tilde{E}}{\gamma mc^2}=\frac{hf}{\gamma mc^2}=1-\sqrt{1-(\frac{v}{c})^2}=1-\frac{1}{\gamma}=\frac{\tilde{E}}{E_{total}},\ E_{total}=\gamma mc^2=\gamma E_0,\\ \frac{\tilde{E}}{E_k}=\frac{\tilde{E}}{(\gamma-1)mc^2}=\frac{hf}{(\gamma-1)mc^2}=1,\ E_{total}=E_0+E_k=E_t\ ,\\ p^2c^2+E_0^2=E_t^2,\quad p^2v-pE_t+p_0E_0=0,\\ \tilde{E}=pu=-E_0\pm\sqrt{E_0^2+p^2c^2}=E_k\left\{=-E_0+\sqrt{E_0^2+p^2c^2}=E_0\left[\sqrt{1+(\frac{pc}{E_0})^2}-1\right]\right\},\\ \begin{Bmatrix}\vec{p}+\tilde{\vec{p}}=\vec{P}=\overrightarrow{const}\Rightarrow d\vec{p}=-d\tilde{\vec{p}}\\ d\tilde{E}=hdf=d(pu)=dE_k=vdp=c^2d(\gamma m)=-c^2d\tilde{m}=-d(\tilde{p}u)=\\ =-vd\tilde{p}\cdot\cos(\vec{p},\tilde{\vec{p}})=\{vd\tilde{p}\text{ or }-vd\tilde{p}\}=hvdf_s\end{Bmatrix}\\ \Rightarrow\Delta E_k=-\Delta\tilde{E},\ \Delta p=-\Delta\tilde{p},\ \Delta L=-\Delta\tilde{L},\ \Delta q=-\Delta\tilde{q},\Delta\dot{p}=-\Delta\dot{\tilde{p}},\Delta\dot{L}=-\Delta\dot{\tilde{L}},...\end{Bmatrix} \quad (4.0.81)$$

The *Wave-packet and Particle Equivalency or Analogy* gets clearer if we place corresponding expressions in the same table T.4.0.2, as for instance,

T.4.0.2	Wave-Packet	Particle
Motional Energy	$\tilde{E} = \int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt = \tilde{m}c^2 = hf =$ $= \hbar\omega = \tilde{m}vu = pu = \tilde{p}u =$ $= \frac{1}{\pi}\int_{0}^{\infty} A^2(\omega)\, d\omega = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)\, dt$	$E_k = (\gamma - 1)mc^2$ $(E_{tot.} = \gamma mc^2 = E_0 + E_k,$ $E_0 = mc^2 = const.)$
Mass	**Photons** $\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{1}{c^2}\int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt =$ $= \frac{1}{\pi c^2}\int_{0}^{\infty} A^2(\omega)\, d\omega =$ $= \frac{1}{2c^2}\int_{-\infty}^{+\infty} a^2(t)\, dt$ **Any Wave-Packet** $\tilde{m} = \frac{\tilde{E}}{vu} =$ $= \frac{1}{\gamma c^2}\int_{-\infty}^{+\infty} \lvert\Psi(t)\rvert^2 dt$ $= \frac{1}{\gamma\pi c^2}\int_{0}^{\infty} A^2(\omega)\, d\omega$	$m_{mot.} = \frac{E_k}{c^2} = (\gamma - 1)m$ $(m_{tot.} = \frac{E_{tot.}}{c^2} = \gamma m =$ $= m + m_{mot.},$ $m = const.)$
Momentum	$\tilde{p} = \tilde{m}v = \tilde{E}/u =$ $= \frac{1}{\pi c^2}\int_{0}^{\infty} \frac{d\omega}{dk}\cdot [A(\omega)]^2 \cdot d\omega =$ $= \frac{1}{2c^2}\int_{-\infty}^{+\infty} v(t)\cdot a^2(t)\cdot dt$	$p = \gamma mv = \frac{h}{\lambda}$
Group Velocity	$v = \frac{d\tilde{E}}{d\tilde{p}} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk}$, $\bar{v}_g = \bar{v} = \frac{\int_{0}^{\infty} \frac{d\omega}{dk}\cdot [A(\omega)]^2\, d\omega}{\int_{0}^{\infty} [A(\omega)]^2\, d\omega} =$ $= \frac{\int_{-\infty}^{+\infty} v(t)\cdot a^2(t)\, dt}{\int_{-\infty}^{+\infty} a^2(t)\, dt} = \frac{\delta\omega}{\delta k}$	$v = \frac{dE_k}{dp}$ $(= \frac{dE_{tot.}}{dp})$
Phase Velocity	$u = \frac{\tilde{E}}{\tilde{p}} = \frac{\tilde{E}}{p} = \frac{\omega}{k} = \lambda f$, $\bar{v}_f = \bar{u} = \frac{\int_{0}^{\infty} \frac{\omega}{k}\cdot [A(\omega)]^2\, d\omega}{\int_{0}^{\infty} [A(\omega)]^2\, d\omega} =$ $= \frac{\int_{-\infty}^{+\infty} u(t)\cdot a^2(t)\, dt}{\int_{-\infty}^{+\infty} a^2(t)\, dt} = \frac{\omega_0}{k_0}$	$u = \frac{E_k}{p} = \lambda f$ $(\lambda = \frac{h}{p})$

See later “T.4.0. Photon – Particle Analogies”, in the next chapter 4.1, as the supporting and complementary insight to T.4.0.2 .

4.0.11. Generalized Wave Functions and Unified Field Theory

Variety of waving phenomena, oscillations, vibrations, and wave-like motions known in all domains of physics should be covered by the same mathematical modeling, and all of that belong to matter waves (including Quantum Theory waves). Mathematical forms or functions describing wave motions generally belong to wave functions that can be presented as complex phasors and power related functions. Power related functions are usually products of mutually coupled Phasors. Complex Phasors modeling is initially developed in electromagnetic theory for easier operating with simple harmonic wave functions (like currents and voltages are). Later, familiar mathematical modeling has been established for all energy-finite and band-limited, arbitrary-shaped functions, as Complex Analytic Signals modeling and analysis. Such modeling is creating generalized, analytic, and complex Phasors (applicable to all kind of matter waves). Again, products of mutually related and generalized Phasors (of arbitrary shaped signals) can create generalized power-related wave functions (that are also complying with Parseval's theorem concerning mutually conjugate domains). Division between mutually related phasors is producing complex Impedances and Admittances (using electromagnetic-theory terminology and definitions). If power-related wave functions are conveniently normalized (becoming non-dimensional functions; - see more in the chapter 4.3, under "4.3.3. Probability and Conservation Laws"), and appropriately merged with the framework of Statistics and Probability theory, we can assemble presently known Orthodox Quantum-mechanical wave functions.

Particle-Waves Duality concept, as favored in this book, creates sufficiently clear frontier between a stable particle (which has constant rest mass in its center of mass reference system), and wave or particle-wave phenomena that belong only to different states of motions. Contrary to such statement, we also know that internal structure of any stable particle is composed of waves and dualistic particle-waves constituents, which are on some stable way "geared and fitted" producing (looking only externally) a stable particle (**internally assembled as ensembles of standing waves and resonant field-structures**). We could also say that *particle presents a stable packing-format for a specific group of matter waves (see (4.0.35) - (4.0.44)).* What is missing in such conceptual picture of particle-waves duality is to explain conditions when and how specific dynamic combination (superposition and interferences) of waves transforms into a stable particle (and when particles are decomposed, again in matter waves). For instance, when motional-wave-mass as one found in T.4.0.1, will create a stable particle. Here favored concept is that relatively free and open-space propagating wave-packets, under certain circumstances self-close their waveforms, and become spatially localized objects, being internally structured as certain standing and self-stabilized waves, or like a vortex formation being in a stable resonance. The continuity and energy coupling manifestations between internal particle-wave structures and externally detectable matter waves (and associated fields of any kind) is producing de Broglie matter waves whenever particles are in motion. Such matter waves are respecting de Broglie – Planck – Einstein expressions for wavelength and wave-packet energy.

Intuitively, we know that a stable particle (one having stable rest-mass) should have specific stationary and waving structure (internally appropriately arranged and balanced), and we also know that in many (motions and scattering related) interactions stable particles are manifesting particle-wave duality properties or could be disintegrated into pure wave-energy constituents. It is also known that convenient superposition of pure wave elements could produce a stable particle (electron-positron creation, for instance). Consequently, the general case of a stable particle should be that its internal constituents are composed of some "wave-mass" elements. The proper internal and dynamic "gearing and fitting" of all particle constituents should produce a stable particle, which has a stable rest mass. This is explicable by applying the rules of Relativity Theory: -such as the Minkowski-Einstein 4-vectors connection between total energy, rest energy, and particle momentum; -see more in Chapter 10., under "10.1 Hypercomplex

Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality". Of course, there are intermediary and mixed particle-wave states, which sometimes behave more as particles or as waves, depending how they are packed and spatially shaped, compared to more stable states of surrounding mases and objects.

The way to establish the grounds of a Unified Field Theory (as suggested in this book) will start (based on Parseval's theorem) from the condition that the square of the wave function should present an active and measurable power-function $\Psi^2(t,r) = P(t,r) = d\tilde{E}/dt = -dE_k/dt$. In other words, if there is certain power or energy content in a certain domain, this is good enough condition to have particles and waves there. Since *particles are only particular states of waves in different packing formats*, we would need to have sufficiently good and generally applicable *matter-waves wave functions* to conceptualize and model different matter-states and interactions between them. We are already used to express power-functions as products between corresponding current and voltage (in electro-technique), or force and velocity (in mechanics), etc. Here we consider the power-functions as one of the most relevant wave functions (see later different options of expressing power in (4.0.82)). This will directly enable us to develop new forms of wave equations (valid for Gravitation, Electromagnetism, Acoustics, and other fields), formally similar or equivalent to Schrödinger and classical wave equations, but now dealing with velocities, forces, currents, voltages... Let us first show what possible forms regarding creating such wave functions are, by showing expressions for power (in electric circuits, mechanics, electromagnetic fields etc.). This way we will operate with empirically and dimensionally tangible, clear, and measurable wave functions, being able to explore many of analogies between them, and to refer to well-developed and proven methods and theories (of Electromagnetic theory and Classical Physics), where most of the facts are non-disputable, and where we really know what we are talking about. In some later steps, such dimensional wave functions could easily be normalized (losing dimensionality). Additional modeling can result in getting isomorphic forms of, in Statistics and Probability common functions, like we have in Orthodox Quantum Theory, but such aspect of modified wave functions would not be profoundly analyzed here (at least not before we get a clear picture of what we are dealing with, and how further generalizations could be made). In Quantum Theory, the wave function $\Psi^2(\mathbf{t})$ is conveniently modeled as a probability function, but effectively it behaves in many aspects like normalized and dimensionless Power function. Here it will be closely related to an effective Power (or power delivered to a load, expressed in watts as its units). Here, an attempt will be made to analogically connect an arbitrary Power Function $\Psi^2(\mathbf{t})$ (which is a product between corresponding current and voltage, or product between force and velocity, or product between any other relevant, mutually conjugate functions creating power), to a wave-function, as known in Quantum Mechanics. This way we will not go out of tangible, deterministic and dimensional frames of such wave functions. Analyzed from energy content, any wave propagation in time and frequency domain can be mutually (time-frequency) related using Parseval's theorem (4.0.4). Consequently, the instantaneous (time-domain) Power-signal can be presented as the square of the wave function $\Psi^2(\mathbf{t})$, which is always the product of two relevant phasors (see (4.0.82)). The analysis of the optimal power transfer (carried by $\Psi^2(\mathbf{t})$) can be extended to any wave-like propagating field (and to arbitrarily shaped signals), and theoretically profit enormously (in booth, directions) after unifying traditional concepts of Active, Reactive and Apparent power with the $\Psi^2(\mathbf{t})$ wave-function mathematics, based on Analytic Signal methodology and PHASORS concept known in electrical sciences.

$$\Psi^2(t,r) = P(t,r) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=)$$

$$(=)\begin{cases} i(t,r)\cdot u(t,r) & (=) & [\text{Current} \cdot \text{Voltage}], \text{ or} \\ f(t,r)\cdot v(t,r) & (=) & [\text{Force} \cdot \text{Velocity}], \text{ or} \\ \tau(t,r)\cdot\omega(t,r) & (=) & [\text{Orb.}-\text{moment}\cdot\text{Angular velocity}], \text{or} \\ (\vec{E}\times\vec{H})\cdot\vec{S} & (=) & \left[\overrightarrow{\text{Pointyng Vector}}\right]\cdot\overrightarrow{\text{Surface}} \\ ----- & (=) & --------- \\ s_1(t,r)\cdot s_2(t,r) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{cases} \quad (4.0.82)$$

Since the Electric Circuits Theory is already very well developed (being an obviously deterministic theory), we can analogically profit from its modeling and methodology, related to handling optimal active-power transfer, to real, imaginary, and complex power, to real and complex impedances, to *complex phasors* notation of basic electric values (like voltages, currents, impedances), etc. Eventually, we will analogically start applying models, structures, and conclusions developed in Electronics, to Classical Mechanics, Gravitation, Quantum Mechanics... We are also familiar with all kinds of measurements regarding electric currents, voltages, electric and magnetic fields, and we know how to produce and use them in many ways. We also know how to transform electric energy into mechanical motion, oscillations, electromagnetic fields, light, sound, etc. Rich mathematical modeling is well developed regarding handling all kinds of electric, magnetic, and electromagnetic phenomena.

As we can see from (4.0.82), electric power is equal to the product between current and voltage measured on the same load (or on the same component). An analogical case known in mechanics, or in any other domain, will be that mechanical power is again equal to a product between a particular pair of mutually coupled (or conjugate) phasors like between an active force and velocity (of the same mechanical element). Since here we accepted (at least in this book) that the square of the wave function corresponds to a relevant power-function, the next idea or proposal is to try to model Quantum Theory matter-wave function as a product between two relevant, mutually coupled (or conjugate) signals. This way, later we can also establish an analogical interpretation of *Orthodox quantum theory wave function* and electrical power (or just power), both being a square of a relevant matter-wave function (see more in the chapter 4.3, under "4.3.3. Probability and Conservation Laws" and in Chapter 10). In this way, the complete methodology, modeling, and complex PHASORS conceptualization, as known in electromagnetism, would naturally be transformed into quantum world wave functions (after applying relevant normalizations, and generally applicable theoretical fitting into rules of Statistics and Probability theory). We can realize this considering all relevant participants, or phasors (not just literally and analogically transforming one set of expressions into another).

Since we already know that the Analytic Signal modeling gives a rich mathematical framework for arbitrary signals and wave functions analyses, it should be directly applicable to any current and voltage signals (or to force and velocity signals). We will make one-step forward, first by transforming all wave and power functions from (4.0.82) into Analytic Signal, or Complex Phasor forms (as products between two mutually related phasors). The advantage here is that none of the functions involved should be only a simple harmonic function (or that voltage and current, that are creating the power function could be arbitrary shaped, and frequency-wise wideband; -of course, simple harmonic waveforms are also included here as the bottom line, simplest and most straightforward cases). The complexity and mathematical richness of such approach, related to complex phasors of electric currents and voltages, will show a lot of advantages and new opportunities regarding analogic application of a similar methodology to any of waveform analyses (when compared to traditional methods). Consequently, we could ask ourselves why not to apply it to quantum-mechanical matter-waves (of course, reasonably, and considering all the specifics and constraints of matter-waves phenomenology, and quantum mechanical, statistical, and probabilistic modeling). Later, it will appear obvious that very convenient ***fields and waves' unification platform*** can be established because variety of mentioned mater-waves phenomenology items would be modeled inside the same and superior mathematical framework of Analytic Signals and Complex (or Hyper-Complex) Phasors framework.

In the next part of this book, it will be exercised what kind of innovative mathematical modeling can be applied for analyzing and characterizing wideband and arbitrary shaped signals.

[♣ COMMENTS & FREE-THINKING CORNER:

4.0.11.0. Wave Function, Energy, Power, and Impedance

We can say that, in this work, the wave function is related to the power function of some movement, i.e., that it directly represents specific energy transfer. To determine the power, it is commonly implied in physics that we know the two functions: $s_1(t)$ and $s_2(t)$, the product of which defines the power, as mentioned in (4.0.82).

The kind of energy transfer from its source to the load depends on the coherence relation between the functions $s_1(t)$ and $s_2(t)$, *which determine the* power*. That is how we can define coherence coefficient between $s_1(t)$* and *$s_2(t)$:*

$$K_t = \frac{\int_{-\infty}^{+\infty} s_1(t)\cdot s_2(t)\cdot dt}{\int_{-\infty}^{+\infty} |s_1(t)|\cdot|s_2(t)|\cdot dt} = \frac{\int_{-\infty}^{+\infty} \Psi^2(t)\cdot dt}{\int_{-\infty}^{+\infty} |\Psi^2(t)|\cdot dt}\ , \qquad (4.0.83)$$

And determine their phase functions:

$$\varphi_1(t) = \text{arc tang}\left[\hat{s}_1(t)/s_1(t)\right],\ \left(s_1(t) = a_1(t)\cos\varphi_1(t) = \frac{1}{\pi}\int_0^{\infty}\left[A_1(\omega)\cos(\omega t + \Phi_1(\omega))\right]d\omega\ ,\right.$$

$$\hat{s}_1(t) = H[s_1(t)] = a_1(t)\sin\varphi_1(t),$$

$$\overline{s}_1(t) = s_1(t) + j\hat{s}_1(t) = a_1(t)\,e^{j\varphi_1(t)}),$$

$$\varphi_2(t) = \text{arc tang}\left[\hat{s}_2(t)/s_2(t)\right],\ \left(s_2(t) = a_2(t)\cos\varphi_2(t) = \frac{1}{\pi}\int_0^{\infty}\left[A_2(\omega)\cos(\omega t + \Phi_2(\omega))\right]d\omega\ ,\right.$$

$$\hat{s}_2(t) = H[s_2(t)] = a_2(t)\sin\varphi_2(t),$$

$$\overline{s}_2(t) = s_2(t) + j\hat{s}_2(t) = a_2(t)\,e^{j\varphi_2(t)}), \qquad (4.0.84)$$

therefore, it is straightforward to find an additional criterion about the mutual position (coherence) of $s_1(t)$ and $s_2(t)$ by simply forming their phase difference,

$$\Delta\varphi(t) = \varphi_2(t) - \varphi_1(t). \qquad (4.0.85)$$

It is also possible to find the amplitude spectral functions for the wave function factors $s_1(t)$ and $s_2(t)$ $A_1(\omega)$ i $A_2(\omega)$ *, as well as to find the functions of the appropriate phase functions $\Phi_1(\omega)$ and $\Phi_2(\omega)$ using* definitions from *(*4.0.3*)* and T.4.0.1*.*

Now we can draw important conclusions concerning coherence criteria for the wave function factors $s_1(t)$ and $s_2(t)$ and connect them with the optimal energy propagation:

-If $K_t = 1$, $s_1(t)$ and $s_2(t)$ are coherent, or mutually *in phase*, *$\Delta\varphi(t) = 0$, we have an optimal energy transf*er. That is the case of active *or resistive* load *impedances.*

-If f $K_t = 0$, $s_1(t)$ and $s_2(t)$ are mutually orthogonal or phase shifted *for $\pi/2$, $|\Delta\varphi(t)| = \pi/2$, no energy circulation from the source to the* load *is possible (i.e., the* load *does not receive any energy).*

-If $K_t = -1$, $s_1(t)$ and $s_2(t)$ are in the counter-phase, then there is a non-optimal energy transfer, i.e., the energy is completely reflected off the load *and returned to its source (the energy has a "-" sign). That is the case of the reactive (or complex)* load *impedances.*

Power *factor, coherence criterion* between *$s_1(t)$ and $s_2(t)$ can be widened by comparing their phase spectral functions, which are defined by* (4.0.3)*, i.e., by forming the difference:*

$$\Delta\Phi(\omega) = \Phi_2(\omega) - \Phi_1(\omega). \qquad (4.0.86)$$

It is equally justified to introduce the coherence factor into the spectral domain, using amplitude spectral functions (4.0.3)*, analogous to the definition* (4.0.83)*,*

$$K_\omega = \frac{\int_0^{+\infty} A_1(\omega)\cdot A_2(\omega)\cdot d\omega}{\int_0^{+\infty} |A_1(\omega)|\cdot|A_2(\omega)|\cdot d\omega} \ . \qquad (4.0.87)$$

While analyzing optimal energy transfer (and introducing the terms like active, reactive, and complex power) great attention should be paid to the fact that, in a general case, Hilbert transformation of wave function elements is not linear, i.e., the following equations are valid:

$$H[\Psi^2(t)] = H[s_1(t)\, s_2(t)] = \mathbf{s_1 s_2} \frac{\mathbf{s_1\hat{s}_2 + \hat{s}_1 s_2}}{\mathbf{s_1 s_2 - \hat{s}_1\hat{s}_2}}, \qquad (4.0.88)$$

$$H[s_2(t) / s_1(t)] = \left(\frac{\mathbf{s_2}}{\mathbf{s_1}}\right)\frac{\mathbf{s_1\hat{s}_2 - \hat{s}_1 s_2}}{\mathbf{s_1 s_2 + \hat{s}_1\hat{s}_2}}. \qquad (4.0.89)$$

4.0.11.1. Generalized Impedances

To analyze the problems related to energy transports, it is necessary that an energy source and its consumer or load exist. The term "load impedances" characterize loads. In physics, electro-technique, electromagnetism, acoustics, and mechanics we can define a universal term "dynamic (time-dependent) load impedance" in the following manner:

$$Z(t) = s_2(t) / s_1(t) = \tilde{S}(t) / s_1^2(t) = s_2^2(t) / \tilde{S}(t) =$$

$$= [\Psi(t) / s_1(t)]^2 = [s_2(t) / \Psi(t)]^2 = \frac{a_2(t)}{a_1(t)} \frac{\cos\varphi_2(t)}{\cos\varphi_1(t)}. \qquad (4.0.90)$$

This time it is advisable to think about the way to treat a complex, time-dependent impedance (taking (4.0.89) into account), defined as a quotient of appropriate analytical (complex) functions:

$$\overline{Z}(t) = \overline{s}_2(t) / \overline{s}_1(t) = \frac{a_2(t)}{a_1(t)} e^{j[\varphi_2(t)-\varphi_1(t)]} = \frac{a_2(t)}{a_1(t)} e^{j\Delta\varphi(t)}, \qquad (4.0.91)$$

because $\overline{Z}(t) = \overline{s}_2(t) / \overline{s}_1(t) \neq s_2(t) / s_1(t) + j\, H[s_2(t)/s_1(t)]$ (except for simple harmonic waveforms).

In a similar manner, we can define frequency dependent, complex impedances, as a quotient of spectral (com*plex) functions of the appropriate wave function factors:*

$$Z(\omega) = \overline{Z}(\omega) = \frac{s_2(\omega)}{s_1(\omega)} = \frac{A_2(\omega)}{A_1(\omega)} e^{j[\Phi_2(\omega)-\Phi_1(\omega)]} = \frac{A_2(\omega)}{A_1(\omega)} e^{j\Delta\Phi(\omega)} . \qquad (4.0.92)$$

In the example of passive electric impedances (or networks made of a load *with the resistance R, capacitance C and inductance L), it is possible to present various ways for a more precise defining of the term impedance.* Later, the *same procedure can be extended to other physics disci*plines as well using "current-force" and "voltage-velocity" analogies (see (1.82)), as for instance:

$$s_1(t) = i(t) = I(t)\cos\varphi_i(t) \ (=) \text{ current}, \ \overline{s}_1(t) = \overline{i}(t) = I(t)\, e^{j\varphi_i(t)}, \ i(\omega) = I(\omega)\, e^{i\Phi_i(\omega)}$$
$$s_2(t) = u(t) = U(t)\cos\varphi_u(t) \ (=) \text{ voltage}, \ \overline{s}_2(t) = \overline{u}(t) = U(t)\, e^{j\varphi_u(t)}, \ u(\omega) = U(\omega)\, e^{i\Phi_u(\omega)}$$
$$\tilde{S}(t) = d\tilde{E}/dt = \Psi^2(t) = u(t)\, i(t) \ (=) \text{ force}. \qquad (4.0.93)$$

4.0.11.2. Resistive Impedances

Let us assume that the current $i_R(t) = i_R$ runs through the electrical resistance R. In that case, voltage $u_R(t) = u_R$ is formed on the resistance R. It is obvious that the following relations are valid:

$$u_R(t) = Z_R\, i_R = R\, i_R, \ \tilde{S}_R(t) = u_R(t)\, i_R(t) = R\, i_R^2 = \Psi_R^2(t),$$
$$K_t = 1, K_\omega = 1, \ \Delta\varphi(t) = \varphi_u - \varphi_i = 0, \ \Delta\Phi(\omega) = \Phi_u - \Phi_i = 0, \qquad (4.0.94)$$

In addition, from this, the resistive dynamic impedance is:

$$Z_R(t) = u_R(t) / i_R(t) = \overline{Z}_R(t) = Z_R(\omega) = u_R(\omega) / i_R(\omega) = R = [\Psi_R(t) / i_R(t)]^2. \qquad (4.0.95)$$

4.0.11.3. Inductive impedances

*The previous procedure can be extended analogously. If the current $i_L(t) = i_L$ runs through the inductance **L**, voltage $u_L(t) = u_L$* will be formed on the inductance ***L***. *The validity of the following relations is obvious (or can be proved):*

$$u_L(t) = Z_L\, i_L = -L(di_L/dt), \ \tilde{S}_L(t) = u_L(t)\, i_L(t) = -L\, i_L(di_L/dt) = \Psi_L^2(t),$$

a) $U(\omega) = -\omega L I(\omega)$,
$\Delta\Phi(\omega) = \Phi_u - \Phi_i = +\pi/2$,

b) $U(\omega) = +\omega L I(\omega)$,
$\Delta\Phi(\omega) = \Phi_u - \Phi_i = \{+3\pi/2, -\pi/2\}$,

$$\Delta\varphi(t) = \varphi_u - \varphi_i = \text{arctg}\left[\frac{\frac{d\varphi_i}{dt}}{\frac{1}{I(t)}\frac{dI(t)}{dt}}\right] +/- \pi \ , \qquad (4.0.96)$$

from this, the inductive impedance is:

$$Z_L(t) = u_L(t) / i_L(t) = \frac{U_L(t)}{I_L(t)} \frac{\cos\varphi_u(t)}{\cos\varphi_i(t)} = (-L / i_L)(di_L/dt) = [\Psi_L(t) / i_L(t)]^2$$

$= (\frac{d\varphi_i(t)}{dt})L\, tg\varphi_i(t) - \frac{1}{I(t)}\frac{dI(t)}{d(t)}L.$

$\overline{Z}_L(t) = \frac{\overline{u}_L(t)}{\overline{i}_L(t)} = \frac{U_L(t)}{I_L(t)}e^{j\Delta\varphi(t)} = -j(\frac{d\varphi_i(t)}{dt})L - \frac{1}{I(t)}\frac{dI(t)}{d(t)}L$,

$Z_L(\omega) = u_L(\omega) / i_L(\omega) = \frac{U(\omega)}{I(\omega)}e^{j\Delta\Phi(\omega)} = \{[+j\omega L = +j\frac{U(\omega)}{I(\omega)}; U(\omega) = -\omega LI(\omega)],$

ili $[-j\omega L = -j\frac{U(\omega)}{I(\omega)}; U(\omega) = +\omega LI(\omega)]\}.$ (4.0.97)

4.0.11.4. Capacitive Impedances

Finally, if current $i_C(t) = i_C$ *runs through capacitance* C*, then voltage* $u_C(t) = u_C$ *will be formed on the capacitance* C*. The validity of the following relations is obvious (or can be proved):*

$$u_c(t) = Z_c i_c = \frac{1}{C}\int_0^t i_c(t)dt,\ \tilde{S}_C(t) = u_C(t)\, i_C(t) = \frac{i_C(t)}{C}\int_0^t i_c(t)dt = \Psi_c^2(t), \quad (4.0.98)$$

$$U(\omega) = \frac{1}{\omega C} I(\omega),$$

$$\Delta\Phi(\omega) = \Phi_u - \Phi_i = -\pi/2,$$

$$\Delta\varphi(t) = \varphi_u - \varphi_i = -\pi/2$$

from this, capacitance impedances are:

$$Z_c(t) = u_c(t) / i_c(t) = \frac{U_C(t)}{I_C(t)}\frac{\cos\varphi_u(t)}{\cos\varphi_i(t)} = \frac{1}{Ci_C(t)}\int_0^t i_c(t)dt = [\Psi_c(t) / i_c(t)]^2 = \ldots,$$

$$\overline{Z}_C(t) = \frac{\overline{u}_c(t)}{\overline{i}_c(t)} = \frac{U_C(t)}{I_C(t)}e^{j\Delta\varphi(t)},$$

$$Z_c(\omega) = u_c(\omega) / i_c(\omega) = \frac{U(\omega)}{I(\omega)}e^{j\Delta\Phi(\omega)} = -j\frac{1}{\omega C} = -j\frac{U(\omega)}{I(\omega)}. \quad (4.0.99)$$

4.0.11.5. R-L-C Impedances for Simple Harmonic Currents

If we consider only the cases when through an impedance a simple harmonic stationary current $i(t) = I_0 \cos\omega t$, ($I_0 = const.$) runs, then the previous generalized impedances are simplified to the following:

$$Z_R(t) = \overline{Z}_R(t) = R,\ Z_R(\omega) = R,$$

$$Z_L(t) = \omega L(\, tg\, \omega t\,) = X_L(tg\, \omega t),\ \overline{Z}_L(t) = -j\omega L = -j X_L,\ Z_L(\omega) = j\omega L = jX_L,$$

(4.0.100)

$$Z_C(t) = (-1/\omega C)(\, tg\, \omega t\,) = -X_C(tg\, \omega t),\ \overline{Z}_C(t) = -j\frac{1}{\omega C} = -jX_C,\ Z_c(\omega) = -j\frac{1}{\omega C} = -jX_C.$$

In the electro-technique of stationary or simple harmonic currents, we can notice the advantage of introducing complex electric impedances, which simplify time-dependent impedances (4.0.100). If we introduce the transformation of *a real current function into a complex current function*, *like this:*

$$i(t) = I_0\cos\omega t \leftrightarrow I_0\, e^{-j\omega t} = \overline{i}(t) = \overline{I}, \quad (4.0.101)$$

and if we use the complex current form (4.0.101) to determine the impedance (4.0.100), we will get so-called complex stationary-current impedance forms, where we (formally) no longer see the difference among so-called time-dependent, complex or frequency dependent impedances (which are evident in (4.0.100)), i.e. for all impedance forms, the following is valid:

$$Z_R = R,$$

$$Z_L = j\omega L = j\,X_L, \qquad (4.0.102)$$

$$Z_C(t) = 1 / j\omega C = -\,j\,X_C\,.$$

The impedance forms are given in (4.0.102), although being practical (useful) and simple, hide within themselves the real identity of the term “impedance” and most of the related facts about it.

The formal way of imaging (4.0.101), which is following the definition of impedance, is nothing else but a replacement of one real function by an appropriate complex, analytical function. Therefore, if the current function i(t) is arbitrary, and not simple harmonic, we can make the replacement (with an appropriate analytical signal function (4.0.1) and (4.0.2), analogous to (4.0.101)):

$$i(t) \leftrightarrow i(t) + j\,H[i(t)] = i(t) + j\,\hat{i}(t) = I_0(t)\,e^{j\varphi(t)} = \bar{i}(t)\,. \qquad (4.0.103)$$

Mathematical formalism (4.0.103), related to simple harmonic (stationary) currents and voltages, is very well developed in electro-technique, whereas the same issue concerning non-periodical and non-harmonic (arbitrary functions of current and voltage) is rather badly dealt with regarding mathematics. It would be good to use the elements of the previously described formalism to analyze the optimal transmission of electrical signals (of the energy, power, currents, etc.) in the cases of their arbitrary, non-periodical waveforms (for arbitrary load impedances). If we do not wish to make a complete redefinition of “complex impedances”, and we want that definition to keep its general meaning within the frame previously established (to preserve the continuity with the usual treatment of that term), we have to limit the meaning of “complex impedance” only to its appropriate frequency-dependent impedance forms, as given in (4.0.100). It is obvious that time-dependent *impedances will be changeable according to the modeling factor tanωt (which is not at all practical for any selection or quantification of impedances).*

For now (in the examples given above), we will rely on the methodology established in electro-technique, because that issue is very comprehensive for us, and, besides, it is extremely well developed, in comparison to the other fields in physics. There is only one small step to the analogous broadening and usage of the same way of thinking to other fields, such as acoustics, fluid mechanics, electromagnetic waves, vibrations, quantum wave mechanics, etc.

4.0.11.6. Power Function

The concept of the complex, active and reactive power, which operates well in the electro-technique of stationary (simple harmonic) currents and voltages, is, in the general case of arbitrary forms of currents and voltages, inapplicable (at least not in any way known so far). Certainly, it is always possible to talk about the active power transfer, when a system operates as an active (resistive) load, and it satisfies the criteria from (4.0.94). All other cases can be treated as cases of the circulation of reactive and complex power. The key criterions for the determination of optimal energy transfer are *coherence relations among the* power *factors (see* (4.0.83*) to (*4.0.99*)). It is good to complement the previous claim with the comparison of the following real and complex wave function forms (by asking the question: which of these relations or parts of the relations, can stand for the complex, and which one/s for the reactive force, and to what extent such a terminology and classification makes sense and practical significance):*

$$\Psi^2(t) = s_1 s_2\ ,$$

$$\bar{s}_1\bar{s}_2 = (s_1 s_2 - \hat{s}_1\hat{s}_2) + j(s_1\hat{s}_2 + \hat{s}_1 s_2)\,,$$

$$\overline{\Psi}^2(t) = s_1 s_2 - \left\{H\left[\sqrt{s_1 s_2}\right]\right\}^2 + 2j\sqrt{s_1 s_2}\, H\left[\sqrt{s_1 s_2}\right], \quad (4.0.104)$$

$$\left|\overline{\Psi}(t)\right|^2 = s_1 s_2 + \left\{H\left[\sqrt{s_1 s_2}\right]\right\}^2,$$

$$s_1 s_2 + jH\left[s_1 s_2\right] = \overline{s}_{12} \ .$$

It is evident that none of the relations from (4.0.104) makes a direct way of introducing and total understanding of the terms: complex, active, and reactive power. To have an appropriate way of looking at the issues related to the term ‘power', we must hold on to a standpoint, which defines that, on the resistive load, the current and voltage functions will be entirely coherent, i.e., in the phase. This will be valid for both time and frequency domain of those functions when the power is entirely active. All other cases indicate the presence of complex or reactive power (or the absence of optimal energy transfer). What is also particularly important is that, if there is any other, more general mathematical analysis of the problems mentioned above, it must consider all cases of the classical treatment of active, reactive, and complex power discussed so far. This is a significant advantage and facility in the study of various wave natural phenomena because nature in a way strives towards creating harmonic oscillations, which can often be very well presented by simple harmonic functions (in most of the cases of interest).

The methodology introduced beginning with (4.0.82) to (4.0.104) opens new (generalized) possibilities for the analysis and measurement of the energy transport from the source to its load and gives the possibility of generalized load-type classification (for arbitrary wave functions). It is of a special significance that, by applying this methodology (with specific scientific extra work on the previously opened questions), we can overcome confusions and delusions that have existed until present.

Naturally, the introduction of the complex analysis into the wave movement analysis, besides other options, plays a significant role in the simplification of the mathematical apparatus used, in the way that, instead of solving integral-differential equations, the whole analysis is based on rather elementary algebra operations. ♣]

Extension of the Electrical Power Definition to Arbitrary (Non-Sinusoidal) Voltage and Current Signals and Consequences Regarding Novel Understanding of Quantum-Mechanical Wave Function

List of Symbols:

$\mathbf{p(t)} = \mathbf{Instantaneous\ Power}$

$\hat{\mathbf{p}}(\mathbf{t}) = \mathbf{Hilbert\ Transform\ of\ Instantaneous\ Power}$

$\overline{\mathbf{p}}(\mathbf{t}) = \mathbf{Instantaneous\ Power\ in\ the\ form\ of\ Complex\ Analytic\ Signal}$

$|\overline{\mathbf{p}}(\mathbf{t})| = \mathbf{Absolute\ Value\ of\ Instantaneous\ Power}$

$\mathbf{P(t)} = \mathbf{Amplitude\ function\ of\ Instantaneous\ (real\ or\ active)\ Power}$

$\mathbf{S(t)} = \mathbf{Amplitude\ function\ of\ Instantaneous\ Apparent\ Power}$

$\mathbf{Q(t)} = \mathbf{Amplitude\ function\ of\ Instantaneous\ Reactive\ Power}$

$\overline{\mathbf{u}}(\mathbf{t}) = \mathbf{Instantaneous\ Voltage\ in\ the\ form\ of\ complex\ Analytic\ Signal}$

$\mathbf{u(t)} = \mathbf{Instantaneous\ Voltage}$

$\hat{\mathbf{u}}(\mathbf{t}) = \mathbf{Hilbert\ Transform\ of\ Instantaneous\ Voltage}$

$\overline{\mathbf{i}}(\mathbf{t}) = \mathbf{Instantaneous\ Current\ in\ the\ form\ of\ Complex\ Analytic\ Signal}$

$\mathbf{i(t)} = \mathbf{Instantaneous\ Current}$

$\hat{\mathbf{i}}(\mathbf{t}) = \mathbf{Hilbert\ Transform\ of\ Instantaneous\ Current}$

$\mathbf{U(t)} = \mathbf{Amplitude\ function\ of\ Instantaneous\ Voltage}$

$\mathbf{I(t)} = \mathbf{Amplitude\ function\ of\ Instantaneous\ Current}$

$\mathbf{U_{rms}} = \mathbf{Effective, RMS\ Voltage}$

$\mathbf{I_{rms}} = \mathbf{Effective, RMS\ Current}$

$\overline{\mathbf{Z}} = \mathbf{R - jX} = \mathbf{Complex\ Impedance},\ |\overline{\mathbf{Z}}| = \mathbf{Z} = \sqrt{\mathbf{R}^2 + \mathbf{X}^2}$

$\mathbf{R} = \mathbf{Resistance},\ \mathbf{X} = \mathbf{Reactance}$

$\mathbf{T} = \mathbf{Time\ interval}$

$\varphi_{\mathbf{p}}(\mathbf{t}) = \mathbf{Phase\ function\ of\ the\ Instantaneous\ Power}$

$\omega_{\mathbf{p}} = \mathbf{Frequency\ of\ the\ Instantaneous\ Power}$

$\varphi_{\mathbf{u}} = \mathbf{Phase\ function\ of\ the\ Voltage\ signal}$

$\omega_{\mathbf{u}} = \mathbf{Frequency\ of\ the\ Voltage\ signal}$

$\varphi_{\mathbf{i}} = \mathbf{Phase\ function\ of\ the\ Current\ signal}$

$\omega_{\mathbf{i}} = \mathbf{Frequency\ of\ the\ Current\ signal}$

$\mathbf{j} = \sqrt{-1} = \mathbf{Imaginary\ unit}$

$\mathbf{H}[\ ]\ (=)\ \mathbf{Hilbert\ Transform}$

$\mathbf{PF} = \cos\theta = \mathbf{Power\ Factor}$

$\Psi(\mathbf{t}) = \mathbf{Wave\ function}$

$\mathbf{a(t)} = \mathbf{Amplitude\ of\ a\ Wave\ function}$

$\varphi(\mathbf{t}) = \mathbf{Phase\ of\ a\ Wave\ function}$

See more in: [57], Michael Feldman, [109], Poularikas A. D., including [110], and [111].

Analytic Signal and Electrical Power Characterization (comparative tables); **Extension of the Electrical Power Definition**

Instantaneous Power (Apparent Power & Analytic Signal)	Averaged Complex Power (Averaged Apparent Power & Phasor notation)	Generalized Complex Power (Generalized Phasor Notation)
$\overline{p}(t) = p(t) + j\cdot\hat{p}(t) = \lvert\overline{p}(t)\rvert\cdot e^{j\varphi_p(t)}$, $\varphi_p(t) = \mathrm{arctg}\dfrac{\hat{p}(t)}{p(t)} = \varphi_u(t) + \varphi_i(t) =$ $= \theta(t) + 2\varphi_u(t) = 2\varphi_i(t) - \theta(t)$, $\theta(t) = \varphi_i(t) - \varphi_u(t)$, $\omega_p = \dfrac{\partial\varphi_p(t)}{\partial t}$, $(j^2 = -1)$	$\overline{S} = \dfrac{1}{2}\overline{U}\cdot\overline{I}^* = P - jQ = Se^{-j\theta} = U_{rms}I_{rms}e^{-j\theta}$ $\theta(t) = \angle[u(t), i(t)] = \varphi_i(t) - \varphi_u(t) = \varphi_i - \varphi_u =$ $= \theta_i - \theta_u = \mathrm{arctg}\dfrac{Q}{P} = \theta$, $\omega = \dfrac{\partial\varphi_u(t)}{\partial t} = \dfrac{\partial\varphi_i(t)}{\partial t}$ $(\varphi_u(t) = \omega t + \theta_u, \varphi_i(t) = \omega t + \theta_i)$	$\overline{S}(t) = \dfrac{1}{2}\overline{u}(t)\cdot\overline{i}^*(t) = P(t) - jQ(t) =$ $= S(t)e^{-j\theta(t)} = \dfrac{1}{2}U(t)I(t)e^{-j\theta(t)}$ $\theta(t) = \varphi_i(t) - \varphi_u(t) = \mathrm{arctg}\dfrac{Q(t)}{P(t)}$
$p(t) = u(t)\cdot i(t) = \lvert\overline{p}(t)\rvert\cdot\cos\varphi_p(t) = \lvert\overline{p}(t)\rvert\cdot\cos(\varphi_u(t) + \varphi_i(t))$ $= U(t)\cdot I(t)\cdot\cos\varphi_u(t)\cdot\cos\varphi_i(t)$ $\lvert\overline{p}(t)\rvert = \sqrt{p(t)^2 + \hat{p}(t)^2} = U(t)I(t)\dfrac{\cos\varphi_u(t)\cos\varphi_i(t)}{\cos\varphi_p(t)} =$ $= \dfrac{p(t)}{\cos\varphi_p(t)} = 2S(t)\cdot\dfrac{\cos\varphi_u(t)\cos\varphi_i(t)}{\cos\varphi_p(t)}$ $\hat{p}(t) = H[p(t)] = \lvert\overline{p}(t)\rvert\cdot\sin\varphi_p(t) = p(t)\cdot\tan(\varphi_u(t) + \varphi_i(t))$, $H[\ ]$ (=) **Hilbert transformation**, $u(t) = U(t)\cos\varphi_u(t)$, $\overline{u}(t) = u(t) + j\cdot\hat{u}(t) = U(t)\cdot e^{j\varphi_u(t)}$, $U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2}$, $\varphi_u(t) = \mathrm{arctg}\dfrac{\hat{u}(t)}{u(t)}, \omega_u = \dfrac{\partial\varphi_u}{\partial t}$, $i(t) = I(t)\cos\varphi_i(t)$, $\overline{i}(t) = i(t) + j\cdot\hat{i}(t) = I(t)\cdot e^{j\varphi_i(t)}$, $I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2}$, $\varphi_i(t) = \mathrm{arctg}\dfrac{\hat{i}(t)}{i(t)}, \omega_i = \dfrac{\partial\varphi_i}{\partial t}$. $ui + \hat{u}\hat{i} = U(t)I(t)\cos\theta(t)$, $u\hat{i} - \hat{u}i = U(t)I(t)\sin\theta(t)$ $ui = U(t)I(t)\cos\varphi_u(t)\cos\varphi_i(t)$, $\hat{u}\hat{i} = U(t)I(t)\sin\varphi_u(t)\sin\varphi_i(t)$	$\overline{U} = \sqrt{2}U_{rms}e^{j\varphi_u}$, $\overline{I} = \sqrt{2}I_{rms}e^{j\varphi_i} = \sqrt{2}I_{rms}e^{j(\varphi_u + \theta)}$, $\overline{I}^* = \sqrt{2}I_{rms}e^{-j\varphi_i} = \sqrt{2}I_{rms}e^{-j(\varphi_u + \theta)}$, $P = U_{rms}I_{rms}\cos\theta$ = Active Power (=) [W] $Q = U_{rms}I_{rms}\sin\theta$ = Reactive Power (=) [VAR] $S = \sqrt{P^2 + Q^2} = U_{rms}I_{rms}$ (=) [VA] $PF = \dfrac{P}{S} = \cos\theta$ = Power Factor, $U_{rms} = \sqrt{\dfrac{1}{T}\int_{[T]} u^2(t)dt}$, $I_{rms} = \sqrt{\dfrac{1}{T}\int_{[T]} i^2(t)dt}$. $\overline{Z} = \lvert Z\rvert e^{-j\theta} = \dfrac{U_{rms}}{I_{rms}}e^{-j\theta} = R - jX =$ $= \sqrt{R^2 + X^2}\cdot e^{-j\theta}$	$P(t) = S(t)\cos\theta(t) = \dfrac{1}{2}(ui + \hat{u}\hat{i})$ $Q(t) = S(t)\sin\theta(t) = \dfrac{1}{2}(u\hat{i} - \hat{u}i)$ $S(t) = \sqrt{P(t)^2 + Q(t)^2} = \dfrac{1}{2}U(t)I(t) =$ $= \lvert\overline{p}(t)\rvert\dfrac{\cos\varphi_p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} =$ $= \dfrac{p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} = \dfrac{P(t)}{\cos\theta(t)}$, $PF(t) = \dfrac{P(t)}{S(t)} = \cos\theta(t) = \dfrac{ui + \hat{u}\hat{i}}{U(t)I(t)}$ $U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2}$, $I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2}$ $\overline{u}(t) = U(t)e^{j\varphi_u(t)}$, $\overline{i}(t) = I(t)e^{j\varphi_i(t)} = I(t)e^{j[\varphi_u(t) + \theta(t)]}$ $\overline{Z}(t) = \dfrac{\overline{u}(t)}{\overline{i}(t)} = \dfrac{U(t)}{I(t)}e^{-j\theta(t)} =$ $R(t) - jX(t) = \sqrt{R^2(t) + X^2(t)}\cdot e^{-j\theta(t)}$

All over this paper are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

4.0.12. Evolution of the RMS concept

Based on the traditional Complex Phasors modeling of sinusoidal currents and voltages we have:

$$\langle p(t)\rangle = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt \ (=) \text{ Average Instantaneous Power}$$

$$\langle p(t)\rangle = \frac{U_{rms}{}^2}{R} = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt = \frac{1}{T}\int_{[T]} u(t)\frac{u(t)}{R}dt =$$

$$= \frac{1}{RT}\int_{[T]} u(t)^2 dt = RI_{rms}{}^2 = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt =$$

$$= \frac{1}{T}\int_{[T]} Ri(t)i(t)dt = \frac{R}{T}\int_{[T]} i(t)^2 dt \ (=) \text{ Average Active Power} \Rightarrow$$

$$\Rightarrow U_{rms} = \sqrt{\frac{1}{T}\int_{[T]} u^2(t)dt}\ ,\ I_{rms} = \sqrt{\frac{1}{T}\int_{[T]} i^2(t)dt}$$

$$P = U_{rms}I_{rms}\cos\theta = \text{Active Average Power}\,(=)\,[W]$$

$$Q = U_{rms}I_{rms}\sin\theta = \text{Reactive Average Power}\,(=)\,[VAR]$$

$$S = \sqrt{P^2 + Q^2} = U_{rms}I_{rms} = \text{Average Apparent Power}\ (=)\,[VA]$$

$$PF = \frac{P}{S} = \cos\theta = \text{Power Factor},$$

$$\bar{U} = \sqrt{2}U_{rms}e^{j\theta_u}, \bar{I} = \sqrt{2}I_{rms}e^{j\theta_i}\ , \bar{I}^* = \sqrt{2}I_{rms}e^{-j\theta_i}\ , \bar{S} = \frac{1}{2}\bar{U}\cdot\bar{I}^*\ .$$

It is important to underline that only the Active Power P is the power delivered to the load, and that the Reactive Power Q is the power reflected from the load and sent back to its energy source. We already know all of that from the basic electro-technique regarding alternative currents and voltages (usually only related to sinusoidal and constant operating frequency signals in electric energy distribution systems). Here we shall extend and generalize the same concept of Active, Reactive and Apparent Power, to the propagation of any, arbitrary shaped and multi-frequency, or large frequency-band signals. The generalization platform for new Active, Reactive and Apparent power definition will be related to Analytic, complex signal (and Hilbert Transform) that gives the possibility to present an arbitrarily shaped time domain signal into the corresponding Complex and Sinusoidal-like signal.

After replacing the Instantaneous Power with its Analytic Signal form, we will find that the traditional concept of RMS currents and voltages remains unchanged:

$$\langle\overline{p}(t)\rangle = \frac{1}{T}\int_{[T]}\overline{p}(t)dt = \frac{1}{T}\int_{[T]}\left|\overline{p}(t)\right|e^{j\varphi_p(t)}dt =$$

$$= \langle p(t)\rangle + j\cdot\langle\hat{p}(t)\rangle \text{ (=) Average Instantaneous Complex Power}$$

$$\langle\overline{p}(t)\rangle = \frac{U_{A-rms}{}^2}{R} + j\frac{U_{R-rms}{}^2}{R} = \frac{1}{T}\int_{[T]}\overline{p}(t)dt = \frac{1}{T}\int_{[T]}p(t)dt + j\frac{1}{T}\int_{[T]}\hat{p}(t)dt =$$

$$= \frac{1}{RT}\int_{[T]}u(t)^2 dt + j\frac{1}{T}\int_{[T]}\hat{p}(t)dt = \frac{U_{rms}{}^2}{R}(1+j) =$$

$$= R\cdot I_{A-rms}{}^2 + jR\cdot I_{R-rms}{}^2 = \frac{R}{T}\int_{[T]}i(t)^2 dt + j\frac{1}{T}\int_{[T]}\hat{p}(t)dt = RI_{rms}{}^2(1+j) \Rightarrow$$

$$\Rightarrow U_{A-rms} = \sqrt{\frac{1}{T}\int_{[T]}u^2(t)dt} = \sqrt{\frac{R}{T}\int_{[T]}p(t)dt} = U_{R-rms} = \sqrt{\frac{R}{T}\int_{[T]}\hat{p}(t)dt} = U_{rms},$$

$$I_{A-rms} = \sqrt{\frac{1}{T}\int_{[T]}i^2(t)dt} = \sqrt{\frac{1}{RT}\int_{[T]}p(t)dt} = I_{R-rms} = \sqrt{\frac{1}{RT}\int_{[T]}\hat{p}(t)dt} = I_{rms},$$

$$\langle\overline{p}(t)\rangle = \langle p(t)\rangle + j\cdot\langle\hat{p}(t)\rangle = \langle p(t)\rangle(1+j) = \langle\hat{p}(t)\rangle(1+j).$$

Based on complex Analytic Signal forms of voltage and current functions we can now generalize the Phasor notation concept by defining Instantaneous, Complex, Apparent, Active and Reactive Power (to apply to arbitrary signal shapes), as follows:

$$\overline{S}(t) = \overline{u}(t)\cdot\overline{i}^*(t) = P(t) - jQ(t) = S(t)e^{-j\theta(t)} = U(t)I(t)e^{-j\theta(t)},$$

$$\theta(t) = \varphi_i(t) - \varphi_u(t) = \operatorname{arctg}\frac{Q(t)}{P(t)} = \operatorname{arctg}\frac{\hat{i}(t)}{i(t)} - \operatorname{arctg}\frac{\hat{u}(t)}{u(t)},$$

$$P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = \text{Power delivered to a load},$$

$$Q(t) = S(t)\sin\theta(t) = \frac{1}{2}(u\hat{i} - \hat{u}i) = \text{Power reflected from a load},$$

$$S(t) = \sqrt{P(t)^2 + Q(t)^2} = \frac{1}{2}U(t)I(t) = \left|\overline{p}(t)\right|\frac{\cos\varphi_p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} =$$

$$= \frac{p(t)}{\cos\varphi_u(t)\cos\varphi_i(t)} = \frac{P(t)}{\cos\theta(t)} = \frac{Q(t)}{\sin\theta(t)},$$

$$PF(t) = \frac{P(t)}{S(t)} = \cos\theta(t) = \frac{ui + \hat{u}\hat{i}}{U(t)I(t)}, \left\{\cos\theta(t) = 1 \Leftrightarrow (u\hat{i} = \hat{u}i)\right\}$$

$$U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2}\left\{= u(t)\sqrt{1 + (\frac{\hat{i}}{i})^2} = u(t)\sqrt{1 + (\frac{\hat{u}}{u})^2}, \text{for } \cos\theta(t) = 1\right\},$$

$$I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2}\left\{= i(t)\sqrt{1 + (\frac{\hat{i}}{i})^2} = i(t)\sqrt{1 + (\frac{\hat{u}}{u})^2}, \text{for } \cos\theta(t) = 1\right\},$$

$$\overline{u}(t) = U(t)e^{j\varphi_u(t)}, \ \overline{i}(t) = I(t)e^{j\varphi_i(t)} = I(t)e^{j[\varphi_u(t)+\theta(t)]},$$

$$\overline{i}^*(t) = I(t)e^{-j\varphi_i(t)} = I(t)e^{-j[\varphi_u(t)+\theta(t)]}$$

$$\overline{Z}(t) = \frac{\overline{u}(t)}{\overline{i}(t)} = \frac{U(t)}{I(t)}e^{-j\theta(t)} = R(t) - jX(t) = \sqrt{R^2(t) + X^2(t)}\cdot e^{-j\theta(t)}$$

$$\left\{= \frac{u(t)}{i(t)}e^{-j\theta(t)}, \text{ for } \cos\theta(t) = 1\right\}.$$

In most energy transfer systems, we do not care too much about immediate, transient, time-domain signals since mathematically it could be complicated to use such functions (especially for arbitrary-shaped signals). What counts much more in the practice of power and energy conversion and distribution systems are effective, rms, mean, averaged and other numerically expressed values (applicable to sufficiently characteristic frequency and time signal intervals).

To avoid time dependence of the above expressions, we shall determine all corresponding average values, as follows:

$$\langle p(t)\rangle = \frac{1}{T}\int_{[T]} p(t)dt = \frac{1}{T}\int_{[T]} u(t)i(t)dt = \frac{1}{T}\int_{[T]} |\overline{p}(t)|\cos\varphi_p(t)dt =$$

$$= \frac{1}{T}\int_{[T]} U(t)I(t)\cos\varphi_u(t)\cos\varphi_i(t)dt =$$

$$= \frac{U_{rms}{}^2}{R} = \frac{1}{RT}\int_{[T]} U(t)^2\cos\varphi^2{}_u(t)dt \;\; = RI_{rms}{}^2 =$$

$$= \frac{R}{T}\int_{[T]} I(t)^2\cos\varphi^2{}_i(t)dt\,(=)\text{ Average Active Power} \Rightarrow$$

$$U_{rms} = \sqrt{\frac{1}{T}\int_{[T]} U(t)^2\cos\varphi^2{}_u(t)dt} = \sqrt{\frac{1}{T}\int_{[T]} u(t)^2 dt}\;,$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_{[T]} I(t)^2(t)\cos\varphi^2{}_i(t)dt} = \sqrt{\frac{1}{T}\int_{[T]} i(t)^2 dt}$$

$$P = U_{rms}I_{rms}\langle\cos\theta(t)\rangle \;\; = \frac{1}{T}\int_{[T]} P(t)dt =$$

$$= \frac{1}{T}\int_{[T]} S(t)\cos\theta(t)dt = \text{Active Average Power}$$

$$Q = U_{rms}I_{rms}\langle\sin\theta(t)\rangle \;\; = \frac{1}{T}\int_{[T]} Q(t)dt =$$

$$= \frac{1}{T}\int_{[T]} S(t)\sin\theta(t)dt = \text{Reactive Average Power}$$

$$\overline{S} = P - jQ = Se^{-j\theta} = \text{ Complex Apparent Power}$$

$$\theta = \operatorname{arctg}\frac{Q}{P} = \operatorname{arctg}\frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle}$$

$$S = \frac{1}{T}\int_{[T]} \sqrt{P(t)^2 + Q(t)^2}dt = U_{rms}I_{rms} =$$

$$= \sqrt{P^2 + Q^2} = \text{ Average Apparent Power}$$

Now it will be possible to extend the meaning of average electrical impedance for the general case of arbitrary voltage and current signals, for instance:

$$\overline{S} = P - jQ = Se^{-j\theta} = ZI^2_{rms}e^{-j\theta} = \frac{U^2_{rms}}{Z}e^{-j\theta} = (R - jX)I^2_{rms}$$

$$\overline{Z} = R - jX = Ze^{-j\theta} = \text{Average Complex Impedance}$$

$$Z = \left|\overline{Z}\right| = \frac{U_{rms}}{I_{rms}} = \sqrt{R^2 + X^2} = \sqrt{\frac{\int_{[T]} u(t)^2 dt}{\int_{[T]} i(t)^2 dt}} = \sqrt{\frac{\int_{[T]} U(t)^2 \cos\varphi^2_u(t)dt}{\int_{[T]} I(t)^2 \cos\varphi^2_i(t)dt}},$$

$$\langle\theta\rangle = \theta = \arctan\frac{X}{R} = \arctan\frac{Q}{P} = \arctan\left[\frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle}\right], \quad \overline{\theta} = \arctan\left[\frac{\overline{\sin\theta(t)}}{\overline{\cos\theta(t)}}\right]$$

$$R = \frac{P}{I^2_{rms}} = Z\cdot\langle\cos\theta(t)\rangle = \text{Average Active (Real) Impedance}$$

$$X = \frac{Q}{I^2_{rms}} = Z\cdot\langle\sin\theta(t)\rangle = \text{Average Reactive (Imaginary) Impedance, or Reactance}$$

$$\frac{X}{R} = \frac{Q}{P} = \frac{\langle\sin\theta(t)\rangle}{\langle\cos\theta(t)\rangle} = \tan\theta = \text{Average Quality Factor}, \quad \tan\overline{\theta} = \frac{\overline{\sin\theta(t)}}{\overline{\cos\theta(t)}}$$

$$\left\{\cos\theta(t), \langle\cos\theta(t)\rangle, \overline{\cos\theta(t)}, \cos\overline{\theta}\right\} = \text{Different Power Factors}$$

Practically, in power management systems (after introducing Analytic Signal methodology) we shall be able to apply innovative concepts based on averaged and rms signal values, easily measurable using existing technology, and valid for arbitrary-shaped signals (without the need to have precise time and frequency expressions). This way, many of traditionally known concepts of power and frequency regulation will be generalized and could be significantly optimized. In addition, Quantum mechanical energy exchanges, quantum states and wave functions can be explained using "*active and reactive wave functions*" (like active and reactive power components). Stable atoms could be described as reactive resonant (multi-dimensional or multi-component) circuit structures, where Quality Factor = $\tan\theta$, approaches infinity ($\theta \to \frac{\pi}{2}$), or where all internal, stationary atom waves and other movements behave (analogically) like currents and voltages in simple loss-less capacitive-inductive resonant-oscillating circuits, without resistive components.

Here we will attempt to connect arbitrary Power Function (product between current and voltage, or product between force and speed, or product between any other relevant, mutually conjugate functions creating power) to a wave function, as known in Quantum Mechanics. Energy-wise analyzed, any wave propagation in time and frequency domain can be mutually (time-frequency) correlated using Parseval's theorem. Consequently, the immediate (time-domain) Power-signal can be presented as the square of the wave-function $\Psi^2(t)$, *and analysis of the optimal power transfer can be extended to any wave propagation field (and to arbitrarily shaped signals), and profit enormously (in booth, directions) after unifying traditional concepts of Active, Reactive and Apparent power with the* $\Psi^2(t)$ *wave-function mathematics, based on Analytic Signal methodology.*

In Quantum Mechanics the wave function $\Psi^2(\mathbf{t})$ is conveniently modeled as a probability function, but in many other aspects, effectively behaves like by total energy normalized, and dimensionless Power function, and here it will be closely related to an effective Power, or power delivered to certain load (expressed in Watts as its units).

Let us start from the immediate electrical power found as a product between corresponding voltage and current signals, where both are arbitrarily shaped (energy, time and frequency limited) functions. We can show that such active-power function (which transfers the power from its source to its load) can be presented as,

$$\Psi^2(\mathbf{t}) = \mathbf{P(t)} = \mathbf{S(t)}\cos\theta(\mathbf{t}) = \frac{1}{2}(\mathbf{ui} + \hat{\mathbf{u}}\hat{\mathbf{i}}) = \mathbf{Q(t)} \cdot \mathbf{cotan}\,\theta(\mathbf{t})\ (=)[\mathbf{W}] .$$

The power reflected from a load, or Reactive Power, can be given as:

$$\mathbf{Q(t)} = \mathbf{S(t)}\sin\theta(\mathbf{t}) = \frac{1}{2}(\mathbf{u}\hat{\mathbf{i}} - \hat{\mathbf{u}}\mathbf{i}) = \Psi^2(\mathbf{t}) \cdot \tan\theta(\mathbf{t}) = \mathbf{P(t)} \cdot \tan\theta(\mathbf{t})\ (=)\ [\mathbf{VAR}]$$

Electric Power and Energy transfer analysis (especially for arbitrary voltage and current signal forms) can be related to a wave-function analysis if we establish the wave function (or more precisely, the square of the wave function) in the following way:

$$\mathrm{P(t)} = \Psi^2(\mathrm{t}) = [\mathrm{a(t)}\cos\varphi(\mathrm{t})]^2 = \text{Wave function}\ ,\ \mathrm{t} \in [\mathrm{T}],$$

$$\Psi(\mathrm{t}) = \mathrm{a(t)}\cos\varphi(\mathrm{t}),\ \hat{\Psi}(\mathrm{t}) = \mathrm{a(t)}\sin\varphi(\mathrm{t}),$$

$$\overline{\Psi}(\mathrm{t}) = \Psi(\mathrm{t}) + \mathrm{j}\hat{\Psi}(\mathrm{t}) = \Psi(\mathrm{t}) + \mathrm{jH}[\Psi(\mathrm{t})] = \mathrm{a(t)}e^{\mathrm{j}\varphi(\mathrm{t})} = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} \mathrm{U}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}d\omega =$$

$$= \frac{1}{\pi^2}\int_0^{+\infty} \mathrm{U}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}d\omega = \frac{1}{\pi}\int_{(0,+\infty)} \mathrm{A}(\omega)e^{-\mathrm{j}\omega\mathrm{t}}d\omega,$$

$$\mathrm{U}(\omega) = \mathrm{U_c}(\omega) - \mathrm{j\,U_s}(\omega) = \int_{(-\infty,+\infty)} \overline{\Psi}(\mathrm{t})\ e^{\mathrm{j}\omega\mathrm{t}}d\mathrm{t} = \mathrm{A}(\omega)e^{-\mathrm{j}\Phi(\omega)}\ ,$$

$$\mathrm{U_c}(\omega) = \mathrm{A}(\omega)\cos\Phi(\omega),\ \mathrm{U_s}(\omega) = \mathrm{A}(\omega)\sin\Phi(\omega),$$

$$\mathrm{a(t)} = \sqrt{\Psi(\mathrm{t})^2 + \hat{\Psi}(\mathrm{t})^2}\ ,\quad \varphi(\mathrm{t}) = \mathrm{arctg}\frac{\hat{\Psi}(\mathrm{t})}{\Psi(\mathrm{t})}\ ,$$

$$\mathrm{A}^2(\omega) = \mathrm{U^2}_c(\omega) + \mathrm{U^2}_s(\omega),\quad \Phi(\omega) = \mathrm{arctg}\frac{\mathrm{U_s}(\omega)}{\mathrm{U_c}(\omega)}\ ,$$

$$\mathrm{T}\cdot\langle \mathrm{P(t)}\rangle = \int_{-\infty}^{+\infty} \mathrm{P(t)dt} = \int_{-\infty}^{+\infty} \Psi^2(\mathrm{t})\mathrm{dt} = \int_{-\infty}^{+\infty} \hat{\Psi}^2(\mathrm{t})\mathrm{dt} = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(\mathrm{t})\right|^2 \mathrm{dt} =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} \left|\Psi(\mathrm{t}) + \mathrm{j}\hat{\Psi}(\mathrm{t})\right|^2 \mathrm{dt} = \mathrm{T}\cdot\langle \hat{\mathrm{P}}(\mathrm{t})\rangle = \int_{-\infty}^{+\infty} \hat{\mathrm{P}}(\mathrm{t})\mathrm{dt} =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} \mathrm{a}^2(\mathrm{t})\mathrm{dt} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |\mathrm{U}(\omega)|^2 d\omega = \frac{1}{\pi}\int_0^{\infty} [\mathrm{A}(\omega)]^2 d\omega,$$

As we can see, any physics-related and finite wave function Ψ has its Hilbert couple $\hat{\Psi}$. Both are presentable as a product of two other functions, since to create an instantaneous (short time) power function $\Psi^2(t)$, (see (4.0.4)) it is essential to make the product of two relevant, mutually conjugate signals, like current and voltage, velocity and force, or some other equally important and analogous couple of conjugate signals (see 4.0.82)).

Consequently, short time active power $P(t)$ in electric circuits is,

$$\overline{P}(t) = \frac{1}{2}\overline{u}(t)\cdot\overline{i}(t)\cdot\cos\theta_{Load}(t) \Rightarrow$$

$$P(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = S(t)\cos\theta_{Load}(t) = \Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t),\ \Psi_1^2 = \frac{ui}{2},\ \Psi_2^2 = \frac{\hat{u}\hat{i}}{2}.$$

Instantaneous current and/or voltage frequency is (see (4.0.2)),

$$\omega(t) = \frac{\partial\varphi(t)}{\partial t} = \dot{\varphi}(t) = \frac{\Psi(t)\dot{\hat{\Psi}}(t) - \hat{\Psi}(t)\dot{\Psi}(t)}{a^2(t)} = \operatorname{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right] = 2\pi f(t) \Rightarrow$$

$$\omega_i(t) = \frac{\partial\varphi_i(t)}{\partial t} = \dot{\varphi}_i(t) = \frac{i(t)\frac{d}{dt}\hat{i}(t) - \hat{i}(t)\frac{d}{dt}i(t)}{I^2(t)} = 2\pi f_i(t),$$

$$\omega_u(t) = \frac{\partial\varphi_u(t)}{\partial t} = \dot{\varphi}_u(t) = \frac{u(t)\frac{d}{dt}\hat{u}(t) - \hat{u}(t)\frac{d}{dt}u(t)}{U^2(t)} = 2\pi f_u(t),$$

$$\sin(\varphi_u - \varphi_i) = \frac{i(t)u(t) - i(t)u(t)}{I(t)\cdot U(t)} = \frac{i(t)u(t) - i(t)u(t)}{\sqrt{i(t)^2 + \hat{i}(t)^2}\cdot\sqrt{u(t)^2 + \hat{u}(t)^2}}.$$

For the same electric circuit with the same source and same load, current and voltage frequencies should be the same.

"Short-Time" Magnitude of the electric-circuit Load Impedance is,

$$M_{stZ(t)} = \frac{U(t)}{I(t)} = \frac{\sqrt{u(t)^2 + \hat{u}(t)^2}}{\sqrt{i(t)^2 + \hat{i}(t)^2}} = \left|\overline{Z}(t)\right|.$$

It should not be too big success to explain quantum-mechanical diffraction causally, or to explain superposition and interference effects, when a "single-wave-object" and/or a single particle (like an electron, or photon) passes a diffraction plate with (at least) two small, diffraction holes, because there is not a single particle or wave object here. There are always minimum two of mutually Hilbert-coupled wave elements and their mixed wavelike products, in some way being force-energy-coupled with their environment, this way extending the number of interaction participants, and we need all of them to find resulting amplitude, phase and frequency functions). What could look like a bit unusual quantum interaction, or interference of a single wave or particle with itself, in fact, presents an interaction of at least two wave entities with some other, third object, here with a plate with diffraction holes ($\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$). Somehow Nature always creates complementary and conjugate couples of essential wave elements (signals, particles, energy states…) belonging to all kind of matter motions, or we can also say that every object (or energy state) in our universe has its non-separable and conjugate, phase-shifted image (defined by an Analytic Signal concept). Consequently, the quantum-mechanical wave function and wave energy should represent only motional energy (or power) composed of minimum two mutually coupled wave functions (see also relevant comments about the meaning of complex functions and Analytic signal components given below T.4.0.1).

Here applied mathematics, regarding wave functions $\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$, after making appropriate normalization/s and generalizations, would start to look as applying Probability Theory laws, like in the contemporary Quantum Theory. Consequently, modern Quantum Theory could also be understood as generalized mathematical modeling of microworld phenomenology, which conveniently unifies all conservation laws of physics in a joint, normalized, and dimensionless, mutually well-correlated theoretical platform, using the framework of Statistics, Probability, and modern Signal Analysis. In this way, a new mathematical theory is created, that is only a bit unusual and strange by its appearance, but it is isomorphic to usual mathematical modeling as known in the deterministic and classical Physics structure.

Also, a kind of generalized analogy with Norton and Thevenin's theorems (known in Electric Circuit Theory) should also exist (conveniently formulated) in all other fields of Physics and Quantum Theory related to wave motions, since the cause or source of a particular action is producing specific effect (or an output), and vice versa and such events are always mutually coupled.

See more in: [57], Michael Feldman, [109], Poularikas A. D., including [110], and [111]

[♣ COMMENTS & FREE-THINKING CORNER:

We should try again (and better than before) to address a complex power in electric circuits, as a product between two complex phasor functions, as for example,

$$\left\{\begin{array}{l} \overline{P}(t) = \frac{1}{2}\overline{u}(t)\cdot\overline{i}(t),\ \overline{u}(t) = Ue^{j[\varphi(t)+\varphi_u]},\ \overline{i}(t) = Ie^{j[\varphi(t)+\varphi_i]},\ j^2 = -1,\ (\varphi_u, \varphi_i) = \text{constants} \\ \overline{P}(t) = \frac{1}{2}\overline{u}(t)\cdot\overline{i}(t) = \frac{1}{2}U\cdot I\cdot e^{j[2\varphi(t)+\varphi_u+\varphi_i]} = \frac{1}{2}U\cdot I\cdot e^{j\theta_{Load}(t)} = \frac{1}{2}U\cdot I\cdot\cos\theta_{Load}(t) + j\frac{1}{2}U\cdot I\cdot\sin\theta_{Load}(t) \\ \theta_{Load}(t) = 2\varphi(t) + \varphi_u + \varphi_i,\ S(t) = \frac{1}{2}U\cdot I = \frac{U}{\sqrt{2}}\cdot\frac{I}{\sqrt{2}} = U_{rms}\cdot I_{rms}, \ldots \varphi(t) = \omega t\ \ or\ \ \varphi(t,x) = \omega t \pm kx \end{array}\right\},$$

$$P(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = \frac{1}{2}U\cdot I\cdot\cos\theta_{Load}(t) = S(t)\cos\theta_{Load}(t) = \Psi^2(t) = \Psi_u(t)\cdot\Psi_i(t).$$

or to present complex power, alternatively and simply as, $\overline{P}(t) = \overline{u}(t)\cdot\overline{i}(t) = U\cdot I\cdot e^{j[2\varphi(t)+\varphi_U+\varphi_I]} = \overline{\Psi}^2(t)$*,* $\overline{P}(t) = \overline{u}(t)\cdot\overline{i}(t) = U\cdot I\cdot e^{j[2\varphi(t)+\varphi_U+\varphi_I]}$*,* $\overline{\Psi}(t) = \left|\overline{\Psi}(t)\right|\cdot e^{j\left[\varphi(t)+\frac{\varphi_u+\varphi_i}{2}\right]}$ *since it is obvious here that a phase functions* $\varphi(t)$*, and* $\theta_{Load}(t) = \varphi(t) + \frac{\varphi_u + \varphi_i}{2}$ *should be more appropriately taken into account … (still working on).*

4.0.12.1. Standard Deviation concerning Load Impedance

*Since the standard deviation is often used to explain Uncertainty relations, let us analyze the meaning of the signal Standard Deviation regarding electric circuits where it is possible to measure voltage, **u**, and current, **i**, on an electric load. An electric load can have entirely resistive (or active) impedance, **R**, or in general case, it can present the complex impedance, **Z** (as the combination of **R**, **L** and **C** elements).*

The Standard Deviation presents the power of the average signal deviation from the mean power, and since we can measure coincidently (by sampling) the load current and voltage (both mutually dependent), we are in the position to formulate the following generalized expression for Standard Deviation, by multiplying voltage and current deviations,

$$\sigma_g^2 = \frac{1}{N}\sqrt{\sum_{(i)}\left|(u_i-\bar{u})(i_i-\bar{i})\right|^2} = \frac{1}{N}\sqrt{\sum_{(i)}(u_i-\bar{u})^2(i_i-\bar{i})^2}$$

$$N\,(=)\text{ number of samples, } \bar{u}=\frac{1}{N}\sum_{(i)}u_i\,,\ \bar{i}=\frac{1}{N}\sum_{(i)}i_i \qquad (4.0.105)$$

$$\bar{u},\ \bar{i}\,(=)\text{ runnung mean values (time evolving)}$$

Only in cases when the load is resistive, above-formulated Standard Deviation, (4.0.105) can be transformed into the following expressions (fully equivalent to the contemporary definition of the Standard Deviation),

$$u = Ri,\ \bar{u} = R\bar{i} \Rightarrow \sigma_g^2 \to \sigma^2 \Rightarrow$$

$$\sigma^2 = \frac{1}{N}\frac{1}{R}\sum_{(i)}\left|u_i-\bar{u}\right|^2 = \frac{1}{N}R\sum_{(i)}\left|i_i-\bar{i}\right|^2 = \frac{1}{N}\frac{1}{R}\sum_{(i)}(u_i-\bar{u})^2 = \frac{1}{N}R\sum_{(i)}(i_i-\bar{i})^2$$

$$\sigma^2 = \frac{1}{N}\frac{\bar{i}}{\bar{u}}\sum_{(i)}(u_i-\bar{u})^2 = \frac{1}{N}\frac{\bar{u}}{\bar{i}}\sum_{(i)}(i_i-\bar{i})^2 = \frac{1}{N}\sum_{(i)}\frac{i_i}{u_i}(u_i-\bar{u})^2 = \frac{1}{N}\sum_{(i)}\frac{u_i}{i_i}(i_i-\bar{i})^2 \qquad (4.0.106)$$

$$\sigma^2 = \frac{1}{N}\sqrt{\sum_{(i)}(u_i-\bar{u})^2\cdot\sum_{(i)}(i_i-\bar{i})^2}$$

*If after sampling of a load current and voltage we find that expressions (4.0.105) and (4.0.106) would produce significantly different results, this should mean that the load impedance has non-resistive, complex character (presenting a combination of **R**, **L** and **C** elements). The factor of "Impedance Complexity" concerning measured standard definition can be defined as,*

$$\frac{\sigma^2}{\sigma_g^2} = \sqrt{\frac{\sum_{(i)}(u_i-\bar{u})^2\cdot\sum_{(i)}(i_i-\bar{i})^2}{\sum_{(i)}(u_i-\bar{u})^2\cdot(i_i-\bar{i})^2}} \begin{Bmatrix} =1\to\text{Resistive or Active Load} \\ \neq 1\to\text{Complex Load}\end{Bmatrix} \qquad (4.0.107)$$

In other words, when the ratio (4.0.107) is not equal 1, contemporary formulation of Standard Deviation (found in all Statistics books) is not the best (and not the most general) representation of the natural signal deviation. If we (analogically) apply the same situation on arbitrary signals, we could say that the complexity of any signal, obtained as a product of two mutually conjugate signals (like found in (4.0.82)), can be tested and additionally classified by (4.0.105), (4.0.106) and (4.0.107). Consequently, often used Standard Deviation in Quantum Theory and Physics in many cases could be very much limited (or in some cases wrong), since not all loads are resistive (electrically, or mechanically, or in some other analogical meaning). Many essential laws of Statistics, Thermodynamics, Quantum Theory, etc., such as Normal Gauss distribution, Black body radiation law, Uncertainty Relations, etc. are already formulated using the traditional definition of the Standard Deviation. Such standard deviation is intrinsically limited to what analogically or directly corresponds to active or resistive loads (where functions, defining the signal power, are in phase), what is influencing that some of the conclusions based on such laws could be wrong (the fact never noticed, because of the present incomplete definition of the Standard Deviation). ♣]

4.0.12.2. New definitions for Average, RMS, and Power functions

Let us take into consideration electric voltage and current on specific load (related to useful measurements and power quantification). Later, when using established systems of electro-mechanical analogies, it will be possible to extend similar definitions to velocity, force, torque, etc. Standard definitions of average and RMS functions are already known from the literature. Current and voltage signals on specific load are mutually dependent (and load impedance dependent), and what should matter in such situations is to consider immediate electrical power, since existing definitions of average and RMS values are not taking directly and explicitly the load power. Here are proposals how to redefine electrical load power, using new RMS functions for voltage and current.

Signal	AVERAGE	RMS
$\mathbf{u(t),\ \hat{u}(t) = H[u(t)]}$, $\mathbf{\bar{u}(t) = u(t) + j\cdot\hat{u}(t) = U(t)e^{j\varphi_u(t)}}$, $\mathbf{U(t) = \sqrt{u(t)^2 + \hat{u}(t)^2}}$ (=) voltage, $\mathbf{j^2 = -1}$	$\langle \mathbf{u} \rangle = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u(t)dt}$, $\langle \hat{\mathbf{u}} \rangle = \frac{1}{T}\int_{t_0}^{t_0+T} \hat{\mathbf{u}}\mathbf{(t)dt}$	$\mathbf{u_{RMS}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{u^2(t)dt}}$, $\hat{\mathbf{u}}_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \hat{\mathbf{u}}^2\mathbf{(t)dt}}$, $\mathbf{U_{RMS}} = \sqrt{\mathbf{u^2_{RMS}} + \hat{\mathbf{u}}^2_{RMS}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T}\left[\mathbf{u^2(t)} + \hat{\mathbf{u}}^2\mathbf{(t)}\right]\mathbf{dt}}$
$\mathbf{i(t), \hat{i}(t) = H[i(t)]}$, $\mathbf{\bar{i}(t) = i(t) + j\cdot\hat{i}(t) = I(t)e^{j\varphi_i(t)}}$ $\mathbf{I(t) = \sqrt{i(t)^2 + \hat{i}(t)^2}}$ (=) current, $\mathbf{j^2 = -1}$	$\langle \mathbf{i} \rangle = \frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{i(t)dt}$, $\langle \hat{\mathbf{i}} \rangle = \frac{1}{T}\int_{t_0}^{t_0+T} \hat{\mathbf{i}}\mathbf{(t)dt}$	$\mathbf{i_{RMS}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \mathbf{i^2(t)dt}}$, $\hat{\mathbf{i}}_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \hat{\mathbf{i}}^2\mathbf{(t)dt}}$, $\mathbf{I_{RMS}} = \sqrt{\mathbf{i^2_{RMS}} + \hat{\mathbf{i}}^2_{RMS}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T}\left[\mathbf{i^2(t)} + \hat{\mathbf{i}}^2\mathbf{(t)}\right]\mathbf{dt}}$
$\mathbf{p(t) = u(t)\cdot i(t) = \Psi^2(t)}$, $\hat{\mathbf{p}}\mathbf{(t) = H[p(t)]}$, $\mathbf{\bar{P}(t) = \hat{p}(t) + j\hat{p}(t) = \bar{\Psi}^2(t)}$, $\mathbf{p^*(t) = \frac{1}{2}[p(t) + \hat{p}(t)] = (\Psi^*(t))^2}$, $\mathbf{\bar{P}^*(t) = \bar{u}(t)\cdot\bar{i}(t) = (\bar{\Psi}^*(t))^2 =}$ $\mathbf{= U(t)\cdot I(t)e^{j[\varphi_u(t)+\varphi_i(t)]}}$	$\langle p \rangle = \langle u \rangle \cdot \langle i \rangle$, $\langle \hat{p} \rangle = \langle \hat{u} \rangle \cdot \langle \hat{i} \rangle$, $\langle P \rangle = \frac{1}{2}(\langle p \rangle + \langle \hat{p} \rangle)$, $\langle P* \rangle = \frac{1}{T}\int_{t_0}^{t_0+T}\left[\frac{u(t)\cdot i(t) + \hat{u}(t)\cdot\hat{i}(t)}{2}\right]dt$	$\mathbf{U_{RMS}\cdot I_{RMS}} = \sqrt{(\mathbf{u^2_{RMS}} + \hat{\mathbf{u}}^2_{RMS})\cdot(\mathbf{i^2_{RMS}} + \hat{\mathbf{i}}^2_{RMS})} =$ $= \mathbf{P_{RMS}}$ (=) **RMS Power**

*For an arbitrary-shaped electric signal, **RMS Power** could be an optimal effective, apparent power, such as,*

$$\mathbf{P_{RMS} = U_{RMS}\cdot I_{RMS}} = \sqrt{(\mathbf{u^2_{RMS}} + \hat{\mathbf{u}}^2_{RMS})\cdot(\mathbf{i^2_{RMS}} + \hat{\mathbf{u}}^2_{RMS})},$$

$$\langle \mathbf{P^*_{RMS}} \rangle = \frac{1}{T}\int_{t_0}^{t_0+T}\frac{\mathbf{p(t)\cdot\hat{p}(t)}}{\mathbf{U_{RMS}\cdot I_{RMS}}}\mathbf{dt}\ . \qquad (4.0.108)$$

Of course, benefits and applicability of such power definitions should be more rigorously tested.

<u>Coherence factors:</u>

Between two signals, $s_1(t)$ and $s_2(t)$ *it is possible to find the measure of their mutual time and spectral coherence, using the following coherence factors definitions:*

$$K_t = \frac{\int_{-\infty}^{+\infty} s_1(t)\cdot s_2(t)\cdot dt}{\int_{-\infty}^{+\infty} |s_1(t)|\cdot|s_2(t)|\cdot dt},\quad K_\omega = \frac{\int_{0}^{+\infty} A_1(\omega)\cdot A_2(\omega)\cdot d\omega}{\int_{0}^{+\infty} |A_1(\omega)|\cdot|A_2(\omega)|\cdot d\omega},$$

$$\tilde{K}_t = \frac{\int_{-\infty}^{+\infty} \tilde{s}_1(t)\cdot \tilde{s}_2(t)\cdot dt}{\int_{-\infty}^{+\infty} |\tilde{s}_1(t)|\cdot|\tilde{s}_2(t)|\cdot dt},\quad \tilde{K}_\omega = \frac{\int_{0}^{+\infty} \tilde{A}_1(\omega)\cdot \tilde{A}_2(\omega)\cdot d\omega}{\int_{0}^{+\infty} |\tilde{A}_1(\omega)|\cdot|\tilde{A}_2(\omega)|\cdot d\omega}, \qquad (4.0.83),\ (4.0.87)$$

$$K_t^* = \sqrt{K_t\cdot\tilde{K}_t}\,,\qquad K_\omega^* = \sqrt{K_\omega\cdot\tilde{K}_\omega}\,.$$

Consequently, for electric signals, we have,

$$K_t = \frac{\int_{-\infty}^{+\infty} u(t)\cdot i(t)\cdot dt}{\int_{-\infty}^{+\infty} |u(t)|\cdot|i(t)|\cdot dt} = \frac{\int_{-\infty}^{+\infty} p(t)\cdot dt}{\int_{-\infty}^{+\infty} |u(t)|\cdot|i(t)|\cdot dt} \cong \frac{\int_{t_0}^{t_0+T} p(t)\cdot dt}{\int_{t_0}^{t_0+T} |u(t)|\cdot|i(t)|\cdot dt}. \qquad (4.0.109)$$

Now we can draw important conclusions concerning coherence criteria for certain signals $s_1(t)$ and $s_2(t)$, or $u(t)$ and $i(t)$ and connect them with the optimal power and energy propagation:

-If $K_t = 1$, $s_1(t)$ and $s_2(t)$ are totally coherent, or mutually *in phase, $\Delta\varphi(t) = 0$. If $s_1(t)$ and $s_2(t)$ are voltage and current on certain electric load, we will have optimal energy (or power) transfer (see (4.0.109)). That is the case of active or resistive load impedances. The total power delivered to a load is active.*

-If $K_t = 0$, $s_1(t)$ and $s_2(t)$ *are mutually orthogonal or phase* shifted *for $\pi/2$, $|\Delta\varphi(t)| = \pi/2$. If $s_1(t)$ and $s_2(t)$ are voltage and current on certain electric load (see (4.0.109)), no energy circulation from the source to the* load *is possible (i.e., the* load *does not receive any energy).*

-If $K_t = -1$, $s_1(t)$ and $s_2(t)$ are in the counter-phase, $|\Delta\varphi(t)| = \pi$. If $s_1(t)$ and $s_2(t)$ are voltage and current on certain electric load (see (4.0.109)), then there is a non-optimal energy transfer, i.e., the energy is completely reflected off the load and returned to its source (the energy has a "-" sign). That is the case of fully reactive load impedances.

....

In any other case when K_t is not equal to 1, 0, or -1, part of power is delivered to a load and part reflected towards its source. Those are cases of complex load impedances (with active and reactive loading components).

Such coherence functions or factors are interesting concerning different signal analysis and measurements strategies, where we need to separate or extract signals (from the specific multicomponent, complex signal) that are mutually dependent and correlated, or we can also extract signals that could be very much mutually independent. For instance, between two Complex Analytic signals, or wave functions, $\overline{\Psi}_1(x,t)$ and $\overline{\Psi}_2(x,t)$ it is possible (on a similar way as already elaborated) to find the measure of their mutual spatial-temporal and spectral coherence, orthogonality, and entanglement, using the following coherence factors definitions:

$$\overline{K}_{r,t} = \frac{\iiint_{(-\infty,+\infty)} \overline{\Psi}_1(r,t)\cdot\overline{\Psi}_2(r,t)\cdot dr\cdot dt}{\iiint_{(-\infty,+\infty)} |\overline{\Psi}_1(r,t)|\cdot|\overline{\Psi}_2(r,t)|\cdot dr\cdot dt},\quad \overline{K}_{k,\omega} = \frac{\iiint_{(-\infty,+\infty)} \overline{U}_1(k,\omega)\cdot\overline{U}_2(k,\omega)\cdot dk\cdot d\omega}{\iiint_{(-\infty,+\infty)} |\overline{U}_1(k,\omega)|\cdot|\overline{U}_2(k,\omega)|\cdot dk\cdot d\omega}$$

*If we give a freedom to certain creative brainstorming and imagination, we could consider coherence factor K_t as a replacement for **Power Factor**, or "cosines-theta" factor (in an analogical relation to a standard, existing definitions of Active, Reactive and Apparent power),*

$$K_t \cong \frac{\int_{t_0}^{t_0+T} p^*(t)\cdot dt}{\int_{t_0}^{t_0+T} |\overline{u}(t)|\cdot|\overline{i}(t)|\cdot dt} (\cong) \cos\theta = PF \Rightarrow \quad (4.0.110)$$

$$\Rightarrow \sin\theta \cong \sqrt{1-(\cos\theta)^2} (\cong) \sqrt{1-\left[\frac{\int_{t_0}^{t_0+T} p^*(t)\cdot dt}{\int_{t_0}^{t_0+T} |\overline{u}(t)|\cdot|\overline{i}(t)|\cdot dt}\right]^2} = \sqrt{1-K_t^2}\ .$$

What is interesting (to explore) in (4.0.109) and (4.0.110), is that Power Factor can take positive values between 0 and 1, and negative values between 0 and -1.

Now, combining Average and RMS Power (4.0.108), and newly introduced Power Factor (4.0.110), we could exercise if a redefinition of Active, Reactive and Apparent power will be defendable, such as,

$$\mathbf{P_{active}} = \mathbf{P_{RMS}\cdot\cos\theta = U_{RMS}I_{RMS}\cdot\cos\theta\ (=)\frac{K_t}{T}\int_{t_0}^{t_0+T}\frac{P^2(t)}{U_{RMS}\cdot I_{RMS}}dt},$$

$$\mathbf{P_{reactive}} = \mathbf{P_{RMS}\cdot\sin\theta = U_{RMS}I_{RMS}\cdot\sin\theta\ (=)\frac{\sqrt{1-K_t^2}}{T}\int_{t_0}^{t_0+T}\frac{P^2(t)}{U_{RMS}\cdot I_{RMS}}dt}, \quad (4.0.111)$$

$$\mathbf{P_{apparent}} = \sqrt{(\mathbf{P_{active}})^2+(\mathbf{P_{reactive}})^2} = \mathbf{P_{RMS} = U_{RMS}I_{RMS}}\ .$$

The meanings of positive and negative power components could be as follows,

-If $\mathbf{P_{active}}$ is taking positive values between 0 and 1, active power is being delivered from its source to its load.

-If $\mathbf{P_{active}}$ is taking negative values between 0 and -1, active power is being reflected or delivered from its load to its source.

-If $\mathbf{P_{reactive}}$ is taking positive values between 0 and 1, reactive power is being reflected from its load to its source.

-If $\mathbf{P^*_{reactive}}$ is taking negative values between 0 and -1, reactive power is being delivered from its source to its load.

Such variety of positive and negative power components can be explicable if we consider that both, energy source and its load, have complex impedance components, and if phasors of electric currents and voltages are arbitrary shaped and wideband (or non-simple harmonic) signals. Coherence factors in original and spectral, temporal, and spatial domains, are basically addressing resonance effects and level of mutual synchronization between two wave functions, signals, or power-function members (like a current and voltage on certain load). This (about resonance and synchronization) can be related to simple harmonic signals, but also to much more complex and spectrally wideband signals, and it is very much linked to entanglement effects (in its wider meaning). The unity of our Universe and all matter wave phenomenology withing the same Universe can be characterized with here introduced coherence factors.

Analyses and operations related to quantum-mechanical wave functions are (theoretically and conceptually) enormously "mutilated" and oversimplified, if we neglect variety of here-presented analytic signals and phasors analysis and qualification options. Methods and models of Statistics and Probability theory can always be particularly useful conclusive and successful if applied in the last instances of modeling, calculations and analyzes based on complex, deterministic and analytic PHASORS (and not in the very first and basic instances, as imposed in the contemporary Orthodox Quantum Theory).

4.1. MATTER WAVES AND QUANTUM MECHANICS

This chapter presents the natural continuation of all ideas and theoretical step-stones about wave-particle duality as already elaborated in Chapter 2 (especially in “2.3.2. Macro-Cosmological Matter-Waves and Gravitation”), and in chapter 4.0 (“Wave functions, wave velocities and uncertainty relations”). It is strongly recommendable, first to read and understand the basics of particle-wave duality found in the chapters 2., 4.0., and 10., and later, many of new ideas and concepts from this chapter would be much easier to understand and accept. The next challenging opportunity (presented in this chapter) will be to extend and upgrade the meaning of de Broglie concept of particle-wave duality (briefly saying, by losing its present duality towards much higher unity, and penetrating its applicability from a subatomic micro-world to a macro-world of planets, stars, and galaxies), thus suggesting the platform for creating new Unified Field Theory. Also, significant conceptual upgrade (and modification) of the foundations of Quantum Theory (that is presently the principal platform where particle-wave duality is scientifically addressed) will be initiated in this chapter. Especially ***we will search what the links between wave and particle properties are, and where, when, or how certain wave-group starts becoming a particle, or a stable mass (and vice versa).***

As we can see from previous chapters, the mainstream elaborations of this book are different attempts to explore:

-***Gravity and Particle–Wave Duality,*** based on specific matter-wave couplings between linear and angular motions (such as rotations and spinning, including coupled or associated electric and magnetic fields and charges). This is analogically applicable to micro and macro world phenomenology (because active and essential natural field’s charges influencing or producing gravitation are electric and magnetic flux, electric charges, and linear and angular moments of motional states (and not at all electromagnetically neutral, static or rest masses); -see tables of analogies in the first chapter, presenting active fields charges). All mentioned natural field charges should be mutually coupled, as governed by Uncertainty relations and wave-particle duality relations.

-***Masses as specific, dominantly, and fundamentally electromagnetic energy packing of energy-momentum, formatted matter-waves states,*** based on self-stabilized agglomerations and couplings of spinning and other matter-wave states and active natural fields charges (structured as self-closed standing-waves resonant states). Static, heavy, electromagnetically neutral, and mechanically stable rest masses are not typical states in our Universe, and not primary sources of Gravitation, but motional masses with (internally and externally) associated linear, angular, and electromagnetic moments (meaning fluxes, charges, moments and dipoles) are.

-***Matter Waves*** as motional energy states are related to space-time proportionality, and energy-momentum exchanges (or communications) between different motional matter states, being equally present in a micro and macro world of Physics. Stable rest masses do not belong (or contribute) to matter waves energy, and only kinetic energy of motional masses (in different forms) presents matter waves energy. All waves and oscillations known in Physics are matter-wave states (like waves in acoustics, mechanical vibrations, waves and turbulences in fluids and plasma, electromagnetic waves...), and all of them should be explained and modeled using the same mathematics based on Analytic Signal framework and Classical wave equation. Uncertainty Relations are also and equally valid and applicable, both for micro and macro world of Physics. Specific and self-stabilized (or self-closed and properly packed) matter-waves and resonant configurations (like standing waves) are creating solid matter, or rest masses states.

-Contemporary knowledge about unique and exclusive existence of four fundamental forces of nature (such as Electromagnetic, Gravitational, Strong and Weak Nuclear forces), will be (one day) conceptually much better explained, significantly modified, or effectively **replaced by forces within (or between) matter-waves and standing-waves structures around nodal or stationary zones of energy and mass-distribution gradients** (showing, most probably, that all of them will have a fundamentally or ontologically, an electromagnetic origin or nature). Particles, within such (standing waves) concept, are mass or energy agglomerations around nodal zones of relevant standing matter waves.

As the historical and state of the art background of matter-waves and wave-particle duality theory, we can read the following citation from [124]:

C Matter waves

"As explained in Introduction, the wave-particle duality is commonly associated with both light and matter, but in the thesis our attention has been restricted to light only. However, in several places (Chapter 3.4 and Chapter 9.3) we are nonetheless forced to refer to the duality of matter. Therefore, for the sake of completeness, in the following appendix we give a strongly abbreviated presentation of the subject, both from the theoretical and from the experimental side.

It was the French physicist and nobleman Louis de Broglie who in his doctoral thesis in 1924 presented the revolutionary idea that all matter had a wavelike nature. This conceptual breakthrough, confirmed in an electron diffraction experiment due to Lester Germer and Clinton Davisson three years later, paved way for the further development of quantum mechanics in the late 20s and the 30s. The so-called de Broglie relations, put in a very simple but strictly mathematical form, assign to every physical particle (like an electron) a wavelength and a frequency. These parameters can then be used to anticipate and describe the diffractive behavior of the particles.

The basic postulate is this: Given a physical object with momentum p and total energy E, we relate to it a wavelength λ and a frequency f given by the formulas [36]:

$$\lambda = h / p \quad (205)$$

$$f = E / h \quad (206)$$

The relativistic effects could be taken into account by introducing the Lorentz factor, $\gamma = 1/\sqrt{1 - v^2 / c^2}$, and setting $p = \gamma m v$ and $E = \gamma m c^2$.

It is not immediately clear what is meant by "relating wavelength and frequency to a physical object". We have seen in Chapter 9.2 that within the Copenhagen interpretation of quantum mechanics one simply perceives physical objects themselves as undulatory phenomena (in specific experimental circumstances), while Bohm's interpretation (see Ch. 9.3) claims that particles are always accompanied by quantum fields responsible for their undulatory behavior.

It is instructive to consider a simple numerical example. An electron with mass me = 9.11×10^{-31} kg and moving with 10% of the speed of light, v = 0.1c, has wavelength $\lambda = 2.4\times10^{-11}$m which is comparable with the size of an atom (≈ 10^{-10}m). Thus, a slowly moving electron will be able to show a diffractive behavior while interacting with matter. On the other hand a car with mass, say, m = 1000 kg and moving with speed v = 100 km/h ≈ 28m/s has wavelength $\lambda = 2.4 \times 10^{-38}$m which is three orders of magnitude smaller than the Planck length $\ell_P \approx 1.6\times10^{-35}$m. The undulatory aspect of the macroscopic physical objects is therefore unobservable and in the everyday life, our senses perceive them just as large "corpuscles".

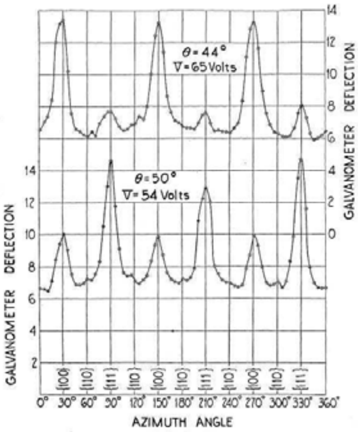


Figure 58: Results of the Davisson-Germer experiment where a block of nickel crystal was bombarded with thermally excited electrons. The crystal scattered the electrons and the authors measured the distribution of the electron intensity behind the target to be periodically dependent on the azimuth angle φ. The diagram shows the measured intensity of the scattered electrons as a function of the angle. Two data series are shown. They differ in the accelerating potential V (which determines the speed of the incident electrons) and the co-latitude of the beam θ. The oscillating pattern, suggesting an interference of some kind, is easily seen. Source: Davisson and Germer [159].

De Broglie's theoretical suggestion that matter in motion could be perceived as a wave with a well-defined wavelength was confirmed experimentally in 1928 by Davisson and Germer [37] [159], and, independently, by Thomson and Reid [160]. The experiments involved scattering narrow electron beams (cathode rays) from a nickel crystal (Davisson and Germer) and a thin celluloid film (Thomson and Reid). The diffraction pattern obtained in both cases (see Fig. 58 for the results of Davisson and Germer) could be easily explained under the assumption that electrons behaved like waves with wavelength given by Eq. (205), and that these waves interfered during their propagation through material just as an ordinary electromagnetic radiation would do. However, the occurrence of these patterns were not predicted by standard corpuscular model combined with knowledge about the atomic structure inside the target.

For some time afterwards, it was not known whether the analogous diffraction phenomena occur with other elementary particles like neutrons and protons, or even with much larger atoms and molecules The second question was settled already in 1930 by Immanuel Estermann and Otto Stern who diffracted a beam of hydrogen and helium atoms using a lithium fluoride crystal [38]. The validity of Eq. (205) was again confirmed. In 1945 Ernest Wollan and R. B. Sawyer carried out the first neutron diffraction experiments using a beam of "monochromatic" neutrons obtained from an atomic reactor [161]. Soon neutron diffraction proved itself to be a fruitful crystallographic technique for the determination of the structure of various materials.

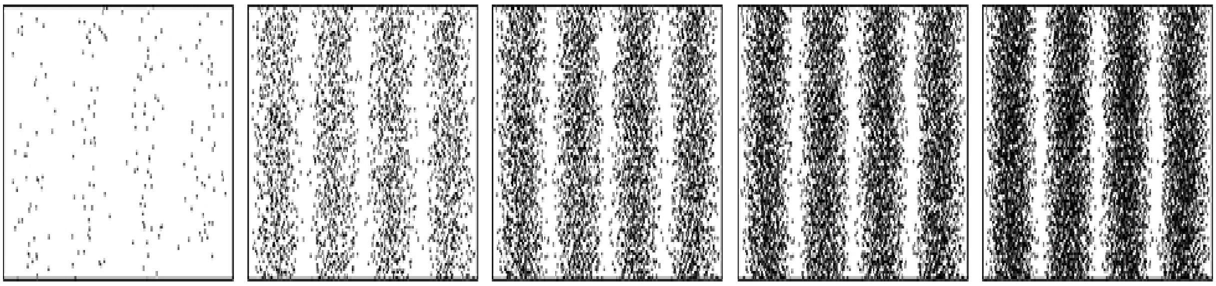


Figure 59 (replaced with a fully equivalent and visually improved picture): The statistical build-up of an interference pattern in the single-electron diffraction experiment due to Tonomura et al. Only the central part of the whole field of view of the detector is shown. The interference fringes are more distinct as the number of single electrons that have hit the detector increases. From left to right, there are respectively 3000, 20000 … and 70000 electron hits. This is a negative of the original picture with increased contrast. Source: Tonomura et al. [5].

A loophole, however, had existed in the matter diffraction experiments so far. In each of them, a continuous flow of particles was considered, and one had to ask whether the diffraction pattern could be explained in terms of some collective behavior of these particles (see the argument from Ch. 3.2 about the corpuscular photons scattering from each other) instead of employing de Broglie waves. The ambiguity would be resolved by performing a diffraction experiment where particles (like an electron or a neutron) travel through the apparatus one by one. If the diffraction pattern would eventually occur, then the case for a matter wave associated with a single particle would be made much stronger.

It was A. Tonomura and his team that in 1989 successfully performed the first precise diffraction experiment with single electrons82 [5]. Moreover, the experiment was also the first exact realization of the famous thought experiment with a single electron passing a double slit (see Introduction) [162]. Tonomura et al. employed an electron microscope equipped with an electron biprism as an equivalent to the double slit, and a position-sensitive electron-counting system as an equivalent to the screen on which the interference pattern could be formed. Fig. 59 presents the pattern they obtained. Their results unambiguously implied that it was a single electron that was able to interfere in a wavelike fashion with itself, and that the phenomenon must not be ascribed to a collective behaviors of a large number of electrons propagating together through system.

So far, the largest material objects that has been shown to exhibit interferential behavior are fullerene C_{60}-molecules. A research team led by Zeilinger obtained in 1999 a diffraction pattern by sending a beam of C_{60}-molecules through a diffraction grating consisting of nominally 50 nm wide slits with a 100 nm period [44]. The velocity distribution of the molecules was measured and fitted; the most probable velocity corresponded to a de Broglie wavelength of 2.5 pm, which is approximately 400 smaller than the diameter of C_{60}. It should be stressed the total mass M of one molecule was used in calculating the de Broglie wavelength, i.e., it was assumed that each interfering de Broglie wave corresponded to a single undivided particle of mass M. Furthermore, the observations supported the view that each C_{60}-molecule interferes with itself alone, even though they did not propagate singly through the apparatus.

Many different experiments confirmed the validity of de Broglie's relation between momentum and wavelength of material objects. Although the relation does not make any explicit distinction between the macroscopic and the microscopic level, it has been verified only in the case of the latter. It remains to be seen if analogous results will be obtained for still larger and more complicated molecules. If not, it will be very interesting to see if it is the size, mass or rather the structure of the physical object under examination that decides when the diffractive behaviour ceases to occur. It is also conceivable that the diffractive behavior of matter will persist, but that the simple de Broglie relation, Eq. (205), will have to be replaced by some other formula, which maybe will give us a better physical insight into the nature of the phenomenon.

Aside from looking for the upper spatial bound, there is another crucial question that could be answered empirically. Imagine that an experiment similar to that of Zeilinger et al. is performed, but with slits in the diffraction grating being considerably smaller than the size of the material object we wish to diffract. Will the diffractive pattern still be obtained? If no, it would imply that there is something intrinsically solid about the matter (in addition to the de Broglie waves) that does not allow a material object to propagate through a slit, which is smaller than the object itself (the size of the object being determined with help of some different means). The persistence of the diffractive pattern, however, would suggest that – at least at the microscopic level and under particular circumstances – the structure of matter is completely undulatory.

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82 Earlier experiments of these kind were conducted by Claus Jönsson in 1961 [163] and P. G. Merli, G. F. Missiroli and G. Pozzi in 1974 [164], but they were less exact and used less sophisticated apparatus.

Wave-Particle duality, Matter-Waves conceptualization and Quantum theory are still part of a continuously evolving process, and we can expect significant updates, and fundamental modifications of the contemporary Orthodox Quantum theory, which already has number of distinctive and useful interpretations. One of such advances will be a creative revitalization of "de Broglie-Bohm theory" and "Many Worlds Interpretation" (*which can be conveniently updated, approaching, and uniting with the Wave-Particle Duality as promoted and favored in this book; -see relevant resume in Chapter 10.*).

Another evolving process will be to show *existence of macrocosmic matter-waves and wave-corpuscular duality, analog to original de Broglie and Schrödinger foundations*, where instead of Planck's constant h it will be relevant another macrocosmic constant H >>>h (especially in cases of self-closed, standing matter-waves formations like in solar systems; -see more in Chapters 2 and 10 of this book).

Wave function concept and its mathematical modeling, including Schrödinger equation will also significantly evolve towards better modeling and more tangible, natural and more deterministic (or not exclusively probabilistic) presentation, based on Analytic Signals and Complex Phasors, as elaborated in this book (see more in Chapters 4.3 and 10.).

As an introduction into such kind of thinking and overall Quantum theory advances, let us read the following Citation from, https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality#cite_note-6

Main article: de Broglie–Bohm theory

Couder experiments,[17] "materializing" the *pilot wave* model.

De Broglie himself had proposed a pilot wave construct to explain the observed wave-particle duality. In this view, each particle has a well-defined position and momentum, but is guided by a wave function derived from Schrödinger's equation. The pilot wave theory was initially rejected because it generated non-local effects when applied to systems involving more than one particle. Non-locality, however, soon became established as an integral feature of quantum theory and David Bohm extended de Broglie's model to explicitly include it.

In the resulting representation, also called the de Broglie–Bohm theory or Bohmian mechanics,[18] the wave-particle duality vanishes, and explains the wave behavior as a scattering with wave appearance, because the particle's motion is subject to a guiding equation or quantum potential.

This idea seems to me so natural and simple, to resolve the wave–particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.[19] - J.S.Bell

Both-particle-and-wave view

The pilot wave model, originally developed by Louis de Broglie and further developed by David Bohm into the hidden variable theory proposes that there is no duality, but rather a system exhibits both particle properties and wave properties simultaneously, and particles are guided, in a deterministic fashion, by the pilot wave (or its "quantum potential") which will direct them to areas of constructive interference in preference to areas of destructive interference. This idea is held by a significant minority within the physics community.[39]

At least one physicist considers the "wave-duality" as not being an incomprehensible mystery. L.E. Ballentine, *Quantum Mechanics, A Modern Development*, p. 4, explains:

When first discovered, particle diffraction was a source of great puzzlement. Are "particles" really "waves?" In the early experiments, the diffraction patterns were detected holistically by means of a photographic plate, which could not detect individual particles. As a result, the notion grew that particle and wave properties were mutually incompatible, or complementary, in the sense that different measurement apparatuses would be required to observe them. That idea, however, was only an unfortunate generalization from a technological limitation. Today it is possible to detect the arrival of individual electrons, and to see the diffraction pattern emerge as a statistical pattern made up of many small spots (Tonomura et al., 1989). Evidently, quantum particles are indeed particles, but whose behaviour is quite different from classical physics would have us to expect.

The Afshar experiment[40] (2007) may suggest that it is possible to simultaneously observe both wave and particle properties of photons. This claim is, however, disputed by other scientists.[41][42][43][44]

"Wave nature of large objects

Since the demonstrations of wave-like properties in photons and electrons, similar experiments have been conducted with neutrons and protons. Among the most famous experiments are those of Estermann and Otto Stern in 1929. [21] Authors of similar recent experiments with atoms and molecules, described below, claim that these larger particles also act like waves.
A dramatic series of experiments emphasizing the action of gravity in relation to wave–particle duality was conducted in the 1970s using the neutron interferometer. [22] Neutrons, one of the components of the atomic nucleus, provide much of the mass of a nucleus and thus of ordinary matter. In the neutron interferometer, they act as quantum-mechanical waves directly subject to the force of gravity. While the results were not surprising since gravity was known to act on everything, including light (see tests of general relativity and the Pound–Rebka falling photon experiment), the self-interference of the quantum mechanical wave of a massive fermion in a gravitational field had never been experimentally confirmed before.
In 1999, the diffraction of C_{60} fullerenes by researchers from the University of Vienna was reported. [23] Fullerenes are comparatively large and massive objects, having an atomic mass of about 720 u. The de Broglie wavelength of the incident beam was about 2.5 pm, whereas the diameter of the molecule is about 1 nm, about 400 times larger. In 2012, these far-field diffraction experiments could be extended to phthalocyanine molecules and their heavier derivatives, which are composed of 58 and 114 atoms respectively. In these experiments the build-up of such interference patterns could be recorded in real time and with single molecule sensitivity. [24] [25]

Importance

Wave–particle duality is deeply embedded into the foundations of quantum mechanics. In the formalism of the theory, all the information about a particle is encoded in its wave function, a complex-valued function roughly analogous to the amplitude of a wave at each point in space. This function evolves according to Schrödinger equation. For particles with mass, this equation has solutions that follow the form of the wave equation. Propagation of such waves leads to wave-like phenomena such as interference and diffraction. Particles without mass, like photons, have no solutions of the Schrödinger equation so have another wave".

Citation from: Scientific American. Ask the Experts: Physics and Math (Kindle Locations 284-286). Scientific American, 2020. Joseph S. Merola, associate dean for research at Virginia Polytechnic Institute, responds:
There are several different ways of approaching this question, but I won't beat around the bush. The simple answer is that wave-particle duality, as it is called, is present in the macroscopic world — but we can't see it. Scientists have developed a number of indirect methods for observing wave/ particle duality. One of the earliest experiments showed that a regular array of atoms could diffract an electron beam. Because diffraction is a property of a wave, this test indicated that particles — electrons in this case — could also behave as waves.
The physicist Louis de Broglie proved that any particle in motion has a wave-like nature. He developed the following relationship: the wavelength of a particle's wave aspect is equal to Planck's constant divided by the momentum of the particle.

Comment from the author of this book: In this book is shown (in the second Chapter, about Gravitation) that macrocosmic objects, like solar or planetary systems can also manifest wave-like nature, where relevant wavelength is equal to certain constant (much bigger than Planck's constant) divided by the momentum of the orbiting planet.

……………………

The main concept of understanding de Broglie matter waves (in this book) can be summarized, with a certain level of creative imagination and intellectual flexibility, by the following statement (that will be more supported later; -see more in Chapter 10. of this book):

"*Regarding structure of particles, the idea favored in this book is that de Broglie matter waves (apart from sometimes being strange regarding indirectly detectable waving manifestations) are always intrinsically incorporated (rooted) inside a structure of every stable particle and atoms in the form of self-sustaining, rotating, stationary and standing-waves (being as certain stable form of an "energy packing"). In cases when particles are moving and interacting with a surrounding environment, mentioned "internally packed" de Broglie matter waves would "unfold", producing externally detectable wave motions, energy, moments, and mass exchange manifestations (i.e., external manifestations of de Broglie waves are only a natural extension or expansion of an intrinsic and internal particle-wave oscillating, spinning, and orbiting structure). The most direct external signs showing existence of such hidden (internally packed) rotation-related behaviors should be spin and orbital moment attributes of all elementary particles, combined with associated electric and magnetic moments, charges and dipoles (and we should not forget that planets, moons, and solar systems* *are in different ways also spinning and rotating). If a rotation has been once in the past involved in the creation of elementary particles and other masses in our Universe, to satisfy the total orbital momentum conservation (including spinning states), some permanent, rotating-like characteristics should be imprinted in, and linked to any-particle size properties, and especially to particles in linear motions. We are presently specifying such properties as some effective, natural, and intrinsic spin attributes (still without clarifying what is internally spinning). From the opposite point of view, considering pure waves (being energy states without rest masses), such as photons, we can say (based on experimental facts) that such wave states could "solidify or sublimate" by entering an already existing particle structure (being "energy-momentum" captured by that particle). It looks that under certain conditions photons (when passing close to other particles or atoms, and most-probably being somehow modulated or influenced by the presence of torsional and spinning field components) can be directly transformed into particles with non-zero rest mass (of course, at the same time satisfying relevant global energy and momentum conservation laws situation). In other words, any motional energy presents a certain state of an effective and spatial mass distribution that is propagating as a kind of spinning and oscillating matter wave. When conditions are met, certain wave packet, peaks and ripples of such motional, distributed mass and energy, can create a space-localized and stable particle that has non-zero rest mass. This is most probably happening when involved torsional and linear motion components fit into specific agreement and coupling by creating some form of "self-closed standing waves", what starts presenting stable mass properties*. For more of supporting arguments, see the Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER *WAVES. Typical examples where we can verify existence of analogical particle-wave dualism and parallelism situations in relation to different forms of motional energy are somewhat innovative analyzes of Compton and Photoelectric effects, including Continuous spectrum of X-rays (or photons), caused by impacts of electrons accelerated in an electrical field, between two electrodes (see such analyzes in Chapter 4.2; under 4.2.2. Example 3: Elastic collision photon-electron (Compton Effect) …).* ***Briefly concluding, we can say that only kinetic or motional energy*** $\tilde{E} = E_k = (\gamma - 1)mc^2$ ***belongs to matter-wave energy states (of course, without rest-mass content,*** $E_0 = mc^2$***), contrary to the erroneous position of the contemporary micro-world physics that a total energy*** $E_t = E_k + E_0 = \gamma mc^2$ ***entirely belongs to matter-waves states.***

A big part of the phenomenology belonging to particle-wave duality is also closely related to Thermodynamics and Fluid dynamics (and not exclusively to Quantum Mechanics). Thermodynamics should not be only the discipline dealing with random motions of mutually isolated particles since all moving particles also have certain undulatory nature and create matter waves. Thermodynamics (excluding its applications to ideal gasses) should also address many possible scatterings, secondary emissions, and impacts between involved and mutually related, electromagnetic, and acoustic fields-coupled particles, (such as photons, electrons, ions, atoms, and molecules). In describing and understanding matter-waves problematic, different fields-interactions, forces, and matter waves between interacting entities should not be neglected, because every moving particle is in a certain relation with its environment and has its matter-wave attributes (especially becoming significant on elevated temperatures when mass of photons and electromagnetic waves, including acoustic and mechanical waves, starts to be involved). De Broglie matter *waves, and all other wave phenomena and oscillations known in Physics, naturally belong to the same family of events and should be formulated and analyzed using the same theory and similar mathematical processing. Also, all fields and wave phenomena in Physics should be considered as a natural extension of micro particles and fluids' (internal and external) states towards diversity of macro "momentum-energy" states, being a kind of "communicating, coupling and gluing substance" in a space between particles and other energy-moments states. Of course, waves are always some oscillations of certain medium, or fluidic and elastic, waving matter states, where energy fluctuates between its kinetic and potential forms. Consequently, in an absolute vacuum state, where electromagnetic waves, neutrinos, solar winds, elementary particles, and various cosmic radiation are propagating, it should anyway exist some exotic, etheric and fluidic matter (maybe still not known, or not well conceptualized in the contemporary physics), as N. Tesla stated many times, [97]. Within the same context, particles could be considered as specifically condensed (packed, formatted or solidified) energy states of self-sustaining, internally folded forms of rotating and spinning matter-waves (where the process of such stabilizing is causally related to internal self-closed standing-waves formations; -see more in Chapter 10.). The same concept is already initiated in the second chapter of this book (about Gravitation; - see equations (2.11.9-1) – (2.11.9-4) and (2.11.10) – (2.11.21)). The situation regarding understanding Particle-Wave duality would become much more straightforward and more evident later, when we start to structure it mathematically, by applying Conservation Laws and particle-wave duality relations.*

The ideas and concepts about Particle-Wave Duality and Quantum Theory are already presenting the significant part of our contemporary micro-world physics (introduced and gradually developed from the beginning of the 20th century of modern human society on the planet Earth). Let us pragmatically, empirically, comparatively, and analogically (*see more in T.1.8, Generic Symmetries and Analogies of the Laws of Physics, in the first chapter of this book*) review and creatively update several starting points to serve as the initial and most relevant background regarding the understanding of particle-wave duality, or better to say unity, here formulated in the following way:

1. *The initially established concept regarding matter waves was based on de Broglie's explanation of stationary electron orbits of Bohr's planetary atom model. The same wave hypothesis was shown to be essential in explanations of Compton and Photoelectric Effects (combined with Planck and M. Maric-Einstein's expression for the energy of a wave packet or photon). Bragg's X-rays (or photons) diffraction from stable crystals is initially explained only considering X-rays as wave phenomena. Later it has been demonstrated, in many variants, that the same Bragg's diffraction law (or mathematical formula) is applicable for describing similar scattering and diffraction of particles like electrons and neutrons, when we treat them as matter waves having de Broglie wavelength. Davisson and Germer demonstrated experimentally, the wave properties of massive motional particles in 1927 in their electron-diffraction experiments. Later, 1930, Estermann and Stern demonstrated diffraction of helium atoms and hydrogen molecules from lithium fluoride crystals (effectively starting the field of atom optics, since they were the first to demonstrate the wave-like properties of atoms). See [92] as an excellent historical resume about foundations of wave-particle duality concepts. Recent development regarding the understanding of particle-wave duality made possible the production of gaseous Bose-Einstein condensate. In such matter-state, macroscopic numbers of atoms occupy the same quantum state, where waves associated with each of the atoms are in phase with one another in a way that is directly analogous to the behaviors of photons in laser devices (see chapter 3 in [27]). If we are convinced that a kind of pure harmonic wave (with predictable and quantifiable wavelength and*

frequency) is associated with particles in motions, we should ask ourselves what and where the source of such waves is. Logically, based on experimental data and applied wave modeling parameters we use in such situations, it should be certain kind of "oscillatory circuit" or "wave generator", or some spinning and helix motion, causally linked to a particle in motion (to its linear and angular moments and involved kinetic energy), producing de Broglie matter waves. Such "oscillatory source", spinning or helix wave motion could be partially related to force and field effects between moving particle and its environment. However, it could also be that de Broglie or matter waves are an intrinsic part of every particle structure, like some internal field-rotation associated with moving particles, externally creating helix waves around the path of linear particle motion, and here actually, this is the starting platform, we will follow in this book.

2. *Briefly, we can safely say that any particle motion is unified (associated, followed, coupled, or at least mathematically presentable) with a corresponding spiraling wave motion and vortices (based on de Broglie hypothesis), and that all wave motions, including de Broglie matter waves, are manifestations of different forms of motional and electromagnetic energy (or power). There should exist a direct correspondence, mutual dependence, and equivalence between matter wave and particle kinetic energy,* $\tilde{\mathrm{E}} \Leftrightarrow \mathbf{E_k}$. *Consequently, every quantitative change or (certain time-space dependent) modulation of motional energy (of any origin) should also be a source of different matter waves, and it should be causally related to action-reaction and inertial forces phenomenology (on some ways analog to electric and magnetic induction phenomenology).*

3. *The same mathematical modeling (related to Signal Analysis) used to describe different (mechanical, electrical, electromagnetic, etc.) oscillations, wave motions and similar phenomena in Physics, is (almost) universally applicable to all of them, regardless of the nature of wave phenomena and regardless of, if applied to microcosms of subatomic entities, or to a macro universe of planets, stars, and galaxies. Uncertainty relations in Physics (or Heisenberg relations) are just a product of the same, generally applicable mathematics, regardless of Physics and the scale and size of analyzed phenomena. Waves' synthesis and analysis, as well as waves superposition and interferences are mathematically and generally explicable and applicable to any size of the universe 8using the same mathematical modeling). There is nothing in mathematical processing what is giving general advantage and uniqueness to a micro world (regarding signals and waves analysis), as Orthodox Quantum Mechanics is implicating. Particle-wave duality theory (or its mathematical modeling) starts from the intuitive concept that velocity of a real particle,* v, *should be the same as the wave-group velocity,* v_g $(= \mathrm{v})$, *of the wave packet or wave group associated to that particle (based on a habitual modeling of a wave group known from Quantum Mechanics). Since every wave group also has its phase velocity,* $\mathbf{u} = \boldsymbol{\lambda}\mathbf{f}$, *and since there is the well-known analytical connection between a group and phase velocity of simple, harmonic, modulated sinusoidal waves* ($\mathbf{v_g} = \mathbf{v} = \mathbf{u} - \lambda \mathbf{du}/\mathbf{d}\lambda$)*, there should also exist one consistent mathematical modeling that will unify all elements of a real particle motion with its associated wave-group replacement.*

4. *Moreover, we should not forget that Planck's wave energy* $\tilde{\mathbf{E}} = \mathbf{hf}$, *combined with relativistic and mechanistic forms of particle energy (Photoelectric and Compton effects,) produces correct results related to wave-mass momentum calculations and transformations. For instance, a photon that has energy* $\tilde{\mathbf{E}} = \mathbf{hf}$, *also has an equivalent (particle-like) momentum* $\tilde{\mathrm{p}} = \mathrm{hf/c}$, *and equivalent (particle-behaving) mass,* $\tilde{\mathrm{m}} = \mathrm{hf/c}^2$, *because the total wave (or motional) energy of the photon is equal to its total relativistic energy and its total kinetic energy (since photon has zero rest mass),* $\tilde{\mathrm{E}} = \mathrm{hf} = \tilde{\mathrm{m}}\mathrm{c}^2 = \gamma\,\mathrm{mc}^2 = \mathrm{pv}/(1+\sqrt{1-\mathrm{v}^2/\mathrm{c}^2})$, $\mathrm{v} = \mathrm{c},\ \tilde{\mathrm{p}} = \mathrm{p} = \mathrm{hf/c}$. *At the same time, the photon has its intrinsic, elementary angular momentum or spin equal to* $\tilde{\mathrm{L}}_{\mathbf{f}} = \tilde{\mathbf{E}}/\omega = \mathbf{hf}/2\pi\mathbf{f} = \mathbf{h}/2\pi$ *(see*

(2.11.3) and T. 4.0). In Chapter 2. ("2.3.2. Macro-Cosmological Matter-Waves and Gravitation") we can also find that similar conceptualization and mathematics applies to planets and solar systems. Practically, only motional energy of a photon analogically corresponds to a motional or kinetic particle energy, and this is one of the best (intuitive) ways for conceptualizing wave-particle duality, but modern Quantum Theory wrongly and ambiguously started from the platform that total particle energy (including rest mass) creates its matter-wave energy.

5. *Every particle or wave motion (in fact, a mathematical function describing certain motion) that can be characterized by some spectral distribution, characteristic frequency, wavelength, oscillating or waving process, etc., should be an integral part of certain visible or hidden rotation, or torsion-field phenomena (in its original or transformation domain). Particles, in the frames of de Broglie Particle-Wave Duality concept, behave (analogically) as photons regarding linear moments, and we know that photons have their intrinsic spinning moments. By applying the same analogy (backward) it should be also valid that all (at least elementary) particles in linear motion should also have certain wave equivalent to angular momentum (or spin,* $L = \frac{c^2}{uv}(\frac{h}{2\pi}), \Delta L = \frac{h}{2\pi}$*: -see the table T.4.0, Photon – Particle Analogies). **In fact, the big secret of particle-wave duality (or unity) is that somehow all forms of wave and/or kinetic energy (or motional energy of elementary particles, quasi-particles, and wave packets) have the same elementary spin unit, equal to*** $\tilde{L} = \frac{h}{2\pi} = \frac{E_{motional}}{\omega} = \frac{\tilde{E}}{\omega} = \frac{E_k}{\omega} = \Delta L$ *(or some integer multiple of* $\frac{h}{2\pi}$*). Consequently, elements of rotation and linear motion should always be intrinsically coupled in all cases of (electrically charged or neutral) particles and waves motions (like* $\frac{L_0}{p_0} = \frac{\Delta L}{\Delta p} = \frac{L}{p} = \lambda \frac{L}{h} = \frac{c^2}{\omega v} = \frac{c^2}{v^2} \cdot r^* = \frac{c^2}{\omega^2} \cdot \frac{1}{r^*}$*). For instance, electrons always have their intrinsic magnetic and orbital moments mutually coupled, meaning that something equivalent to rotation naturally (or intrinsically) implemented in internal electron structure (connecting rotating mass and rotating electric charge, also known in relation to gyromagnetic ratio) should exist. Also, in cases of linear (or circular) macro-motions of electrons, again we should have rotating (or helix) magnetic field components around their paths (see on Internet somewhat similar concept about Henry Augustus Rowland effect of the magnetic field around the rotating conductor, presented by Jean de Climont).* ***Photons also have certain helicity of mutually coupled electric and magnetic field vectors, including resulting spin and angular moments. Consequently, the analogical situation could also be valid for electrically neutral particles, like atoms, that perform the linear motion, producing simultaneously spinning and helical effects of certain field components (belonging to their internal, electrically charged constituents).*** *This will eventually produce externally measurable consequences, such as different and omnipresent rotations in the world of molecules, atoms, and elementary particles, as well as rotations and spinning of astronomic objects. Here is the reason why Gravitation is still not well integrated into the texture of other important theories of Physics, since its coupled, or conjugate field-component, related to certain kind of rotation and/or spinning (coupled with linear mass motion, like coupling between electric and magnetic fields), is still not adequately considered. We could search for such missing rotating components (in the world of Gravitation, and non-charged particle motions) by analyzing Coriolis, Centrifugal and Centripetal forces, and effects associated to gyroscopes and oscillations of a pendulum. We know that accelerated motions of masses are producing effects equivalent to Gravitation. This is easy and simply detectable when we are in certain elevator (as A. Einstein speculated), but in real cases of gravitation, where we do not have real or imaginary Einstein's elevators, we still need to have certain exotic and invisible, continuous mass flow in one direction that will effectively create reactive gravitational force in the opposite direction. Something similar, N. Tesla conceptualized as his never completely finalized and published Dynamic Theory of Gravitation, and he experimentally measured and produced mentioned associated effects of continuous and somewhat exotic mass flow, serving as a **"radiant"** electromagnetic and cosmic-rays' energy, **[97])**. Here*

elaborated particle-wave duality concept is strongly related to the necessity of theoretical unification of linear and rotational motions (in the domain of Gravitation and Electromagnetism, see (2.3) - (2.4-3,) as the first step in such attempts) on a much more profound and explicit level than presently applied in Physics. Without such specific rotation (or spin, vortex, torsional field components, and eddy currents, associated with linear motions…), existence and creation of stable elementary particles would not be possible. Most probably, the future upgraded theory of Gravitation will deal with couples of mutually conjugate fields (analogically like an electromagnetic field), related to linear and rotational motions (of any nature or origin). Eventually, we would be able to find that only motional electromagnetically charged or polarized particles and associated electromagnetic fields (manifesting as currents and voltages) are in the background of all gravitation-related situations. Anyway, electrically neutral masses are internally composed of electrically charged particles (often having internal magnetic domains and electric dipoles). Since there is an enormously big difference between masses of positive and negative electrical charges, it should be easy and natural to experience effects of electrical and magnetic dipole polarizations and attractions based on such effects (when masses are in a mutually relative angular or orbital motions). Laws of electromagnetic induction, Coulomb-Newton laws, and Lenz law would guide such (transient) interactions between mutually moving masses (when they are in a zone of interaction). The situation regarding elementary spin units or orbital and linear moments of photons, atoms, electrons, and other particles could be much more complex than here presented, but what counts here is that linear and rotating or spinning motions (of any kind) are always mutually coupled.

6. *The creation of an electron-positron pair from the energy-momentum content of a sufficiently energetic photon, and the annihilation of an electron-positron pair that produces two photons are very indicative experimental situations explicitly explaining that internal mass content of an electron (or positron) could be just another specific form of **resonant photons-energy packing**.* Of course, additional reaction ingredients are also necessary to be present to satisfy all known conservation laws. In fact, in mentioned examples, we have specific structuring and formatting of electromagnetic field energy. *Of course, such events are respecting the conservation of a total system energy and important orbital and linear moments. Something similar (at least by analogy and symmetry) should also be valid for protons (and anti-protons). Since neutron anyway and dominantly presents specific coupled combination of one electron and a proton (including additional conditions making that all relevant conservation laws are satisfied), we could conclude that quanta of electromagnetic energy (or photons) in different **"packing and resonant formats"** (most probably) create overall masses and atoms diversity in our Universe. The hydrogen atom is also the specific association of an electron and proton, somewhat like a neutron by its internal content since both are composed of an electron and a proton on different ways under favorable conditions (or being differently packed). Conceptualizing this way, we could conditionally and hypothetically exercise that all other atoms are on some specific way composed from hydrogen atoms, (meaning from electrons and protons), what is, at least quantitatively and approximately, close to correct. There are some measurements related insights that neutrons exist only when being outside of atoms. Here is also the explanation of the nature of particle-wave duality or unity between particles and waves. It could eventually happen that we discover that our universe is composed of a variety of matter-forms, waves, particles, and their transients, all of them having profound electromagnetic nature (as N. Tesla too many times stated, [97]).*

7. *In any case, one of possible well-operating, stochastic, and pragmatic, mathematical modeling, based on probability expectations and empirical data fitting regarding particle-wave duality situations is already known. This is the mathematical structure of the Orthodox Quantum Mechanics (OQM). The traditional probabilistic grounds of OQM are well known and presently (officially) recognized and by mainstream official science forged as the "only acceptable", leading and most important picture and framework of the contemporary Physics (regarding matter structure). Only the conceptual picture of the world behind such contemporary OQM is unclear or missing, or not sufficiently compatible with a*

clear intellectual and deterministic visualization and being without strong and causal connections with existing natural body of empirically developed mathematics. Contemporary microworld picture is being rich with assumptions, postulates, patchwork-invented principles, and "exotic or hocus-pocus", ontologically stochastic, and magic-nature events, but anyway, such quantum theory practically works, and has its self-correcting and self-guiding methodology, producing correct results or predictions, sometimes without real explanations. This is something like mass data processing, while applying always good and always working mathematical fittings and conclusions based on probability, statistics, and signal analysis (and this can be equally well applied in any of natural or other sciences, when analyzing events with big number of identical or similar items). This is not a real, independent, original, self-standing, and natural theory about real, experimentally verifiable, and conceptually clear Physics (but it works and produces results). The founders or inventors of such theory (and mass of their non-critical followers) are proposing and imposing way of thinking and concepts and conclusions making (and giving convenient examples with premeditated questions and answers) to reinforce and defend the probabilistic Quantum theory as the only and best option. Here, an effort will be made to "dress" the OQM's mathematical modeling into a conceptually clearer, more logical, more natural, and much more deterministic shell (using the known pictures, models, and analogies from other chapters of Physics, and using improved, more convenient mathematical modeling of wave motions). It will be demonstrated that in parallel to probabilistic OQM, there should exist another (isomorphic, more general, and more natural) level of modeling wave functions and wave equations. Mentioned new (wave functions) modeling will be conceptually clearer, capturing more of tangible reality, and using more natural mathematical models (than present OQM), and it can be used to describe the world of all particles and waves known in physics (based on Analytic Signal and Hilbert integral transform). This would again, in cases when such new wave functions are normalized or "undressed" (losing dimensionality), produce the models and equations already known in OQM. Quantum mechanics or Quantum theory is a misleading title and should be replaced by something like "Theory of interactions, communications, energy exchanges, formatting, and packing, within resonant and standing-waves structures of energy-momentum carrying, matter-waves and motional particle states." Anyway, contemporary Quantum Theory will always have its significant place in Physics, as the rich "mathematical toolbox" useful for any new and more advanced particles and matter-waves theory. Modern OQM is also rich with number of challenging, imaginative, exotic, creative, sometimes improvable, and very motivating and interesting concepts, assumptions, and simplified visualizations, useful for creating kind of explanations how OQM world works.

8. *What should be more general and realistic modeling of particle-wave duality is the framework of Analytic Signal functions (established by D. Gabor; -see much more in chapters 4.0, 4.3 and 10). Using the Analytic Signal model (creatively, and with convenient mathematical and dimensional arrangements), we should be able to present every matter-wave function of certain motion, field and force as a couple of mutually phase shifted and Hilbert transform related wave functions* $\Psi(t)$ ***and*** $\hat{\Psi}(t)$*, which create complex analytic signal function* $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1 + jH)\,\Psi\,(t)$*. Here, both* $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ *present real, natural, and detectable items, like combination and coupling of electric and magnetic field vectors that are creating an electromagnetic field. On a similar way, every linear motion of particles and waves presented with a wave function* $\Psi(t)$*, should be automatically followed by synchronously created analytic signal couple* $\hat{\Psi}(t)$*, again creating a complex analytic signal* $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t)$*, which has all matter-wave properties that are products of Analytic Signal modeling (such as de Broglie wavelength, frequency, phase etc.). Both* $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ *should present detectable material items. For instance, an excellent example of* $\Psi(t)$ ***and*** $\hat{\Psi}(t)$

coupled matter waves is when $\Psi(t)$ presents specific linear motion, and $\hat{\Psi}(t)$ presents a spiral, spinning or helix, de Broglie matter wave of a field existing in a space around the path of the original linear motion of $\Psi(t)$. De Broglie matter waves have the frequency and phase described by the properties of the corresponding Analytic Signal model, as defined in (4.0.2). From the Analytic Signal frequency and phase, we can determine matter wave wavelength, $\lambda = h/p = u/f$ and belonging group and phase velocity $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$. Relevant Analytic Signal wave functions that are naturally describing de Broglie or matter waves are power and motional energy-related functions, including corresponding field and force functions. By creating normalized, non-dimensional (power-related) wave functions, we will have similar as Quantum theory approach to the same problematic. This will be the mathematical background for explaining matter-waves and particle-wave duality in this book. Most of the contributions of Louis de Broglie, A. Einstein, Max Planck, E. Schrödinger, and W. Heisenberg, concerning matter waves and particle-wave duality, will be perfectly well and naturally described, and (integrally) considered, when being mathematically modeled with Analytic Signal wave functions.

9. *In conclusion, we can safely say that the real and most-significant discovery of matter waves and particle-wave duality historically belongs to Jean-Baptiste Joseph Fourier, French mathematician, and physicist (21 March 1768 – 16 May 1830). From Fourier integral transformation (that is the predecessor of D. Gabor Analytic Signal; -find much more in Chapter 4.0) we can clearly see that every time-domain function or signal is presentable with an integral superposition of simple harmonic, elementary waves, or components (in other words, meaning that we consider only motional states as matter waves states). In addition, based on many analyses and experiments (from Fourier time until present), it was proven that the Nature really respects (without any exception) Fourier spectral analysis, or signals' time and frequency domains presentations, meaning that almost any arbitrary, time-domain signal can really and most-naturally be decomposed on simple-harmonic elementary wave functions. Dennis Gabor realized the most significant theoretical improvement of Fourier spectral analysis (5 June 1900 – 9 February 1979). He defined the Analytic Signal and associated time-frequency analysis, producing or extracting important, generalized wave parameters (from any wavefunction), like amplitude, frequency, phase, power, impedance etc. This kind of characterization is also applicable in all cases of arbitrary shaped and finite-time-duration signals.*

   Real historical predecessors and on some way contributors to Fourier spectral analysis were Christiaan Huygens, (14 April 1629 – 8 July 1695), and Isaac Newton (25 December 1642 – 20 March 1726/27). Isaac Newton practically exercised with the first spectrum analyzer that was an optical prism, and he separated wideband, daily (white) light on its colors or spectral components. Later came Augustin-Jean Fresnel, the French engineer, and physicist (10 May 1788 – 14 July 1827), and Thomas Young (13 June 1773 – 10 May 1829) who was an English polymath and physician, with much more explanations about wave nature of light.

Citation from: https://en.wikipedia.org/wiki/Huygens%E2%80%93Fresnel_principle,

Huygens' Theory and the Modern Photon Wavefunction

Huygens' theory served as a fundamental explanation of the wave nature of light interference and was further developed by Fresnel and Young but did not fully resolve all observations such as the low-intensity double-slit experiment that was first performed by G. I. Taylor in 1909, see the double-slit experiment. It was not until the early and mid-1900s that quantum theory discussions, particularly The Feynman Lectures on Physics as well as early discussions at the 1927 Brussels Solvay Conference, where Louis de Broglie proposed his de Broglie hypothesis that a wave function guides the photon.[7] The wave function presents a much different explanation of the observed light and dark

bands in a double slit experiment. Feynman partially explains that a photon will follow a predetermined path, which is a choice of one of many possible paths. These chosen paths form the pattern; in dark areas, no photons are landing, and in bright areas, many photons are landing. The path of the photon, or its chosen wave function is determined by the surroundings: the photons originating point (atom), the slit and the screen, the wave function is a solution to this geometry. The wave function approach was further proven by additional double-slit experiments in Italy and Japan in the 1970s, and 1980s with electrons see [8].
Huygens' principle and quantum field theory

Huygens' principle can be seen as a consequence of the homogeneity of space—the space is uniform in all locations.[9] Any disturbance created in a sufficiently small region of homogenous space (or in a homogeneous medium) propagates from that region in all geodesic directions. The waves created by this disturbance, in turn, create disturbances in other regions, and so on. The superposition of all the waves results in the observed pattern of wave propagation. Homogeneity of space is fundamental to quantum field theory (QFT) where the wave function of an object propagates along all available unobstructed paths. When integrated along all possible paths, with a phase factor proportional to the action, the interference of the wave functions correctly predicts observable phenomena. Every point on the wavefront acts as the source of secondary wavelets that spread out in the light cone with the same speed as the wave. The new wavefront is found by constructing the surface tangent to the secondary wavelets.

Only much later, (during the 20th century...) will appear (and disappear) several of inventors and founders of the contemporary particle-wave duality and matter-waves concepts, like Luis de Broglie, Niels Bohr, Albert Einstein, Max Planck, Compton, Heisenberg, Schrödinger, and others. All of them theoretically and experimentally, in fact unintentionally, proved (in many ways) that Jean-Baptiste Joseph Fourier created excellent and stable mathematical foundations of wave-particle duality, and time-frequency analysis.

Let us summarize (and extend) the grounds and significance of Fourier, and Dennis Gabor Analytic Signal and spectral analysis (see more in the chapters 4.0 and 10.):

A) *Any time-dependent signal (in its original time domain) has its natural mathematical and Physics-related, also experimentally verifiable, spectrum in its spectral domain, and vice versa.*

B) *Any spatial-domain signal has its spectral or spatial-frequency domain, and vice versa (like known and practiced in Statistical and Fourier Optics).*

C) *The structure of our Universe manifests as certain kind of symmetry and direct proportionality between its temporal and spatial (both mutually original and spectral) domains, and this is the broadest and most significant mathematical framework of particle-wave duality and matter waves concepts. Uncertainty relations, group and phase wave-packet velocity, and A. Einstein Relativity theory are supporting direct mutual proportionality between temporal and spatial domains of our Universe (see more in Chapter 10.). Fourier and Dennis Gabor integral transformations are eventually leading to the definition of Analytic Signal concept, which is naturally applicable and suitable for modelling all matter-waves and wave-particle-duality signals, wave functions and motions, thanks to formulations of essential **PWDC**-facts, discovered in Physics during the 20th century. In this book, essential particle-wave duality facts are summarized under **PWDC** (=) **P**article-**W**ave **D**uality **C**ode) as the most valuable content of the contemporary Matter-Waves and Wave-Particle-Duality theory (see more about **PWDC** in the next pages of this chapter, and in Chapter 10.).*

Statistics and Probability theory applied on wave-particle duality problematic (as known within contemporary Quantum theory) have mostly accessorial, auxiliary, and mathematical-modeling toolbox significance (whenever such methods are naturally and mathematically well applicable. It is almost useless to say that Statistics and Probability theory are equally applicable in any science or situation appearing in our Universe when conditions for good mathematical applicability are met.

4.1.1. Particle-Wave Duality Code or PWDC

The fact is that Physics community initially discovered number of direct, indirect, and indicative manifestations of Particle-Wave Duality and Matter Waves such as Compton and Photoelectric effects, creation of electron-positron couples from photons, or annihilation of mentioned particles and creation of new photons, different secondary emissions, impacts and scattering transformations, Bragg's diffractions of X-rays, electrons, and neutrons… All of that is mathematically manageable with A. Einstein, or better to say Minkowski 4-vectors, mass-energy relations, applicable to moving masses, photons, and other matter waves. In early days of foundation of particle-wave duality and matter waves concepts, nobody had a ready-made, easily applicable and generally valid theory to explain such phenomenology, that will be sufficiently and smoothly connected with the Classical Physics, as known until the end of the 20-th century of humans' history on the planet Earth. Something similar is possible to say for A. Einstein Relativity Theory, which was also introduced in parallel with Particle-Wave Duality and Matter Waves concepts, and all of them are still not well united. Large-scale searching and theoretical trial-and-error testing started, and eventually, modern Quantum Theory was mathematically constructed, partially axiomatically postulated and eventually, by consensus accepted. The ontological, deterministic, and essential understanding of Particle-Wave Duality and Matter Waves has never been established and clarified sufficiently profound, but mathematically operational theory to handle such problematic was formulated. With such theory, we can explain many or all (mater-waves related) experimental results. We operate successfully with matter-waves phenomenology and have methods to manage and predict experimental results mathematically. In this book, we will start from experimental facts, using well-known mathematical framework, and indicative and striking analogies, and step by step, we will come to an explanation of Particle-Wave Duality and Matter Waves, which will be more tangible, logical, and more natural than an artificial, probabilistic, and axiomatic Quantum theory is.

What we are searching to find is the uniting zone, where classical mechanics and physics, waves and oscillating motions theory, fluid mechanics, relativistic and quantum theory, including electromagnetic theory have non-doubtful, common areas, and were all of them, are using similar, mutually complementary, and compatible mathematics and conceptualization. Quantum theory name is misleading, since there is nothing in Physics randomly and stochastically created, but at the same time, uniquely, intrinsically, and ontologically quantized, without big and clear deterministic reason, or without certain larger conceptual frame, and clear causal and natural foundations. Quantization is relevant when we address spatially limited structures with stable resonant and standing waves, or self-closed matter-waves formations, and when we can count involved integer numbers of half-wavelengths, or energy amounts belonging to mentioned resonant formations. Consequently, such stochastic, and artificially imposed (but still oversimplified) quantization cannot be a dominant and uniting factor here. The unifying, mutually complementary, analog, and/or equivalent and mutually transformable mathematical models, and concepts (regarding wave-particle duality and matter waves) will be found in universally valid and complementing relations between linear, rotational, and spinning motions. Linear motions are creating matter-waves characterized with relevant group and phase velocity $u = \lambda f$, $v = u - \lambda(du/d\lambda)$, having de Broglie matter-waves wavelength $\lambda = h/p$, and Planck-Einstein wave energy $\tilde{E} = h \cdot f$, being comparable to moving particles when using the concept of wave packets. Associated complexity in elaborating such **PWDC** problems is that mentioned items are mutually so much related, multilevel connected, and analyzed using the same mathematical tools of modern signal analysis. It is often necessary to make precious intellectual and mathematical efforts, differentiation, and classifications, to extract what is innovative and significant there. For instance, enormously significant mathematical modeling advantages are obtainable by replacing Fourier-based signal analysis with the Analytic Signal analysis (based on using Hilbert transform), but this is not immediately noticeable (because involved mathematics of Analytic Signals superficially looks like Fourier-based analysis; -see more about Analytic Signal concept in Chapter 4.0).

The success of Quantum Mechanics (or its mathematical and predictive power) is mostly related to the proper and practical integration (or hybridization) between the essential **P**article-**W**ave **D**uality **C**oncept, and widely applicable, generally valid mathematics of modern Signals Analysis, Statistics and Probability theory, combined with matter waves equations (or with Classical and Schrödinger Wave Equations). Anyway, nobody is using such simplified formulation regarding **PWDC** and foundations of Quantum Theory. The central part of the **PWDC** was initially (almost by chance, unintentionally, gradually, and implicitly) formulated and applied by L. de Broglie, Max Planck, Niels Bohr, A. Einstein, and his wife Mileva Maric, W. Heisenberg, and E. Schrödinger (including number of other contributors). In this book, the same situation regarding **PWDC** will be more naturally addressed, slightly upgraded, and more profoundly explained, than the case in today's Quantum Theory is.

Briefly, PWDC is the set of common rules (properties, mathematical relations, equations, and formulations) that govern and explain how micro and macro particles and matter waves are mutually coupled. PWDC formulates how linear, circular, and torsional motions are mutually related, how matter waves can be transformed into particles and vice versa, and how energy and crucial moments can be redistributed between particles and waves. The best matter-wave mathematical model for defining, exploring, and uniting all PWDC items is the Analytic Signal model. *See more about **PWDC** in Chapter 10. of this book.*

One of the fundamental wave equations, applicable to micro-world matter-wave states and in a close relation to **PWDC**, first time constructed (or postulated) by E. Schrödinger, will also be step by step developed, upgraded, and generalized (based on Analytic Signal wave functions). We will demonstrate a much higher level of applicability of generalized Classical wave equations, to any possible wave phenomena (or, at least, new differential wave equations will become more general than Schrödinger's Equation as known from contemporary Quantum Mechanics). In fact, it will become evident (in this book) that well known Schrödinger Equation (***which is originally and analogically formulated as modeling standing waves on an oscillating string***) is just a simple mathematical consequence of Classical wave equation and generalized wave function formulated in the form of an Analytic Signal, which will be introduced later in Chapter 4.3. It can be shown that Schrödinger equation is not so strong and unique starting platform of Quantum

Mechanics that should be postulated as one of the primary sources and step-stones of wave mechanics (since it can be simply and logically derived from other, more general mathematical and classical Physics wavefunctions and equations, not-related to Quantum theory assumptions). The significance and success of "Schrödinger family of equations" is also related to the fact that condensed matter that has stable rest mass is usually a consequence of various standing-waves energy packing. It will also be shown that new forms of universally valid wave equations (such as (4.9), (4.10), chapter 4.3, WAVE FUNCTION AND GENERALIZED SCHRÖDINGER EQUATION), after proper merging with **PWDC,** can describe any wave motion in electromagnetism, acoustics, hydrodynamics, including "*quantum mechanical waves", and motions of planets, and gravitation related phenomenology*. This would also lead to formulation of different forms of generalized Schrödinger's wave equation (or the equation of all matter waves).

The updated Schrödinger-like equation merged with **PWDC** (the one that will be generalized in this book; -see equations starting from (4.9) in Chapter 4.3.) will reinforce applicability of renewed Particle-Wave Duality Theory. This is the theory that is more conceptually tangible, precise, natural, and richer in comparison with the Orthodox Quantum Mechanical picture of the Nature (see much more in Chapter 10.).

To answer the question how matter waves have been initially created is one of the objectives in this book (and will be specifically addressed, in the next paragraph, 4.1.2.). As an easy introduction (into conceptual understanding of the **PWDC**), let us start analyzing an idealized, a "sufficiently isolated (self-confined) and mutually interacting, two-body" or two-particle system. In such two-body system, the first body is an ordinary, known parameters particle, in a rectilinear motion (still and temporarily considered as being without elements of rotation), having linear momentum p_1. Here, the second "particle" effectively presents surrounding environment or universe, having some resulting linear momentum p_2. We can assume that certain force or interaction (explicable by the existence of an intermediary field, or spatial energy-distribution) should always exist inside and between mentioned particles. Here, all interactions are treated traditionally (from the platform of Classical Mechanics) manifested by forces having the ability to change the magnitude and direction of linear moments of involved moving particles. We still exclude options of having other types of forces manifesting between rotating and spinning objects, that could change orbital and spin moments of particles having such attributes (just to simplify this analysis in its very first steps). Since we intend to treat this case (two particles interaction) like an isolated system, the general law of total momentum conservation (First Newton Law) should be satisfied: $\vec{P} = \vec{p}_1 + \vec{p}_2 = \overrightarrow{\text{const}}$. The central forces effectively acting on each entity of the two-body system can be found by applying the Second Newton Law: $\frac{dP}{dt} = \frac{dp_1}{dt} + \frac{dp_2}{dt} = \vec{F}_1 + \vec{F}_2 = 0,\quad \frac{dp_1}{dt} = \vec{F}_1 = \vec{F}_{12},\quad \frac{dp_2}{dt} = \vec{F}_2 = \vec{F}_{21},\ \vec{F}_{12} = -\vec{F}_{21} \Leftrightarrow \vec{F}_1 = -\vec{F}_2$. The ordinary-like real particle (the first body) is characterized by its linear motion particle momentum p_1 and performs the force $\vec{F}_{12}$, acting on its couple (which is surrounding universe). The second body has its resulting linear moment p_2, and performs an action $\vec{F}_{21}$ on the first particle characterized with p_1. Later in this book, conceptualization of forces and fields between involved particles ($p_1 = p$, and resulting, <u>surrounding-space momentum</u> $p_2 = \tilde{p}$ $\Rightarrow$ $P = p + \tilde{p} = \text{const.},\ dp = -d\tilde{p}$) will be the starting spot for explanation of the **PWDC**. In other cases, if we were dealing with two (known and mutually interacting), moving particles "immersed" in the surrounding environment (what would effectively become, at least, a three-body system), the previous situation evolves as, $\vec{P} = \vec{p}_1 + \vec{p}_2$, $\frac{d\vec{P}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \vec{F}_1 + \vec{F}_2$, $\frac{d\vec{p}_1}{dt} = \vec{F}_1 + \vec{F}_{12}$, $\vec{F}_{12} = -\vec{F}_{21}$, $\frac{d\vec{p}_2}{dt} = \vec{F}_2 + \vec{F}_{21}$. The significant **PWDC** elements (in this new case) would be related to "internal" central forces between particles $\vec{F}_{12} = -\vec{F}_{21}$, and to the work produced by such forces (acting on the path r_{12}, which connects two moving particles): $E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = E_{k1} + E_{k2}$, $\vec{F}_{12} = -\overrightarrow{\text{grad}(U_{12})}$, $E_{k1} + E_{k2} + U_{12} = 0$. (see (4.3)-(4.8); in this case forces $\vec{F}_1, \vec{F}_2$ are external forces).

The understanding of **PWDC** (regarding forces between mutually approaching particles) could also be complemented with the generalized Newton-Coulomb force laws, as presented in Chapter 2. (see equations from (2.3) to (2.9), and later from (2.11.10) to (2.11.21)). The important message here is that forces between two interacting objects could be composed of different, static, and dynamic force components, since "Newton-Coulomb force laws" are not enough to capture the complexity of all possible forces between two objects.

Again, briefly summarizing, in this book will be shown that general Two-Body problem still has not well exposed and not entirely analyzed aspects related to coupling between linear motions and intrinsically associated rotating, orbital, and spinning motions, what should reveal the nature and origins of Particle-Wave Duality of matter (see later in the same chapter: "4.1.2. De Broglie Matter Waves, Unity of Linear and Angular Motions, and Fluid Dynamics"). Since matter (or masses) in our universe consist of atoms and molecules with internally embedded electric charges and magnetic or spinning states, this already presents an electromagnetic background and elements of the unification platform between matter waves, gravitation, and electromagnetic fields and forces.

How and why the forces between interacting and approaching objects create matter waves is another question to answer. If we have two mutually approaching particles, without having any of "force-field" interaction between them, the particles will pass the zone of interaction (or, more correctly, pass the zone of non-interaction) and disappear in an open space in front of them. In cases of plastic or any other collision (between energy-momentum states that are in linear motions), we would get (in classical mechanics well known) results predictable by generally applicable conservation laws. In other cases, if in the process of approaching, and in a near interaction zone, additional central forces between participants affect involved particles on a way that initially rectilinear motions of both particles get associated elements of angular motion, torsional-fields, and spinning or vortex elements, the situation will become much more interesting and challenging. Rotating motion elements will create matter waves, and we will get a chance to associate the wavelength and frequency to such motions, what De Broglie mathematically mastered regarding wavelength, and what is also the concept being favored in this book, but wider elaborated. If interacting particles already had linear, orbital, and spinning moments (including electric charges and magnetic properties), before starting the interaction, the same situation regarding forces and associated matter waves would become richer with options and more naturally explicable. ***In this book, a step further is made, suggesting that linear and angular moments can also be "active field charges" for mentioned***

interacting forces including gravitation (see the second chapter: equations from (2.3) to (2.9) and later from (2.11.13-1) to (2.11.13-5)).

*We could also imagine that all interacting particles and objects (including their environment) behave like being mutually connected with some (invisible) springs. This is creating a multidimensional and elastic, spatial standing-waves structure (or matrix), able to oscillate, or to produce matter waves, if any of its nodes or elements are excited, activated or set in-motion, where **PWDC** describes essential elements of such waving behaviors and couplings. Furthermore, all nodal elements (particles, interacting objects, atoms, molecules, etc.) of the mentioned elastic space-matrix structure, themselves (internally) are presenting stabilized, oscillating and/or resonant structures, where **PWDC** is the framework describing how such coupled oscillators are internally structured and connected, and how they mutually communicate and synchronize. Mentioned universal, spatial background matrix, of an ever-existing texture of our Universe should also behave like some exotic and fluidic media, as an ether, or carrier of matter waves (whatever that means, and serving until we find certain better and more precise conceptualization). Tesla, [97], also favored the similar concept of space and ether, which has tangible and measurable properties like dielectric and magnetic permeability,* $\varepsilon, \mu, c = 1/\sqrt{\varepsilon\mu}$.

Contemporary physics deals very much traditionally, one-sided, and straightforwardly when analyzing two-body relations, mostly from classical mechanics of rigid particles interactions, and this is one of the problems we shall meet when we start introducing and explaining the full meaning of **PWDC**. In electric circuit theory and electromagnetic theory, we analyze electric currents, voltages, field components, and electric power in an electric network, and we apply concepts about active, reactive, and apparent power. We should be able to create an analogical concept for mechanical and other forces, velocities, and power, associated to moving particles, matter waves and their interactions (as initiated in the first chapter of this book). This is still not a widely accepted practice. In such (analogical) way, we could introduce the meaning of Complex, Active, Reactive and RMS forces and velocities (of any origin and nature), as well as exploit Active, Reactive and Apparent mechanical power. This way we will realize that any motion (in Physics) has vibrant and balanced nature between intrinsic, mutually interacting corpuscular and wave properties of its constituents (which will be addressed in the following chapters of this book).

*The message of this book is that only **"ENERGY IN MOTION"**, or **"CURRENT OF ENERGY" (=) ENERGY FLOW** (regardless of its origin), or simply **POWER (= dE/dt,** applying the analogy with alternating, electric currents and/or voltages, or with electromagnetic waves), presents almost everything we should relate to matter waves or de Broglie waves. Here, the total time-space fluctuating and **alternating POWER** can have electromagnetic, mechanical, linear motion, rotational and other mutually coupled components. Later, it will be shown that the famous Schrödinger's equation and Quantum Wave Mechanics were also and effectively formulated using similar ideas. By normalizing and averaging various power-related wave functions (making them dimensionless), and conveniently applying rules of universally valid Probability Theory, Statistics and Signal spectrum analysis on such wave equations* *and functions, while respecting all Conservation Laws known in Physics, we can formulate most of equations of contemporary Quantum Wave Mechanics.*

The initial **formulation and explanation of the PWDC** we shall start based on specific set of analogies (whenever applicable). Much more about **PWDC** can also be found in Chapter 10. of this book. For instance, we can start with "*T.1.8 Generic Symmetries and Analogies of the Laws of Physics*", from the first chapter, combined with principal formulas and results already known, proven, and used in all contemporary presentations of particle-wave duality concepts, (see T.4.0), such as:

A) De Broglie matter-wavelength, $\lambda = h/p = 2\pi/k$, and Einstein-Planck's expression for wave packet (or photon) energy, $\tilde{E} = hf$,

$$\left\{\lambda = \frac{h}{p},\ u = \lambda f,\ \tilde{E} = hf\right\} \Leftrightarrow \left\{\frac{\lambda}{v} = \frac{h}{pv} \Leftrightarrow \frac{\lambda f}{vf} = \frac{u}{vf} = \frac{h}{pv}\right\} \Leftrightarrow \left\{\frac{u}{v} = \frac{hf}{pv} = \frac{\tilde{E}}{pv}\right\}. \quad (4.1)$$

In (4.1), $\mathbf{v}$ and $\mathbf{u} = \boldsymbol{\lambda}\mathbf{f}$ are group and phase velocities, $\gamma mv = p = hk/2\pi$ is the particle linear momentum, $\gamma = (1 - v^2/c^2)^{-1/2}$, $f_t = f = \omega/2\pi$ is the time-domain frequency of associated de Broglie wave, $f_s = k/2\pi$ is its space-domain frequency, $\mathbf{h}$ is Planck's constant, and $\tilde{E} = pu = hf \Leftrightarrow E_k$ is the (motional or wave) energy of the involved mater wave packet, where $E_k = pv/(1 + \sqrt{1 - v^2/c^2})$ is the kinetic energy of the particle in linear motion. We often use relations and expressions given in (4.1) without thinking how de Broglie, A. Einstein and M. Planck came to such simple and useful relations. *The basic particle-wave duality relations (4.1) are formulated or postulated (or mathematically fitted, but still not completely explained) by searching for the best possible solutions, or missing mathematical links, to explain some dualistic and matter-wave phenomena, experiments, and models in microphysics. This was initially based on analogies between a photon as a matter wave-packet, and an equivalent moving particle; -see more in 4.1.1.1. Photons and Particle-Wave Dualism. For instance, we are using photon-particle analogies to better explain Bohr's hydrogen atom model and thermal radiation of a black body, and to serve similar purposes in explaining many other micro-world phenomena, such as Photoelectric, Compton Effect, Bragg's diffraction (or scattering) of X-rays, the structure of electrons and neutrons, etc. In chapter 2. (concerning Gravitation), it is successfully revealed that similar, or mutually equivalent models, wave equations and concepts from the world of microphysics, are also applicable to planetary systems, and motions of macrocosmic objects (without involving Statistics and Probability theory concepts).*

B) For a moment we do not need to state explicitly what are all possible mutual relations (or quantitative connections) between the wave energy $\tilde{\mathbf{E}}$, wave momentum $\tilde{\mathbf{p}}$, kinetic (particle) energy $\mathbf{E_k}$, and its (particle) momentum $\mathbf{p}$, except to consider that specific analogy, equivalency and coupling between them exists, and that both entities (particle and wave group) have the same (group) velocity, $\mathbf{v} = \mathbf{v_g}$. Later, the exact mathematical connections between all of them will be found; -see (4.2). Currently, we shall give priority to equivalency relations $\tilde{\mathbf{E}} = \mathbf{pu} = \mathbf{hf} \Leftrightarrow \mathbf{E_k}$, since any wave energy is also a kind of kinetic energy). One of objective here is to explore if matter-wave entities are something that could exist independently, or only associated with particles in motions. Of course, here we are again using A. Einstein and Minkowski 4-vectors relativistic relations between involved masses, moments, and energy. In the explanation of Photoelectric effect, where energy and momentum of particles are analogically associated with similar photon properties, we also have in mind that, in a larger (mathematical and experimental) frame, similar concepts are successfully applied on explanations of Compton effect, electron-positron creation and annihilation, and Bragg's scattering of X-rays, electrons and neutrons…

For a moving particle, we know the differential of its relativistic kinetic energy in the form $\mathbf{dE_k = vdp}$. By analogy with a photon (see later **T.4.0.)**, for wave energy of an associated or equivalent wave-group, similar relation should also be valid: $d\tilde{E} = dE_k = vdp$. Differential kinetic or wave energy amounts should have the same mathematical forms in Classical and Relativistic Mechanics since there is only one, united or the same physics reality, what is producing,

$$\left\{\begin{pmatrix} E_k = \frac{1}{2}mv^2 = \frac{pv}{2} \\ m = m_0 = const. \\ p = mv \end{pmatrix} \Rightarrow \boxed{\begin{matrix} \boxed{dE_k = vdp} \\ dE_k = pdv \\ dE_k = \frac{1}{2}(vdp + pdv) \\ vdp = pdv \end{matrix}}\right\} \Leftrightarrow \left\{\begin{pmatrix} E_k = mc^2(\gamma - 1) \\ m = m_0 = const. \\ p = \gamma mv \\ E_{tot.} = \gamma mc^2 \end{pmatrix} \Rightarrow \boxed{dE_k = vdp} = mc^2 d\gamma = dE_{tot.}\right\}.$$

Consequently, we will use $d\tilde{E} = dE_k = vdp$ in differential equations of interest (for instance in the equation that connects group and phase velocity, (4.2). Practically, we will (analogically and intuitively) assume and accept that both for classical mechanics and relativistic mechanics cases, the differential of kinetic energy should be $dE_k = vdp = d\tilde{E}$, regardless we see that in Classical Mechanics case we also have other options.

C) Let us now try to connect and unify all (above mentioned) energy forms regarding particle and its corresponding wave group. From the last, right-hand part of the equation (4.1), we can come closer to the conclusion that de Broglie wavelength is not the most-significant qualification of particle-wave duality, and that fuller qualification should be the relation between phase and group velocity of de Broglie wave packet. Practically, we shall exploit and merge all relations from (4.1) with $\mathbf{\tilde{E} = hf = pu \Leftrightarrow E_k, dE_k = vdp \Leftrightarrow d\tilde{E} = hdf = d(pu)}$, and $v = u - \lambda(du/d\lambda)$, and come to conclusions as given in (4.2). It is not necessary, but we could also apply in (4.1) conclusions based on analogies (from the first chapter), using the last part of (1.19), to create more explicit relations between particle and wave characteristics of de Broglie or matter wave packet, as shown in (4.2). Modern Quantum Theory supports that a total particle energy (including its rest mass energy) is the content of an equivalent matter-wave packet or group, what is not the case in this book. Here, we will exclusively identify motional or kinetic, or matter-wave energy as the content of an equivalent, matter-wave object, or wave packet (without rest mass).

Presence of inertial effects (in mechanics and electromagnetism), we can detect whenever some sudden and non-uniform changes of certain energy-flow happen (meaning when some currents, voltages, forces, velocities, and moments of specific stationary state would suddenly change). Our conceptual problem related to mathematical modeling of transient inertial effects is that we understand well what that means in an electromagnetic environment (described under different induction laws), but in mechanics, our deeper understanding of similar problematic stops with Newton laws. Usually, we do not search there for the field components that should complement Gravitation, like in mutual relations between electric and magnetic fields. It is also not excluded that all transient and inertia-related effects (in mechanics) have their deep roots in electromagnetic induction laws (since mass is composed of atoms, and atoms have electrically and magnetically charged content). Anyway, it should be clear that inertia is not only a tendency to keep a steady state of constant velocity of uniform, linear or rectilinear motion. Place for "inertial rotational, and spinning", as well as for other more complex, stable, periodical, and inertial motions (including specific accelerated motions, and standing waves states) should also be properly established in Physics.

If corpuscular and wave momentum $(\mathbf{p}, \tilde{\mathbf{p}})$ are mutually collinear or parallel (even identical) vectors ($\cos(\mathbf{p}, \tilde{\mathbf{p}}) = \mathbf{1} \text{ or } -\mathbf{1}$) we will find that there is only one consistent and unifying result, given by (4.2), which extends relations (4.1), relevant for the basic understanding of particle-wave duality. Let us support such matter-waves concept by a very simplified mathematical procedure (by listing initial statements and going directly to their implications), what is intuitively improvised, and summarized as follows (see more in [5], and in Chapter 10. Especially under "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"):

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{array}{l}\left\{\begin{array}{l}E_k \Leftrightarrow \tilde{E}=pu\,,\; pv=E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right]=\frac{\gamma^2-1}{\gamma}mc^2, \\ E=E_0+E_k=\gamma mc^2=mc^2+\frac{\gamma mv^2}{1+\sqrt{1-(\frac{v}{c})^2}}=mc^2+\frac{1}{2}mv^2+... \\ dE_k=vdp=d\tilde{E}=hdf\end{array}\right\}, \\ \left\{(\frac{u}{v})=\frac{\tilde{E}}{pv}=\frac{pu}{E_k\left[1+\sqrt{1-(\frac{v}{c})^2}\right]}=\frac{1}{1+\sqrt{1-(\frac{v}{c})^2}}=\frac{\gamma}{\gamma+1}\right\}, \\ \left\{\begin{array}{l}\lambda=\frac{h}{p}, k=\frac{2\pi}{\lambda}=\frac{2\pi}{h}p, \omega=2\pi f\,, \\ \frac{d\lambda}{\lambda}=-\frac{dp}{p}=-\frac{dk}{k}=-\frac{df}{f}, u=\frac{\omega}{k}, v=\frac{d\omega}{dk}\end{array}\right\}\end{array}\right\} \Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}v=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=u+p\frac{du}{dp}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=h\frac{df}{dp}=\frac{df}{df_s}=\frac{2u}{1+\frac{uv}{c^2}}\,, \\ u=\lambda f=\frac{\omega}{k}=\frac{\tilde{E}}{p}=\frac{hf}{p}=\frac{f}{f_s}=\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}=\frac{E_k}{p},\; f_s=k/2\pi \quad\Rightarrow \\ \Rightarrow\quad 0\le 2u\le\sqrt{uv}\le v\le c\,, \\ d\tilde{E}=hdf=mc^2d\gamma=dE_k=dE,\quad \frac{df}{f}=(\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}}\Rightarrow\frac{\Delta f}{\bar{f}}=(\frac{\Delta v}{\bar{v}})\cdot\frac{1+\sqrt{1-\frac{\bar{v}^2}{c^2}}}{1-\frac{\bar{v}^2}{c^2}}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l}\frac{\tilde{E}}{mc^2}=\frac{hf}{mc^2}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1=\gamma-1=\frac{\tilde{E}}{E_0},\; E_0=mc^2=const., \\ \frac{\tilde{E}}{\gamma mc^2}=\frac{hf}{\gamma mc^2}=1-\sqrt{1-\frac{v^2}{c^2}}=1-\frac{1}{\gamma}=\frac{\tilde{E}}{E_{total}},\; E_{total}=\gamma mc^2=\gamma E_0=E, \\ \frac{\tilde{E}}{E_k}=\frac{\tilde{E}}{(\gamma-1)mc^2}=\frac{hf}{(\gamma-1)mc^2}=1,\; E_{total}=E_0+E_k=E_t\,, \\ p^2c^2+E_0^2=E_t^2,\quad p^2v-pE_t+p_0E_0=0, \\ \tilde{E}=pu=-E_0\pm\sqrt{E_0^2+p^2c^2}=E_k\left\{=-E_0+\sqrt{E_0^2+p^2c^2}=E_0\left[\sqrt{1+(\frac{pc}{E_0})^2}-1\right]\right\}, \\ \left\{\begin{array}{l}\vec{p}+\tilde{p}=\vec{P}=\overrightarrow{const}\Rightarrow d\vec{p}=-d\tilde{p} \\ d\tilde{E}=hdf=d(pu)=dE_k=vdp=c^2d(\gamma m)=-d(\tilde{p}u)= \\ =-vd\tilde{p}\cdot\cos(\vec{p},\tilde{p})=\left\{vd\tilde{p}\text{ or }-vd\tilde{p}\right\}=h\cdot df_s\end{array}\right\} \\ \Rightarrow\Delta E_k=-\Delta\tilde{E},\;\Delta p=-\Delta\tilde{p},\;\Delta L=-\Delta\tilde{L},\;\Delta q=-\Delta\tilde{q},\;\Delta\dot{p}=-\Delta\dot{\tilde{p}},\;\Delta\dot{L}=-\Delta\dot{\tilde{L}},...\end{array}\right\},\quad (4.2)$$

As we can see (from (4.2) and later in this chapter), here we are also dealing with the generalized concept of "action-reaction", transient and inertial forces (as an equivalent to the Third Newton Law, being similar to electric and magnetic induction laws: ($\Delta \mathbf{p} = -\Delta \tilde{\mathbf{p}}$, $\Delta \mathbf{L} = -\Delta \tilde{\mathbf{L}}$, $\Delta \mathbf{q} = -\Delta \tilde{\mathbf{q}}$ ...) $\Rightarrow$ ($\Delta \dot{p} = -\Delta \dot{\tilde{p}}$, $\Delta \dot{\mathbf{L}} = -\Delta \dot{\tilde{\mathbf{L}}}$, $\Delta \dot{\mathbf{q}} = -\Delta \dot{\tilde{\mathbf{q}}}$ ...)). This becomes explicitly evident after implementing the time differentiation on involved momentum and charge properties, regardless of their field nature, being applicable to gravitation, electromagnetic fields, rotation, etc. See more in Chapter 4.2; -equations from 4.8-2 to 4.8-4.

Later (in Chapters 5. and 10.) we will see that PWDC relations (4.2), and Uncertainty relations (5.1), $\Delta x \cdot \Delta p = \Delta t \cdot \Delta E = h \cdot \Delta t \cdot \Delta f \geq h/2 \Leftrightarrow \Delta x \cdot \Delta \tilde{p} = \Delta t \cdot \Delta \tilde{E} = h \cdot \Delta t \cdot \Delta f \geq h/2$, are effectively describing mutual space-time proportionality, or coupling, since group velocity (of a specific narrow-band signal, or wave group) is also equal to,

$$v = \frac{\Delta x}{\Delta t} = \frac{\Delta \tilde{E}}{\Delta p} = \frac{\Delta \omega}{\Delta k} = h\frac{\Delta f}{\Delta p} \Leftrightarrow v\Delta t = \Delta x = \frac{\Delta \omega}{\Delta k}\Delta t = h\frac{\Delta f}{\Delta p}\Delta t .$$

*Practically, it will become much more evident (later, in this and other chapters) that equations and relations found in (4.1) and (4.2) describe the essential **P**article-**W**ave **D**uality **C**ode, or **PWDC** (see also: (4.3), tables T.4.0 and T 5.3, Uncertainty relations from Chapters 4.0., 5., and 10., as the important complements to **PWDC** understanding). **PWDC** relations should also include mathematical formalism with Einstein-Minkowski 4-vectors, conveniently united with the Analytic Signal concept (see more in Chapter 10.). The most direct, exact, and explicit (experimental and mathematical) confirmation of **PWDC** relations can be found in Chapter 4.2. of this book (see 4.2.2., Example 3: Elastic collision photon-electron; -Compton Effect). What we formulated here as the **PWDC** also belongs to natural properties of a relevant Complex (and hypercomplex) Analytic Signal, or Phasor, presenting the most general mathematical model and framework for operating with matter wavefunctions (or with de Broglie matter waves).*

4.1.1.1. Photons and Particle-Wave Dualism

We already know about number of very convincing and successful wave-particle duality related experiments and analyses regarding interactions between electrons, photons, neutrons, other elementary particles, and matter (such as Compton and Photoelectric effect, electron-positron creation and/or annihilation, absorption, and emissions of photons by atoms, continuous spectrum X-rays generators, Bragg's diffraction of X-rays, electrons, and neutrons…). This should help in understanding the nature of photons and origins of Particle-Wave Duality (see examples and analyzes of experiments in Chapter 4.2).

Matter waves concepts (formulated by L. de Broglie, M. Planck, A. Einstein, N. Bohr...) initially and historically were partially established based on mechanical analogies between photons and moving particles. During such analogical formulations of matter-waves properties, certain smaller omissions, negligence, and contradictions were introduced (related to improper and incomplete, or unidirectional analogical conclusions regarding photons, particles, and matter waves). ***The biggest and most problematic mistake here (meaning in the contemporary quantum theory) has been that a total energy of a photon is analogically compared or linked to a total particle energy***. Any photon has only motional energy (without rest mass), and its total energy is its total motional energy (and nothing else), meaning that photon should be analogically compared only with motional-energy states of particles (excluding states of rest energies and rest masses). This, first step-stone mistake created incomplete (and wrong) foundations of matter waves and particle-wave dualism, and later, it was necessary to introduce additional correcting steps and concepts to comply with the laws and experimental manifestations of Nature (all of that belonging to modern Quantum Theory). For instance, the following relations, $\mathbf{hf} = \mathbf{mc}^2$ or $\mathbf{hf} = \gamma\mathbf{mc}^2, \mathbf{uv} = \mathbf{c}^2$, between certain mass and corresponding matter wave, are not at all generally applicable or correct, but we can still find such relations in modern Physics books when explaining particle-wave duality concepts. In addition, nobody made sufficiently selective and completed analyses of continuous X-rays and Compton and Photoelectric effects spectral components produced by high-speed electrons. From such analyses, we can clearly find that motional energy and mechanical moments of involved or produced photons are equal to motional energies of involved electrons, or being directly exchangeable between motional interaction participants, without any participation of involved rest masses. In this book, mentioned analyses of X-rays generation, Compton and Photoelectric effects will be presented later (see Chapter 4.2 Interactions modeling), showing explicitly and clearly that matter-wave energy of interacting participants corresponds only to a relevant kinetic or motional particle energy, without rest masses participation. By considering rest-mass energy as the part of matter-wave energy, founders of particle-wave duality and Quantum theory (and their followers) did not contribute on the best way to a proper description of matter waves and wave-particle duality. Matter waves (or de Broglie waves) are only motional energy-momentum states without rest-mass energy states, but this is still not, explicitly, clearly, and affirmatively, stated in Quantum Theory, and here we are going to rectify this situation. Of course, in certain situations, photons can be fully transformed into particles or particles impacts can generate only photons, but this is not the generally valid situation. In addition, Probabilities, Statistics, and Uncertainties (or Uncertainty Relations) are not the unique, natural, and best support and answers to mentioned step-stone mistakes (except being well applicable for mathematical modeling when we have big sets of identical members). ***Photons also have structural helicity of involved electric and magnetic field vectors, including spin moment, and we should analogically conclude that particles in linear motion should also have certain matter-waves, spin-related helicity*** (or spiraling, solenoidal field effects), meaning that analogies "Photon-Moving-Particle" should be equally applicable in both directions (regarding linear motion and associated helical or spinning effects; see similar situation regarding spinning disk and "linear-moment thrust" around equations (2.11-4), in Chapter 2, and in Chapter 10. see "10.02 Meaning of natural forces"). For instance, an electron in linear motion should create helical, solenoidal, or spiral, spinning magnetic field, around its path of motion. Anyway, for electrons we already know that they have intrinsic magnetic and mechanical spin moments, implicating that an electron should be certain complex, electromagnetic field structure, where photons or electromagnetic field vectors are, on some specific way, conveniently structured (of course, while all conservation laws are satisfied, and other measurable facts are taken properly into an account). For instance, self-closed standing matter waves are often associated to creation of stable micro end elementary particles; -*see on Internet similar concept about Henry Augustus Rowland effect of the magnetic field around a rotating conductor, as presented by Jean de Climont*. Something similar (regarding matter waves helicity) should also and analogically be valid for protons and neutrons, and if we consider that atoms and other neutral macro particles are composed of electrons, protons, neutrons, photons... and their structured, standing-waves agglomerations, again analogically, we can conclude that electromagnetically neutral micro and macro particles in linear motions have associated and complementing matter-waves helicity and spin moments. Contemporary physics (and Faraday-Maxwell theory), regarding electrons in linear motions, is not saying that electrons have associated helical magnetic field components, and that matter-waves (or moving masses) also respect analogy with photon spinning or helicity, meaning that a matter wave helical spinning about a moving electromagnetically neutral particle, is still not properly explained.

We know that photons carry measurable energy and linear momentum (for instance, pressure of photons' radiation creates mechanical motion or rotation), analogically implicating those photons are comparable to a stream of particles. If photon has certain unknown rest-mass $\mathbf{m_o}$, then its relativistic energy and momentum should be,

$$\begin{bmatrix} \tilde{E}_p = \gamma m_o c^2 = m_o c^2 / \sqrt{1 - v^2/c^2} = hf_p \neq 0 \\ \tilde{p}_p = \gamma m_o v = m_o v / \sqrt{1 - v^2/c^2} \neq 0 \\ \tilde{p}_p / \tilde{E}_p = v/c^2, \ v = c \end{bmatrix} \Rightarrow \begin{bmatrix} m_o = 0, \\ \tilde{p}_p = \tilde{E}_p / c = hf_p / c = h/\lambda_p = \tilde{m}_p c, \\ \tilde{E}_p = \tilde{m}_p c^2 = hf, \\ \lambda_p f_p = c \end{bmatrix},$$

and we can find that its rest mass should be equal to zero (or being nonexistent), implicating that any particle with non-zero rest mass can never reach speed v = c.

Since photons also have certain spin, let us additionally exploit such particle-matter-wave-photon analogy (see below). Such analogy was not originally considered useful when de Broglie formulated his hypothesis about matter-waves duality, and can be formulated as follows,

Wave-Particle Duality and analogical PHOTON-PARTICLE comparison, extended from linear motion towards associated helicity

$$\left\{ \begin{array}{l} \text{PHOTON} \\ \text{(accounting only its linear motion)} \\ \tilde{E}_p = hf_p = \tilde{m}_p c^2 = \tilde{p}_p c, \ \tilde{p}_p = \tilde{E}_p / c = hf_p / c, \\ f = \tilde{p}_p c / h, \ \lambda_p f_p = c \Rightarrow \boxed{\lambda_p = c / f_p = h / \tilde{p}_p} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \text{Particle} \\ \text{matter} \\ \text{wave} \\ \text{wavelength} \\ \boxed{\lambda = h/p} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \text{PHOTON (acounting its spinning)} \\ \tilde{L}_p = h/2\pi, \ h = 2\pi \tilde{L}_p, \\ \tilde{E}_p = hf = \omega \tilde{L}_p = \tilde{m}_p c^2 = \tilde{p}_p c, \\ \lambda_p f_p = c \Rightarrow \lambda_p = c/f_p = h/\tilde{p}_p = 2\pi \tilde{L}_p / \tilde{p}_p, \\ \boxed{\tilde{L}_p = \lambda_p \tilde{p}_p / 2\pi = h/2\pi} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \text{Particle spin-moment} \\ \text{as a matter wave} \\ \boxed{\begin{array}{l} \tilde{L} = \lambda p / 2\pi = h/2\pi \\ \tilde{L}\omega = pv \end{array}} \end{array} \right\}$$

Effectively, particle in linear motion should also have certain associated spinning moment and vice versa (by the analogy with a photon properties), and this spinning is related to de Broglie matter waves. ***In other words, spinning object is also (based on analogical conclusions) creating "linear momentum thrust" or force, which should be casually related to the force of Gravitation (of course considering spinning and rotation-associated effects of magnetic field).***

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

A) To initiate new insights in relation to photons and wave-particle duality, we can again summarize our knowledge about photons and wave-particle duality starting from reasonable assumptions and facts in relation to photons, such as,

1. *What we understand as a photon is certain spatially-&-temporally localizable (and finite) structure of an electromagnetic field, which has an energy amount of* $\tilde{E} = h \cdot f = \frac{h}{2\pi} \cdot \omega$, $h = \text{const.}$ *It is likely that some of different forms of electromagnetic waves are not shaped and do not carry energy like photons.*

2. *Photon has zero rest mass. It presents only a wave, and it has motional or kinetic energy, which is at the same time its total energy* $\tilde{E}_t = \tilde{E}_k = \tilde{E} = hf = \tilde{m}c^2$.

3. *Effective motional mass of a photon is* $\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{h \cdot f}{c^2} = \frac{h}{2\pi c^2} \cdot \omega$, $(h, c = \text{consts.})$.
Linear moment of a photon is $\tilde{p} = \tilde{m}c = \frac{\tilde{E}}{c} = \frac{hf}{c} = \frac{h}{2\pi c}\omega$, $(h, c = \text{consts.})$. *We know that such statements and mathematical expressions are correct since in number of impact and scattering interactions, like Compton and Photoelectric effects are, we are successfully explaining experimentally verifiable results using mentioned expressions for photon mass, momentum, and energy …*

4. *We could now safely consider that photon, when moving in an isotropic and homogenous, non-dissipative and non-dispersive space (like in a vacuum), presents certain kind of uniform and inertial, self-maintaining linear and spinning motion, or motional energy state, and effectively manifests some particle properties (interacting with other real, non-zero rest mass particles, like particle interactions are treated or behaving in mechanics).*

5. *Consequently, photons should have limited spatial, temporal, and spectral sizes and lengths (or durations). For instance, if a photon is propagating along certain linear path s, the relations* $ds = c \cdot dt \Rightarrow \Delta s = c \cdot \Delta t$ are naturally valid, where Δs and Δt are spatial and temporal durations or lengths of the same photon. Since energy of a photon is $\tilde{E} = hf = \tilde{m}c^2$, we can extend the specification of a photon size or its finite spectral and energy content as, $d\tilde{E} = h \cdot df = c^2 \cdot d\tilde{m} \Rightarrow \Delta\tilde{E} = h \cdot \Delta f = c^2 \cdot \Delta\tilde{m}$. ***Since photons are obviously space-time-energy limited, energy finite and localizable electromagnetic matter-wave packets in all mentioned domains, we should conclude that amplitude or envelope function of a photon is like Gaussian pulse (because only Gaussian pulses can be well defined in both, temporal and spatial domains).*** The next consequence of such situation is that here we can approximately apply relations of "*elementary certainty*" (5.3) of *matter-states domains*, as described in Chapter 5. of this book, which are here presenting relations between finite (total, non-statistical) durations of certain photon in all of its domains,

$$\left(\Delta q_{mag.} \cdot \Delta q_{el.}\right)_{min.} = \left(\Delta\alpha \cdot \Delta L\right)_{min.} = h \cdot \left(\Delta t \cdot \Delta f\right)_{min.} = \left(\Delta x \cdot \Delta \tilde{p}\right)_{min.} = c\left(\Delta t \cdot \Delta \tilde{p}\right)_{min.} =$$
$$= \left(\Delta t \cdot \Delta \tilde{E}\right)_{min.} = c^2 \left(\Delta t \cdot \Delta \tilde{m}\right)_{min.} \geq h/2,\ \Delta \tilde{E} = h \cdot \Delta f = c^2 \cdot \Delta \tilde{m},\ \Delta x = c \cdot \Delta t.$$

Furthermore, in Chapter 9. of this book, under "9.1. Wave Function of a Single Photon", we can find number of additional relations between temporal, spatial and spectral durations, amplitudes, and other important characteristics describing a *single photon wave function*.

Consequently, here we could imaginatively speculate with the concept that photons are specifically structured, elementary matter-wave packets and building blocks of all matter in our Universe, and that different matter-states are mutually communicating and interacting involving photons.

B) *Based on such initial grounds (from 1. to 5.), we can create number of conclusions and new concepts, such as,*

6. *In mechanics, the closest analogical state of motion, or motional energy, analogical to a photon, as just described, is a spinning body, which could be a disk, certain toroidal form, spheroidal or ellipsoidal body, and which has spinning motional energy expressed as,* $E_s = L \cdot \omega = 2\pi L \cdot f$ *(as gyroscopes). If mentioned spinning body is in a self-maintaining, uniform and inertial motion, we can consider that its angular moment is constant,* $L = const.$ *Now, we are coming to the direct conceptual and mathematical analogy between a photon and a spinning (or orbiting) body, since photon energy is* $\tilde{E} = h \cdot f = hf_s = \frac{h}{2\pi} \cdot \omega = \tilde{L} \cdot \omega = \tilde{m} c^2,\ \tilde{L} = \frac{h}{2\pi} = const.$*, and spinning body energy is also* $E_s = L \cdot \omega = hf_s,\ L = \overline{const.}$*. Based on such analogy, we could conceptually consider that photon really presents certain spinning, electromagnetic, spatial formation. If there are other, isomorphic, equivalent and 1:1 functional or spatial mapping configurations or topologies, we can say that photon could also take forms of such spatial formations.*

7. *Photon energy we can now express as,* $\tilde{E} = h \cdot f = \frac{h}{2\pi} \cdot \omega = \tilde{L} \cdot \omega = \tilde{m}c^2,\ \tilde{L} = \frac{h}{2\pi} = \frac{\tilde{m}c^2}{2\pi f} = \frac{\tilde{E}}{\omega} = const.$ *We could now speculate that not all electromagnetic waves are like photons, and that* $\tilde{E} = h \cdot f = \frac{h}{2\pi} \cdot \omega,\ h = const.$ *is probably not applicable to all forms of electromagnetic waves. Only photonic electromagnetic formations that are part of interactions within and between atoms and elementary particles, are conceptually and energetically presentable as spinning formations, or as what we consider being photons (and in some cases behaving like particles), since only in such experimental situations we create and operate with photons. In fact, for electromagnetic waves being like photons, or like other waveforms, there is a decisive question how such waves are being created. L-C antenna-*

circuits-created radio, telephone and TV electromagnetic waves are most probably different wave formations when compared to spinning photons that are spatially limited and finite by energy. Other forms of electromagnetic waves could have different, not bounded spatial shapes (being not at all analogue to photonic, inertial, and spinning particles). Photons have constant spinning moments, $\tilde{L} = \frac{h}{2\pi} = \text{const.}$, *and other electromagnetic waves (created by L-C antenna circuits), most probably, have variable angular moments.*

8. *If we replace spinning photon with an equivalent rotating toroidal (or ring) form with standing waves structure of electromagnetic waves, which could be considered as an isomorphic, 1:1 functional mapping (or transformation from a spinning disk or ring to a spinning torus), we will again have similar angular or orbital, equivalent to mechanical moment quantizing, as for example,* $2\pi r = n\lambda = n\frac{h}{\tilde{p}} \Leftrightarrow \tilde{L} = \tilde{p}r = n\frac{h}{2\pi}, n = 1, 2, 3....$ *Bohr-Sommerfeld quantization and development of Schrödinger equation is also familiar approach to here-initiated torus quantization based on stable, self-closed standing waves. Always valid mathematical Uncertainty relations are also valid and applicable here, regardless of Heisenberg formulations, since such relations are the product of mathematical, generally valid signal analysis and relations between total (non-statistical) durations of mutually original and spectral domains. This kind of imaginative excursions is showing how stable particles (with non-zero rest masses) could be created starting from standing-waves electromagnetic formations combined with an inertial and stable rotation, with elements of certain intrinsic periodicity, and with constant spinning or orbital moment properties. Electrons are an example of specific toroidal and resonant formations of standing electromagnetic waves. Solid matter, atoms and stable rest masses most probably have photons and similar electromagnetic-waves formations as their primary constituents like nucleation states (combined or formatted on a way that all relevant conservation laws are satisfied). For instance, we can also show that a single photon has a zero rest-mas* $\tilde{m}_0 = 0$ *based on its 4-vector moment, and inertia-systems invariance relation, as follows,*

$$\tilde{P}_4 = (\tilde{p}, \frac{\tilde{E}}{c}) \Rightarrow \tilde{p}^2 - \left(\frac{\tilde{E}}{c}\right)^2 = -\left(\frac{\tilde{E}_0}{c}\right)^2 \Rightarrow 0 = -\left(\frac{\tilde{E}_0}{c}\right)^2 \Rightarrow$$

$$\Rightarrow \tilde{E}_0 = \tilde{m}_0 c^2 = 0 \Leftrightarrow \tilde{m}_0 = 0, \left(\tilde{E} = hf = pc\ , \tilde{m} = \frac{hf}{c^2}, p = \frac{hf}{c}\right).$$

However, we can also show on a similar way that ***conveniently superimposed group of non-collinear and favorably spatially focused photons, and/or other matter wave-packets without rest masses, could create a particle with non-zero rest mass*** M_0, *as follows,*

$$\tilde{P}_4 = (\sum_{(n)} \tilde{p}_i, \frac{\sum_{(n)} \tilde{E}_i}{c}) = (\sum_{(n)} \tilde{p}_i, c \cdot \sum_{(n)} \tilde{m}_i) \Rightarrow \left[\sum_{(n)} \tilde{p}_i\right]^2 - \left[c \cdot \sum_{(n)} \tilde{m}_i\right]^2 = -\left(\frac{\tilde{E}_0}{c}\right)^2 = -\left(M_0 c\right)^2 \Rightarrow$$

$$\Rightarrow M_0 = \sqrt{\left[\sum_{(n)} \tilde{m}_i\right]^2 - \left[\frac{1}{c}\sum_{(n)} \tilde{p}_i\right]^2} = \frac{L_o \omega_0}{c^2} = \frac{Hf_0}{c^2} \neq 0,\ \tilde{m}_i = \frac{hf_i}{c^2},\ \tilde{p}_i = \frac{hf_i}{c},\ H = const.$$

Of course, all conservation laws should be properly addressed if we intend to describe creation of real particles starting from wave-packets without rest masses. Based on similar grounds we could start developing ideas about new mass propulsion or trust methods, about targeted particles projection, about new ways of motion and energy transfer, etc.

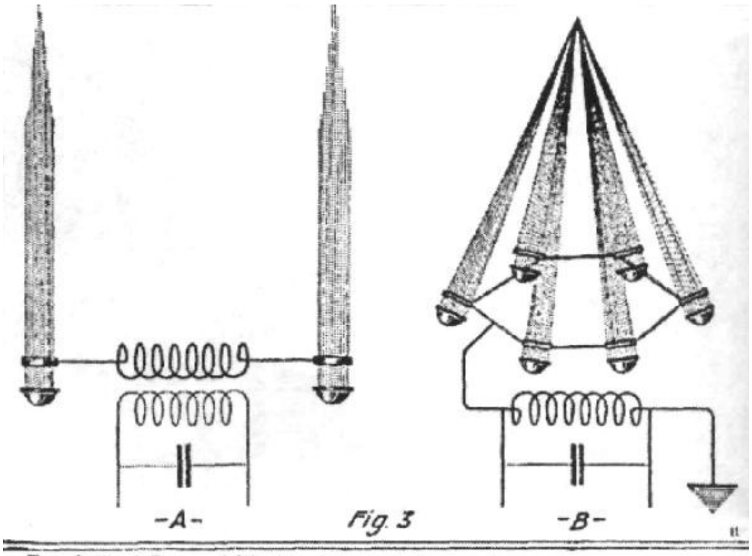
What is essential and ontological here, are standing waves formations of electromagnetic waves on self-closed spatial structures, orbits or paths, and expectations that stable, solid, non-zero rest-masses could be created starting from self-closed, structural spatial formations and combinations of photons and other wave packets.

As an exciting brainstorming and imaginative excursion towards creating real masses from wave packets, we could revamp some familiar, still innovative creations of Nikola Tesla. The biggest ever-born, creative and visionary inventor, as well as a visionary and realistic science fiction dreamer on our planet was Nikola Tesla, who invented on his exotic, magic, and very imaginative way everything that still presents important electromagnetic, electrotechnical and electromechanical grounds of our contemporary technological civilization. Tesla was mostly using pure imagination and spiritual or mental visualization, going almost directly to final inventions formulation, like taking his inventions from certain "universal data bank or common cosmic knowledge library".

*Tesla also gave number of descriptions, some of them still not well understood or documented, almost like science fiction predictions and looks-like inventions, where proper theoretical and factual grounds or terminology to give better and more appropriate explanations and formulations have been still missing. Modern science and technological community are presently considering such spiritual inventions indeed as creative but as not well founded, almost as an arbitrary dreaming or brainstorming and magic visions. Tesla also has number of such (mental) inventions, visions and patents that are completely verified and presenting fundaments of our modern planetary and technological civilization. We could now ask ourselves if other Tesla inventions and dreaming, being still not verified, confirmed, or technically materialized, also have certain predictive merits or potentially significant meaning, if properly addressed, and slightly reformulated with updated modern scientific language, proper mathematics, and with contemporary scientific achievements. One of such, still not materialized, and not well or completely formulated mental invention is about Tesla's "**death-rays**". We could imaginatively and creatively, connect and transform (or revitalize) mentioned "**death-rays**" idea with **photons transformation, from zero-rest-mass electromagnetic wave-packets to real non-zero-rest-mass particles (like electrons)**, as already discussed (see citation below).*

*One of still not realized Tesla's invention about "**death-rays**". Citation from internet: Proposing the "death ray" for defense - Philadelphia Inquirer - October 20, 1940:*

*"It is based on an entirely new principle of physics that nobody ever has dreamed of. **It is different from the principle embodied in my inventions relating to the transmission of electrical power from a distance**, for which I hold a number of basic patents."*



*Illustrations from an article in the March 1920 issue of Electrical Experimenter entitled "Wireless Transmission of Power Now Possible". The illustrations show Tesla's prototype devices for "directed ionized beam transmissions," a "**deathray**—searchlight" device. But according to Tesla, the results of tests did not justify the hope of important practical applications in large distance.*

Comment from the author of this book: The same Tesla's invention, when slightly modified and combined with proper electrodes and light sources to stimulate Photoelectric and Compton effects, has big chances to produce results estimated by Nikola Tesla.

Let us now compare, analogically, dimensionally, and mathematically, essential characteristics, equations, and formulas applicable to a photon (which here we consider as a ***Gaussian-envelope wave-packet***, of course without rest mass), with a moving particle that has a non-zero rest mass. The comparison photon-particle will be presented in the table T.4.0, based on data from (2.11.3), (4.1), (4.2) and T.1.8. Specific formulas and symbols, found in T.4.0, are introduced there only for making simpler and more indicative (dimensionally correct) mathematical analogies and comparisons, to initiate and support new ideas about wave-particle duality that would appear in this book later. ***It will become apparent, concluding based on analogical comparisons found in T.4.0, and in T.4.0.2, that photon properties can be compared only with motional or kinetic energy states of a particle (or with a particle-equivalent matter-wave packet).*** Rest mass is not the part of matter waves (except in situations with annihilation of matter and antimatter particles, and in certain impact interactions in high-energy particle accelerators when rest masses are being decomposed or annihilated). Similar conclusions can be obtained when we selectively analyze several simple interactions, mentioned earlier (see also in this chapter: "4.1.3.1. Example 2: X-ray Spectrum and Reaction Forces" and see examples from Chapter 4.2).

Since any photon has its spin, by analogy, an equivalent matter wave (or motional particle) should also have a spin (because good and complete analogy should work in both directions, considering all photon properties).

Citation from [147] : **Electrons are spin 1/2 charged photons generating the de Broglie wavelength**
Richard Gauthier. Proceedings Volume 9570, The Nature of Light: What are Photons? VI; 95700D (2015). 10 September 2015. https://doi.org/10.1117/12.2180345 Event: SPIE Optical Engineering + Applications, 2015, San Diego, California, United States

Abstract
The Dirac equation electron is modelled as a helically circulating charged photon, with the longitudinal component of the charged photon's velocity equal to the velocity of the electron. The electron's relativistic energy-momentum equation is satisfied by the circulating charged photon. The relativistic momentum of the electron equals the longitudinal component of the momentum of the helically-circulating charged photon, while the relativistic energy of the electron equals the energy of the circulating charged photon. The circulating charged photon has a relativistically invariant transverse momentum that generates the z-component of the spin ħ / 2 of a slowly-moving electron. The charged photon model of the electron is found to generate the relativistic de Broglie wavelength of the electron. This result strongly reinforces the hypothesis that the electron is a circulating charged photon. Wave-particle duality may be better understood due to the charged photon model—electrons have wavelike properties because they are charged photons. New applications in photonics and electronics may evolve from this new hypothesis about the electron.

INTRODUCTION
In his Nobel Prize lecture Paul Dirac[1] said: "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

One surprising result of the Dirac equation was that the electron moves at the speed of light and has a characteristic associated length $R_o = \hbar / 2mc = 1.93 \times 10^{-13}$m, where $\hbar = h / 2\pi$ and h / mc is the Compton wavelength $\lambda_{Compton} = 2.426\times10^{-12}$m. A second and related surprise was that the electron has a "trembling motion" or *zitterbewegung* in addition to its normal linear motion. Dirac's finding of light-speed for the electron is particularly problematical because electrons are experimentally measured to travel at less than the speed of light. Dirac did not offer a spatially extended model of the electron to correspond to these results, though his results did contain the characteristic length R_o.

Various researchers such as Hestenes[2], Williamson and van der Mark[3], Rivas[4], Hu[5], and Gauthier[6-8] have suggested spatially-extended electron models where the internal energy of the electron circulates at light-speed with the characteristic Dirac radius $R_o = \hbar / 2mc$. Both the Dirac equation's light-speed motion and its *zitterbewegung* are reflected in these models.

Hestenes and Rivas independently analyzed the Dirac equation for spatial and dynamical characteristics of the electron's motion. Based on these analyses, they independently proposed that the trajectory of a moving free electron is a helix along which the electron's charge moves at light-speed. When the linear speed and momentum of the electron are zero, the helix becomes a circle of radius Ro = ħ / 2mc. Neither author associates this circulating light-speed electric charge with a photon.

Another idea regarding the comparison between a photon and a real particle in motion is to show that internal (or intrinsic) particle-wave properties (such as its characteristic wavelength and frequency) are closely related and connected to similar external matter-wave attributes of a particle (such as de Broglie wavelength and frequency), originating from certain associated spinning. Oversimplifying this situation, we will show (later) that ontologically, particles are electromagnetic fields-folded, self-stabilized, standing matter-waves structures, or saying the same more imaginatively and hypothetically, structures assembled from photons (or from electromagnetic wave packets). All of that is also in the function of an analogical and indicative explanation of the meaning of **PWDC** (see more in Chapters 4,0 and 10.).

T.4.0. Photon – Particle Analogies (see also T.4.0.2, from Chapter 4.0)

	PHOTON	MOVING PARTICLE		
	Expressions related to wave or motional energy	Expressions related to states of rest	Expressions related to motional energy states	Expressions related to a total energy content
Energy	$\tilde{E} = hf$	$E_0 = mc^2$	$E_k = (\gamma - 1)mc^2 = \frac{pv}{1+\sqrt{1-v^2/c^2}}$	$E_t = E_0 + E_k = \gamma mc^2$
	$d\tilde{E} = hdf$	$dE_0 = 0$	$dE_k = vdp = hdf = d\tilde{E}$	$dE_t = dE_k = vdp = hdf$
Frequency	$f = \frac{\tilde{E}}{h}$ Mean frequency of the photon wave group	$f_0 = \frac{E_0}{h} = \frac{mc^2}{h}$	$f = \frac{E_k}{h} = \frac{\tilde{E}}{h} = f_t - f_0 = (\gamma - 1)f_0 = \frac{\gamma(\Delta f)^*}{\gamma - 1} = \frac{c^2}{uv}(\Delta f)^*$	$f_t = f_0 + f = \frac{E_t}{h} = \frac{\gamma mc^2}{h} = \gamma f_0$
	$df = \frac{d\tilde{E}}{h}$	$df_0 = 0$	$df = \frac{dE_k}{h} = \frac{d\tilde{E}}{h}$	$df_t = df = \frac{dE_t}{h} = \frac{dE_k}{h}$
Mass	$\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{hf}{c^2}$	$m = \frac{E_0}{c^2}$	$m_k = \Delta m = \frac{E_k}{c^2} = (\gamma - 1)m = \frac{\tilde{E}}{c^2} = \tilde{m}$	$m_t = \frac{E_t}{c^2} = \gamma m = m + \Delta m$
	$d\tilde{m} = \frac{d\tilde{E}}{c^2} = \frac{hdf}{c^2}$	$dm = 0$	$dm_k = \frac{dE_k}{c^2} = d\tilde{m}$	$dm_t = \frac{dE_t}{c^2} = \frac{dE_k}{c^2} = dm_k$
	$\frac{\tilde{E}}{\tilde{m}} = c^2$	$\frac{E_0}{m} = \frac{E_k}{\Delta m} = \frac{E_t}{m_t} = \frac{dE_k}{dm_k} = \frac{dE_t}{dm_t} = \frac{d\tilde{E}}{d\tilde{m}} = c^2$		
Linear Momentum & Energy	$\tilde{p} = \frac{\tilde{E}}{c^2}c = \frac{h}{\lambda} = \frac{hf}{c} = \tilde{m}c$	$p_0 = \begin{cases} \frac{E_0}{c^2}v = mv, \text{ or} \\ \frac{h}{\lambda_0} = \frac{E_0}{c} = mc\frac{h}{\lambda_0} = \\ = \frac{E_0}{c} = mc \end{cases}$	$\Delta p = \frac{E_k}{c^2}v = (\Delta m)\cdot v = \frac{h}{\Delta\lambda} = (\gamma - 1)mv$	$p = \frac{E_t}{c^2}v = \frac{h}{\lambda} = \frac{\Delta p}{\gamma - 1} = p_0 + \Delta p = \gamma p_0 = \gamma mv$

<table>
<tr>
<td rowspan="2"></td>
<td rowspan="2">$\lambda = \frac{h}{\tilde{p}} = \frac{h}{\tilde{m}c} = \frac{c}{f}$</td>
<td>$\lambda_0 = \frac{h}{p_0} = \frac{h}{mc} = \frac{u_0}{f_0}$</td>
<td>$(\Delta\lambda)^* = \frac{h}{\Delta p} = \frac{h}{(\Delta m)\cdot v} = \frac{\lambda_0}{\gamma - 1} = \frac{\lambda\lambda_0}{\Delta\lambda} = (\frac{c}{f})(\frac{c}{v}) = \frac{u}{(\Delta f)^*} = \lambda\frac{f}{(\Delta f)^*}$</td>
<td>$\lambda = \frac{h}{p} = \frac{h}{\gamma m v} = \frac{\lambda_0}{\gamma} = \frac{u}{f}$</td>
</tr>
<tr>
<td colspan="3">$\frac{1}{\lambda} = \frac{1}{\lambda_0} + \frac{1}{(\Delta\lambda)^*}$</td>
</tr>
<tr>
<td></td>
<td>$u = \lambda f = c = v$</td>
<td>$u_0 = \lambda_0 f_0 = c$</td>
<td>$u^* = (\Delta\lambda)^*(\Delta f)^* = u$</td>
<td>$u = \lambda f$</td>
</tr>
<tr>
<td></td>
<td>$uv = c^2$</td>
<td>$u_0 v_0 = c^2$</td>
<td colspan="2">$uv = c^2(1 - \sqrt{1 - v^2/c^2}) = c^2(1 - 1/\gamma) \le c^2$
$0 \le 0.5v \le u < v \le c$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{E} = \tilde{m}uv = \tilde{m}c^2 = hf$</td>
<td>$E_0 = mu_0 v_0 = mc^2$</td>
<td>$\tilde{E} = m^* uv = \gamma m uv = E_k = (\gamma - 1)mc^2 = hf$</td>
<td>$E_t = \gamma mc^2$</td>
</tr>
<tr>
<td></td>
<td>$d\tilde{E} = hdf = c^2 d\tilde{m} = cd\tilde{p}$</td>
<td>$dE_0 = 0$</td>
<td colspan="2">$d\tilde{E} = dE_k = dE_t = c^2 d\tilde{m} = hdf = vdp$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{E} = hf = \tilde{m}c^2 = \tilde{p}c$
$(c = \lambda f = u = v)$,
$\tilde{E}\cdot\Delta t = \tilde{p}\cdot\Delta s = h$,
$u = v = \Delta s/\Delta t = c$
(photon is Gaussian-envelope wave-group)</td>
<td colspan="3">$p^2c^2 + E_0^2 = E_t^2 = (E_0 + E_k)^2$, $p = h/\lambda$, $\lambda = hc/\sqrt{E_t^2 - E_0^2}$
$\gamma = (1 - v^2/c^2)^{-1/2} = (1 - uv/c^2)^{-1}$, $u = v/(1 + 1/\gamma) = \lambda f$
$(\frac{\gamma - 1}{\gamma})\lambda_0 f_0 = (\Delta\lambda)^*(\Delta f)^* = \lambda f = u$, $\frac{(\Delta f)^*}{f} = \frac{uv}{c^2} = \frac{\gamma - 1}{\gamma} = \frac{\lambda}{(\Delta\lambda)^*}$
$p\lambda = p_0\lambda_0 = (\Delta p)(\Delta\lambda)^* = h$, $\frac{p}{p_0} = \frac{\lambda_0}{\lambda} = \gamma$,
$\Delta\lambda = \lambda_0 - \lambda = \lambda(\gamma - 1) = \frac{\lambda\lambda_0\Delta p}{h} = \frac{h\Delta p}{pp_0} = \lambda\frac{\Delta p}{p_0} = \frac{\lambda\lambda_0}{(\Delta\lambda)^*}$</td>
</tr>
<tr>
<td rowspan="2">Angular Momentum

see (2.9.5)</td>
<td>$\tilde{L} = \frac{\tilde{E}}{\omega} = \frac{hf}{\omega} = \frac{h}{2\pi}$</td>
<td>$L_0 = \frac{E_0}{\omega} = \frac{h}{2\pi}$</td>
<td>$\Delta L = \frac{E_k}{\omega} = \frac{h}{2\pi}$
$\tilde{E} = \frac{L\omega}{1 + \sqrt{1 - v^2/c^2}}$</td>
<td>$L = \frac{E_t}{\omega} = L_0 + \Delta L = \frac{c^2}{uv}(\frac{h}{2\pi}) = n(\frac{h}{2\pi})$</td>
</tr>
<tr>
<td>$\frac{\tilde{L}}{\tilde{p}} = \frac{\lambda}{2\pi} = \frac{c}{\omega} = \lambda\frac{\tilde{L}}{h}$
$\tilde{L}\omega = \tilde{p}c$</td>
<td colspan="3">$\frac{L_0}{p_0} = \frac{\Delta L}{\Delta p} = \frac{L}{p} = \lambda\frac{L}{h} = \frac{v}{\omega} \le \left(\frac{c^2}{v^2}\cdot r^* = \frac{c^2}{\omega^2}\cdot\frac{1}{r^*}\right)$, $L\omega = pv$</td>
</tr>
<tr>
<td colspan="5">(The meaning of analogies and terms found in this table will be explained and analyzed later: see (4.1) - (4.8))</td>
</tr>
</table>

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

Here is the resume additionally describing photons:

1. Photon is a narrow-band electromagnetic wave packet, composed of simple-harmonic elementary (sinusoidal) electromagnetic waves, concentrated around certain mean frequency f and it has the energy equal to $\tilde{E} = E_p = hf = \tilde{m}c^2$ (h = Planck constant).

2. Photon propagates linearly, while oscillations of electric and magnetic field components of a photon are transversal.

3. Photon has finite energy and finite spatial and temporal durations in both original and conjugate spectral domains, meaning that its amplitude envelopes are Gaussian curves, both in its original and spectral domains. Single photon wave function could be formulated in its spatial-temporal domain as, $\overline{\psi}(x,t) = |\overline{\psi}(x,t)| \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} e^{I(\omega t \pm kx)}$.

4. When photon is propagating trough an isotropic space with dielectric constant ε, and magnetic permeability μ, its propagation, group, and phase speed are mutually equal and constant, and can be calculated as $c = 1/\sqrt{\varepsilon\mu}$. Photon group and phase velocity (v and u) in isotropic and non-dispersive media are always mutually related or connected as, $\mathbf{v = u - \lambda du / d\lambda = -\lambda^2 (df / d\lambda) = c, u = \lambda f = c}$, $c = 1/\sqrt{\varepsilon\mu}$.

5. Photon is like a moving particle without rest mass, since it has all mechanical moving-particle properties, such as motional energy $\tilde{E} = E_p = hf = \tilde{m}c^2$, linear momentum $\tilde{p} = hf/c = \tilde{m}c$, and an effective motional mass $\tilde{m} = \tilde{p}/c = hf/c^2$. Photon also has a spinning moment $\tilde{L}_f = \mathbf{\tilde{E} / \omega = hf / 2\pi f = \tilde{m}c^2 / 2\pi f = h / 2\pi}$, $\tilde{L}_f / \mathbf{\tilde{p} = c / \omega}$.

6. Photon is an energy-finite and spatially-temporally concentrated and limited wave-group within its domains, like real particles are, and interacts (mechanically and electro-mechanically) with other particles. Mechanical moment and mass of a photon changes during interactions with other particles, and motional mass of a photon does not have the property of inertia (see similar situation regarding an equivalence between a spinning object and "***linear-moment thrust force, produced by spinning***", $L\omega = pv$, around equations (2.11-4), in Chapter 2, and in "10.02 Meaning of natural forces", in Chapter 10.).

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Taken from the Internet: http://en.wikipedia.org/wiki/Matter_creation .

"Because of momentum conservation laws, the creation of a pair of fermions (matter particles) out of a single photon cannot occur. However, matter creation is allowed by these laws when in the presence of another particle (another boson, or even a fermion) which can share the primary photon's momentum. Thus, matter can be created out of two photons.

The law of conservation of energy sets minimum photon energy required for the creation of a pair of fermions: this threshold energy must be greater than the total rest energy of the fermions created. To create an electron-positron pair, the total energy of the photons must be at least $2m_ec^2 = 2 \times 0.511$ MeV = 1.022 MeV (m_e is the mass of one electron and c is the speed of light in vacuum), an energy value that corresponds to soft gamma-ray photons. The creation of a much more massive pair, like a proton and antiproton, requires photons with an energy of more than 1.88 GeV (hard gamma ray photons).

Lev Landau did first calculations of the rate of e^+-e pair production in the photon-photon collision in 1934.[1] It was predicted that the process of e^+–e^- pair creation (via collisions of photons) dominates in the collision of ultra-relativistic charged particles—because those photons are radiated in narrow cones along the direction of motion of the original particle greatly increasing photon flux.

In high-energy particle colliders, matter creation events have yielded a wide variety of exotic massive particles precipitating out of colliding photon jets (see two-photon physics). Currently, two-photon physics studies the creation of various fermion pairs both theoretically and experimentally (using particle accelerators, air showers, radioactive isotopes, etc.).

As shown above, to produce ordinary baryonic matter out of a photon gas, this gas must not only have a very high photon density but also be very hot – the energy (temperature) of photons must exceed the rest mass energy of the given matter particle pair. The threshold temperature for production of electrons is about 10^{10} K, 10^{13} K for protons and neutrons, etc. According to the Big Bang theory, in the early universe, photons and fermions (massive particles of matter) would inter-convert freely. As photon gas expanded and cooled, some fermions would be left over (in tiny amounts ~10^{-10}) because low energy photons could no longer break them apart. Those leftover fermions would have become the matter we see today in the universe around us".

…………………

Here is also the convenient place to remember (and underline) that photons, electrons, and positrons are on a specific electromagnetic way mutually similar, familiar, and complementary, based on the following facts:

-Photons entering or being absorbed by an atom can move electrons (or electron wave groups) inside an atom (or better to say change energies of internal electron states). Incident photons can also produce that atom will emit electrons and new photons, in relation to Photoelectric, Compton and Secondary emissions effects, where is clear that photons are directly interacting with electrons and positrons, and that electrons could be kind of specific electromagnetic wave groups. Emissions and absorptions of photons follow electrons changing energy levels inside an atom.

-Sufficiently high-energy photon (under specific conditions, when passing in close vicinity of an atom), can be fully transformed, or converted into a couple of an electron and a positron. Something similar is happening under convenient "collision" between high-energy photons. A collision of photons could also produce many matter-waves energy-momentum states (including elementary particles and quasi-particles). Such effects should be possible thanks to more complex interactions with associated spinning and torsional effects of locally involved electromagnetic fields.

-Spinning and spiraling electrons (in a strong magnetic field) are creating synchrotron radiation or radiation of Gamma photons (meaning that electrons also have photonic energy content, specially structured).

-Photonic "Bremsstrahlung" of electrons that are passing by another electrically charged particle, are emitted Gamma photons, because electrons are losing energy in such process, meaning that photons are naturally belonging to the content of electrons, and electrons are naturally emitting photons when conditions are met.

-Inverse Compton Effect is also known as the product of "collision" or interaction between moving electron and incoming low energy photon, when high energy Gamma photons can be produced, meaning that photons, positrons, and electrons are mutually and naturally reacting and recombining on different ways.

-Collisions or contacts between an electron and a positron will produce high-energy photons (and annihilate initial reaction participants). Annihilation or contact between any particle and its antiparticle is also producing photons and number of matter-waves and other energy-momentum states (with linear and angular momentum properties). If particle and its antiparticle have kinetic energies before impact, this energy will become the part of mater wave products after annihilation (generally higher energy photons would be created).

-All the mentioned complementary participants (photon, electron, and positron) have spin characteristics (of course including protons, neutrons and many energy-momentum states known in microphysics).

-Electron, positron, and photon are examples of wave packets (or wave groups), and depending on how we use, observe, or detect them, they behave either as particles or waves. Something similar should be valid for protons and neutrons. Neutron is anyway (still officially unrecognized as such), specifically coupled pair of an electron and a proton. Supporters of contemporary Quantum Theory very strongly and angrily deny this.

-Based on Bragg's scattering, we could draw analogical and universal matter-wave behaviors between electrons and neutrons (because in both cases we successfully apply de Broglie wavelength and the same Brag's diffraction formula).

-Electron microscope is a particularly good example where high-speed electrons, are used only as de Broglie matter-waves or like beams of photons, with short wavelengths (much shorter than in light beams of optical microscopes), this way realizing exceptional amplification and resolution, on some intuitive way confirming that electrons are specifically formatted or packed photons.

Electron microscope is an excellent example or direct confirmation of Wave-Particle Duality. We initially consider electrons as particles, and such accelerated particles should have matter waves, or de Broglie waves properties. Since we know that electron microscopes are perfectly working, producing results of magnification as any other optical or photon-microscope, but with enormously higher resolution, we can conclude that accelerated electrons are behaving as waves or photons with noticeably short wavelengths, respecting de Broglie wavelength formula.

Electron microscope: *Citation from Wikipedia: https://en.wikipedia.org/wiki/Electron_microscope . Electron microscope constructed by Ernst Ruska in 1933.*

*An **electron microscope** is a microscope that uses a beam of accelerated electrons as a source of illumination. As the wavelength of an electron can be up to 100,000 times shorter than that of visible light photons, electron microscopes have a higher resolving power than light microscopes and can reveal the structure of smaller objects. A scanning transmission electron microscope has achieved better than 50 pm resolution in annular dark-field imaging mode[1] and magnifications of up to about 10,000,000× whereas most light microscopes are limited by diffraction to about 200 nm resolution and useful magnifications below 2000×.*
Electron microscopes use shaped magnetic fields to form electron optical lens systems that are analogous to the glass lenses of an optical light microscope.
Electron microscopes are used to investigate the ultrastructure of a wide range of biological and inorganic specimens including microorganisms, cells, large molecules, biopsy samples, metals, and crystals. Industrially, electron microscopes are often used for quality control and failure analysis. Modern electron microscopes produce electron micrographs using specialized digital cameras and frame grabbers to capture the images.

-Majority of other, exotic particles of micro world of Physics, known from Quantum theory Standard Model and impacts reactions, are short living, not self-standing, transient, and not stable, dualistic matter-waves packets, or quasi-particles. Some of them are not directly detectable or measurable and being only conditional, mathematical products or assumptions of specific modeling (quarks, for instance). More of common supporting arguments can be found in the Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.

-Structure of photons presents certain specific combination and coupling of (mutually orthogonal and orthogonal to a direction of photon propagation) rotating vectors of an electric and magnetic field. If such photon structure is externally influenced or modified by presence of specific electric and magnetic fields, sufficiently high energy photon could be transformed into a couple electron-positron, meaning that electron and positron are also specific packing formats of mutually coupled vectors of an electric and magnetic field like in cases of Complex Analytic signals (still not officially accepted in modern Quantum Theory as the fact).

-Protons should not be too much different regarding their internal electromagnetic field structure. All of them (meaning elementary particles) are typical examples of dualistic, particle-wave behaving objects, except that photon do not have rest mass, and electrons, positrons, protons, and neutrons have effective rest masses based on their total electromagnetic energy content, and on the way of internal packing. The way how involved vectors of electric and magnetic fields are mutually coupled and packed (structured or formatted inside electrons, positrons, protons, neutrons… and eventually atoms and molecules) is producing the property we specify as a rest mas.

-Modern and extremely precise content analyzes of materials are based on electromagnetic (or light) emissions produced by electric discharges or by high-temperature excitation of material under such testing, and we know that every atom or molecule has its very exact, specific, unique, and repeatable spectral content. Consequently, we can conclude that internal atom, or molecules, or masses structures and energy states are unique ensembles of specific electromechanical elements having resonant or oscillatory matter-waves states and their harmonics. The characteristic property of such resonant states or structures should be to have an essentially electromagnetic nature (because of intrinsic and structural involvements of photons and electrically charged particles). Since such resonant states are in the same time wave-particle, dualistic objects, and have certain motional properties, we can conceptualize self-closed, standing, and stationary matter-waves formations, and this is the platform where modern microphysics and quantum theory should have common foundations. Rudjer Boskovic, [6], already paved initial historical grounds for such concepts, (see also familiar elaborations in the chapter 8. of this book).

-Citation: "***Light-by-light scattering*** is a very rare phenomenon in which two photons – particles of light – interact, producing another pair of photons. This process was among the earliest predictions of quantum electrodynamics (QED), the quantum theory of electromagnetism, and is forbidden by classical physics theories (such as Maxwell's theory of electrodynamics). Direct evidence for light-by-light scattering at high energy had proven elusive for decades, until the Large Hadron Collider (LHC) began its second data-taking period (Run 2). Collisions of lead ions in the LHC provide a uniquely clean environment to study light-by-light scattering…" end of citation from: https://home.cern/news/news/physics/atlas-observes-light-scattering-light.

-Here we should always have in mind that photons and elementary particles are spatial-temporal formations of standing matter waves (or electromagnetic waves) where temporal and spatial dimensions are mutually coupled and velocity dependent (meaning that scattering, diffractions, superpositions and interferences of mentioned objects would depend on involved wave-packets geometry and size or durations in all relevant domains, as well as on spatial geometry of the surrounding environment).

-There is an overwhelming analogy between photons, electrons and positrons, and any other motional particle, regarding associated properties that are common, both for wave and particle motions, as loosely exercised in T.4.0. From mentioned analogies, we can say that only motional or kinetic energy completely corresponds to, or creates the appropriate matter-wave packet, without taking energy content of rest masses (what is not the position of modern Quantum theory). From the mathematical and modeling point of view, we could also say that all mentioned matter-waves should have finite spatial, temporal and frequency localizations or concentrations in the form of Gaussian wave packets or pulses and should be mathematically processed by windowed Fourier transform or Gabor transform; see [79]. For more of common supporting arguments, see the Appendix: Chapters 8., 9. and 10.

-In conclusion, we could say that all matter we know is on different structural ways composed of resonant or oscillatory, matter-waves-vibrating, more elementary states (or specific elementary particles). This is on some way familiar approach to String theory concepts, except those elementary particles, here mentioned, are specific combinations of some very elementary and simplest possible, resonating strings.

*Citation from https://en.wikipedia.org/wiki/String_theory. In physics, **string theory** is a theoretical framework in which the point-like particles of particle physics are replaced by one-dimensional objects called strings. It describes how these strings propagate through space and interact with each other. On distance scales larger than the string scale, a string looks just like an ordinary particle, with its mass, charge, and other properties determined by the vibrational state of the string. In string theory, one of the many vibrational states of the string corresponds to the graviton, a quantum mechanical particle that carries gravitational force. Thus string theory is a theory of quantum gravity.*

String theory is a broad and varied subject that attempts to address a number of deep questions of fundamental physics. String theory has been applied to a variety of problems in black hole physics, early universe cosmology, nuclear physics, and condensed matter physics, and it has stimulated a number of major developments in pure mathematics. Because string theory potentially provides a unified description of gravity and particle physics, it is a candidate for a theory of everything, a self-contained mathematical model that describes all fundamental forces and forms of matter. Despite much work on these problems, it is not known to what extent string theory describes the real world or how much freedom the theory allows in the choice of its details.

String theory was first studied in the late 1960s as a theory of the strong nuclear force, before being abandoned in favor of quantum chromodynamics. Subsequently, it was realized that the very properties that made string theory unsuitable as a theory of nuclear physics made it a promising candidate for a quantum theory of gravity. The earliest version of string theory, bosonic string theory, incorporated only the class of particles known as bosons. It later developed into superstring theory, which posits a connection called supersymmetry between bosons and the class of particles called fermions. Five consistent versions of superstring theory were developed before it was conjectured in the mid-1990s that they were all different limiting cases of a single theory in eleven dimensions known as M-theory. In late 1997, theorists discovered an important relationship called the AdS/CFT correspondence, which relates string theory to another type of physical theory called a quantum field theory.

Let us now address “double-slit” experiments that are too often taken as a proof or strongest argument confirming probabilistic and wave-particle duality nature of photons and other elementary and micro particles, including atoms and molecules.

Explanations of double-slit experiments from the point of view favored in this book**:**

Citation (lit. [124]): "The famous double-slit experiment, different versions of which we will come back to in the course of the thesis, serves as the canonical illustration of the wave-particle duality. Let us here present its simplified description: A light beam emerges from a source, propagates through two very small slits and impinges on a screen. We can reduce the intensity of the beam in such a way that according to a standard concept of quantum mechanics there will be only one quantum of light (photon) present in the apparatus at any given time. If we now place a detector behind each slit, we will see that they do not respond simultaneously, and thus we will be led to the conclusion that the photons behave like tiny corpuscles moving through either the first or the second slit. However, if we choose not to disturb the light with measurement before it reaches the screen, an interference pattern will emerge on it. This pattern is most easily predicted and explained by claiming that light is in fact an electromagnetic wave. The double-slit experiment can be also conducted with electrons (or other material particles) instead of light, and the same conclusions would be reached. In the words of Richard Feynman, this extraordinary phenomenon "is impossible, absolutely impossible, to explain in any classical way, and (...) has in it the heart of quantum mechanics. In reality, it contains the only mystery" [4]. Thus, claimed Feynman, the wave-particle duality problem is one of the central features of quantum mechanics".

1. Photon is always and only a limited-frequency-band, electromagnetic wave group, agglomerated around certain mean frequency. In other words, photon presents limited-bandwidth superposition of elementary sinusoidal, electromagnetic wave elements (as known from spectral measurements and from Kotelnikov-Shannon-Whittaker-Nyquist signal analysis and signals reconstruction). For such photon we can say that it has certain time duration T, certain spatial length, width or duration L, certain total, motional energy $\tilde{E}$, and linear and angular moments $\tilde{P}$ and $\tilde{L}$. Relation between all total domains-durations of mentioned photon is $T \cdot \tilde{E} = L \cdot \tilde{P}$ (see more in Chapter 10.). Analogically, and respecting de Broglie matter waves concept regarding wave-particle duality, we can give the same description for a moving particle, electrons, protons, atoms, and molecules (which are also narrow band superpositions of elementary sinusoidal wave components). On the similar way as a total photon-energy $\tilde{E}$ can be presented as superposition of energies of sinusoidal, time-dependent wave components, its total momentum $\tilde{P}$ can be presented as a superposition of elementary moments being sinusoidal wave components dependent of relevant spatial coordinate x, (as for example,

$$\tilde{\mathbf{E}} = \int_{[T]} d\tilde{\mathbf{E}} = \int_{[T]} \psi^2(\mathbf{t}) \cdot d\mathbf{t} = \int_{[T]} \tilde{\mathbf{P}}(\mathbf{t}) \cdot d\mathbf{t}, \quad \tilde{\mathbf{P}} = \int_{[T]} d\tilde{\mathbf{p}} = \int_{[L]} \frac{1}{\mathbf{v}} d\tilde{\mathbf{E}} = \int_{[L]} \frac{\psi^2(\mathbf{x})}{\mathbf{v}^2} \cdot d\mathbf{x} = \frac{d\tilde{\mathbf{E}}}{d\mathbf{t}},$$

$$\psi(x,t) = a(x,t) \frac{\sin(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\ x)} \cos(\omega t \pm kx)).$$

2. Similar relations and conclusions are also applicable (from a spectral, geometrical, or spatial point of view) on a single, double, or multiple slits diffraction and interference plates, meaning that wave patterns, sinusoidal wave components, and corresponding spatial periodicity can be associated to a *spatial-spectral periodicity related geometry* of mentioned diffraction plates. In other words, temporal periodicity, and elementary, time-dependent sinusoidal waves of incident photons (electrons, atoms etc.) will interact or interfere with spatial (or geometrical) periodicity of standstill plates with slits, since temporal and spatial domains are mutually coupled, phase shifted, and mutually transformable (see more about spatial-temporal proportionality in Chapter 10.). Or saying the same differently, here we always have mutually interacting, coupled and synchronously present temporal and spatial periodicities, with spectral characteristics and waving properties in all mentioned domains. For certain photon propagating in the x direction, temporal-spatial proportionality (or coupling) is described as, $dx = c \cdot dt$. Here, we should not forget or completely neglect possible "***extended-meaning-of-entanglement relations***" among and between incident photons and surrounding objects, as well as effects of resonant synchronizations and resonant couplings between spectral components of photons and diffraction plate. Within Orthodox Quantum theory it is seriously mystified understanding of any situation when we monitor double or multiple slits diffraction. For instance, without an observer, sensor, detector, or measurements device, we get diffraction patterns as in cases of typical waves diffractions. Contrary to this situation, if we introduce an observer/monitor/sensor, we will see that previous waves-diffraction picture would disappear, and on the back screen it will be registered only the situation typical for localized particles imprints. The reason for such

behaviors is that a presence of an observer, which is a physical body with certain geometry and mass, meaning with certain spatial spectral and energy content (obviously being dominant mass, much bigger and heavier, compared to photons, electrons or atoms involved in analyzed diffraction), is disturbing, interfering, modifying and destroying the initial spatial and/or geometrical periodicity or spectral patterns (of initial particle beams, and of double or multiple-slits diffraction plates), and this is significantly changing the picture on the back-screen. There is no other magical and inexplicable, stochastic influence, and "*wave function collapsing"* situation here (see more about links between spatial and temporal spectral functions in Chapter 10.). Orthodox Quantum theory is simply (on certain isomorphic way) respecting energy and moments conservation laws, while an instantaneous, real time-space-dependent signal phase is not known, because of the limitations of applied mathematical modeling, and because of presumed stochastic, ontological nature of mentioned phenomenology (without taking into account that involved elements of condensed matter also have spontaneous tendency to communicate and mutually synchronize, internally and externally). This is producing "magical effects" that after certain time of playing with isolated particles (regarding diffraction experiments) we get results like experimenting with waves in fluids. All such situations we should conceptualize and analyze as fully closed energy/power/moments flow-circuits with known front-end, or source elements, and last-end load elements (see more about such circuits in the first chapter of this book). Hanging, not well fixed open-ends circuits, in analyzing double or multiple holes (or diffraction slits) will not produce correct results. ***In addition, if we admit that the whole diffraction setup (waves or particles source, diffraction plate with holes or slits, and back screen) is submersed or encircled within certain fine, material fluid or ether (which has electromagnetic properties, as an ideal vacuum has), the explanation of here-mentioned, strangely and stochastically behaving diffraction effects will be much more deterministic, logical, tangible and normal, since spatial-temporal, large zone waving, scattering and interferences in such delicate fluid will be immediately and synchronously created when we send some particles, electrons or photons (which are also wave groups) towards diffraction holes, and all of diffraction holes would be synchronously affected.***

3. When specific photon (or an electron, atom…) is interacting with certain static (standstill) object or narrow slit plate, which important physical size is larger than photon mean wavelength, than such photon has big chances to behave dominantly as a small particle (when passing such narrow slit).

4. Anyway, a photon (or an electron, atom…) propagating in a direction of certain physical object (like single or multiple slits solid plate) will become a part of the "two-body-system" with mutual mechanical, electromechanical and electromagnetic field relations, and some electromagnetic field coupling with double-slits physical object should not be neglected. Effectively, it will be created certain (by the photon/electron/atom modulated) central-mass and corresponding reduced-mass in the locally, and mathematically defined center of mass system (before a final or expected reaction, interference, superposition, impact, or scattering, between the photon/electron/atom and double-slits object happens). This way, center-of-mass-system would gradually evolve becoming more important (following the photon/electron/atom propagation towards its back-screen target), compared to the observer's, more natural, (but not more relevant) local Laboratory system of coordinates. Here we should not forget that any energy flow, or flow of entities with angular and linear moments, should be a part of certain larger picture, where we obligatorily need to have *closed-circuits-flow* with front, or source-elements, and with back or load-elements (as elaborated in the first chapter of this book). Again, to underline, hanging, not closed, open-ended energy-flow circuits do not exist, meaning that source of photons and *interference-patterns standstill backscreen* should be an integral and active part of the single, double, and multiple slits experiments, this way additionally supporting detected interference and superposition effects. Of course, effects of such "two-body-coupling" system could be small, but not negligible. ***However, to underline again, if we accept that the whole diffraction setup (waves or particles source, diffraction plate with holes or slits, and back screen) is submersed inside certain fine, material fluid, or ether, which is carrier of electromagnetic waves, the same diffraction-related situation will be much easier to understand and explain, being more deterministic, logical, tangible, and normal (like in fluids), compared to what Orthodox Quantum theory is proposing.***

5. Most of double-slits "*dualistic wave-particle diffraction experiments*" are carried using photons, electrons, single atoms, and some molecular or nanoparticles. We should not forget that photons are electromagnetic wave packets, and electrons are specific, structural packing format effectively composed of photons, meaning that both (photons and electrons) intrinsically have dualistic wave and particle, electromagnetic behaviors. In addition, atoms are composed of electrons protons and neutrons, where neutron is on some specific structural way composed of an electron and a proton. Most probably and analogically, protons are also specific structural (and standing waves (formatting of electromagnetic field waves or photons, meaning that moving atoms could locally create ***dipole-structured electric charges***, and naturally present wave-particle objects on a similar way as photons and electrons are (by manifesting diffraction patterns as known for waves in fluids). If we analyze double-slits diffraction as a two or multiple body (electromechanical) system, we should always account acting and surrounding electromagnetic forces and fields, what in such cases naturally contributes to dualistic matter-wave and particle-wave manifestations.

6. If we detect incident photon on some way, before or after interaction, just after passing a narrow slit (using certain standstill sensor, instrument, detector…, which has much larger spatial size and mass compared to the incident photon mean-wavelength, and to its wave-packet total length), we will conclude that such photon is registered (or detected) as a particle.

7. Since wave packet also has certain phase characteristic (being spatial and temporal coordinates dependent), it will be important how such wave packet (photon) would arrive at the place of interaction (meaning what would be its immediate phase when passing diffraction slits). That means, different photons (even from the same source, and being initially maximally identical) would arrive with different spatial-temporal phase values, producing, after certain time, typical, oscillatory, interference patterns on some large standstill screen behind (since natural waving effects, including thermal motions and oscillations of all interaction participants should gradually accumulate after sufficiently long time, because of spontaneous and natural, gradually evolving, tendency for mutual communications and synchronizations between moving objects with overlapping resonant and spectral characteristics). Contemporary Quantum theory is neglecting the immediate phase function, since probability wave function naturally has no meaningful and useful immediate phase (contrary to Analytic Signal modeling of a wave function, which is still not widely applied in quantum theory). In addition, every motional state, wave-packet, or moving particle, can be kinematically (as a wave function) modeled using the Analytic signal model (see more in Chapters 4.0. and 10.). Every Analytic signal has at least two wave function components (its real and its imaginary part), and combination of mention functions is creating or defining the same signal phase and amplitude. From the point of mathematics of wave motions, real and imaginary part of an Analytic signal should both exist synchronously and continuously, meaning that we already have at least two wavefunction members, or two waves interacting with diffraction holes, this way conceptually and additionally supporting unusual (stochastically behaving) wave patterns behind diffraction plate, when we are sending single particles towards diffraction plate.

8. If we are not using photon detector or another sensor (to prematurely stop the photons or electrons), then in a "two-body-system" we will have only the wave-packet properties of the reduced-photon-mass (which is still a narrow-band wave packet), and by photon (slightly) modulated (or vibratory excited), common central mass. This would create some favorable conditions for superposition and interference effects, and (after certain time, with continuously advancing synchronization) we will start to see corresponding interference patterns on the relatively remote, large and standstill screen. Of course, with a single, double, or multiple slits diffraction barrier we will always see different interference patterns on a standstill and large backscreen.

9. *The two-hole or two-slits diffraction of a single particle (including electrons and other electromagnetically charged particles) is also a clear examples of de Broglie matter waves diffraction, resulting in a dominant part of the particle passing through one of two diffraction holes, while the interaction field (and associated unfolding matter wave with number of elementary sinusoidal waves) between the particle and both diffraction holes will "flash" two holes at the same time (being very much similar to what was just explained for photons). This is also similar to the pilot wave model, originally developed by Louis de Broglie and further*

developed by David Bohm. Involved time-space-dependent phase functions (as parts of two-body systems) are continuously evolving, making that a single particle (or its spectral and radiative energy components) effectively interacts with both diffraction holes (or slits) long before it has the chance to pass to the opposite diffracting plate side. At the opposite screen-side, the same interaction field (or oscillations produced in an everywhere existing ether) will again make channeling or guiding-influence on the spatial-temporal phase of the passing particle (as the consequence of temporal-spatial proportionality and satisfying energy and momentum conservation laws in two-body system relations). Here, we always have at least two or three-body interaction cases, where incident particle, before entering the proximity of diffraction slits acts as a very much independent object, which gradually becomes a part of the bigger and mutually coupled (incident particle + diffracting plate) system. Here, system center-mass, and reduced-mass start to be parts of a new interacting process that becomes dominant (in the near field of interaction, where synchronization effects are increasing). As we can see, the real cause of stochastically behaving particle distributions in cases of diffractions should be related to the fact that certain (transitory and waving) interaction field is always created between the incident particle and its target (or its diffraction slits and holes, since such objects are composed of atoms and molecules, having their own spectral characteristics and elementary sinusoidal waves content), including radiant energy flow of local electromagnetic fields and thermal photons emissions). Such situations are appearing sufficiently long before interaction (diffraction, scattering, interference, etc.), and remaining effective sufficiently long after an interaction happens (and such interacting or coupling field has axial and rotational, oscillatory and vortex components, manifesting in certain surrounding carrier fluid or ether). The appearance of such transitory and torsion fields (between moving particle and its target) can also be partially related to "multi-component electromagnetic dipole-like polarizations". For instance, an electrically neutral particle in the state of rest will have a stable center of mass, stable center of its total electrical charge, stable center (or axis) of its total orbital moments, stable center (or axis) of its magnetic moments etc. On the contrary, a moving particle, especially during acceleration, experiences a "multi-component, electromagnetic, dipole-like evolving polarizations" and forces, where every (above-mentioned) stable center (or axis) of certain involved material field, generates its oscillating "dipole-like moment" (not necessarily all of them acting along the same axis). Such dynamically created and transitory "multi-component dipole moments" will start interacting with their environment, and with particle's target, creating new de Broglie matter waves (originating from the extended Newton-Coulomb force laws as presented in the second chapter of this book; -see equations (2.3) - (2.4-3) - (2.9)). In Chapter 5. it will also be shown that certain spatial periodicity and atomization (or standing-waves related quantizing) coincidently exist in relation to all linear and angular motions (as well as in associated electromagnetic fields), meaning that a moving microparticles (or wave packets) in some cases of standing matter-waves cannot freely and smoothly select any angular position in front of them. Quantum Mechanics effectively mastered "in-average" presentable (artificially, formally, and mathematically) an operating mathematical model of such (phase unpredictable) multi-component interactions, mostly using the methodology of Statistics and Probability Theory (sacrificing a qualitative and immediate spatial-temporal identity of interaction participants), by effectively randomizing the uncounted motional-phase values. Since the immediate rotating phase of some particle motion, after a sufficiently long time, anyway, covers a full circle ($-\pi \leq \varphi(t, x) \leq +\pi$), precisely calculable probability that particles will or can be found in certain areas should become an experimental fact (but only in-average, and after sufficiently long time, when effects of synchronizations are also advancing). Strictly speaking, if we know all the essential elements and shapes of specific, sufficiently isolated wave motion, we should be able to predict when, where, and how interaction of some wave groups, particles and waves would materialize (see more among equations (4.42) - (4.45)). ***But, if we (in addition) accept that the whole diffraction setup (waves or particles source, diffraction plate with holes or slits, and back screen) is submersed in certain fine, material fluid or ether (which has electromagnetic properties, as Nikola Tesla concluded), the explanation of here-mentioned, stochastically behaving diffraction effects will be much more deterministic, logical, tangible, and natural. All mutually interacting entities will be always mutually connected by mentioned fluidic matter, and all diffraction slits or holes will be agitated in real time by single or multiple wave-packets and particles, which are anyway creating new matter waves in such etheric, electromagnetic-waves carrier. With presence of an ether, we do not need to imagine how single electron or photon is passing two or many diffraction slits at the same time.***

10. *Here, we can again summarize all relevant background regarding understanding wave-particle duality and matter waves in relation to double slit experiments. For photons we already have too many experimental and theoretical facts and results confirming that photons have dualistic wave and particle nature. If instead of photons we start radiating electrons, protons, neutrons, atoms, and molecules, we will again use similar theoretical concept, experimental setup, and supporting explanations, producing similar results, like with photons, because moving electrons, protons, neutrons, atoms etc. are still presenting matter-wave packets, characterized with certain mean frequency, mean wavelength, and spatial-temporal evolving-phase function. The other reason for appearance of interference patterns could be the fact that mentioned micro-particles (electrons, protons, neutrons, atoms…) are also specifically packed formats of electromagnetic energy or photons (which is not strictly or sharply localized within spatial boundaries of a mechanical body of mentioned particles). Of course, moving particles with bigger masses (and with rest-mass properties) will naturally behave much more as particles than waves, when compared to photons, but the logic (and experimental setup) in explaining interference and diffraction effects would stay the same. Atoms are composed of electrically charged particles, having magnetic and spin moments, and such electromagnetic charged particles are anyway some kind of electromagnetic field or photons formations, specifically packed as self-closed standing waves, meaning that for bigger particles we again have diffraction, superposition, and interferences, partially influenced by internally involved photons, electrons, and other electromagnetic field forms. The only mystery here is the man-created confusion and incomplete or unclear (over-simplified) mathematical modeling, and poor conceptualization, which was later accepted, unconditionally, and non-critically, by the mass of well obeying followers of such teaching. For instance, photons and electrons are always like relatively narrow-band electromagnetic wave packets, and it is our mistake if we are wrongly forging such items to be particles seen on our oversimplified, mechanistic way. Under favorable (and by geometry and size determined) conditions, photons (or electrons) in certain interactions, would behave more as particles, and under other experimental conditions, the same photons would behave more as waves.* ***Anyway, we need to face the reality that here we always have an omnipresent and fine fluid with electromagnetic properties, or ether, and the whole diffraction setup (waves or particles source, diffraction plate with holes or slits, and back screen) is submerged inside such material fluid, which is the carrier of electromagnetic and other matter-waves and fields. Incident photons, electrons or microparticles, and number of, by spatial configuration produced, secondary waves would be continuously created, and all diffraction holes will be on some way synchronously agitated (with some phase shifts), because mentioned ether is behaving as a fluid. This is the natural, deterministic, logical, and tangible explanation of seemingly strangely and stochastically behaving diffraction results.***

In fact, most of experiments and analyses of double or multiple slits diffraction (we can find in different publications) are related to electrons, where we simply consider electrons as being almost ordinary particles (with non-zero rest mass) that are (anyway and naturally) in some situations (like in diffractions, impacts, scattering etc.) manifesting matter-waves behaviors. The first and biggest conceptual mistake that should be corrected here is that electron is not an ordinary particle, but kind of active resonator, or standing electromagnetic waves composed body, in a shape of certain torus, thin disk or ring, meaning that electron is already a matter-wave formation composed of photons, and such spatial forms naturally have different spectral or resonant frequencies. In other words, we are coming back to all discussions, explanations and conclusions as already elaborated and being valid for single photons diffraction (including embedded and natural synchronization and entanglement effects between identical or similar resonators). Effects of ***"extended meaning of entanglement relations"*** *and resonant synchronizations are well explicable when we are using modeling of involved matter-waves with Complex Analytic Signal functions. Something similar about electromagnetic matter-waves packing we could also find in other more complex micro particles composed of atoms and molecules. Certain particle could be externally electromagnetically compensated and neutral, but internally still has electromagnetic formations and couplings (meaning it has spatially structured and stabilized electromagnetic energy, or it has some photonic content). During motion (when emitting such particles towards diffraction slits), some electric and magnetic dipole polarization could be created, producing spatially separated electromagnetic resonators (such as electrons and protons). In addition, we should know that all kind of natural resonators are described as wave sources respecting Classical second order, differential Wave Equation, while excited, natural resonators or their wave equations always have* *solutions being as coupled, in-pairs created waves, propagating in mutually opposite, or inwards and outwards directions. Also, the Analytic wave function model is always composed of two wave functions being mutually phase shifted and being relevant for spatial-temporal wave function properties. All of that is also enriching two-body complexity of spatial-temporal evolving interactions, being combined with already mentioned entanglement and resonant synchronization effects, producing number of interesting, unusual, and strange diffraction effects, and superficially unusual and strange (still not well interpreted) interactions with surrounding observers and sensors. There is no need to wonder too much and to glorify Quantum theory statistical and probabilistic conceptualization based on such phenomenology (which simply was not properly conceptualized and mathematically modeled, since Complex Analytic Signal modeling was established more than 10 years later, after statistical Quantum theory was already canonized and dogmatized; -see more in Chapter 10. of this book). Here is the real background of **wave-particle Duality**, which can be now understood as the **Wave-Particle Unity**.*

…………………

Unusual, stochastically behaving diffraction and interference effects of micro particles and photons (regarding one and two-hole diffraction) misled the creators of Quantum Mechanics to associate only the probability nature to matter wave function $\Psi(t)$. *In fact, the secret why there are virtually unpredictable (or only statistically predictable) wave-like diffraction positions (spots on the screen, or on a photographic plate) regarding single, incident micro particle or photon, with effective mass m, passing the diffraction hole (target, M >> m), is rectilinear motion that is always accompanied by immanent (and somehow hidden) mass-spinning (what creates matter waves). Effectively, in such situations, we deal with a moving particle and its associated field components (see (2.3) – (2.9), chapter 2., and (4.3-0)-a,b,c…k, from this chapter). Such associated field components create "coupling and guiding channels" with all diffraction holes (at once), long before a micro-particle reaches any of the holes, somewhat similar to the pilot wave model, originally developed by Louis de Broglie and further developed by David Bohm into the hidden variable theory. Between identical or similar resonators (which could be different masses, atoms, elementary particles, and formations of photons) it is often easy to create effects of spatial-temporal synchronizations and entanglements, what is most probably happening in cases of matter waves diffraction and interferences. Here, in addition, we could hypothetically include an influence of certain revitalized ether with participation of higher and hidden dimensions of certain multidimensional Universe. The local time and space dependent motional wave-phase* ($-\pi \leq \varphi(t, x) \leq +\pi$) *of such complex field phenomena, now when the particle m "hits" and passes any of the diffraction holes belonging to a target mass M, dominantly contributes to stochastic-like scattering picture on the back screen. This is eventually creating typical wave diffraction and interference patterns with concentric circles or white and black zones (or other wave-like, linear patterns), but we see such picture only after certain time, when many single particles (or photons) pass through a diffraction hole. Since we are almost unable to know the local and real time-domain wave-phase of a particle's rotation, it seems (in the beginning of a particle diffraction experiment) that micro particles randomly choose to hit arbitrary positions on the one-hole (or many-holes) diffraction screen. In all such experiments "that virtually confirm stochastic backgrounds of the Copenhagen Quantum theory" the existence of spinning field components has been systematically omitted, since it is not present as the modeling concept, and nobody has been taking such (still hypothetical) possibility into account. Consequently, this way Orthodox Quantum Theory established its probability grounds, mathematically compensating the missing knowledge about all (immediate) motional field-phase components. Newly constructed mathematical fitting and processing has been so good, eventually producing precise and numerically verifiable results, that founders of Probabilistic Quantum Theory concluded that everything else, regarding imposed and artificially established basements, step stones, and founding concepts, should also be correct. To avoid and suppress possible critical and damaging analyzes, and negative insights (of course unintentionally and non-consciously), founders of Orthodox, Probabilistic Quantum theory (and mass of their non-critical followers) are proudly prescribing the ways of good talking, thinking, analyzing, and experiments setup, that should be applied in such (wave-particle duality-related) situations, that nobody would start thinking and analyzing such situations differently. Typical and mathematically explicable wave patterns or geometrical shapes, circles, or lines with certain periodicity (in different diffraction, scattering and interference experiments) are only reinforcing existence of a fine and ideal fluid or ether. This also indicates that here we have certain deterministic and causal nature or background of mentioned phenomenology, contrary to the probabilistic and statistical modeling of Orthodox Quantum theory.*

Effectively, contemporary Quantum Theory, in some of its foundations has been the implicit (never admitted), quiet victory of illogical concepts, unintentionally mixed with hidden metaphysical ingredients, *effectively being against causality, objectivity, and reality in natural sciences. Somehow, the deep and subconscious soul of Western world society (as the founder of Quantum Theory) seems to be "infected by an intellectual virus" that indirectly (and wrongly) teaches that "random choice guided miracles" could happen (as it happens in some ideological and religious concepts). Many looks-like scientists, philosophers, people in some power, mystics, and their naïve followers published almost countless number of (in some cases almost worthless) books sitting partially and unknowingly, or not-intentionally, on mentioned wrong and mystic grounds. They are ironically expressing themselves in the name of an objective and neutral science (mixing an artificial mathematical objectivity with wrong and incomplete, or only approximate concepts, and illusions about Physics of wave-particle duality). However, this (about Quantum theory foundations) has also been a period of enormous prosperity of mathematics that found an exact and fantastic way to describe and represent such unclear, ambiguous, illogical, and challenging teachings, while producing correct (experimentally verifiable) results.*

On a similar way, we could modify or update our understanding of **Photoelectric effect**.

Citation (taken from https://en.wikipedia.org/wiki/Photoelectric_effect): "The **photoelectric effect** is the emission of electrons or other free carriers when light falls on a material. Electrons emitted in this manner can be called *photoelectrons*. This phenomenon is commonly studied in electronic physics, as well as in fields of chemistry, such as quantum chemistry or electrochemistry".

1. First, we will consider that good and mainstream-accepted explanation about photoelectric effect already exist (thanks to Mileva Maric, A. Einstein, Max Planck, and L. de Broglie foundations). Here, we will not criticize or go against such healthy explanations.

2. Let us now imagine that metals (or other materials) susceptible to emit photoelectrons when radiated with sufficiently energetic photons, are internally structured as spatially distributed sets of electromagnetic or electromechanical resonators, covering certain bands of frequencies. To have a reference to existing atom models, we already call such spatially distributed resonators electron clouds, electron states, or electron orbits. Anyway, mentioned electron resonators are specifically formatted or structured electromagnetic waves. In addition, atoms (or metals and other masses) internally also have another set (or ensemble) of electromagnetic resonators within opposite electric-charge polarity, presenting atom-nucleus formations or states, covering different and much higher frequency or energy ranges (compared to electrons).

3. Electrons and nucleus state or set of mentioned resonators also consume certain amount of binding or coupling (electromagnetic) energy, keeping such sets or resonators united, meaning, being specifically, spatially distributed (on internal electron states, and nucleus states).

4. If we irradiate mentioned sets of electromagnetic resonators with low energy photons, and if incident photons have lower energy or frequency compared to the range of frequencies of internal electron states, we will not excite such resonators, and we will not detect emitted photoelectrons.

5. If incident photons have frequencies higher than certain minimal threshold, entering the interval of frequencies of internal electron states, emission of photoelectrons would start. This means that electron states are formations of electromagnetic waves (or specific formations of photons), and if we overexcite such resonator with an external (incident) photon, resonator will briefly absorb such photon, becoming internally (and electromagnetically) excited, restructured, or perturbed, and it will eventually stabilize (after certain time) by expulsing one electron. Mileva Maric and A. Einstein already explained energy and moments balance of such impact interactions. Electromagnetic and electromechanical, or only mechanical resonators, are dominantly sensitive and reactive (or responsive) only when being excited on their natural resonant frequencies. Since nucleus states cover much higher frequency bands, it would be too difficult to produce photoelectric effects by interacting with nucleus states.

Consequently, situation described here is on some way like in double-slit experiments. If incident photons, matter-waves or moving particles, have lower (de Broglie) frequency range (meaning longer wavelengths, and longer total duration or length), compared to natural resonance or eigenfrequency and mean wavelength belonging to interacting object, such situations would dominantly manifest as dealing with waves. If incident (mater-wave) items cover higher frequency range, such wave formations, or wave packets (meaning incident photons, electrons, and other micro particles) would manifest as dominantly being particles. Incident matter waves (photons or narrow-band wave-packets) are characterized by certain mean-frequency and mean-wavelength, including total wave-packet temporal and spatial durations. Interacting object could also have some internal resonant properties, or if it is only a static body with specific geometry, it would have its spatial spectrum with spatial periodicities (meaning crystalline structure), or spatial frequencies, what presents certain lengths, or distances between spatial positions, crystals, peaks (or slits). In such cases, we will compare incident wave-packet mean-wavelength (or its total spatial length) with spatial periodicities of the interacting object. If an incident wave packet has the mean-wavelength larger *than mean-spatial-periodicity length (of the interacting object), such incident wave packet will behave dominantly as a wave. If it has much shorter mean-wavelength, or shorter total spatial length (compared to interacting object periodicities and size), it will dominantly manifest corpuscular properties (see more in Chapter 10.). There is no place for any mystery, here (except in situations of mutually similar wavelengths and total lengths of all interaction participants, when we could experience mixed and ambiguous effects).*

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The unity of corpuscular and undulatory or matter-waves nature of moving objects, rather than mutual exclusivity, is already becoming an experimentally confirmed fact (contrary to the position of the contemporary Quantum theory.

See the following Citation from [124], Borys Jagielski:

“The aforementioned events stimulated the interest in the general properties of light, but several experiments conducted quite recently touched directly upon the problem of the wave-particle duality. In 1989 the team of Akira Tonomura conducted for the first time the famous double-slit experiment with electrons in carefully controlled laboratory conditions (see Appendix C) [5]. In 1999 Anton Zeilinger and his colleagues performed another experiment of the same kind, but with fullerene molecules instead of electrons [44]. The interference pattern was again obtained, confirming that also particles much larger than electrons can behave in accordance with de Broglie relations.

The somewhat controversial Afshar experiment was one of the most recent words uttered on the matter of the wave-particle duality of light [45]. It was first conceived and carried out by Shahriar Afshar in 2005 (but later repeated in an improved form). The experiment is assumed by some to demonstrate a paradox of the wave-particle duality, because it seemingly allows to observe both the corpuscular and the undulatory behavior of light at once [46]. However, a consensus has not been yet reached. The Afshar experiment and its possible implications will be analyzed in Chapter 8”.

References:

[5] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, H. Ezawa, Demonstration of single-electron buildup of an interference pattern, American Journal of Physics 57 (1989), pp. 117-120.

[44] M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw, A. Zeilinger, Wave-particle duality of C60 molecules, Nature 401, pp. 680-682 (1999).

[45] Shahriar Afshar, Sharp complementary wave and particle behaviours in the same “welcher Weg” experiment, Proceedings of SPIE 5866, pp. 229-244 (2005)

[46] Shahriar Afshar, Eduardo Flores, Keith McDonald, Ernst Knoesel, Paradox in Wave-Particle Duality, Foundation of Physics 37, pp. 295-305 (2007)

See also in [124], Chapter 8, The Afshar experiment, and under C; -Matter waves; -pages 179 – 182.

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Citation (from [91], Common Sense Science, Charles W. Lucas, Jr. Statements related to some ideological items in the same text are omitted merely):

Experimental Evidence to Support Boscovich’s Atomic Model. ... It started with the discovery of solitons ... Solitons are long lasting semi-permanent standing wave structures with a stable algebraic topology. [9] The soliton can exist in air or water as a toroidal ring. Solitons in water are usually formed in pairs known as a soliton and anti-soliton. Their structure is weak, and they decay away after 10 or 20 minutes.

Bostick’s Plasmons. Winston Bostick (1916-1991), the last graduate student of Nobel Prize winner Arthur Compton (1892-1962), experimentally discovered how to make “plasmons” or solitons from the electromagnetic field within an electromagnetic plasma. [10] These structures were robust, compared to solitons in air and water. They had long lifetimes and could not be destroyed by normal processes in nature. Bostick proposed that electrons were quite simple solitons and positrons were contrary or anti-solitons. All other elementary particles were built of more complex, geometrical structures such as dyads, triads, quatrads, etc. All plasmons or solitons in the electromagnetic field are of the same shape, i.e. a toroidal ring. The plasmon was of extremely high strength. Bostick tried to create a bottle from plasmons to hold controlled thermo-nuclear fusion. All materials known to man up to that time slowly disintegrated when exposed to controlled thermonuclear fusion. Only the plasmon was strong enough, but Bostick failed to succeed in building a bottle from plasmons...

Hooper’s Electromagnetic Field Experiments. The nature of the plasmon, electromagnetic soliton was more completely revealed by another modern-day scientist, William J. Hooper [11]. He discovered that charged elementary particles, such as the electron, were not only made from the electromagnetic field, but variations in the field around them due to their structure extends to great distances. This same feature is also observed about solitons in water.

Hooper [11] also discovered that there are three types of electric and magnetic fields. One of these types is due to velocity effects from Lenz's Law causing it to have the property that it cannot be shielded. Thus portions of the electromagnetic field exist everywhere in the universe.

References:

9. May, J. P., **Stable Algebraic Topology**, p. 1 (1966). http://www.math.uchicago.edu/~may/PAPERS/history.pdf
10. Bostick, Winston H., "Mass, Charge, and Current: The Essence and Morphology," **Physics Essays, Vol. 4, No. 1**, pp. 45-59 (1991).
11. Hooper, W. J., **New Horizons in Electric, Magnetic, and Gravitational Field Theory** (Electrodynamic Gravity, Inc., 543 Broad Blvd., Cuyahoga Falls, OH 44221, 1974), preface. http://www.rexresearch.com/hooper/horizon.htm

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The message of the author of this book is: "If certain theory, or concept, or unusual and abstract mathematical construction, starts producing many of strange and unanswered questions, dilemmas, contradictions, multi-plausible, illogical, or ambiguous explanations (confirmable mostly based on axiomatic statements, imaginatively introduced names and labels without explanation, and with added postulations, including probability and statistical modeling), it would be better, safer, more natural, more correct and more scientifically productive to search for new theory and new or better conceptualization. Some particularly good example-candidates of such imaginative, irrational, and exotic creations are, Schrödinger's cat elaborations, Black or Dark Energy and Mass labels, nuclear forces and Gravitation related titles without essential explanations and deep insights, Neutron Stars assumptions, Black Holes speculations and oversimplifications, Wave Function Collapse and Observer's magic influence on Measurements and Reality… It is expected by the author of this book that all of mentioned and other familiar items would evolve or be replaced with a significantly updated content".

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Motional or kinetic particle energy that should be causally related to matter-wave energy (and analogically matching photon energy), could be treated as usual (in mechanics), having any positive (velocity dependent) value, if this energy is measured "externally", in the space where a particle is in motion. If we attempt to solve the relativistic equation that connects all energy aspects of a single and moving particle ($p^2c^2 + E_0^2 = E_t^2 = (E_0 + E_k)^2$), we will find that one of the solutions for kinetic energy could be the negative energy amount that here corresponds to the particle rest mass energy, for instance,

$$\begin{Bmatrix} E^2 = E_0^2 + p^2c^2 = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2 \quad , \\ E_0 = mc^2 \quad , \quad E = \gamma mc^2, \quad E_k = E - E_0 = (\gamma - 1)mc^2 \quad , \quad \gamma = (1 - v^2/c^2)^{-1/2} \end{Bmatrix} \Rightarrow$$
$$E_k^2 + 2E_0E_k - p^2c^2 = 0 \;\Rightarrow$$
$$E_k = -E_0 \pm \sqrt{E_0^2 + p^2c^2} = -E_0 \pm E \Rightarrow$$
$$\Rightarrow E_k = \begin{Bmatrix} +E_k \\ -E_0 \end{Bmatrix} = \begin{Bmatrix} (\gamma - 1)mc^2 \\ -mc^2 \end{Bmatrix} \quad (=)$$
$$(=) \quad \begin{Bmatrix} \text{motional particle energy "in external space"} = (\gamma - 1)mc^2 \\ \text{motional particle energy internaly captured or "frozen" by its rest mass} = -mc^2 \end{Bmatrix}.$$

Such a result ($E_k = -E_0$) could look illogical and could be neglected or considered unrealistic. If we consider that internal particle structure (that creates its rest mass) is also composed of motional-field energy components (or well packed, self-stabilized and internally closed, standing matter waves), we could consider that negative motional energy belongs to the ordinary motional energy that is internally "frozen" or captured by the particle rest mass.

*[♣ **COMMENTS & FREE-THINKING CORNER**: How to address properly rotating motions, spin, and orbital moment's characteristics (concerning matter-wave properties) is important for understanding the **PWDC**.*

To get an idea of how to express linear motion energy component and orbital, rotational, or spin-related energy components let us again use analogical thinking.

1. *For instance, if a moving particle is only in a linear motion (without having mechanical elements of rotation), its relativistic kinetic energy is:*

$$\begin{Bmatrix} E_k = mc^2(\gamma - 1) \\ m = m_0 = \text{const.} \\ p = \gamma m v \\ E_{tot.} = \gamma mc^2 \end{Bmatrix} \Rightarrow \left\{ dE_k = vdp = mc^2 d\gamma = dE_{tot.} = d\tilde{E} \right\}$$

The same conclusion regarding the infinitesimal value of kinetic energy should be valid for classical-mechanic kinetic energy, since we already demonstrated that one of the possible options is,

$$\begin{Bmatrix} E_k = \frac{1}{2}mv^2 = \frac{1}{2}pv \\ m = m_0 = \text{const.} \\ p = mv \end{Bmatrix} \Rightarrow \left\{ dE_k = pdv = d\tilde{E} \right\} .$$

2. *By the same analogy, if the same particle only spins (no linear or other motion), we will see that differential of its kinetic energy is,* $dE_k = \omega \, dL_s$, *since we already established that it should be analogically valid,* $\{dE_k = vdp\}_{linear-motion} \Leftrightarrow \{dE_k = \omega dL_s\}_{spinning}$. *Here we apply the following analogies:* ***linear velocity** replaced by **angular, spinning velocity**,* $v \leftrightarrow \omega$ *and **linear moment** replaced by **spinning angular moment**,* $dp \leftrightarrow dL_s$.

3. *By extending the same analogies (as under 1 and 2) to an orbital motion when a small mass* m *is orbiting much bigger mass, while not spinning, (where the radius of such orbiting is very long compared to both particle diameters), we will have,*

$$\begin{Bmatrix} E_k = mc^2(\gamma - 1) = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{L\omega}{1+\sqrt{1-v^2/c^2}} \\ dE_k = vdp = \omega dL = d\tilde{E} \end{Bmatrix} \Rightarrow E_k = \begin{Bmatrix} \begin{bmatrix} \left(\frac{pv}{2} = \frac{L\omega}{2}\right) \Leftrightarrow (pv = L\omega), v \ll c \\ (pv = L\omega), v \approx c \end{bmatrix} \end{Bmatrix} \Leftrightarrow \begin{pmatrix} pv = L\omega \\ \text{always} \end{pmatrix}$$

$$\Rightarrow dE_k = \begin{cases} \frac{1}{2}(pdv + vdp) = \frac{1}{2}(Ld\omega + \omega dL), (v \ll c) \rightarrow (vdp = pdv = \omega dL = Ld\omega) \\ pdv + vdp = Ld\omega + \omega dL = vdp = \omega dL, (v \approx c) \rightarrow pdv = Ld\omega = 0, \ v = \text{const.}, \ \omega = const. \end{cases}$$

where L *and* ω *are orbiting particle momentum and angular velocity, being mutually parallel, while* $pv = L\omega$ *is always valid since we present the same kinetic energy of the same particle on two different ways.*

4. *Now, let us imagine that the same small particle in orbital motion also has specific spinning. This will produce the following equivalency between involved linear and orbital motions,*

$$\begin{Bmatrix} E_k = m^*c^2(\gamma^* - 1) = \frac{p^*v^*}{1+\sqrt{1-v^{*2}/c^2}} = \frac{(L+L_s)\omega^*}{1+\sqrt{1-v^{*2}/c^2}} \\ dE_k = v^*dp^* = \omega^* d(L+L_s) = d\tilde{E} \end{Bmatrix} \Rightarrow \left(p^*v^* = (L+L_s)\omega^*\right) ,$$

where symbols with asterisks indicate that particle spin is changing particle linear and angular velocity.

5. Here we could also hypothesize with the concept that resulting, effective spinning motion (of the number of elementary mass constituents) is somehow captured and "frozen" by a total particle rest mass what (hypothetically) indicates that rest mass is created from some properly packed spinning and torsional-motion energy domains. What is important here is to start familiarizing with the idea or concept that every motion has linear and torsional (or spinning) kinetic energy components but mentioned spinning in cases of particles could be hidden by the formation of particles rest masses. Internal and intrinsic matter-waves rotation and spinning, is captured by a particle rest mass (becoming externally "non-visible" and not measurable because the number of elementary domains inside a macro-particle is randomly packed and mutually coupled, neutralizing the total, resulting spin moment, as well as compensating and neutralizing distributed electromagnetic moments and dipoles). Of course, we know that real macro-particles have many atoms, and that each atom has the number of more elementary particles (all of them having constant and closely related magnetic, spin and orbital moments). All that makes this situation more complicated, but the conceptual understanding would stay simple and clear as, Number of mutually coupled elementary domains that create rest mass, captures internal, hidden, and spinning-related motional matter-waves components. Later we will have enough mathematical and modeling margins to be able to pay attention to details and proper equations in mentioned conceptualization. For better understanding see (2.5-1) to (2.11), (4.18), (4.5-1) - (4.5-3), (4.42) to (4.45) and (5.4.1) to (5.10), (4.3-0), (4.3-0) -a,b,c,d,e,f,g,h,i… and Fig.4.1.1, Fig.4.1.1a, Fig.4.1.2, T.4.4, dealing with the unity of linear and rotational motions). ♣]

The much more natural and conceptually more precise explanation of **PWDC** (compared to earlier formulated, analogical, and mathematical merging of particle and wave properties (4.1), (4.2) and T.4.0) can be formulated as:

a) First, we should know (or accept) that there is not a case of a single particle (within our universe), fully isolated and without any interaction with its vicinity. Any single particle or energy state belongs to a much more general case of (at least) two-body relations, where the first body is that single particle, and the second body is its vicinity (or surrounding environment). This way (in any possible interaction, or when describing any possible particle or quasi-particles states), we effectively deal with at least a two-body system (or with interactions between two bodies). The primary, natural, and very tangible matter-waves related conceptualizations of two-body, or binary systems properties, are already elaborated in the second chapter and should be taken as an introduction in any **PWDC** foundation (see: "2.3.2. Macro-Cosmological Matter-Waves and Gravitation", -equations from (2.11.21) to (2.11.21)). When we analyze the general two-body problem traditionally (or only mechanically), we can formulate the most important relations describing such interaction, given by results from (4.5) until (4.8). Effectively, by analyzing how we came to all results and relations (4.5) - (4.8), we can conclude, later, that **PWDC** from (4.1) and (4.2), is an equivalent mathematical modeling way to explain the general two-body problem *by creating direct parallelism between particle (or its mechanical) and wave properties*. Differently formulated, we will see that energy-momentum complexity of two-body or two-particle relations presents the real source of **PWDC** (which will become more understandable after we make an equivalent wave presentation of the mechanical two-body situation). When analyzing the two-body problem in the light of **PWDC** we can also formulate the generalized, Newton-like "action-reaction", inertial forces (valid for all possible field situations: in gravitation, in electric and magnetic fields, etc.). Of course, the statements above are still a bit speculative before we see the development of all mentioned relations, (4.3) - (4.8), which is just temporarily postponed before creating a more natural, clear, and still introductory **PWDC** formulation.

b) The second, also an essential element of particle-wave duality is just coming from the fact that two bodies, mentioned in a), mutually interact even before any mechanical contact between them is established (again, see the development of (4.3) - (4.8) …, as a support). We can call this interaction a "field-channel, or particle-wave field connection", ***and this is just a modus of conserving (balancing and redistributing) energy and momentum (both linear and orbital, including possible spinning and wave energy of participants)***. By introducing the idea that a specific field or wave connection should always exist (in any two or multiple-body situation), we can mathematically analyze what happens when two (or much bigger number of) elementary waves interact. For instance, this is the example of a simple superposition of elementary (sinusoidal) waves to create the wave group, or a wave packet model, well analyzed in almost any modern Physics and Quantum Mechanics book). What we create this way is called the wave packet or group. For such waving form, we are in the position to find its group and phase velocity, and the essential relation between them, given by the equation, $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$. When we analyze this situation (see also (4.0.73) - (4.0.76) from the chapter 4.0), we can find that de Broglie wavelength $\lambda = h/p$, Einstein-Planck's relation for wave energy $\tilde{\mathbf{E}} = \mathbf{hf}$, and Einstein's mass-energy relation, $\mathbf{E_{total}} = \boldsymbol{\gamma}\mathbf{mc^2}$ are simply (mathematically) integrated in the structure of the equation that connects group and phase velocity (this way supporting and/or deriving every one of them using the combination of others), and all of them are shown to be mutually compatible, complementary and essential to explain (non-mechanically) the two-body situation a), this way automatically explaining the **PWDC** relations from (4.1) and (4.2), **as well as putting new light into a necessity of novel understanding of Max Planck's law of black-body thermal radiation, as briefly summarised, below**:

$$\left\{\begin{array}{c} \left[\lambda = \frac{h}{p}, k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p,\right] \\ \\ \left[\tilde{E} = hf\right] \\ \\ \left[\begin{array}{l} \overline{P}_4 = (p, \frac{E}{c}) \Rightarrow p^2 - \frac{E^2}{c^2} = -m^2c^2 \\ p = \gamma mv, \gamma = (1 - v^2/c^2)^{-0.5} \\ dE_k = vdp = mc^2 d\gamma = dE_{tot} = d\tilde{E} \\ E = E_{tot} = \gamma mc^2,\ E_k = (\gamma - 1)mc^2 \end{array}\right] \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} dE = dE_k = d\tilde{E} = vdp = d(pu) = pdu + udp = hdf = c^2 d(\gamma m) = mc^2 d\gamma, \\ E_k = hf = pu, \omega = 2\pi f,\ \lambda \frac{du}{d\lambda} = -p\frac{du}{dp} = -k\frac{du}{dk}, \frac{dp}{p} = \frac{dk}{k} = -\frac{d\lambda}{\lambda}, \\ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = \frac{dE}{dp} = \frac{d\omega}{dk} = u + p\frac{du}{dp} = u + k\frac{du}{dk}, \\ u = \lambda f = \frac{\omega}{k} = \frac{E_k}{p} = \frac{\tilde{E}}{p} = \frac{v}{1 + \sqrt{1 - v^2/c^2}},\ \frac{d\lambda}{\lambda} = -\lambda\frac{df}{v} \\ 0 \le 2u \le \sqrt{uv} \le v \le c \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} \lambda = \frac{h}{\tilde{p}} = \frac{h}{p},\ \tilde{E} = E_k = hf = \hbar\omega = m^* vu = pu = h\frac{\omega}{\mathbf{2}\pi} = \frac{h}{\tau} = (\gamma - \mathbf{1})mc^{\mathbf{2}},\ k = \frac{\mathbf{2}\pi}{\lambda} = \frac{\mathbf{2}\pi}{h}p, \\ \omega = \frac{\mathbf{2}\pi}{T} = \frac{\mathbf{2}\pi}{h}\tilde{E},\ \ u = \lambda f = \frac{\tilde{E}}{p} = \frac{\omega}{k}, v = v_g = u - \lambda\frac{du}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk}, m^* = \gamma m, p = \gamma mv = \tilde{p} \end{array}\right\} \Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} dE = d(pu) = pdu + udp = pd\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right) + \frac{vdp}{1+\sqrt{1-v^2/c^2}} = \\ = pd\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right) + \frac{dE}{1+\sqrt{1-v^2/c^2}} \Rightarrow \\ \left(\frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)dE = pd\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right) \Leftrightarrow \\ \Leftrightarrow \left(\frac{v\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)\frac{dp}{p} = d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right) \Rightarrow \\ \Rightarrow \boxed{\frac{dp}{p} = \frac{dk}{k} = -\frac{d\lambda}{\lambda} = \frac{d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)}{\left(\frac{v\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)}} = \left(\frac{1}{1+\sqrt{1-v^2/c^2}}\right)\frac{df}{f} = \\ = \frac{\lambda}{v}df = \frac{u}{v}\frac{df}{f} = \frac{hdf}{pv}, \boxed{\frac{df}{f} = \frac{\left(1+\sqrt{1-v^2/c^2}\right)^2}{v\sqrt{1-v^2/c^2}} d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)} \end{array}\right\} \Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} \frac{dp}{p} = \frac{dk}{k} = -\frac{d\lambda}{\lambda} = \frac{d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right)}{\left(\frac{v\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right)} = \left(\frac{1}{1+\sqrt{1-v^2/c^2}}\right)\frac{df}{f} \cong \begin{cases} \frac{dv}{v} = \frac{1}{2}\frac{df}{f}, & \text{for } v << c \\ \frac{df}{f}, & \text{for } v \approx c \end{cases} \\ \frac{df}{f} = \frac{d\tilde{E}}{\tilde{E}} = -(\frac{v}{u})\cdot\frac{d\lambda}{\lambda} = (\frac{dv}{v})(1+1/\gamma)\gamma^2 = -(1+\frac{1}{\gamma})\cdot\frac{d\lambda}{\lambda} = \frac{\left(1+\sqrt{1-v^2/c^2}\right)^2}{v\sqrt{1-v^2/c^2}} d\left(\frac{v}{1+\sqrt{1-v^2/c^2}}\right) \cong \\ \cong \begin{cases} 2\frac{dv}{v} = -2\frac{d\lambda}{\lambda}, & \text{for } v << c \\ -\frac{d\lambda}{\lambda}, & \text{for } v \approx c \end{cases} \end{array}\right\} \Rightarrow \quad (4.2\text{-}1)$$

⇒(After integration) ⇒

$$\Rightarrow\left\{\begin{array}{l} p = p(v) = m\gamma v,\ k = k(v) = \frac{2\pi}{h}p(v) = \frac{2\pi}{h}m\gamma v, \\ \lambda = \lambda(v) = \frac{h}{p(v)} = \frac{h}{m\gamma v}, u(v) = \lambda(v)f(v) = \frac{v}{1+\sqrt{1-v^2/c^2}} \end{array}\right\} \Rightarrow \boxed{f = \frac{u}{\lambda} = \frac{up}{h} = f(v)}.$$

We know that by explaining all experimental situations regarding particle-photon interactions (such as Compton, Photoelectric, electron-positron creation from a high-energy photon, or their annihilation, Bragg's diffraction of X-rays, electrons and neutrons, and other familiar effects, such as examples in Chapter 4.2), the equality between wave and particle energy ($\tilde{E} = hf = \tilde{m}c^2 = E_k = mc^2(\gamma - 1),\ p = \tilde{p} = \frac{hf}{c},\ \lambda = \frac{h}{\tilde{p}}$), is entirely provable, and related only to motional energies of all interacting participants. We can find familiar elaborations of **PWDC** situations including rectifications of original de Broglie foundations of wave-particle duality in [105]).

In other words, if we accept such approach, the real sources of particle-wave duality become much more realistic (tangible and deterministic) than a contemporary probability platform of Quantum Theory in explaining the same phenomena (for more of complementary explanation of **PWDC** see also the chapter 5, especially T.5.3, and chapter 10).

In conclusion, moving particles analogically behave as photons (or waves), and vice versa, related to their "linear and angular or spinning moments", kinetic energy, and masses. Presently we only exploit the analogy between linear moment of a particle and photon. Since photon in addition has an intrinsic angular or spinning moment ($L = \frac{c^2}{uv}(\frac{h}{2\pi})$), analogically this *should also be valid for a particle in linear motion (and this is what creates wave-particle duality and de Broglie matter-waves)*. ***Consequently, mentioned analogies between photons and moving particles should be naturally applicable in both directions***. It is also indicative that photon spin, and the spin belonging to a mooving particle, are both equal to $\frac{h}{2\pi}$ (see (2.11.3), T 4.0 and (4.8)). Now we can say that rotation (or spinning) ***and linear motion (of the same object) should always be intrinsically coupled in all cases of particle motions***. For instance, moving electrons create rotating (or helix) magnetic field around their paths of propagation (*see on Internet very similar concept about Henry Augustus Rowland effect of magnetic field of electrons around rotating metal conductors, as presented by Jean de Climont*). Something similar should also be valid for all cases of electromagnetically neutral particles that perform linear motion. In such situations, we should (analogically and coincidently) expect effects of rotation (or helix spinning) of specific matter-wave field components, what is eventually producing de Broglie matter waves, and other measurable consequences such as omnipresent rotations in the world of atoms, elementary particles, and rotations of astronomic objects. For instance, planetary rotation around the sun could be quantifiable as: $L = \frac{c^2}{uv}(\frac{H}{2\pi}) \cong \frac{2c^2}{v^2}(\frac{H}{2\pi}), u \cong \frac{v}{2} << c, h \to H = const.$). Whenever we have certain dynamic coupling of linear motion with rotation, this means that associated matter waves have been created along specific helix line of two complementary field components, what presents the content of a particle motional energy. Here, de Broglie wavelength and frequency should be in a direct relation with mentioned spinning-helix field structure. **The most direct externally measurable signs regarding the existence of natural rotation-related behaviors of all elementary particles should be their gyromagnetic, spin and orbital moment attributes.** In cases of stationary (or stabilized) inter-atomic, self-closed and circular motions, the much more general picture (regarding the same situation) is given by Wilson-Sommerfeld rules (see 5.4.1).

c) **Matter Waves & Rotation**: In cases of rotation, particle angular velocity (or mechanical rotating frequency around specific center) should not be mixed with angular velocity (or frequency) of belonging matter waves, since mentioned frequencies are mutually connected by certain velocity dependent function, where both group and phase velocity should be considered. For instance, it is often the situation that in many analyses regarding spin and orbital moments, authors do not make sufficiently clear differences between mentioned frequencies (related to important group and phase velocity), making a conceptual confusion regarding understanding particle-wave duality (or unity).

d) The next significant part of understanding the **PWDC** is already realized based on the possibility to create an equivalent and isomorphic mathematical structure that "in-average" very well represents (or replaces) deterministic concepts given under a) and b), using the framework of Probability, Statistics, and Signal Analysis (as successfully realized in the contemporary Quantum theory). This (about an isomorphic mathematical formulation) is officially still not admitted, or recognized qualification of today's Orthodox Quantum Mechanics, which found particularly good way how to construct its imposing mathematical structure, and which is working perfectly well (without exceptions), especially when loudly and repeatedly formulated by an army of non-conditional, and non-critical believers in such Quantum Theory. In fact, such Quantum theory still works very well, that other (different) voices and ideas could be easily suppressed and eliminated. Here, we will show that if once in the past, when Quantum Theory got its foundations, we had a better, more natural, and more general mathematical wave functions modeling, many of present Quantum Mechanical, ad hoc theoretical discoveries, postulates, and rules, would have been just elementary (mathematical) products of that, more general and better mathematical modeling (for instance, if based on Complex, Analytic Signal Phasors). Later, it will be shown that Schrödinger's and other wave equations, known as big achievements of Orthodox Quantum Theory, are quite simple mathematical consequences and results of using the much more general model of Complex Classical Wave Equation. Such generalized wave equation applies to any wave function, if interpreted as the complex Analytic Signal, based on using Hilbert transform (see equations from (4.9) to (4.28), and foundations of Analytic Signals in Chapter 4.0). It will also be shown that all forms of Uncertainty Relations (or Heisenberg relations), presently placed in the basket of big achievements of Quantum Theory, are simple products of universally applicable Signal Analysis theory, equally valid for micro and macro world (resulting when we analyze relations between two of mutually conjugate, original, and spectral domains of the same event). ***The place of probability and stochastic modeling and concepts in such situations can only be non-essential, secondary, as curves-fitting, or "in-average" evidence keeping, and non-ontological***, but it is mathematically operational (and equivalent to non-dimensional normalizations and statistical averaging techniques, whenever applicable). Of course, characteristic wave functions (describing motions in a micro and macro world) should be correctly treated and presented within the same mathematical framework (see Chapter 5). It is natural that all fundamental laws of (present and future, more advanced) physics should be equally valid and applicable on all sizes or scales of our Universe (what is still not the case in contemporary physics). In fact, we already know that Orthodox (or probabilistic) Quantum Theory mathematically works correctly, but we could also try to see it as an equivalent and alternative, or isomorphic mathematical modeling, which after big and fruitful modeling, tricky and innovative efforts, replaces much more tangible particle-wave platform of a) and b). One of the objectives in this book is to show how such equivalent (or isomorphic) modeling has been successfully realized, and to replace it later with more advanced and more general concepts. The current mainstream tendency of modern physics is to present Probabilistic Quantum Theory as the unique, most general and the most correct, theoretical, practical, and ontological vision of a micro world, almost predestinated to be never significantly changed (contrary to the message of this book). Of course, we should be conscious that Statistics and Probability theory are irreplaceable and essentially important only when we analyze sets and events containing a virtually countless number of identical or mutually similar elements. *We could also ask in which tangible domains of physics (apart from not so conceptually tangible Quantum Theory), Probability and Statistics play the essential (and irreplaceable) role, and we will find that this is Thermodynamics.* ***For Thermodynamics, we could say that it is "probabilistically deterministic" theory, and for present Quantum Theory, we could say that it is "probabilistically non-deterministic".*** *It is maybe too early to say that Quantum Theory and Thermodynamics should be united into one single theory (as many other Physics domains, too), but it is just the time to start being creatively suspicious, innovative, and* *curious when answering the question why both theories are essentially and intrinsically merged with Statistics and Probability. Regarding Thermodynamics, nobody seriously claims that there is not experimentally verifiable reality there, opposite to a situation with Quantum Theory.* Whichever the correct case is, we know that the present Quantum Theory, after many years of enormous mathematical fitting and research investments, and involvement of generations of scientific brain and workforce in building it, is now working very well. Anyway, such Quantum theory is overloaded with mathematical and other artificial complexity. The reasons are related to the fact that modeling, fitting, and matching between the theory and relevant experimental results are made operational and mathematically correct (but from the conceptual point of view much more questions are left open than answered). This book is addressing an innovated conceptual picture of the particle-wave duality. It indirectly offers kind of neo-deterministic alternative to Probabilistic Quantum Theory teaching. Of course, traditional, bottom-line, old-fashion mechanistic determinism is not directly and literally applicable to the matter-waves Physics we are addressing here.

e) In our early steps, in a process of building new wave-particle duality theory, we still do not have a better choice than to compare it with corresponding results of Orthodox Quantum Theory. The admirers and defenders of present-days Quantum Theory would say that it is almost forbidden, useless, and worthless to search for a new and better theory (because the old or contemporary one is already operational, working well, and this will bring nothing new, since everything relevant is already published and fixed, etc.), but the objective of this book is to open such process.

f) We can also find that several elements of here formulated **PWDC** are also incorporated in the N. Bohr's hydrogen atom model (although artificially and with challenging assumptions; -see Chapter 8.), which sufficiently and surprisingly, but correctly operates in the frames of its definition and within already known applications (in spectroscopy, for instance). In Chapter 2. about Gravitation, we can also find that all elements of **PWDC** concepts and modeling are present in describing planetary systems (see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation").

Most of the wave-particle, dualistic behaviors of matter in motion have been noticed and analyzed based on photons, electrons, positrons, protons, and neutrons, mutual interactions, such as scattering and impacts with atoms and other particles. Photoelectric and Compton effects (including electron-positron creation and annihilation) are suggesting that electrons and photons are on a certain way (under favorable circumstances) mutually transformable, or at least internally composed of similar electromagnetic matter, having particle-like, and/or wave-like properties (while all involved conservation laws should be satisfied). Bragg's diffraction of X-rays (meaning photons, or electromagnetic waves) is entirely explicable only with wave properties of photons, but the most indicative and significant is the fact that similar diffraction of particles like electrons and neutrons is also explicable with the same Bragg's formula (which is established only for wave motions). Consequently, we again have the situation that motional particles like electrons, protons and neutrons also behave like de Broglie matter waves, like photons (having de Broglie wavelength and Einstein-Planck wave energy and respecting other **PWDC** relations). Consequently, the conclusion is that we could creatively search for a concept that is innovatively explaining neutron as an "***exotic atom***" composed from one-electron-shell (or envelope) and one-proton, as its internal core, where electron spin is dominant (See such conceptual contemplations in "**[88],** David L. Bergman (2001). Notions of a Neutron. *Foundations of Science* 4 (2):1-8. Science, Logic, and Mathematics"). ***In other words, we have matter-waves and particle-wave duality manifestations (complying with PWDC) only in several specific cases when different electromagnetic energy formations with spin and magnetic moment attributes are involved (such as interactions between photons, electrons, positrons, protons, neutrons, bigger electrically charged particles, with atoms and other neutral or electrically charged particles...).*** Instead of frequently evoking wave-particle duality and matter-wave properties of all matter in motion, we could say that only photons, electrons, positrons, protons, and neutrons are presenting specific structural packing of electromagnetic energy that is ontologically and essentially creating all matter in our Universe. It should be evident that such electromagnetic packets or wave-groups (in different packing forms) also have specific wavelengths and energy formulated using the same mathematical (de Broglie) expressions (here summarized as **PWDC**). Consequently, wave-particle duality is equally valid and applicable to both electrically charged and electrically neutral micro and macro entities (but here we also consider that electromagnetically neutral masses are internally, intrinsically and ontologically composed of electromagnetic entities). Nevertheless, wave-particle duality is primarily, essentially, and overwhelmingly, related to electromagnetic and mechanical, motional energy-momentum manifestations. Consequently, "***no electromagnetic fields and electrically charged entities with magnetic spin attributes is equal to no matter waves***". This is equally applicable to microphysics and astronomic objects (see Chapter 2., "2.3.3. Macro-Cosmological Matter-Waves and Gravitation", and [71], from Jovan Djuric; -"Magnetism as Manifestation of Gravitation").

*Quantum theory (QT) is considering that **the total particle energy** should be entirely presentable with an equivalent matter-wave packet (what is philosophically correct, but only on a specific way, as explained in this book). Also, when kinetic energy is considered (within modern QT), these are mostly the cases of Classical Mechanics, low velocity moving particles, v<<C, what is producing that relevant phase velocity of an associated matter-wave will be higher than universal constant C, u > C, what is illogical and wrong. See the more complete explanation in (4.10-5), from Chapter 4.3.*

What follows in the next part of this chapter (and later) is kind of step-by-step explanation of a), b), c), d), ... statements, or a deeper explanation of the **PWDC** and its integration with generalized wave equations, *effectively transforming the Particle-Wave Duality concept into Particle-Wave Unity concept.*

[♣ COMMENTS & FREE-THINKING CORNER:

4.1.1.1. The resume regarding PWDC

*A brief and simplified summary of all above-given explanation/s regarding **PWDC** can also be presented in the following way:*

1. *Relativity theory shoved or implicated that there is a simple relation of direct proportionality between any mass and its total energy that could be produced by fully transforming that mass into radiation,* $E_0 = mc^2$, $E_{tot.} = \gamma mc^2 = E_0 + E_k = E_0 + (\gamma - 1)mc^2 = \sqrt{E_0^2 + p^2c^2}$.

2. *The most important conceptual understanding of frequency dependent matter wave energy, which is entirely equivalent to particle motional, or kinetic energy, is related to the fact that total photon energy can be expressed as the product of the Planck's constant and frequency of the photon wave packet,* $E_f = hf$. *Consequently (since there is a known proportionality between mass and energy), the photon momentum was correctly found as* $p_p = hf / c = m_p c$ *(and proven applicable and correct in analyzes of different interactions between photons as waves, or quasi-particles and real particles).*

3. *Since a photon has a specific energy, we should be able to present this energy in two different ways, for instance,* $E_p = hf = \sqrt{E_{0p}^2 + p_p^2 c^2} = p_p c = E_{kp}$. *Since photon rest mass equals zero, there is only a photon kinetic energy* $E_p = hf = p_p c = E_{kp} = m_p c^2$, *and in many applications, this concept (and all equivalency relations for photon energy and momentum) showed to be correct.*

4. *Going backward, we can apply the same conclusion, or analogy, to any real particle (which has a rest mass), accepting that particle kinetic energy is presentable as the product between Planck constant and characteristic particle's matter wave frequency* $E_k = (\gamma - 1)mc^2 = \tilde{E} = hf$. *Doing it that way, we can find the frequency of de Broglie matter waves as,* $f = E_k / h = (\gamma - 1)mc^2 / h = \tilde{E} / h$. *Now, we can find the phase velocity of matter waves as* $u = \lambda f = \frac{h}{p} f = \frac{E_k}{p} = \frac{(\gamma - 1)mc^2}{\gamma m v} = \frac{v}{1 + \sqrt{1 - v^2/c^2}}$.

5. *The relation between phase and group velocity of a matter wave packet is also known in the form* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$. *Combining two latest forms of phase and group velocities, we can get* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u(1 + \sqrt{1 - v^2/c^2})$, *implicating validity of the following differential relations:* $d\tilde{E} = d\left[(\gamma - 1)mc^2\right] = mc^2 d\gamma = hdf = vdp = d(pu)$, *and practically confirming mathematical consistency of all above introduced equivalency and analogy based relations (named in this book as **PWDC** = **P**article-**W**ave **D**uality **C**ode).*

6. *It is even more interesting to notice that in the same framework (from 1 to 5) the spin of motional energy (photon spin and kinetic energy spin, T 4.0) is always equal to* $\frac{\tilde{E}}{\omega} = \frac{E_k}{\omega} = \frac{h}{2\pi}$ *(or to a multiple of* $\frac{h}{2\pi}$*, see (4.8)).*

7. *Since the above addressed particle wave equivalency relations are found valid only if we use the wave packet model (as a replacement for a particle in motion), consequently, we have an argument more to say that matter waves (or corresponding wave functions) should exist as forms of harmonic, modulated sinusoidal signals (naturally satisfying the rules of Fourier signal and spectrum analysis). It is of essential importance to notice that a rest mass (or rest energy) does not belong directly to matter-wave energy (opposite to many current presentations in modern physics books, regarding matter wave properties), but certainly, when in motion, rest mass presents the source of matter waves. Analyzing Compton Effect and many other elementary interactions known in Quantum Mechanics, we can easily prove the statement that only kinetic or motional energy presents the active matter wave energy. The next consequence of this concept is that the rest mass of the particle itself should present (internally and intrinsically) a "passive matter wave energy" in the form of a closed, self-sustainable, stationary, and standing wave structure, which only externally looks like a stable and closed shell particle. This*

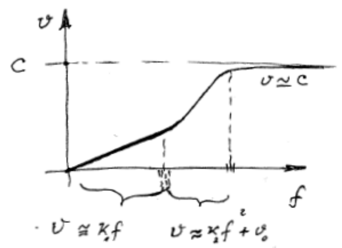
internally closed matter wave energy would become externally measurable (directly or indirectly) in all cases when a particle interacts with its environment participating in any motion.

*8. We can also rethink about the meaning of phase velocity **u**, based on (4.2), because modern interpretations of this velocity in most Quantum Mechanics and Physics books are different from (4.2), (however, in some basic literature we can also find the relation **v = 2u**, for **v** << **c**, which agrees with (4.2)). For instance, thermal blackbody radiation can also be addressed using group and phase velocity relations from (4.2). We can try to estimate what happens inside a black body cavity where we have a complex, random motion of hot gas particles, atoms and molecules, and random light emission, absorption, and photons scattering. We only know from Planck's blackbody radiation formula the resulting spectral distribution of outgoing light emission, in the case when we make a small hole on the surface of the black body, and let photons be radiated and detected in an external free space of the blackbody. Such external light radiation is characterized (in Planck's modeling) by free photons (with standing waves formations) where each photon has (almost) the same phase and group velocity **v = u = c = constant**. This is not the case inside the black-body cavity, since inside there are many mechanical and field interactions between photons, gas particles, electrically and magnetically charged particles, particles with spinning and magnetic properties, matter waves and cavity walls, and there we should have a broad range of spectral dependency between group and phase velocities,* $0 \le 2u \le \sqrt{uv} \le v \le c$. *A significant number of wave packets (de Broglie matter wave groups with mutually united or coupled mechanical and electromagnetic properties) inside the blackbody cavity permanently interact (among themselves, as well as with the cavity and gas particles) and we cannot consider them being free (mutually independent) wave groups, or stable and/or standing wave formations. It is logical (as the starting point in an analysis of such case), to imagine that mean particle or group velocity of such wave groups is directly proportional to the blackbody temperature. The average kinetic energy of involved particles is proportional to certain temperature, and when the gas temperature (inside a blackbody radiator) is relatively low, then we should dominantly have motions with non-relativistic particle velocities (*$v << c \Rightarrow v \approx 2u$*). When the temperature is sufficiently (or remarkably) high, we should dominantly have the case of relativistic particle motions with high velocities (*$v \approx c \Leftrightarrow v \approx u$*), according to (4.2). **Practically, we can say that Planck's blackbody radiation formula (see chapter 9.) is very useful and correct, but the way in which it is assembled (or fitted) is not sufficiently correct.** Let us create the following table, T.4.1., comparing relativistic and non-relativistic wave-group energies of an internal blackbody situation, searching for a better background and development of the blackbody radiation formula, and exercising the practical meaning of the **PWDC**, as established in (4.1) and (4.2).*

T.4.1. Interacting and coupled wave groups inside the black body cavity

$\left\{ \begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, \\ u = \frac{v}{1+\sqrt{1-v^2/c^2}} = \frac{u - \lambda \frac{du}{d\lambda}}{1+\sqrt{1-v^2/c^2}} = \frac{-\lambda^2 \frac{df}{d\lambda}}{1+\sqrt{1-v^2/c^2}} \end{array} \right\} \Rightarrow$	$\left\{ \begin{array}{l} \frac{du}{u} = -\left(\frac{d\lambda}{\lambda}\right)\sqrt{1-v^2/c^2} = \frac{df}{f} \cdot \frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \\ \frac{df}{f} = -\left(\frac{d\lambda}{\lambda}\right)(1+\sqrt{1-v^2/c^2}) \end{array} \right\}$
Nonrelativistic group velocities: $v << c \Rightarrow v \cong 2u,\ \sqrt{1-v^2/c^2} \cong 1$ (Lower temperatures)	Relativistic group velocities: $v \approx c \Rightarrow v \approx u \approx c,\ \sqrt{1-v^2/c^2} \cong 0$ (Very high temperatures)
$\left\{ \frac{du}{u} \cong -\frac{d\lambda}{\lambda} \cong \frac{df}{2f} \right\} \Rightarrow \left\{ \ln\left\|\frac{u}{u_0}\right\| \cong -\ln\left\|\frac{\lambda}{\lambda_0}\right\| \cong \frac{1}{2}\ln\left\|\frac{f}{f_0}\right\| \right\}$ $\Rightarrow v \cong 2u = 2\lambda f \cong \frac{2u_0\lambda_0}{\lambda} = \frac{C_1}{\lambda}, f \cong \frac{C_2}{\lambda^2}$ $\left[\tilde{E}\right]_{Low\ temp.} = hf \approx \frac{C_3}{\lambda^2} = C_4 \frac{f^2}{u^2} \approx C_5 \left(\frac{f}{v}\right)^2,$ $\left[\frac{d\tilde{E}}{d\lambda}\right]_{Low\ temp.} \approx -\frac{C_6}{\lambda^3} = -C_7 \frac{f^3}{u^3} \approx -C_8 \left(\frac{f}{v}\right)^3,$ $C_i = \text{constants}$ $\left\{ \begin{array}{l} \text{Vortex flowmeter relation:} \\ \text{If } v \cong k \cdot f \Rightarrow \left[\tilde{E}\right]_{Low\ temp.} = hf \approx \frac{C_5}{k^2} = \text{const.} \end{array} \right\}$	$\left\{ \frac{du}{u} \cong 0,\ \frac{df}{f} \cong -\frac{d\lambda}{\lambda} \right\} \Rightarrow$ $\Rightarrow \left\{ \ln\left\|\frac{u}{u_0}\right\| \cong 0,\ -\ln\left\|\frac{\lambda}{\lambda_0}\right\| \cong \ln\left\|\frac{f}{f_0}\right\| \right\} \Rightarrow$ $\Rightarrow (v \cong u = \lambda f \cong u_0 = \lambda_0 f_0) \cong c, f \cong \frac{c}{\lambda} \Rightarrow$ $\left[\tilde{E}\right]_{High\ temp.} = hf \approx h \cdot \frac{c}{\lambda} = \frac{hf}{u} \cdot c \approx hc\left(\frac{f}{v}\right) \ge \frac{hf}{c} \cdot c,$ $\left[\frac{d\tilde{E}}{d\lambda}\right]_{High\ temp.} = -h \cdot \frac{c}{\lambda^2} \approx -hc\left(\frac{f}{v}\right)^2 \le -\frac{hf^2}{c^2} \cdot \text{const.}$

It is difficult to say what is the real situation inside a black body cavity, but we can imagine that numbers of Secondary Emission, Photoelectric and Compton events are permanently created, and combined with thermal motion, and mutual electromagnetic interferences, and couplings of gas particles, etc. The analyzed situation is not directly comparable with external blackbody radiation (Planck's result), since (in the above-given table) we have only internal wave energy states in the black body cavity, but the results are very much indicative and provoking. For instance, we see that in the case of lower temperatures, we have the wave energy function $\left[\tilde{E}\right]_{\text{Low temp.}} \approx A\left(\frac{f}{v}\right)^2,\ v << c,\ A = \text{const.}$ $\left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{Low temp.}} \approx -C_8\left(\frac{f}{v}\right)^3$ *, and in the case of very high temperatures, the same energy is* $\left[\tilde{E}\right]_{\text{High temp.}} \approx B\left(\frac{f}{v}\right), v \to c, B = hc = \text{const.}$ $\left[\frac{d\tilde{E}}{d\lambda}\right]_{\text{High temp.}} \approx -hc\left(\frac{f}{v}\right)$*, meaning that internal wave groups are decreasing energy with temperature increase (which is not contradictory to the conclusions of Planck's radiation law). Here we see that there is a big difference between free wave groups, like free photons in open space, and mutually interacting (de Broglie) matter waves and particles (inside of a limited space of a cavity). On the contrary, in most analyzes of similar situations in modern Quantum Mechanics, we do not find that such differentiation is explicitly underlined and adequately addressed (mostly we find that de Broglie matter waves are treated similarly to free photons or some other free wave groups, or artificial probability waves). Also, in mathematical development of Planck's radiation law, we can only find specific particularly suitable (idealized, poor, and artificial) modeling and curve fitting situations where the phase velocities of a black-body (internal) photons always have the velocity of free (externally radiated) photons,* $u = \lambda f = v = c = \text{Constant}$*. The results are given in table T.4.1. are also applicable to any other situation when we analyze relativistic or non-relativistic motions. Especially interesting is to clarify group- velocity-frequency relation (based on results found in T.4.1.). Presently we do not have enough arguments to make such conclusions, but we could estimate that for a small particle, or group velocities, velocity-frequency curve is linear, for middle range velocity is second order (parabolic) curve, and for remarkably high velocities is asymptotically approaching to the speed of light c (see the picture below).*



Estimated velocity-matter-waves-frequency curve

Another instructive exercise (to understand the meaning of matter waves) is to exploit the symmetry with mutual replacements of corpuscular and wave momentum and motional and wave energy, ($\vec{p} \leftrightarrow \tilde{p}$*), (*$E_k \leftrightarrow \tilde{E}$*) presented in the following table, T 4.1.1.*

T 4.1.1

Symmetry-based on equivalence: $\vec{p} \leftrightarrow \tilde{p}$	
$E_k = \frac{\vec{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}}$	$\tilde{E} = pu$
$\tilde{E} = \tilde{p}u$	$E_k = \frac{\tilde{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}}$

$$\Rightarrow \left\{ \begin{array}{l} (\frac{\tilde{E}}{E_k})^2 = \left[\frac{\tilde{p}}{p}\cos(\vec{p},\tilde{p})\right]^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \\ = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v\cos(\vec{p},\tilde{p})}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = \cos(\vec{p},\tilde{p}) \end{array} \right\}$$
$$\Rightarrow \left\{ \lambda = \frac{h}{p} = (\frac{h}{\tilde{p}})\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})} = (\frac{h}{\tilde{p}})\frac{u(1+\sqrt{1-\frac{v^2}{c^2}})}{v\cos(\vec{p},\tilde{p})} = (\frac{v}{f})\frac{\sqrt{\cos(\vec{p},\tilde{p})}}{1+\sqrt{1-\frac{v^2}{c^2}}} \right\}$$
$$\{\cos(\vec{p},\tilde{p})=1\} \Rightarrow \left\{ \begin{array}{l} (\frac{\tilde{E}}{E_k})^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = 1, \\ E_k = \tilde{E},\ p = \tilde{p}, u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ \lambda = \frac{h}{p} = \frac{h}{\tilde{p}} \end{array} \right\}, \{\cos(\vec{p},\tilde{p})=-1\} \Rightarrow \left\{ \begin{array}{l} (\frac{\tilde{E}}{E_k})^2 = (\frac{p}{\tilde{p}})^2 = (\frac{m}{\tilde{m}})^2 = \left[\frac{u}{v}(1+\sqrt{1-\frac{v^2}{c^2}})\right]^2 = \left[\frac{v}{u(1+\sqrt{1-\frac{v^2}{c^2}})}\right]^2 = -1 \\ E_k = -j\tilde{E},\ p = j\tilde{p}, u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ j^2 = -1 \end{array} \right\}$$

From the table T 4.1.1 we can conclude that corpuscular and wave momentum of a moving particle (which is also represented as a matter-wave object) should act in the same direction, since the only logical and reasonable result is that the angle between them should stay in the following limits: $0 < \cos(\vec{p},\tilde{p}) \leq 1 \Rightarrow -\frac{\pi}{2} \leq [\angle(\vec{p},\tilde{p})] \leq \frac{\pi}{2}$.

The conclusion we could draw from all elaborations in this chapter (starting from (4.1) until T4.1. and T4.1.1) is that group, and phase velocities of any wave motion (or wave group) can be presented on two mutually equivalent (not contradictory, and mixed) ways, from two very much different theoretical and conceptual platforms (see A and B, below). The first of mentioned platforms (A) almost exclusively belongings to the ordinary wave-motions physics from Classical Mechanics and Acoustics (as wholly explained in the chapter 4.0. around equations (4.0.6) until (4.033)), such as,

(4.1.1.1.)-a

A) $$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, u = \lambda f = \frac{\omega}{k}.$$

The second point of view or platform (regarding group and phase velocity), not in conflicts with A), is on some way related to Relativity theory, producing the following results,

(4.1.1.1.)-b

B) $$u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}},\ v = \frac{2u}{1+\frac{uv}{c^2}},\ 0 \leq 2u \leq \sqrt{uv} \leq v \leq c,$$

Also, at the same time both (A & B) have mutually mixed (not contradictory) uniting relations, and relations with energy concepts of Quantum Mechanics, such as,

(4.1.1.1.)-c

All over this book are scattered small comments placed inside the squared brackets, such as: [♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

(A & B)

$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}},$$

$$u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p},\ f_s = k/2\pi,$$

$$d\tilde{E} = hdf = c^2 d(\gamma m) = mc^2 d\gamma,\quad \frac{df}{f} = (\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \Rightarrow \frac{\Delta f}{\bar{f}} = (\frac{\Delta v}{\bar{v}})\cdot\frac{1+\sqrt{1-\frac{\bar{v}^2}{c^2}}}{1-\frac{\bar{v}^2}{c^2}},$$

$$\left(\lambda = \frac{h}{p},\ u = \lambda f,\ \tilde{E} = hf,\ \frac{u}{v} = \frac{hf}{pv} = \frac{\tilde{E}}{pv},\ \gamma = 1/\sqrt{1-\frac{v^2}{c^2}}\right).$$

*The fact is that one quant of elementary wave-packet energy is equal to $\tilde{E} = hf$. This relation is correct, but it is on some way only intuitively and empirically postulated, or fitted to get results in relation to Blackbody radiation, and later successfully used to explain Photoelectric and Compton Effect, Bragg's diffraction, etc. Here, we see that $\tilde{E} = hf$ it is a consequence, and part of relations found in **(4.1.1.1.)-a,** and **(4.1.1.1.)-b**, meaning that it also has specific much more significant theoretical background than just useful assumption.*

*When presenting specific particle in motion, the most significant and probably only relevant links to Relativity theory (without covariant or Lorentz invariant misrepresentations), are definitions of particle "**proper time** τ", "**proper mass** m_0" and "**proper energy** E_0", such as,*

(4.1.1.1.)-d

C)

$$dt = \gamma d\tau,\ d\tau^2 = dt^2 - dr^2/c^2 = \text{invariant}$$

$$(dr^2 = dx^2 + dy^2 + dz^2)$$

$$\begin{bmatrix} m = \gamma m_0 \\ E = \gamma E_0 = \gamma m_0 c^2 \\ p = \gamma p_0 = mv = \gamma m_0 v \end{bmatrix} \Rightarrow p^2 - \frac{E^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2}.$$

*Proper time τ (measured with a single co-moving clock, linked to the particle in question) has the same meaning as well elaborated and clarified by Thomas E. Phipps, Jr. in [35], and proper mass and proper energy m_0, E_0 are only analogical names introduced here to underline the analogy with proper time ($dt = \gamma d\tau \leftrightarrow m = \gamma m_0, E = \gamma E_0$), but in reality meaning rest mass and rest energy. Everything else what we know from Einstein-Minkowski 4-vectors formalism should be explicable based on here introduced "**proper parameters**". Inertial "frame time" t is the time measured by a spatially extended set of clocks at rest in that inertial frame (and something similar is valid for inertial "frame mass" and inertial "frame energy", m and E (see T.2.2-3 and (2.4-17.1) in the second chapter of this book).*

The equation that connects group and phase velocity (4.1.1.1.)-b, (initially developed by analyzing superposition of simple-harmonic, elementary sinusoidal waves; -see equations from (4.06) until (4.0.33)), we can quickly get from much more of "corpuscular and mechanical thinking" as elaborated in the chapter 4.0., around equations from (4.0.73) to (4.0.76). Moreover, in the second chapter of this book is documented that the same relations of group and phase velocity, like in (4.1.1.1.)-c, are fully applicable to planetary systems (see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation") … all of that working well without any need to involve stochastic and probability concepts.

These are the platforms and areas (as summarized under (4.1.1.1.)-a,b,c,d...) where we should search for understanding the wave-particle duality or unity. If something structural and essential could be wrong with Relativistic and Quantum theory, mentioned relations between group and phase velocity are still valid and working well (see more of supporting and familiar elaborations in the second chapter of this book around equations (2.4-11) - (2.4-17.1)). Such mixed and mutually supporting relations (like above underlined in (4.1.1.1.)-a,b,c,d...) are also supporting introductory energy and wave-packets concepts of Quantum Mechanics. Since we know that Classical Mechanical case (of group and phase velocity *concept) is always valid, simple, and clear, we could conclude that Relativistic and Quantum theory extensions and connections within the same problems is, surprisingly and sufficiently well formulated. We should also notice that here we did not touch the ideas and concepts regarding statistics and probabilities (since such an approach is still not necessary). We can also notice that Relativistic theory and Classical Mechanics premises regarding velocities and motions are mutually different, challenging, and not distinct, bottom-line clear (as in Relativistic theory). Here, (with A), B) and C)) we demonstrated specific analogical and mathematical match, mutual compatibility and hybridization that works sufficiently well in agreement with Physics (regardless of mentioned differences). One could complain, or criticize explanations and elaborations, presented here as somewhat chaotic, not well organized, and based on simplified analogical conclusions. This is correct (if a reader does not like to be intellectually and intuitively flexible and open-minded), but anyway mentioned conclusions and results are very indicative and motivating for opening a project that will address the problematic of matter-waves velocities and other properties much better.*

If we need to imagine what matter waves are, we could say that all waves and oscillations known in the world of Physics (including acoustic and electromagnetic waves) are matter waves (not only probability and possibility, wavelike functions, and different events distributions). Mathematics for describing and analyzing all matter waves is the same and belongs to Analytic Signal functions (see more in chapter 4.0). Particles are also specific and stabilized matter waves formations. The opinion of the author of this book is that essential and unavoidable, necessary ingredient for a stable particle creation, starting from a specific combination of complex matter waves, is that some of the electromagnetic components of matter waves are also involved in such process. Electromagnetic matter waves are serving as a kind of "binding and gluing medium" connecting all other (non-electromagnetic) energy-momentum entities in the process of stable particles creation.

*One of the enormous obstacles and wrongly addressed items within Relativity theory is a unique position and fixed properties of a light speed **c** in a vacuum. Reasons, why this is the case, are well explained in [35], [80] and [81]. Briefly summarizing, Relativity theory is considering that a photon speed (in a vacuum) is always constant regardless of the motion of its source or its sink or observer, and this is not universally correct. All other elaborations in Relativity theory resulting in specific mathematics forms of velocity additions are also not universally valid and applicable. Light waves or wave packets like photons are behaving similarly as other waves, being the source or sink (or emitter and receiver), motions dependent and within certain limits variable. Let us compare the case when a specific light source is approaching receiver with velocity v_s, and the case when the same light source is moving in an opposite direction (on the same line) with the same velocity. Theory of Relativity is explaining or postulating that in both cases photons should have the same (group) velocity equal to constant **c**. If this is correct, we should have,*

$$\left\{\begin{pmatrix} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = h\frac{df}{dp} \\ u = \lambda f = \frac{\omega}{k} = \frac{hf}{p} \end{pmatrix} \wedge \begin{bmatrix} (v+v_s) = (v-v_s) = c \\ v_s = \text{const.} << c \end{bmatrix}\right\} \Rightarrow$$

$$\Rightarrow \left[u + v_s - \lambda\frac{d(u+v_s)}{d\lambda} = u - v_s - \lambda\frac{d(u-v_s)}{d\lambda}\right] \Leftrightarrow v_s = 0, v \neq c$$

And

$$\left\{\begin{pmatrix} v = \dfrac{2u}{1+\dfrac{uv}{c^2}} \\ u = \dfrac{v}{1+\sqrt{1-\dfrac{v^2}{c^2}}} \end{pmatrix} \wedge \begin{bmatrix} (v+v_s) = (v-v_s) = c \\ v_s = \text{const.} << c \end{bmatrix}\right\} \Rightarrow$$

$$\left[\frac{v+v_s}{1+\sqrt{1-\frac{(v+v_s)^2}{c^2}}}+v_s=\frac{v-v_s}{1+\sqrt{1-\frac{(v-v_s)^2}{c^2}}}-v_s\right]\Leftrightarrow\left[v+2v_s\cong v-2v_s\right]\Leftrightarrow v_s=0, v\neq c \qquad \textbf{(4.1.1.1.)-e}$$

The conclusion from (4.1.1.1.)-e is clear and distinct: only when source velocity v_s is equal to zero mentioned light, or any wave group velocities are mutually equal.

We can now test the statement that all monochromatic photons (with different frequencies; -for instance f_1 and f_2, $f_1 \neq f_2$*) should have the same speed equal to **c**, as follows,*

$$\left(\begin{array}{c} v=-\lambda^2\frac{df}{d\lambda}, \; u=\lambda f \\ \wedge \\ v_1=v_2=v, \; u_1=u_2=u \end{array}\right)\Rightarrow\left(-\lambda_1^2\frac{df_1}{d\lambda_1}=-\lambda_2^2\frac{df_2}{d\lambda_2}, \; \lambda_1 f_1=\lambda_2 f_2\right)\Rightarrow\frac{d\lambda_2}{\lambda_2^2}=\frac{df_2}{df_1}\frac{d\lambda_1}{\lambda_1^2}, \; \frac{1}{f_1^2}\frac{df_1}{d\lambda_1}=\frac{1}{f_2^2}\frac{df_2}{d\lambda_2}\Rightarrow$$

$$\Rightarrow\left(\frac{\lambda_2^2\lambda_1^2}{f_1^2}\frac{df_1}{d\lambda_1}=\frac{\lambda_1^2\lambda_2^2}{f_2^2}\frac{df_2}{d\lambda_2}\right)\Rightarrow\left(\frac{\lambda_2^2}{f_1^2}v_1=\frac{\lambda_1^2}{f_2^2}v_2\right)\Leftrightarrow\left(u_2^2 v_1=u_1^2 v_2\right)\Leftrightarrow f_1=f_2=f, \; \lambda_1=\lambda_2=\lambda. \qquad \textbf{(4.1.1.1.)-f}$$

The result of (4.1.1.1.)-f is that such generally valid solution does not exist, and that only mutually identical, monochromatic photons (or other matter-wave groups) could have the same group and phase velocity.

*Now we can safely say that all different photons have a different group and phase velocities, and what we consider as the universal physical constant, **c = 299792458 m/s**, is the only specific mean value of the "white" light speed in a vacuum, but anyway extremely relevant in our world of physics. From experimental practices, we also know that light speed standard deviation or variance, considering all different light sources and photons (in a vacuum), is small. Real universal constant **c** should be more adequately linked only to vacuum properties, since* $c=1/\sqrt{\varepsilon_0\mu_0}$ *, and consequences of such situation are that speed of light in a vacuum has very narrow velocity distribution curve around* $c=1/\sqrt{\varepsilon_0\mu_0}$. *Repeated astronomic and laboratory measurements of light speed **c** are showing that it is not possible to find non-doubtful, clear, unique, stable, and constant value, regardless statements that measured deviations are expected as methodological and instrumentation related errors. The same situation, with similar measurements' facts and conclusions, is also applicable and valid for Newton gravitational constant G. Both **c** and G are not entirely constant, stable, and independent from dynamic and geometry conditions related to measurements, and Relativity theory postulates about **c** are not entirely correct (but numerical variations in different measurement conditions are sufficiently small that we can say that **c** and G are very much constant, at least within our solar system).*

*There should be certain much stronger and more essential (ontological) relation and direct coupling between **c** and G, still not exposed in contemporary physics. Let us try to create such G and **c** hypothetic connections using analogies exercised in the first and second chapter of this book (see T.1.2 until T.1.8, from the first chapter, and T.2.2-1, T.2.2-2, T.2.8., (2.4-13), (2.4-5.1), (2.11.13-1)-(2.11.13-5), (2.11.23), (2.11.24), (2.11.14-4), from the second chapter).*

$$\left\{\begin{array}{l} F_{e/1,2} = k\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \\ F_{g/1,2} = G\frac{m_1 m_2}{r^2} = \frac{1}{4\pi g_0}\frac{m_1 m_2}{r^2} = \frac{1}{4\pi g_0^{'}}\frac{m_1 c\cdot m_2 c}{r^2} = \frac{1}{4\pi g_0^{'}}\frac{p_1^*\cdot p_2^*}{r^2} = \frac{c^2}{4\pi g_0^{'}}\frac{m_1 m_2}{r^2}, \\ q_{1,2}\begin{pmatrix}\leftrightarrow \\ \text{analog to}\end{pmatrix} p_{1,2}^* = m_{1,2}c,\ k = \frac{1}{4\pi\varepsilon_0}\begin{pmatrix}\leftrightarrow \\ \text{analog to}\end{pmatrix} G = \frac{1}{4\pi g_0} = \frac{c^2}{4\pi g_0^{'}}, \\ \left[c = 1/\sqrt{\varepsilon_0\mu_0}\begin{pmatrix}\leftrightarrow \\ \text{analog to}\end{pmatrix} c = 1/\sqrt{g_0\rho_0}\right] \Rightarrow \varepsilon_0\mu_0 = g_0\rho_0, \varepsilon_0 = \frac{1}{\mu_0 c^2}, \end{array}\right\} \Rightarrow \quad \textbf{(4.1.1.1.)-g}$$

$$\Rightarrow g_0 = \frac{1}{4\pi G} = \frac{g_0^{'}}{c^2} = \frac{1}{\rho_0 c^2},\ G = \frac{1}{4\pi g_0} = \frac{\rho_0 c^2}{4\pi} = \frac{\rho_0}{4\pi\varepsilon_0\mu_0} = \frac{c^2}{4\pi g_0^{'}},\ \rho_0 = \frac{4\pi G}{c^2} = \frac{1}{g_0 c^2}$$

*Also, here is a place to give expressions for the "fine structure constant" α and to ask the question about possible relations between α, **c** and G,*

$$\alpha = \frac{2\pi k e^2}{hc} = \frac{e^2}{2\varepsilon_0 hc} = \frac{\mu_0 c e^2}{2h} = \frac{e^2}{2h}\sqrt{\frac{\mu_0}{\varepsilon_0}} \cong \frac{1}{137}$$

$$\text{e-electron charge}, k = \frac{1}{4\pi\varepsilon_0}, h = \text{Planck constant}.$$

Of course, here we are dealing with intuitive and brainstorming, hypothetical options, expecting that once we will create specific physics-relevant concept and theory as a better replacement. ***♣]***

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ **COMMENTS & FREE-THINKING CORNER...** ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

4.1.2. Matter Waves Unity and Complementarity of Linear, Angular and Fluid Motions

As a very natural introduction into matter-waves rotation-related conceptualization it is necessary first to see two-body, binary systems analysis, already elaborated in the second chapter (see equations from (2.4-11) to (2.4-18) and "2.3.2. Macro-Cosmological Matter-Waves and Gravitation"; -equations from (2.11.11) to (2.11.23)). The position of the author of this book (formulated on a conceptual and common-sense level) is that linear particle motion (especially when we analyze micro and elementary particles) should also have (or create) certain elements of particle rotation, spinning, and associated torsion field components (followed by certain kind of vortices and turbulence), coupled with similar electromagnetic fields manifestations. Such torsion filed components could be of different origins and manifested in different ways, including spinning of participants. Some of the possibilities that could support such a concept will be addressed here. Since certain (at present, mathematical) analogy between electric and gravitation fields is obvious, as well as the analogy between magnetic field and "field of mass rotation" (see T.1.6, T.1.8, T.2.2, **[3]** and **[4]**), this can open the way towards establishing a new, Maxwell-like General Theory of Gravitation (at least by suggesting initial brainstorming ideas, as follows). Presently known phenomenology of de Broglie's matter waves strongly confirms that any rectilinear motion of some micro-particle with momentum (quantity of motion) can be "dualistically" presented by a certain wave, the wavelength of which is directly dependent on its linear momentum. The nature of waving (or oscillations in general) is usually connected to some background rotation that is associated with a wave source, but such initial rotation is not always and easily recognizable concerning Matter Waves. Indirectly, we could conclude that any rectilinear motion is likely to be accompanied by a "field of effective mass and electromagnetic energy rotation, including spinning". In some cases, we should be able to see visible elements of mechanical mass rotation (since de Broglie matter waves are oscillations linked to linear motion). Here, we will explain the origins of such hidden and associated spinning. This could mean that some associated form of field and distributed mass rotation follows any particle in linear motion (that cannot always be quickly and clearly detectable, but it certainly exists on a certain phenomenological, or space-time dependent energy-distribution level. This is often not well understood, because we know that de Broglie waves indirectly and eventually produce measurable and mathematically predictable experimental consequences. We could also imagine that our universe has more dimensions than we are presently able to detect, and that mentioned associated waving has some (or all) of its components linked to such multidimensional universe (but this would not be the best theory-saving concept at this time).

At least, we know that all subatomic microparticles and quasi-particles (in any motions) have their very real (measurable) spin and orbital moments characteristics linked to associated electromagnetic moments and dipoles and behave as de Broglie matter waves. Quantum Theory usually avoids addressing real particles rotation or spinning using the terminology such as associated intrinsic spin and orbital moment's attributes, probability guided electron clouds, energy states, statistical distributions, etc.). We also know that rotation and spinning are very much typical in motions of planets, solar systems, galaxies, and other astronomic entities, meaning that rotation (on all scales and

levels) cannot be just a hazard or random play of Nature. Of course, our Universe has plenty of examples of chaotic, non-circular motions, since astronomical objects are mutually impacting, scattering, and exploding, creating random motions. Only relatively quiet, stationary, stabilized, and periodic motions tend to create orbital (circular and elliptic) motions, usually in the form of planetary and solar systems. Matter waves manifestations should be directly detectable in fluid motions, or particle motions in the specific fluid in the form of vortices, turbulences and spinning effects, often coupled with similar electromagnetic complexity. On a macro scale, we can safely hypothesize that frequency of such matter waves (with spinning properties) is explicitly defining locally dominant time clock, or time train (linked to a dominant center of mass). For instance, our dominant center of mass system is linked to our planet Earth, and our local (and, for us most relevant) time scale is related to the rotation of our planet about our Sun.

From modern Quantum Mechanics, de Broglie waves are effectively treated as probability or "possibility" waves (conceptually and empirically not meaning too much). We also know that this is just one of today's most successful mathematical modeling of the micro world (at least officially publicized by its followers like being the most successful). In this book, it will be shown that de Broglie matter waves should have much more general, empirically tangible, and deterministic nature, causally linked to the fact that all linear particles motions are, or effectively segments of larger-scale orbital motions (see [36], Anthony D. Osborne, & N. Vivian Pope). ***Matter waves are some extension (or energy-momentum, wave-energy exchanges, balancing and compensation) of initial orbital and spinning moments of interaction participants.***

In almost all known situations when de Broglie wave phenomenology has been considered being a relevant event (directly or indirectly measured or detected), we can find the interactions of two bodies, particles/quasi-particles in a mutually relative motion, where one (test) moving particle or its energy mass-equivalent, $m_1 = m$, *is significantly smaller than the other particle,* $m_2 = M >> m$. We could also say that in a laboratory coordinate system $m_1 = m$ *has velocity* v_1 *and* $m_2 = M$ *has velocity* v_2, *where* $v_1 >>> v_2$. A "bigger particle", **M**, is usually a scattering target, a diffraction plate, atom nucleus, instrument in a specific laboratory, etc., (to be closer to many commonly known experimental situations regarding de Broglie matter waves).

Let us now analyze the same situation in its center of mass coordinate system. It can be shown that we can always (dynamically and mathematically) transform rectilinear motion of the test particle $m_1 = m$ to a kind of equivalent, a rotation-similar motion of a reduced mass, $\mu = m_r = mM/(m + M) \approx m$ around its center of gravity (that has a significantly more significant mass, $m_c = m + M \approx M >> m_r \approx m$). The rotating circle-radius of $\mathbf{m_r}$ around $\mathbf{m_c}$, will be equal to the real distance between interacting particles, $\mathbf{r_{12}}$. As a general case, $\mathbf{r_{12}}$ would have a space-time evolving value, until eventually stabilizing on specifically closed orbit, respecting Kepler, and Newton-Coulomb laws, including possible electromagnetic interactions and couplings (when and if this is applicable, and when optimal energy packing rules based on standing-waves relations can be satisfied; -see chapter 10). The natural and spontaneous tendency of a small mass $\mathbf{m_r}$ to develop its motion towards certain spiral or eventually circular motion, around bigger mass $\mathbf{m_c}$, (especially indicative in their center of mass reference system), should be familiar to an analogy and unity of Newton and Kepler's laws of gravitation. This could be the consequence of global orbital and spin momentum conservation. Here is particularly useful to see the brainstorming example in the second chapter with expressions (2.4-11) - (2.4-18), which can be considered as a natural and straightforward introduction to matter waves understanding. To support the same idea better, we can notice that Newton attractive force, between two masses, is equally valid, both in Laboratory and in Center of Mass System. This is leading to equal force expressions (in cases of non-relativistic motions), for instance, $F_{12} = G\frac{m_1 m_2}{r_{12}^2} = G\frac{m_c m_r}{r_{12}^2} \Leftrightarrow m_1 m_2 = m_c m_r = mM$. Kepler laws are also the consequence of Newton laws (regarding gravitation) and vice versa, and Kepler laws are describing circular and elliptic (orbital or rotational) motions. Here is a part of the understanding why there should exist a natural tendency of a small particle (which could also be some planet) to rotate around big particle (which could be its local Sun). Tendency towards creating stable circular, orbital and spinning motions is well known in micro-world of atoms and elementary particles where electromagnetic forces are dominant (especially concerning new atom and elementary particles modeling from Bergman, Lucas, and others; - see [16] to [25]). Here (in relation to Gravitation) we are specifically addressing electrically (and magnetically) neutral masses, but in cases if specific free or dipole-like electromagnetic charges are present (in two-body interactions), the same rotation-related tendency should be even more present (supported by direct and striking analogies between Newton and Coulomb's laws; -see the second chapter: equations (2.4-7) to (2.4-10) and 2.3.3. Macro-Cosmological Matter-Waves and Gravitation, around equations (2.11.13-1) - (2.11.13-5)). Based on analogies "**Photon-Moving-Particle**" (see "4.1.1.1. Photons and Particle-Wave Dualism", and T.4.0, at the beginning of this chapter), we could conclude that particles in linear motion should have associated helical or spinning matter-waves components. It is also interesting to see ideas about intrinsic nature of all motions to be globally presentable as combinations of rotating and spinning motions in: [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces".

Until present (respecting all valid approximations, and mentioned analogies), we can express the balance of total kinetic energy of two particles (or two mutually interacting bodies) in question as,

$$E_{k1} + E_{k2} = E_{km} + E_{kM} = E_{kc} + E_{kr} = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v^2, E_{kr} = \frac{1}{2}m_r v_r^2 \cong \frac{1}{2}mv^2 = E_{k1} \cong \frac{1}{2}J\omega_m^2,$$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_r v_r^2 \cong \frac{1}{2}Mv_c^2 + \frac{1}{2}mv^2 \left\{ = \frac{1}{2}Mv_c^2 + \frac{1}{2}J\omega_m^2 \right\},$$

or (analogically and by extrapolation concluding, "velocity-upwards", based on always applicable approximations when $v_i << c$) it should also be valid (for higher and relativistic velocities),

$$\left\{\begin{array}{l}\left(E_{ki}=\frac{1}{2}m_i v_i^2=\frac{1}{2}p_i v_i\right)_{v\uparrow}\cong\frac{\gamma_i m_i v_i^2}{1+\sqrt{1-v_i^2/c^2}}=\frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}},\, p_i=\gamma_i m_i v_i\\ \left(E_{kr}=\frac{1}{2}m_r v_r^2\right)_{v\uparrow}\cong\frac{p_r v_r}{1+\sqrt{1-v_c^2/c^2}}=\frac{m_r v_r^2}{1+\sqrt{1-v_c^2/c^2}},\, p_r=m_r v_r,\, \gamma_r=1\end{array}\right\}\Rightarrow$$

$$\Rightarrow\frac{\gamma_1 m v_1^2}{1+\sqrt{1-v_1^2/c^2}}+\frac{\gamma_2 M v_2^2}{1+\sqrt{1-v_2^2/c^2}}\cong\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}+E_{kr}\left\{\cong\frac{\gamma_c M v_c^2}{1+\sqrt{1-v_c^2/c^2}}+\frac{J\omega_m^2}{1+\sqrt{1-v_c^2/c^2}}\right\}.$$

Another analogical and equally possible strategy would be to account all relevant, total energies, for instance,

$$\left(mc^2+\frac{\gamma_1 m v_1^2}{1+\sqrt{1-v_1^2/c^2}}\right)+\left(Mc^2+\frac{\gamma_2 M v_2^2}{1+\sqrt{1-v_2^2/c^2}}\right)=\left(m_c c^2+\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}\right)+\left(m_r^*\cdot c^2+E_{kr}\right)\Leftrightarrow$$

$$\Leftrightarrow\gamma_1 mc^2+\gamma_2 Mc^2=\gamma_c m_c c^2+\left(m_r^*\cdot c^2+E_{kr}\right)\Rightarrow m_r^*=m_{or}=\gamma_1 m+\gamma_2 M-\gamma_c m_c-\frac{E_{kr}}{c^2}\cong 0,$$

$$E_{kr}=\frac{m_r v_r^2}{1+\sqrt{1-v_c^2/c^2}}=\frac{p_r v_r}{1+\sqrt{1-v_c^2/c^2}},\ \gamma_i=\frac{1}{\sqrt{1-v_i^2/c^2}},\ p_i=\gamma_i m_i v_i, p_r=m_r v_r, \gamma_r=1.$$

Obviously that here we could also speculate with sporadic existence of certain initial-velocities dependent, distributed and *"**hidden rest or reduced mass**"* $m_{ro}=m_r^*\neq 0$, which in some way could have specific internally balanced dynamics, but macroscopically it has resulting zero linear and zero angular moments, because $\vec{p}_m+\vec{p}_M=\vec{p}_1+\vec{p}_2=\gamma_1 m\vec{v}_1+\gamma_2 M\vec{v}_2=\vec{p}_c=\gamma_c m_c\vec{v}_c=\gamma_c(m+M)\vec{v}_c$.

This is only possible if spatially distributed linear and angular moments belonging to m_r are reacting as couples of mutually opposite and equal-intensity rotating vectors (this way generating certain self-balanced torque-couple, and performing kind of self-compensated rotation around m_c, with mutually opposite (and mutually canceling) spin moments, directly related to creation of de Broglie matter waves. Practically, a particle m_r is adequately represented by two matter waves propagating in mutually opposite ways. We know that the general solution of Classical Wave equations is always two wave functions propagating in mutually opposite directions. *Here we could also think about finding possible analogical connections of rotating and spinning m_r with Falaco Solitons; -see [15] and [40]).*

Here we are intentionally introducing somewhat hypothetical, dynamically equivalent, or possible and effective rotational or orbital motion, causally related to the kinetic energy of the reduced mass m_r. ***Such mass has only its kinetic energy (since its rest mass does not exist), meaning that this is something like a pure wave, or like a photon.*** For better conceptual understanding, see more in the second chapter, around equations (2.11.13-1) - (2.11.13-5)) to describe de Broglie matter waves, where specific frequency and wavelength should start to be relevant associated parameters (see later (4.3) for more details). Of course, we know that m_r and m_c are artificial and invisible (but effectively or mathematically existing) masses, being mathematical products related to Center of Mass reference system. We always know mathematically where such masses are effectively placed, we know their active energies, moments and velocities, and the main idea here is to show that such artificial masses configuration is the generator or source of matter waves (being extraordinarily significant and noticeable when mentioned conceptualization and approximations are applicable).

In our laboratory system, we can see only initial masses m_1 and m_2, but we also know (mathematically) the exact positions where m_r and m_c should effectively exist. In such zones of spatial and temporal (active) presence of m_r and m_c we could expect real and measurable existence of particular fields, forces, and energy-momentum (phantom) matter waves states that could have an influence on motions of nearby passing objects and waves (since here, both linear and angular or spinning moments should be conserved).

It will be shown that kinetic energy belonging to m_r is the energy of the relevant (associated) matter wave (see later, at the end of this chapter "4.1.4. Matter Waves and orbital motions", where the same rotation related concept is becoming clearer and more comfortable to accept). If initial two-body situation participants already have certain angular and orbital moments before interaction (presenting some orbital motion in a larger scale), when they start creating two-body coupled pair, whatever created that way (during and after interaction), would also have certain angular, orbital and/or spinning moment, to satisfy global angular moments and energy conservation laws. Such orbital and spinning moments are elements of de Broglie matter waves.

Practically, we are gradually elaborating that ***matter wave nucleus or source is created from the kinetic energy belonging to a reduced mass*** (in any two-body-problem situation when one mass is much smaller than the other mass), what we should be able to present as,

$$\left(E_{kr} = \frac{p_r v_r}{1+\sqrt{1-v_c^2/c^2}} = \tilde{E} = hf\right) \Rightarrow \begin{pmatrix} dE_{kr} = v_r dp_r = d\tilde{E} = hdf \\ v_1 dp_1 + v_2 dp_2 = v_r dp_r + v_c dp_c \end{pmatrix}$$

$$\left(u_r = u = \lambda_r f_r = \lambda f\,,\ \lambda = \frac{h}{p}\right) \Rightarrow du = fd\lambda + \lambda df = fd\lambda + \lambda\frac{v_r dp_r}{h} = fd\lambda + v_r\frac{dp_r}{p_r} = u\frac{d\lambda}{\lambda} + v_r\frac{dp_r}{p_r} \Rightarrow$$

$$\frac{du}{u} = \frac{d\lambda}{\lambda} + \frac{v_r}{u_r}\frac{dp_r}{p_r} = \frac{d\lambda}{\lambda} + \frac{v}{u}\frac{dp_r}{p_r} = \frac{d\lambda}{\lambda} + \left(\frac{v}{u}\right)\frac{d\left(\frac{h}{\lambda}\right)}{\left(\frac{h}{\lambda}\right)} = \frac{d\lambda}{\lambda} - \left(\frac{v}{u}\right)\frac{d\lambda}{\lambda} = \left(1-\frac{v}{u}\right)\frac{d\lambda}{\lambda},$$

$$\left((u,v) << c,\ v = 2u \Rightarrow \frac{du}{u} \cong \frac{d\lambda}{\lambda} + 2\frac{dp_r}{p_r} = \frac{dp_r}{p_r}\right), \left((u,v) \approx c \Rightarrow \frac{du}{u} \cong \frac{d\lambda}{\lambda} + \frac{dp_r}{p_r} = 0\right)$$

Using simplified terms, here we are hypothesizing that within any of possible "Two-Body Relations" mass m_r has a natural tendency to enter in a stable, inertial, and orbital motion around m_c. From the chapter 2., we already know that natural, stable, non-forced orbital motions are inertial motions, potentially hosting stable, standing matter waves; -see familiar elaborations around equations (2.11.13-1) - (2.11.13-9) from the chapter 2). Here we neglected possibilities that Two-Body interacting participants could initially have angular, mechanical, and electromagnetic, spin and dipole moments, what should be the case facilitating creation (and understanding) of matter waves.

Also, we know that all stable, uniform, stationary and linear (or inertial) motions could be treated as cases of circular or orbital motions with arbitrary large radiuses of rotation, where motions in a central force field, also belong. Consequently, all linear and stable or inertial motions are on some way orbital motions (implicitly meaning that specific center of rotation should exist). We also know that all stable, inertial, and linear motions (and motions in a central force field) can be very much spatially stabilized when being combined with gyroscopic stabilizers, or gyroscopic, self-spinning effects.

Briefly summarizing, our Universe is holistically (and locally or microscopically) rotating and spinning on many ways (what presents accelerated motions). This is producing effects of centrifugal mass separations and oriented polarizations of associated electric and magnetic dipoles, being spatially organized, that forces between such dipoles are of Coulomb (or Newton) attractive types (since masses of electrons and protons are enormously different, and atoms in accelerated motions are spatially deformable).

All of that is introducing the idea or hypothetic concept that all stable, inertial, linear, and orbital motions always have associated self-spinning moments, or kind of matter-waves, this way creating helix or spiral shaped matter-waves, or spatial-temporal structure with periodic and repetition-related (spinning) elements. From such imaginative, stabilized standing matter waves' concept, we will be able to deduce ideas about matter-wave wavelength, energy, and resonant quantization. The most convenient mathematical modeling for such matter waves is already known in the form of Analytic Signals (see more in chapter 4.0).

Let us now try to visualize de Broglie's matter waves as an associated rotating field manifestation (that has circular and spiral or spinning wave shape around the particle path) present in the space around mowing particle $m_1 \cong m_r \cong m$, which has the speed $\vec{v}_r = \vec{v}_1 - \vec{v}_2 \cong \vec{v}_1 = \vec{v}$. De Broglie wavelength could be visualized as certain (periodically repetitive) length measured along that spiral or helix line, and as a periodical interval between any of neighboring quasi-circles (see illustrations on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5 and equations under (4.3) and later).

The present concept found in Quantum Theory is that de Broglie matter waves are only indirectly detectable, and there is no explicit statement about the nature of such waves. Matter waves (in a micro world) are generally related to relative motions, impacts, diffractions and different interactions between electrons, protons, neutrons, atoms molecules, and other subatomic and micro-particles. Most of the mentioned particles also have measurable spin and orbital moment properties (but nobody is seriously considering that such spinning properties could have any relation to de Broglie matter waves). Such situation (about spin and orbital moment properties) is indicative, supporting the hypothesis that some associated form of spinning field (like the helix path on Fig.4.1, Fig.4.1.2, Fig.4.1.3, and Fig.4.1.4) should naturally accompany every linear motion (much in the same way as magnetic fields accompany electrical currents and fields). We know that in many cases of matter forms charge-to-mass and gyro-magnetic ratios are or should be constant numbers (see chapter 2, equations (2.4-6) – (2.4-10)). Consequently, we could be very much sure that de Broglie matter waves are closely related to some form of electromagnetic fields and waves, of course, coupled with associated inertial, spinning, and orbiting effects. As we know, photons also have spin and helical wave properties and shapes.

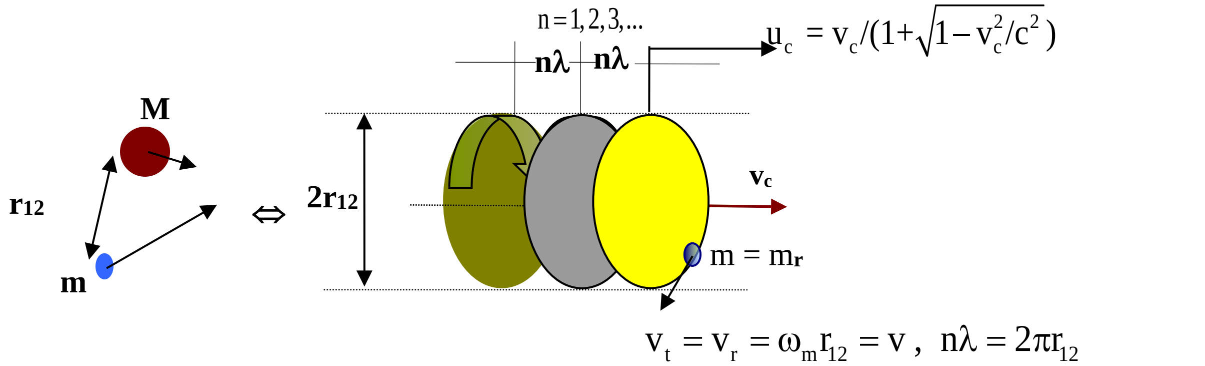


Fig.4.1. The two-body problem presented in the equivalent center of mass system

Let us now analyze in details the above-introduced common sense concept of de Broglie waves (as presented with Fig.4.1 and based on equations (4.1) and (4.2)), by analyzing the dynamically equivalent quasi-rotation of a small particle ($\mu = m_r = mM/(m+M) \approx m$) around a much bigger particle (**M**), or around its center of mass ($\mathbf{m_c} = m + M \approx M$). ***We will effectively analyze the kind of elastic impact situation between two mutually interacting masses with evolving elements of angular motions and rotation (appearing in a suitable center-of-mass reference system). In the same transitory process, we will also get certain spatial (or micro-volumetric) electromagnetic charges and dipoles polarization where Coulomb forces could be relevant.*** In the Laboratory System of coordinates, practically only mass **m** with velocity **v** moves towards mass **M**, and that mass **M** is almost standstill in the same system. The results of the same analysis will not change if we consider that a Laboratory System in question is fixed to the big mass **M**. To comply with de Broglie matter-waves hypothesis (or to rediscover expression for de Broglie wavelength), we should be able to show that de Broglie matter-wave wavelength (in the Laboratory System) originates from the effective or dynamically equivalent particle rotation around its center of mass (in its Center of mass system). This agrees with, $\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{2\pi r_{12}}{n}$, $n = 1, 2, 3, ...$. Here we (hypothetically) say that the small particle $\mu = \mathbf{m_r} \approx \mathbf{m}$ effectively rotates in its Center of Mass System (around mass **M**) because from the analysis of the two-body problem we know that the total, (internal) angular momentum of such system exists, and can be expressed as: $\mathbf{L} = \mathbf{L}_m = \mathbf{J}_m \omega_m$

$= m_r r_{12} \times v_{12} \cong m_r r_{12} \times v \cong m r_{12} \times v$. Now is possible to find the value that should be equivalent to the particle angular velocity $\omega = \omega_m = v_{12} / r_{12} \cong v / r$ (valid for the Center of Mass System). Since the small "rotating particle", $\mathbf{m}$ at the same time makes:

a) **Linear motion in its Laboratory System,**
b) **As well as a kind of (at least one revolution) circular path motion in its Center of Mass System** (having in both situations the same linear speed equal $\mathbf{v}$, because of valid approximations and initial conditions: $m_r = mM/(m+M) \approx m$, $v_1 >>> v_2$, and because mass $\mathbf{M}$ is considered almost standstill),

we could express the particle kinetic energy in two usual ways for linear and rotational motions. For instance, in Classical Mechanics, the kinetic energy of linear particle motion is given by $E_k = \frac{1}{2} mv^2$, and if a particle is rotating, we have an analog kinetic energy expression $E_k = \frac{1}{2} \mathbf{J} \omega_m^2$, where ω_m is the circular, "mechanical" frequency (of rotation), and $\mathbf{J}$ is particle moment of inertia. Since in this example, the same particle is in linear motion and a kind of dynamically equivalent rotation (depending on the point of view), and because of valid approximations, which make particle kinetic energy in a Laboratory and Center of mass system quantitatively (almost) equal, we will have,

$E_k = \frac{1}{2} mv^2 = \frac{1}{2} pv = \frac{1}{2} \mathbf{J} \omega_m^2 = \frac{1}{2} \mathbf{L} \omega_m$, $\Rightarrow mv^2 = \mathbf{J} \omega_m^2 \Leftrightarrow \mathbf{J} = \frac{mv^2}{\omega_m^2} = \frac{m(\omega_m r_{12})^2}{\omega_m^2} = m r_{12}^2$ (see also similar elaborations around equations (2.11.13-1) - (2.11.13-9) in the chapter 2). In a differential form relationship between the linear and rotational aspect of the same motion should be $dE_k = vdp = \omega_m d\mathbf{L} = hdf = c^2 d(\gamma m)$. *Also, because of mentioned approximations and considering only cases when r_{12} is significantly larger than any other space dimension of the rotating particle in question, we have seen that particle's important moment of inertia is $\mathbf{J} = m r_{12}^2$ (which is the result also known for any single rotating particle)*. ***We can also get the same result if the rotating particle is (mathematically) replaced by certain closed-space distributed mass, the thin-walls rotating torus of an equivalent wave energy formation.*** *All of that is confirming* that linear motion of the certain particle, relative to another particle could be effectively presented as rotation around their common center of mass. The other message from here elaborated concept is that ***matter waves are consequence or products of interactions between (at least two) masses. If interacting masses are mutually surrounded (and communicating) with electromagnetic fields, created matter waves will also have an electromagnetic nature. If there is another field around mentioned two or multi-body interacting states, we will have matter waves of other nature.*** Here, we still assume that test particle is not spinning (it is only making rotational or orbital motion about other, much bigger particle). The same kinetic energy equivalency between linear and rotational nature of a test particle motion in question can be (analogically) expressed or extended as:

$$E_k = E_{km} = \tilde{E} = hf = pu = \begin{Bmatrix} \frac{1}{2}mv^2 \\ \frac{1}{2}J\omega_m^2 \end{Bmatrix}_{v<<c} \Leftrightarrow \begin{Bmatrix} \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{J\omega_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{L\omega_m}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} E_{km} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{L\omega_m}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{J\omega_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = (\frac{J4\pi^2 f_m^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{J4\pi^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{v^2}{(2\pi r_{12})^2} \\ = \frac{Jv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{1}{(r_{12})^2} = \frac{\gamma m r_{12}^2 v^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{1}{(r_{12})^2} = \frac{\gamma m v^2}{1+\sqrt{1-\frac{v^2}{c^2}}}) = (\frac{2\pi L}{1+\sqrt{1-\frac{v^2}{c^2}}}) \cdot f_m = (\frac{2\pi L}{n}) \cdot f \Rightarrow \\ \Rightarrow \frac{2\pi L}{n} = h \Leftrightarrow L = L_m = n\frac{h}{2\pi} = \frac{E_{km}}{\omega_m}\left(1+\sqrt{1-\frac{v^2}{c^2}}\right) \end{Bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{Bmatrix} J = \gamma m r_{12}^2 = J_m,\ v = \omega_m r_{12},\ \gamma = (1-v^2/c^2)^{-0.5}, \\ 2\pi r_{12} = n\lambda = n\frac{h}{p} = \frac{v}{f_m} = n\frac{u}{f},\ n = 1,2,3,...,\ p = \gamma mv = h\frac{k}{2\pi} = \hbar k,\ k = \frac{2\pi}{\lambda},\ \hbar = \frac{h}{2\pi}, \\ \tilde{E} = hf = \hbar\omega = h\frac{\omega}{2\pi} = pu = \gamma mvu = E_k,\ u = \lambda f = v/(1+\sqrt{1-v^2/c^2}) = \frac{\omega}{k}, \\ dE_k = vdp = \omega_m dL = hdf = c^2 d(\gamma m). \end{Bmatrix}, \quad (4.3)$$

$$f_m = \omega_m / 2\pi = \frac{f}{n}(1+\sqrt{1-\frac{v^2}{c^2}})\ (=)\ \text{frequency of mechanical rotation},\ f_m \neq f,$$

$$f = \omega/2\pi = u/\lambda = nf_m/(1+\sqrt{1-v^2/c^2})\ (=)\ \text{de Broglie} - \text{matter} - \text{wave frequency},$$

$$1 \leq \left[\frac{v}{u} = \frac{pv}{\tilde{E}} = \frac{n \cdot f_m}{f} = 1+\sqrt{1-\frac{v^2}{c^2}}\right] \leq 2,\ n = 1,2,3,...\ ,$$

$$\frac{1}{2} \leq \left[\frac{u}{v} = 1+\frac{\lambda}{v}\frac{du}{d\lambda} = 1+(\frac{h}{pv})\cdot(\frac{du}{d\lambda}) = \frac{hf}{pv} = \frac{\hbar\omega}{pv}\right] \leq 1 \Rightarrow$$

In cases when test particle is making orbital motion and spinning (in the same time), where E_{km} is kinetic, orbital or rotational, mechanical particle energy (like energy of linear motion), E_{ks} is the spinning kinetic particle energy, and S is the test particle spin moment, analogically concluding (as in (4.3)), we will have,

$$\left\{\begin{array}{l} E_k = E_{k(m+s)} = \tilde{E} = h\cdot f = \dfrac{(\vec{L}+\vec{S})\vec{\omega}_m}{1+\sqrt{1-\left(\dfrac{v^*}{c}\right)^2}} = (\dfrac{2\pi\left|\vec{L}+\vec{S}\right|\cdot\cos\varphi}{1+\sqrt{1-\left(\dfrac{v^*}{c}\right)^2}})\cdot f_m = (\dfrac{2\pi\left|\vec{L}+\vec{S}\right|}{n})\cdot f = \\ = \dfrac{\vec{p}^*\vec{v}^*}{1+\sqrt{1-\left(\dfrac{v^*}{c}\right)^2}} = hf^*,\ \vec{p}^* = \gamma(m+\Delta m)\vec{v}^*,\ \Delta m = \dfrac{E_{ks}}{c^2} = \dfrac{S\omega_s}{1+\sqrt{1-\left(\dfrac{v^*}{c}\right)^2}} \end{array}\right\} \Rightarrow \qquad (4.3)^*$$

$$\Rightarrow \frac{2\pi\left|\vec{L}+\vec{S}\right|}{n} = h \Leftrightarrow \left|\vec{L}+\vec{S}\right| = \mathbf{L}_{(m+s)} = n\frac{h}{2\pi},\ \lambda^* = \frac{h}{p^*},\ f = \frac{\tilde{E}}{h},\ u^* = \frac{\tilde{E}}{p^*} = \frac{v^*}{1+\sqrt{1-\left(\dfrac{v^*}{c}\right)^2}}.$$

Here is a big part of the explanation regarding the real (orbital and spinning motions related) origins of Planck's energy formula $\tilde{E} = hf$, which should be complemented with (4.0.50) - (4.0.53) from the chapter 4.0. Practically, mechanical particle spinning is changing its matter-wave frequency and wavelength (meaning instead of $\lambda = \frac{h}{p}$ we will have $\lambda^* = \frac{h}{p^*}$).

We could also dynamically present the motion of the big mass $\mathbf{M}$ as performing a kind of rotation (with a much smaller radius than r_{12}) around the same center of mass, in its Center of mass system, similarly as we did for the small mass $\mathbf{m}$. However, considering already accepted approximations this would not bring us any new conceptual or mathematical benefit in the case analyzed here.

In most of the analyses of two-body situations, we are merely neglecting (or omitting) that participants (m_1, m_2), or (m, M) may carry initial, orbital and spin moments, as well as be electromagnetically dipole-polarized, or carry certain amounts of free electrical charges. Considering such (new) elements in two-body problem analysis will directly generate conclusions about the necessity of matter waves rotation and spinning. ***Of course, if initial spinning (of moving particles participants) does not exist, we will still have (self-generated) matter waves spiral spinning. This way we justify matter-waves wavelength and frequency, with (at least) two mutually canceling spinning moments (propagating in opposite directions, very much associating on solutions of second order Classical and Matter Waves differential equation), since conservation-laws are always valid.***

See later (in this chapter): Fig.4.1.1, Fig.4.1.1a, Fig.4.1.2, Fig.4.1.3 and Fig.4.1.4, the equations (4.3-0), (4.3-0) -a,b,c,d,e,f,g,h…, (4.3-1) until (4.3-3), and equations from (5.4.1) until (5.4.10) from the Chapter 5, where similar and equivalent concept of matter waves is additionally elaborated from a little bit different perspective. Particle-Wave Duality of the matter is so important that it should be completely explained and demystified. As long we have only complicated, unclear, probabilistic, artificial, and non-tangible concepts (regardless how well mathematically operational), this cannot be a good explanation or picture about certain Physics related phenomenology (this could only be a sufficiently acceptable, or good modeling and fitting with flexible mathematical processing).

As we can see, from (4.3), combined with results from (4.2) and later from (4.3-1), we do not even need to have a real, visible (continuous and full-circle) rotation of two particles in the same plane (in the Laboratory System) to "generate" de Broglie matter-wave with specific wavelength and frequency. Any of two (somehow interacting) particles (**m** and **M**, where **m** could be a photon), in linear motion, can be presented as a sort of rotation of a mass $m_r = mM/(m+M)$ and mass $\mathbf{m_c} = \mathbf{m} + \mathbf{M}$ around their common center of mass. Effectively this kind of quasi-rotation is on some ways linked to associate de Broglie matter waves since it shows that the following fundamental relation could be valid: $2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = 2\pi\frac{v_i}{\omega_m}$ (what secures structural and orbital standing-waves stability and continuity of described motion). Of course, mentioned particle rotation, visible in the plane of the Center of mass (perpendicular to the velocity of the center of mass), can also be certain angular swing, without creating full circle in a Laboratory System, where kind of associated spinning field and wave motion around it will create matter waves in question. If **M** presents the big *mass in our Laboratory System*, and if $(\mathbf{M} >>> \mathbf{m}) \Rightarrow (\mathbf{m_r} \cong \mathbf{m})$, consequently, the motion of every small particle **m** in such system can be treated (approximately) as the motion of $\mathbf{m_r} \approx \mathbf{m}$ in the Center of mass system, satisfying relations given in (4.3). **This way, it becomes clear that the binding and surrounding (mutually interacting) fields in the space between m and M create de Broglie matter waves as a way of energy-momentum exchange and coupling between them**, also satisfying the following differential energy balance: $dE_k = vdp = \omega_m dL = hdf = c^2 d(\gamma m)$ (see the second chapter: Tables T.2.4, T.2.5 and T.2.6 and equations from (2.1) to (2.11.5)). Now it also becomes clear when and why de Broglie relation $\lambda = h/p$ is valid and applicable. We need to be careful in noticing a significant difference between mechanical rotating or revolving frequency of a rotating particle and spinning matter waves frequency that is a field related parameter (see (4.3) and (4.3-1)).

Now is a right place to mention an analogy between here-introduced concepts (of couplings and equivalency between linear and angular motions), as presented in Chapter 2. around equations (2.11-4), and on illustrations on Fig.4.1, Fig.4.1.2, Fig.4.1.3, Fig.4.1.4, Fig.4.1.5, and equations under (4.3) and later in this chapter, and prof. Eric's Laithwaite demonstrations of unusual and extraordinary spinning gyroscope effects (ref. [102]).

We should also underline that for the time internally interacting nature of structural elements of a particle is entirely neglected regarding its intrinsic orbital and linear moments (and regarding all other electric and magnetic properties, standing waves, spinning, and rotating states; -see equations (2.11.3), (2.11.4) and (2.11.5) from the second chapter).

De Broglie matter waves should belong to all other wave phenomena already known in Physics, including some of (hypothetical) fields proposed in the second chapter of this book by force laws (2.1) and (2.2). The challenging question that appears here is whether all matter waves (of different nature, regarding how we see and measure them) have their profound, hidden, or recognizable origins in the world of electromagnetism. **Since Maxwell equations (analogically) are the ones of hydrodynamics, fluid-flow type of**

equations, this should lead to the conclusion that theory dealing with electromagnetic phenomena is necessarily a part of classical mechanic's concepts. We are now starting to see and know, or at least have a clear concept regarding **where, when, why and how** de Broglie matter waves are produced (see: Fig.4.1.2, T.4.4 and equations (2.5.1), (4.18), (4.5-1) - (4.5-3), (4.3-0), (4.3-0)-a,b,c,d,e,f,g,h,i… dealing with unity of linear and rotational motions).

We can now find the quantitative meaning of de Broglie wavelength concerning center-of-mass axial (or linear) motion. Let us determine the shortest axial distance, Δx, between two successive quasi-circles (on the helix line) from the Fig. 4.1, as $\Delta S = v_c \cdot \Delta t \cong \frac{mv}{M} \cdot \frac{1}{f_m} \cong \frac{mv}{M} \cdot \frac{1}{f_m} \cong \frac{mv}{M} \cdot \frac{n\lambda}{v} = n\frac{m}{M}\lambda <<< \lambda,\ n = 1, 2, 3, ... M >>> m$. It is clear that de Broglie matter waves are related only to a (kind of) circular motion since for the distance ΔS in the axial direction (when particle m would make one full circle, during the time interval $\Delta t = 1/f_m$) we shall get the length that is much shorter than actual de Broglie wavelength ($n\frac{m}{M}\lambda <<< \lambda$).

The laboratory (or "kitchen"), where de Broglie matter waves are created is the situation described by "virtual objects" interactions in the Center of Mass System since Physics and our Universe consider such effective reference systems as principal, dominant and most important. The real (and initial) interaction participants (in the frames of the above-introduced concept, Fig.4.1) are masses $m_1 = m$ and $m_2 = M$, but these are our perceptual, sensorial, or visual, laboratory reference system parameters. *In the Center of Mass System we replace them with "virtual reaction participants" m_r and m_c, because only in the Center of Mass System we can associate rotational motion to such situation (m_r rotating around m_c, or more correct is to say that both of them are rotating around their common center of mass point). After introducing the concept of such "virtual or equivalent rotation", which is in fact mathematically and from the point of view of Conservation Laws fully equivalent to the real situation in the Laboratory System (with real interaction participants), we can answer the questions regarding what should be the frequency and wavelength associated to such rotation. Later, we find that de Broglie matter waves have the same frequency and wavelength as conceptualized by such mathematically equivalent rotation.* ***This way, a little bit indirectly, we can conclude that what counts dominantly, regarding the creation of matter waves, is a complex of conditions, forces, and fields between mutually approaching objects in their Center of Mass System (see more in the chapter 2. of this book).*** *Such potentially interacting objects are already creating mutual couplings and new interaction participants (long before being united or experiencing any impact and scattering), what is becoming mathematically explicable when Two-Body problem is analyzed in the Center of Mass System. The same situation is explained with much more mathematical details at the end of this chapter. Here is the proper place to consult Fig.4.1.3 and T.4.4 briefly and give some new options. We could (for instance) imagine our reality as we are the part of a multidimensional universe, and that there are many mutually phase-shifted universes. Of course, this is too hypothetical, and like a science fiction concept. We could also imagine that in mentioned two-body situations, we could have other exotic, virtual, "magic and rare reactions and interactions", what Quantum Mechanics covers with probabilities that certain energy-momentum states will suddenly appear and disappear...*

*Another, significant theoretical and imaginative, challenging support, when solving two and multi-body problems could be analogically borrowed from Electro and Magneto-Statics. We know that "***Method of mirror imaging of electric and magnetic charges, fields and currents***" got its non-doubtful legitimacy in solving many problems in Electromagnetics. Effectively and mathematically, looks like that certain real electromagnetic charge (or other electromagnetic entity) is virtual, immediately and synchronously (like in situations of permanently coupled entangled photons) interacting with its mirror image. Of course, this is the mathematical method, but results when solving real problems using such methodology are also real and fully correct. We could easily imagine that* ***every motional energy-momentum state or particle has such mirror imaging and entanglement connections with its surrounding environment.*** *Certainly, mirror surface (here) could also be something very unusual and arbitrarily shaped, but anyway, at least mathematically, we could organize useful strategies to handle such unusual images. If we find a way to optimize and implement such imaging strategy to two and multi-body interacting situations (and to atoms structure), we will significantly enrich our understanding of the number of Wave-Particle Duality and Matter Waves situations. For instance,* ***two-slit interference and diffraction experiments (with photons, electrons, and other elementary and micro particles), could be analogically treated as a mirror imaging, and kind of entanglement situation. In such cases, all mutually interacting objects are synchronously and permanently connected, even much before interference happens, while respecting the framework of "two-body concept" with laboratory and center of mass reference systems mathematics*** *(see similar proposals in [36], IMMEDIATE DISTANT ACTION AND CORRELATION IN MODERN PHYSICS). Of course, it will be necessary to find a way how to apply a mirror-imaging concept and to differentiate the entities susceptible to have practically useful mirror images. Probably, besides electromagnetic charges, all the matter states with spinning properties are also convenient objects for described mirror-imaging, mathematical strategies (based on action-reaction and induction effects, and moments' conservation laws).*

The situation related to the center of mass and laboratory systems, as elaborated here, is only approximately or qualitatively correct. We know that macro masses could be (macroscopically) electromagnetically neutral, but internally masses have molecules, atoms, electrons, protons, neutrons... and micro-spatial distribution of internal electric and magnetic fields, forces and dipoles are not globally and electromagnetically compensated, or neutralized in every point of mass distribution. We could always search where centers of electric and magnetic neutrality of internal electric and magnetic dipoles inside certain mass are (or also where centers of positive and negative electric charges or magnetic fluxes are). Such centers of electromagnetic neutrality are not exactly overlapping with the important center of mass, a center of inertia, or center of self-gravitation. Such situation is (naturally or additionally), creating specific torque and angular moments (between masses), potentially supporting the existence of global rotation within our Universe. The future theoretical advances regarding similar questions (related to macroscopically compensated masses and active centers of most relevant coordinate systems) will be to redefine universal, global, or effective center of masses, including centers of linear and angular momenta, and centers of electromagnetic neutrality. If masses, also, have non-compensated electromagnetic charges, fluxes, angular and spin moments macroscopically, the same situation is becoming more complex. The existence of mentioned, not-completely overlapping and different dipoles, *moments and mass centers is (most probably), very much related to gravitation. See similar ideas in [33], from Dr. Jovan Djuric.*

We could connect the total angular moment of the two-body configuration in the Laboratory System, with the angular moment of the same configuration found in the Center of mass system, as follows:
$\mathbf{L}_{Lab.} = \mathbf{L}_m + \mathbf{L}_c = \mathbf{J}_m \omega_m + \mathbf{J}_c \omega_c = \mathbf{L}_m + m_c \, r_c \times v_c$. This approach could lead to recognition of de Broglie matter wave frequency and wavelength, as we have found in the case of equations (4.3), where principal rotation was linked only to an angular movement of the reduced mass in its Center of mass system.

We already know that all elementary particles and photons have their intrinsic spin characteristics (or attributes), but we are often not able to notice the presence of a visible mechanical rotation there. In this book we have been saying that regardless of not being able to see mechanical or some other kind of rotation (in cases where spin is involved), something that is by externally measurable effects equivalent to rotation (or to spinning) should internally exist (belonging to the internal particle structure and associated electromagnetic phenomenology). Being more explicit, based on ideas presented in this book, we are already familiar to the concept that relatively stable particles (and quasiparticles) are nothing else than specific "packing or parking formats" of certain fields in the form of self-closed and rotating standing waves. Such internal matter wave rotation is externally measurable as particle spin attribute (meaning that at the same time an effective, and unusual rotation exists, but not externally visible). In other words, without spin attributes (or without matter wave rotation) we would not have stable particles, because there should be a cause that would bend and place certain wave in a self-closed circular area, and make it stable there, by creating standing waves on such self-closed circular area (see chapter 2, equations (2.11.3), (2.11.4) and (2.11.5). The same situation (regarding rotation and spin characteristics of elementary particles) is well explained (conceptually and quantitatively) by Ph. M. Kanarev, [44] C. Lucas, David L. Bergman and his colleagues, [16] - [22], "Common Sense Science". See also excellent modeling of the helical electron in [108], Oliver Consa. "g-factor and Helical Solenoidal Electron Model".

We also know that every electrically charged particle moving in magnetic field will follow a similar spiral (or helix) line as presented in Fig.4.1, caused by Lorentz force (in case of electrically charged particles we only have stronger interacting fields between such particles than between electrically neutral particles, and most of above-given mathematics and logic is again applicable). Dominant and stable elementary constituents of all atoms and other known particles in our universe are electrons and protons, (or electrically charged particles, also including their combinations in the form of neutrons, as well as positrons and antiprotons). We can safely say that dominant interaction environment responsible for the creation of de Broglie matter waves should have an electromagnetic origin, and that the most important coordinates-frame for modeling such interactions is the Center of mass system.

Above-described quasi-rotational movement and rotation-like field structure separates, and under certain conditions could become a self-sustaining object. For instance, like rotating the ring, or rotating toroidal, closed space form; -see more in [94], Classical

Mechanics; - chapter 14), which has certain rest mass and spin as its permanent characteristics, it should depend on kinetic energies and respective positions, paths, and forces between interaction participants. To be more explicit, it is worthwhile to underline that every motion of mutually approaching objects (particles, quasiparticles, particles, and waves, etc.), naturally creates additional energy-carrying, couples of rotating elements with associated orbital moments in the zone of their interaction, this way establishing specific conditions for creating new particles and/or waves or producing different interference and diffraction effects. If interacting objects already have specific spin moment characteristics, long before an interaction happens, the appearance of additional rotating elements in the interaction zone would be an even much more apparent consequence (see T.2.4, T.2.5 and T.2.6) and equations (2.11.3), (2.11.4), (2.11.5), and (2.11.13-1)-(2.11.13-5), where is clearly shown how matter waves in two-body problems are being created). After the necessary motional energy threshold (in an interaction) is reached, we expect the creation of electrons, positrons, photons and other particles and quasiparticles (which is the known experimental fact in particles' physics). Most probably the presence of radial or central attractive force components (between interacting elements) is somehow, conveniently balanced with specific centrifugal force components (when interacting objects are mutually close enough), this way of producing self-sustaining (non-collapsing or non-expanding) closed spinning structures, such as electrons, positrons, protons, etc. In such situations, usually a couple of mutually complementary and rotating fields are intrinsically involved, like cases of complementarities and coupling between electric and magnetic fields, and of course, every closed circular field structure, to be stable and self-sustaining, should automatically create standing wave structures: $2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = 2\pi\frac{v_i}{\omega_m}$*), known as de Broglie matter waves. Elementary particle models based on rotating ring or toroidal field structures are already well established and conveniently explained (conceptually and theoretically supported, like in [94], Classical Mechanics, from Tom W. B. Kibble and Frank H. Berkshire; - chapter 14). Mentioned models are mathematically tested, to produce exact results, previously known (in Quantum Mechanics) only after measurements, without being conceptually explained. For instance, see works of C. Lucas, David L. Bergman, and his colleagues [16] - [22], "Common Sense Science"). It should not be just a matter of hazard, and probability, in any sense, that everything we see, find, analyze, and measure in our universe rotates, like galaxies, stars, solar systems, planets, fluid motions, atoms, elementary particles, photons, etc. Even in cases when it seems that certain objects experience only linear motion (for instance rockets sent from the earth to outer space), linear motion is only an external, visible, and dominant manifestation, but internally, inside the moving solid mass structure everything rotates, keeping only the external 3-D object-frame and structural object-matrix stable.*

After understanding where the hidden place of rotation in rectilinear motion is, we can conceptually visualize what de Broglie waves phenomenology means (Fig.4.1, mathematically formulated by (4.2) and (4.3)), and we also leave the door open for introducing "Field/s of Rotation" or Torsion fields in the (new) Theory of Gravitation united with Faraday-Maxwell Electromagnetic Theory. The following table of extended analogies (based on de Broglie matter wavelength of linear particle motion), T.4.2, is sufficiently illustrative to support introduced ideas, at least, "in-average", and conceptually (regarding intrinsic coupling and periodical, standing waves packing between linear motion and spinning. Also, we could extend the same concept, analogically and hypothetically, to an electric and magnetic field; -See in chapter 5. for more, as T.5.3 and equations (5.1) – (5.4-1). More of familiar, supporting ideas we can find in the Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.

T.4.2. Wavelength analogies in different frameworks

Matter Wave Analogies	Linear Motion	Rotation	Electric Field	Magnetic Field
Characteristic Charge	**Linear Momentum p**	**Orbital Momentum** $L = pR$	**Electric Charge** $q_e = q$	**"Magnetic Charge"** $q_m = \Phi$
Matter Wave Periodicity	***Linear Path Periodicity*** $\lambda = \frac{h}{p}$ **(Linear motion Wavelength)**	***Angular Motion Periodicity*** $\theta = \frac{h}{L}$ **(Angular motion Wavelength)**	***"Electric Periodicity"*** $\lambda_e = \frac{h}{q_e} = q_m$	***"Magnetic Periodicity"*** $\lambda_m = \frac{h}{q_m} = q_e$
Standing Waves on a circular self-closed zone	$n\lambda = 2\pi R$, $p = n\frac{h}{2\pi} \cdot \frac{1}{R}$	$n\theta = 2\pi$, $L = n\frac{h}{2\pi}$	$\lambda_e \lambda_m = q_e q_m = h$	
	$\theta L = \lambda p = h$, $\theta = \frac{\lambda}{R} = \frac{2\pi}{n}$			

(Periodicity – here invented, unifying formulation, $q_m = \Phi$ *is not a free and independent magnetic charge)*

Simplifying the same situation, we can say that any rectilinear motion of a particle should always be accompanied by certain angular, rotating and oscillating (like helix vortices) field components (including belonging electromagnetic field components). In many cases, a certain level of real mass rotation could also appear (for instance: rotation and spinning of planets around the sun, rotation of galaxies, rotation and spinning of electrons and other elementary particles, etc.). Since a single, isolated, and very free particle cannot exist, practically we always have a two-particle system: a test particle, **m**, and the rest of its surrounding universe, **M** >> **m**. Even particles like the planet Earth are tiny comparing to our Sun, and practically, we can also say that in our universe, there is no pure linear and straight-line uniform motion, except in some more or less approximate and limited (laboratory or mathematical) conditions (see [4]).

We should not immediately conclude that the frequency of mechanical rotation, $\mathbf{f_m}$, of the particle **m** (meaning number of full particle revolutions per second) directly corresponds to associate de Broglie wave frequency, **f**, since any rotating particle, surrounded by specific fields, presents the source of matter waves, making wave-like perturbations (or wave groups) in its vicinity. When we come to waves propagation, we should not forget that every wave-group has its group and phase velocity, and that mathematical connection between group and phase velocity is given by the following non-linear, mathematical relations, found in (4.2), $v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} =$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$= \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = 2u / (1 + \frac{uv}{c^2})$. This is producing the clear difference between (a mechanical) frequency of the particle rotation f_m and frequency of its (associated, de Broglie) rotational field components, f, such as: $1 \leq n\frac{f_m}{f} \leq 2$, (see (4.3)).

From another point of view, we can say that every linear motion of the specific particle, is an approximate case of certain orbital rotation (with arbitrary long radius) and belongs to a more general case of *"quasi-elastic collision between that particle and its vicinity"*. In cases of elastic collisions, the rest mass of the system does not change, and conservation of kinetic energy is satisfied. In such cases, an "elastic collision" system could be considered as an interaction of the (relatively free) moving particle with the rest of a surrounding universe (where rotation of the particle along its path of movement becomes immanent and evident for an observer in the local Center of a mass system). Every moving particle, or common mass in a rotation, or just particle transient angular swings, should be synchronously coupled with corresponding inertial torsion-field swing, creating de Broglie matter waves around it.

We know (based on experiments in particles' physics) that all elementary particles, atoms, and molecules behave as dualistic particle-wave objects, and that all of them have intrinsic wave structure (and that all of them have intrinsic spin attributes). De Broglie waves and torsion fields should be a natural (external) extension of particles' intrinsic (internal) wave properties. When a change in particle motion is created, this will influence the internal (and elastic) field perturbation of the particle intrinsic field structure, and such wave perturbation will have important, externally manifesting consequences, with properties of spinning de Broglie waves (see also T. 5.3 to understand a more general nature of de Broglie Waves and elementary particles). The most logical conclusion is that de Broglie matter waves already (or always) exist, creating natural, internally rotating form of self-closed, standing wave particles structure, when particles are in states of relative rest, long before we can (externally) detect anything regarding ***PWD*** *phenomenology. Such effects are becoming externally (directly or indirectly) measurable in all situations when particles mutually interact and change their previous states of motion and energy (see [16] - [22]). In other words, certain "packing" or folding of de Broglie stationary (or standing) waves is everything that we have under the (internal) content and fundamental structure of stable particles. When such particles move, this internal wave content starts "unfolding", producing (radiating, or "leaking") de Broglie waves, externally (creating energy flow, coupling forces, and communicating channels between interacting particles). Classical Mechanic's analysis of two-body interactions (only partially given here) completely neglects internal matter-wave particle's structure and neglects the associated external matter wave manifestations (what we are trying to address in this book). Oversimplifying, we could say that stable particles in states of relative rest (with non-zero rest masses) present "parking, packing or folding domains" of their internal de Broglie matter waves, manifesting as resonating, rotating, and standing waves, spatial formations of electromagnetic energy. Such internally captured de Broglie waves get their "external promotions" in all cases of particle interactions (by unfolding internally captured matter waves).*

The author of this book supports and promotes the concept that complementary rotational or spinning motion energy-member of total motional particle energy is causally related to de Broglie matter waves. Saying the same differently: de Broglie or matter waves present the manifestation of an intrinsic nature and coupling between linear and rotational motional energy components including associated electromagnetic phenomenology. In other words, total motional or kinetic energy of a particle is equivalent to its matter wave energy (excluding the amount of energy locked inside of the particle rest mass). For instance, we know that moving particle, which has linear motion moment $\mathbf{p}$*, also has de Broglie wavelength* $\lambda = h/p$*, phase velocity* $u = \lambda f = \frac{E_k}{p}$*, group velocity* $v = u - \lambda \frac{du}{d\lambda} = \frac{dE_k}{dp} = -\lambda^2 \frac{df}{d\lambda}$ *and angular velocity* $\omega = 2\pi f = 2\pi \frac{E_k}{h}$ *, and complementary rotational particle motional energy components should also be related to the same (matter wave) parameters. We should only be cautious not to immediately identify matter waves frequency* $\omega = 2\pi f = 2\pi \frac{E_k}{h}$ *with a mechanical revolving frequency of a particle* $\omega = \mathbf{L}/\mathbf{J}_0 = \omega_m$*, (around its center of rotation). Such frequencies are mathematically connected with certain velocity dependent function, and generally not equal, because one of them is typical mechanical rotation (around the certain center) and the other is a spinning frequency around particle's path of propagation. Also, the specific functional relationship between the involved group and phase velocity is making this situation more complex. This will be explained later much better (see (4.3-0), (4.3-0) -a,b,c,d,e,f,g,h,i… and* ***PWDC*** *with equations (4.2) and (4.3)).*

The origins of mechanical rotation of astronomic size macro-objects are only an extension of rotation and torsion field properties of its micro-particle constituents, and well explained as strongly coupled to atomic scale magnetic moment perturbations caused by the attractive force of gravitation inside of big astronomic masses. For more information, see works of P. Savic and R. Kasanin, **[51],** published by Serbian Academy of Sciences, SANU, Department for Natural and Mathematical Sciences, in the period 1960-1980, and summarized in the book: “Od atoma do nebeskih tela”, author Pavle Savic, publisher “Radnicki univerzitet Radivoj Cirpanov”, Novi Sad, 1978, Yugoslavia - Serbia).

[♣ COMMENTS & FREE-THINKING CORNER:

The information and conceptual background that is explaining and defending the unity of linear motion, mechanical rotation and associated matter waves spinning (as presented on Fig.4.1., around equations (4.3)), should also be included in the equation that is connecting group and phase velocity.

*Let us imagine that specific particle is rotating (around its center of rotation **C**), having tangential velocity* $v = v_g = \omega_g R$*, where the radius of rotation is **R**, and* ω_g *is the mechanical, angular particle velocity (number of full, mechanical rotations per second around the center of rotation **C**). Here vectors* $\vec{v}_g, \vec{\omega}_g$ *are mutually orthogonal, and particle velocity is equal to its group velocity (*$v = v_g$*), if we associate wave group to such particle. Let us find all particle and matter-wave parameters for such a moving particle. Here, we will apply the following indexing (to make intuitive associations to mechanical particle motions and equivalent wave motions): **m** (=) mechanical motion, **p** (=) phase, **g** (=) group, **c** (=) center of rotation, **s** (=) self-rotation or spinning, **gc** (=) wave-group related to center **C**, …*

Practically, the same concept of a wave packet, which has its group and phase velocity, in cases of linear motions should be analogically extendable to the rotating wave packet that has the group and phase angular velocity ($v \text{ and } u$*), for instance,*

$$\left\{ \mathbf{v} = \mathbf{v}_{gc} = \mathbf{u}_c - \lambda \frac{\mathbf{du}_c}{\mathbf{d}\lambda} = -\lambda^2 \frac{\mathbf{df}}{\mathbf{d}\lambda} = \omega_{gc}\mathbf{R} = \frac{\mathbf{d}\omega_s}{\mathbf{dk}_s} (=) \textbf{wave group velocity} (=) \textbf{particle, tangential velocity} \right\} \Leftrightarrow$$

$$\left\{ \begin{array}{l} \omega_{gc} = \dfrac{\mathbf{v}_{gc}}{\mathbf{R}} = \dfrac{\mathbf{u}_c}{\mathbf{R}} - \dfrac{\lambda}{\mathbf{R}} \dfrac{\mathbf{du}_c}{\mathbf{d}\lambda} = -\dfrac{\lambda^2}{\mathbf{R}} \dfrac{\mathbf{df}}{\mathbf{d}\lambda} = \dfrac{\mathbf{1}}{\mathbf{R}} \dfrac{\mathbf{d}\omega_s}{\mathbf{dk}_s} = \mathbf{2}\pi\mathbf{f}_{gc} = \left(\omega_p - \lambda \dfrac{\mathbf{d}\omega_p}{\mathbf{d}\lambda} \right)_{\mathbf{R}=\mathbf{const.}}, \mathbf{u}_c = \lambda\mathbf{f} = \mathbf{u}, \ \omega_p = \mathbf{2}\pi\mathbf{f} = \dfrac{\mathbf{u}_c}{\mathbf{R}}, \omega_s = \mathbf{2}\pi\mathbf{f}_s \\ \mathbf{f}_{gc} = \dfrac{\omega_{gc}}{\mathbf{2}\pi} = -\dfrac{\lambda^2}{\mathbf{2}\pi\mathbf{R}} \dfrac{\mathbf{df}}{\mathbf{d}\lambda} = \dfrac{\mathbf{1}}{\mathbf{2}\pi\mathbf{R}} \dfrac{\mathbf{d}\omega_s}{\mathbf{dk}_s} = \dfrac{\mathbf{v}_{gc}}{\mathbf{2}\pi\mathbf{R}} (=) \textbf{frequency of mechanical particle rotation around center C.} \end{array} \right\} \Rightarrow$$

$$\mathbf{f}_{gc} = \mathbf{f} - \lambda \frac{\mathbf{df}}{\mathbf{d}\lambda} - \frac{\lambda\mathbf{f}}{\mathbf{R}} \frac{\mathbf{dR}}{\mathbf{d}\lambda} = -\frac{\lambda^2}{\mathbf{2}\pi\mathbf{R}} \frac{\mathbf{df}}{\mathbf{d}\lambda} = \frac{\mathbf{1}}{\mathbf{2}\pi\mathbf{R}} \frac{\mathbf{d}\omega_s}{\mathbf{dk}_s} = \frac{\mathbf{1}}{\mathbf{R}} \frac{\mathbf{df}_s}{\mathbf{dk}_s} = -\frac{\lambda_s^2}{\mathbf{2}\pi\mathbf{R}} \frac{\mathbf{df}_s}{\mathbf{d}\lambda_s} \Rightarrow$$

$$\mathbf{u} = \mathbf{u}_c = \lambda\mathbf{f} = \lambda_s\mathbf{f}_s = \frac{\omega_s}{\mathbf{k}_s} = \omega_p\mathbf{R} = \omega_{gc}\mathbf{R} = \mathbf{v} / (\mathbf{1} + \sqrt{\mathbf{1} - \frac{\mathbf{v}^2}{\mathbf{c}^2}}),$$

$$\mathbf{k}_s = \frac{\mathbf{2}\pi}{\lambda_s} = \frac{\mathbf{2}\pi}{\mathbf{H}} \mathbf{p} (=) \textbf{ wave vector, } \mathbf{H} = \textbf{const.},$$

$$\Rightarrow \mathbf{f} = \frac{\lambda\mathbf{f}}{\mathbf{R}} \frac{\mathbf{dR}}{\mathbf{d}\lambda} + \lambda(\mathbf{1} - \frac{\lambda}{\mathbf{2}\pi\mathbf{R}}) \frac{\mathbf{df}}{\mathbf{d}\lambda} = \left(\frac{\lambda\mathbf{f}}{\mathbf{R}} \frac{\mathbf{dR}}{\mathbf{d}\lambda} \right)_{\lambda = \mathbf{n}\lambda_s = \mathbf{2}\pi\mathbf{R}}, \ \tilde{\mathbf{E}} = \mathbf{Hf} (=) \textbf{wave-group energy},$$

$$\text{If } \{\lambda = 2\pi R = n\lambda_s\} \Rightarrow \omega_p = \frac{\omega_s}{n} = 2\pi f = \frac{2\pi f_s}{n},\ n = 1, 2, 3...,$$

$$\omega_p = 2\pi f = \frac{2\pi R}{\lambda}\frac{\omega_{gc}}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{2\pi R}{\lambda}\frac{\omega_{gc}}{1+\sqrt{1-\frac{\omega_{gc}^2}{\omega_c^2}}} = \left(\frac{\omega_{gc}}{1+\sqrt{1-\frac{v^2}{c^2}}}\right)_{\lambda = n\lambda_s = 2\pi R},$$

$$\frac{\omega_{gc}}{\omega_p} = \frac{\lambda}{2\pi R}\left(1+\sqrt{1-\frac{v^2}{c^2}}\right) = \frac{\lambda}{2\pi R}\left(1+\sqrt{1-\frac{\omega_{gc}^2}{\omega_c^2}}\right) = \frac{\lambda}{2\pi R}\frac{v}{u},$$

$$\vec{p} = \frac{\omega_s}{v}\vec{L}_s = \gamma m\vec{v},\ \vec{L}_s = \frac{v}{\omega_s}\vec{p} = \frac{v}{\omega_s}\gamma m\vec{v},$$

$$\begin{pmatrix} 0 \le v \le c \\ \lambda = 2\pi R = n\lambda_s \end{pmatrix} \Rightarrow 1 \le \frac{\omega_{gc}}{\omega_p} = \frac{f_{gc}}{f} = n\frac{f_{gc}}{f_s} = \frac{v}{u} \le 2$$

$$\begin{pmatrix} v << c \\ \lambda = 2\pi R = n\lambda_s \end{pmatrix} \Rightarrow v = 2u \Rightarrow \frac{\omega_{gc}}{\omega_p} = \frac{f_{gc}}{f} = n\frac{f_{gc}}{f_s} = 2$$

$$\begin{pmatrix} v \cong c \\ \lambda = 2\pi R = n\lambda_s \end{pmatrix} \Rightarrow (v = u) \cong c \Rightarrow \frac{\omega_{gc}}{\omega_p} = \frac{f_{gc}}{f} = n\frac{f_{gc}}{f_s} = 1.$$

*Matter waves associated with any particle in motion are practically defined by **PWDC** relations (equations (4.2) - (4.3)), and not equal to particle mechanical rotating parameters, and we should be cautious in making the difference between mechanical rotation (mechanical angular speed around specific center of rotation) and orbital frequency of associated, spinning (and helix) matter waves.*

Usually, such precise differentiation between mechanical and matter wave parameters has never been made, influencing that many ad hoc and exotic postulates and suspicious, or theory-correcting and supporting statements (regarding elementary particles and atom structure) have gained legitimacy in Orthodox Quantum Mechanics. For instance, situations where we find familiar conflicting situations are related to gyro-magnetic ratio, spin attributes, correspondence principle, orbital, and magnetic moments etc. In cases when radius related to the rotation is not constant, the example above should be appropriately modified (and accorded with Wilson-Sommerfeld quantizing rules: see (5.4.1)). It would be very fruitful to elaborate more profoundly the ideas mentioned here. ♣***].***

Based on the above-presented intrinsic nature of matter related to strong couplings between linear motion and rotation (including spinning), and coupling between electric and magnetic fields, we are now in the position to propose a significantly simplified conceptualization of the particle-wave duality, illustrated on Fig.4.1.1, applied on a moving particle (also familiar to the situation presented on the Fig.4.1). If we accept that some matter-wave field is rotating (or spinning) around a particle in a linear motion (to have a chance to visualize the process on a simplified way), all elements of Particle-Wave duality will mutually fit into the united and simple picture, as illustrated on the Fig.4.1.1. Particle and its wave-group envelope would have the same group velocity v, and helix tail, or spirally rotating field path (behind such energy state) would have phase velocity u. *Effectively, we will now conceptualize motional particle (or its mass) as an energy packing or formatting state that is realized by specific rotation or spinning (expressed by equivalency of kinetic energies of the associated liner and rotational*

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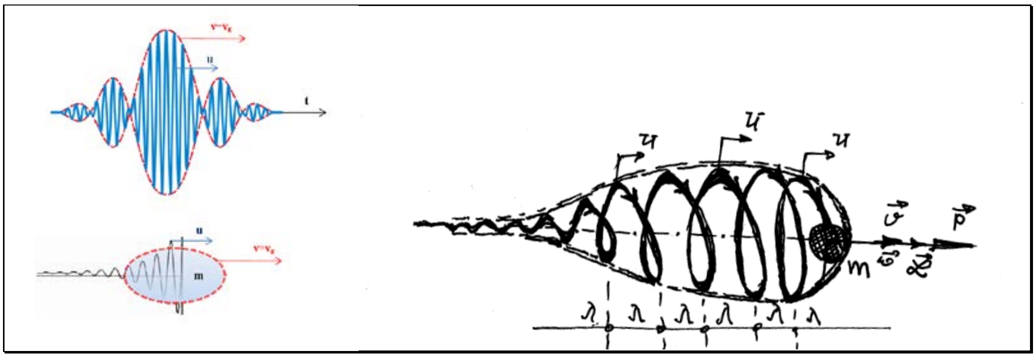
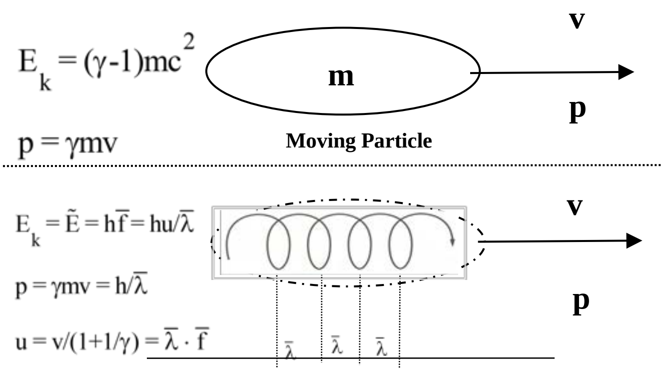
motions; -see (4.3-0)). This is also presenting the content and meaning of already introduced **PWDC** (**P**article-**W**ave-**D**uality-**C**ode).





Wave Group or Wave Packet, as an equivalent of moving the particle

Fig.4.1.1. Different ways of presenting a moving particle and its matter wave

The chain of conclusions, after such conceptualization (supported by many elaborations introduced earlier in the same chapter; -see (4.3)) follows,

$$\left\{\begin{array}{l} u = \dfrac{v}{1+\sqrt{1-v^2/c^2}} = \lambda f = \dfrac{h}{p} f \\ v = u - \lambda\dfrac{du}{d\lambda} = \dfrac{d\tilde{E}}{dp} = \dfrac{\tilde{E}}{p} - \dfrac{h}{p} d(\dfrac{\tilde{E}}{p})/d(\dfrac{h}{p}) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \text{Particle, or its matter-wave packet in linear motion} \\ E_k = (\gamma-1)mc^2 = \dfrac{pv}{1+\sqrt{1-v^2/c^2}} = hf = \tilde{E} \end{array}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} \text{Orbiting particle about its center of rotation} \\ v << c \Rightarrow E_k = \dfrac{mv^2}{2} = \dfrac{pv}{2} = \dfrac{I\omega_m^2}{2} = \dfrac{L_m\omega_m}{2} \end{array}\right\} \Rightarrow \{pv = L_m\omega_m\} \Leftrightarrow \left\{\vec{p} = \dfrac{\omega_m}{v}\vec{L}_m = \gamma m\vec{v}, \vec{L}_m = \dfrac{v}{\omega_m}\vec{p} = \dfrac{v}{\omega_m}\gamma m\vec{v}\right\} \Rightarrow \qquad (4.3\text{-}0)$$

$$\left\{\begin{array}{l} \text{Particle motional energy as: linear motion, orbiting motion, and matter-wave packet} \\ E_k = (\gamma-1)mc^2 = \dfrac{pv}{1+\sqrt{1-v^2/c^2}} = \dfrac{L_m\omega_m}{1+\sqrt{1-v^2/c^2}} = \dfrac{2\pi L_m f_m}{1+\sqrt{1-v^2/c^2}} = hf = hf_{mw} = \tilde{E} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\dfrac{h}{2\pi} \le L_m \le \dfrac{h}{\pi}, 0 \le v \le c\right\},$$

$$\omega_m = 2\pi f_m - \text{orbital, mechanical, angular velocity},\ f = f_{mw} - \text{matter-wave, spinning frequency.}$$

*Here, (in (4.3-0)), we consider that specific particle is revolving around its center of rotation, having angular rotating speed ω_m. Associated matter wave may have another angular or spinning speed $\omega_{mw} = 2\pi f_{mw}$, which is not equal to ω_m, and may have another, different nature (does not make simple rotating motion around particle's center of rotation). In this book, **we will conceptualize associated matter wave-***

packet as a motional-energy spinning, or self-spinning state (or certain spinning waves' formation) around the motional particle. *Since moving particle is kind of orbital motion, its matter-wave spinning-state, is effectively creating helicoidally shaped tail behind the particle, and we are now able to say what should be the wavelength, group, and phase velocity of such state (as in (4.3-0)). The same, spinning and helix matter-waves concept will be more precisely elaborated a bit later, around equations (4.3-0) -a,b,c…,h. What spins and oscillates in such situations, where externally, often we do not see oscillations, in cases when moving particle is in a certain fluid, should be related to Orbital and Spin, Angular Moments, and associated electromagnetic dipole moments of moving particles (followed by vortex and turbulence manifestations). Effectively, the internal structure of matter constituents (atoms) has significant electromagnetic nature, where internal electric charges in some ways rotate and spin on stationary orbits, creating mutually coupled electric dipoles, magnetic and orbital moments. In addition, we know that masses of an electron and proton are enormously different, what should stimulate creations of electric dipoles in cases of curvilinear, accelerated motions. Any external matter-waves field surrounding a particle macro-motion should also be coupled and synchronized with resulting internal spinning, orbital motions, and moments, being adequately described by de Broglie matter-waves concept.* ***If motional particles in question do not have initial spinning moments, helix matter waves spinning will anyway be self-generated, but because of angular moments conservation, we will get (at least) two of mutually cancelling spinning matter-wave states.***

The simplest, primary, and most essential existence of helically spinning matter waves should be detectable around electrons and all electrically charged particles in linear motions in the form of associated magnetic-field spinning (or electromagnetic waves spinning). This is still not adequately addressed or correctly experimentally verified situation in present electromagnetic theory (not predictable by present-days equations and laws of electromagnetic theory; -see on Internet somewhat similar concept regarding Henry Augustus Rowland effect of magnetic field of electrons around the rotating conductor, presented by Jean de Climont). Such simple matter waves are on some way principal sources of all other matter-waves manifestations (and, of course, this is closely related to matter-waves concepts being promoted in this book).

Another reality of matter waves and particle-wave duality (as promoted in this book) is that all of them are waves of matter (not at all being probability and other exotic, virtual and imaginary waves). Such waves are everywhere around us and should be especially visible or detectable in all elastically deformable fluids and plasma states of matter. Matter waves in question are, electromagnetic waves, acoustic waves, mechanical vibrations, waves in plasma states, different radiations, cosmic and particle beams…

An incredible success of particle-wave analogies (regarding their mixed, dual nature) between photons and particles, in connection to Minkowski Energy-Momentum 4-vectors, is indicating that all matter in our Universe could be composed of photons or being specific agglomeration and formatting states of electromagnetic and photonic formations.

Mathematics already created very convenient and most general, uniting model or framework for addressing all matter waves, formulated as an Analytic Signal (or complex Phasor) function. If we know specific state of motion which has its energy, momentum, and wave function in our real, spatial and time domain, then associated matter wave which belongs to such motion is created by Hilbert-transform producing an imaginary component of relevant Analytic signal or Phasor (see about Analytic signals in chapter 4.0 of this book). Also, vector fields belonging to real and imaginary components of mentioned Analytic signal should behave as elements of a complex field, which has one potential and one solenoidal vector field (see similar elaborations around equations (3.5.1)- (3.5.4) in chapter 3. and see the picture below).

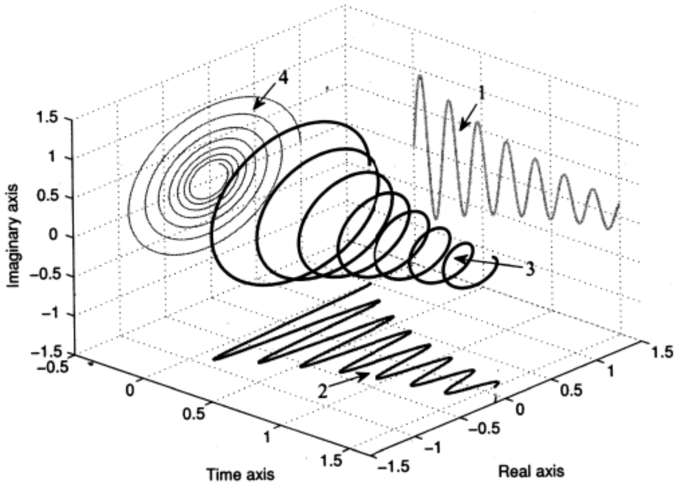


Figure 2.3 The HT projection (1), the real signal (2), the analytic signal (3), and the phasor in complex plain (4) (Feldman, ©2011 by Elsevier)

Anyway, spiraling and vortex-like de Broglie matter waves in different forms and based on different fields should be on some way directly or indirectly detectable in a medium where waves are created and propagate. Specific fluid-like carrier medium should always exist in the background of matter waves. Often, we do not know what such background fluidic media is, but we are directly and indirectly able to measure its electromagnetic properties (like dielectric permeability and magnetic susceptibility constants). We should not deny the existence of mentioned space-time texture, which was imaginatively specified (a long time ago) as an ether, whatever that means. ***Luckily, "imprints, traces and shadows" of different matter waves are also detectable in liquids, if we do not know what to do with ether concepts****. Many recent research results are showing that it is possible to produce different acoustic, mechanic and electromagnetic agitations and excitations of liquids (effectively creating different matter waves inside), and when such agitated liquid is being frozen or solidified, it is possible to notice the presence of different crystalline formations. Every different kind of liquid agitation has its specific, unique, and experimentally verifiable and repeatable crystalline structure, meaning that liquids have some structural memory, being able to record certain agitation, and later, after such liquid is being frozen, it is possible to see different, but agitation-specific crystals. Of course, an ether situation is also part of any liquid media, and here we effectively detect matter waves structure indirectly as crystalline (or molecular formations) imprints on certain structural way memorized by liquids.*

4.1.2.1 Matter Waves and Vortex Flow Meter

Let us now try to materialize (or verify) introduced matter-waves concept by analyzing a specific sufficiently convincing example. Every particle submersed in some fluid, which is in relative motion related to that fluid, should also manifest (or produce) some helix-like rotating tail, or vortices on the downstream side (for instance phenomenon known as Karman, Vortex Street). This is well known (but not sufficiently theoretically explained) effect used in vortex flow meters (here is recommendable to read readily available literature about flow metering based on vortex shedding effects to get familiar with such practices and terminology). The frequency f_v *of such vortex shedding (in a tube which has internal diameter* $D_c = \text{const.}$*) is experimentally related to the fluid flow velocity* v *(or here, to particle velocity, which is, here, the relative velocity between a particle and fluid) respecting empirically established formula,* $f_v = \frac{1}{D_c} S \cdot v \cong \text{Const} \cdot v$.

Here S is *the Strouhal number, which is about constant across a wide range of the Reynolds' numbers (10*^{2} *~ 10*^{7}*). The intention here is to show that matter-waves (associated with motional particles), in the form of some helix vortices, could be detected in liquids. Let us verify this using expression for motion or wave energy from* (4.3-0).

$$\left\{ \begin{array}{l} f_v \cong \frac{1}{D_c} S \cdot v \cong \text{Const} \cdot v \\ \tilde{E} = hf = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_k \\ f = f_v,\ \lambda = \frac{h}{p},\ v << c \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} f = f_v \cong \frac{p}{2h} \cdot v = \frac{v}{2\lambda} \\ \frac{S}{D_c} = \frac{p}{2h} = \frac{1}{2\lambda} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} f_v \lambda = u = \frac{v}{2} \\ \lambda = \frac{2\pi}{k} = \frac{D_c}{2S} \\ \omega_v = 2\pi f_v = \frac{v}{2} k \end{array} \right\}. \qquad \text{(4.3-0)-a}$$

The result of such unusual (and innovative) analogical associations is that we got well known, correct relation between group and phase velocity (for non-relativistic fluid velocities) $u = \frac{v}{2}$*, which is confirming that matter waves tail, familiar to one illustrated on Fig.4.1.1., should exist. Of course, the real place and significance of Planck constant* $\boldsymbol{h}$ *in flow-metering is doubtful (probably nonexistent), but we can safely say and verify that instead of Planck constant "*$\boldsymbol{h}$*" in (4.3-0)-a we should anyway have specific analogous constant* H*, and come to the same conclusions, for example,*

$$\left\{ \begin{array}{l} f_v \cong \frac{1}{D_c} S \cdot v \cong \text{Const} \cdot v \\ \tilde{E} = Hf = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_k \\ f = f_v,\ \lambda = \frac{H}{p},\ v << c \\ h \to H = \text{Const.} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} f = f_v \cong \frac{p}{2H} \cdot v = \frac{v}{2\lambda} \\ \frac{S}{D_c} = \frac{p}{2H} = \frac{1}{2\lambda} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} f_v \lambda = u = \frac{v}{2} \\ \lambda = \frac{D_c}{2S} = \frac{2\pi}{k} = \frac{H}{p} \\ p = \frac{2HS}{D_c} \\ \omega_v = 2\pi f_v = \frac{v}{2} k \end{array} \right\}. \qquad \text{(4.3-0)-b}$$

Effective expression for "fluidic linear momentum" p*, related to a liquid flow in pipelines (from (4.3-0)-b), is* $p = \frac{2HS}{D_c} = \frac{\text{const.}}{D_c}$ *, meaning that if an internal pipe diameter* D_c *is* *smaller, useful fluid momentum is more significant, what looks logical (since fluid velocity will increase, and everything else will stay unchanged).*

Fluids' masses (in relative motion to specific particle) should present sensitive sensor bodies (or antennas) for detecting and visualizing matter waves, vortices, spinning and turbulence, including possibility of analogical detecting of astronomical macro matter waves (for instance, if we consider water in the oceans as a sensor). In the world of Physics, everything that is around particles, are fluids, fields, waves, and other particles, but in some cases, we still do not know how to describe certain fluid states. Present conceptualization related to Fluid dynamics and Navier-Stokes's equations should be enriched and optimized by considering here elaborated matter-waves manifestations. All kind of fluid motions where we see turbulence and vortices conceptually should be connected and explicable as matter waves manifestations and de Broglie matter-waves concepts. Also, we could also extend matter-waves association and conceptualization to orbital planetary motions in solar systems (see Chapter 2. Gravitation; ***2.3.3. Macro-Cosmological Matter-Waves and Gravitation,*** around equations (2.11.13-1) - (2.11.13-5), where is clearly shown how matter waves in two-body problems are being created*).*

Until here, regarding fluids (Fig.4.1.1 and equations (4.3-0)-a, (4.3-0)-b), we could say that relative motion of specific particle has been considered on different ways, to show that fluids could detect imprints and paths of associated matter waves.

Let us now more systematically decompose the same relative motion situation in **three steps** of understanding and conclusions, as follows:

FIRST, we imagine that the same linear motion is a segment of larger scale rotational motion (around the specific center of rotation C, as initiated in (4.3-0)), where a radius of rotation R can be arbitrarily large (see Fig.4.1.1a). We will practically extend and elaborate the concept of the unity of different motions, as summarized in (4.3-0). This is giving the chance to present kinetic energy of the same particle in different terms. For instance, the rotating particle has kinetic energy expressed regarding liner-motion parameters, which is

$$E_k = (\gamma - 1)mc^2 = \frac{pv}{1+\sqrt{1-v^2/c^2}} . \qquad (4.3\text{-}0)\text{-c}$$

SECOND, the assumption is that the same particle is anyway, always rotating (mechanically revolving around specific center C, with an arbitrary, sufficiently large radius of rotation). Such particle has its moment of inertia $J_m = mR^2$, and orbital moment $\vec{L}_m = J_m\vec{\omega}_m$ (being perpendicular to $\vec{v}$ and $\vec{R}$) about the center of rotation C. Now we can add another expression for the same kinetic (now revolving or orbiting) energy of the same particle,

$$E_k = \frac{L_m\omega_m}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}}, \vec{v} = \vec{\omega}_m \times \vec{R} . \qquad (4.3\text{-}0)\text{-d}$$

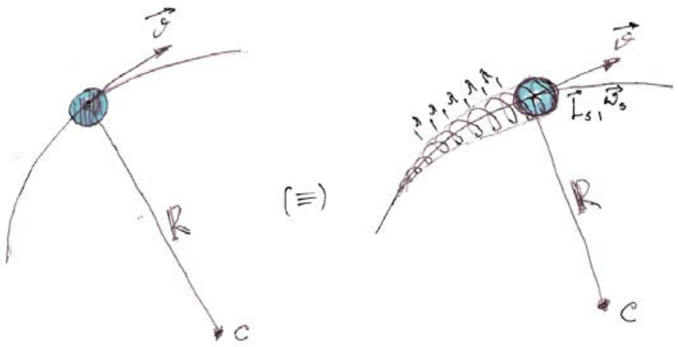


Fig.4.1.1a

The THIRD step is to creatively apply the conceptualization elaborated by Fig.4.1.1 and equations (4.3-0), (4.3-0)-a, (4.3-0)-b. Let us find another, **third** (and somewhat hypothetical) possibility, to express the same kinetic energy of the same particle by introducing an equivalent self-spinning particle which still has the same kinetic energy (see also equations (2.9.5-9) from the second chapter). We will say that particle is not only rotating around its center of rotation C; -it is also on some way spinning around its propagation path (around its velocity vector $\vec{V}$), as illustrated on Fig.4.1.1 and Fig.4.1.1a. Here $\vec{\omega}_{mw}$ is certain *matter-waves spinning,* angular, or spinning velocity, collinear with particle velocity $\vec{V}$, and $\vec{L}_{mw}$ is its *matter-wave spinning* moment, collinear with linear particle moment $\vec{p}$ (a bit analogical to eddy currents and Lenz law situations in electromagnetism). Since this still has speculative and hypothetic meaning, we can say that, at least, introduced *matter-wave spinning* or helix motion (whatever it is) should have the same kinetic energy as one the particle had before,

$$E_k = \frac{\vec{L}_{mw}\vec{\omega}_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{\vec{L}_m\vec{\omega}_m}{1+\sqrt{1-v^2/c^2}} = \frac{\vec{p}\vec{v}}{1+\sqrt{1-v^2/c^2}},$$
$$\vec{p} = \left(\frac{\omega_{mw}}{v}\right)\frac{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}{\cos(\vec{p},\vec{v})}\vec{L}_{mw} = \gamma m\vec{v},\ \vec{L}_{mw} = \left(\frac{v}{\omega_{mw}}\right)\frac{\cos(\vec{p},\vec{v})}{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}\vec{p}\ . \qquad (4.3\text{-}0)\text{-e}$$

Here, we can say (or hypothesize) that such imaginative helix and spinning motion should be linked to de Broglie matter waves that are representing moving particle in question. ***Whatever is about wave motion around the particle in question, is a certain kind of its kinematic equivalent, or replacement, or specific waveform behaving as an equivalent wave-group or wave-packet. When solving Classical Wave equation, we will always get two wave functions propagating in mutually opposite directions, and here we should have a similar situation, meaning that two matter-wave functions, or wave groups (with energy $E_k = \tilde{E}$) should be on some way present, having mutually opposed (and mutually canceling) angular spin moments. Consequently, a total orbital particle moment will stay equal to $\vec{L}_m = J_m\vec{\omega}_m$, and two opposed vectors L_{mw} (of helicoidally matter-wave spin moments) will be mutually canceled.*** Such helix waveforms should have their group and phase velocity. Group velocity will naturally be the same as the particle velocity.

The universally applicable equation that is connecting such group, v, and phase velocity, u, is already known as,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}, u = \lambda f,\ \lambda = \frac{H}{p}, H = const. \qquad (4.3\text{-}0)\text{-f}$$

Now we can again express the same kinetic or motional energy (of the same particle) on three different ways including new ***spinning wave-motion*** terms as,

$$\begin{aligned}
&E_k = \tilde{E} = Hf_{mw} = Hf = \frac{2\pi \mathbf{L}_{mw}}{1+\sqrt{1-v^2/c^2}} f_{mw} = \frac{\mathbf{L}_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbf{L}_m\omega_m}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}},\\
&\vec{p} = \left(\frac{\omega_{mw}}{v}\right)\frac{\cos(\vec{\mathbf{L}}_{mw}, \vec{\omega}_{mw})}{\cos(\vec{p}, \vec{v})}\vec{\mathbf{L}}_{mw} = \gamma m\vec{v},\ \vec{\mathbf{L}}_{mw} = \left(\frac{v}{\omega_{mw}}\right)\frac{\cos(\vec{p}, \vec{v})}{\cos(\vec{\mathbf{L}}_{mw}, \vec{\omega}_{mw})}\vec{p},\ \mathbf{L}_{mw} = \gamma mvR = \mathbf{J}_{mw}\omega_{mw},\ \vec{v} = \vec{\omega}_m \times \vec{R},\\
&v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, u = \lambda f,\ \lambda = \frac{H}{p}, H = const., (\text{for micro and subatomic world}: H \to h = \text{Planck const.}),\\
&\frac{H}{2\pi} \le \mathbf{L}_{mw} = \left\{\frac{H}{2\pi}(1+\sqrt{1-v^2/c^2}) = \frac{H}{2\pi}\frac{v}{u}\right\} \le \frac{H}{\pi},\ (\vec{\omega}_m \text{ and } \vec{\omega}_{mw} \text{ are mutually orthogonal vectors}),\\
&\boxed{pv = \mathbf{L}_m\omega_m = \mathbf{L}_{mw}\omega_{mw}},\ \boxed{dE_k = vdp = \omega_m d\mathbf{L}_m = \omega_{mw} d\mathbf{L}_{mw} = d\tilde{E}},\\
&\mathbf{J}_m = \gamma mvR = \frac{H}{2\pi}\cdot\frac{d\mathbf{L}_m}{d\mathbf{L}_{mw}}(1+\sqrt{1-v^2/c^2}) = \frac{H}{2\pi}\cdot\frac{\omega_{mw}}{\omega_m}(1+\sqrt{1-v^2/c^2}).
\end{aligned} \qquad (4.3\text{-}0)\text{-g}$$

Effectively, all initial, analogical, intuitive, and mathematical elaborations introduced in the very beginning of this chapter, and especially in "**4.1.1. Particle-Wave Duality Code**", around equations (4.1), (4.2), including analogies T.4.0., is getting its full and richer meaning in connection with this **THIRD** step until formulating (4.3-0)-g. *What has been the most important and essential here is to understand that motional energy has different forms and different ways to manifest and to be presented, and that matter wave's energy is the same motional or kinetic energy (see the more supporting background in chapter 4.2). Our Universe or Nature is presenting its unity by different and coincidental manifestations of motional energy states (while mathematically we make appropriate distinctions).*

Specific analyses of Compton, Photoelectric effects, Creation of electron-positron couple from high energy photon, or Annihilation of an electron-positron couple and creation of photons (as in examples given in Chapter 4.2), can explicitly and without any doubts confirm that Kinetic particles energy is fully transformable to, or equivalent to matter-waves motional energy, and vice versa. The process which is opposite to (or inverse of) Compton Effect is the continuous spectrum of X-rays (or photons) emission, caused by impacts of electrons (accelerated in the electrical field between two electrodes) with the anode as their target. The emission of x-ray photons starts when the electrons are abruptly stopped on the anode surface. If the final, impact electron speed is non-relativistic, $\mathbf{v} << \mathbf{c}$, *the maximal frequency of the emitted X-rays is found from the relation:* $\mathbf{hf}_{max.} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{mv}^2$, *and in cases of relativistic electron velocities, we have* $\mathbf{hf}_{max.} = (\gamma - \mathbf{1})\mathbf{mc}^2$ (and both are experimentally confirmed to be correct). *If we now consider electrons (before the impact happen) as matter waves, where the electron matter wave energy corresponds only to kinetic electron energy, without rest-mass energy content, we will be able to find de Broglie, matter wave frequency of such electrons (just before their impact with the anode). With impact realization, the* *electrons are entirely stopped, and the energy content of their matter waves is fully transformed and radiated in the form of X-ray photons (or into another form of waves), whose frequency corresponds to the matter wave frequency of electrons in the moment of the impact. This equality of the frequencies of radiated X-ray photons and electron matter-waves (in the moment of impact) explains us the essential nature of electron matter waves (eliminating the possibility that the rest mass belongs to matter-wave energy content). In all such analyses, we are using relations, which agree with* ***PWDC;*** *-see "4.1.1. Particle-Wave Duality Code", around equations (4.1), (4.2) from the beginning of this chapter.*

Let us again go back to the bottom-line basics, related to fluids-flow and associated matter waves manifestations. We know (presently only experimentally) that in cases of vortex flow meters, fluid speed and frequency of generated vortices are mutually directly and linearly proportional. Nobody developed such relations mathematically, systematically, and starting from more elementary and generally valid step stones. Strouhal, only empirically, established principal flow metering relation, $f_v \cong Const \cdot v$, between relevant fluid velocity v and frequency f_v of Karman Vortex Street, and this relation has been successfully tested, and since very long time utilized in producing vortex flow meters (and looks like nobody is seriously asking how and why it works).

Here, we have a chance to develop the vortex flow metering, Strouhal relation mathematically. We can start from matter-waves relations (4.3-0)-a,b,c,d,e,f,g and get,

$$\left\{\begin{array}{l} \vec{p} = \frac{\omega_{mw}}{v}\frac{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}{\cos(\vec{p},\vec{v})}\vec{L}_{mw} = \gamma m\vec{v}, \\ \vec{L}_s = \frac{v}{\omega_{mw}}\frac{\cos(\vec{p},\vec{v})}{\cos(\vec{L}_{mw},\vec{\omega}_{mw})}\vec{p} \\ vp = \omega_{mw}L_{mw} \\ vdp = \omega_{mw}dL_{mw} \end{array}\right\}_{\substack{initial \\ relations}} \Rightarrow \left\{\frac{v}{\omega_s} = \frac{L_s}{p} = \frac{dL_s}{dp}\right\} \Rightarrow_{\substack{after \\ integration}} \left\{\begin{array}{l} \frac{L_s}{L_{so}} = \frac{p}{p_o} \Rightarrow L_{mw} = L_{mwo}\frac{p}{p_o} = \frac{v}{\omega_{mw}}p \\ (L_{mwo}, p_o)(=)constants \end{array}\right\} \Leftrightarrow \quad (4.3\text{-}0)\text{-h}$$

$$\Leftrightarrow \omega_{mw} = 2\pi f_{mw} = \frac{p_o}{L_{mwo}}v \Leftrightarrow \underline{f_{mw} = \frac{p_o}{2\pi L_{mwo}}v = Const \cdot v}\ .$$

As we can see in (4.3-0)-h, the result of combined initial relations is $f_{mw} = \frac{p_o}{2\pi L_{mwo}}v = Const \cdot v$, which is the same vortex-flow-meters' relation, initially and still known only empirically. The conceptual picture of de Broglie matter waves (exercised by (4.3-0), (4.3-0) -a,b,c,d,e,f,g,h) is gradually getting stronger grounds, becoming self-supporting and internally consistent (and supportable from different points of view). Of course, we still need to "digest" unusual analogies between micro and macro quantum worlds (as we can find in chapter 2. *Gravitation;* ***2.3.3. Macro-Cosmological Matter-Waves and Gravitation***), but initial results are challenging. Especially fruitful relations for future elaborations (from many aspects) are $\{vp = \omega_{mw}L_{mw}, vdp = \omega_{mw}dL_{mw}\}$. In other words, to be simple and bottom line clear, here we are exploiting the analogy between de Broglie matter waves associated to a moving particle, and vortex flowmeter (turbulent) waves that are generated when fluid is in relative motion to certain flow-barrier (which is inside the flow-meter pipe). In case of de Broglie matter waves, we should imagine that moving particle is also making relative motion related to certain kind of surrounding fluid (as in case of vortex flowmeter where the surrounding fluid is moving). What the nature of the surrounding fluid in cases of (other) de Broglie matter waves is, we still do not know (such conclusions cannot be generalized, because different fields and forces will have different manifestations). What we know is that if we apply such fluids-related analogy, we can develop all equations for vortex flow metering (known only empirically, until present). If we now extend such analogical thinking and say that planets of specific solar system, in their orbital motions, are also making a motion that is relative to specific surrounding fluid (or space), we should analogically expect to have some vortices, or "Planetary, Karman Streets", behind planets. As we know, such an analogy is already well exercised in the second chapter, documenting the existence of macro cosmological matter waves that are respecting de Broglie wavelength and quantization like in case of Bohr atom modeling. Going backwards, if we know orbital velocities of planets, we should be able to find frequencies of "planetary vortex waves" behind planets (based on $f_{mw} = Const \cdot v$, $vp = \omega_{mw} L_{mw}$, $vdp = \omega_{mw} dL_{mw}$; -(see (2.11.13-1)-(2.11.13-5), and (2.11.14)-a, (2.11.13-1)-(2.11.13-5). *Chapter 2. Gravitation;* ***2.3.3. Macro-Cosmological Matter-Waves and Gravitation***). Here is a place to say, that the same analogies, and conceptualization, is directly applicable to micro-world of subatomic and elementary particles (but still differently formulated and explained in a contemporary Quantum Theory). Many situations (in Physics) that are looking mutually distinct are showing as analogically and analytically connected with the same conceptual thinking, but we have been considering them a too long time as distinct and not connected. *Much more general topology or geometry platform in the background of here conceptualized links between linear and spinning or rotational motions most probably belongs to Möbius transformations and Riemann Sphere concepts, where such links are something evident and natural.*

There are simple examples of familiar rotations associated with every liquid that is in the specific tank, which has a circular sink (draining hole) at the bottom. When draining hole is open, we will always notice that free surface of the liquid is rotating and helicoidally funneling around the circular outlet (while a liquid is going out of the tank). Whatever we try, to make perfectly circular, axially symmetric, and polished sink (to avoid liquid rotation caused by some sink-geometry related reasons), we will always detect mentioned liquid rotation (because linear motions are intrinsically coupled with associated rotational or spinning motions). Most turbulent and vortex type phenomenology associated with different fluids flow situations should be until certain level related to intrinsic coupling between linear and spinning motions (even hurricanes and galactic rotations). Fluids are very suitable media for creating and detecting (or visualizing) couplings between linear motions and spinning or helix-vortex phenomena (for instance, among such effects are "Falaco vortex Solitons"). Most probably, new sensors for detecting gravitational and inertial effects, spatial linear and torsional accelerations can be made based on exploring relevant matter-waves phenomenology in liquids. *In [52], (Rainer W. Kühne: Gauge Theory of Gravity Requires Massive Torsion Field), we can also find the theory, which is promoting the necessity of linear, torsion and spinning motions coupling.* ***Fluid dynamics (and Navier-Stokes's framework) will be significantly updated by considering here elaborated matter-waves manifestations.***

The matter-waves concept described around equations (4.3-0)-a,b,c,d,e,f,g,h… is showing that linear motions, if considered in a large scale as parts of rotational motions, potentially have other levels of associated spinning and rotating motions. Moreover, if we extend such concept, each (new) level of associated spinning could carry another higher level of (much higher frequency) helical spinning, around its helical path, converging towards finer and finer, spatially packed, multiple helical spinning structures (like twisted, toroidal solenoids, and mutually overlapping layers). This will present structural multilevel quantizing, and every higher-level helical motion, around its primary helical spinning path, should cover the interval of higher angular (or rotating) frequencies, until we rich terminal or maximal tangential velocity of certain, highest, and finest helical path. Turbulent fluid manifestations should be somewhat motional, spatial perturbations belonging to different associated spinning paths levels.

Another level of complexity that can be associated to the same situation is to imagine or accept that any linear motion is only a reasonable approximation of specific rotational or orbital motion (with an arbitrary large radius of rotation R). Here we can consider as globally valid conservation of associated orbital and spin moments (see [36], Anthony D. Osborne, & N. Vivian Pope, "An Angular Momentum Synthesis of 'Gravitational' and 'Electrostatic' Forces"). Now it will be possible to apply generalized Arnold Sommerfeld quantization rules (for such arbitrary circular and closed orbits) as,

$$\oint pdR = n_R H,\ \oint \mathbf{L}_c d\alpha = n_\alpha H,\ \oint \mathbf{L}_s d\varphi = n_\varphi H,\ H = \text{const.}, \{n_R, n_\alpha, n_\varphi\} \in [1, 2, 3, ...). \quad \text{(4.3-0)-i}$$

In equations (4.3-0)-f,g,h,i we can see that micro-world Planck constant **h** is not generally applicable to all other macro motions. Planck constant is (most probably) universally applicable to a micro-world of atoms and subatomic entities (complying with different self-closed, matter-waves formations). In other macro-world situations (where we deal with stable orbital motions) looks that instead of **h**, series of new, much bigger **H** constants are relevant, depending on the situation. See similar elaborations in the second chapter around equations (2.9.1) and (2.9.2). This way thinking, we conclude that an innovative conceptual platform for a more natural understanding of matter waves should be established.

Let us now go to another extreme, starting with kinetic energy equivalency relations (4.3-0)-g. We can exercise that rest mass, as an energy packing format, could be created (once in its past) when a certain amount of kinetic energy was packed and stabilized that way (with significant participation of spinning). This is producing the following chain of results,

$$\left\{\begin{array}{l} E_k = \tilde{E} = Hf_{mw} = HF = \dfrac{\mathbf{L}_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \dfrac{pv}{1+\sqrt{1-v^2/c^2}} = \\ = mc^2(\gamma-1),\ E_{tot.} = \gamma mc^2 = mc^2 + E_k, \\ \vec{p} = \left(\dfrac{\omega_{mw}}{v}\right)\dfrac{\cos(\vec{\mathbf{L}}_{mw}, \vec{\omega}_{mw})}{\cos(\vec{p}, \vec{v})}\vec{\mathbf{L}}_{mw} = \gamma m\vec{v},\ \vec{\mathbf{L}}_{mw} = \left(\dfrac{v}{\omega_{mw}}\right)\dfrac{\cos(\vec{p}, \vec{v})}{\cos(\vec{\mathbf{L}}_{mw}, \vec{\omega}_{mw})}\vec{p}, \vec{\mathbf{L}}_{mw} = \mathbf{J}_{mw}\vec{\omega}_{mw}, \\ v = u - \lambda\dfrac{du}{d\lambda} = -\lambda^2\dfrac{df}{d\lambda}, u = \lambda f,\ \lambda = \dfrac{H}{p}, H = \text{const.}, \\ \dfrac{H}{2\pi} \leq \mathbf{L}_{mw} = \dfrac{H}{2\pi}\dfrac{v}{u} \leq \dfrac{H}{\pi} \end{array}\right\}_{\left(\substack{v \to c \\ H \to h}\right)} \Rightarrow$$

$$\Rightarrow\begin{Bmatrix} mc^2 = J_c\omega_c^2 = L_c\omega_c, \\ \lambda_c = \lambda_c = \frac{h}{mc}, \omega_c = 2\pi f_c, \lambda_c f_c = u_c = c \\ ----------------- \\ mv^2 = J_{mw}\omega_{mw}^2 = L_{mw}\omega_{mw}, \\ \lambda = \frac{h}{\gamma mv} \cong \frac{h}{mv}, \ v << c \end{Bmatrix} \Rightarrow \begin{Bmatrix} L_{mwc} = \frac{h}{2\pi} \\ ----- \\ L_{mw} = \frac{h}{2\pi}\frac{v}{u} \end{Bmatrix} \Rightarrow m = \frac{hf_c}{c^2} = \frac{h}{c\lambda_c}. \quad (4.3\text{-}0)\text{-j}$$

With (4.3-0)-j we have an essential part of explanation what Compton wavelength $\lambda_c = h/mc$ presents. Until here, Compton wavelength was only a significant or indicative numerical quantification without real conceptual meaning (but appearing in many analyses related to particle interactions). More of common supporting arguments can be found in the Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.

In cases of inertial motions, we will have the following situation,

$$\begin{Bmatrix} E_k = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{L_m\omega_m}{1+\sqrt{1-v^2/c^2}} \\ \vec{p} = \frac{\omega_{mw}}{v}\vec{L}_{mw} = \gamma m\vec{v}, \ \vec{L}_{mw} = \frac{v}{\omega_{mw}}\vec{p} = \frac{v}{\omega}\gamma m\vec{v} \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} pv = L_{mw}\omega_{mw} = L_{mwc}\omega_{mwc} = L_m\omega_m, \frac{\omega_{mw}}{v} = \frac{p}{L_{mw}} \\ vdp = \omega_{mw}dL_{mw} = \omega_m dL_m \\ F = \frac{dp}{dt} = \frac{\omega_{mw}}{v}\frac{dL_{mw}}{dt} = \frac{\omega_m}{v}\frac{dL_m}{dt} (=) \text{force} \\ \tau_{mw} = \frac{dL_{mw}}{dt} = \frac{v}{\omega_{mw}}\frac{dp}{dt} = \frac{\omega_m}{\omega_{mw}}\frac{dL_m}{dt} = \frac{\omega_m}{\omega_{mw}}\tau_m (=) \text{torque} \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \{\text{INERTIAL STATE}\} \Leftrightarrow \begin{Bmatrix} F = 0 \\ \tau = 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} p = \text{const.} \\ L_{mw} = \text{Const.}, \ L_m = \text{constant} \\ \frac{v}{\omega_{mw}} = \frac{L_{mw}}{p} = \frac{dL_{mw}}{dp} = \text{Constant} \\ \frac{v}{\omega_m} = \frac{L_m}{p} = \frac{dL_m}{dp} = \text{Const.} \end{Bmatrix}. \quad (4.3\text{-}0)\text{-k}$$

Naturally, from (4.3-0)-k we can conclude that constant linear velocity is not the best and unique identifier of inertial motions, and that **rotational or spinning inertial states are among other inertial states** (see in the second chapter "*2.3.1. Extended Understanding of Inertia*").

Unity of Linear and Orbital or Angular motions should also be presentable in a Minkowski 4-vectors framework of Relativity Theory. Let us briefly summarize

Momentum 4-vectors of a specific moving particle in linear, circular motion in two cases:

Case A) Particle is in linear motion and rotating around a remote fixed point,

and

Case B) Particle is in a linear motion (as in the Case A)), rotating around a fixed point, and in the same time mechanically spinning (with angular spinning moment $\vec{L}_s = \vec{S}$) around the specific axis (belonging to the same particle).

For the **Case A)** we will have the following situation (based on already elaborated expressions from (4.3-0)-c until (4.3-0)-j):

$$\overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) = \left(\frac{\omega_{mw}}{v}\vec{L}_{mw}, \frac{E}{c}\right)$$
$$\vec{p} = \gamma m\vec{v},\ E = \gamma mc^2 = E_0 + E_k,\ E_0 = mc^2$$
$$E_k = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} = \frac{L_m\omega_m}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = Hf_s,$$
$$L_{mw}\omega_{mw} = L_c\omega_c = L_m\omega_m = pv, \vec{p} = \frac{\omega_{mw}}{v}\vec{L}_{mw} = \gamma m\vec{v},\ \vec{L}_{mw} = \frac{v}{\omega_{mw}}\vec{p}\ ,$$
$$\lambda_{mw} = \frac{H}{p} = \frac{h}{\gamma mv},\ u = \lambda_{mw}f_{mw} = \frac{v}{1+\sqrt{1-v^2/c^2}},$$
$$\vec{L}_{mw} = \left(\frac{H}{2\pi}\right)\cdot(1+\sqrt{1-v^2/c^2}),\ \frac{H}{2\pi} \le L_{mw} = \frac{H}{2\pi}\frac{v}{u} \le \frac{H}{\pi}.$$

For the **Case B)** we will have an additional kinetic energy member coming from mechanical spinning (here indexed with "s", as in (4.3)). Particle linear and orbital moments will also change (here marked with asterisks " * "), as follows,

$$\overline{P}_4^* = \left(\vec{p}^*, \frac{E^*}{c}\right) = \left(\frac{\omega_{mw}^*}{v^*}\vec{L}_m^*, \frac{E^*}{c}\right),\ \vec{L}_m^* = \vec{L}_m + \vec{L}_s, v \to v^*$$
$$\vec{p}^* = \gamma^*(m+\Delta m)\vec{v}^*,\ E^* = \gamma^*(m+\Delta m)c^2 = E_0 + E_k + E_s,\ E_0^* = mc^2 + E_s,$$
$$\Delta m = \frac{E_s}{c^2}, E_s = \frac{L_s\omega_s}{1+\sqrt{1-v^2/c^2}}$$
$$E_k = \frac{L_{mw}^*\omega_{mw}^*}{1+\sqrt{1-v^2/c^2}} = \frac{L_m^*\omega_m^*}{1+\sqrt{1-v^2/c^2}} = \frac{p^*v}{1+\sqrt{1-v^2/c^2}},$$
$$L_{mw}^*\omega_{mw}^* = L_c^*\omega_c^* = p^*v^* = \gamma(m+\Delta m)v^2,\ \vec{p}^* = \frac{\omega_{mw}^*}{v}\vec{L}_{mw}^* = \gamma(m+\Delta m)\vec{v}^*,\ \vec{L}_{mw}^* = \frac{v^*}{\omega_{mw}^*}\vec{p}^*,$$
$$\omega_{mw}^* = \frac{2\pi}{H}\cdot\frac{\gamma^*(m+\Delta m)v^{*2}}{1+\sqrt{1-v^{*2}/c^2}} = \frac{2\pi}{h}\cdot\frac{p^*v^*}{1+\sqrt{1-v^{*2}/c^2}} = 2\pi f_{mw}^*\ ,$$
$$\lambda_{mw}^* = \frac{H}{p^*} = \frac{h}{\gamma^*(m+\Delta m)v^*},\ u^* = \lambda_{mw}^*f_{mw}^* = \frac{v^*}{1+\sqrt{1-v^{*2}/c^2}} = \lambda_{mw}f_{mw} = \lambda_m f_m = \frac{v}{1+\sqrt{1-v^2/c^2}} = u,$$
$$\vec{L}_{mw}^* = \left(\frac{H}{2\pi}\right)\cdot(1+\sqrt{1-v^2/c^2}),\ \frac{H}{2\pi} \le L_{mw}^* = \frac{H}{2\pi}\frac{v^*}{u} \le \frac{H}{\pi}.$$

As we can see, in the case **under B)**, particle matter wave wavelength λ^*_{mw} is shortened (compared to the case **under A)**), and matter wave spinning frequency f^*_{mw} is higher, but their product, or phase velocity $\mathbf{u}$, did not change. Mechanical spinning ($\vec{\mathbf{L}}_s = \vec{S}$), which is motional energy E_s , is merely contributing to the rest mass of the moving particle. The significance of here presented cases A) and B) is to exercise how spinning is intrinsically (or naturally) already present and integrated into a rest mass, in the framework of Minkowski 4-vectors of Relativity Theory, showing one of the aspects how unity between linear and orbital motions is working. Similar conclusions or concepts could be drawn from Analytic Signal modeling and even much better from the unification between Minkowski 4-vector and Analytic Signal concepts (see more in chapter 10).

To support the picture about the unity of linear and rotational (or orbital) inertial motions additionally, let us create the table T.4.2.1 with mutually analogical elements of linear and rotational motions seen from an observer in a convenient Laboratory System of coordinates. Certain of analogical expressions in T.4.2.1 are unusual and created only to satisfy mathematical symmetry, respecting Mobility table of mutually analogical values (see about such analogies in T.1.2, T.1.4, and T.1.8 from the first chapter). We will accept that all inertial motions (in larger picture observations) are also belonging to rotational, orbital motions, as postulated in [36], by Anthony D. Osborne, & N. Vivian Pope. In case when we analyze (or describe) inertial motions of number of moving particles (or bodies), mutually related (like specific planetary or solar system), we will also consider that each particle could perform mechanical spinning (and be in a linear motion that is just a reduced-picture part of a larger scale rotation). In such situations, it will become evident that strong coupling between all orbital and spinning moments (of participants) should be globally (holistically) satisfied. For instance, in T.4.2.1 we can find a total angular moment $\vec{\mathbf{L}}_{ct}$ (as the sum of rotating orbital and mechanical spinning moments of participants),

$$\vec{\mathbf{L}}_{ct} = \sum_{(i)} (\vec{\mathbf{L}}_i + \vec{\mathbf{L}}_{i-s}) = \vec{\mathbf{L}}_c + \vec{\mathbf{L}}_{c-s} = \text{const} \;\Big|\; \underset{\text{differentiation}}{\Rightarrow} \; \dot{\vec{\mathbf{L}}}_c + \dot{\vec{\mathbf{L}}}_{c-s} = \vec{\tau}_c + \vec{\tau}_{c-s} = \vec{0},$$

meaning that spinning forces, or torques, are mutually coupled and real-time balanced with orbiting forces ($\vec{\tau}_c = -\vec{\tau}_{c-s}$), where index "s" is indicating mechanical spinning. In other words, planets and moons of a specific solar system are in permanent orbital (and inertial) rotation around its sun, but mutually communicating mechanical rotations and spinning of planets and all satellites secure the stable balance.

__Every sudden change in any of orbital or spin moments will be detected and real-time balanced by (opposite sign) equivalent orbital moments change in the rest of the system__. Of course, since all linear motions are just segmenting of larger-scale rotational motions, we do not need to be strictly linked to solar systems, and consequently, here elaborated conclusions are becoming universally valid for every system of mutually approaching and mutually related particles and bodies (especially knowing that electromagnetic and gravitation related properties are on some way coupled). Briefly, *rotational motions are mutually balanced with spinning (of participants), and in cases when conventional mechanical spinning is not visible, there is still a spinning of matter waves around moving particles*. Therefore, our universe is creating matter waves (and, of course, not only in the world of microphysics). The phenomenology known in Quantum theory as **Entanglement** belongs to here elaborated, global and immediate balancing between orbital and spin moments of a specifically coupled system of particles and matter waves (see more in Chapter 4.3, including equations (4.10-12)).

T.4.2.1 Analogies Between n-Body Coupled Inertial Motions in a Laboratory System

Linear Inertial Motion	**Orbital inertial rotation around center C**	**Inertial spinning** (in addition to orbital rotation)
Center of mass $\vec{r}_c = \frac{\sum_{(i)} m_i \vec{r}_i}{\sum_{(i)} m_i} = \frac{\sum_{(i)} m_i \vec{r}_i}{m_c}$	**Effective orbital phase** $\theta_c^* = \frac{\sum_{(i)} \mathbf{J}_i \theta_i^*}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \mathbf{J}_i \theta_i^*}{\mathbf{J}_c}$?!	**Effective mechanical spinning phase** $\theta_{c-s}^* = \frac{\sum_{(i)} \mathbf{J}_{i\text{-}s} \theta_{i-s}^*}{\sum_{(i)} \mathbf{J}_{i\text{-}s}}$?!
Center of mass velocity $\vec{v}_c = \dot{\vec{r}}_c = \frac{\sum_{(i)} m_i \dot{\vec{r}}_i}{\sum_{(i)} m_i} = \frac{\sum_{(i)} m_i \vec{v}_i}{m_c}$	**Center of Inertia angular velocity** $\vec{\omega}_c = \dot{\vec{\theta}}_c^* = \frac{\sum_{(i)} \mathbf{J}_i \vec{\omega}_i}{\sum_{(i)} \mathbf{J}_i} = \frac{\sum_{(i)} \vec{L}_i}{\mathbf{J}_c}$	**Effective mechanical spinning, inertial velocity** $\vec{\omega}_{c-s} = \dot{\vec{\theta}}_{c-s}^* = \frac{\sum_{(i)} \vec{L}_{i-s}}{\sum_{(i)} \mathbf{J}_{i-ss}} = \frac{\vec{L}_{c-s}}{\mathbf{J}_{c-s}}$
Central mass $m_c = \sum_{(i)} m_i$	**Central moment of Inertia** $\mathbf{J}_c = \sum_{(i)} \mathbf{J}_i$	**Effective mechanical spinning, a moment of Inertia** $\mathbf{J}_{c-s} = \sum_{(i)} \mathbf{J}_{i-s}$
Linear moment $\vec{p}_c = \sum_{(i)} \vec{p}_i = \sum_{(i)} m_i \vec{v}_i = = m_c \vec{v}_c = \text{Const.}$ $\vec{F}_c = \dot{\vec{p}}_c = \vec{0}$	**Angular moment** $\vec{L}_c = \sum_{(i)} \vec{L}_i = \sum_{(i)} \mathbf{J}_i \vec{\omega}_i = \sum_{(i)} \vec{r}_i \times \vec{p}_i = I_c \vec{\omega}_c = \vec{r}_c \times \vec{p}_c$	**The effective mechanical spinning moment** $\vec{L}_{c-ms} = \sum_{(i)} \vec{L}_{i-s} = \sum_{(i)} \mathbf{J}_{i-s} \vec{\omega}_{i-s} = (\sum_{(i)} \vec{r}_{i-s} \times \vec{p}_{is} = \mathbf{J}_{c-s} \vec{\omega}_{c-s} = \vec{r}_{cs} \times \vec{p}_{cs} = \vec{r}_c \times \vec{p}_{cs})$?!
	$\vec{L}_{ct} = \sum_{(i)} (\vec{L}_i + \vec{L}_{i-s}) = \vec{L}_c + \vec{L}_{c-s} = \text{const}, \ \dot{\vec{L}}_c + \dot{\vec{L}}_{c-s} = \vec{\tau}_c + \vec{\tau}_{c-s} = \vec{0}$	
Kinetic Energy $E_k = \frac{p_c v_c}{1+\sqrt{1-v_c^2/c^2}} \cong \frac{p_c v_c}{2}$ (for v<<c)	**Angular Kinetic Energy** $E_{kr} = \frac{L_c \omega_c}{1+\sqrt{1-v_c^2/c^2}} \cong \frac{L_c \omega_c}{2}$ (for $\omega_c << \omega_{c-max.}$)	**Mechanical Spinning Motional Energy** $E_{k-ms} = \frac{L_{c-s}\omega_{c-s}}{1+\sqrt{1-v_c^2/c^2}} \cong \frac{L_{c-s}\omega_{c-s}}{2}$ (for $\omega_{c-s} << \omega_{c-s-max.}$)
Total Kinetic Energy = Orbiting + Mechanical Spinning Energy $E_k = E_{kr} + E_{k-s}$, $p_c v_c + \mathbf{L}_{c-s}\omega_{c-s} = (\mathbf{L}_c \omega_c + \mathbf{L}_{c-s}\omega_{c-s})$, $p_c v_c = \mathbf{L}_c \omega_c = \mathbf{L}_{cs}\omega_{cs}$, $\vec{L}_{total} = \vec{L}_c + \vec{L}_{c-s}$; $dE_k = dE_{kr} + dE_{k-s} = v_c dp_c + \omega_{c-s} d\mathbf{L}_{c-s} = \omega_c d\mathbf{L}_c + \omega_{c-s} d\mathbf{L}_{c-s} = \omega_{cs} d\mathbf{L}_{cs} + \omega_{c-s} d\mathbf{L}_{c-s}$; $v_c dp_c = \omega_c d\mathbf{L}_c = \omega_{cs} d\mathbf{L}_{cs}$		

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

- $\theta_c^*, \theta_{cs}^*$ (=) effective, composite, spatial angular values (composed of 2 angles), when the analyzed system of orbiting particles is not in the same plane.
- Index "s" (=) mechanical spinning.
- ?! (=) the value that has only formal, dimensional meaning to satisfy used analogical presentation.
- L_{cs}, ω_{cs} (=) spinning matter wave angular moment and angular velocity when orbiting around C.

To focus our attention on Energy-Momentum conservation laws, let us address the same n-Body inertial situation from T.4.2.1 using 4-vectors (of Minkowski space) in the same Laboratory System. The first case, (4.3-0)-l, will be when a system of n particles is performing only mechanical orbital (and linear) motions, without mechanical self-spinning of participants (just to start with, for simplicity and more natural understanding of the next case, (4.3-0)-m, when participants are mechanically spinning). We will at the same time attempt to define particle orbital, 4-vector momentum:

$$\left\{\begin{array}{l} \overline{P}_{4-i} = \left(\vec{p}_i, \frac{E_i}{c}\right),\ \overline{P}_4 = \sum_{(i)} \overline{P}_{4-i} = \left(\sum_{(i)} \vec{p}_i, \frac{\sum_{(i)} E_i}{c}\right) \Rightarrow \\ \vec{p}_i^{\,2} - \frac{E_i^2}{c^2} = -\frac{E_{0i}^2}{c^2},\ \left(\sum_{(i)} \vec{p}_i\right)^2 - \frac{\left(\sum_{(i)} E_i\right)^2}{c^2} = -\frac{\left(\sum_{(i)} E_{0i}\right)^2}{c^2} \\ \vec{p}_i = \gamma_i m_i \vec{v},\ E_i = \gamma_i m_i c^2 = E_{0i} + E_{ki},\ E_{0i} = m_i c^2 \end{array}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} \overline{L}_{4-i} = \vec{r}_i \times \overline{P}_{4-i} = \vec{r}_i \times \left(\vec{p}_i, \frac{E_i}{c}\right) = \left(\vec{r}_i \times \vec{p}_i, \frac{E_i}{c}\vec{r}_i\right) = \left(\vec{L}_i, \frac{E_i}{c}\vec{r}_i\right), \\ \overline{L}_4 = \sum_{(i)} \overline{L}_{4-i} = \left(\sum_{(i)} \vec{L}_i, \frac{\sum_{(i)} E_i \vec{r}_i}{c}\right) = \vec{r}_c \times \overline{P}_4 = \left(\vec{r}_c \times \sum_{(i)} \vec{p}_i, \frac{\sum_{(i)} E_i}{c}\vec{r}_c\right) \end{array}\right\} \Rightarrow \qquad (4.3\text{-}0)\text{-l}$$

$$\left\{ \vec{L}_i^2 - \frac{E_i^2}{c^2}\vec{r}_i^{\,2} = -\frac{E_{0i}^2}{c^2}\vec{r}_i^{\,2},\ \left(\sum_{(i)} \vec{L}_i\right)^2 - \frac{\left(\sum_{(i)} E_i \vec{r}_i\right)^2}{c^2} = -\frac{\left(\sum_{(i)} E_{0i} \vec{r}_i\right)^2}{c^2} \right\}$$

The same case when particles are (also) mechanically spinning ($\vec{L}_{i-ms} \neq \vec{0}$), having certain motional, spinning energy ($E_{i-s} = m_{i-s}c^2$), will become,

$$\left(\begin{array}{l} \vec{p}_i = \gamma_i m_i \vec{v} \rightarrow \gamma_i (m_i + m_{i-s})\vec{v} = \vec{p}_i + \vec{p}_{i-s} = \vec{p}_i^{\,*} \\ \vec{L}_i \rightarrow \vec{L}_i + \vec{L}_{i-s} = \vec{L}_i^* \\ E_i = \gamma_i m_i c^2 \rightarrow E_i + E_{i-s} = E_{0i} + E_{ki} + E_{i-s} = E_{0i} + E_s + E_{i-s} = E_i^* \\ E_{0i} = m_i c^2 \rightarrow E_{0i} + E_{i-s} = (m_i + m_{i-s})c^2 = E_{0i}^* \\ E_{ki} = (\gamma_i - 1)m_i c^2 = E_s, E_{i-s} = \text{mechanical spinning energy} \end{array}\right) \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} \overline{P}_{4-i} = \left(\vec{p}_i^*, \frac{E_i^*}{c}\right), \ \overline{P}_4 = \sum_{(i)} \overline{P}_{4-i} = \sum_{(i)} \left[\vec{p}_i^*, \frac{\sum_{(i)} E_i^*}{c}\right] \Rightarrow \\ (\vec{p}_i^*)^2 - \frac{(E_i^*)^2}{c^2} = -\frac{(E_{0i}^*)^2}{c^2}, \ (\sum_{(i)} \vec{p}_i^*)^2 - \frac{\left(\sum_{(i)} E_i^*\right)^2}{c^2} = -\frac{\left(\sum_{(i)} E_{0i}^*\right)^2}{c^2} \end{array} \right\}. \quad \text{(4.3-0)-m}$$

Mechanical particle spinning ($\vec{L}_{i-s}$) from (4.3-0)-m should not be mixed with de Broglie, matter-waves helix spinning ($\vec{L}_{mw} = \frac{v}{\omega_{mw}} \frac{\cos(\vec{p},\vec{v})}{\cos(\vec{L}_{mw}, \vec{\omega}_{mw})} \vec{p}, \vec{L}_{mw} = J_{mw}\vec{\omega}_{mw}$), which is directly and only related to linear particle motion (when a particle has certain linear momentum $\vec{p}$, collinear with matter waves spin moment $\vec{L}_{mw}$). Mechanical self-spinning vector ($\vec{L}_{i-s}$) could have any spatial orientation around the specific axis of the moving body (not related to linear momentum $\vec{p}$).

New, total orbital or angular moment (analog to 4-vector (4.3-0)-l), which is considering additional mechanical self-spinning of participants, is:

$$\left\{ \begin{array}{l} \overline{L}_{4-i} = \left(\vec{L}_i + \vec{L}_{i-s}, \frac{E_i^*}{c}\vec{r}_i\right), \sum_{(i)} (\vec{L}_i + \vec{L}_{i-s}) = \vec{r}_c \times \sum_{(i)} \vec{p}_i^*, \frac{\sum_{(i)} E_i^* \vec{r}_i}{c} = \frac{\sum_{(i)} E_i^*}{c} \vec{r}_c \\ \overline{L}_4 = \sum_{(i)} \overline{L}_{4-i} = \left(\sum_{(i)} (\vec{L}_i + \vec{L}_{i-s}), \frac{\sum_{(i)} E_i^* \vec{r}_i}{c}\right) = \vec{r}_c \times \overline{P}_4 = \left(\vec{r}_c \times \sum_{(i)} \vec{p}_i^*, \frac{\sum_{(i)} E_i^*}{c} \vec{r}_c\right) \end{array} \right\} \Rightarrow \quad \text{(4.3-0)-n}$$

$$\left\{ (\vec{L}_i + \vec{L}_{i-s})^2 - \frac{(E_i^*)^2}{c^2}\vec{r}_i^2 = -\frac{(E_{0i}^*)^2}{c^2}\vec{r}_i^2, \left(\sum_{(i)} (\vec{L}_i + \vec{L}_{i-s})\right)^2 - \frac{\left(\sum_{(i)} E_i^* \vec{r}_i\right)^2}{c^2} = -\frac{\left(\sum_{(i)} E_{0i}^* \vec{r}_i\right)^2}{c^2} \right\}.$$

The conclusion from (4.3-0)-l and (4.3-0)-n is showing that *mechanical spinning energy contribution is effectively entering or increasing the rest mass of the system, while any spin moment is increasing total orbital moment (as a vector)*. The same situation can be easily extended to account the angular velocity of matter waves spinning (ω_{is}, ω_{cs}) if we conveniently (and creatively) apply (4.3-0)-n, combined with (4.3-0)-m,

$p_i v_i = (L_i\omega_i + L_{i-s}\omega_{i-s}) = L_{is}\omega_{is}$ and $p_c v_c = (L_c\omega_c + L_{c-s}\omega_{c-s}) = L_{cs}\omega_{cs}$. (4.3-0)-o

Elaborations from and around equations (4.3-0)-l until (4.3-0)-o are still presenting kind of preliminary brainstorming and somewhat loosely imaginative thinking, but touching particularly challenging problematic, which will evolve towards more natural and richer concepts.

Let us now focus on a linear motion of an electron (or electrically charged particle) in an electric field. Of course, at the same time, the electron is permanently spinning (or just has its spinning moment). Contemporary Electromagnetic theory is not predicting the appearance of helical magnetic (or electromagnetic) field structure around moving electron. Anyway, the concept of helix matter waves, as elaborated here, is telling us that we should be able to detect presence of specific helix field structure around moving electron (similar as on Fig.4.1.1, and *somewhat like Henry Augustus Rowland effect of the magnetic field around rotating conductor, presented by Jean de Climont*). In this situation, somehow by default, we are already familiar with association of de Broglie matter wave wavelength $\lambda_e = h/p = h/m_e v_e$ to a moving electron, since this idea is experimentally verifiable when we experiment with diffraction of electron beams from crystalline structures, but nobody is going more in-depth to associate such wavelength to an electron, helix (and electromagnetic) wave periodicity. This (helix wave) situation will stay as the subject for an additional, future experimental verification (*see on Internet somewhat similar concept about Henry Augustus Rowland effect of the magnetic field around the rotating conductor, presented by Jean de Climont*).

Before any of such future revelations, we already know that the force law between equidistant, parallel paths of uniformly moving electrical charges (q_1 , q_2), while being at the same height, is going out of the frames of Coulomb law predictions (see (2.4-4) in the second Chapter of this book). Such force is:

$$F_{1,2} = \frac{\mu}{4\pi}\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2} = K\frac{(q_1 v_1)\cdot(q_2 v_2)}{r^2},\ K = \frac{\mu}{4\pi} = \text{const.}. \tag{2.4-4}$$

Here, the original Coulomb's law (like in cases of static electric charges) cannot directly explain magnetic force between them. For the same sign on the products $(q_1 v_1)$ and $(q_2 v_2)$ the charges are drawn closer (attractive force), and for opposite signs the charges are drawn apart (repulsive force). The most likely, still intuitive explanation is that moving charged particles are creating helicoidally spinning electromagnetic fields around paths of motion. Where such electromagnetic fluxes are mutually not cancelling, electric charges will experience an attractive force, and in cases of mutually opposed magnetic fields, the repulsive force between them will appear. Experimental verification of such spiral field phenomenology could be expected in electric plasma discharges in rarefied gases.

Direct analogical (and still hypothetical) force expression between two moving masses, based on (2.4-4) and on Mobility analogies from the first chapter is,

$$\begin{aligned} F_{12} &= K'\frac{(p_1 v_1)\cdot(p_2 v_2)}{r^2} = K''\frac{E_{k1}\cdot E_{k2}}{r^2} = K''\frac{(E_1 - E_{01})\cdot(E_2 - E_{02})}{r^2} = \\ &= \frac{K''}{r^2}\left(E_1E_2 - E_1E_{02} - E_{01}E_2 + E_{01}E_{02}\right) = \frac{K''}{r^2}\left(\gamma_1\gamma_2 m_1 m_2 C^4 - \gamma_1 m_1 m_2 C^4 - \gamma_2 m_1 m_2 C^4 + m_1 m_2 C^4\right) = \\ &= K'''\left(\gamma_1\gamma_2\frac{m_1 m_2}{r^2} - \gamma_1\frac{m_1 m_2}{r^2} - \gamma_2\frac{m_1 m_2}{r^2} + \frac{m_1 m_2}{r^2}\right) = \boxed{K'''\frac{m_1 m_2}{r^2}}\cdot\left(\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1\right) = \boxed{\gamma^*\cdot K'''\frac{m_1 m_2}{r^2}} \end{aligned} \tag{2.4-4-1}$$

In order to support the existence of such attractive force, that will be equal to Newton force of gravitation, it will be necessary that masses $\mathbf{m_1}$ and $\mathbf{m_2}$ are in permanent motion or rotation,

$$\boxed{K''' \frac{m_1 m_2}{r^2}} \cdot (\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) = \gamma^* \cdot K''' \frac{m_1 m_2}{r^2} \cong G^* \frac{m_1 m_2}{r^2} \Rightarrow \gamma^* = (\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1) > 0, \quad \text{(2.4-4-2)}$$
$$G^* = G(\gamma_1, \gamma_2) = G(v_1, v_2) \cong Const., \quad \gamma_{1,2} = (1 - v_{1,2}^2 / c^2)^{-0.5} .$$

We could imagine that our Universe is in permanent (holistic and intrinsic) motion or rotation to support the existence of Newtonian gravitation. Here we can also see why Newton constant of gravitation can never be measured very accurately, and be always the same, constant number, since it is related to involved velocities of relative motions between mutually attracting masses.

[♣ COMMENTS & FREE-THINKING CORNER: still in preparation...

4.1.2.1. Hypercomplex functions interpretation of energy-momentum vectors and time

In parallel to 4-vectors (in the Minkowski-Einstein space of Relativity Theory), we could try to formulate equivalent Hypercomplex Phasor functions or vectors (see chapter 4.0 where Analytic Signal, complex and Hypercomplex functions are introduced, and Chapter 10, where the same idea is much more elaborated as the support to de Broglie mater waves hypothesis). On the example of the linear momentum vector, it is possible to demonstrate how we can create the Hypercomplex momentum and its complex Phasor, as follows,

$$\overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \Leftrightarrow \left(\vec{p}_x, \vec{p}_y, \vec{p}_z, I\frac{E}{c}\right) = \left(\vec{p}, I\frac{E}{c}\right) \Rightarrow \vec{p}^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c^2})^2 \Leftrightarrow E_0^2 + p^2c^2 = E^2 = (E_0 + E_k)^2 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \overline{E} = E_0 \pm I \cdot pc = E \cdot e^{\pm I\theta} = (E_0 + E_k) \cdot e^{\pm I\theta} = \sqrt{E_0^2 + p^2c^2} \cdot e^{\pm I \cdot \mathrm{arctg}\frac{pc}{E_0}} = \gamma mc^2 \cdot e^{\pm I\theta} = \gamma \overline{m} c^2 = \\ = E \cdot \cos\theta \pm I \cdot E \cdot \sin\theta, \; E \cdot \sin\theta = H[E \cdot \cos\theta], \; H(=) \text{Hilbert transform}, \\ E_0 = mc^2, \; \overline{m} = m \cdot e^{\pm I\theta} = m \cdot \cos\theta \pm I \cdot m \cdot \sin\theta = m_r \pm I \cdot m_i, \; I^2 = -1, \\ \gamma mc^2 = \sqrt{E_0^2 + p^2c^2} = E_0 + E_k = E \Leftrightarrow (\gamma m)^2 = (\frac{E_0}{c^2})^2 + (\frac{p}{c})^2, \gamma = 1/\sqrt{1 - (\frac{v}{c})^2}, \\ \theta = \mathrm{arctg}\frac{pc}{E_0} = \mathrm{arctg}(\gamma \frac{v}{c}) = \mathrm{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}}, \; 0 \le \theta \le \frac{\pi}{2}, \; \overline{m} = \begin{cases} m, & \theta = 0 \\ (1 \pm I)\frac{\sqrt{2}}{2} m = m \cdot e^{\pm I\pi/4}, & \theta = \frac{\pi}{4} \\ \pm I \cdot m, & \theta = \frac{\pi}{2} \end{cases} \end{array}\right] \Rightarrow$$

$$\Rightarrow \overline{P} = \gamma \overline{m} \vec{v} = \gamma m \vec{v} \cdot e^{\pm I\theta} = \vec{p} \cdot e^{\pm I\theta} \Rightarrow$$

$$\Rightarrow \begin{cases} \overline{L}_{mw} = \frac{v}{\omega_{mw}} \overline{P} = \frac{v}{\omega_{mw}} \vec{p} \cdot e^{\pm I\theta} = \vec{L}_{mw} \cdot e^{\pm I\theta} = \mathbf{J}_{mw} \vec{\omega}_{mw} \cdot e^{\pm I\theta} = \overline{\mathbf{J}}_{mw} \vec{\omega}_{mw}, \\ \vec{L}_{mw} = \frac{v}{\omega_{mw}} \vec{p} = \mathbf{J}_{mw} \vec{\omega}_{mw}, \; \omega_{mw} = 2\pi f_{mw}, \lambda = H/p, u = \lambda f_{mw} \end{cases} \quad \textit{(4.3-0)-p}$$

The same case as in (4.3-0)-p, when the motional particle will get certain mechanical spinning moment ($\vec{L}_{i-s} \neq \vec{0}$) will become (like (4.3-0)-m),

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

$$\left\{\begin{array}{l}\left(\begin{array}{l}\vec{p}=\gamma m\vec{v}\to\gamma(m+m_s)\vec{v}=\vec{p}+\vec{p}_s=\vec{p}^*\\ \vec{L}\to\vec{L}+\vec{L}_s=\vec{L}^*\\ E\ =\gamma mc^2\to E+E_s=E_0+E_k+E_s=E_0+E_{mw}+E_s=E^*\\ E_0=mc^2\to E_0+E_s=(m+m_s)c^2=E_0^*\\ E_k=(\gamma-1)mc^2=E_{mw}, E_s=\text{mechanical spinning energy}\end{array}\right)\Rightarrow\\ \overline{P}_4=\left(\vec{p},\frac{E}{c}\right)\to\left(\vec{p}^*,\frac{E^*}{c}\right)\Leftrightarrow\left(\vec{p}^*,I\frac{E^*}{c}\right)\Rightarrow(\vec{p}^*)^2-(\frac{E^*}{c})^2=-(\frac{E_0^*}{c^2})^2\Leftrightarrow(E_0^*)^2+(\vec{p}^*)^2c^2=(E^*)^2=(E_0^*+E_k^*)^2\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left[\begin{array}{l}\overline{E}^*=E^*\cdot e^{\pm I\theta^*}=(E_0^*+E_k^*)\cdot e^{\pm I\theta^*}=\sqrt{(E_0^*)^2+(\vec{p}^*)^2c^2}\cdot e^{\pm I\cdot\mathrm{arctg}\frac{p^*c}{E_0^*}}=\gamma m^*c^2\cdot e^{\pm I\theta^*}=\gamma\overline{m}^*c^2,\\ E_0^*=m^*c^2,\ \overline{m}^*=m^*\cdot e^{\pm I\theta},\ I^2=-1,\\ \gamma m^*c^2=\sqrt{(E_0^*)^2+(\vec{p}^*)^2c^2}=E_0^*+E_k^*=E^*,\\ \theta^*=\mathrm{arctg}\frac{p^*c}{E_0^*}=\mathrm{arctg}(\gamma\frac{v}{c})=\mathrm{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}}=\theta,\ \ 0\le(\theta=\theta^*)\le\frac{\pi}{2}\end{array}\right]\Rightarrow \qquad (4.3\text{-}0)\text{-}q$$

$$\Rightarrow\overline{P}^*=\gamma\overline{m}^*\vec{v}=\gamma m^*\vec{v}\cdot e^{\pm I\theta^*}=\vec{p}^*\cdot e^{\pm I\theta^*}=\vec{p}^*\cdot e^{\pm I\theta}$$

Hypercomplex Phasors interpretation of energy-momentum vectors is revealing energy-momentum phase function " θ *". The same Phase function will naturally define or support de Broglie matter waves wavelength and frequency (see kore in Chapter 10.).*

Here, it looks like we come closer to understanding a natural and proper time flow (time scale, or time dimension) concerning a mass in motion and its associated helix matter-wave, if we find a connection between energy-momentum phase function " θ ***" and its proper, local time "t". Here, certain (local and dominant) matter-wave, spinning frequency*** ω_s ***, associated to linear momentum of a motional particle, could serve as the time clock, or time-reference signal for registering (measuring or representing) real-time flow.*** *For instance, (advancing with a loosely presented brainstorming exercise), we could explore the following possibility,*

$$\theta=\mathrm{arctg}\frac{pc}{E_0}=\mathrm{arctg}(\gamma\frac{v}{c})=\mathrm{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}}\ (=)\ \omega_s t\mp kx,$$

$$\vec{L}_{mw}=\frac{v}{\omega_{mw}}\vec{p}=I\vec{\omega}_{mw},\overline{L}_{mw}=\frac{v}{\omega_{mw}}\overline{P}=\frac{v}{\omega_{mw}}\vec{p}\cdot e^{\pm I\theta}=\vec{L}_{mw}\cdot e^{\pm I\theta}=\vec{L}_{mw}\cdot e^{\pm I\vec{\omega}_{mw}t}, \qquad (4.3\text{-}0)\text{-}r$$

$$\vec{p}=\frac{\omega_{mw}}{v}\vec{L}_{mw}\cdot e^{\pm I\vec{\omega}_{mw}t},\ \overline{P}=\gamma\overline{m}\vec{v}=\gamma m\vec{v}\cdot e^{\pm I\theta}=\vec{p}\cdot e^{\pm I\vec{\omega}_{mw}t},\ \overline{P}_4=(\vec{p},I\frac{E}{c}),$$

$$I^2=-1,\ I=(i,j,k),\ i^2=j^2=k^2=-1, ij=k,\ jk=i, ki=j...$$

In (4.3-0)-p,q,r we could recognize certain combination between complex and vector functions. Spinning circular frequency $\vec{\omega}_s$ *is a vector collinear with the particle linear moment* $\vec{p}$*, and hyper-complex imaginary unit* ***I*** *also has the structure similar to vectors, since it is composed of three, mutually orthogonal, more elementary imaginary units* ***i, j, k*** *(see more in the chapter 6., around equations (6.8) – (6.13) and in Chapter 10.). Ordinary vectors can naturally be related (or fixed) to specific observer system of reference, but hyper-complex imaginary units, here naturally coupled with linear particle momentum and associated spinning, could get other dynamic and structural assignments and meanings, and have specific couplings with observer's system of reference.*

Much more promising evolution of energy-momentum 4-vector from Relativity theory is to extend it (analogically, towards Hypercomplex and Phasor functions), and at the same time to stay very close to present formulation of 4-vectors, as follows:

$$\left.\begin{cases}\overline{P}_4=\left(\vec{p},\dfrac{E}{c}\right)\Leftrightarrow\left(\vec{p},I\dfrac{E}{c}\right)=\left(\vec{p}_i,i\dfrac{E_i}{c_i}\right)+\left(\vec{p}_j,j\dfrac{E_j}{c_j}\right)+\left(\vec{p}_k,k\dfrac{E_k}{c_k}\right)\Leftrightarrow\\ \Leftrightarrow\left(\vec{p},I\dfrac{E}{c}\right)=\left(\vec{p}_i+\vec{p}_j+\vec{p}_k,\dfrac{iE_i+jE_j+kE_k}{c_i}\right)=\left(\vec{p}_i+\vec{p}_j+\vec{p}_k,I\dfrac{E}{c}\right)\\ \overline{P}_4=\left(\vec{p},\dfrac{E}{c}\right)=\text{invariant}\Rightarrow\vec{p}^2-(\dfrac{E}{c})^2=-(\dfrac{E_0}{c^2})^2\Leftrightarrow E_0^2+p^2c^2=E^2=(E_0+E_k)^2,\\ \vec{p}=\vec{p}_i+\vec{p}_j+\vec{p}_k=\vec{p}_t\,,E=E_{0t}+E_{kt},\\ E_i=E_{0i}+E_{ki},E_j=E_{0j}+E_{kj},E_k=E_{0k}+E_{kk},\\ I^2=i^2=j^2=k^2=-1,ij=k,\ jk=i,ki=j,ji=-k,kj=-i,ik=-j\end{cases}\right\}\Rightarrow(\vec{p}_i+\vec{p}_j+\vec{p}_k)^2-(\frac{E}{c})^2=-(\frac{E_0}{c^2})^2$$

$$\left.\begin{cases}I\dfrac{E}{c}=i\dfrac{E_i}{c_i}+j\dfrac{E_j}{c_j}+k\dfrac{E_k}{c_k},\\ \left(\dfrac{E}{c}\right)^2=\left(\dfrac{E_i}{c_i}\right)^2+\left(\dfrac{E_j}{c_j}\right)^2+\left(\dfrac{E_k}{c_k}\right)^2\\ \left(\dfrac{E_0}{c}\right)^2=\left(\dfrac{E_{0i}}{c_i}\right)^2+\left(\dfrac{E_{0j}}{c_j}\right)^2+\left(\dfrac{E_{0k}}{c_k}\right)^2\\ \left(\dfrac{E_{kt}}{c}\right)^2=\left(\dfrac{E_{ki}}{c_i}\right)^2+\left(\dfrac{E_{kj}}{c_j}\right)^2+\left(\dfrac{E_{kk}}{c_k}\right)^2\end{cases}\right\}\Rightarrow\left\{\begin{array}{l}p^2=\dfrac{E^2-E_0^2}{c^2}=\dfrac{E_i^2-E_{0i}^2}{c_i^2}+\dfrac{E_j^2-E_{0j}^2}{c_j^2}+\dfrac{E_k^2-E_{0k}^2}{c_k^2}=\\ =\vec{p}_i^2+\vec{p}_j^2+\vec{p}_k^2\end{array}\right\}\qquad(4.3\text{-}0)\text{-}s$$

In (4.3-0)-s, constants c_i, c_j, c_k that have dimensions of speed (like the universal speed constant $c \cong 3\cdot10^8 m/s$), could also be equal to $c = c_i = c_j = c_k$ (but involved mathematics is giving chances for other options). Energies indexed with "0" and "k" in (4.3-0)-s are effectively paving the ways to explain creations of significant numbers of products in impact reactions (zeros are indicating particles with rest masses, and k-indexing stands for kinetic energy states). Such extended energy-momentum framework can be later merged with universal Complex and Hypercomplex Analytic Signal representation of wave functions (leading to all famous wave equations of Quantum Theory), and with different foundations of multidimensional Universe (see Chapter 4.3 and chapter 6, equations (6.10) - (6.13)). Presence of three imaginary units in (4.3-0)-s is intuitively igniting ideas about mutually coupled energy triplets such as three quarks, three anti-quarks, neutrinos etc., what could create another, the more general and more precise concept of modeling of Super-symmetry in the world of microphysics (and significantly or fundamentally enrich and simplify the Standard Model). See much more about similar items in chapter 10.

Regarding the time reference signal, as initiated in (4.3-0)-r, we could try to understand it in the following way: For us, humans (and all living species on the planet Earth), dominant and natural time flow is related to the orbital motion of our planet around local Sun and around its local axis of self-spinning, or about local center of mass. Planet Earth is at the same time self-rotating and has certain associated, helix matter-wave, characterized by spiraling frequency $\vec{\omega}_s$. This frequency and associated wavelength are parameters of associated de Broglie matter-waves (see much more in Chapter 10.). Since our masses (related to humans and other living species) are negligible compared to the local planetary mass (and local center of mass), it is logical that our proper or real-time flow is dominated by the time flow belonging to our planet. We also know that our SI units of time are (since a long time) indeed linked and extracted from the parameters of orbital and spinning motion of our planet Earth.

♣]

Let us now briefly address the relevant wave functions that can be associated to rotational motions as presented on Fig.4.1.1, Fig.4.1.1a, and elaborated around equations (4.3.0) and (4.3.0)-a,b,c,d,e,f,g,h. Based on factorized, Analytical wave function illustrated in chapter 6, around equations (6.18) until (6.23), we will be able to give an idea how different levels of rotation and spinning could be mathematically modeled, as for instance,

$$E_k = \tilde{E} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} = E_c = E_s$$

$$\tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega, \qquad (4.3\text{-}0)\text{-}t$$

$$\Psi(t) = a_0(t)\cos\varphi_0(t) = a_1(t)\cos\varphi_1(t)\cos\varphi_0(t) = a_2(t)\cos\varphi_2(t)\cos\varphi_1(t)\cos\varphi_0(t),$$

$$E_c = \frac{L_c\omega_c}{1+\sqrt{1-v^2/c^2}} \rightarrow \Psi_c^2 = \frac{dE_c}{dt} = [a_0(t)\cos\varphi_0(t)]^2 = [a_0(t)\cos(\omega_c t+\varphi_c(t))]^2,$$

$$E_{mw} = \frac{L_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} \rightarrow \Psi_c^2 = \frac{dE_c}{dt} = [a_1(t)\cos\varphi_1(t)\cos\varphi_0(t)]^2 = [a_1(t)\cos(\omega_s t+\varphi_s(t))\cdot\cos(\omega_c t+\varphi_c(t))]^2.$$

*[♣ **COMMENTS & FREE-THINKING CORNER:** There could be one great comparison between photon (or some wave group) and fluid vortex-shedding phenomenology, known from fluid flow measurements, that supports the existence of torsion field components in connection with the linear motion. It is experimentally found that fluid velocity is directly proportional to the vortex-shedding frequency* **f** *(when a non-moving obstacle or "bluff-body" is placed in a moving fluid). In such a case the fluid flow velocity can be measured based on Strouhal relation* $\mathbf{v_{fluid} = s \cdot f}$*, where the proportionality parameter,* **s = constant** *(**dimensionally equal to certain wavelength**), is essentially constant over wide velocity ranges and independent of fluid density (see [**12**]). Since motional or kinetic energy is directly proportional to the square of relevant velocity* $E_k \sim mv^2$*, then vortex-shedding waves should have an energy proportional to their squared frequency* $s^2\mathbf{f}^2$*. There are wave-phenomena, like flexural waves, where wave group velocity is proportional to the square root of appropriate frequency,* $v \approx \sqrt{f}$ *, what agrees with Planck wave energy that is proportional to frequency* $\tilde{E} = hf\ (\approx v^2)$*. Other matter-waves creating (analogical) situation are waves on a quiet water surface created by some moving object (a boat), where the water surface is visualizing matter waves associated with a moving object. Something similar regarding matter waves understanding could be associated with a pendulum motion if we observe the pendulum from another inertial reference system that is in relative motion to the pendulum system of reference. Let us imagine (by analogy with the above-described situation of vortex shedding), that any object from our 4-dimensional universe, is "immersed" or moving in some fluid. Mentioned fluid presents for us still non-detectable hyperspace, or multidimensional universe, but it is conceptually useful since we could easily create an association with appearance of de Broglie matter waves, by comparing them to the vortex-shedding phenomenology of real objects (from our detectable universe), being in an unknown and still undetectable "fluid". An interesting reference regarding similar subjects (vortices in fluids and Schrödinger Equation) can be found in the article [**15**] written by R. M. Kiehn. It is also evident that wave energy (or waves velocities) in cases of different waves, could be on a different way dependent on relevant waves' frequency, proportional either to* f or f^2*, or also to* f/v or f^2/v^2*, like shown in T.4.1. The micro-world of atoms, subatomic particles and states, and photons are dominantly respecting Planck's, Einstein, de-Broglie energy and wavelength formulations, where* $\tilde{E} = hf,\ \lambda = h/p,\quad u = \lambda f = \omega/k,\ v = d\omega/dk$*, and other wave phenomena from a Marco-objects world could have different wave energy to frequency relations. The intention here is to initiate thinking that for macro-objects, like planets and other big objects, also exists certain relevant and characteristic wavelength, analog to de Broglie matter-waves wavelengths, but no more proportional to Planck's constant. The problem in defining de Broglie type of wavelength for macro-objects is that such wavelength (*$\lambda = h/p$*) is meaningless and extremely small, and since macro-objects in motion should also create associated waves, like any other micro-world object, the macro-wavelength in question should be differently formulated. ♣].*

In brief, this book favors the conceptual model that any motional, or Particle-Wave object could have, a) elements of linear motion (as a moving particle), b) elements of spinning around the line of its linear motion, and c) associated de Broglie matter waves. Associated forms of matter-waves are kind of helix and oscillating field perturbation, which is composed of axial and torsion field components, where torsion field components are coupled with particle rotation (where the particle kinetic or motional *energy corresponds to the energy of a matter wave associated to that particle). If we do not directly see, measure, or detect de Broglie waves in the space around moving Particle-Wave object, this only means that de Broglie matter waves are still inside the internal and intrinsic waving structure of that object (and will become externally measurable in case of some interaction, impact, diffraction, scattering, interference, etc.).*

Matter waves and particle-wave duality concept presented in this book precisely states that kinetic or motional energy (of microparticles and/or matter waves) can be expressed in two different and mutually fully equivalent ways, such as $\mathbf{E_k = (\gamma - 1)mc^2 = hf}$ ***(while using the wave packet model as an equivalent replacement for a moving particle, apart from the particle rest mass). The analogy of the statement above with properties of stationary electron states in an atom is almost total. When an electron's stationary wave changes its state from a higher energy level,*** E_{s1}***, state (1), to a lower energy level,*** E_{s2}***, state (2), the surplus of energy will be radiated as a photon that will have the frequency,*** f_{1-2}***, equal to a frequency difference between corresponding frequencies of electron stationary waves:*** $E_{s1} - E_{s2} = E_{k1} - E_{k2} = hf_1 - hf_2 = h\Delta f = hf_{1-2}$***.***

Practically, a very similar situation happens (regarding de Broglie waves) when any moving particle (previously) in an energy state E_1***, passes to another energy state*** E_2***, where*** E_1 ***and*** E_2 ***could be both total or kinetic particle energy (because differences between total or only kinetic energy states of the same particle are mutually equal):***

$$\Delta E = E_1 - E_2 = E_{total-1} - E_{total-2} = E_{k1} - E_{k2} = hf_1 - hf_2 = h\Delta f = hf_{1-2} = c^2 \Delta m$$

$$\lambda_1 = h/p_1, \lambda_2 = h/p_2, \ \ \lambda_{1,2} \cdot f_{1,2} = u_{1,2} = v_{1,2} / (1 + \sqrt{1 - v_{1,2}^2 / c^2})$$

$$E_{k(1,2)} = (\gamma_{1,2} - 1)mc^2 = hf_{1,2} \ , E_{total-1,2} = \gamma_{1,2} mc^2 \ .$$

Somehow, the particles' macro-universe reacts (regarding de Broglie matter waves) very much in the same way as a micro-universe of inter-atomic stationary states of elementary particles (except that in the world of macroparticles we do not need to have only stationary and standing matter waves). Here could be the starting point of new understanding of Quantum Nature of all matter, fields, and waves in our universe: "Quantum properties should be the optimal integer-number packing rules related to stable objects, standing waves (resonant) structures, and their energy-momentum "communications". There should also be a lot of other non-quantum phenomenology related to forces, matter and waves in our Universe". For instance, an electron inside an atom has different shape and size parameters compared to a relatively free-moving electron in an open space, but to both cases, we can associate different de Broglie matter wavelengths and frequencies. The only essential difference could be that a stable inter-atomic state of an electron should also have stable and constant matter wave parameters, and freely moving electron could have time-evolving matter wave characteristics. Briefly, when waves fold, particles could be

created, and similarly, wave sources are often related to unfolding the waves captured by "particle shells".

An entirely different question in such situations is if we also deal (in the frames of the same problematic situation) with some presently undetectable multidimensional media (say a kind of fluid, or ether, where de Broglie waving is a natural phenomenon). Mathematically more pragmatic has been the platform that this undetectable field or fluid with de Broglie waves is modeled as the "possibility or probability" distribution of specific event realization. As we know, Orthodox Quantum Mechanics has been successfully exploiting this platform (as a perfect modeling replacement for something we do not see or measure directly), losing completely any common-sense conceptual platform.

If we do not consider rotation, spinning and rotational fields as naturally associated phenomena to all particles in linear motion/s, and if something like that should always exist (regardless of our ignorance), indeed we have a big conceptual problem. There will be a missing link in explaining particle interactions (and we do not know that we have such a problem, because we do not see the problem, and we cannot apply our commonsense logic). Consequently, we see the results of particle interactions (particle diffraction and interference, for instance) only as random distributions, because we are unable to consider an essential element of particle motion (its intrinsic rotation), or just we do not know the motional, immediate, real-time particle phase function $\{-\pi \leq [\varphi(t, x) = \varphi(\omega t - kx)] \leq +\pi\}$. *To compensate this missing link (missing rotation and phase), we can just assume that after a long time, after a countless number of experimentally realized interactions (or diffractions), we should get smooth and wave-like probability distribution, or interference picture of what really happens, and this was the case (or assumption) of Orthodox Quantum Mechanics. It looks like a miracle happens, because on average (statistically) we can predict the exact future shape of a resulting diffraction picture (event distribution). At the same time, we are unable to predict the immediate, real space-time position of a single event, and Quantum Mechanics concluded that nature (of micro-world) is primarily guided by probability (and not real-time) distributions in the form of waves, saying (and documenting, in addition), that a countless number of experiments permanently confirms such position. Consequently, the founders of Quantum Theory convinced their non-critical followers that all strange assumptions and mathematics in Quantum Mechanics should be correct, even if we do not have full and common-sense explanation for most of them!? Closely related to the above-presented concepts are the Uncertainty relations, which if not well understood and properly applied, can make Particle-Wave Duality picture only more confusing (see chapter 5. of this book). How to understand multi-plausible Quantum Reality beyond today's Quantum Mechanics is well analyzed in [13], but if a more common-sense logic were used in the very beginning, as proposed here, this multi-plausible reality would certainly lose many elements of its magic.*

Anyway, until the present, all over this book, it has been attempted to explain the unknown origins of the associated effects of effective mass rotation and its direct connection to de Broglie matter waves. In other words, a mass in linear motion should have some hidden spinning (matter wave) properties, like a photon (see T.4.0.). We can show that, if kinetic particle-energy is equal to its (hypothetical) matter-wave spinning energy $E_{mw} = E_k$, and if such hidden spinning energy is equal to

$E_{mw} = \frac{J\omega^2}{\sqrt{1-v^2/c^2}} = L\omega = E_k$, then we will get the same results as already found in (4.2), (4.3), (4.3-0)-a,b,c,d,…. Consequently, particle in motion can be modeled as a wave-packet that has the wave energy equal to a particle kinetic energy:

$$\left\{\begin{matrix} E_k = (\gamma-1)mc^2 = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{mv^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\cdot\sqrt{1-\frac{v^2}{c^2}}} = hf = \tilde{E}\ , \\ E_{mw} = \frac{J\omega^2}{\sqrt{1-\frac{v^2}{c^2}}} = L\omega = E_k\,,\ \lambda = \frac{h}{p},\quad \omega = 2\pi f\,,\ L = \frac{J\omega}{\sqrt{1-\frac{v^2}{c^2}}}. \end{matrix}\right\} \Leftrightarrow$$

$$\Leftrightarrow \frac{E_k}{hf} = \frac{\frac{pv}{hf}}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{v}{u}}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{L\omega}{hf} = \frac{E_{mw}}{hf} = \frac{\tilde{E}}{hf} = \frac{L}{(\frac{h}{2\pi})} = 1 \Rightarrow \tag{4.3-1}$$

$$\Rightarrow E_k = hf = \tilde{E},\ L = \frac{h}{2\pi},\ u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \lambda f = \frac{\omega}{k},\ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\omega}{dk}$$

$$\omega = \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{\hbar}{m}k^2 = 2\pi f\,,\ p = \frac{h}{\lambda} = \frac{h}{2\pi}k = \hbar k.$$

First, here we need to consider that any linear motion is just a particular case of specific rotational (or orbital) motion where the radius of rotation could be appropriately large. For idealized rectilinear motion radius of rotation will be infinite. The meaning of (4.3-1) is that any particle in linear motion should have some associated ***rotating (or spinning) field****, and energy of such* ***spinning field*** *is equal to the kinetic particle energy.* ***Waves created by such spinning field (toroidal, helix or spiral wave structure caused by linear particle motion) are de Broglie matter waves*** *(see Fig.4.1.1, Fig.4.1.1a and Fig.4.1.2). The same concept is already introduced in the second chapter of this book; -see equations from (2.11.2) to (2.11.22)* and very indicative examples in the second chapter, around equations (2.11.13-1)-(2.11.13-5), where is clearly shown how matter waves in two-body problems are being created. *Here we should make a difference between the two kinds or forms of rotational or spinning motions. The kinetic energy of a particle in linear motion can be presented as* $E_k = (\gamma-1)mc^2 \cong \left(\frac{1}{2}mv^2\right)_{v<<c}$ *. If we could say that the same particle is in the same time making specific circular (or orbital) motion around specific center of such rotation (having a radius of rotation* ***r****) we can present the same kinetic energy as* $E_k = E_{rot.} \cong \left(\frac{1}{2}J_r\omega_r^2\right)_{v<<c}, v = \omega_r r$*, where moment of inertia of the particle is* I_r *(related to mentioned center of rotation). This is the first, natural or ordinary, visible mechanical particle rotation, which has not too much to do with de Broglie matter waves. The second (hidden) rotational or spinning motion associated with the same particle is a matter-waves', spiral or helical motion surrounding the particle along the particle orbit (see Fig.4.1.4). Again, we can introduce another alternative formulation for the same particle kinetic energy (as before),* $E_k = E_{spinning} \cong \left(\frac{1}{2}J_{mw}\omega^2\right)_{v<<c} = E_{mw} = \tilde{E}$*. Of course, ordinary mechanical rotation frequency* $\omega_r = 2\pi f_r$ *will not be comparable to matter waves*

spinning frequency $\omega = 2\pi f$, and anyway here we have mutually orthogonal vectors ($\omega_r \neq \omega$). Now we can unite all the introduced kinetic energy aspects and start conceptualizing particle-wave duality as,

$$\left\{\begin{array}{l} E_k \cong \left(\frac{1}{2}mv^2 = \frac{1}{2}pv\right)_{v<<c} = \\ = E_{rot.} = \left(\frac{1}{2}J_r\omega_r^2 = \frac{1}{2}L_r\omega_r\right)_{v<<c} = \\ = E_{spinning} = \left(\frac{1}{2}J_{mw}\omega^2 = \frac{1}{2}L_{mw}\omega\right)_{v<<c} = E_{mw} = \tilde{E}, v = \omega_r r \end{array}\right\} \Rightarrow \qquad (4.3\text{-}1.1)$$

$$\Rightarrow mv^2 = J_r\omega_r^2 = J_{mw}\omega^2 = pv = L_r\omega_r = L_{mw}\omega$$

In cases of high-speed motions (relativistic motions), we could establish analogical formulations for more generally valid kinetic energy expressions, for instance,

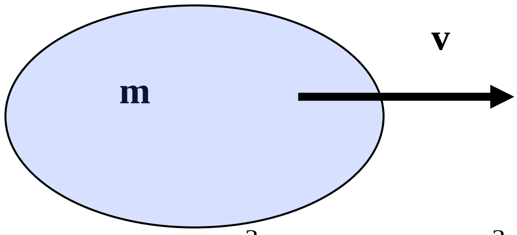
$$\left\{\begin{array}{l} E_k = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}(1+\sqrt{1-\frac{v^2}{c^2}})} = \frac{m*v^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{p*v}{1+\sqrt{1-\frac{v^2}{c^2}}} \cong \left(\frac{1}{2}mv^2 = \frac{1}{2}pv\right)_{v<<c} \\ m \to m* = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m, p \to p* = m*v = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mv, \frac{1}{2} \to \frac{1}{1+\sqrt{1-\frac{v^2}{c^2}}}, \\ mv^2 = J_r\omega_r^2 = J_{mw}\omega^2 = pv = L_r\omega_r = L_{mw}\omega,\ \omega = 2\pi f. \end{array}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} J_r \to J_r^* = \frac{J_r}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_r, J_{mw} \to J_{mw}^* = \frac{J_{mw}}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_{mw}, \\ L_r = J_r\omega_r \to L_r^* = J_r^*\omega_r = \frac{J_r\omega_r}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_r\omega_r = \gamma L_r, L_{mw} = J_{mw}\omega \to L_{mw}^* = J_{mw}^*\omega = \frac{J_{mw}\omega}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma J_{mw}\omega = \gamma L_{mw} \end{array}\right\} \Rightarrow \qquad (4.3\text{-}1.2)$$

$$\left\{\begin{array}{l} E_k = \frac{p*v}{1+\sqrt{1-v^2/c^2}} = \frac{L_r^*\omega_r}{1+\sqrt{1-v^2/c^2}} = \frac{L_{mw}^*\omega}{1+\sqrt{1-v^2/c^2}} = E_{rot.} = E_{spinning} = E_{mw} = \tilde{E} = hf* = p*u*, \\ pv = L_r\omega_r = L_{mw}\omega = mv^2 = J_r\omega_r^2 = J_{mw}\omega^2, p*v = L_r^*\omega_r = L_{mw}^*\omega = m*v^2 = J_r^*\omega_r^2 = J_{mw}^*\omega^2, \\ v = \omega_r r,\ \lambda \to \lambda* = h/p* = \frac{h}{L_{mw}^*}(\frac{v}{\omega}) = u*/f*, p* = L_r^*\frac{\omega_r}{v} = L_{mw}^*\frac{\omega}{v} = m*v = J_r^*\frac{\omega_r^2}{v} = J_{mw}^*\frac{\omega^2}{v}, \\ u = \lambda*f* = v/(1+\sqrt{1-v^2/c^2}),\ L_{mw}^* = \frac{h}{2\pi}(1+\sqrt{1-v^2/c^2}) \end{array}\right\}$$

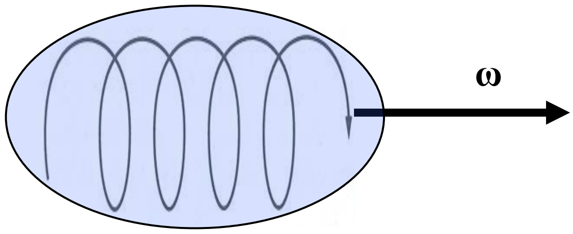
What is interesting here is that $\vec{p}*$ and $\vec{L}_{mw}^*$ are mutually collinear vectors ($\vec{p}* = \vec{L}_{mw}^*\frac{\omega}{v}$), but in cases when moving particle is additionally spinning (this time mechanically), we should consider that new, resulting orbital moment of such particle will change. ***Also, we could conclude that many misinterpretations of contemporary particle-wave duality are related to improper linking and mixing between spinning $\vec{\omega} = \vec{\omega}_s$ and orbiting $\vec{\omega}_r$ (that are mutually orthogonal vectors).***

If we now imagine that matter waves are being created as stable, stationary, and standing waves on a closed circular path (see Fig.4.1.4), we could start modeling *periodical, cosmological or atom-world orbital motions and structures of elementary particles. For additional conceptual understanding see (2.11.10) – (2.11.23) from the second chapter, as well as books from Ph. M. Kanarev [44], C. Lucas and David L. Bergman dealing with innovative modeling of subatomic and elementary particles [16] - [22]; -"Common Sense Science".*

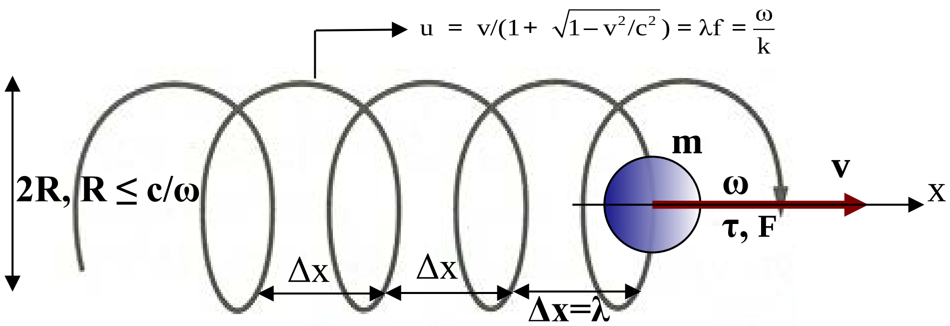


Moving particle: $E = mc^2 + E_k = \gamma mc^2$

⇓ (Equivalent) ⇑



Progressive matter-wave packet: $E = mc^2 + \tilde{E} = \gamma mc^2$



$$E_k = E_k(p,v) = \frac{pv}{1+\sqrt{1-v^2/c^2}},\ p = \gamma mv,\ \gamma = \frac{1}{\sqrt{1-v^2/c^2}},$$

$$\tilde{E} = \tilde{E}(\mathbf{L},\omega) = hf = h\frac{\omega}{2\pi} = \frac{\mathbf{L}\omega}{1+\sqrt{1-v^2/c^2}} = \frac{pv}{1+\sqrt{1-v^2/c^2}} = E_{mw} = E_k,$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p,\ \omega = 2\pi f = ku = \frac{kv}{1+\sqrt{1-v^2/c^2}} = \frac{2\pi}{h}\frac{pv}{1+\sqrt{1-v^2/c^2}} = \frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\cdot\frac{\hbar}{m}k^2,$$

$$\mathbf{L} = \mathbf{L}_{mw} = \frac{h}{2\pi}(1+\sqrt{1-v^2/c^2}) = \mathbf{J}\omega = \frac{pv}{\omega} = \frac{\gamma mv^2}{\omega},\ \mathbf{J} = \frac{pv}{\omega^2} = \frac{\gamma mv^2}{\omega^2}, \omega = \textit{spinning}.$$

Fig.4.1.2 Moving particle and de Broglie matter wave

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

*In (4.3-1) and on Fig.4.1.2 particle linear velocity $\mathbf{v}$ and angular velocity ω, which belongs to its associated, spinning matter-waves field, **should be mutually collinear vectors**, and any (axial) force F acting on the same moving particle should be collinear to the torque τ related to the same spinning, as shown in (4.3-2).*

$$
\begin{aligned}
&\frac{dE_k}{dt} = v\frac{dp}{dt} = \omega\frac{d\mathbf{L}}{dt} = v\cdot F = \omega\cdot\tau = \psi^2 \ (=)\ [W],\ p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{h}{\lambda},\ \mathbf{L} = \frac{J\omega}{\sqrt{1-\frac{v^2}{c^2}}},\\
&dE_k = vF\cdot dt = \omega\tau\cdot dt = F\cdot dx = \tau\cdot d\theta = h\cdot df = vdp = \omega d\mathbf{L},\ Z_m = \frac{v}{F} = \frac{\omega}{\tau}\left(\frac{d\mathbf{L}}{dp}\right)^2,\\
&F = \frac{\omega}{v}\cdot\tau = \frac{v}{\omega}\left(\frac{dp}{d\mathbf{L}}\right)^2\cdot\tau = \frac{d\theta}{dx}\cdot\tau = h\cdot\frac{df}{dx} = \frac{1}{v}\cdot\frac{dE_k}{dt} = \frac{dp}{dt} = -\frac{h}{\lambda^2}\frac{d\lambda}{dt}\ (=)\ [N],\\
&\omega = \frac{2\pi m}{h}\frac{v^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}}} = \frac{2\pi m}{h}\frac{uv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2\pi}{h}E_k = 2\pi f = \frac{2\pi}{T} \le \frac{c}{R},\\
&\Rightarrow \left\{\begin{array}{l} d\mathbf{L} = \frac{h}{2\pi}\cdot\frac{df}{f} = \frac{h}{2\pi}\cdot\frac{dE_k}{E_k} = \frac{v}{\omega}dp \Rightarrow \mathbf{L} = \mathbf{L}_c - \frac{h}{2\pi}\cdot\left(\frac{E_c}{E_k}\right)^2 = \mathbf{L}_c - \frac{h}{2\pi}\cdot\left(\frac{F_c}{f}\right)^2, *?!?!* \\ (E_c, F_c, \mathbf{L}_c) = \text{constants},\ h = E_c / F_c = \text{Planck constant.}\ , \text{*to be more analyzed*} \end{array}\right\}.
\end{aligned}
\tag{4.3-2}
$$

If we consider that maximal tangential velocity of the spinning matter waves v_t (which is perpendicular to the particle linear velocity v) should not be higher than c, $v_t = \omega R \le c$, where R is the "activity radius" captured by the spinning field (Fig.4.1.2). From (4.3-2) we can find R as,

$$
\left\{\begin{array}{l}
v_t T = 2\pi R = \frac{v_t}{f} = \omega R\cdot\frac{1}{f},\ E_k = \frac{mv^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}}} = hf,\\
\Delta x = uT = \frac{u}{f} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{f} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{h}{E_k} = \frac{h}{p} = \lambda\\
v_t = \omega R = 2\pi R f = 2\pi R\cdot\frac{E_k}{h} = \frac{2\pi R m}{h}\cdot\frac{v^2}{(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}}} \le c
\end{array}\right\} \Rightarrow
$$

$$\Rightarrow R \le \frac{hc}{2\pi m v^2}(1+\sqrt{1-\frac{v^2}{c^2}})\sqrt{1-\frac{v^2}{c^2}} = \lambda\frac{c}{2\pi v}(1+\sqrt{1-\frac{v^2}{c^2}}) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{l} v << c,\ u \cong \frac{v}{2},\ R \cong \frac{hc}{\pi m v^2} = \frac{\lambda}{\pi}\frac{c}{v} = \frac{\Delta x}{\pi}\frac{c}{v},\ uv \cong \frac{h}{mT}(1-\frac{1}{2}\frac{v^2}{c^2}) \\ \quad v \to 0,\ \ u \cong \frac{v}{2} \to 0,\ \ \ R \to \infty,\ \ uv \to \frac{v^2}{2} = \frac{h}{mT} \\ \quad v \to c,\ \ u \to c\,,\ \ \ \ R\ =\ \frac{\lambda}{2\pi} = \frac{\Delta x}{2\pi} \to 0 \end{array} \right). \qquad (4.3\text{-}3)$$

Interesting conclusion from (4.3-3) is that ***for a particle which has remarkably high linear speed,*** $v \to c$***, de Broglie matter waves are converging inside of a particle,*** *since* $\text{Lim} R|_{v \to c} = 0$ *, as illustrated on the Fig.4.1.3.*

$$\Delta x = \lambda = \frac{h}{p} = \frac{2\pi}{k},$$
$$v \to c,\ \Delta x \to 0,\ R \to 0$$

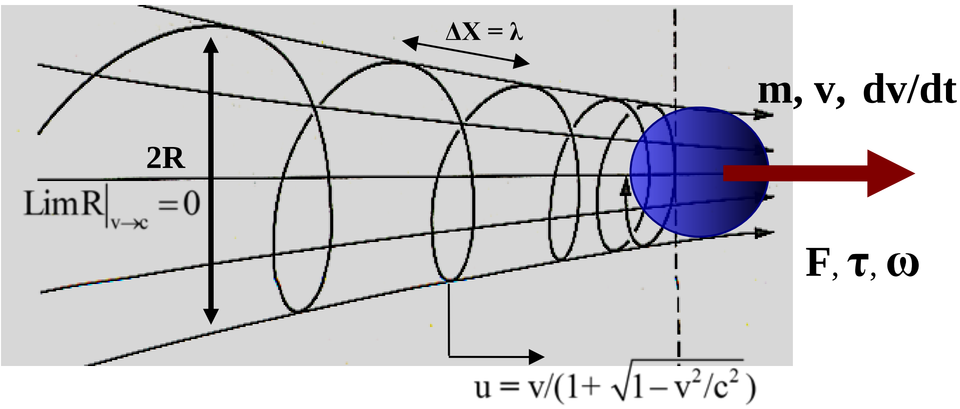


Fig.4.1.3 De Broglie matter waves around accelerating particle

Of course, the remaining task here is to unify concepts illustrated by Fig.4.1 and Fig.4.1.2, Fig.4.1.3 and Fig.4.1.4, and by equations (4.3), (4.3-0)-a,b,c,d,…, (4.3-1), (4.3-2) and (4.3-3), as well as to eliminate possible discrepancies between them. It is already clear that de Broglie matter waves are becoming less mysterious than ever before (see also the second chapter with equations (2.5.1-4) – (2.5.1-6) addressing the place of spinning in total particle energy). Analogically concluding, knowing that electromagnetic field presents specific unification or coupling of electric and magnetic field components, we could establish the idea that every (mechanical) linear motion should have a certain similar coupling with a specific spinning motion. Now we can realize that such associated spinning field component would be causally related to de Broglie matter waves (like illustrated on Fig.4.1.2 and with relations (4.3-1) – (4.3-3)). Such (associated) matter wave field should present some axially spinning field structure around the particle. This is very much analog to a photon that can analogically be presented (and treated in relevant interactions with other particles or energy-momentum states) as a moving particle, or as an equivalent wave packet ($m_p = hf/c^2, p_p = m_p c = hf/c, \tilde{E}_p = hf, \lambda_p = h/p_p = c/f$). At the same time, from

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electromagnetic theory we also know that photon is rotating (or saying the same more precisely, its mutually complementary and mutually orthogonal electric and magnetic field vectors are rotating along the common propagation path…).

Especially interesting situations are ones when a particular particle (or matter wave which has effective mass **m**) is performing the orbital motion, which is creating stationary and standing waves (toroidal) field structure along the path or perimeter of its rotation like illustrated on the Fig.4.1.4. This is giving a chance to conceptualize structure of atoms, elementary particles, and planetary motions in a specific solar system in relation with surrounding matter waves (*conceptually and theoretically well supported, like in [94], Classical Mechanics; - chapter 14*). For instance, if we consider that mass **m** on Fig.4.1.4 is our planet Earth, then its surrounding toroidal (or helix) matter-wave complement could be related to the lunar path of our Moon. Of course, in such astronomic framework where relevant objects are much bigger than atoms and other subatomic entities, analogous relations for matter-waves wavelength, energy, etc. could be somewhat different compared to what we know in connection with original de Broglie, Max Planck, Schrödinger and Einstein micro-world matter-waves conceptualization (see also the chapter 2 dealing with "**2.3.2. Macro-Cosmological Matter-Waves and Gravitation**", equations (2.11.10) until (2.11.20)).

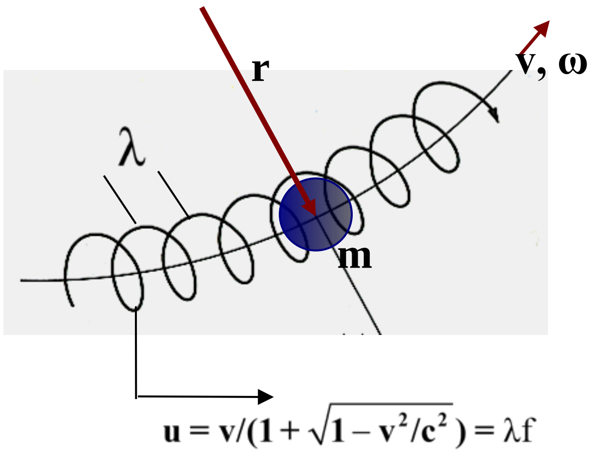
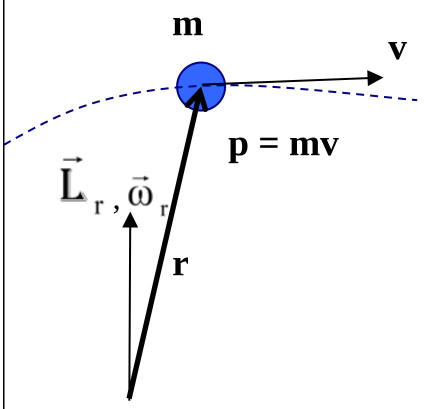
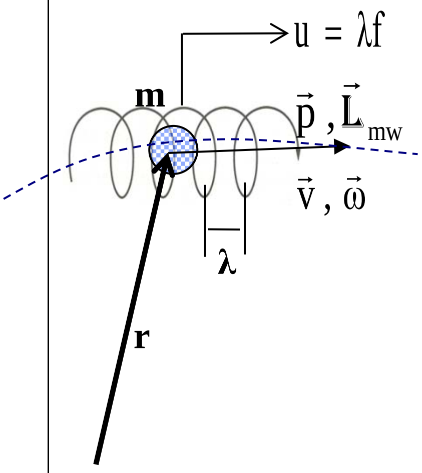


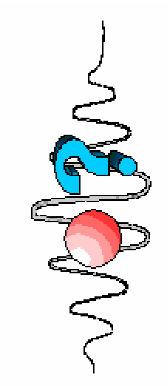
Fig.4.1.4 De Broglie matter wave on a circular path

As we can see, particle-wave duality is also related to how we are presenting a particle or energy state in motion. The dualistic way of presentation is dominantly related to a motional or kinetic energy formulation, and it is mostly a matter of convenience (regarding specific problems analyses) if we would apply typically corpuscular or wave-motions methodology. For making this clearer, let us imagine that specific non-zero-rest-mass particle is (only) in a state of linear motion and that another similar particle is in a similar motion, but performing mechanical (externally visible, or measurable) spinning also. We will try to present and compare different energy states of mentioned moving particles insisting on dualistic particle-wave presentations. Until present, many efforts are invested in showing that matter waves (or de Broglie waves) are representing only motional energy, and that structure (or shape) of such waves looks like helix spinning of specific (hidden) field form. Apparently, in the example taken here, if the second particle is performing linear motion and externally detectable spinning, such mechanical spinning should have specific coupling or relation with the associated matter waves spinning field. The table T.4.3 is summarizing the most relevant, comparative, corpuscular and dualistic or particle-wave qualifications of such moving particles (see also Fig.4.1.2, and similar elaborations in Chapter 10). **T.4.3.**

Comparative particle-wave presentations of a motional particle

	The particle is only in linear motion (which is a particular case of certain rotating or orbital motion)	**The same particle is presented as a matter wave train** (where matter wave is spinning around the particle)
Simplified conceptualization of corpuscular and wave nature related to the same particle		

Particle-wave duality relations



$$E = mc^2 + E_k = mc^2 + \tilde{E} = \gamma mc^2,$$

$$E_k = E_{mw} = E_k(p,v) = \frac{pv}{1+\sqrt{1-v^2/c^2}} = (\frac{v}{c})\frac{\sqrt{\tilde{E}(\tilde{E}+2E_0)}}{1+\sqrt{1-v^2/c^2}} =$$

$$= \frac{\mathbb{L}_r\omega_r}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbb{L}_{mw}\omega}{1+\sqrt{1-v^2/c^2}} = \tilde{E} = hf = h\frac{\omega}{2\pi},$$

$$P_4^2 = (\vec{p}, \frac{E}{c})^2 = inv. \Leftrightarrow p^2c^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, p^2c^2 + m^2c^4 = E^2,$$

$$dE_k = vdp = \omega_r d\mathbb{L}_r = \omega d\mathbb{L}_{mw} = hdf = d\tilde{E} = dE_{mw}, v = \omega_r r,$$

$$p = \gamma mv = \mathbb{L}_{mw}\frac{\omega}{v} = \mathbb{L}_r\frac{\omega_r}{v} = \frac{\mathbb{L}_r}{r} = \pm\frac{1}{c}\sqrt{E^2 - E_0^2} =$$

$$= \pm\sqrt{(E-E_0)(E+E_0)}/c = \pm\sqrt{\tilde{E}(\tilde{E}+2E_0)}/c,$$

$$\mathbb{L}_{mw}\omega = \mathbb{L}_r\omega_r = pv, \mathbb{L}_{mw} \neq \mathbb{L}_r, \omega \neq \omega_r, \mathbb{L}_{mw}d\omega = \mathbb{L}_r d\omega_r,$$

$$\mathbb{L}_{mw} = \mathbb{L} = \mathbb{L}_r\frac{\omega_r}{\omega} = p\frac{v}{\omega} = \frac{\gamma mv^2}{\omega} = \frac{h}{2\pi}(1+\sqrt{1-v^2/c^2}) = \mathbb{J}\omega,$$

$$\left\{\begin{array}{l} d\mathbb{L} = \frac{h}{2\pi}\cdot\frac{df}{f} = \frac{h}{2\pi}\cdot\frac{dE_k}{E_k} = \frac{v}{\omega}dp \Rightarrow \mathbb{L} = \mathbb{L}_c - \frac{h}{2\pi}\cdot\left(\frac{E_c}{E_k}\right)^2 = \mathbb{L}_c - \frac{h}{2\pi}\cdot\left(\frac{F_c}{f}\right)^2, *?!?!* \\ (E_c, F_c, \mathbb{L}_c) = \text{constants}, \ h = E_c/F_c = \text{Planck constant.}, \text{*to be more analyzed*} \end{array}\right\},$$

$$\mathbb{L}_r = \gamma L_r = p\frac{v}{\omega_r} = pr = \mathbb{L}_{mw}\frac{\omega}{\omega_r} = \frac{\gamma mv^2}{\omega_r},$$

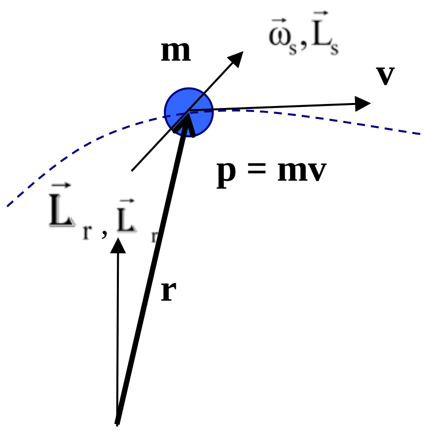
$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \left(\frac{h}{\mathbb{L}_{mw}}\right)\frac{v}{\omega} = \left(\frac{h}{\mathbb{L}_r}\right)\frac{v}{\omega_r} = \left(\frac{h}{\mathbb{L}_r}\right)r, \left(\frac{h}{\mathbb{L}_i}\right) = \theta_i,$$

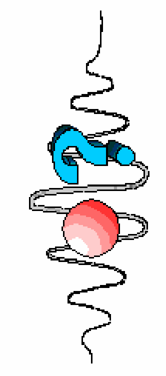
$$u = \lambda f = \frac{v}{1+\sqrt{1-v^2/c^2}}, \ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda},$$

$$\Psi^2 = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} = \omega\frac{d\mathbb{L}_{mw}}{dt} = \omega_r\frac{d\mathbb{L}_r}{dt} = vF = \omega_r\tau_r = \omega\tau.$$

The particle is in linear motion and mechanically spinning

(having externally visible or measurable spinning: $\vec{\omega}_{ms}, \vec{L}_{ms}$)



Particle-wave duality relations 	$E = mc^2 + E_k = mc^2 + \tilde{E} = \gamma Mc^2, p^2c^2 + M^2c^4 = E^2, M = m + \frac{E_s}{c^2},$ $E_k = E_{mw} = E_k(p,v) + E_k(L_s,\omega_s) = \frac{pv + L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \frac{L^*_{mw}\omega_{mw}}{1+\sqrt{1-v^2/c^2}} =$ $= \frac{\mathbf{L}_r\omega_r + L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \frac{\mathbf{L}_{mw}\omega_{mw} + L_s\omega_s}{1+\sqrt{1-v^2/c^2}} = \tilde{E} = h(f+f_s) = h\frac{\omega}{2\pi},$ $dE_k = vdp + \omega_s dL_s = \omega_r d\mathbf{L}_r + \omega_s dL_s = \omega d\mathbf{L}_{mw} + \omega_s dL_s = hd(f+f_s) = d\tilde{E} = dE_{mw},$ $E_{ms} = E_k(\mathbf{L}_s,\omega_s) = (M-m)c^2, v = \omega_r r, p = \gamma Mv, \mathbf{L}_r = \frac{pv}{\omega_r} = pr,$ $\omega_r d\mathbf{L}_r = \omega d\mathbf{L} = vdp, \mathbf{L}_{mw}\omega = \mathbf{L}_r\omega_r = pv, \mathbf{L}_{mw} \neq \mathbf{L}_r, \omega \neq \omega_r, \mathbf{L}_{mw}d\omega = \mathbf{L}_r d\omega_r,$ $\lambda = \frac{h}{p}, u = \lambda f = \frac{v}{1+\sqrt{1-v^2/c^2}}, v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, \omega_{mw} = \omega = 2\pi f,$ $\Psi^2 = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} + \omega_s\frac{dL_s}{dt} = \omega\frac{d\mathbf{L}_{mw}}{dt} + \omega_s\frac{dL_s}{dt} = \omega_r\frac{d\mathbf{L}_r}{dt} + \omega_s\frac{dL_s}{dt} =$ $= vF + \omega_s\tau_s = \omega_r\tau_r + \omega_s\tau_s = \omega\tau + \omega_s\tau_s.$

We also know that our universe is hosting many inertial and uniform motions. The same problem is partially addressed by Newton Law of Inertia (related to linear momentum conservation and force…). Many other, familiar, or complementary motions complying the primary meaning of inertial events are still "*waiting to enter the domain of inertial motions*". For instance, many rotating or spinning motions in planetary systems and inside of atoms, or angular and spin moments related states of elementary particles, states with stable Gyromagnetic ratios, or phenomenology described by Faraday-Lenz laws of electromagnetic induction … It appears natural that experiences of Inertia and inertial motions can be generalized and united by particle-wave duality concepts.

If we consider that linear motion is only a case of rotation with the large (or infinite) radius of rotation, consequently, law of orbital or angular momentum conservation should dominate and cover (or represent) linear momentum conservation. Let us exercise the meaning of conservation laws applied on a specific uniform and inertial particle/s motion (which has elements of linear and rotational motions) using here elaborated particle-wave duality concepts (see appropriate mathematical expressions from (4.3-1) to (4.3-3), and **T.4.3.**). If we (also) imagine that uniform state of motion (as one from **T.4.3.**) has many mutually related particles (and other energy states), and if we attempt to apply linear and orbital momentum conservation laws strictly, we will get,

$$\left\{\begin{array}{l} \vec{P} = \sum_{(i)} \vec{p}_i = \sum_{(i)} \left(\overrightarrow{L_{mw} \frac{\omega}{v}} \right)_i = \sum_{(i)} \left(\overrightarrow{L_r \frac{\omega_r}{v}} \right)_i = \sum_{(i)} \left(\overrightarrow{\frac{L_r}{r}} \right)_i = \overrightarrow{Const.}, \\ \vec{L}_r = \sum_{(i)} \left(\overrightarrow{p \frac{v}{\omega_r}} \right)_i = \sum_{(i)} \left(\overrightarrow{r \times p} \right)_i = \sum_{(i)} \left(\overrightarrow{L_{mw} \frac{\omega}{\omega_r}} \right)_i = \sum_{(i)} \left(\overrightarrow{\frac{\gamma m v^2}{\omega_r}} \right)_i = \overrightarrow{const.}, \\ \vec{L}_{mw} = \sum_{(i)} \left(\overrightarrow{L_r \frac{\omega_r}{\omega}} \right)_i = \sum_{(i)} \left(\overrightarrow{p \frac{v}{\omega}} \right)_i = \sum_{(i)} \left(\overrightarrow{\frac{\gamma m v^2}{\omega}} \right)_i = \overrightarrow{CONST.}, \\ P_4^2 = (\vec{P}, \frac{E}{c})^2 = inv. \Leftrightarrow P^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, E_k = E - E_0 = \tilde{E} = ... \end{array}\right\}. \quad (4.3\text{-}3.1)$$

Obviously, it will be a challenging task to show (under certain assumptions and approximations) if, how and when all linear and orbital moments from (4.3-3.1), mutually dependent and related to the same inertial motion, could be conserved (for instance as, $\vec{P} = \overrightarrow{Const.}, \vec{L}_r = \overrightarrow{const.}, \vec{L}_{mw} = \overrightarrow{CONST.}$, like elaborated in the second chapter 2.3.1. with extended meaning of inertia, shown under (2.9.1), (2.9.2) and later). ***If we can consider that relevant set of objects (4.3-3.1) is in a specific complex state of stable, inertial, uniform, circular and stationary motion (like planets in a solar system), and if we consider all involved (relevant) orbits hosting stable, standing matter-waves formations, we will coincidently satisfy conservation of all involved linear and orbital moments.*** *See chapter 2, about the extended meaning of inertia under 2.3.1, what we are anyway universally applying in analyzing and explaining all kind of interactions and motions.*

Most probably, the conceptualization of matter waves (as given until here, in T.4.3 and on Fig.4.1.2, Fig.4.1.3, Fig.4.1.4) should be additionally supported and upgraded (applying a lot of open-minded, creative, and positive intellectual flexibility), but it is already clear where the fundamental and ontological essence of matter waves is.

Now, we are closer than ever to answering what de Broglie or matter waves are. All presently known waves phenomenology (in our universe) should in some ways belong to de Broglie matter waves. It is just a question of our ignorance, misinterpretation, or incomplete and particularly deviated modeling, why we still do not see de Broglie matter waves as the part of the picture of all other waves and oscillations known in physics. The other important fact (more contributing to the confusion than giving real conceptual insight into matter waves) is that our present mathematical processing and engineering of such (micro-world) problematic is also producing sufficiently acceptable results. Such results are presented statistically and in-average, unintentionally neglecting or masking the physical and conceptual side of the same problematic by, for the time being, successful probabilistic postulating, fittings, and framing (because of applied unnatural modeling). The hidden tendency of the contemporary Quantum Theory is to present its mathematical framework and processing as the ultimate and only allowed or only acceptable and the best point of view, and it could pass a long time until such position will be significantly corrected.

4.1.2.1. Example 1: Bohr's Hydrogen Atom Model

As an illustration of here introduced platform for treating de Broglie waves, let us briefly analyze a hydrogen atom in the center-of-mass coordinate system (not going too far from the original Bohr's model). Let us apply (4.1), (4.2) and (4.3) to describe the movement of an electron and a proton around their common center of gravity (in the center of mass system), and to describe their associated de Broglie waves, assuming that the electron and atom nuclei are treated as real rotating charged bodies (slightly modified Bohr's model). The following terms are involved here:

$\mathbf{m_e}$ -electron mass,

$\mathbf{m_p}$ -the nucleus or proton mass,

$\mathbf{v_e = \omega_{me} r_e}$ -electron velocity around the common center of gravity,

$\mathbf{v_p = \omega_{mp} r_p}$ -proton velocity around the common center of gravity,

$\mathbf{r_e}$ -the radius of the revolving electron,

$\mathbf{r_p}$ -the radius of the revolving proton,

$\mathbf{\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m}$ -the mechanical, revolving frequency of the electron and the proton,

$\lambda_e = h/\gamma_e m_e v_e = h/p_e$ -de Broglie wavelength of the electron wave,

$\lambda_p = h/\gamma_p m_p v_p = h/p_p$ -de Broglie wavelength of the proton wave,

$\mathbf{f_e}$ -de Broglie frequency of the electron wave,

$\mathbf{f_p}$ -de Broglie frequency of the proton wave,

$\mathbf{u_e = \lambda_e f_e}$ -the phase velocity of de Broglie electron wave and

$\mathbf{u_p = \lambda_p f_p}$ -the phase velocity of de Broglie proton wave.

In order to satisfy structural stability and non-dissipative nature of a hydrogen atom, the internal angular momentum of the electron and proton orbital motion should also be conserved: $\gamma_e m_e r_e^p \omega_m = \gamma_p m_p r_p^2 \omega_m \Rightarrow \gamma_e m_e v_e r_e = \gamma_p m_p v_p r_p$, and electron and proton stationary wave structure should be stable and satisfy relations, $\mathbf{n\lambda_e = 2\pi r_e}$, $\mathbf{n\lambda_p = 2\pi r_p}$ (see **[4]** regarding the same situation). We can also imagine that an electron and proton have the forms of rotating, electrically charged rings, with spiral current paths on their toroids, like solenoids because of similar reasons already described with results (4.3) and Fig. 4.1. This will only proportionally change their moments of inertia, for specific multiplicative constant on both sides of equation ($J_e \omega_{me} = A \cdot \gamma_e m_e r_e^2 \omega_m = J_p \omega_{mp} = A \cdot \gamma_p m_p r_p^2 \omega_m$, $A = \text{Const.}$), not changing the results (see (4.4)). In a few steps, implementing the conditions mentioned above in the framework of the original Bohr's atom model (and using data from (4.1) - (4.3)), we can find:

$$\sqrt{\frac{\gamma_p m_p}{\gamma_e m_e}} = \frac{r_e}{r_p} = \frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} = \frac{\gamma_p m_p v_p}{\gamma_e m_e v_e} = \sqrt{\frac{\gamma_p}{\gamma_e} 1836.13} = 42.8503386217\sqrt{\frac{\gamma_p}{\gamma_e}},$$

$$f_m = f_{me} = f_{mp} = f_{m(e,p)} = \omega_m / 2\pi, \quad \gamma_{e,p} = (1 - v_{e,p}^2 / c^2)^{-0.5}$$

$$\text{or for } (v_e, v_p, u_e, u_p << c) \Rightarrow$$

$$\sqrt{\frac{m_p}{m_e}} \cong \frac{r_e}{r_p} = \frac{\lambda_e}{\lambda_p} = \frac{p_p}{p_e} \cong \frac{m_p v_p}{m_e v_e} \cong \frac{v_e}{v_p} \cong \frac{u_e}{u_p} \cong \sqrt{1836.13} = 42.8503386217$$

$$f_m \cong \frac{m_e e^4}{4n^3 h^3 \varepsilon_0^2}, \quad \frac{v_p}{v_e}\sqrt{\frac{m_p}{m_e}} \cong \frac{E_{kp}}{E_{ke}} = \frac{\tilde{E}_p}{\tilde{E}_e} = \frac{m_p u_p}{m_e u_e} = 1,$$

$$f_e = n\frac{f_m}{2}(1 + \frac{u_e^2}{c^2}) \cong n\frac{m_e e^4}{8n^3 h^3 \varepsilon_0^2}(1 + \frac{u_e^2}{c^2}) \cong n\frac{m_e e^4}{8n^3 h^3 \varepsilon_0^2} \cong f_p,$$

$$f_p = n\frac{f_m}{2}(1 + \frac{u_p^2}{c^2}) \cong n\frac{m_e e^4}{8n^3 h^3 \varepsilon_0^2}(1 + \frac{u_p^2}{c^2}) \cong n\frac{m_e e^4}{8n^3 h^3 \varepsilon_0^2} \cong f_e,$$

$$\frac{f_e}{f_p} \cong 1 + 1836.13\frac{u_p^2}{c^2} \cong 1$$

$$1 < \frac{v_e}{u_e} = n \cdot \frac{f_m}{f_e} = \frac{2}{(1 + \frac{u_e^2}{c^2})} = 1 + \sqrt{1 - \frac{v_e^2}{c^2}} \le 2,$$

$$1 < \frac{v_p}{u_p} = n \cdot \frac{f_m}{f_p} = \frac{2}{(1 + \frac{u_p^2}{c^2})} = 1 + \sqrt{1 - \frac{v_p^2}{c^2}} \le 2, \; n = 1, 2, 3, ... \tag{4.4}$$

It is important to underline that the revolving mechanical frequency of the electron and the proton around their common center of gravity, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$ should not be mixed with de Broglie wave frequency of stationary electron and proton waves, $\omega_m = 2\pi f_m \neq (\omega_e = 2\pi f_e, \, \omega_p = 2\pi f_p)$, and that relationship between them is given by $f_{e,p} \le n \cdot f_{m\text{-}e,p} = f_{e,p} \cdot (1 + \sqrt{1 - v_{e,p}^2 / c^2}) \le 2f_{e,p}$.

From (4.4) we can also conclude that (wave) energy of a stationary electron wave ($hf_e = (\gamma_e - 1)m_e c^2 = \gamma_e m_e v_e u_e = p_e u_e = E_k$) is fully equal to electron's motional or kinetic energy, meaning that the rest electron mass or its rest energy has no direct participation in this part of the energy (see much more about Bohr's atom model in Chapters 2., and 8).

*[♣ **COMMENTS & FREE-THINKING CORNER**: Obviously, relations equivalent to (4.4) should be valid for planets rotating around their suns, except that we would have the dominance of Gravitation related field/s instead of electromagnetic field, and relevant mass ratio will be a different number (meaning that planets in their solar systems should also have their associated de Broglie waves). For instance, we know that our planet Earth rotates around the Sun and in the same time rotating around its planetary axis, performing similar motion as presented on Fig.4.1 (and that our complete solar system rotates around our Galaxy center, and that our Galaxy also rotates…).*

Bohr's hydrogen atom model is a straightforward one, very much experimentally tested and proven applicable in all frames of its definition (of course, also having many known limitations). By combining

Bohr's planetary model with here introduced concept of de Broglie waves (Fig.4.1 and equations (4.1), (4.2) and (4.3)), we are indirectly testing and proving the hypothesis (of this book), claiming that every rectilinear motion should be accompanied with rotation (and such rotation naturally creates de Broglie waves, producing correct results found in (4.4)). Bohr's hydrogen atom model could also be used to prove all elements of **PWDC** (**P**article-**W**ave **D**uality **C**ode) found in (4.1) - (4.3).

Wave Theory originates from the need to explain Bohr's postulates, and that was its first meaningful application. Atom stability and experimentally verifiable spectral series of hydrogen atom postulated modifications of Rutherford's dynamic atom model, resulting in Bohr's atom model. Bohr was the founder of Quantum Theory concepts. In his time, it was a practical need to explain the orbital hydrogen atom structure and the nature of quantized emissions and absorptions of electromagnetic energy. Bohr created his explanation introducing certain postulates, without giving any better explanation regarding his postulates. The unclear and unexplained situation (in Bohr's atom model) has been the one concerning why and how electron does not emit or absorb light while rotating on its stationary orbit/s. For creating an operating mathematical formulation, electron momentum needed to be quantized, and the logical connection between the frequency of periodic orbital electron motion and frequency of radiated (or absorbed) photon/s should be mutually related (in classical Electrodynamics, the two mentioned frequencies are mutually equal). In Bohr's hydrogen atom model, those frequencies are different, except in cases of orbits with integer main quantum number, where such frequencies are approximately equal (which is additionally supported by formulating or postulating the "correspondence principle").

Emission or absorption of photons appears possible only when electrons pass between two stationary orbits, and energy of such photon (directly proportional to its frequency multiplied by Planck's constant) is precisely equal to the difference of corresponding orbital electron energies (this way additionally legitimizing the Planck expression for photon energy). The total stationary orbit electron energy is equal to the sum of its kinetic and potential energy. It looks that Bohr only made a simple hybrid merging between Quantum Theory concepts, Classical Mechanics and Electrodynamics to explain already known phenomenology, without profoundly elaborating his theory.

*Luis de Broglie electron wavelength has also been perfectly fitting in the concept of stationary electron orbits, explaining atom stability in a quite simple and pragmatic way, saying that the perimeter (or total length) of one stationary orbit should be equal to an integer multiple of the electron matter wave wavelength (like standing waves on a string). We could say that Bohr was the first one who applied the **PWDC**, although without having such intention (and without being conscious, what he was doing regarding the **PWDC**).* ***Unfortunately, Bohr's hydrogen atom model (and planetary atom concept) looks already as an ancient and unnatural concept, comparing it to the atom constituents modeling of Kanarev, [44], C. Lucas, David L. Bergman and colleagues [16] - [22]; -"Common Sense Science"). Also, the concept of electron and its associated magnetic field should be significantly updated:*** *see on Internet one of such innovative concepts related to Henry Augustus Rowland effect of the magnetic field around rotating conductor, presented by Jean de Climont ♣]*

4.1.3. Matter Waves and Conservation Laws

De Broglie relation, $\lambda = h/p$ **, for matter wave wavelength, (4.1), is not fully explained and systematically developed starting from energy and momentum conservation laws. De Broglie found it by fitting the most logical solution that makes orbital electron wave stable, by supporting postulates of Bohr's hydrogen atom model (and creating self-closed standing waves)**. Later, the same de Broglie relation was successful in supporting the wave properties of different particles (diffraction and interference experiments, Compton, and Photoelectric effect…). Consequently, nobody asked the question how this relation could generally be proved valid and developed from a more independent platform (than Bohr's hydrogen atom model, which was in certain aspects fundamentally wrong; -see Kanarev [44], and Bergman-Lucas [16] - [22]). The same situation was with Planck's expression for the photon energy $\tilde{\mathbf{E}} = \mathbf{hf}$, (4.1). **The proper expression for wave-packet energy was found (mathematically combined with certain assumptions) by the best curve fitting to explain measured data regarding the spectrum of the black body radiation.** Later, the same relation, combined with de Broglie wavelength, and energy-momentum conservation laws, was quietly generalized and accepted to become valid for any matter wave. This was successfully applied in explaining many quantum interactions (Compton Effect, Photoelectric Effect, etc. See more in chapter 10. Of this book).

Let us briefly exercise compatibility between de Broglie wavelength and Planck's wave energy with energy and momentum conservation laws, analyzing the situation on Fig.4.1.

We will start from a two-body interaction when two particles $\mathbf{m_1}$ and $\mathbf{m_2}$, ($m = m^{rel.} = m_o/\sqrt{1-v^2/c^2} = \gamma m_0$, $m_0 = const.$) move relative to each other, respectively having velocities $\mathbf{v_1}$ and $\mathbf{v_2}$, linear moments $\mathbf{p_1}$ and $\mathbf{p_2}$, orbital moments $L_{c1} = L_1$ and $L_{c2} = L_2$, and kinetic energies $\mathbf{E_{k1}}$ and $\mathbf{E_{k2}}$. By presenting them in the Laboratory and Center of mass systems, and applying total Energy and Momentum conservation laws (using already established analogies in earlier chapters and the same symbolic, definitions and designations as in (4.1) - (4.4) and on Fig.4.1, where $\mathbf{E_{oi}}$ and $\mathbf{m_{oi}}$ are particle energies and masses in the state of rest) we will have:

$$E_{tot.} = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{0C} + E_{kc} + E_{kr},$$
$$E_0 = E_{01} + E_{02} = E_{0C},\ E_{k1} + E_{k2} = E_{kc} + E_{kr}, E_{01} = m_{01}c^2, E_{02} = m_{02}c^2,\ E_{oc} = m_{oc}c^2,$$
$$m_{oc} = m_1 + m_2,\ \mu = m_r = \frac{m_1 m_2}{m_1 + m_2},\ E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = E_{kr}, F_{12} = F_r = \frac{dp_r}{dt},$$

We already know from (4.1) and (4.2) that de Broglie wavelength and Planck wave packet energy in connection with group and phase velocity equation are fully mutually integrated, cross-related, and compatible, for instance,

$$\left\{\lambda = \frac{h}{p}\,,\ u = \lambda f,\ \tilde{E} = hf\right\} \Leftrightarrow \left\{\frac{u}{v} = \frac{hf}{pv} = \frac{\tilde{E}}{pv}\right\} \Leftrightarrow \left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = \frac{2u}{1+\frac{uv}{c^2}}\,, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p} \\ \text{For microworld: } h = \text{Planck constant} \\ \text{For macroworld: } h \rightarrow H = \text{constant} \end{array}\right\}.$$

Since only motional (or kinetic) energy could be equal to (de Broglie) matter-wave energy, $E_k = \vec{p}\vec{u} = Hf = \tilde{E}$, there is no other possibility than treating de Broglie matter waves as motional energy forms. Also, matter waves wavelength cannot be conceptually and logically explained if we do not consider that all particle motions are cases of certain orbital motions, having associated spinning fields (see (4.3-0)-g). Taking all of that into account, we have,

$$\left\{\begin{array}{l} \vec{P}_4 = (\vec{P}, \frac{E}{c}) \Rightarrow \vec{P}^2 - \frac{E^2}{c^2} = \vec{P}'^2 - \frac{E'^2}{c^2} = -\frac{E_0^2}{c^2} = -\frac{(E_{01}+E_{02})^2}{c^2} = \text{invariant}. \\ \vec{P} = \vec{p}_1 + \vec{p}_2 = \gamma_1 m_1\vec{v}_1 + \gamma_2 m_2\vec{v}_2 = \vec{p}_c = \frac{E}{c^2}\vec{v}_c = \frac{E_1+E_2}{c^2}\vec{v}_c = \frac{E_c+E_r}{c^2}\vec{v}_c = \gamma_c m_{oc}\vec{v}_c\,, \\ E = E_{tot.} = E_1 + E_2 = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = E_c + E_r = E_{01}+E_{k1}+E_{02}+E_{k2} = E_{0C}+E_{kc}+E_{kr}\,, \\ \vec{v}_c = \frac{c^2}{E}\vec{p}_c = \frac{c^2(\gamma_1 m_1\vec{v}_1+\gamma_2 m_2\vec{v}_2)}{\gamma_1 m_1 c^2+\gamma_2 m_2 c^2} = \frac{\gamma_1 m_1\vec{v}_1+\gamma_2 m_2\vec{v}_2}{\gamma_1 m_1+\gamma_2 m_2},\ \vec{v}_r = \vec{v}_1 - \vec{v}_2\,,\ u_i = \frac{v_i}{1+\sqrt{1-v_i^2/c^2}} = \lambda_i f_i\,, \\ E_{ki} = (\Delta m_i)c^2 = \tilde{E}_i = Hf_{si} = \frac{L_{si}\omega_{si}}{1+\sqrt{1-v_i^2/c^2}} = \frac{L_{ci}\omega_{ci}}{1+\sqrt{1-v_i^2/c^2}} = \frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}} = p_i u_i\,, \\ \vec{p} = \frac{\omega_{si}}{v_i}\frac{\cos(\vec{L}_{si},\vec{\omega}_{si})}{\cos(\vec{p}_i,\vec{v}_i)}\vec{L}_{si} = \gamma m\vec{v}_i\,,\ \vec{L}_{si} = \frac{v_i}{\omega_{si}}\frac{\cos(\vec{p}_i,\vec{v}_i)}{\cos(\vec{L}_{si},\vec{\omega}_{si})}\vec{p}_i\,, v_i = u_i - \lambda_i\frac{du_i}{d\lambda_i} = -\lambda_i^2\frac{df_i}{d\lambda_i}, u_i = \lambda_i f_i,\ \lambda_i = \frac{H}{p_i}, \\ \frac{H}{\pi} \le L_{si} = \left\{\frac{H}{2\pi}(1+\sqrt{1-v_i^2/c^2}) = \frac{H}{2\pi}\frac{v_i}{u_i}\right\} \le \frac{H}{2\pi}, \\ (\vec{\omega}_{ci} = 2\pi f_{ci} \text{ and } \vec{\omega}_{si} = 2\pi f_{si} \text{ are mutually orthogonal vectors}). \end{array}\right\}$$

$$\Leftrightarrow \left\{\begin{array}{l} \left[\begin{array}{l} \tilde{E}_1 + \tilde{E}_2 = \tilde{E}_c + \tilde{E}_r \\ \vec{v}_c = \frac{\gamma_1 m_1\vec{v}_1+\gamma_2 m_2\vec{v}_2}{\gamma_1 m_1+\gamma_2 m_2} \end{array}\right] \Rightarrow \left[\begin{array}{l} [f_1 + f_2 = f_c + f_r], \\ |\vec{v}_r| = |\vec{v}_1 - \vec{v}_2|,\ |\vec{u}_r| = |\vec{u}_1 - \vec{u}_2| = \lambda_r f_r \end{array}\right] \Rightarrow \\ \Rightarrow \left[|\vec{u}_c| = \left|\frac{\gamma_1 m_1\vec{u}_1+\gamma_2 m_2\vec{u}_2}{\gamma_1 m_1+\gamma_2 m_2}\right| = \lambda_c f_c = \frac{v_c}{1+\sqrt{1-v_c^2/c^2}}\right] \end{array}\right\} \Rightarrow \tag{4.5}$$

$$\Rightarrow \left\{\begin{array}{l} \lambda_1 = \frac{H}{p_1},\ \lambda_2 = \frac{H}{p_2},\ \lambda_c = \frac{H}{p_c},\ \lambda_r = \frac{H}{p_r} = \lambda_1\frac{v_1}{v_r} + \lambda_2\frac{v_2}{v_r}, v_i = u_i - \lambda_i\frac{du_i}{d\lambda_i} = -\lambda_i^2\frac{df_i}{d\lambda_i} \\ (\vec{p}_1+\vec{p}_2 = \vec{p}_c) \Leftrightarrow (\frac{\vec{p}_1}{H}+\frac{\vec{p}_2}{H} = \frac{\vec{p}_c}{H}) \Rightarrow \left[\begin{array}{c} \frac{1}{\lambda_1^2}+\frac{1}{\lambda_2^2}+2\frac{\cos(\vec{p}_1,\vec{p}_2)}{\lambda_1\lambda_2} = \frac{1}{\lambda_c^2} \\ \lambda_2^2\lambda_c^2+\lambda_1^2\lambda_c^2+2\lambda_1\lambda_2\lambda_c^2\cos(\vec{p}_1,\vec{p}_2) = \lambda_1^2\lambda_2^2 \end{array}\right] \\ \frac{Hv_1}{\lambda_1(1+\sqrt{1-v_1^2/c^2})} + \frac{Hv_2}{\lambda_2(1+\sqrt{1-v_2^2/c^2})} = \frac{Hv_c}{\lambda_c(1+\sqrt{1-v_c^2/c^2})} + E_r, \\ \vec{v}_c = \frac{\gamma_1 m_1\vec{v}_1+\gamma_2 m_2\vec{v}_2}{\gamma_1 m_1+\gamma_2 m_2} = \frac{\gamma_1 m_1\vec{u}_1+\gamma_2 m_2\vec{u}_2}{\gamma_1 m_1+\gamma_2 m_2} - \lambda_c\frac{d\left(\frac{\gamma_1 m_1\vec{u}_1+\gamma_2 m_2\vec{u}_2}{\gamma_1 m_1+\gamma_2 m_2}\right)}{d\lambda_c}(=) - \lambda_c^2\frac{df_c}{d\lambda_c} \end{array}\right\}. \tag{4.6}$$

Motional energies and group and phase velocities of mutually approaching objects (as well as all values in (4.5) and (4.6)) are coupled, time and position dependent, and continuously evolving, (since objects somehow communicate by the presence of surrounding fields. See later (4.5-3)).

More informative and useful relations unifying energy and momentum conservation (and avoiding differences between Relativistic and Classical Mechanics mass interpretation) can be given in the following differential form (as in (4.2)),

$$\left\{\begin{array}{l} E_{k1}(t,x)+E_{k2}(t,x)=E_{kc}(t,x)+E_{kr}(t,x) \Leftrightarrow \tilde{E}_1+\tilde{E}_2=\tilde{E}_c+\tilde{E}_r, \\ dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr} \Leftrightarrow d\tilde{E}_1+d\tilde{E}_2=d\tilde{E}_c+d\tilde{E}_r, dE_{kr}=v_r dp_r=\omega_r dL_r \\ v_i=v_i(t,x)=u_i-\lambda_i\dfrac{du_i}{d\lambda_i}=-\lambda_i^2\dfrac{df_i}{d\lambda_i}=\dfrac{d\tilde{E}_i}{d\tilde{p}_i},\ u_i=u_i(t,x)=\lambda_i f_i \end{array}\right\} \Rightarrow \tag{4.7}$$

$$\Rightarrow \left\{\begin{array}{l} d\tilde{E}_i=Hdf_i=v_i dp_i=d(p_i u_i)=dE_{ki}=c^2 d(\gamma m_i)=-c^2 d\tilde{m}_i=-d(\tilde{p}_i u_i)= \\ =-v_i d\tilde{p}_i\cdot\cos(\vec{p}_i,\tilde{p}_i)=\left[v_i d\tilde{p}_i \text{ or } -v_i d\tilde{p}_i\right] \end{array}\right\},$$

since after applying integration (when solving such equations) we can take into consideration boundary conditions and all stationary, motional, or state of rest parameters of specific interaction (see also (4.9-0)).

Here we can also address the idea of how to treat forces acting between two mutually approaching particles ($\vec{F}_1, \vec{F}_2, \vec{F}_c, \vec{F}_r$), for instance:

$$\begin{array}{l} \{dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr}\}/dt \Rightarrow \vec{v}_1\dfrac{d\vec{p}_1}{dt}+\vec{v}_2\dfrac{d\vec{p}_2}{dt}=\vec{v}_c\dfrac{d\vec{p}_c}{dt}+\vec{v}_r\dfrac{d\vec{p}_r}{dt} \Leftrightarrow \\ \Leftrightarrow \vec{v}_1\vec{F}_1+\vec{v}_2\vec{F}_2=\vec{v}_c\vec{F}_c+\vec{v}_r\vec{F}_r\ ;\ \vec{F}_1+\vec{F}_2=\vec{F}_c\ ,\ E_{12}=\int_{(1)}^{(2)}\vec{F}_{12}d\vec{r}_{12}=\int_{(1)}^{(2)}\vec{F}_r d\vec{r}_{12}=E_{kr}. \end{array} \tag{4.7.1}$$

By considering the existence of initial orbital moments (including spinning) of interaction participants, we can analogically (just for brainstorming exercising) create another torques-balancing equation,

$$\begin{array}{l} \{dE_{k1}+dE_{k2}=dE_{kc}+dE_{kr}\}/dt \Rightarrow \vec{\omega}_1\dfrac{d\vec{L}_1}{dt}+\vec{\omega}_2\dfrac{d\vec{L}_2}{dt}=\vec{\omega}_c\dfrac{d\vec{L}_c}{dt}+\vec{\omega}_r\dfrac{d\vec{L}_r}{dt} \Leftrightarrow \\ \Leftrightarrow \vec{\omega}_1\vec{\tau}_1+\vec{\omega}_2\vec{\tau}_2=\vec{\omega}_c\vec{\tau}_c+\vec{\omega}_r\vec{\tau}_r\ ;\ \vec{\tau}_1+\vec{\tau}_2=\vec{\tau}_c\ ,\ E_{12}=\int_{(0)}^{(2\pi)}\vec{\tau}_{12}d\theta=\int_{(0)}^{(2\pi)}\vec{\tau}_r d\theta=E_{kr}. \end{array} \tag{4.7.1-1}$$

Particularly interesting cases are when the force $\vec{F}_r=\vec{F}_{12}$ between mutually approaching objects becomes balanced with the centrifugal force of quasi-rotational movement of the same objects in their Center of mass system, $F_r=\dfrac{dp_r}{dt}=\dfrac{m_r v_r^2}{r_{12}}(=G\dfrac{m_r m_c}{r_{12}^2}\pm ...?!)$. This could create conditions for stable and self-sustaining helix spinning-toroid formation, leading eventually to stable particle/s formation. See also force expressions (2.1) to (2.9) from the second chapter, to understand conceptually that such forces should have several static, dynamic, and mixed, linear, and rotational or spinning components. Also, see very indicative examples in the second chapter, around equations (2.11.13-1) - (2.11.13-5), where is clearly shown how matter waves in two-body problems are being created). Giving a little bit more freedom to our thinking and imagination, we could make direct relations and analogies between Ruđer Josip Bošković Force, (see literature references under [6]), and two-body particle-matter-wave interactions around equations from (4.5) to (4.7.1-1).

***[♣ COMMENTS & FREE-THINKING CORNER**:* Ruđer Josip Bošković (18 May 1711 – 13 February 1787), was born in the independent, multiethnic Republic of Ragusa (presently town Dubrovnik, in Croatia) in a time period when the state and nation of Croatia did not exist (as geopolitically and ideologically promoted in recent years), and when Dubrovnik (in historical time frame) was not a part of recently created Croatia. The Republic of Ragusa was established as the Adriatic harbor town within a bigger geographical area known by the name Dalmatia. This was the name of the old Roman empire province (originally not in any significant and exclusive ontological, religious, geopolitical, and ethnic relation with much later invented Croatians, but being effectively or genetically populated with Serbs and Italians). Dubrovnik was the part of the medieval Serb state in the 11th century (of course, dominantly populated with Serbs). "Croatia", also a tiny part of old Roman Dalmatia, as a name of the country is derived from the word "cratkia" which was first used in 702 (not at all by Croats). The word "cratkia" meant short in old Slav's language (when Croatian as the language did not exist in any other form, except being naturally the Serbian, or a version of an old and common south Slavic language; -not to mention that the complete Slavic and Balkan population still speaks the same language without the need for interpreters and dictionaries). Cratkia eventually became mistyped, and word reordered (Read more about Croatia here, http://wiki.answers.com/Q/Origin_of_name_of_country_croatia#ixzz1FlFxs8Rf). The dominant part of the population of such old Roman province Dalmatia is originally created by Slavic, or mostly Serbian ethnicities, later continuously increasing with (newly arriving) Serbs gradually escaping Turkish (Ottoman) invasion, during few centuries (when Croatia and Croatian nation and language did not exist, except if we consider geographic and regional names as being some grounds for new national entities creation, what is presently an increasing strategy of Western world countries, used to dilute, destroy and disorganize other parts of the world). Slowly, such Slavic and Serbian population, which was originally escaping territories with Byzantine, Orthodox Christian heritage, was forced, motivated, or indoctrinated to switch from the Orthodox Christianity brand to Roman-catholic "copy-right" modus of Christianity, to benefit advantages and opportunities of new habitats of surrounding catholic states. This happened mostly because of social, political, existential, and survival-related reasons, created as hidden, tricky, and in some recent cases genocidal strategies elaborated in the Vatican and Germanic workshops, and mostly executed by indoctrinated Serbs hypnotically convinced that they are this way becoming some certified, God-blessed, and privileged Arians, Croats, and great Catholics. With such ambitious objectives (and with catholic and fascist mentor's support) so-called Croats (or former Serbs) enthusiastically exterminated several hundred thousand (close family) members of other part of mostly Serbian population, during the Second World War. Just in case, to be on a good side, they also captured and killed available members of Nikola Tesla's family. Presently, such indoctrinated, dominantly Serbian-origin population (living in a newly recomposed and falsely assembled Croatia) is almost consciously (anyway ideologically, hypocritically, and intentionally reprogrammed) glorifying any minor shadow or glimpses pointing towards reinforcing newly created Croatian origins, with overwhelming support from Vatican and Germanic mentors. Such imaginative Croats are still speaking (at least 98% of them) a mixture of an old and modern Serbian language (since language heritage is difficult to change in a matter of few decades). Anyway, it is good to know that Serbs born and educated in Serbia, can speak, write, and understand Croatian, without a dictionary and interpreter, and it is vice versa, naturally valid for Croats. Genetically, there are more of original Serbs in Croatia than in modern days Serbia. Of course, this could be slowly changed in the next few centuries, since dark linguistic-laboratory forces in modern Slavistics (manipulated and falsified by German and Vatican geopolitical and ideological activities, objectives, and inventors) are permanently creating and updating new Croatian language with a tendency to deviate it, as much as possible, from its Serbian and Slavic grounds. This old strategy of tricky national transformations could be described as "transformation of a geographic, territorial and religious entity into newly-invented and artificially imposed national identity, reinforced by newly and gradually implanted or imposed ideological and geopolitical attributes", what is still happening in relation to Croatia, Bosnia, Ukraine, Romania, Bulgaria, Greece, Poland, Russia, Belorussia, and many other countries established or dominated (long time ago) by original Serbs, where German and Vatican-inspired projects are slowly and "democratically" penetrating (with significant background investments and other hidden support). Even German population or nation in its past has been heavily mixed with Slavic population, culture, and genetic material, much more than 50% of their total population (but such facts are being politically, ideologically, and systematically suppressed during last few centuries by the Vatican and associated geopolitical manipulations). Even Turkey has about eight million people directly originating from Serbs. The fact is that Ruđer Josip Bošković was born in the Republic of Ragusa (or Dubrovnik) from an Italian mother and Serbian-origin father (not to mention here that big part of south and central Italy, including old Etruria, already had dominant Serbian origins, genetic, including Serbian cultural and historical roots, and that about half or old Roman imperators (18 of them) were Serbs, meaning in wider scope that Italy or Italian population is also a Serbian derivative). Even Christianity, as the officially accepted Roman state religion, was legally established in the middle of Serbia (in the town Naisus, Mediana, Niš) by the Roman imperator Constantin, who was born from Serbian or Slavic parents. Ruđer Bošković was educated in catholic schools (since nothing else or better existed there, and such schools were free of charges and easily accessible or available, especially for reprograming Serbs and other cultural flavors), where he learned about his (in that time almost inexistent) Croatian identity (probably as a part of ongoing Vatican-supported strategy to expand its ideological brand, and to offer new psychotherapy and soul-saving identity to assimilated Serbs), and this way, unintentionally and unknowingly supporting much later invented, falsely decorated, and magnified Croatian facts. Anyway, Ruđer Bošković gave his extraordinary cosmopolitan and scientific contribution to Physics, overwhelmingly spreading his scientific messages too far from such Serbo-Croatian and Italian, or Vatican and German orchestrated framework. All of that is good to mention as a contribution to the future awakening of Balkan and South Slavic, effectively Serbian population, if they manage to survive in sufficient numbers to reclaim something of their real, common, and (at least) several thousands of years old heritage, history, culture, and deep, glorious origins, purified from ideological, geopolitical, and naive religious traps. The same (about searching imaginative and false or virtual identity) is valid for some parts of population in Slovenia, Bosnia, Montenegro, Macedonia, Ukraine, Romania, Bulgaria, Greece, Poland, Russia, Belorussia… Even until couple of centuries ago, an older version of Serbian language was the official diplomatic and multinationally accepted, common language on an exceptionally large territory (from Egypt, Arabia, including Europe and Russia). Most of modern European and Western-culture languages are (in a historical frames) relatively recently, starting from twelfth century of our present time counting, modified, remodeled, and intentionally or forcefully transformed, evolving to their present forms, to become much different and unrecognizable regarding linguistic and other links to the old Serbian and Slavic language. Here we could add the fact that there is an enormous linguistic overlapping between Sanskrit and old Slavic language. Rectification of mentioned ideologically inspired, linguistic and geopolitical practices, could happen if some of more open-minded and more conscious, not too much indoctrinated Serbs or Slavic origin population (from all of existing brands) start following directions paved by their cosmopolitan predecessors like Ruđer Bošković, Nikola Tesla, Milutin Milanković, Mihailo Pupin, Mileva Maric, Pavle Savic… instead of pathologically participating in old and obscure, secret projects of Germanic and Vatican-elaborated geopolitical, social, cultural, ethnic and linguistic laboratory experiments. We could ask ourselves how many of mentioned, national, genetic, historical, archeological, and cultural identity items are still interesting, required,

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

relevant and worth to be rediscovered, accumulated and revitalized, when we know that contemporary, planetary, and mainstream tendency is to melt and unify all such good and wrong, imaginative, false, virtual, ideologically transformed, and other cultural creations of different "*mental energy-dissipating warriors*", in a multinational, multicultural planetary society. We could also briefly say that biological hardware (including genetic background) of all of us (who are sufficiently educated, open-minded and ready to learn, including in proper frames most of mammals and primates) is very much the same, but we also know that the software (or cultural content and educational level) is what counts, and creates big differences among mentioned species. ♣]

Let us imagine that only one particle moves relatively towards the other (that is in a state of rest) and that the moving particle is much smaller than another particle (see equations (4.3), (4.5) and (4.6)). We can again demonstrate that the rotation of a reduced mass $\mathbf{m_r}$ in the Center of mass system is responsible for creating de Broglie matter waves.

$$
\begin{gathered}
m_1 = m << m_2 = M,\ v_1 = v, v_2 = 0, E_{k1} = \tilde{E}_1 = \tilde{E}_r = pu = hf,\ E_{kc} \cong 0, \\
m_{oc} = m + M \cong M,\ m_r \cong m_1 = m,\ v_c \cong \frac{m}{M} v \cong 0, v_r = v_1 = v, u_c \cong \frac{m}{M} u \cong \frac{1}{2} v_c \cong 0, \\
(\vec{p}_1 + \vec{p}_2 = \vec{p}_c) \Leftrightarrow (\frac{\vec{p}_1}{h} + \frac{\vec{p}_2}{h} = \frac{\vec{p}_c}{h}) \Rightarrow \begin{bmatrix} \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + 2\frac{\cos(\vec{p}_1, \vec{p}_2)}{\lambda_1 \lambda_2} = \frac{1}{\lambda_c^2} \\ \lambda_2^2\lambda_c^2 + \lambda_1^2\lambda_c^2 + 2\lambda_1\lambda_2\lambda_c^2 \cos(\vec{p}_1, \vec{p}_2) = \lambda_1^2\lambda_2^2 \end{bmatrix}, \\
p = \gamma m v = p_1 \cong p_r, p_2 = 0, \cos(\vec{p}_1, \vec{p}_2) = \cos\pi = -1, \\
u_r = u = u_1 = \lambda f = \frac{hf}{p} = \frac{v}{1 + \sqrt{1 - v^2/c^2}}, u_2 = 0, \\
f = f_1 \cong f_r = \frac{\mathbf{J}_r \omega_{mr}^2}{h(1 + \sqrt{1 - v_c^2/c^2})} = \frac{nf_m}{1 + \sqrt{1 - v_c^2/c^2}} \cong \frac{nf_m}{2}, \omega_{mr} = 2\pi f_m, \\
f_m = nh/4\pi^2 \mathbf{J}_r, \mathbf{L}_r = \mathbf{J}_r \omega_m = nh/2\pi = n\hbar,\ n \in N \Leftrightarrow \{1,2,3...\}, \\
f_2 \cong f_c \cong \frac{m}{M} f \cong 0,\ \lambda_1 = \frac{h}{p_1} = \frac{h}{\gamma m v} = \frac{h}{p} = \lambda \cong \lambda_r \cong \lambda_c,\ \lambda_2 \cong \frac{1}{2}\lambda.
\end{gathered}
\tag{4.8}
$$

What is interesting in (4.8) is that wavelength $\lambda_2 \cong \frac{1}{2}\lambda = \frac{1}{2}\lambda_1$ also exists, even $\mathbf{p_2} = \mathbf{0}$, probably as the consequence of a kind of "mirror imaging effect" of the incident particle $\mathbf{m}$ that creates coupling and interactive field with its (big mass) target, meaning that specific oscillating perturbation should also be measurable on/in the mass $\mathbf{M}$.

Since in the Center of mass system both moving particles $\mathbf{m_r}$ and $\mathbf{m_c}$ can also be presented as rotating around their common center of inertia, this "rotation" directly creates associated de Broglie waves. Such waves are mathematically recognizable on an energetic or spectral level, and "visible" in the Center of mass system, but not necessarily and directly recognizable and "visible" in the original time-space domain, in the Laboratory System. The above-analyzed example, summarized by (4.8), can also be applicable in the case of hydrogen Bohr's atom model (4.4), where an electron rotates around atom nucleus.

Here we have been talking more about transient, motional elements like a mechanical rotation (in the Center of a mass system), than about real, full-circle rotational movement (of interacting particles). When particles are in linear motion and approaching each other (without previously having their orbital moments), it is imaginable that specific transitory angular and vortex field components could be created between them (producing de Broglie matter waves), and both particles will feel (or get) equal, mutually opposite (mechanical) orbital moments. Consequently, the resulting orbital moment (of all mutually interacting objects) equals zero, but energy (or spectral) component associated with such orbital moments could be higher than zero. Generalizing this situation, we can always say that every single particle in linear motion should have a certain level of associated rotational components (orbital moments, spin, torsion field structure, etc.) since it always creates a two-body system with the rest of the surrounding universe. Implicitly, here we always assume that between mutually interacting particles there should exist specific field, force and certain wave carrier or some physical medium, even though we are sometimes not able to detect or explain what kind of material wave carrier we are dealing with.

This situation can also be modeled as a dynamic and transient "dipole-formation" (between the moving particle and its vicinity, or its target), where such dipoles could have electrical, magnetic, gravitational, inertial, or some other composite nature. The above-mentioned "dipole states" effectively rotate or produce transitory torque swings in a local Center of mass system, because the observed particle moves, and consequently produces angular and vortex field components.

The next consequence could be that Einstein Special Relativity Theory (SRT) is much more limited than it is currently considered to be (valid only under certain assumptions and for uniform, non-accelerated and rectilinear motion, which effectively does not exist without elements of rotation), and that something similar should also be valid for Maxwell electromagnetic field components. It becomes evident that SRT, Gravitation and linear motions should be upgraded for the missing rotational (or spinning) field components. Typical examples of situations that are sources of torsion field components should be all cases of elastic collisions (of course, any other collision types should also create torsion field components and de Broglie matter waves).

When we consider initial particle/s attributes, long before the interaction, and final attributes long after interaction happen, we address the totality of possible interactions between mutually approaching objects $\mathbf{m_1, m_2}$, or more correctly between their moments $\mathbf{p_1, p_2}$. We should also consider specific coupling (or binding) energy $\mathbf{U_{12}}$, or potential energy $\mathbf{U(r_{12})}$ between them, especially in cases of plastic collisions when after collision we get only one object. Until the point when collision starts and during the short process of collision (before colliding objects separate again or stay united) we can treat all collisions in the same way. Later, if objects separate, this could be the case of elastic collisions, but if objects stay fully united, we shall have an ideal plastic collision (and we could also have some other intermediary cases).

This way, the ideal plastic collision (realized or not realized) becomes like an asymptotic guiding and modeling frame for treating all collision types (as well as for treating all other interactions between two objects). Here, the idea favored is that the most essential and decisive elements of one collision process are parameters of that process ($\mathbf{m_c, m_r, v_c, v_r}$), related to its Laboratory and Center of Inertia or Center of Mass reference system. The mutually closer interacting objects with masses ($\mathbf{m_1, m_2}$) and moments ($\mathbf{p_1, p_2}$) are, the more dominant, and more relevant become (new and calculated) equivalent parameters ($\mathbf{m_c, m_r, v_c, v_r}$) = (central mass, reduced mass, center mass speed, reduced mass speed). The two-body problem between mutually non-interacting, neutral objects (like between two neutral masses) in this book is also treated as kind of interaction between them, underlining that in the near proximity of mutually approaching objects certain specific, dynamic, and transitory conditions are created which are making such bodies as mutually interacting (or energy exchanging and generating matter waves).

[♣ COMMENTS & FREE-THINKING CORNER:

Effectively, in case of plastic collision of two mutually non-interacting, electrically neutral particles (if we take care only about input-output energy balance), the equations (4.5) - (4.8) could be modified accordingly, for instance:

$$\begin{aligned}
&E = E_{tot.} = E_1 + E_2 = E_{01} + E_{k1} + E_{02} + E_{k2} = E_{oc} + E_{kc} + E_{kr},\\
&E_{01} + E_{02} = E_{oc} = (m_1 + m_2)c^2 = m_{oc}c^2,\ E_k = E_{k1} + E_{k2} = E_{kc} + E_{kr},\\
&E_{01} = m_1c^2, E_{02} = m_2c^2,\\
&E_{kr} = E_{k1} + E_{k2} - E_{kc}.
\end{aligned} \tag{4.8-1}$$

Let us briefly mention the number of mathematical options related to well-known or most probable formulations of involved energies and moments (to initiate thinking about the same problem from different platforms).

$$E_{k1} = \frac{\gamma_1 m_1 v_1^2}{1+\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{(p_1)^2}{\gamma_1 m_1\left[1+\sqrt{1-\frac{v^2_1}{c^2}}\right]} = \frac{p_1 p_c}{\gamma_1 m_{oc}\left[1+\sqrt{1-\frac{v^2_1}{c^2}}\right]} + \frac{p_1 p_r}{\gamma_1 m_1\left[1+\sqrt{1-\frac{v^2_1}{c^2}}\right]} = E_{k1c} + E_{k1r} =$$

$$= (\gamma_1 - 1)m_1c^2 = p_1v_1 / \left[1+\sqrt{1-\frac{v^2_1}{c^2}}\right] = p_1c\sqrt{\frac{\gamma_1 - 1}{\gamma_1 + 1}},\ p_1 = \gamma_1 m_1 c\sqrt{\gamma_1{}^2 - 1} = \gamma_1 m_1 v_1\ ,$$

$$E_{k2} = \frac{\gamma_2 m_2 v_2^2}{1+\sqrt{1-\frac{v^2_2}{c^2}}} = \frac{(p_2)^2}{\gamma_2 m_2\left[1+\sqrt{1-\frac{v^2_2}{c^2}}\right]} = \frac{p_2 p_c}{\gamma_2 m_c\left[1+\sqrt{1-\frac{v^2_2}{c^2}}\right]} - \frac{p_2 p_r}{\gamma_2 m_2\left[1+\sqrt{1-\frac{v^2_2}{c^2}}\right]} = E_{k2c} - E_{k2r} =$$

$$= (\gamma_2 - 1)m_2c^2 = p_2v_2 / \left[1+\sqrt{1-\frac{v^2_2}{c^2}}\right] = p_2c\sqrt{\frac{\gamma_2 - 1}{\gamma_2 + 1}},\ p_2 = \gamma_2 m_2 c\sqrt{\gamma_2{}^2 - 1} = \gamma_2 m_2 v_2\ ,$$

$$E_{kc} = \frac{\gamma_c m_{oc} v_c^2}{1+\sqrt{1-\frac{v^2_c}{c^2}}} = \frac{(p_c)^2}{\gamma_c m_{oc}\left[1+\sqrt{1-\frac{v^2_c}{c^2}}\right]} = \frac{p_1 p_c}{\gamma_c m_{oc}\left[1+\sqrt{1-\frac{v^2_c}{c^2}}\right]} + \frac{p_2 p_c}{\gamma_c m_{oc}\left[1+\sqrt{1-\frac{v^2_c}{c^2}}\right]} = E_{kc1} + E_{kc2} =$$

$$= \frac{\gamma_1}{\gamma_c}\cdot\frac{m_1}{m_c}E_{k1} + \frac{\gamma_2}{\gamma_c}\cdot\frac{m_2}{m_c}E_{k2} + \frac{p_1p_2}{\gamma_c m_c} = (\gamma_c - 1)m_{oc}c^2 = p_cv_c / \left[1+\sqrt{1-\frac{v^2_c}{c^2}}\right] = p_cc\sqrt{\frac{\gamma_c - 1}{\gamma_c + 1}},$$

$$p_c = \gamma_c m_c c\sqrt{\gamma_c{}^2 - 1} = \gamma_c m_{oc} v_c = \left|\vec{p}_1 + \vec{p}_2\right|,\ \vec{p}_c + \vec{p}_r = \vec{p}_c = \vec{p}_1 + \vec{p}_2,$$

$$E_{kr} = \frac{p_r v_r}{1+\sqrt{1-\frac{v^2_c}{c^2}}} = J_r\omega^2_r / \left[1+\sqrt{1-\frac{v^2_c}{c^2}}\right] = \frac{\gamma_2}{\gamma_c}\cdot\frac{m_2}{m_{oc}}E_{k1} + \frac{\gamma_1}{\gamma_c}\cdot\frac{m_1}{m_{oc}}E_{k2} - \frac{p_1p_2}{\gamma_c m_{oc}} \quad ?!$$

$$p_r = \left\|\vec{p}_r\right\|_{eff.} = m_r v_r \text{ (= effective value)}, ?!$$

$$\vec{p}_r = \iiint d\vec{p}_r = \vec{0} \text{ (= total, resulting vectorial field, ??!!), ?!}$$

$$E_{k1} + E_{k2} = E_{kc} + E_{kr} \Rightarrow v_1 dp_1 + v_2 dp_2 = v_c dp_c + v_r dp_r ,$$

$$\Leftrightarrow \frac{\gamma_1 m_1 v_1^2}{1+\sqrt{1-\frac{v_1^2}{c^2}}} + \frac{\gamma_2 m_2 v_2^2}{1+\sqrt{1-\frac{v_2^2}{c^2}}} = \frac{p_c v_c}{1+\sqrt{1-\frac{v_c^2}{c^2}}} + \frac{p_r v_r}{1+\sqrt{1-\frac{v_c^2}{c^2}}}, ?!$$

$$\Leftrightarrow E_{k-Lab.} = [E_{k1} + E_{k2}]_{Lab.} = [E_{k1}]_{Lab.} + [E_{k2}]_{Lab.} =$$

$$= [E_{k\text{-Translat.}}] + [E_{k\text{-Rotat.}}] = \frac{(p_1 + p_2)v_c}{1+\sqrt{1-\frac{v_c^2}{c^2}}} + \frac{\mathbf{J}_r \omega_r^2}{1+\sqrt{1-\frac{v_c^2}{c^2}}} ,$$

$$[E_{k\text{-Translat.}}] = \frac{(p_1 + p_2)v_c}{1+\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{\mathbf{J}_c \omega_c^2}{1+\sqrt{1-\frac{v_c^2}{c^2}}} , \qquad (4.5\text{-}1)$$

$$[E_{k\text{-Rotat.}}] = \frac{\mathbf{J}_r {\omega^2}_r}{1+\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{p_r v_r}{1+\sqrt{1-\frac{v_c^2}{c^2}}},$$

$$2\pi r_i = n\lambda_i = n\frac{h}{p_i} = \frac{v_i}{f_m} = n\frac{u_i}{f_i}, \; u_i = \lambda_i f_i , \; \omega_r = \omega_c = 2\pi f_m ,$$

where indexing "**Lab.**" represents energy states in a Laboratory coordinate system, "**Translat.**" energy states of translation (or linear motion) and index "**Rotat.**" represents energy states of rotation. Based on (4.5-1) we could also upgrade (4.6) - (4.8) similarly. In the process of particles' mutual approaching and an impact, and just after the impact happens, all energies and moments from (4.5-1) should be presented by time-space evolving functions.

*What is very characteristic in (4.5-1) is that every particle (m_1, m_2, m_{oc}) in the Laboratory System can have certain non-zero momentum $\vec{p}_1, \vec{p}_2, \vec{p}_c$ (in a vector form), except the particle $\mathbf{m_r}$. The resulting linear macro moment of $\mathbf{m_r}$, as a vector equals zero $\vec{p}_r = 0$ (in the Laboratory System), but its effective (**eff.**) non-vector moment (that makes contribution in energy E_{kr}) is different from zero, $p_r = \left\|\vec{p}_r\right\|_{eff.} \neq 0$... (?!). This implicates that $\mathbf{m_r}$ should be distributed around m_{oc} (in some cases maybe like a toroid) performing rotation, which would create $\vec{p}_r = \vec{0}$. What should be distributed around m_{oc} is an inertial and waiving field that has torsion components and motional energy $\mathbf{E_{kr}}$.* It is also important to notice that energy conservation of two-body interactions, like (4.5-1), in the physics of particle interactions, is usually analyzed without highlighting its direct relation to Particle-Wave Duality and Torsion Fields. Certain expressions in (4.5-1) should be still considered only as "temporarily valid" (as a starting brainstorming initiation). Relevant elaborations should be reconfirmed and developed from a much more general platform, such as the one given by T.4.4 See, also, very indicative examples in the second chapter, around equations (2.11.13-1) - (2.11.13-5), where is clearly shown how matter waves in two-body problems are being created.

In parallel with here given conceptualization, we could also say that de Broglie matter waves could be presentable as products of certain "equivalent to antenna, or resonant circuit oscillations", where moving mass, force-coupled with its environment, intrinsically creates a kind of mass-spring or inductance-capacitance oscillating circuit (where missing oscillatory circuit elements belong to the particle environment). Here we could also apply a much wider analogy with electric or mechanical oscillatory circuits to deduce what should be the unknown oscillatory circuit elements that complement the motion *of the known mass since we already know some of the important parameters of de Broglie waves. In other words, we know certain results, and we would search what produces such results. We know from electrical oscillatory circuits that total energy circulates between inductive and capacitive elements (following certain sinusoidal function, and periodically being either electrical or fully magnetic, but having constant, a total amount of energy. Analogically thinking, we would be able to conclude that total motional particle energy should fluctuate from the kind of linear motion kinetic energy to its complementary rotational motion energy and vice versa. This could be the reason why de Broglie waves are detectable only as consequences or final acts of certain interactions of particles (behaving as well-hidden waves). This theoretical matter wave concept regarding "equivalent oscillatory, resonant and antenna type circuits" should be much better elaborated later and connected with concepts and results from (4.3).*

If we now imagine (staying in the frames of mechanics) that between interacting objects could exist specific coupling or binding energy U_{12}*, and that there should be specific energy exchange between energy states of translation and rotation (for the amount* $\delta m \cdot c^2 = U_{12} / c^2 \Rightarrow$ $E_{Lab.} = E_{CI} = [E_c]_{Translat.} + [E_r]_{Rotat.} \cong \{[E_c]_{Translat.} - \delta m \cdot c^2\} + \{[E_r]_{Rotat.} + U_{12} / c^2\}$*), instead of (4.5-1) we can create the following (very much speculative, but unusual to think about) energy conservation form:*

$$E_{tot.} = E_1 + E_2 = E_c + E_r = (E_c - \delta m \cdot c^2) + (E_r + \delta m \cdot c^2), E_c = E_{oc} + E_{kc}, E_r = E_{0r} + E_{kr},$$
$$E_{tot.} = (E_{01} + E_{k1}) + (E_{02} + E_{k2}) = (E_{oc} + E_{kc}) + (E_{0r} + E_{kr}),$$

$$E_{01} + E_{02} = E_{oc} + E_{0r},\ E_{k1} + E_{k2} = E_{kc} + E_{kr},$$
$$E_{01} = m_1c^2, E_{02} = m_2c^2, E_{oc} = (m_{oc} - \delta m_c)c^2,\ E_{0r} = (m_r + \delta m_r)c^2 = \delta m_c c^2,$$

$$E_{k1} = (\gamma_1 - 1)m_1c^2 = E_1 - E_{01}, E_{k2} = (\gamma_2 - 1)m_2c^2 = E_2 - E_{02},$$
$$E_{kc} = (\gamma_c - 1)(m_{oc} - \delta m_c)c^2, E_{kr} = (\gamma_r - 1)(m_r + \delta m_r)c^2 = (\gamma_r - 1)\delta m_c c^2,$$

$$\frac{\delta m_c}{\delta m_r} = (\frac{v_r}{v_c})^2,\ \delta m_r = m_r / \left[(\frac{v_r}{v_c})^2 - 1 \right] = \frac{v_c}{v_r} \cdot \frac{U_{12}}{c^2} = \delta m_c - m_r \cong \frac{v_c}{v_r} \cdot \delta m,$$

$$\delta m_c = m_r (\frac{v_r}{v_c})^2 / \left[(\frac{v_r}{v_c})^2 - 1 \right] = \frac{v_r}{v_c} \cdot \frac{U_{12}}{c^2} = m_r + \delta m_r \cong \frac{v_r}{v_c} \cdot \delta m\ , \qquad (4.5\text{-}2)$$

$$m_{oc} = m_1 + m_2 = \frac{m_1 m_2}{\delta m} \cdot (\frac{v_r v_c}{v^2_r - v^2_c}), \delta m \cong \sqrt{\delta m_r \delta m_c} = \frac{v_r}{v_c} \delta m_r = U_{12}/c^2,$$
$$m_r = m_1 m_2 / (m_1 + m_2) = \frac{U_{12}}{c^2} \cdot (\frac{v^2_r - v^2_c}{v_r v_c}) = \delta m \cdot (\frac{v^2_r - v^2_c}{v_r v_c}) = \delta m_c - \delta m_r.$$

If we now compare the first relation from (4.4) for non-relativistic velocities, $\frac{m_p}{m_e} = (\frac{v_e}{v_p})^2 = (\frac{r_e}{r_p})^2 = (\frac{\lambda_e}{\lambda_p})^2 = (\frac{u_e}{u_p})^2$*, with similar mass relation from (4.5-2),* $\frac{\delta m_c}{\delta m_r} = (\frac{v_r}{v_c})^2$*, it becomes obvious where are (hidden) elements of rotation associated to linear motions.*

To reveal the secret about what could happen in the close vicinity of approaching objects is not an easy task because the full picture can be created only if we consider that specific (known or unknown) carrier medium (fluid or coupling field) should exist between them. In such carrier-medium, particle-wave phenomenology should be detectable. This can be analyzed by solving characteristic wave equations *describing such process (see (4.25) - (4.37)). Before we develop universal wave equations (suitable to treat such situations), we will try to use simplified modeling and to create clear conceptual picture related to collision processes. The principal objective in presenting different expressions for energy conservation in (4.5) - (4.8), (4.5-1) and (4.5-2) is to show that all two-body interactions create a particular near field, a transitory interaction zone, where all interacting members (real and virtual) mutually "communicate" producing inertial and particle-wave duality effects.*

The wave-to-particle transformation, or particle creation, should be a process related to two-body, mutually approaching objects interactions. Their relative energy (in their Center of a mass system), $E_{kr} = E_{12} = \int_{(1)}^{(2)} \vec{F}_{12} d\vec{r}_{12} = p_r u_r$*, would become a part of internal energy (and rest mass) of the unified object* $\mathbf{m_c}$*, in case of an ideally plastic impact. The other possibility is that before such impact happens, the same relative energy* $\mathbf{E_{kr}}$ *will reach certain energy level (and satisfy other necessary conditions, related to relevant conservation laws), sufficient for generating new particles (which are initially not present in the same interaction). The typical example of such interactions is when a very high-energy photon passes close to an atom, generating a couple of electron-positron particles (practically transforming the quasi-rotating wave energy content* $\mathbf{E_{kr}}$*, of a high-energy incident photon, into (new) real particles with non-zero rest masses). ♣]*

Let us again summarize two-body relations and underline how and where matter waves are being created (see the illustration on the Fig. 4.1.5. and details from T.4.4).

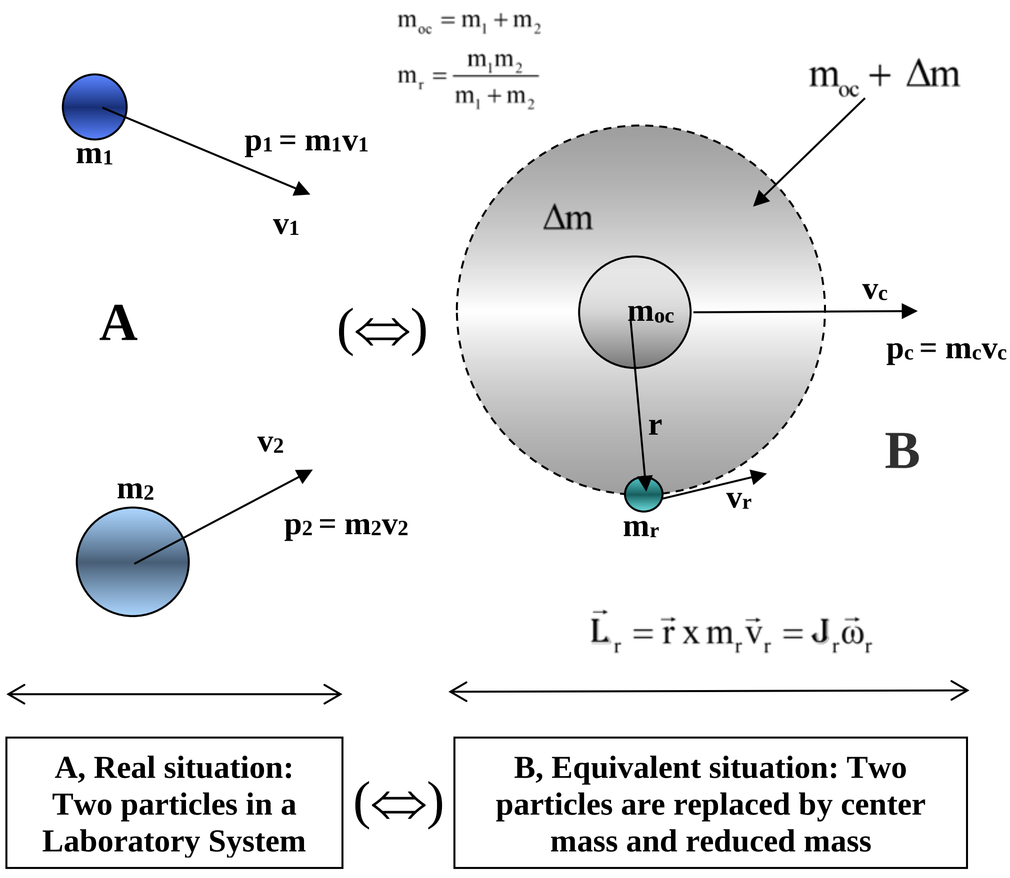


Fig.4.1.5 Mutually equivalent presentations of a two-body system: A ⇔ B
A (=) before interacting, B (=) in the process of interacting
(The plain where m_r performs rotation-like motion around m_c should be considered being perpendicular to the center of mass velocity v_c; -see T.4.4

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

The same two-body system from Fig.4.1.5, which is equivalent to the situation from Fig.4.1, can be analyzed (or described) from energy and momentum conservation laws, as follows (see the table below; T.4.4). Under two-body interactions, here we would understand: Elastic and/or Inelastic Impacts, Particle/s Creation and/or Disintegration/s, Annihilation, Compton, and Photoelectric effect etc. See, also, very indicative examples in the second chapter, around equations (2.11.13-1) - (2.11.13-5), where is clearly shown how matter waves in two-body problems are being created.

From the total energy conservation (comparing the states in a Laboratory system given under **A** and **B**), the two-body situation from Fig. 4.1.5 could be described as,

$$E_{total} = E = E(A) = E_0(A) + E_k(A) = (m_1 + m_2)c^2 + (\gamma_1 - 1)m_1c^2 + (\gamma_2 - 1)m_2c^2 =$$
$$= E(B) = E_0(B) + E_k(B) = m_c c^2 + E_{0r} + (\gamma_c - 1)m_c c^2 + E_{kr} = m_c c^2 + m_r c^2 + (\gamma_c - 1)m_c c^2 + E_{kr} =$$
$$= \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = \gamma_c m_c c^2 + E_r = \gamma_c M c^2 \Rightarrow$$
$$M_{total} = M = m_c + \frac{E_r}{\gamma_c c^2} = \frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c},\ (m_r = \frac{m_1 m_2}{m_1 + m_2},\ m_c = m_1 + m_2),$$
$$E_r = E_{0r} + E_{kr} = m_r c^2 + E_{kr} = \gamma_c (M - m_c)c^2 = (\gamma_1 m_1 + \gamma_2 m_2 - \gamma_c m_c)c^2, E_{0r} = m_r c^2,$$
$$E_{kr} = \left[\gamma_c (M - m_c) - m_r\right]c^2 = \left[\gamma_c (\frac{M - m_c}{m_r}) - 1\right] m_r c^2 = (\gamma_1 m_1 + \gamma_2 m_2 - \gamma_c m_c - m_r)c^2,$$
$$E_c = \gamma_c m_c c^2 = E_{oc} + E_{kc} = m_c c^2 + (\gamma_c - 1)m_c c^2, E_{oc} = m_c c^2, E_{kc} = (\gamma_c - 1)m_c c^2.$$

Since situations A and B are describing two of mutually (dynamically) equivalent states, it should be valid,

$$E_0 = Mc^2 = \left(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c}\right)c^2 = \left(m_c + \frac{E_r}{\gamma_c c^2}\right)c^2 = \left(m_c + \Delta m\right)c^2 = E_0(A) \ = \ E_0(B),$$
$$E_0(A) = E_{01} + E_{02} + \Delta E_A = m_1 c^2 + m_2 c^2 + \Delta E_A = m_c c^2 + \Delta E_A = Mc^2,$$
$$E_0(B) = E_{0r} + E_{oc} + \Delta E_B = m_r c^2 + m_c c^2 + \Delta E_B = Mc^2,$$
$$\Delta E_A = \left[\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c} - (m_1 + m_2)\right]c^2 = c^2 \Delta M_A,$$
$$\Delta E_B = \left[\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c} - (m_r + m_c)\right]c^2 = c^2 \Delta M_B,\ \Delta m = \frac{E_r}{\gamma_c c^2}.$$

$$E_k = (\gamma_c - 1)Mc^2 = (\gamma_c - 1)(m_c + \frac{E_r}{\gamma_c c^2})c^2 = (\gamma_c - 1)(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c})c^2 = E_k(A) \ = \ E_k(B),$$
$$E_k(A) = E_{k1} + E_{k2} + \delta E_A = (\gamma_1 - 1)m_1 c^2 + (\gamma_2 - 1)m_2 c^2 + \delta E_A,$$
$$E_k(B) = E_{kc} + E_{kr} + \delta E_B = (\gamma_c - 1)m_c c^2 + E_{kr} + \delta E_B = \left[(\gamma_1 m_1 + \gamma_2 m_2) - (m_r + m_c)\right]c^2 + \delta E_B,$$
$$\delta E_A = (\gamma_c - 1)(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c})c^2 - \left[(\gamma_1 - 1)m_1 + (\gamma_2 - 1)m_2\right]c^2 = c^2 \delta M_A,$$
$$\delta E_B = (\gamma_c - 1)(\frac{\gamma_1 m_1 + \gamma_2 m_2}{\gamma_c})c^2 - \left[(\gamma_1 m_1 + \gamma_2 m_2) - (m_r + m_c)\right]c^2 = c^2 \delta M_B.$$

$$\left\{\begin{array}{l}\left[\vec{P}(A)\right]^2-\frac{1}{c^2}[E(A)]^2=\left[\vec{P}(B)\right]^2-\frac{1}{c^2}[E(B)]^2=\text{invariant}=M^2c^2,\\ \vec{P}(A)=\vec{p}_1+\vec{p}_2=\gamma_1 m_1\vec{v}_1+\gamma_2 m_2\vec{v}_2,\\ \vec{P}(B)=\vec{P}(A)=\gamma_c m_c\vec{v}_c+\delta\vec{p}=\gamma_c(m_c+\delta m)\vec{v}_c,\\ E(A)=\gamma_1 m_1c^2+\gamma_2 m_2c^2,\ E(B)=\gamma_c m_c c^2+\delta E=\gamma_c m_c c^2+E_r.\end{array}\right\}\Rightarrow$$

$$\Rightarrow[E(A)]^2=[E(B)]^2=M^2c^4\Rightarrow[\gamma_1 m_1c^2+\gamma_2 m_2c^2]^2=[\gamma_c m_c c^2+\delta E]^2=M^2c^4\Rightarrow \quad (4.5\text{-}3)$$

$$\Rightarrow(\gamma_1 m_1c^2)^2+(\gamma_2 m_2c^2)^2+2(\gamma_1 m_1c^2)(\gamma_2 m_2c^2)=(\gamma_c m_c c^2)^2+2(\gamma_c m_c c^2)(\delta E)+(\delta E)^2=M^2c^4$$

Here we are paving or testing the concept that in the near zone of interaction, at least certain time, interacting objects would create a virtually united object $\mathbf{m_c}$ with energy coupling and energy-exchange events (here presented with: $\Delta E_A, \Delta E_B,\ \delta E_A, \delta E_B,$ $c^2\Delta m$, δE , $\delta\vec{p}$), where the matter waves are being created. Another message in formulating (4.5-3), and later (4.5-4), regardless if in some future mathematical revision (4.5-3) and (4.5-4) would be corrected or upgraded, is to show that the part of the motional energy of $\mathbf{m_r}$ is "effectively injected" into the "effective rest mass" of the central mass, as $m_c+\Delta m = m_c + \frac{E_r}{\gamma_c c^2}$. Motional energy associated with reduced mass $\mathbf{m_r}$ is dynamically equivalent to the energy of rotation (where effectively $\mathbf{m_r}$ is rotating around $\mathbf{m_c}$). Externally (from the Laboratory system) we can see only a linear particles motion, as a motion of their common center of mass (since rotational motions would be "mathematically captured" by internal content of the equivalent rest mass; -For additional conceptual clarification see chapter 2, equations (2.5.1-4) until (2.5.1-6), (2.11.1) until (2.11.9), and T.2.4, T.2.5, and T.2.6). For instance, if the specific particle is spinning around its axis and performing a rectilinear motion at the same time, it looks evident that its total motional energy should have two different components: $E_{rot.}+E_{linear-motion}=E_{rot.}+E_k$. In order to follow the message from (4.5-3) and to be more explicit, we could say that the same particle without having any element of rotation (or spinning) would have the total and kinetic energy equal to: $E_{tot.}=\gamma mc^2$, $E_k=(\gamma-1)mc^2$, and if elements of rotation or spinning are present the total and kinetic energy would become $E_{tot.}=\gamma(m_0+\frac{E_{rot.}}{c^2})c^2$, $E_k=(\gamma-1)(m_0+\frac{E_{rot.}}{c^2})c^2$, $\mathbf{m}=\mathbf{m_0}+\frac{\mathbf{E_{rot.}}}{\mathbf{c^2}}$. In other words, here, all elements of spinning are treated as a specific equivalent contribution to the rest mass. In cases when we have many particles passing from one complex motional state (state 1) to the other (state 2), where particles could have linear and rotational motion components, the same situation would be presentable as given in (4.5-4),

$$\overline{P}^2 = p_1^2 - \frac{E_1^2}{c^2} = p_2^2 - \frac{E_2^2}{c^2} = -m^2c^2 = \text{Invariant} \quad ,$$
$$\vec{L} = J\vec{\omega} = \vec{L}_1 = J_1\vec{\omega}_1 = \vec{L}_2 = J_2\vec{\omega}_2 \quad (= \text{total orbital momentum conservation}), \qquad (4.5\text{-}4)$$
$$p_{1/2} = \gamma_{1/2}m_{1/2}v_{1/2} = \gamma_{1/2}(m_{0\text{-}1/2} + \frac{E_{rot\text{-}1/2}}{c^2})v_{1/2} \quad , \quad E_{rot\text{-}1/2} = E(\vec{L}_{1/2})$$
$$E = \gamma_{1/2}m_{1/2}c^2 = \gamma_{1/2}(m_{0\text{-}1/2} + \frac{E_{rot\text{-}1/2}}{c^2})c^2,$$
$$m_{1/2} = m_{0\text{-}1/2} + \frac{E_{rot.\text{-}1/2}}{c^2} \quad .$$

*In all cases, given by expressions in T.4.4 and (4.5-1) - (4.5-3), the real and initial interaction participants (**m_1**, **m_2**) have only linear motion moments (no rotation, no spinning). When applying the law of orbital moments conservation, it should be clear that the sum of all (initial) orbital moments before interaction will stay equal to the sum of all orbital moments appearing after interaction (in this case equal zero).*

*There is only a transitory period in the near zone of interaction when two interacting or mutually approaching bodies (**m_1, m_2**) effectively create additional elements of rotation, such as **m_r** rotates around **m_c** or **m_1** and **m_2** both rotate around their common center-of-mass point. Such additional elements of rotation should also be balanced, producing that important, total orbital moment (including spinning) in every moment during the interaction equals to the total, initial orbital moment. In other words, if the initial total orbital moment of (**m_1, m_2**) equaled zero (measured from the Laboratory System, before the interaction started), in the transitory, near zone of interaction we should have only interaction products or participants with mutually balanced orbital moments that as vectors cancel each other. This is extremely important to consider if we want to understand the nature of rotation associated to the Center of the mass system (that is at the same time the source of de Broglie matter waves). In cases when initial particles (**m_1**, **m_2**) have non-zero orbital moments and spin attributes, like in (4.5-4), the same situation becomes much more complex and mathematically more productive (because we need to apply the Orbital Moments Conservation Law and find all possible distributions and redistributions of orbital moments and spinning during the process of interaction, and after interaction).*

This time we did not address the possibility that between two initial masses m_1, m_2 (entering interaction) exist some electromagnetic or other binding energy couplings (such as U_{12} in (4.5-1)), what would make previous mathematical elaboration more complicated, but without diminishing conclusions regarding motional or rotational energy transformation into a total, equivalent rest mass. Here could also be a part of the answer about understanding hidden or "dark matter" of our universe (what is a question of proper mathematical interpretation of known conservation laws).

*Effectively, any two-body situation in the process of interaction evolving creates a kind of transitory, compound system where resulting (and equivalent) central rest mass **m_c** is increased for the rest mass amount of **Δm**, becoming **m_c** + **Δm** (where $\Delta m = \frac{E_r}{\gamma_c c^2}$).*

*This way, the "rotation-like motion" of a reduced mass **m_r** around **m_c** is effectively considered by the amount of **Δm**, and a new, transitory compound system is presented only as a linear motion of the mass **m_c+Δm** with the velocity **v_c**. If **Δm** eventually became a real particle with a rest mass, the final rest mass increase or reduction (after the interaction is ended) would depend on many other factors, still not introduced here,* *to give the advantage to clear, global, and conceptual thinking (without too many details). The mass* Δm *is presented only in the function to show how rotation related motional energy component could be "mathematically injected in or extracted" from the rest mass. This situation would become more challenging if interacting particles with masses* m_1 *and* m_2 *present one real particle (with non-zero rest mass) and a photon (for instance), or if both are photons. Here we are paving the way to a new understanding of how particles are created (or disintegrated), and where the place of rotation in such a process is (regardless that what we have here could not be only an ordinary kind of rotation, but it is a state that can have orbital and magnetic moments). De Broglie, matter waves are created inside the interaction zone between* m_c *and* m_r*, and such matter waves present the "communicating channel" for all energy exchanges and mass transformations that would happen there. The parameters of mentioned matter waves, de Broglie wavelength, and frequency are also products of the same interacting zone between* m_c *and* m_r*. Both objects (*m_c *and* m_r*) could effectively be presented as having some aspects of rotational motions (for instance, as rotating around their common center of inertia, having orbital moments: see also Fig.4.1 and equations (4.3), and complementary elaborations in chapter 10. Of this book).*

[♣ COMMENTS & FREE-THINKING CORNER:

The real origins of matter waves in this book are related to energy-momentum coupling forces or fields between (at least) two bodies that are mutually in relative motion. In the following table (that has many possible and critical items, being still in development, but well enough for initiating productive brainstorming) we will present natural kinetic, and total energy balance for a two-body system.

	E_{k1}, E_1 (Particle m_1)	E_{k2}, E_2 (Particle m_2)	E_{kc}, E_c ($m_{co} = m_1 + m_2$)	E_{kr}, E_r ($m_r = \frac{m_1 m_2}{m_1 + m_2}$)
Kinetic Energy	$E_{k1} = \frac{m_1 v_1^2}{2}, v_1 << c$	$E_{k2} = \frac{m_2 v_2^2}{2}, v_2 << c$	$E_{kc} = \frac{m_{co} v_c^2}{2}, v_c << c$	$E_{kr} = \frac{m_r v_r^2}{2}, v_r << c$
	$E_{k1} = (\gamma_1 - 1) m_1 c^2$	$E_{k2} = (\gamma_2 - 1) m_2 c^2$	$E_{kc} = (\gamma_c - 1) m_{co} c^2$	$E_{kr} = \frac{m_r v_r^2}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}}$
Total Energy	$E_1 = E_{k1} + m_1 c^2 = \gamma_1 m_1 c^2$	$E_2 = E_{k2} + m_2 c^2 = \gamma_2 m_2 c^2$	$E_c = E_{kc} + m_{co} c^2 = \gamma_c m_{co} c^2$	$E_r = E_{kr} = \frac{p_r v_r}{1 + \sqrt{1 - \frac{v_c^2}{c^2}}}$
	$E = E_1 + E_2 = E_c + E_r \Leftrightarrow \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = \gamma_c m_{co} c^2 + E_r \Rightarrow E_r = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 = c p_r$ $E_r = E_{kr}, dp = h \frac{d\lambda}{\lambda^2}, E_{k1} + E_{k2} = E_{kc} + E_{kr} \Rightarrow v_1 dp_1 + v_2 dp_2 = v_c dp_c + v_r dp_r \Rightarrow v_1 \frac{d\lambda_1}{\lambda_1^2} + v_2 \frac{d\lambda_2}{\lambda_2^2} = v_c \frac{d\lambda_c}{\lambda_c^2} + v_r \frac{d\lambda_r}{\lambda_r^2}$ $\vec{p}_c = \frac{E}{c^2} \vec{v}_c = \frac{E_1 + E_2}{c^2} \vec{v}_c = \frac{E_c + E_r}{c^2} \vec{v}_c = \gamma_c m_{co} \vec{v}_c = \vec{p}_1 + \vec{p}_2 = \vec{p}_{c,r} = \vec{p}_c = \vec{p},$ $\vec{v}_c = \frac{c^2}{E} \vec{p}_c = \frac{c^2 (\gamma_1 m_1 \vec{v}_1 + \gamma_2 m_2 \vec{v}_2)}{\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2}, v_i << c \Rightarrow \gamma_i \cong 1 \Rightarrow E_i = m_i c^2, \vec{p}_i = m_i \vec{v}_i \Rightarrow$ $\vec{v}_c = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}, m_c = m_1 + m_2, m_r = \frac{m_1 m_2}{m_1 + m_2}, E_r \cong \frac{1}{2} m_r v_r^2, E_c = m_c c^2 + \frac{1}{2} m_c v_c^2.$			

4-vectors **First option (in the process !?)**	$\overline{P}_1 = (\vec{p}_1, \frac{E_1}{c}),$ $\vec{p}_1 = \gamma_1 m_1 \vec{v}_1,$ $\overline{P}_1^2 = \vec{p}_1^2 - \frac{E_1^2}{c^2} = -m_1^2 c^2$	$\overline{P}_2 = (\vec{p}_2, \frac{E_2}{c}),$ $\vec{p}_2 = \gamma_2 m_2 \vec{v}_2,$ $\overline{P}_2^2 = \vec{p}_2^2 - \frac{E_2^2}{c^2} = -m_2^2 c^2$	$\overline{P}_c = (\vec{p}_c, \frac{E_c}{c}),$ $\vec{p}_c = \gamma_c m_{co} \vec{v}_c,$ $\overline{P}_c^2 = \vec{p}_c^2 - \frac{E_c^2}{c^2} = -m_{co}^2 c^2$	$\overline{P}_r = (\vec{p}_r, \frac{E_r}{c}), m_{ro} = 0,$ $\overline{P}_r^2 = \vec{p}_r^2 - \frac{E_r^2}{c^2} = 0 \Rightarrow E_r = cp_r =$ $= \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 \Rightarrow$ $p_r = \gamma_1 m_1 c + \gamma_2 m_2 c - \gamma_c m_{co} c = \frac{E_r}{c}.$
	$(\overline{P}_1 + \overline{P}_2)^2 = (\overline{P}_c + \overline{P}_r)^2 = -m_{co}^2 c^2 \Leftrightarrow \overline{P}_1^2 + 2\overline{P}_1\overline{P}_2 + \overline{P}_2^2 = \overline{P}_c^2 + 2\overline{P}_c\overline{P}_r + \overline{P}_r^2 = -m_{co}^2 c^2 \Rightarrow \overline{P}_c\overline{P}_r = 0,...$ $\left\{ \begin{array}{l} \overline{P}_i = (\vec{p}_i, \frac{E_i}{c}), \overline{P}_1^2 = \vec{p}_i^2 - \frac{E_i^2}{c^2} = -m_i^2 c^2, \vec{p}_i = \gamma_i m_i \vec{v}_i, \\ \overline{P}_i \cdot \overline{P}_j = \vec{p}_i \cdot \vec{p}_j - \frac{E_i \cdot E_j}{c^2}, \overline{P}_1 \cdot \overline{P}_2 = \overline{P}_c \cdot \overline{P}_r \end{array} \right\} \Rightarrow (\vec{p}_1, \frac{E_1}{c}) \cdot (\vec{p}_2, \frac{E_2}{c}) = (\vec{p}_c, \frac{E_c}{c}) \cdot (\vec{p}_r, \frac{E_r}{c}) \Leftrightarrow$ $\Leftrightarrow \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = \vec{p}_c \cdot \vec{p}_r - \frac{E_c \cdot E_r}{c^2} = \vec{p}_c \cdot \vec{p}_r - \frac{\gamma_c m_{co} c^2 \cdot cp_r}{c^2} = \vec{p}_c \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$ $\vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = \vec{p}_c \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2) \Rightarrow \vec{p}_c \cdot \vec{p}_r = \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} + \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$ $\overline{P}_4^2 = (\gamma_c m_c \vec{v}_c)^2 - \frac{(E_c + E_r)^2}{c^2} = (\vec{p}_1 + \vec{p}_2)^2 - \frac{(E_1 + E_2)^2}{c^2} = inv., \vec{p}_c = \gamma_c m_c \vec{v}_c, E_c = \gamma_c m_c c^2, m_c = m_1 + m_2, m_r = \frac{m_1 m_2}{m_1 + m_2},$ $E_r = \sqrt{(\gamma_c m_c \vec{v}_c)^2 c^2 - (\vec{p}_1 + \vec{p}_2)^2 c^2 + (E_1 + E_2)^2} - \gamma_c m_c c^2, E_c + E_r = \gamma_c (m_c + \Delta m) c^2, E_r = \gamma_c (\Delta m) c^2 = \gamma_r^* m_r c^2,$ $\Delta m = m_c \left(\sqrt{\frac{\vec{v}_c^2}{c^2} - \frac{(\vec{p}_1 + \vec{p}_2)^2}{m_c^2 c^2} + \frac{(E_1 + E_2)^2}{m_c^2 c^4}} - 1 \right) = \frac{\gamma_r^*}{\gamma_c} m_r, \gamma_r^* = \gamma_c \frac{m_c}{m_r} \left(\sqrt{\frac{\vec{v}_c^2}{c^2} - \frac{(\vec{p}_1 + \vec{p}_2)^2}{m_c^2 c^2} + \frac{(E_1 + E_2)^2}{m_c^2 c^4}} - 1 \right).$			

<table>
<tr>
<td rowspan="2">4-vectors

Second option (in the process !?)</td>
<td>$\overline{P}_1 = (\vec{p}_1, \frac{E_1}{c}),$
$\vec{p}_1 = \gamma_1 m_1 \vec{v}_1,$
$\overline{P}_1^2 = \vec{p}_1^2 - \frac{E_1^2}{c^2} = -m_1^2 c^2$</td>
<td>$\overline{P}_2 = (\vec{p}_2, \frac{E_2}{c}),$
$\vec{p}_2 = \gamma_2 m_2 \vec{v}_2,$
$\overline{P}_2^2 = \vec{p}_2^2 - \frac{E_2^2}{c^2} = -m_2^2 c^2$</td>
<td>$\overline{P}_c = (\vec{p}_c - \vec{p}_r, \frac{E_c}{c}),$
$\vec{p}_c = \gamma_c m_{co} \vec{v}_c,$
$\overline{P}_c^2 = (\vec{p}_c - \vec{p}_r)^2 - \frac{E_c^2}{c^2} = -m_{co}^2 c^2$</td>
<td>$\overline{P}_r = (\vec{p}_r, \frac{E_r}{c}), m_{ro} = 0,$
$\overline{P}_r^2 = \vec{p}_r^2 - \frac{E_r^2}{c^2} = 0 \Rightarrow E_r = cp_r =$
$= \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 \Rightarrow$
$p_r = \gamma_1 m_1 c + \gamma_2 m_2 c - \gamma_c m_{co} c = \frac{E_r}{c}.$</td>
</tr>
<tr>
<td colspan="4">

$$(\overline{P}_1 + \overline{P}_2)^2 = (\overline{P}_c + \overline{P}_r)^2 = -m_{co}^2 c^2,$$

xxxxxxxxx

$$\left\{ \begin{array}{l} \overline{P}_i = (\vec{p}_i, \frac{E_i}{c}), \overline{P}_1^2 = \vec{p}_i^2 - \frac{E_i^2}{c^2} = -m_i^2 c^2, \vec{p}_i = \gamma_i m_i \vec{v}_i, \\ \overline{P}_i \cdot \overline{P}_j = \vec{p}_i \cdot \vec{p}_j - \frac{E_i \cdot E_j}{c^2}, \overline{P}_1 \cdot \overline{P}_2 = \overline{P}_c \cdot \overline{P}_r \end{array} \right\} \Rightarrow$$

$$\Rightarrow (\vec{p}_1, \frac{E_1}{c}) \cdot (\vec{p}_2, \frac{E_2}{c}) = (\vec{p}_c - \vec{p}_r, \frac{E_c}{c}) \cdot (\vec{p}_r, \frac{E_r}{c}) \Leftrightarrow$$

$$\Leftrightarrow \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \frac{E_c \cdot E_r}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \frac{\gamma_c m_{co} c^2 \cdot cp_r}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$$

$$\vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} = (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r - \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2) \Rightarrow$$

$$\Rightarrow (\vec{p}_c - \vec{p}_r) \cdot \vec{p}_r = \vec{p}_1 \cdot \vec{p}_2 - \frac{E_1 \cdot E_2}{c^2} + \gamma_c m_{co} \cdot (\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2)$$

</td>
</tr>
</table>

(in the process !?)	$\overline{P}_{1,2} = \overline{P}_1 + \overline{P}_2 = (\vec{p}, \frac{E}{c}) = (\vec{p}_1 + \vec{p}_2, \frac{E_1 + E_2}{c})$ $\vec{p} = \vec{p}_1 + \vec{p}_2,$ $\overline{P}_{1,2}^2 = \left(\overline{P}_1 + \overline{P}_2\right)^2 = \overline{P}_1^2 + \overline{P}_2^2 + 2\overline{P}_1 \cdot \overline{P}_2 =$ $= \vec{p}_1^2 - \frac{E_1^2}{c^2} + \vec{p}_2^2 - \frac{E_2^2}{c^2} + 2\vec{p}_1 \cdot \vec{p}_2 - 2\frac{E_1 \cdot E_2}{c^2} =$ $= -m_{co}^2 c^2$	$\overline{P}_{c,r} = (\vec{p}_{c,r}, \frac{E_{c,r}}{c}) = (\vec{p}_c + \vec{p}_r, \frac{E_c + E_r}{c}),$ $\vec{p}_{c,r} = \vec{p}_c + \vec{p}_r = \vec{p}_1 + \vec{p}_2 = \gamma_c m_{co} \vec{v}_c = \vec{p},$ $\overline{P}_{c,r}^2 = \left(\overline{P}_c + \overline{P}_r\right)^2 = \overline{P}_c^2 + \overline{P}_r^2 + 2\overline{P}_c \cdot \overline{P}_r =$ $= \left(\vec{p}_c^2 - \frac{E_c^2}{c^2}\right) + \left(\vec{p}_r^2 - \frac{E_r^2}{c^2}\right) + 2\left(\vec{p}_c \cdot \vec{p}_r - \frac{E_c \cdot E_r}{c^2}\right) =$ $= \left(\vec{p}_r^2 + 2\vec{p}_c \cdot \vec{p}_r + \vec{p}_c^2\right) - \left(\frac{E_c^2}{c^2} + 2\frac{E_c \cdot E_r}{c^2} + \frac{E_r^2}{c^2}\right) =$ $= \left(\vec{p}_r + \vec{p}_c\right)^2 - \left(\frac{E_r}{c} + \frac{E_c}{c}\right)^2 = -m_{co}^2 c^2 \Leftrightarrow$ $\Leftrightarrow \left(\vec{p}_1 + \vec{p}_2\right)^2 - \left(\frac{E_r + E_c}{c}\right)^2 = -m_{co}^2 c^2 \Leftrightarrow \vec{p}^2 - \frac{E}{c}^2 = -m_{co}^2 c^2$

4-Vectors Momentum & Energy during and after interaction (in the process !?)	$\overline{P}_{1,2} = (\vec{p}, \frac{E}{c}) = (\vec{p}_1 + \vec{p}_2, \frac{E_1 + E_2}{c}) \Leftrightarrow \overline{P}_{1,2}^2 = \vec{p}^2 - \frac{E^2}{c^2} = -(m_1 + m_2)^2 c^2$, $\overline{P}_{1,2}^2 = (\vec{p}_1 + \vec{p}_2)^2 - \frac{(E_1 + E_2)^2}{c^2} = -m_{co}^2 c^2$, $E^2 = \vec{p}^2 c^2 + m_{co}^2 c^4 = (\vec{p}_1 + \vec{p}_2)^2 c^2 + m_{co}^2 c^4 = (\gamma_1 m_1 + \gamma_2 m_2)^2 c^4 = \gamma_c^2 (m_1 + m_2 + \Delta m)^2 c^4$, $E^2 = [E_{k1} + m_1 c^2 + E_{k2} + m_2 c^2]^2 = (\vec{p}_1 + \vec{p}_2)^2 c^2 + m_{co}^2 c^4$ $E = E_1 + E_2 = E_c + E_r = \gamma_c (m_{co} + \Delta m) c^2 = \gamma_c m_c c^2, \vec{p} = \vec{p}_1 + \vec{p}_2$, $m_c = m_{co} + \Delta m$, $\gamma_c \Delta m = \gamma_1 m_1 + \gamma_2 m_2 - \gamma_c (m_1 + m_2)$, $E_k = E_{k1} + E_{k2} = E_{kc} + E_{kr} = \left[1 \pm 2\sqrt{1 - \left(\frac{p}{m_{co} c} \right)^2} \right] m_{co} c^2 = (\gamma_c - 1) m_{co} c^2 + \gamma_c \Delta m \cdot c^2 = E_{kc} + \Delta E$ $\left\{ = \frac{(\vec{p}_1 + \vec{p}_2)^2}{(\gamma_c + 1)(m_{co} + \Delta m)} \right\}$, $E_r = E_{kr} = \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 - \gamma_c m_{co} c^2 = (\gamma_c - 1) m_{co} c^2 + \gamma_c \Delta m \cdot c^2 - E_{kc} = \gamma_c \Delta m \cdot c^2 = \Delta E$, $\{ = (\gamma_r - 1) m_r c^2 \}$, $\overline{P}_c = \overline{P}(\vec{p}_c, \frac{E_c}{c}), \Leftrightarrow \overline{P}_c^2 = \vec{p}_c^2 - \frac{E_c^2}{c^2} = -m_{co}^2 c^2$, $\overline{P}_{c,r} = (\vec{p}_{c,r}, \frac{E_{c,r}}{c}) \Leftrightarrow \overline{P}_{c,r}^2 = \vec{p}_{c,r}^2 - \frac{E_{c,r}^2}{c^2} = -(m_{co} + \Delta m)^2 c^2$, $\overline{P}_{c\text{-}r}^2 = [\gamma_c (m_{co} + \Delta m) \vec{v}_c]^2 - \frac{[\gamma_c (m_{co} + \Delta m) c^2 + (E_r - \Delta E)]^2}{c^2} = -(m_{co} + \Delta m)^2 c^2$, $E_{c,r}^2 = [\gamma_c (m_{co} + \Delta m) c^2 + (E_r - \Delta E)]^2 = [\gamma_c (m_{co} + \Delta m) \vec{v}_c]^2 c^2 + (m_{co} + \Delta m)^2 c^4 = (E_1 + E_2)^2$, $\vec{p}_{c,r} = \gamma_c (m_{co} + \Delta m) \vec{v}_c$, $E_{c,r} = \gamma_c (m_{co} + \Delta m) c^2 + (E_r - \Delta E)$, $\Delta E = \gamma_c \Delta m \cdot c^2$.

As another way of analyzing the same situation, let us make mutually equivalent and analog mathematical descriptions of the same two-body system (moving masses $\mathbf{m_1}$ and $\mathbf{m_2}$, which are electrically and magnetically neutral, or without other active charges) in two different systems of references: - Laboratory System and Center of mass system (A and/or B, and A* and/or B*). This will be presented in four different ways, or four mutually linked (or mutually dependent) systems of references. The principal (original, real) and initial "mass-momentum players" in A are particles $\mathbf{m_1}$ and $\mathbf{m_2}$. The dynamically (or mathematically) equivalent, "mass-momentum players" in B, and B* will be new "virtual particles" $\mathbf{m_r}$ and $\mathbf{m_c}$. In fact, parallel to real interaction participants $\mathbf{m_1}$ and $\mathbf{m_2}$, that are introduced in A (where everything is conceptually obvious and clear), we are introducing additional and a little bit artificial (but dynamically equivalent), "virtual two-body situations", placed in A* and B*, or B, with mutually interacting masses $\mathbf{m_r}$ and $\mathbf{m_c}$, which have known mathematical relations with $\mathbf{m_1}$ and $\mathbf{m_2}$. A* and B*, as well as A and B, are mutually linked Laboratory and Center of mass systems (where A and A* are dealing with $\mathbf{m_1}$ and $\mathbf{m_2}$, and B and B* are dealing with $\mathbf{m_r}$ and $\mathbf{m_c}$). The objective here is to present all possible mathematical relations between moving objects $\mathbf{m_1}$, $\mathbf{m_2}$, $\mathbf{m_r}$ and $\mathbf{m_c}$ in different referential systems A, B, A* and B*, by respecting relevant conservation laws (See table T.4.4). The idea behind all of that is to show that moving objects are entering certain "mass-momentum communication" initiating (dynamically equivalent) elements of rotation, which are sources of matter-waves phenomenology. From the Laboratory System, we see (mutually approaching) objects $\mathbf{m_1}$ and $\mathbf{m_2}$, but how $\mathbf{m_1}$ is noticing $\mathbf{m_2}$ and vice versa is related to "energy-momentum" coupling/s between them, and this part of the analysis should explain the background of matter-waves and particle-wave duality. Let us first make more precise descriptions or definitions of all referential systems, A, B, A* and B*, as:

1° Laboratory System **A**: Is presented with moving and Real Interaction Participants $\mathbf{m_1}$ and $\mathbf{m_2}$.

2° Laboratory System **B**: Is presented with moving and "Virtual Interaction Participants" such as, Center mass $\mathbf{m_c}$, Reduced Mass $\mathbf{m_r}$, Center Mass Velocity $\mathbf{v_c}$, and Reduced Mass Velocity $\mathbf{v = v_r}$, etc., all of them measured by the observer from the Laboratory System. Here we need to imagine that the same observer, as in the case of the Laboratory system A, would start seeing only motions of $\mathbf{m_c}$ and $\mathbf{m_r}$, instead of seeing $\mathbf{m_1}$ and $\mathbf{m_2}$, and all mathematical relations should be established to make presentations in A and B mutually equivalent.

3° Center of mass system **A***: The observer is linked to the center of mass, seeing only two, primary and initial particles $\mathbf{m_1}$ and $\mathbf{m_2}$, which are also found in the Laboratory System A, now having different velocities and moments. Here, system A* is moving with relative velocity $\mathbf{v_c}$, measured from A.

4° Center of mass system **B***: The observer is linked to the center of mass, seeing only "Virtual Interaction Participants" such as: Center mass and Reduced mass, $\mathbf{m_c}$ and $\mathbf{m_r}$, analog to the situation in the Laboratory System under B. This time only $\mathbf{m_r}$ is moving around $\mathbf{m_c}$ and $\mathbf{m_c}$ is in the state of rest (from the point of view of the observer in B*). Here we need to imagine that the same observer, as in the case of the Center of mass system A*, would start seeing only motions of $\mathbf{m_c}$ and $\mathbf{m_r}$, instead of seeing $\mathbf{m_1}$ and $\mathbf{m_2}$, and all mathematical relations should be established to make presentations in A* and B* mutually equivalent.

The main idea here is to show that particle-wave duality should have its roots and explanation in relation with the Laboratory System B and Center of mass system B*. It has already been explained (in the beginning of this chapter) that the "kitchen" where matter waves (de Broglie wavelength and frequency) are created is causally related to what happens between $\mathbf{m_r}$ and $\mathbf{m_c}$ in B and B*, since this is the best and maybe the only way to show that $\mathbf{m_r}$ performs rotation around $\mathbf{m_c}$. Of course, here we are talking about something that is mathematically presentable as equivalent to the rotation while paying attention to satisfy all conservation laws and to make mutually similar or mutually compatible descriptions between states of motions of real particles ($\mathbf{m_1}$, $\mathbf{m_2}$) and their effective replacements ($\mathbf{m_c}$, $\mathbf{m_r}$). The concept of rotation is causally linked to a concept of frequency. There is just a small step, to imagine the creation of a certain kind of waves, which would have a specific wavelength (that is de Broglie, matter waves wavelength). If we can find elements of rotation (frequency and wavelength) related to virtual particles ($\mathbf{m_c}$, $\mathbf{m_r}$), it would be necessary to make just a small step to determine how such elements would appear to an observer from the Laboratory System. This is what L. de Broglie, A. Einstein, M. Planck, and other founders of Wave Quantum Mechanics established, apparently using different methodology; in this book, we are using the abbreviated name **PWDC** = **P**article – **W**ave – **D**uality – **C**ode, to encircle the same domain).

1° With the data presented in T.4.4, **as the first step**, we intend to make this situation mathematically and conceptually much clearer. The table T.4.4 is created by exploiting the complete, formal, or mathematical symmetry for all expressions that are related to energies and moments of (m_c, m_r), by making them look like analogous expressions of energies and moments of (m_1, m_2), in all systems of reference (A, A*, B, B*). We will soon realize by analyzing mutual mathematical consistency and compatibility of data from T.4.4 that certain energy-momentum relations in T.4.4 are mathematically non-sustainable and not compatible. Especially challenging are expressions in connection with reduced mass m_r. The biggest mathematical and conceptual challenge in realizing here elaborated strategy would be the question how to address or associate quantity of motion (linear momentum) to the Reduced Mass m_r, and to the Center mass m_c, or saying differently, what would mean "vector" quantities $\vec{p}_r = m_r\vec{v}_r$ and $\vec{p}_c = \gamma_c m_c \vec{v}_c$, found in T.4.4. The most probable case is that the total initial quantity of motion, or linear motion momentum of both particles (m_1, m_2), would be "given" only to m_c. This way, since m_r has a certain amount of motional energy, it would be shown that this is only the rotational-motional energy (and that m_r has nothing related to linear motion; consequently, m_r should only have the specific orbital moment or spin).

2° Then, **as the second step**, for the Laboratory System B, we would introduce the assumption that m_c should be the carrier of the total quantity of rectilinear motion, and that m_r has only a certain amount of rotational motional energy, without having any rectilinear motion momentum, $\vec{\mathbf{p}}_r = \mathbf{0}$ (in other words, m_r can only make rotation or spinning around m_c). Doing this way, we should be able to correct/modify all mutually not-compatible expressions in T.4.4, and exactly explain the origin and meaning of de Broglie matter waves, and the nature of unity between linear and rotational motions (obviously, this would be a voluminous mathematical task, well started but still not finalized in this book).

T.4.4. Laboratory System (A)	Laboratory System (B)	Center of the mass system (A*)	Center of the mass system (B*)
$\mathbf{m_1}$ $\mathbf{m_2}$	$\mathbf{m_c = m_1 + m_2}$ $\mathbf{m_r = \frac{m_1 m_2}{m_1 + m_2}}$	$\mathbf{m_1}$ $\mathbf{m_2}$	$\mathbf{m_c = m_1 + m_2}$ $\mathbf{m_r = \frac{m_1 m_2}{m_1 + m_2}}$
$\vec{\mathbf{v}}_1$ $\vec{\mathbf{v}}_2$	$\vec{\mathbf{v}}_c = \frac{c^2(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)}{\mathbf{E_1 + E_2}}$ $\vec{\mathbf{v}}_r = (\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1) = \vec{\mathbf{v}}$	$\vec{\mathbf{v}}_1^* = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_c$ $\vec{\mathbf{v}}_2^* = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_c$	$\mathbf{v}_c^* = \mathbf{0}$ $\mathbf{v}_r^* = \mathbf{v}_r = \mathbf{v}$
$\vec{p}_1 = \gamma_1 m_1 \vec{v}_1$ $\vec{p}_2 = \gamma_2 m_2 \vec{v}_2$ $\vec{P}(A) = \vec{p}_1 + \vec{p}_2$	$\vec{p}_c = \gamma_c m_c \vec{v}_c$ $\vec{p}_r = \vec{0},\ \lvert\vec{p}_r\rvert = m_r v_r$ (?!) $\vec{P}(B) = \vec{p}_c + \vec{p}_r$	$\vec{p}_1^* = \gamma_1^* m_1 \vec{v}_1^* = \gamma_1^* m_1 (\vec{v}_1 - \vec{v}_c) =$ $= -m_r(\vec{v}_2 - \vec{v}_1) = -m_r \vec{v}$ $\vec{p}_2^* = \gamma_2^* m_2 \vec{v}_2^* = \gamma_2^* m_2 (\vec{v}_2 - \vec{v}_c) =$ $= -m_r(\vec{v}_2 - \vec{v}_1) = +m_r \vec{v}$ $\vec{P}(A^*) = \vec{p}_1^* + \vec{p}_2^* = 0$	$p_c^* = 0$ $p_r^* = m_r v_r^*$ $P(B^*) = p_r^*$
$E_1 = E_{01} + E_{k1} = \gamma_1 m_1 c^2$ $E_{01} = m_1 c^2$, $E_{k1} = (\gamma_1\text{-}1) m_1 c^2$ $E_2 = E_{02} + E_{k2} = \gamma_2 m_2 c^2$ $E_{02} = m_2 c^2$, $E_{k2} = (\gamma_2\text{-}1) m_2 c^2$ $E(A) = E_1 + E_2 = E(B) =$ $= \gamma_c E(A^*) \geq E(A^*)$ $E_0(A) = E_{01} + E_{02} = E_0(A^*) =$ $= (m_1 + m_2)c^2 = m_c c^2$ $E_k(A) = E_{k1} + E_{k2} =$ $= \gamma_c E_k(A^*) \geq E_k(A^*)$	$E_c = E_{0c} + E_{kc} = \gamma_c m_c c^2$ $E_{0c} = m_c c^2$, $E_{kc} = (\gamma_c\text{-}1) m_c c^2$ $E_r = E_{kr}, E_{0r} = 0$, $E(B) = E_c + E_r = E(A) =$ $= \gamma_c E(B^*) \geq E(B^*)$ $E_0(B) = E_{0c} + E_{0r} = E_0(B^*) =$ $= (m_c + m_r)c^2$ $E_k(B) = E_{kc} + E_{kr} =$ $= \gamma_c E_k(B^*) \geq E_k(B^*)$	$E_1^* = E_{01}^* + E_{k1}^* = \gamma_1^* m_1 c^2$ $E_{01}^* = m_1 c^2$, $E_{k1}^* = (\gamma_1^*\text{-}1) m_1 c^2$ $E_2^* = E_{02}^* + E_{k2}^* = \gamma_2^* m_2 c^2$ $E_{02}^* = m_2 c^2$, $E_{k2}^* = (\gamma_2^*\text{-}1) m_2 c^2$ $E(A^*) = E_1^* + E_2^* = E(B^*) =$ $= E(A)/\gamma_c \leq E(A)$ $E_0(A^*) = E_{01}^* + E_{02}^* = E_0(A) =$ $= (m_1 + m_2)c^2 = m_c c^2$ $E_k(A^*) = E_{k1}^* + E_{k2}^* =$ $= E_k(A)/\gamma_c \leq E_k(A)$	$E_c^* = E_{0c}^* + E_{kc}^* = \gamma_c^* m_c c^2 = m_c c^2$ $E_{0c}^* = m_c c^2$, $E_{kc}^* = (\gamma_c^*\text{-}1) m_c c^2 = 0$ $E_r^* = E_{0r}^* + E_{kr}^*$ $E_{0r}^* = m_r c^2$, $E_{kr}^* =$ $E(B^*) = E_c^* + E_r^* = E(A^*) =$ $= E(B)/\gamma_c \leq E(B)$ $E_0(B^*) = E_{0c}^* + E_{0r}^* = E_0(B) =$ $= (m_c + m_r)c^2$ $E_k(B^*) = E_{kc}^* + E_{kr}^* = E_{kr}^* =$ $= E_k(B)/\gamma_c \leq E_k(B)$
$\gamma_1 = (1 - \frac{v_1^2}{c^2})^{-0.5}$ $\gamma_2 = (1 - \frac{v_2^2}{c^2})^{-0.5}$	$\gamma_c = (1 - \frac{v_c^2}{c^2})^{-0.5}$	$\gamma_{1,2}^* = (1 - \frac{(v_{1,2}^*)^2}{c^2})^{-0.5}$	$\gamma_c^* = 1$

The same situation presented with 4 vectors in the Minkowski Space.			
$\overline{P}_1 = \overline{P}_1(\vec{p}_1, \frac{E_1}{c})$ $\overline{P}_2 = \overline{P}_2(\vec{p}_2, \frac{E_2}{c})$ $\overline{P}_{1,2}^2 = p_{1,2}^2 - \frac{E_{1,2}^2}{c^2} = -m_{1,2}^2 c^2$ $\left[\overline{P}(A)\right]^2 = (\overline{P}_1 + \overline{P}_2)^2 = \left[\overline{P}(A^*)\right]^2 =$ $= \left[\overline{P}(A)\right]^2 c^2 - \frac{\left[E(A)\right]^2}{c^2} =$ $= -\frac{\left[E_0(A)\right]^2}{c^2} = -(m_1 + m_2)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_1\overline{P}_2 = \overline{P}_1^*\overline{P}_2^*$	$\overline{P}_c = \overline{P}_c(\vec{p}_c, \frac{E_c}{c})$ $\overline{P}_r = \overline{P}_r(\vec{p}_r, \frac{E_r}{c})$ $\overline{P}_{c,r}^2 = p_{c,r}^2 - \frac{E_{c,r}^2}{c^2} = -m_{c,r}^2 c^2$ $\left[\overline{P}(B)\right]^2 = (\overline{P}_c + \overline{P}_r)^2 = \left[\overline{P}(B^*)\right]^2 =$ $= \left[\overline{P}(B)\right]^2 c^2 - \frac{\left[E(B)\right]^2}{c^2} =$ $= -\frac{E_0(B)}{c^2} = -(m_c + m_r)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_c\overline{P}_r = \overline{P}_c^*\overline{P}_r^*$	$\overline{P}_1^* = \overline{P}_1^*(\vec{p}_1^*, \frac{E_1^*}{c})$ $\overline{P}_2^* = \overline{P}_2^*(\vec{p}_2^*, \frac{E_2^*}{c})$ $(\overline{P}_{1,2}^*)^2 = (p_{1,2}^*)^2 - \frac{(E_{1,2}^*)^2}{c^2} = -m_{1,2}^2 c^2$ $\left[\overline{P}(A^*)\right]^2 = (\overline{P}_1^* + \overline{P}_2^*)^2 =$ $= \left[\overline{P}(A^*)\right]^2 c^2 - \frac{\left[E(A^*)\right]^2}{c^2} = \left[\overline{P}(A)\right]^2 =$ $= -\frac{\left[E_0^*(A)\right]^2}{c^2} = -(m_1 + m_2)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_1\overline{P}_2 = \overline{P}_1^*\overline{P}_2^*$	$\overline{P}_c^* = \overline{P}_c^*(\vec{p}_c^*, \frac{E_c^*}{c})$ $\overline{P}_r^* = \overline{P}_r^*(\vec{p}_r^*, \frac{E_r^*}{c})$ $(\overline{P}_{c,r}^*)^2 = (p_{c,r}^*)^2 - \frac{(E_{c,r}^*)^2}{c^2} = -m_{c,r}^2 c^2$ $\left[\overline{P}(B^*)\right]^2 = (\overline{P}_c^* + \overline{P}_r^*)^2 = \left[\overline{P}(B)\right]^2 =$ $= \left[\overline{P}(B^*)\right]^2 c^2 - \frac{\left[E(B^*)\right]^2}{c^2} =$ $= -\frac{E_0(B^*)}{c^2} = -(m_c + m_r)^2 c^2 =$ $= \overline{P}(A)\overline{P}(B) = \overline{P}(A^*)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(A^*) = \overline{P}(B)\overline{P}(B^*) =$ $= \overline{P}(A)\overline{P}(B^*) = \overline{P}(A^*)\overline{P}(B) = ...$ $\overline{P}_c\overline{P}_r = \overline{P}_c^*\overline{P}_r^*$

T.4.4 is created mostly using mathematical analogies and generally known methodology from 4-vector relativistic relations in the Minkowski space (without paying too much attention to whether all details regarding newly introduced concepts about “virtual interaction participants” are already fully correct and defendable). Also, it would be necessary to double-check and test all critical relations listed there, to be able to draw final and relevant conclusions. Practically all relevant results, expressions and relations starting from (4.2), (4.3), (4.5) – (4.8) until (4.5-1) and (4.5-2) should be compared with similar, identical, or equivalent results and relations that could be developed from T.4.4, and should be mutually compatible, or if not, should be corrected and made compatible). An early, and still non-finalized “experimental” attempt, without considering 4-vectors in the Minkowski space, to develop a similar concept, as T.4.4 is presently paving, has been initialized by formulating expressions for energies and moments given in (4.5-1) and (4.5-2). The real remaining task and challenge in this situation, which would give the correct picture about the unity of rectilinear and rotational motions, would be to make all colons (of energies, moments, velocities, etc.) found in T.4.4 mutually compatible and correctly formulated in all details, what could still not be the case. Presently, the most important activity here has been to establish the concept of matter-waves creation regarding real and virtual interaction participants in different systems of reference. If we continue such process, not all possibly missing or incorrect mathematical details will escape being arranged later (*in other words the significance of the concept that is presently being introduced here is much higher than still unfinished mathematical works around it*). We should also not forget that interacting (real and virtual) objects would mutually create certain forces or fields that should agree with generalized Newton-Coulomb force expressions given from (2.4) until (2.4-3); see the second chapter of this book (Gravitation). We should not exclude the possibility of having “exotic” interactions between real ($\mathbf{m_1}$, $\mathbf{m_2}$), and virtual ($\mathbf{m_r}$, $\mathbf{m_c}$) objects (that are presently still brainstorming options). ***…………………♣]***

4.1.3.1. Example 2: X-ray Spectrum and Reaction Forces

As an illustration regarding the extension of the particle-wave duality concept, in this example, we shall analyze the generation of X-rays in an x-ray tube. Let us imagine that there is a potential difference **U** between two stable metal electrodes (in an x-ray tube). If the potential difference between electrodes is sufficiently high, this will pull and accelerate electrons from the negative electrode towards the positive electrode. Now when the electron leaves the negative electrode (since there are an electrical voltage and field between electrodes, and the electron has its mass and charge), both electrodes will "feel" certain reactive force in the form of a small "electromechanical" shock/s, and certain transient electric current (or current pulse) will be measured in the external electrode circuit. Also, now when the flying electron strikes the surface of the positive electrode, this will again create a certain electromechanical shock or waving perturbation inside the positive electrode, and X-rays will be radiated from the impact surface. Of course, the negative electrode will also "feel" the impact event because there is an electrical field between the positive and the negative electrode and a certain amount of current will flow. The acoustical activity (mechanical vibrations in electrodes) will also be generated when the electron leaves or strikes an electrode because the electron behaves as a particle that has its mass, spin, and charge. This situation is sufficiently complex to explain the nature and appearance of de Broglie matter waves.

In this process, effectively we have three distinct time intervals with sets of different (waves and particles) energy and momentum states. To make a difference between them (on a time scale), we will introduce the next indexing:
-index “**0**” - will characterize all states of rest (or electrical non-activity) before the electron leaves the negative electrode (and before voltage between electrodes is switched on),
-index “**1**” will characterize all states covering the time interval when the electron flies between the two electrodes, and
-index “**2**” will characterize all states (in both electrodes and between them) after the electron strikes the positive electrode.
-For marking electron states, we shall use index “**e**”,
-for negative electrode states, index “**ne**”,
-for positive electrode states index “**pe**” and
-for X-rays index “**x**”
-adding indexes **0**, **1** or **2** for indicating which time interval we are considering.
-For kinetic energy we will use the index “**k**”, and
-to indicate that a certain state is a kind of wave, vibration, or oscillation (that is, in fact, a state of motional, or kinetic energy), we will use the symbol "**~**”.

We will apply the energy and momentum conservation laws, if before an electric field accelerated the electron, we had only the electron in its (relative) state of rest (inside the negative electrode with energy: $\mathbf{E_{eo} = mc^2}$). When the electrical voltage was switched on, we got a moving electron under the influence of the electrical field (which will give the motional energy to the same electron of, $E_{electric field} = -eU = E_{ek1} = (\gamma_1 - 1)mc^2$).

We can also consider that the electron and both electrodes of the X-rays tube, in the state of rest (just until the moment the electron leaves the negative electrode, before the voltage was switched on, in the time interval marked with “**0**”) had negligible amounts of (internal, average, equilibrium state) wave energy and wave momentum ($\tilde{\mathbf{E}}_{eo} \approx 0, \tilde{\mathbf{p}}_{eo} \approx 0, \tilde{\mathbf{E}}_{neo} \approx 0, \tilde{\mathbf{p}}_{neo} \approx 0, \tilde{\mathbf{E}}_{peo} \approx 0, \tilde{\mathbf{p}}_{peo} \approx 0$). Also, it is obvious that the negative and the positive electrode (both being relatively big masses in the permanent state of rest) cannot have (macroscopically) any kinetic energy, or momentum ($\mathbf{E}_{neko} = \mathbf{0}$, $p_{neo} = 0$, $\mathbf{E}_{nek1} = \mathbf{0}$, $\mathbf{p}_{ne1} = \mathbf{0}$, $\mathbf{E}_{nek2} = \mathbf{0}$, $\mathbf{p}_{ne2} = \mathbf{0}$, $\mathbf{E}_{peko} = \mathbf{0}$, $\mathbf{p}_{peo} = \mathbf{0}$, $\mathbf{E}_{pek1} = \mathbf{0}$, $\mathbf{p}_{pe1} = \mathbf{0}$, $\mathbf{E}_{pek2} = \mathbf{0}$, $\mathbf{p}_{pe2} = \mathbf{0}$), providing that they are well fixed to the walls of an X-rays tube. This is important to underline, since we have already established (in this book) that every kinetic energy of a particle automatically corresponds to the same amount of its wave energy, and since the electrodes do not move (looking externally from the position of the Laboratory System), there is no wave energy belonging to them, too. Since the electrodes are parts of a closed electric circuit, a certain amount of wave energy could be created internally, inside the electrodes. The same electrodes act as a carrier medium for electric currents (or electric waves and oscillations), and a carrier for mechanical vibrations and such internal electrode states having a certain content of wave energy will be marked using the symbol "**~**".

Let us clarify the same situation more precisely. Usually, when we analyze a moving particle (in free space), its kinetic energy is equal to its wave energy $\mathbf{E}_{ek} = (\gamma - \mathbf{1})\mathbf{mc}^2 = \tilde{\mathbf{E}} = \mathbf{hf}$, which is the case regarding an electron in the state "**1**", flying between two electrodes (accelerated by an electric field). When the same electron strikes the positive electrode (being absorbed in the state "**2**"), we shall say that the electron as a particle is stopped (losing its kinetic energy and momentum: $(\mathbf{E}_{ek2} = \tilde{\mathbf{E}}_{e2}) \cong \mathbf{0}$, $(\mathbf{p}_{e2} = \tilde{\mathbf{p}}_{e2}) \cong \mathbf{0}$), but the positive electrode itself becomes the carrier of specific transient electric current pulse, and carrier of certain mechanical vibration, being characterized by non-zero internal wave states $(\tilde{\mathbf{E}}_{pe2}, \tilde{\mathbf{p}}_{pe2}) = (\tilde{\mathbf{E}}_{ne2}, \tilde{\mathbf{p}}_{ne2})$. Of course, a similar situation regarding internal electrode states, when the electron flies between them, will also make $(\tilde{\mathbf{E}}_{pe1}, \tilde{\mathbf{p}}_{pe1}) = (\tilde{\mathbf{E}}_{ne1}, \tilde{\mathbf{p}}_{ne1})$ because both electrodes are permanently a part of the externally closed electric circuit. To make this situation even more apparent, in the following table, all (particle and wave) energy and momentum states of electrodes, electron, and x-ray photons are classified.

	States just before electron left negative electrode (indexing: 0)	**States after electron left the negative electrode, before striking positive electrode (indexing: 1)**	**States just after electron stroke a positive electrode (indexing: 2)**
electron (index: e)	$(\mathbf{E}_{ek0} = \tilde{\mathbf{E}}_{e0}) \cong \mathbf{0}$, $(\mathbf{p}_{e0} = \tilde{\mathbf{p}}_{e0}) \cong \mathbf{0}$	$\mathbf{E}_{ek1} = \tilde{\mathbf{E}}_{e1}$ $\mathbf{p}_{e1} = \tilde{\mathbf{p}}_{e1}$	$(\mathbf{E}_{ek2} = \tilde{\mathbf{E}}_{e2}) \cong \mathbf{0}$, $(\mathbf{p}_{e2} = \tilde{\mathbf{p}}_{e2}) \cong \mathbf{0}$
negative electrode (index: ne)	$(\mathbf{E}_{neko} = \tilde{\mathbf{E}}_{neo}) = \mathbf{0}$, $(\mathbf{p}_{neo} = \tilde{\mathbf{p}}_{neo}) = \mathbf{0}$	$\mathbf{E}_{nek1} = 0, \mathbf{p}_{ne1} = 0$ $\tilde{\mathbf{E}}_{ne1}, \tilde{\mathbf{p}}_{ne1}$	$\mathbf{E}_{nek2} = 0, \mathbf{p}_{ne2} = 0$ $\tilde{\mathbf{E}}_{ne2}, \tilde{\mathbf{p}}_{ne2}$
a positive electrode (index: pe)	$(\mathbf{E}_{peko} = \tilde{\mathbf{E}}_{peo}) = \mathbf{0}$, $(\mathbf{p}_{peo} = \tilde{\mathbf{p}}_{peo}) = \mathbf{0}$	$\mathbf{E}_{pek1} = 0, \mathbf{p}_{pe1} = 0$ $\tilde{\mathbf{E}}_{pe1}, \tilde{\mathbf{p}}_{pe1}$	$\mathbf{E}_{pek2} = 0, \mathbf{p}_{pe2} = 0$ $\tilde{\mathbf{E}}_{pe2}, \tilde{\mathbf{p}}_{pe2}$
x-ray photons	**n/a**	**n/a**	$\tilde{E}_{x2} = hf_x, \tilde{p}_{x2} = hf_x / c$

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

We are now in the position to generalize and explicitly formulate another aspect of particle-wave duality regarding the internal wave energy content (not discussed in earlier chapters of this book), practically summarizing facts mentioned above, as follows:

In a Laboratory System of coordinates, we can characterize a moving particle by its kinetic energy. At the same time, the same kinetic energy can be conceptually presented in two different ways, such as $E_{ek} = (\gamma - 1)mc^2 = \tilde{E} = hf$, (producing experimentally, directly, or indirectly verifiable effects of de Broglie matter waves, relative to its Laboratory System). If the same body is in a state of relative rest (not moving macroscopically), its kinetic and wave energy (relative to the Laboratory System) are again mutually equal, and equal zero, $E_{ek} = \tilde{E} = \tilde{E}_{external} = 0$ (looking externally). Since the same macro-body (electrodes in this example) presents a complex material structure, it can serve (internally) as the carrier of electric currents, and mechanical signals, meaning that inside the body we could also have a certain kind of wave propagation, or specific wave energy content, which is precisely the case found in this example. This is the reason why total motional energy of specific body should be presented as the sum of its (external) kinetic or wave energy (if the body moves relative to its Laboratory System), and its internally captured wave energy. This is particularly interesting if somehow this body is excited and becomes the carrier of mechanical, electrical and any other kind of signals (apart from counting rest mass energy as its internal wave energy content).

For mathematical modeling, the appearance of any wave energy (and action-reaction forces) will be generally related to the cases of sudden changes of electron's motional energy. The first time, when the electron leaves the negative electrode, and the second time when the electron strikes the positive electrode, we can expect some transient electric current is waiving and acoustic perturbation and radiation effects on/in electrodes, or the space around them. We also know that when the electron strikes the positive electrode, x-ray photon/s will be emitted from the positive electrode surface ($\tilde{E}_{x2} = hf_x$, $\tilde{p}_{x2} = hf_x/c$), and at the same time the external electrical circuit between the two electrodes will indicate the presence of corresponding, transient current pulse (here represented by internal electrode states with corresponding wave energies and momentum: $\tilde{E}_{ne1}, \tilde{p}_{ne1}$, $\tilde{E}_{ne2}, \tilde{p}_{ne2}$, $\tilde{E}_{pe1}, \tilde{p}_{pe1}$, $\tilde{E}_{pe2}, \tilde{p}_{pe2}$).

*[♣ **COMMENTS & FREE-THINKING CORNER:** Going a little bit further in generalizing, we can see that in all of the particle mentioned above wave events or interactions, we deal with closed circuits of energy flow (like in electric circuits' analysis). For instance, an x-ray tube creates (at least) two of such, mutually coupled, closed circuits. One of them is an electrical circuit, where external voltage **U** is connected to electrodes, causing the flow of electrons between electrodes, and the second circuit is a photonic one, starting (or branching) from the point where the electrons strike the positive electrode and start generating x-ray photons. Photons propagate using the external space as a carrier, and again, in some way, a closed circuit of electromagnetic energy flow goes back to the x-ray tube (not necessarily in the form of the electromagnetic waves). All particle and wave interactions and wave motions, current/s and different signal propagations are, in one or the other way, a part of a local or broader area, closed circuits of the energy-moments flow. We are used to such concepts in Electric Circuit Theory, but this concept can be (analogically) extended and applied to all kind of motions, oscillations, waving, and to all particle and wave interactions known in physics, where forces and torques are involved. Operating with (correctly established) wave functions can help to generalize the energy flow analysis (in the frames of closed-circuit analysis), regardless of the energy origin. Any theoretical analysis of individual interaction, where particle-wave duality is involved, without understanding (or considering) how this interaction closes its energy flow*

circuit (of course, including other conservation laws) indicates that this interaction is still not entirely explained, like in case of Gravitation. Action and reaction forces and concepts of inertial forces should be in a direct connection with channels that create closed circuits of the energy-moments flow. Without closed circuit energy-flow concepts (see Fig. 4.1.4), our theories also "float in a foggy space of uncertainty and probability", presenting very much locally accurate modeling, or limited-sets data fitting.

Whenever we analyze closed circuits of some fluid, electricity, particles, etc. (or flow of some entities carrying energy and moments), implicitly we should consider that the flow of one sort of such motional matter is usually coupled with inertial and/or induction effects of its complementary (or conjugate) matter couples. This situation is analog to a flow of electrically charged particles that are accompanied by an electrical field, and characterized by electrical current, causing the appearance of a corresponding magnetic field, including "transient inertia" effects explained by Faraday, Maxwell, and Lorenz laws of electromagnetic induction, etc. Also, the Newton laws of inertia and action-reaction forces, judging by analogy, belongs to the same generally valid concept of universally applicable Inertia and Induction laws.

For illustrating closed circuit concept of an energy-moments flow, let us imagine that a particle, which initially in its state of rest has a mass $\mathbf{m}$*, is moving under an action of certain active force* $\mathbf{F_a}$*; see Fig. 4.1.6, and read 1.1. in the first chapter of this book (about Inertia, Inertial systems, and Inertial motions). In the space around the moving particle, we could have a flow of other particles, and presence of different fields (waves and forces), making our moving particle affected, irradiated, and internally excited in many ways, increasing its internal energy, temperature, or its rest mass. Such situations are always present in real particle motions and should be in some way considered to supplement our conceptual understanding of particle-wave duality. To have a more straightforward framework, we will for the time being neglect the possible presence of external and internal rotational elements in a particle motion (such as orbital moments of any kind) and consider that our particle is moving by dominant influence of an active force* $\mathbf{F_a}$*. The energy balance in such a case must account for the existence of the particle initial rest mass and motional (or environmental) contribution to particle rest mass caused by all possible external influences that increase the internal particle energy. In other words, the moving particle, besides its principal and closed energy flow circuit, also has specific additional energy flow (or exchange) because of couplings with its environment, which in many practical situations (regarding calculations) could be neglected but should not be forgotten entirely.*

$\mathbf{F_a} = \mathbf{dp} / \mathbf{dt}$ *(=) Principal, active force that makes particle moving (externally),*

$\tilde{\mathbf{F}}_{\mathbf{int.}}$ *(=) Forces of external energy flow that internally excite rest mass states, such as heating, various external radiations, vibrations, a flux of elementary particles, etc.*

$\mathbf{E_0} = \mathbf{m_0 c^2}$ *(=) Real, a minimal level of particle rest energy,*

$\tilde{E}_{int.} = \tilde{m}_{int.} c^2 = \int_{[r]} \tilde{F}_{int.} dr$ *(=) Energy of rest mass (internally) excited states, caused by some external influence/s,*

$\mathbf{E}_0 + \tilde{\mathbf{E}}_{\mathbf{int.}} = \mathbf{mc}^2 = (\mathbf{m}_0 + \tilde{\mathbf{m}}_{\mathbf{int.}})\mathbf{c}^2$ *(=) Total particle rest energy,*

$E_{total} = E_0 + E_k = \gamma mc^2 = \gamma (m_0 + \tilde{m}_{int.})c^2$ *(=) Total particle energy in motion,*

$E_k = \tilde{E} = (\gamma - 1)mc^2 = (\gamma - 1)(m_0 + \tilde{m}_{int.})c^2 = \int_{[r]} F_a dr$ *(=) Motional particle energy.*

All over this book are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

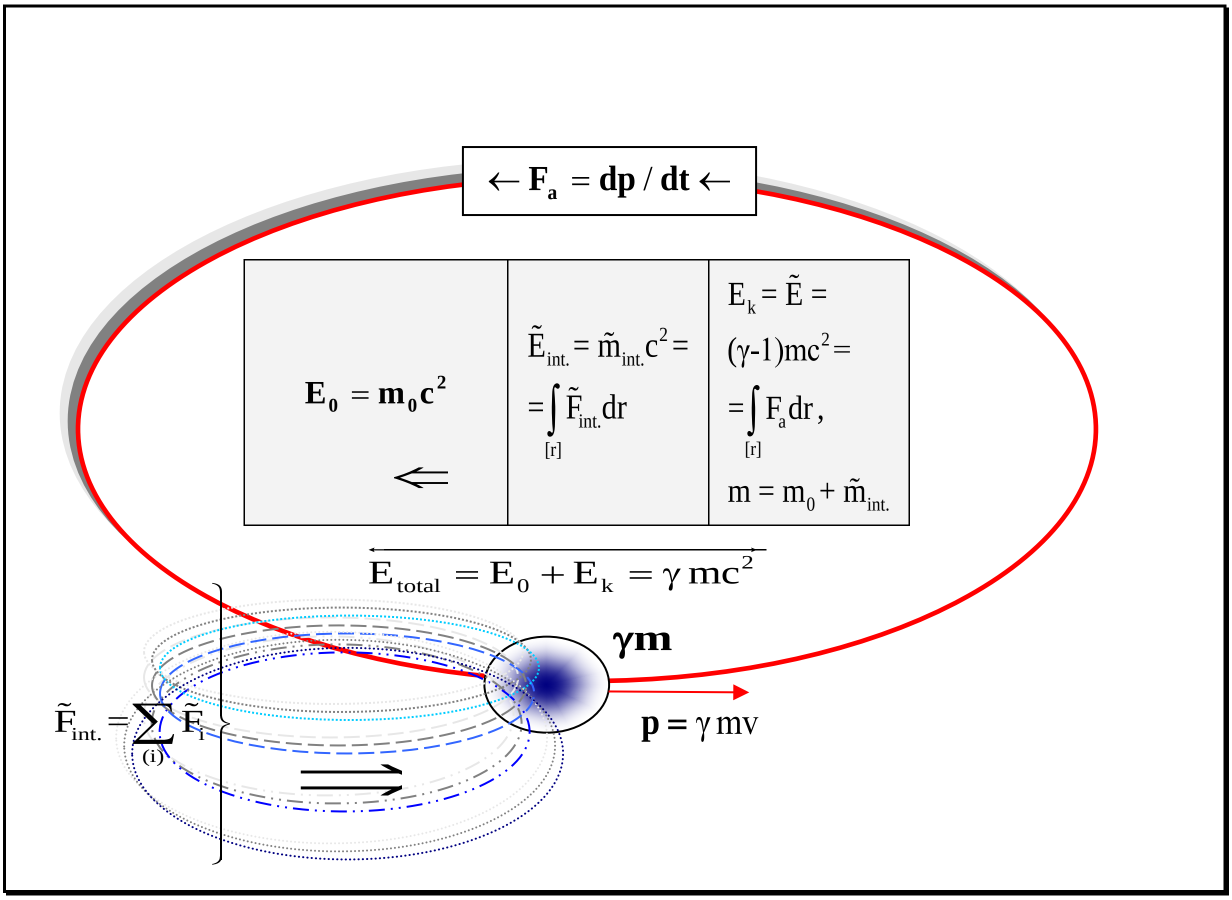


Fig. 4.1.6 An Illustration of the Closed Circuit Energy Flow ♣]

In other words, we should always have the entities like energy or signal source (voltage or current source, velocity, or force source, etc.), and certain kind of receiver or load. Such simple source-load, fully closed circuits structures could be coupled and chained with many other source-loads closed segments, but anyway, we should be able to identify and characterize them (like in electric circuits theory and practice). Gravitation theory is an example where such conceptualization is still missing. Also, electron and nucleus states of an atom (and their bidirectional electromagnetic energy or photons exchanges) inside an atom, as well as similar extended, bidirectional external electromagnetic exchanges, should create fully closed energy-mass-moments-flow circuits. This way we will create the background for understanding Gravitation on a way how R. Boskovic, [6] and N. Tesla, [97], conceptualized; -see more about such gravitation in the chapters 2., 8., and 9.). N. Tesla's and R. Boskovic ideas about gravitation, universal natural force, and radiant energy are implicating that all atoms in our Universe should mutually communicate externally and internally (outwards and inwards) to have closed circuits of involved energy, mass, moments, currents, and voltage components. Laws related to Newton action-reaction forces, and similar electromagnetic induction laws, including quantum entanglement effects, and solutions of classical wave equations where we see that mutually opposed and in different directions propagating waves are always being created, are supporting closed circuits concepts. See more about such concepts and analogies in the first chapter of this book, as well as in the chapters 4.3, 8, 9 and 10.

Let us go back to the previously analyzed example of x-ray radiation. Apparently, in the situation when analyzing X-ray radiation, we have sufficient number of tangible, measurable and visible waving and radiation (electrical and acoustic) events. Consequently, we cannot characterize de Broglie electron matter waves only as "phantom probability waves", since here we always have a closed electrical circuit where we are in a natural (and fully deterministic) position to know, see, calculate, and measure what and where really waves and produces electrical currents, voltages, and photons), in real time.

Considering the differences between electron group and phase velocities, when an electron flies between two electrodes, the effects of associated retarded potentials will make this situation a little bit analytically more complex.

Let us now calculate the outgoing kinetic energy $\mathbf{E_{ek1}}$ and speed $\mathbf{v_{e1}}$ of a single electron, $\mathbf{m = m_e}$, $\mathbf{q_e = -e}$ (just now of leaving the surface of a negative electrode; time interval "**1**"). The total energy conservation law applied, in this case, will give,

$$\left\{E_{eo} = E_{ek1} + E_{eo} + E_{electric\,field}\right\} \Rightarrow$$

$$\left\{\begin{array}{l} mc^2 = (\gamma_1 - 1)mc^2 + mc^2 - eU\,, \\ \gamma_1 = 1 + eU/mc^2 = (1 - v_{e1}{}^2/c^2)^{-1/2} \Rightarrow v_{e1} = c\left[1 - 1/(1 + eU/mc^2)^2\right]^{1/2}, \\ E_{ek1} = (\gamma_1 - 1)mc^2 = eU = \gamma_1 m v_{e1}{}^2 / \left[1 + (1 - v_{e1}{}^2/c^2)^{-1/2}\right],\ p_{e1} = mv_{e1} \end{array}\right\}$$

We know that when the electron leaves the negative electrode (since the electron has a certain mass, moment, spin, and charge), the negative electrode will "feel" small electrical and mechanical shock. A certain amount of energy ($\delta\tilde{E} \geq 0$) will be dissipated in such (transient) process (in a closed electrical circuit), reducing outgoing electron speed (in fact, the outgoing electron speed will be: $v_{e1} \leq c\left[1 - 1/(1 + eU/mc^2)^2\right]^{1/2}$). Applying the law of total energy conservation again, we can consider this correction in the following way:

$$\left\{\left[E_{eo} = E_{ek1} + E_{eo} + E_{electric\,field} + \delta\tilde{E}\right], \left[E_{ek1} = (\gamma_1 - 1)mc^2 = \tilde{E}_{e1} = \tilde{p}_{e1} \cdot u_{e1} = -eU\right]\right\} \Rightarrow$$

$$\left\{\begin{array}{l} mc^2 - \delta\tilde{E} = (\gamma_1 - 1)mc^2 + mc^2 - eU\,, \\ \gamma_1 = 1 + (eU - \delta\tilde{E})/mc^2 = (1 - v_{e1}{}^2/c^2)^{-1/2} \\ v_{e1} = c\left\{1 - 1/\left[1 + (eU - \delta\tilde{E})/mc^2\right]^2\right\}^{1/2} \leq c\left\{1 - 1/\left[1 + eU/mc^2\right]^2\right\}^{1/2}, \\ E_{ek1} = (\gamma_1 - 1)mc^2 = -eU - \delta\tilde{E} = \gamma_1 m v_{e1}{}^2 / \left[1 + (1 - v_{e1}{}^2/c^2)^{-1/2}\right] \leq -eU\,,\ p_{e1} = mv_{e1} = \tilde{p}_{e1} \end{array}\right\}$$

Since an external voltage source electrically connects negative and positive electrodes, making a closed electrical circuit, any (wave energy or electrical current) perturbation in one electrode will coincidently produce a similar effect in the opposite electrode ($(\tilde{\mathbf{E}}_{\mathbf{pe1}}, \tilde{\mathbf{p}}_{\mathbf{pe1}}) = (\tilde{\mathbf{E}}_{\mathbf{ne1}}, \tilde{\mathbf{p}}_{\mathbf{ne1}}), (\tilde{\mathbf{E}}_{\mathbf{pe2}}, \tilde{\mathbf{p}}_{\mathbf{pe2}}) = (\tilde{\mathbf{E}}_{\mathbf{ne2}}, \tilde{\mathbf{p}}_{\mathbf{ne2}})$). Apparently, in this case, energy and momentum conservation laws should be applied in a very general way, considering (internal electrical and acoustical) states in electrodes, flying-electron energy, and energy of x-ray radiation from the positive electrode.

To have a complete energy conservation picture of this process we should apply the principal relations (4.2), between (total) kinetic and wave energy and their momentum, $\Delta E_k = -\Delta\tilde{E}$ and $\Delta\mathbf{p} = -\Delta\tilde{\mathbf{p}}$. Let us first apply (4.2) between the states **(0 → 1),** when the electron was in the state of rest (on the negative electrode surface) and just after it left the negative electrode,

$$\Delta E_k = -\Delta\tilde{E} \Leftrightarrow \Delta\left[\sum_{(i)} E^i{}_k\right] = -\Delta\left[\sum_{(i)} \tilde{E}^i\right] \Rightarrow$$

$$\left[(E_{ek1}+E_{nek1}+E_{pek1})-(E_{eko}+E_{neko}+E_{peko})\right] = -\left[(\tilde{E}_{e1}+\tilde{E}_{ne1}+\tilde{E}_{pe1})-(\tilde{E}_{eo}+\tilde{E}_{neo}+\tilde{E}_{peo})\right] \Rightarrow$$
$$E_{ek1} = -(\tilde{E}_{e1}+\tilde{E}_{ne1}+\tilde{E}_{pe1}) = (\gamma_1-1)mc^2 = -eU-\delta\tilde{E} \le -eU\ ,\ \delta\tilde{E}\ge 0$$
$$\Delta p = -\Delta\tilde{p} \Leftrightarrow \Delta\left[\sum_{(i)} p^i\right] = -\Delta\left[\sum_{(i)} \tilde{p}^i\right] \Rightarrow$$
$$\left[(p_{e1}+p_{nek1}+p_{pe1})-(p_{eo}+p_{neo}+p_{peo})\right] = -\left[(\tilde{p}_{e1}+\tilde{p}_{ne1}+\tilde{p}_{pe1})-(\tilde{p}_{eo}+\tilde{p}_{neo}+\tilde{p}_{peo})\right] \Rightarrow$$
$$p_{e1} = -(\tilde{p}_{e1}+\tilde{p}_{ne1}+\tilde{p}_{pe1}) = mv_{e1}$$

Since electrical circuit between electrodes, when the electron passes from the negative to the positive electrode, is always closed, internal waving phenomena (or currents) in one electrode will be at the same time present in the opposite electrode. By applying the essential relations between kinetic again and wave energy and their momentum, $\mathbf{\Delta E_k = -\Delta\tilde{E}}$ and $\mathbf{\Delta p = -\Delta\tilde{p}}$, between the states **(1 → 2),** when the electron was flying between two electrodes, and just after it stroke the positive electrode, we get:

$$\Delta E_k = -\Delta\tilde{E} \Leftrightarrow \Delta\left[\sum_{(i)} E^i{}_k\right] = -\Delta\left[\sum_{(i)} \tilde{E}^i\right] \Rightarrow$$

$$\left[(E_{ek2}+E_{nek2}+E_{pek2})-(E_{ek1}+E_{nek1}+E_{pek1})\right] =$$
$$= -\left[(\tilde{E}_{e2}+\tilde{E}_{ne2}+\tilde{E}_{pe2}+\tilde{E}_{x2})-(\tilde{E}_{e1}+\tilde{E}_{ne1}+\tilde{E}_{pe1})\right] \Rightarrow$$
$$\Rightarrow \begin{Bmatrix} 0 \le \tilde{E}_{x2} = hf_x = E_{ek1} - (\tilde{E}_{ne1}+\tilde{E}_{pe1}) - (\tilde{E}_{ne2}+\tilde{E}_{pe2}) \le E_{ek1} \\ 0 \le \tilde{E}_{x2} - E_{ek1} = hf_x - E_{ek1} = -(\tilde{E}_{ne1}+\tilde{E}_{pe1}) - (\tilde{E}_{ne2}+\tilde{E}_{pe2}) \le 0 \\ -(\tilde{E}_{ne1}+\tilde{E}_{pe1}) \le (\tilde{E}_{ne2}+\tilde{E}_{pe2}) \\ E_{ke1} - (\tilde{E}_{ne1}+\tilde{E}_{pe1}) \le hf_x + (\tilde{E}_{ne2}+\tilde{E}_{pe2}) \end{Bmatrix}$$
$$\Delta p = -\Delta\tilde{p} \Leftrightarrow \Delta\left[\sum_{(i)} p^i\right] = -\Delta\left[\sum_{(i)} \tilde{p}^i\right] \Rightarrow$$
$$\left[(p_{e2}+p_{ne2}+p_{pe2})-(p_{e1}+p_{nek1}+p_{pe1})\right] = -\left[(\tilde{p}_{e2}+\tilde{p}_{ne2}+\tilde{p}_{pe2}+\tilde{p}_{x2})-(\tilde{p}_{e1}+\tilde{p}_{ne1}+\tilde{p}_{pe1})\right] \Rightarrow$$
$$\left[0-p_{e1}\right] = -\left[(0+\tilde{p}_{ne2}+\tilde{p}_{pe2}+\tilde{p}_{x2})-(\tilde{p}_{e1}+\tilde{p}_{ne1}+\tilde{p}_{pe1})\right], p_{e1} = \tilde{p}_{e1} \Rightarrow$$

$$\tilde{p}_{x2} = hf_x / c = p_{e1} - (\tilde{p}_{ne2} + \tilde{p}_{pe2}) - (\tilde{p}_{e1} + \tilde{p}_{ne1} + \tilde{p}_{pe1}) = -(\tilde{p}_{ne2} + \tilde{p}_{pe2}) - (\tilde{p}_{ne1} + \tilde{p}_{pe1})$$
$$c\tilde{p}_{x2} = hf_x = \tilde{E}_{x2} = -c(\tilde{p}_{ne2} + \tilde{p}_{pe2}) - c(\tilde{p}_{ne1} + \tilde{p}_{pe1}) =$$
$$= E_{ek1} - (\tilde{E}_{ne1} + \tilde{E}_{pe1}) - (\tilde{E}_{ne2} + \tilde{E}_{pe2}) \leq E_{ek1} = (\gamma_1 - 1)mc^2 = -eU - \delta\tilde{E} \leq -eU$$
$$0 < f_x \leq \frac{E_{ek1}}{h} = \frac{(\gamma_1 - 1)mc^2}{h} = \frac{-eU}{h} .$$

Eventually, when the electron (as a particle) strikes the positive electrode, its final kinetic energy ($\mathbf{E_{ek1}}$) will be (partially or fully) transformed into radiation of X-rays (and the part of the same energy would create transient electric current and mechanical oscillations in electrodes circuit). The maximal frequency of radiated x-ray photons will be,

$$(f_x)_{max.} = -eU/h = \frac{(\gamma_1 - 1)mc^2}{h} = \frac{E_{ek1}}{h} > \frac{m_e v_{el}^2}{2} .$$

We got the well-known frequency of X-rays $(\mathbf{f_x})_{max.} = -\mathbf{eU/h}$, explaining this way that (**only and exclusively**) the relativistic motional energy of a particle $\mathbf{E_{ek1}}$, which is equal to the particle-wave energy $\tilde{\mathbf{E}}_{\mathbf{e1}}$, is wholly or partially, radiated in the form of X-rays, $\mathbf{hf_x}$. If accelerated electrons have sufficiently high striking speeds, there is no way to show that x-ray energy could be calculated using classical mechanics kinetic energy expression $\mathbf{m_e v_e^2 / 2}$, which indirectly indicates that traditional (non-relativistic) Schrödinger equation would also be inapplicable to this case (since in Schrödinger's equation, particle kinetic energy is treated as $\mathbf{m_e v_e^2 / 2}$). We also see that contemporary quantum mechanical concept (or model) of a particle, which includes its rest mass and rest energy as an integral part of its wave packet, is unacceptable (at least in this case). We can calculate and measure that only relativistic motional energy is transformed into X-rays and waving perturbations in electrodes (and all of them belong to de Broglie or matter waves, being easily measurable and with deterministic nature).

The typical X-rays spectrum has a form of continuous spectral distribution because of many reasons such as:

-the negative electrode of the x-ray tube is heated to facilitate electrons emission (modulating the speed of electrons),

-external electrical circuit presents resistive and reactive electric impedance, producing specific energy dissipation and oscillating-current effects,

-there are also associated acoustic phenomena in electrodes, and such process is also different in many other (x-ray tube design) details in comparison with an idealized case of this example.

The only common valid conclusion for all x-ray devices is that the maximal experimentally measured x-ray frequency is precisely equal to the frequency calculated in this example $\mathbf{f_x} \leq -\mathbf{eU/h}$, confirming that de Broglie matter waves present only a form of kinetic energy of ordinary vibrations (of electromagnetic, mechanical and/or any other nature), without any participation of rest masses.

4.1.4. Matter Waves and orbital motions

The two-body problem (widely exploited until here) is offering one of the explanations regarding matter waves formation, after involving few of assumptions and a certain level of creative imagination (see very indicative examples in the second chapter, around equations (2.11.13-1) - (2.11.13-5)). In most of the cases presented in different literature (and here), the two-body problem is being analyzed from relations and interactions between linear particle motions and linear moments of participants. We also know that every linear motion is a small segment of a picture of a more substantial rotation or orbital motion picture (where the radius of rotation could be arbitrarily long). The natural tendency of all motions in our universe is, eventually to stabilize as some form of rotating or orbital motion (in Gravitation related to Kepler and Newton laws, and in micro-world related to atoms and subatomic particles). This is presenting an additional opportunity to analyze the two-body problem (regarding matter waves), just by respecting global conservation of involved orbital and spin moments.

Let us consider the same two-body situation (see Fig.4.1. and Fig.4.1.5) as already introduced at the beginning of this chapter under 4.1.2., but now, analogically, and equivalently presented from orbital and spin moments conservation. This should always be applicable (see the significant background to the familiar approach regarding consequences of global conservation of orbital and spin moments in [36], Anthony D. Osborne, & N. Vivian Pope), as follows,

$$\left[\begin{array}{l}
\left\{\begin{array}{l}
E_{k1}+E_{k2}=E_{km}+E_{kM}=E_{kc}+E_{kr}=\frac{1}{2}m_c v_c^2+\frac{1}{2}m_r v^2, E_{kr}=\frac{1}{2}m_r v_r^2\cong\frac{1}{2}mv^2=E_{k1}\cong\frac{1}{2}J\omega_m^2,\\
\frac{1}{2}m_1 v_1^2+\frac{1}{2}m_2 v_2^2=\frac{1}{2}mv_1^2+\frac{1}{2}Mv_2^2=\frac{1}{2}m_c v_c^2+\frac{1}{2}m_r v_r^2\cong\frac{1}{2}Mv_c^2+\frac{1}{2}mv^2\left[=\frac{1}{2}Mv_c^2+\frac{1}{2}J\omega_m^2\right]
\end{array}\right\},\\
\text{and/or}\left\{\begin{array}{l}
E_{ki}=\frac{1}{2}m_i v_i^2=\frac{1}{2}p_i v_i\cong\frac{\gamma_i m_i v_i^2}{1+\sqrt{1-v_i^2/c^2}}=\frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}}, E_{kr}=\frac{1}{2}m_r v_r^2\cong\frac{p_r v_r}{1+\sqrt{1-v_r^2/c^2}},\\
\frac{\gamma_1 mv_1^2}{1+\sqrt{1-v_1^2/c^2}}+\frac{\gamma_2 Mv_2^2}{1+\sqrt{1-v_2^2/c^2}}\cong\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}+E_{kr}\left[\cong\frac{\gamma_c Mv_c^2}{1+\sqrt{1-v_c^2/c^2}}+\frac{J\omega_m^2}{1+\sqrt{1-v_c^2/c^2}}\right],\\
\left(mc^2+\frac{\gamma_1 mv_1^2}{1+\sqrt{1-v_1^2/c^2}}\right)+\left(Mc^2+\frac{\gamma_2 Mv_2^2}{1+\sqrt{1-v_2^2/c^2}}\right)=\left(m_c c^2+\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}\right)+\left(m_r^*\cdot c^2+E_{kr}\right)\\
\Leftrightarrow\gamma_1 mc^2+\gamma_2 Mc^2=\gamma_c m_c c^2+\left(m_r^*\cdot c^2+E_{kr}\right)\Rightarrow m_r^*=\gamma_1 m+\gamma_2 M-\gamma_c m_c-E_{kr}\cong 0, \gamma_i=\frac{1}{\sqrt{1-v_i^2/c^2}}
\end{array}\right\},\\
\text{and}\left\{m_c=m_1+m_2,\ m_r=\frac{m_1 m_2}{m_1+m_2}, \vec{v}_c=\frac{m_1\vec{v}_1+m_1\vec{v}_1}{m_1+m_2}, v_i=\omega_i r_i,\ m_i v_i^2=J_i\omega_i^2,\ p_i v_i=L_i\omega_i\right\}
\end{array}\right] \&$$

$$\& \left\{ \begin{array}{l} \vec{L} = \sum_{(i)} \vec{L}_i = \vec{\omega}_c \sum_{(i)} J_i = \text{const.},\ \vec{\omega}_c = \dfrac{\sum_{(i)} J_i \vec{\omega}_i}{\sum_{(i)} J_i} = \dfrac{\sum_{(i)} \vec{L}_i}{\sum_{(i)} J_i} = \dfrac{J_1\vec{\omega}_1 + J_2\vec{\omega}_2}{J_1 + J_2} = \dfrac{\vec{L}_1 + \vec{L}_2}{J_1 + J_2}, \\ \vec{L} = \vec{L}_1 + \vec{L}_2 = J_1\vec{\omega}_1 + J_2\vec{\omega}_2 = \vec{\omega}_c(J_1 + J_2), \vec{L}_i = J_i\vec{\omega}_i, J_c = J_1 + J_2, J_r = \dfrac{J_1 J_2}{J_1 + J_2} \\ \vec{\omega}_r = \vec{\omega}_1 - \vec{\omega}_2 = \vec{\omega}_m,\ \vec{L}_r = \dfrac{J_1 J_2}{J_1 + J_2}(\vec{\omega}_1 - \vec{\omega}_2) = J_r\vec{\omega}_m = J_r\vec{\omega}_r, \omega_r = \omega_m . \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} E_{k1} + E_{k2} = E_{km} + E_{kM} = E_{kc} + E_{kr} = \\ \frac{1}{2}m_1 v_1 r_1 \frac{v_1}{r_1} + \frac{1}{2}m_2 v_2 r_2 \frac{v_2}{r_2} = \frac{1}{2}m_c v_c r_c \frac{v_c}{r_c} + \frac{1}{2}m_r v_r r \frac{v_r}{r} \\ = \frac{1}{2}J_1\omega_1^2 + \frac{1}{2}J_2\omega_2^2 = \frac{1}{2}J_c\omega_c^2 + \frac{1}{2}J_r\omega_r^2 = \\ = \frac{1}{2}L_1\omega_1 + \frac{1}{2}L_2\omega_2 = \frac{1}{2}L_c\omega_c + \frac{1}{2}L_r\omega_r,\ E_{kr} = \dfrac{J_r\omega_r^2}{1 + \sqrt{1 - v_c^2/c^2}} = hf_s = \tilde{E}_s \end{array} \right\}. \qquad (4.1.4\text{-}1)$$

Based on such analogical conceptualization (and accepted assumptions), here involved two-body interaction participants are rotating or being in specific orbital motion long before interaction starts (both in Laboratory and Center of the mass system). The kinetic energy member (which is a product of such two-body interaction),

$$E_{kr} = \frac{J_r\omega_r^2}{1 + \sqrt{1 - v_c^2/c^2}} = hf_s = \tilde{E}_s \cong \frac{1}{2}J_r\omega_r^2 = \frac{1}{2}L_r\omega_r = \frac{1}{2}m_r v_r^2 , \qquad (4.1.4\text{-}2)$$

should present kind of rotation, and at the same time (as promoted in this book) this is the crucial, spinning matter wave energy ($\tilde{E}_s = hf_s$). What is strange regarding such rotation ($\vec{L}_r$, $\vec{\omega}_r$) is related to the total orbital momentum conservation, which is producing, $\vec{L} = \vec{L}_1 + \vec{L}_2 = \text{const.} \Rightarrow \vec{L}_r = \vec{0}$. Since $E_{kr} = \frac{1}{2}L_r\omega_r \neq 0,\ \vec{\omega}_r = \vec{\omega}_1 - \vec{\omega}_2 \neq 0$ one of the solutions is that $\vec{L}_r$ could present "orbiting-spinning" motion (with spiral, toroidal, or rotating ring envelope; see [94], Classical Mechanics; - chapter 14) composed of minimum two spinning objects with equal and mutually opposed spin moments:

$$\vec{L}_r = \vec{L}_r^{+} + \vec{L}_r^{-} = \frac{\vec{L}_r}{2} - \frac{\vec{L}_r}{2} = \vec{S} - \vec{S} = \vec{0}, , \vec{L}_r^{+} = +\vec{S} = +\frac{\vec{L}_r}{2},\ \vec{L}_r^{-} = -\vec{S} = -\frac{\vec{L}_r}{2}$$

$$\Leftrightarrow \vec{L}_r^{+} = -\vec{L}_r^{-} \Rightarrow E_{kr} = \frac{1}{2}L_r\omega_r = \frac{1}{4}L_r^{+}\omega_r + \frac{1}{4}L_r^{-}\omega_r . \qquad (4.1.4\text{-}3)$$

Much more about similar items and familiar elaborations, given in chapter 10. Of this book.

[♣ COMMENTS & FREE-THINKING CORNER:

Until here, all attempts are made to prove that matter wave energy (of de Broglie waves) belongs to kinetic or motional energy of particles, quasi-particles including different aspects of waves and oscillations, and that the rest mass does not belong directly to matter wave energy (but, within certain limits also presents an absorber or emitter of a wave energy). We also know that going deeper into the matter and particle structure, we gradually find a more complex field and wave structures, which only conditionally present particles (with possible content of rest mass). Again, by analyzing them structurally, we find new energy content in the form of some other waves and fields inside (somehow self-stabilized in a closed form of standing waves). What should be the rest mass, if internal building constituents of every particle are waves and fields, or motional energy in the form of stationary, standing waves, or other kind of self-resonant states? Most probably, this is the place for understanding intrinsic, self-sustaining, rotational field nature of elementary wave ingredients of our universe. Just, somehow, all simple matter structures, called elementary particles (electrons, protons, neutrons, etc.) create stable, closed, limited-space domains (like a toroid, rotating rings, etc.) of internally rotating, stationary, and standing wave field formations. Once, when such self-sustaining, space-limited vortex domain is created, it behaves like an elementary particle (or if wave "sublimating and solidifying" process is not finished, we have a quasi-particle, a photon, etc.). In our universe it is natural that such closed, rotating wave structures can be created, remaining stable during a time (or during an exceedingly long time). Inside such structures there is always specific energy content, giving a chance to associate the rest mass to such objects (knowing the exact proportionality between mass and energy, from the Relativity Theory). The visible external signs or marks indicating that such rotating structures are the reality of our world are orbital moments and spin characteristics of all elementary particles (as well as de Broglie matter waves, spontaneous radioactivity, different fields, and forces). Of course, this is still an intuitive and speculative concept, but sufficiently useful as a starting platform for understanding the meaning of the rest mass. Nevertheless, it was a conceptual mistake of quantum mechanical matter waves and wave function modeling to include the stable rest mass of a particle unconditionally into a matter wave modeling, since only a space-time variable and non-stationary energy flow creates free-propagating matter wave.

4.2. INTERACTIONS MODELING

One of the cases of common interactions where involved participants could be particles, quasiparticles, waves, wave-groups, and photons, is certain scattering (see the picture below, Fig.4.2.0). For instance, a moving particle, or photon, or matter-waves packet from the conditions under "Medium 1" is colliding with the surface of a "Medium 2". In such situation, an energy-momentum entity could be reflected, refracted, diffused, or scattered, producing new particles and/or waves in a "Medium 1", and also could be partially refracted into a "Medium 2" in a form of matter waves (as photons, phonons, some particles, and/or mechanical waves, for instance). Of course, all energy and momentum conservation laws should be satisfied (for interaction participants) to correctly describe such events. ***In this chapter, we will address similar interactions between particles and waves, with an intention to show that a total matter-waves energy is equal only to the relevant, incident particle (or wave packets) kinetic or motional energy, and that involved <u>rest mass energy</u> (if exist) is not a part of an active, only motion-related matter-wave energy.***

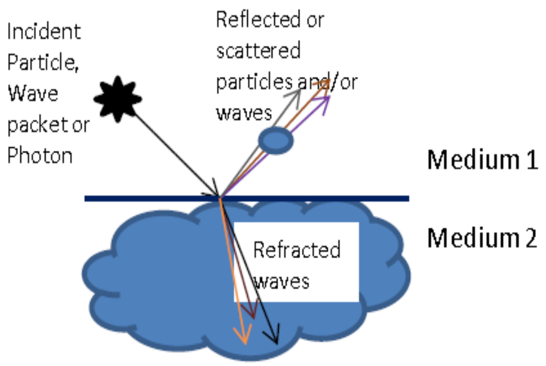


Fig. 4.2.0 Scattering Interactions

Let us now explore the limits of traditional mechanical analysis regarding conservation laws and interactions where matter wave phenomenology is involved. The most general case of interactions between particles and waves or wave-groups in any mutual combination of participants is presented in the table T.4.2.1.

T.4.2.1.

Elements entering interaction	Interaction zone	Resulting elements after interaction
<u>**Particles**</u>: $\{E_k^i, p^i\}, E_0^i$ $i \in [1,2,3,...P]$ →	**?!?!**	→ <u>**Particles**</u>: $\{E_k^m, p^m\}, E_0^m$ $m \in [1,2,3,...M]$
<u>**Waves**</u>: $\{\tilde{E}^j, \tilde{p}^j\}$ $j \in [1,2,3,...Q]$ →		→ <u>**Waves**</u>: $\{\tilde{E}^n, \tilde{p}^n\}$ $n \in [1,2,3,...N]$

<u>Moving</u> **Particles** are "energy-momentum" dualistic matter-wave states with non-zero rest masses, $\{E_k^i, p^i\}, E_0^i > 0$. Not moving (standstill) particles with non-zero rest masses are pure particle states (of course inside certain inertial reference system), $\{E_k^i = 0, p^i = 0\}, E_0^i > 0$.

All over this paper are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER... ♣]. The idea here has been to establish intuitive and brainstorming, not-confirmed and freethinking corners for making fast comments, and presenting challenging ideas, that could be some other time developed towards something much more meaningful and more properly integrated with Physics.

Pure **Waves** are energy-momentum states with zero rest masses (like photons), $\left\{\tilde{E}^j, \tilde{p}^j\right\}$, $E_0^j = 0$.

Energy conservation for cases from T.4.2.1 can be presented as follows:

$$
\begin{aligned}
&E_{total} = E = E_k + E_0 = \sum_{(i)}(E_k^i + E_0^i) + \sum_{(j)} \tilde{E}^j = \sum_{(m)}(E_k^m + E_0^m) + \sum_{(n)} \tilde{E}^n \Leftrightarrow \\
&\Leftrightarrow (\sum_{(i)} E_k^i - \sum_{(m)} E_k^m) + (\sum_{(i)} E_0^i - \sum_{(m)} E_0^m) = -\left[\sum_{(j)} \tilde{E}^j - \sum_{(n)} \tilde{E}^n\right] \Leftrightarrow \\
&\Leftrightarrow \Delta E = \Delta E_k + \Delta E_0 = -\Delta \tilde{E} \Leftrightarrow \Delta E + \Delta \tilde{E} = 0, \\
&\Delta E_k = \sum_{(i)} E_k^i - \sum_{(m)} E_k^m = \Delta\left[\sum_{(i,m)} E_k^{i,m}\right], \ \Delta E_0 = \sum_{(i)} E_0^i - \sum_{(m)} E_0^m = \Delta\left[\sum_{(i,m)} E_0^{i,m}\right] \\
&\Delta \tilde{E} = \sum_{(j)} \tilde{E}^j - \sum_{(n)} \tilde{E}^n = \Delta\left[\sum_{(j,n)} \tilde{E}^{j,n}\right] \\
&(E_{total} = E = \gamma mc^2, E_k = (\gamma - 1)mc^2 = E - E_0, \ E_0 = mc^2, \\
&\tilde{E} = E_k = pu = \gamma mvu = cp\sqrt{\frac{\gamma-1}{\gamma+1}} = (\gamma-1)mc^2 \ (=) \text{ real particles kinetic energy}, \\
&\tilde{E} = hf = pu = cp \ (=) \text{ Photon energy}).
\end{aligned}
\tag{4.8-1}
$$

Planck constant **h** is applicable and relevant in cases of photons and elementary or subatomic microparticles (where conservation of angular moments on closed orbits, or where self-closed standing wave structures are on some way involved). In other cases of (bigger) microparticles, and/or involvements of macro masses, including astronomic masses (like in solar or planetary systems), certain new and different $H >> h$ constant would be applicable (again in cases of orbital, self-closed and periodical motions).

Momentum conservation in cases from T.4.2.1 can be presented as follows:

$$
\begin{aligned}
&\vec{P} = \sum_{(i)} p^i + \sum_{(j)} \tilde{p}^j = \sum_{(m)} p^m + \sum_{(n)} \tilde{p}^n \Leftrightarrow \\
&\Leftrightarrow \sum_{(i)} p^i - \sum_{(m)} p^m = -\left[\sum_{(j)} \tilde{p}^j - \sum_{(n)} \tilde{p}^n\right] \Leftrightarrow \Delta p = -\Delta \tilde{p} \Leftrightarrow \Delta p + \Delta \tilde{p} = 0, \\
&\Delta p = \sum_{(i)} p^i - \sum_{(m)} p^m = \Delta\left[\sum_{(i,m)} p^{i,m}\right], \quad \Delta \tilde{p} = \sum_{(j)} \tilde{p}^j - \sum_{(n)} \tilde{p}^n = \Delta\left[\sum_{(j,n)} \tilde{p}^{j,n}\right]
\end{aligned}
\tag{4.8-2}
$$

$$
\vec{P}^2 - \frac{E_{total}^2}{c^2} = \vec{P}'^2 - \frac{E'^2_{total}}{c^2} = \vec{P}''^2 - \frac{E''^2_{total}}{c^2} = inv. = -m^2c^2.
$$

If the objects entering an interaction have angular moments (either external macro moments, $\mathbf{L} = \mathbf{L}_{external}$, and/or internal, intrinsic, or spin moments, $\mathbf{L}_0 = \mathbf{L}_{internal}$), we can again (analogically) present the conservation of angular moments on a similar way as we did for energy and momentum conservation. By summarizing and analogically generalizing conservation laws (4.8-1) and (4.8-2) in a condensed form, including involved angular moments, the unity and complementarity of particle and wave aspects of certain motion will be presentable as:

$$\left\{\begin{array}{l} \left[\begin{array}{l}\Delta E = \Delta E_k + \Delta E_0 = -\Delta\tilde{E} \\ \Delta p = -\Delta\tilde{p}\end{array}\right] \Rightarrow \\ \Rightarrow \Delta \mathbf{L} = \Delta \mathbf{L}_{external} + \Delta \mathbf{L}_{internal} = -\Delta\tilde{\mathbf{L}} \\ \dots \end{array}\right\} \Leftrightarrow \{\Delta X = -\Delta\tilde{Y}\}_{(when\ applicable)} . \tag{4.8-3}$$

If we now consider only the case of a single object in a certain phase of transformation, we can address the meaning of involved reaction forces by transforming (4.8-3) into (4.8-4), as for instance:

$$\{\Delta \to d\} \Rightarrow \left\{\begin{array}{l} dE = dE_k + dE_0 = dE_k = -d\tilde{E} \\ dp = -d\tilde{p} \\ d\mathbf{L} = d\mathbf{L}_{external} + d\mathbf{L}_{internal} = -d\tilde{\mathbf{L}} \\ \dots\dots\dots \end{array}\right\} \Leftrightarrow \{dX = -d\tilde{Y}\} \Rightarrow$$
$$\Rightarrow \{dX = -d\tilde{Y}\}/dt \Leftrightarrow \frac{dX}{dt} = -\frac{d\tilde{Y}}{dt} \Leftrightarrow \{action \equiv reaction\ in\ opposite\ direction\},$$
$$\Rightarrow \mathbf{F} = \frac{dp}{dt} = -\frac{d\tilde{p}}{dt} (=) Linear\ force,\ \tau = \frac{d\mathbf{L}}{dt} = -\frac{d\tilde{\mathbf{L}}}{dt} (=) Torque,$$

where,

$$\left\{\begin{array}{l} v = \frac{dE}{dp} = \frac{d\tilde{E}}{d\tilde{p}} = \frac{\gamma+1}{\gamma} u (\equiv)\ linear\ group\ velocity, \\ \left[v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda},\ \gamma = (1 - v^2/c^2)^{-0.5}\right], \\ u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = c\sqrt{\frac{\gamma-1}{\gamma+1}} = \frac{\gamma}{\gamma+1} v (\equiv)\ linear\ phase\ velocity,\ \lambda = \frac{h}{p}, \\ \Rightarrow \left[\begin{array}{l} \omega_g = \frac{dE}{dL} = \frac{d\tilde{E}}{d\tilde{L}} = 2\pi f_g (\equiv)\ angular\ group\ velocity, \\ \omega_{ph.} = \frac{\tilde{E}}{\tilde{L}} = 2\pi f_{ph} = 2\pi f\ (\equiv)\ angular\ phase\ velocity \end{array}\right] \Rightarrow \\ \Rightarrow \omega_g = \omega_{ph.} - \lambda_{ph}\frac{d\omega_{ph.}}{d\lambda_{ph}} = -\lambda_{ph}^2\frac{df}{d\lambda_{ph}},\ \lambda_{ph} = \frac{h}{L} \\ \omega_g = \omega_{ph.} - \frac{h}{L}\frac{d\omega_{ph.}}{d\lambda_{ph}} \leftrightarrow 2\pi f_g = 2\pi f - \frac{h}{L}\frac{d\omega_{ph.}}{d\left(\frac{h}{L}\right)} \leftrightarrow f_g = f + L\frac{df}{dL}. \end{array}\right\}. \tag{4.8-4}$$

The idea in developing (4.8-4), regardless using an oversimplified procedure (based on analogies), is to underline the unity between particle and wave nature of all objects carrying motional energy and angular momenta. It is just a matter of our interpretation (and operational mathematical advantages and preferences) how to expose corpuscular or wave nature of moving objects since both exist at the same time (see **PWDC** foundations in Chapters 4.1, and 10.). Of course, later, along this analysis, we need to include involved electromagnetic reality (charges, electric dipoles, multipoles, currents, voltages and magnetic moments), because angular, orbital, spinning and circular motions could be easily coupled with relevant electric currents, magnetic moments, and electromagnetic fields.

The proposal how to address and analyze any kind of possible binary interactions, as mentioned until here, is that we imagine and exercise (conceptually, experimentally, and theoretically) that any of such events will be creatively and analytically placed between an idealized plastic and an idealized elastic impact scenario. *The meaning here is that imaginable and mathematically and theoretically mastered frames and skeleton of ideally plastic and ideally elastic collisions will serve as a guiding boundary (or asymptotic) case, and any real interaction (between waves and particles in any combination) should be somewhere inside or between such frontiers (regarding our mathematical modelling and expected experimental results). Natural, structural, and theoretical frameworks of such frontiers are all conservation laws of Physics. This way we can avoid statistical and probabilistic strategies, too often being used in such analyzes, and use stochastic methods only when necessary for better mathematical representation of results, or for certain instructive modeling, when conditions for the statistical approach are naturally present.*

4.2.1. Elastic Collisions

Let us now analyze an <u>ideally elastic collision</u> of two objects (for instance between two particles, or between one particle and a photon, like in the case of Compton Effect). Since rest masses of interacting particles do not change, here we only have a certain exchange of kinetic energies between collision participants. Because the elastic collision should not be dependent on time-axis direction, we will create the kinetic energy balance in case when particles $\mathbf{m}_1, \mathbf{m}_2$ enter the collision, producing $\mathbf{m}_3, \mathbf{m}_4$ (T.4.3.1), and when particles $\mathbf{m}_3, \mathbf{m}_4$ go backwards, producing $\mathbf{m}_1, \mathbf{m}_2$, (by reversing the time-axis direction, and assuming that we could have the total temporal process reversibility in all of its aspects, as presented with T.4.4).

T.4.3.1.

Elements entering interaction	Interaction zone	Resulting elements after interaction
<u>Object-1</u> $\{E_{k1}, p_1 = m_1 v_1\}$ or $\{\tilde{E}_1, \tilde{p}_1\}$	→ **?!?!** →	**<u>Object-3</u>** $\{E_{k3}, p_3 = m_3 v_3\}$ or $\{\tilde{E}_3, \tilde{p}_3\}$
<u>Object-2</u> $\{E_{k2}, p_2 = m_2 v_2\}$ or $\{\tilde{E}_2, \tilde{p}_2\}$		**<u>Object-4</u>** $\{E_{k4}, p_4 = m_4 v_4\}$ or $\{\tilde{E}_4, \tilde{p}_4\}$

T.4.4. (The same interaction as in T.4.3.1, but time-reversed)

Elements entering interaction	Interaction zone	Resulting elements after interaction
<u>Object-1</u> $\{E_{k1}, p_1 = m_1 v_1\}$ or $\{\tilde{E}_1, \tilde{p}_1\}$	← **?!?!** ←	**<u>Object-3</u>** $\{E_{k3}, p_3 = m_3 v_3\}$ or $\{\tilde{E}_3, \tilde{p}_3\}$
<u>Object-2</u> $\{E_{k2}, p_2 = m_2 v_2\}$ or $\{\tilde{E}_2, \tilde{p}_2\}$		**<u>Object-4</u>** $\{E_{k4}, p_4 = m_4 v_4\}$ or $\{\tilde{E}_4, \tilde{p}_4\}$

If we now apply the energy conservation law on T.4.3.1 and T.4.4, for non-relativistic velocities, it will be:

$$\begin{Bmatrix} E_{k1}+E_{k2}=E_{kr-12}+E_{kc-12}=E_{k3}+E_{k4} \\ E_{k3}+E_{k4}=E_{kr-34}+E_{kc-34}=E_{k1}+E_{k2} \end{Bmatrix} \Rightarrow \{E_{kr-12}+E_{kc-12}=E_{kr-34}+E_{kc-34}\} \Rightarrow$$

$$\begin{Bmatrix} \frac{m_1 v^2{}_1}{2}+\frac{m_2 v^2{}_2}{2}=\frac{m_1 m_2}{m_1+m_2}\frac{(\vec{v}_1-\vec{v}_2)^2}{2}+\frac{(m_1\vec{v}_1+m_2\vec{v}_2)^2}{2(m_1+m_2)}=\frac{m_3 v^2{}_3}{2}+\frac{m_4 v^2{}_4}{2} \\ \frac{m_3 v^2{}_3}{2}+\frac{m_4 v^2{}_4}{2}=\frac{m_3 m_4}{m_3+m_4}\frac{(\vec{v}_3-\vec{v}_4)^2}{2}+\frac{(m_3\vec{v}_3+m_4\vec{v}_4)^2}{2(m_3+m_4)}=\frac{m_1 v^2{}_1}{2}+\frac{m_2 v^2{}_2}{2} \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} E_{kr}=\frac{m_1 m_2}{m_1+m_2}\frac{(\vec{v}_1-\vec{v}_2)^2}{2}=\mu_{r(1-2)}\frac{v^2_{r(1-2)}}{2}=\frac{m_3 m_4}{m_3+m_4}\frac{(\vec{v}_3-\vec{v}_4)^2}{2}=\mu_{r(3-4)}\frac{v^2_{r(3-4)}}{2}, \\ E_{kc}=\frac{(m_1\vec{v}_1+m_2\vec{v}_2)^2}{2(m_1+m_2)}=\frac{(m_3\vec{v}_3+m_4\vec{v}_4)^2}{2(m_3+m_4)}=m_c\frac{v_c^2}{2}=\frac{\vec{P}\vec{v}_c}{2},\ E_{kr}=\mu_r\frac{v_r^2}{2}=\frac{p_r v_r}{2}, \\ v_c=\frac{\vec{P}}{m_c},\ m_c=m_1+m_2=m_3+m_4\ ,\ \mu_r=m_r=\frac{m_i m_j}{m_i+m_j} \\ \vec{P}=\vec{p}_1+\vec{p}_2=\vec{p}_3+\vec{p}_4=m_1\vec{v}_1+m_2\vec{v}_2=m_3\vec{v}_3+m_4\vec{v}_4,\ p_r=\mu_r v_r \\ v^2_{r(1-2)}=(\vec{v}_1-\vec{v}_2)^2,\ v^2_{r(3-4)}=(\vec{v}_3-\vec{v}_4)^2, \\ \frac{m_1 m_2}{m_1+m_2}(\vec{v}_1-\vec{v}_2)^2=\frac{m_3 m_4}{m_3+m_4}(\vec{v}_3-\vec{v}_4)^2\ ,\frac{d\vec{p}_1+d\vec{p}_2}{m_1+m_2}=\frac{d\vec{p}_3+d\vec{p}_4}{m_3+m_4}=\frac{d\vec{P}}{m_c} \end{Bmatrix}$$

Alternatively, and analogically created, equivalent (at least indicative) mathematical processing in case of relativistic velocities could be:

$$\Rightarrow \begin{Bmatrix} E_{kr}=\frac{\gamma_1\gamma_2 m_1 m_2}{\gamma_1 m_1+\gamma_2 m_2}\left[\frac{(\vec{v}_1-\vec{v}_2)^2}{1+\sqrt{1-v_c^2/c^2}}\right]=\frac{\gamma_3\gamma_4 m_3 m_4}{\gamma_3 m_3+\gamma_4 m_4}\left[\frac{(\vec{v}_3-\vec{v}_4)^2}{1+\sqrt{1-v_c^2/c^2}}\right]= \\ =\frac{\gamma_c\mu_r v_r^2}{1+\sqrt{1-v_c^2/c^2}}=E_{k1}+E_{k2}-E_{kc}=E_{k3}+E_{k4}-E_{kc}=\int_{(1)}^{(2)}\vec{F}_r d\vec{r}_{12}=\int_{(3)}^{(4)}\vec{F}_r d\vec{r}_{34}, \\ v^2_{r(1-2)}=(\vec{v}_1-\vec{v}_2)^2,\quad v^2_{r(3-4)}=(\vec{v}_3-\vec{v}_4)^2,\ \vec{F}_r=\frac{d\vec{p}_{r(1-2)}}{dt}=\frac{d\vec{p}_{r(3-4)}}{dt}, \\ E_{kc}=\frac{P_c v_c}{1+\sqrt{1-v_c^2/c^2}}=\frac{P_c^2}{(1+\sqrt{1-v_c^2/c^2})\gamma_c m_c}=\frac{\gamma_c m_c v_c^2}{1+\sqrt{1-v_c^2/c^2}}= \\ =E_{k1}+E_{k2}-E_{kr}=E_{k3}+E_{k4}-E_{kr} \\ \vec{v}_c=\frac{(\sum\vec{p})c^2}{\sum E}=\frac{\vec{P}}{\gamma_c m_c},\quad \frac{\sum E}{c^2}=\gamma_c m_c=\gamma_1 m_1+\gamma_2 m_2=\gamma_3 m_3+\gamma_4 m_4, \\ \vec{P}=\vec{p}_1+\vec{p}_2=\vec{p}_3+\vec{p}_4=\gamma_1 m_1\vec{v}_1+\gamma_2 m_2\vec{v}_2=\gamma_3 m_3\vec{v}_3+\gamma_4 m_4\vec{v}_4=\gamma_c m_c\vec{v}_c=\vec{P}_c, \\ E_{ki}=\frac{\gamma_i m_i v^2{}_i}{1+\sqrt{1-v_i^2/c^2}}=\frac{p_i v_i}{1+\sqrt{1-v_i^2/c^2}}=\frac{p_i^2}{(1+\sqrt{1-v_i^2/c^2})\gamma_i m_i}=p_i u_i=\tilde{E}_i, \\ \vec{P}^2-\frac{(\sum E)^2}{c^2}=\vec{P}'^2-\frac{(\sum E')^2}{c^2}=\vec{P}''^2-\frac{(\sum E'')^2}{c^2}=inv.=-M^2c^2. \end{Bmatrix} \tag{4.8-5}$$

As we can see from all energy and momentum conservation relations valid for Classical-Mechanics case (or for small velocities), and for cases with relativistic velocities (4.8-5), it is relatively complicated to use such mathematical processing, and find general solutions for elastic scatterings. The other opportunity is to use energy conservation in its differential form, since mathematically we have the same and quite simple expressions, both for low and high-velocity (or for non-relativistic and relativistic) motions. Instead of kinetic energies, we can also take total energies, as for instance:

$$\begin{aligned}
&d\{E_{k1}+E_{k2}=E_{kr}+E_{kc}=E_{k3}+E_{k4}\}\Leftrightarrow d\{E_1+E_2=E_{kr}+E_{kc}+E_{oc}=E_3+E_4\}\Leftrightarrow\\
&\Leftrightarrow v_1dp_1+v_2dp_2=v_cdp_r+v_cdp_c=v_3dp_3+v_4dp_4,\\
&dE_{kr}=\vec{v}_r d\vec{p}_r=(\vec{v})_c^2 d(\gamma_c\mu_{r(1-2)})+\vec{p}_{r(1-2)}d\vec{v}_{r(1-2)}=(\vec{v})_{r(3-4)}^2 d(\gamma_c\mu_{r(3-4)})+\vec{p}_{r(3-4)}d\vec{v}_{r(3-4)},\\
&v_{r(1-2)}^2=(\vec{v}_1-\vec{v}_2)^2,\ v_{r(3-4)}^2=(\vec{v}_3-\vec{v}_4)^2,\\
&E_i=E_{0i}+E_{ki}, E_{0i}=\text{const.}, dE_{0i}=0,\ p_i=\gamma_i m_i v_i\ ,\ \gamma_i=(1-v_i^2/c^2)^{-0.5},\ p_{ri}=\gamma_i\mu_i v_{ri},
\end{aligned} \tag{4.8-6}$$

Doing that way, we can address the forces acting in every phase of scatterings,

$$\begin{aligned}
&\{v_1dp_1+v_2dp_2=v_rdp_r+v_cdp_c=v_3dp_3+v_4dp_4\}/dt\Rightarrow\\
&\vec{v}_1\frac{d\vec{p}_1}{dt}+\vec{v}_2\frac{d\vec{p}_2}{dt}=\vec{v}_r\frac{d\vec{p}_r}{dt}+\vec{v}_c\frac{d\vec{p}_c}{dt}=\vec{v}_3\frac{d\vec{p}_3}{dt}+\vec{v}_4\frac{d\vec{p}_4}{dt}\Rightarrow\\
&\Rightarrow \vec{v}_1\vec{F}_1+\vec{v}_2\vec{F}_2=\vec{v}_r\vec{F}_r+\vec{v}_c\vec{F}_c=\vec{v}_3\vec{F}_3+\vec{v}_4\vec{F}_4.
\end{aligned} \tag{4.8-7}$$

For instance, only mutual forces between moving masses m_1 and m_2, and m_3 and m_4, in their Center of Mass System (including Newton-Coulomb attractive forces) will be:

$$\begin{aligned}
&\vec{F}_{r(1-2)}=\frac{d\vec{p}_{r(1-2)}}{dt}=-\vec{F}_{r(2-1)}=\frac{d\vec{p}_{r(2-1)}}{dt}\ ,\ \vec{F}_{r(3-4)}=\frac{d\vec{p}_{r(3-4)}}{dt}=-\vec{F}_{r(4-3)}=\frac{d\vec{p}_{r(4-3)}}{dt},\\
&E_{kr}=\int_{(1)}^{(2)}\vec{F}_{r(1\text{-}2)}d\vec{r}_{12}=\int_{(3)}^{(4)}\vec{F}_{r(3\text{-}4)}d\vec{r}_{34}.
\end{aligned}$$

The force acting in the center of mass (analyzed from a Laboratory System) where the mass m_c is, will be,

$$\vec{F}_c=\frac{d\vec{p}_c}{dt}=\frac{d\vec{P}}{dt}=\vec{F}_1+\vec{F}_2.$$

And, of course, external forces acting on each particle (from the Laboratory System point of view), will be, respectively,

$$\vec{F}_1=\frac{d\vec{p}_1}{dt},\vec{F}_2=\frac{d\vec{p}_2}{dt},\vec{F}_3=\frac{d\vec{p}_3}{dt},\vec{F}_4=\frac{d\vec{p}_4}{dt}.$$

From (4.8-5) - (4.8-7), we can see that the most important “transient-time laboratory place” where one interaction happens is the Center of Mass System, regarding evolving transformations of reduced mass μ_r and relative velocity v_r is as follows,

$$\begin{aligned}
&\vec{v}_r \frac{d\vec{p}_r}{dt} = \vec{v}_r \vec{F}_r = (\vec{v})^2_{r(1-2)} \frac{d(\gamma_c \mu_{r(1-2)})}{dt} + \vec{p}_{r(1-2)} \frac{d\vec{v}_{r(1-2)}}{dt} = \\
&= (\vec{v})^2_{r(3-4)} \frac{d(\gamma_c \mu_{r(3-4)})}{dt} + \vec{p}_{r(3-4)} \frac{d\vec{v}_{r(3-4)}}{dt} \Leftrightarrow \\
&\Leftrightarrow \frac{dE_{kr}}{dt} = \vec{v}_r \vec{F}_r = (\vec{v})^2_{r(1-2)} \frac{d(\gamma_c \mu_{r(1-2)})}{dt} + \vec{p}_{r(1-2)} \vec{a}_{r(1-2)} = \\
&= (\vec{v})^2_{r(3-4)} \frac{d(\gamma_c \mu_{r(3-4)})}{dt} + \vec{p}_{r(3-4)} \vec{a}_{r(3-4)} \\
&\vec{p}_r = \gamma_c \mu_r \vec{v}_r \, , \, \vec{a}_{r(i-j)} = \frac{d\vec{v}_{r(i-j)}}{dt} \, (=) \, \text{acceleration} \; .
\end{aligned} \tag{4.8-8}$$

In reality, reduced mass μ_r is a kind of virtual object, or kind of de Broglie matter wave packet (placed in the space between interacting objects), and presents an evolving wave-energy group in process of transformation ($dE_{kr} = \vec{v}_r d\vec{p}_r = \vec{v}_r \vec{F}_r dt = c^2 d(\gamma_c \mu_r v_r) = h df_r = -d\tilde{E}_r$), created by mutually approaching objects, which has at least two different field or force components, $dE_{kr} = (\vec{v})^2_r d(\gamma_c \mu_r) + \vec{p}_r d\vec{v}_r$. Here is the part of the explanation why and how a single object (electron, or photon, etc.) coincidently passes two slits "making interference and diffraction with itself" on the opposite side of a diffraction plane, being also "energy-momentum" coupled with a diffraction plate, since all the participants here have a joint reduced-mass (without involving any of Quantum Mechanical exotics). In addition, in cases when energy $\mathbf{E_{kr}}$ reaches certain threshold level, we could experience the creation of new particles (not initially entering relevant reaction), but such cases (for the time being) are outside presently analyzed elastic collisions.

It is important to notice that in cases of ideally plastic collisions, after the collision, when initial objects create only one (united) object, the energy E_{kr} will be there as an "injected", transient matter-wave packet, or field energy. We will first consider the initial (input) energy in the close temporal and spatial vicinity, just before the act of collision. After the realized plastic collision, mentioned energy E_{kr} is becoming fully absorbed or injected into an internal rest mass, or state of a rest energy of the newly created (single) object, which remains (as a stable product) after the plastic collision (here represented as m_c). This should be kind of "direct wave energy to mass transformation" example ($\mu_r \rightarrow m_c$), which has not been elaborated from that point of view in traditional analyses of collisions.

[♣ COMMENTS & FREE-THINKING CORNER:

The relativity theory also implicates that, there is a simple relation of direct proportionality between any mass and total energy that could be produced by fully transforming that mass into radiation $E = mc^2, E_{tot.} = \gamma mc^2 = E_0 + E_k = E_0 + (\gamma - 1)mc^2 = \sqrt{E_0^2 + p^2c^2}$. The most important conceptual understanding of frequency-dependent matter wave energy, which is fully equivalent to particle motional, or kinetic energy, is related to the fact that total photon energy can be expressed as a product between Planck's constant and a mean frequency of the photon wave packet, $E_f = hf$. Consequently (since there is known proportionality between mass and energy), the photon momentum *was correctly found as* $p_f = hf / c = m_f c$ *(and proven being applicable and correct in analyzes of different interactions). Since a photon has certain energy, we should be able to present this energy in two different ways, as for instance,* $E_f = hf = \sqrt{E_{0f}^2 + p_f^2 c^2} = p_f c = E_{kf}$. *The photon rest mass equals zero, and there is only photon kinetic energy* $E_f = hf = p_f c = E_{kf} = m_f c^2$. *In number of applications and interactions (such as analyses of Compton and Photoelectric effects), this concept (and all equivalency relations for photon energy and momentum) showed to be fully correct. Going backward, we can apply the same conclusion, or analogy, to any real particle (which has a rest mass), accepting that particle kinetic energy is presentable as the product between Planck constant and characteristic particle's matter wave frequency* $E_k = (\gamma - 1)mc^2 = \tilde{E} = hf$. *Doing that way, we can find the frequency of de Broglie matter waves as,* $f = E_k / h = (\gamma - 1)mc^2 / h = \tilde{E} / h$. *Now, we can find the phase velocity of matter waves as,* $u = \lambda f = \frac{h}{p} f = \frac{E_k}{p} = \frac{(\gamma - 1)mc^2}{\gamma m v} = \frac{v}{1 + \sqrt{1 - v^2/c^2}}$. *The relation between phase and group velocity of a matter-wave-packet is also known in the form,* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$. *By combining given forms of phase and group velocities we can get:* $v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda}$ = $u(1 + \sqrt{1 - v^2/c^2})$, *implicating validity of the following differential relations:* $d\tilde{E} = d\left[(\gamma - 1)mc^2\right] = mc^2 d\gamma$ = $hdf = vdp = d(pu)$, *and practically confirming mathematical correctness and consistency of all above introduced equivalency and analogy-based relations (named in this book as* ***PWDC*** *=* ***P****article-****W****ave* ***D****uality* ***C****ode). Since the above-given equivalency relations are found valid only if we use the wave packet model (as a replacement for a particle in motion), consequently, we have an argument more to say that matter waves (or wave functions) should exist as forms of harmonic, modulated sinusoidal signals (naturally satisfying the framework of Fourier signal and spectrum analysis).*

It is of essential importance to notice that a rest mass (or rest energy) does not belong to matter-wave energy (opposite to many current interpretations in modern physics books, regarding matter wave properties). Analyzing the Compton Effect and many other elementary interactions known in Quantum Mechanics can easily prove this statement (that only kinetic or motional energy presents the matter wave energy), as follows later in this chapter. ♣]

Anyway, all objects and energy states in our universe are always in some of states of mutually relative motions. In most such cases, we have a kind of dominant (case by case) binary-interactions between any two objects in mutual vicinity. For instance, one of them is the place where we are and where it is fixed our observatory or our laboratory coordinate system, and the second object is either some real and big (single) body or complex system of many objects and energy states that can be effectively presented as an equivalent (mathematically defined) object.

The choices where to place the most relevant (or most common and most practical) coordinate or observer system should respect the following:

a) Either our coordinate system will be fixed to the place where we belong, meaning to one of the bodies in mutual relative motion, or

b) It will be fixed to the second body of the same binary system, or

c) It could also be fixed to a certain satellite orbiting one or the other body of the binary system or,

d) We place (mathematically) the observatory or relevant coordinate system in the center of mass of the binary system in question (of course established mathematically).

e) All of mentioned (and practically good) observer options should and can be spatially-temporally and spectrally, mathematically synchronized and mutually related with relevant space-time-velocity dependent functions (as already practiced in modern GPS systems, not necessarily using something from Relativity theory assumptions and mathematics). After establishing certain of relevant space-time, reference-observer place, or coordinate system, and after knowing how all of such equally good reference systems are mutually related (how they can be mutually synchronized), our analyses of two-body, or binary interactions will become much more meaningful.

f) In all practical cases of observer systems (in relation to certain energy-momentum process), we should always have in mind the questions like where and what dominant front-end and last-end elements are, or where energy-source and sink elements of the process in question (like already elaborated in the end of the first chapter of this book) are. In Physics literature, we often find that such concepts, questions, or answers are unclear and neglected, but we should know that complete and good insights (in describing certain process and involved interactions) are impossible without answering such questions (related to closed loops of an energy and relevant moments flow).

4.2.2. Example 3: Elastic collision photon-electron (Compton Effect)

The principal objective of this exercise (or Compton Effect analysis) is to show that electrons (as particles) can be equally treated as matter waves, and photons as particles, and vice versa (like elaborated in Chapters 4.1 and 10. of this book). ***In addition, here will be proven, that rest mass of a stable particle does not belong to its matter-wave energy, and that only motional, or kinetic energy is equivalent to matter-waves energy (contrary to what we often find in Orthodox Quantum theory).***

Let us apply the energy and momentum conservation laws (T.4.2.1, T.4.3.1, T.4.4 and equations (4.2), (4.3), (4.8-3), (4.8-5)), in the case when a photon elastically collides with an electron, which is in a state of rest (see Fig.4.2).

A) ***<u>First, we will treat all interaction participants as particles</u>*** (see "T.4.0. Photon – Particle Analogies" in chapter 4.1).

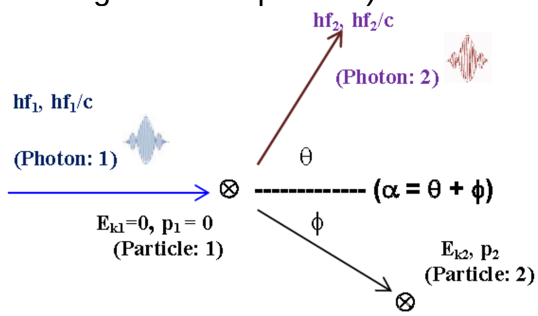


Fig. 4.2 Photon-Particle collision

After an elastic collision, the incident photon **(hf_1, hf_1/c)** loses a part of its energy, being transformed into a new photon **(hf_2, hf_2/c)**, and the electron that initially was in the state of rest (**$E_{k1}=0$, $p_1=0$**) gets certain kinetic energy (**E_{k2}, p_2**). All particle and photon states before collision will be marked using index "1" and after collision using index "2", as presented on the Fig.4.2 and in table T.4.5.

T.4.5.

	States long before the collision (indexing: 1)	**States just after the elastic collision (indexing: 2)**
Photon wave energy & wave momentum (for waves in states 1 & 2)	$\tilde{E}_{f1} = hf_1$, $\left[\tilde{p}_{f1} = \frac{hf_1}{c}\right]$	$\tilde{E}_{f2} = hf_2$, $\left[\tilde{p}_{f2} = \frac{hf_2}{c}\right]$
Electron kinetic energy and linear momentum (for particles in states 1 & 2)	$E_{k1} = E_{e1} = 0$ $[p_1 = p_{e1} = 0]$	$E_{ke} = E_{k2} = \underline{(\gamma-1)m_e c^2 = h(f_1 - f_2) = h\Delta f = \tilde{E}_e = p_2 u_2}\big\rvert_{v_e \ll c} \cong m_e v_e^2/2$, $\left[\vec{p}_2 = \vec{p}_e = \tilde{p}_2 = \gamma m \vec{v}_2 = \frac{\overrightarrow{hf_1}}{c} - \frac{\overrightarrow{hf_2}}{c} = \underline{\vec{p}_e = \gamma m_e \vec{v}_e}\big\rvert_{v_e \ll c} \cong m_e \vec{v}_e ;\ (v_2 = v_e \cong 2u_2 = 2u_e) \ll c,\ \gamma = (1 - v_2^2/c^2)^{-0.5}\right]$
Total motional energy (for electron and photon in states 1 & 2)	$hf_1 + E_{k1} = hf_1$	$hf_2 + E_{k2}$
Total energy, (only for an electron in states 1 & 2)	$E_1' = mc^2$	$E_2' = E_{e2} = \underline{m_e c^2 + hf_1 - hf_2 = \gamma m_e c^2 = E_{e-total}}\big\rvert_{v_e \ll c} \cong$ $\cong m_e c^2 + m_e v_e^2/2$
Total energy & momentum (for electron & photon)	$mc^2 + hf_1$ $\left[\frac{\overrightarrow{hf_1}}{c} = \frac{hf_1}{c}\vec{e}_1\right]$	$\gamma mc^2 + hf_2$ $\left[\vec{p}_2 + \frac{\overrightarrow{hf_2}}{c} = \gamma m \vec{v}_2 + \frac{hf_2}{c}\vec{e}_2\right]$
Total energy and linear moments balance between states 1 & 2	$mc^2 + hf_1 = \gamma mc^2 + hf_2,\ hf_1 = hf_2 + E_{k2}$, $\left[\frac{hf_1}{c}\vec{e}_1 = m_2\vec{v}_2 + \frac{hf_2}{c}\vec{e}_2,\ (v_2 = v = v_e, m_2 = m = m_e);\ \frac{hf_1}{c} = m_2 v_2 \cos\Phi + \frac{hf_2}{c}\cos\theta,\ 0 = m_2 v_2 \sin\Phi + \frac{hf_2}{c}\sin\theta\right]$	
Differential energy balance between states 1 & 2	$dE_{k1} + d\tilde{E}_{f1} = dE_{kc} + d\tilde{E}_{kr} = dE_{k2} + d\tilde{E}_{f2}$	

From relevant energy-momentum conservation relations in T.4.5 we can find:

$$\left[\begin{array}{l} h^2(f_1^2 + f_2^2 - 2f_1 f_2 \cos\theta) = m^2 v_2^2 c^2 \\ h^2(f_1^2 + f_2^2 - 2f_1 f_2) = \frac{1}{4} m^2 v_2^4 \\ 2h(f_1 - f_2) = m v_2^2 = m_e v_e^2 \end{array}\right] \Rightarrow$$

$$2h^2 f_1 f_2 (1 - \cos\theta) = \underline{m^2 v_2^2 c^2 \left(1 - \frac{v_2^2}{4c^2}\right)}\Big\rvert_{v_2 \ll c} \cong m^2 v_2^2 c^2 = 2hmc^2 (f_1 - f_2) \Rightarrow \tag{4.8-9}$$

$$hf_1 f_2 (1 - \cos\theta) \cong mc^2 (f_1 - f_2) = mc^2 \Delta f,\ \Delta f \cong \frac{mv_2^2}{2h} = \frac{h}{mc}\frac{f_1 f_2}{c}(1 - \cos\theta) = \lambda_c \frac{f_1 f_2}{c}(1 - \cos\theta).$$

B) Let us now consider that ***all interaction participants can be equally treated as matter-waves***, by considering wavelengths of all interaction participants, including de Broglie matter-wave wavelength of the moving electron (analogically, on the same way as summarized in "T.4.0. Photon – Particle Analogies" in chapter 4.1) as,

$$\left\{\begin{array}{l} (\text{Incident photon}), \lambda_1 = h/p_{f1} = h/(\frac{hf_1}{c}) = \frac{c}{f_1}, \\ (\text{Scattered or diffused photon}), \lambda_2 = h/p_{f2} = h/(\frac{hf_2}{c}) = \frac{c}{f_2}, \\ (\text{Moving electron}), \lambda_e = h/p_2 = h/m_2v_2 = h/m_e v_e, \\ (\text{Compton wavelength}), \lambda_c = h/mc = h/m_e c = h/m_e v_e, \\ hf_1f_2(1-\cos\theta) \cong mc^2(f_1-f_2) = mc^2\Delta f,\ f_c = c/\lambda_c,\ \lambda_c \cdot f_c = c, \\ hf_1 = hf_2 + hf_e \cong hf_2 + \frac{mv_2^2}{2}, \tilde{E}_e = E_{k2} \cong \frac{mv_2^2}{2} = hf_e = h(f_1-f_2) = h\Delta f \\ \boxed{u = u_2 = u_e = \lambda_e f_e \cong v_e/2},\ u_e = \frac{\omega_e}{k_e} = \frac{E_{ke}}{p_e},\ \omega_e = 2\pi f_e,\ k_e = \frac{2\pi}{\lambda_e}, \\ v_e = u_e - \lambda_e \frac{du_e}{d\lambda_e} = -\lambda_e^2 \frac{df_e}{d\lambda_e} = \frac{d\omega_e}{dk_e} = \frac{dE_{ke}}{dp_e}. \end{array}\right\} \Rightarrow \quad (4.8\text{-}10)$$

From previous results, we can easily prove the validity of the following relations,

$$\begin{array}{l} h\frac{c^2}{\lambda_1\lambda_2}(1-\cos\theta) \cong mc^2(\frac{c}{\lambda_1}-\frac{c}{\lambda_2}) = mc^3\frac{\lambda_2-\lambda_1}{\lambda_1\lambda_2} = mc^3\frac{\Delta\lambda}{\lambda_1\lambda_2} \Rightarrow \\ \Delta\lambda = \lambda_2-\lambda_1 \cong \frac{h}{mc}(1-\cos\theta) = \lambda_c(1-\cos\theta) = \frac{c\Delta f}{f_1f_2} = c\frac{f_1-f_2}{f_1f_2} = c\frac{f_e}{f_1f_2}. \end{array} \quad (4.8\text{-}11)$$

Experimentally measured results for $\Delta\lambda$ are equal to calculated results using the formula,

$$\boxed{\Delta\lambda = \lambda_2-\lambda_1 \cong \lambda_c(1-\cos\theta) = c\frac{\Delta f}{f_1f_2} = c\frac{mv_2^2}{2hf_1f_2},}$$

and this is exactly confirming (theoretically and experimentally) that we can analyze the same Compton Effect, or elastic scattering situation both as interaction of particles or matter-wave states (including mixed particle-waves states), where ***only kinetic or motional energy and linear moment of any interaction participant, at the same time is presenting its matter-wave energy and has wave properties such as wavelength and frequency.*** After the impact between incident photon and an electron, the electron will get certain velocity and kinetic energy (like in impacts between two particles), and here, we just proved that electron could also be treated as an "***electron matter-wave packet***", having certain mean wavelength and frequency, as for example,

$$\boxed{E_{k2} \cong mv_2^2 / 2 = hf_e = \tilde{E}_e, u_2 = u_e = \lambda_e f_e \cong v_e / 2, (v_e, u_e) << c, \lambda_e = h / mv_e}. \quad (4.8\text{-}12)$$

Effectively, it has been proven that in case of Compton Effect ***photon can be treated as a particle, and an electron treated as a matter wave (and vice versa)***, satisfying and respecting all **PWDC** relations (in both directions) as described in Chapters 4.1 and 10. of this book. In this case, relevant **PWDC** relations are given by (4.8-13) and/or as formulated in Chapter 10., by (10.1),

$$\begin{aligned} & u_e = \frac{\omega_e}{k_e} = \frac{E_{ke}}{p_e} = \lambda_e f_e, \ \omega_e = 2\pi f_e, \ k_e = \frac{2\pi}{\lambda_e}, \lambda_e = h / mv_e \\ & v_e = u_e - \lambda_e \frac{du_e}{d\lambda_e} = -\lambda_e^2 \frac{df_e}{d\lambda_e} = \frac{d\omega_e}{dk_e} = \frac{dE_{ke}}{dp_e}, E_{ke} = \tilde{E}_e = hf_e, \end{aligned} \quad (4.8\text{-}13)$$

and we can verify that, if we satisfy (4.8-13), we will get results as in (4.8-12).

We can also exercise (just to satisfy our curiosity and to prove what is correct) that a rest energy, or total electron energy are creating electron matter wave (what is the contemporary and wrong consideration often practiced in the official Quantum Theory), and we will find that results for both options are either not realistic, being mutually contradictory, or mathematically illogical, as follows,

$$\begin{aligned} & \left[hf_e = \tilde{E}_e = m_e c^2, \lambda_e = h / \gamma m_e v_e, f_e = m_e c^2 / h, u_e = \lambda_e f_e = c^2 / \gamma v_e, v_e \leq c\right] \Rightarrow u_e >> c ... \\ & \left[hf_e = \tilde{E}_e = \gamma m_e c^2, \lambda_e = h / \gamma m_e v_e, f_e = \gamma m_e c^2 / h, u_e = \lambda_e f_e = \left(c^2 / v_e\right) > c\right] \Rightarrow v_e \leq c \Rightarrow u_e \geq c, \end{aligned} \quad (4.8\text{-}14)$$

We can find that there is not an agreement and mutual compatibility of (4.8-14) with basic **PWDC** relations (4.8-13), that are valid for any group and phase velocity of matter-waves. Consequently, we can draw the conclusion that a rest mass or rest energy is not at all belonging to matter-waves energy. Total particle energy is also not equal to corresponding matter wave energy. ***What remains is that only kinetic or motional energy entirely belongs to, and/or presents matter wave energy***, as we can see in (4.8-12) and (4.8-13). Of course, a photon is an exception, because its total, wave and motional energy are mutually the same, since it has zero rest energy (or zero rest mass).

It is also obvious that probability and statistics still have no place in here elaborated analysis of Compton Effect. Of course, later we can show that innovated, ***deterministic electron matter wavefunction is related only to "motional electron-wave power" and to its kinetic energy*** (see more in Chapter 4.3 and Chapter 10.).

C) Let us now make kind of mathematical experiment. We will address the electron and photon's total energy, including its kinetic or motional energy and momentum, using relativistic momentum-energy 4-vectors. Since this is an almost elastic scattering (or we are approximately considering it as a "totally elastic impact"), the rest energy or rest mass of the system in states, 1 & 2, (before and after scattering) should be approximately the same.

$$\begin{Bmatrix} \left[P_4^{(1)} = (\frac{\overrightarrow{hf_1}}{c}, \frac{mc^2 + hf_1 - W_1}{c}) \right] (\Leftrightarrow) \left[P_4^{(2)} = (\frac{\overrightarrow{hf_2}}{c} + \vec{p}_e, \frac{\gamma mc^2 + hf_2 + W_2}{c}) \right] \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} \left(\frac{\overrightarrow{hf_1}}{c}\right)^2 - \left(\frac{mc^2 + hf_1 - W_1}{c}\right)^2 = -\left(\frac{mc^2 - W_1}{c}\right)^2 \\ \left(\frac{\overrightarrow{hf_2}}{c} + \vec{p}_e\right)^2 - \left(\frac{\gamma mc^2 + hf_2 + W_2}{c}\right)^2 = -\left(\frac{mc^2 + W_2}{c}\right)^2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \left(\overrightarrow{hf_1}\right)^2 - \left(mc^2 + hf_1 - W_1\right)^2 = -\left(mc^2 - W_1\right)^2 \\ \left(\overrightarrow{hf_2} + c\vec{p}_e\right)^2 - \left(\gamma mc^2 + hf_2 + W_2\right)^2 = -\left(mc^2 + W_2\right)^2 \end{Bmatrix} \Rightarrow \quad (4.8\text{-}15)$$

$$\begin{Bmatrix} W_1 = mc^2, W_2 = -mc^2 \\ hf_1 = \gamma mc^2 + hf_2 + W_2 = (\gamma - 1)mc^2 + hf_2 = E_{ke} + hf_2 \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} \left[P_4^{(1)} = (\frac{\overrightarrow{hf_1}}{c}, \frac{mc^2 + hf_1 - mc^2}{c}) \right] (\Leftrightarrow) \left[P_4^{(2)} = (\frac{\overrightarrow{hf_2}}{c} + \vec{p}_e, \frac{\gamma mc^2 + hf_2 - mc^2}{c}) \right] \end{Bmatrix}$$

Here, W_1, W_2 should be relevant rest energies in the states 1 & 2, introduced to create mutually equivalent 4-vectors. As we can see later, ***equations of Compton Effect could also describe Photoelectric Effect***, since in case of ideal elastic collisions we have $hf_1 = (\gamma - 1)mc^2 + hf_2 = E_{ke} + hf_2$, but if incident photon is fully captured (or absorbed) by an atom, we will have $hf_2 \to 0$, and only an electron will be expulsed, what presents Photoelectric effect situation, which is much closer to a plastic collision case. Consequently, we will have new energy and moments balance (between involved particles, and matter waves), such as, $hf_1 = E_{ke} + hf_2 + W*$, where $W*$ is an internally captured, coupling, or absorbed energy (inside atoms), what is equivalent to M. Maric and A. Einstein explanation of the Photoelectric effect. On a similar way, we could extend the same analysis to explain a "pair of electron-positron creation" (and later annihilation) based on sufficiently high-energy incident photon interaction with an atom. ***Analyzes elaborated in this chapter are selectively addressed, or conveniently decomposed, to show that typical moving-particles could be dualistically presented as equivalent matter-waves, and that typical waves have properties of moving-particles*** (this way giving the theoretical and experimental support to Wave-Particles Duality concepts, in this book summarized as **PWDC**).

D) Let us now exploit ***classical (Minkowski-Einstein) 4-vector, relativistic invariance relations*** applied to an electron involved in the Compton Effect scattering, and we will again get the same results as in the case under A).

$$\begin{Bmatrix} \left[P_{4e}^{(1)} = (\vec{0}, \frac{mc^2}{c}) \right] (\Leftrightarrow) \left[P_{4e}^{(2)} = (\vec{p}_e, \frac{\gamma mc^2}{c}) \right], \\ \underline{\gamma m_e c^2 = hf_1 - hf_2 + m_e c^2 = E_{e-total}}\Big|_{v_e << c} \cong m_e c^2 + \frac{m_e v_e^2}{2}, \\ \vec{p}_e = \vec{p}_2 = \vec{p}_e = \tilde{p}_2 = \gamma m \vec{v}_2 = \frac{\overrightarrow{hf_1}}{c} - \frac{\overrightarrow{hf_2}}{c} = \underline{\gamma m_e \vec{v}_e}\Big|_{v_e << c} \cong m_e \vec{v}_e = m \vec{v}_2 \end{Bmatrix} \Rightarrow$$

$$(\vec{p}_e)^2 - \left(\frac{\gamma mc^2}{c}\right)^2 = -\left(\frac{mc^2}{c}\right)^2 \Rightarrow \left(\gamma mc^2\right)^2 - (\vec{p}_e)^2 c^2 = m^2c^4 \Rightarrow$$
$$(hf_1 - hf_2 + mc^2)^2 - (\frac{hf_1}{c}\vec{e}_1 - \frac{hf_2}{c}\vec{e}_2)^2 c^2 = m^2c^4 \Rightarrow$$
$$h^2 f_1 f_2 (1-\cos\theta) = h(f_1 - f_2)mc^2 = h\cdot\Delta f\cdot mc^2 \Rightarrow$$
$$\frac{h}{mc}\frac{f_1 f_2}{c}(1-\cos\theta) = \lambda_c \frac{f_1 f_2}{c}(1-\cos\theta) = f_1 - f_2 = \Delta f \cong \frac{mv_e^2}{2h},$$
$$\lambda_1 = c/f_1, \lambda_2 = c/f_2, x = \frac{hf_1}{mc^2} = \frac{\lambda_c}{\lambda_1},$$
$$\frac{\lambda_2 - \lambda_1}{c} = \frac{\Delta\lambda}{c} = \frac{1}{f_2} - \frac{1}{f_1} = \frac{f_1 - f_2}{f_1 f_2} = \frac{\Delta f}{f_1 f_2} = \frac{h}{mc^2}(1-\cos\theta) = \frac{\lambda_c}{c}(1-\cos\theta), \tag{4.8-16}$$
$$hf_2 = \frac{hf_1}{1 + x(1-\cos\theta)}, \lambda_2 = \lambda_1\left[1 + x(1-\cos\theta)\right], \frac{f_1}{f_2} = \frac{\lambda_2}{\lambda_1} = 1 + x(1-\cos\theta),$$

and finally, we can again find the same wavelength and frequency differences (between incident and diffused photons), as before in (4.8-11),

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta) = \lambda_c(1-\cos\theta) = c\frac{\Delta f}{f_1 f_2},$$
$$\Delta f = f_1 - f_2 = \frac{f_1 f_2}{c}\lambda_c(1-\cos\theta) = \frac{f_1 f_2}{c}\Delta\lambda. \tag{4.8-17}$$

For instance, if an incident photon is totally reflected (back to its source, meaning, $\theta = \pi, \cos\theta = -1$), we can find theoretically maximal values for the same wavelength and frequency differences,

$$(\Delta\lambda)_{max.} = \frac{2h}{mc} = 2\lambda_c, \ (\Delta f)_{max.} = \frac{2hf_1 f_2}{mc^2} = 2\frac{\lambda_c}{c}f_1 f_2 = 2\frac{f_1 f_2}{f_c} \cong \frac{mv_{e-max.}^2}{2h}. \tag{4.8-18}$$

We can again show that a moving electron, after an elastic impact with a photon, can equally be treated as a particle, or as an equivalent matter-wave packet (analogically, on the same way as summarized under A) and in “T.4.0. Photon – Particle Analogies" in chapter 4.1), as for example,

$$\left\{\begin{aligned} &E_{k2} = (\gamma-1)mc^2 = hf_1 - hf_2 = hf_1\frac{x(1-\cos\theta)}{1+x(1-\cos\theta)} = hf_1\frac{2x\cdot\cos^2\phi}{(1+x)^2 - x^2\cdot\cos^2\phi} = \\ &= p_2 u_2 = \frac{p_2 v_2}{1+\sqrt{1-v_2^2/c^2}} = \left.\frac{m_2 v_2^2}{(1+\sqrt{1-v_2^2/c^2})\sqrt{1-v_2^2/c^2}}\right|_{v_2 << c} \cong \frac{m_2 v_2^2}{2} = \frac{m_2 v_e^2}{2} = E_{ke} = \tilde{E}_e = hf_e \end{aligned}\right\} \Rightarrow$$
$$f_e = \frac{m_2 v_2^2}{2h}, \lambda_e = \frac{h}{p_e} = \frac{h}{m_2 v_2}, u_2 = u_e = \lambda_e f_e = \frac{v_2}{2} = \frac{v_e}{2}, hf_e = \frac{m_2 v_2^2}{2} = E_{k2} = E_{ke} = \tilde{E}_e = h(f_1 - f_2) = h\Delta f, \tag{4.8-19}$$
$$\left(\cos\theta = -1, \theta = \pi\right) \Rightarrow (E_{k2})_{max.} = hf_1\frac{2x}{1+2x}.$$

We also know (from different experimental results) that photons and electrons are manifesting typically wave properties in cases of interference, superposition, wave scattering, and refractions, and here we simply presented how photons and electrons could be treated as particles and/or waves, supporting and proving the intrinsic wave-particle duality of matter in motion (based on **PVDC**).

[♣ Free thinking corner -------------------- (presentation in process of evolving)

E) Usual analyses of the Compton Effect do not consider any ***wave energy or wave momentum being present in the transition zone of the collision process***. The objective of this example is to show that there is certain (hidden) phenomenology, which deals with involved fields and forces, here causally related to the wave energy and wave momentum, existing in a close temporal and spatial vicinity of a collision event. Let us first apply conservation laws (4.5), (4.8-3) and (4.8-5), to get all important energy members,

$$E_{tot.} = hf_1 + mc^2 = E_{0C} + E_{kc} + E_{kr} = hf_2 + \gamma mc^2,$$
$$hf_1 = E_{kc} + E_{kr} = hf_2 + (\gamma - 1) mc^2 = hf_2 + E_{k2},$$
$$E_{0C} = m_c c^2,\ E_c = E_{0C} + E_{kc} = \gamma_c m_c c^2,$$
$$\tilde{p}_{f1} = \frac{hf_1}{c}\vec{e}_1 = \frac{hf_2}{c}\vec{e}_2 + \vec{p}_2,\ |\vec{e}_1| = |\vec{e}_2| = 1$$

$$\vec{v}_c = \frac{(\sum \vec{p})c^2}{\sum E} = \frac{(\frac{hf_1}{c}\vec{e}_1)c^2}{hf_1 + mc^2} = \frac{\frac{hf_1}{c}\vec{e}_1}{m + \frac{hf_1}{c^2}} = \frac{c\vec{e}_1}{1 + \frac{mc^2}{hf_1}} = \frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}} c\vec{e}_1 = \frac{\vec{v}_2 + \frac{hf_2}{\gamma mc^2} c\vec{e}_2}{1 + \frac{hf_2}{\gamma mc^2}} \Rightarrow$$

$$\Rightarrow \cos\alpha = \frac{\left[\frac{hf_1(1 + \frac{hf_2}{\gamma mc^2})}{hf_1 + mc^2}\right]^2 - \left(\frac{hf_2}{\gamma mc^2}\right)^2 - (\frac{v_2}{c})^2}{2\frac{v_2}{c}\frac{hf_2}{\gamma mc^2}} \qquad (4.8\text{-}20)$$

$$\gamma_c m_c = \frac{\sum E}{c^2} = \frac{hf_1 + mc^2}{c^2} = m + \frac{hf_1}{c^2} = \frac{hf_2 + \gamma mc^2}{c^2} = \gamma m + \frac{hf_2}{c^2},$$
$$m_c = m\sqrt{1 + 2\frac{hf_1}{mc^2}} = m\left[1 + \frac{hf_2}{mc^2}\sqrt{1 - v_2^2/c^2}\right]\sqrt{\frac{1 - v_c^2/c^2}{1 - v_2^2/c^2}},\quad E_{oc} = m_c c^2$$

$$\gamma_c = \frac{1}{\sqrt{1 - \frac{v_c^2}{c^2}}} = \frac{1 + \frac{hf_1}{mc^2}}{\sqrt{1 + 2\frac{hf_1}{mc^2}}} = \frac{1 + x}{\sqrt{1 + 2x}},\ \gamma = \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$\vec{p}_c = (\sum \vec{p}) = \frac{\sum E}{c^2}\vec{v}_c = (m + \frac{hf_1}{c^2})\vec{v}_c = \gamma_c m_c \vec{v}_c =$$
$$= \frac{\gamma mc^2 + hf_2}{c^2}\vec{v}_c = \gamma m(\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2) = \vec{p}_2 + \frac{hf_2}{c}\vec{e}_2 = \frac{hf_1}{c}\vec{e}_1 = \tilde{p}_{f1},$$
$$p_2 = p_e = \frac{1}{c}\sqrt{E_{k2}(E_{k2} + 2mc^2)} = \frac{1}{c}\sqrt{E_{k2}(E_{k2} + \frac{2hf_1}{x})} =$$
$$= \frac{2hf_1}{c}\frac{(1+x)\cos\phi}{(1+x)^2 - x^2\cos^2\phi}, \; x = \frac{hf_1}{mc^2}. \tag{4.8-20}$$

$$E_{kc} = (\gamma_c - 1)m_c c^2 = hf_1 - E_{kr} = \frac{p_c v_c}{1 + \sqrt{1 - v_c{}^2/c^2}} = \frac{p_c^2}{\gamma_c m_c(1 + \sqrt{1 - v_c{}^2/c^2})} =$$
$$= hf_1 \frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2} + \sqrt{1 + 2\frac{hf_1}{mc^2}}} = \left[1 + \frac{hf_1}{mc^2} - \sqrt{1 + 2\frac{hf_1}{mc^2}}\right]mc^2 = p_c u_c = hf_c =$$
$$= \frac{(\frac{hf_2}{c}\vec{e}_2 + \gamma m\vec{v}_2)^2}{(1 + \sqrt{1 - v_c^2/c^2})(\gamma m + \frac{hf_2}{c^2})} = (\frac{\gamma mc^2}{1 + \sqrt{1 - v_c^2/c^2}})\frac{(\frac{hf_2}{\gamma mc^2})^2 + \frac{v_2^2}{c^2} + 2\frac{v_2}{c}(\frac{hf_2}{\gamma mc^2})\cos\alpha}{1 + \frac{hf_2}{\gamma mc^2}}$$

$$E_{kc(1)} = E_{kc(2)} \Rightarrow$$
$$\left\{ m\sqrt{1 + 2\frac{hf_1}{mc^2}}(\frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}}c\vec{e}_1)^2 \cong m\left[1 + \frac{hf_2}{mc^2}\sqrt{1 - v_2^2/c^2}\right]\sqrt{\frac{1 - v_c^2/c^2}{1 - v_2^2/c^2}}(\frac{\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2}{1 + \frac{hf_2}{\gamma mc^2}})^2 \right\} \Leftrightarrow \tag{4.8-21}$$
$$\Leftrightarrow \left\{ \sqrt{1 + 2\frac{hf_1}{mc^2}}(\frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}}c\vec{e}_1)^2 \cong \left[1 + \frac{hf_2}{mc^2}\sqrt{1 - v_2^2/c^2}\right]\sqrt{\frac{1 - v_c^2/c^2}{1 - v_2^2/c^2}}(\frac{\vec{v}_2 + \frac{hf_2}{\gamma mc^2}c\vec{e}_2}{1 + \frac{hf_2}{\gamma mc^2}})^2 \right\}$$

$$E_{kr} = hf_1 - E_{kc} = hf_2 + E_{k2} - E_{kc} = \frac{p_r v_r}{1 + \sqrt{1 - v_c{}^2/c^2}} = hf_r =$$
$$= \frac{p_r^2}{\gamma_c \mu_r(1 + \sqrt{1 - v_c{}^2/c^2})} = p_r u_r = \frac{m(\frac{hf_1}{c^2})}{m + \frac{hf_1}{c^2}}\left[\frac{(\vec{0} - \vec{c})^2}{1 + \sqrt{1 - c^2/c^2}}\right] = mc^2\frac{\frac{hf_1}{mc^2}}{1 + \frac{hf_1}{mc^2}} =$$

$$= \frac{\gamma m(\frac{hf_2}{c^2})}{\gamma m + \frac{hf_2}{c^2}}\left[\frac{(\vec{v}_2-\vec{c})^2}{1+\sqrt{1-v_c^2/c^2}}\right] = (\frac{mc^2}{1+\sqrt{1-v_c^2/c^2}})\frac{\frac{hf_2}{mc^2}(1+\frac{v_2^2}{c^2}-2\frac{v_2}{c}\cos\alpha)}{1+\frac{hf_2}{\gamma mc^2}} =$$

$$= hf_2 + \frac{p_2 v_2}{1+\sqrt{1-v_2^2/c^2}} - (\frac{\gamma mc^2}{1+\sqrt{1-v_c^2/c^2}})\frac{(\frac{hf_2}{\gamma mc^2})^2+\frac{v_2^2}{c^2}+2\frac{v_2}{c}(\frac{hf_2}{\gamma mc^2})\cos\alpha}{1+\frac{hf_2}{\gamma mc^2}},$$

$$v_r^2 = v_{r1}^2 = (0-\vec{c})^2 = c^2,\ v_{r2}^2 = (\vec{v}_2-\vec{c})^2 ,$$

$$E_{kr(1)} = E_{kr(2)} \Rightarrow \left\{\frac{m\frac{hf_1}{c^2}}{m+\frac{hf_1}{c^2}}(0-c)^2 \cong \frac{\gamma m\frac{hf_2}{c^2}}{\gamma m+\frac{hf_2}{c^2}}\left[\frac{(\vec{v}_2-\vec{c})^2}{1+\sqrt{1-(\vec{v}_2-\vec{c})^2/c^2}}\right]\right\} \Leftrightarrow \quad (4.8\text{-}22)$$

$$\Leftrightarrow \left\{\begin{array}{l} v_2 << c \Rightarrow \frac{f_1}{\left(1+\frac{hf_1}{mc^2}\right)} \cong \frac{f_2}{\left(1+\frac{hf_2}{mc^2}\right)}, \\ v_2 \binom{\approx}{\leq} c \Rightarrow \frac{f_1}{\left(1+\frac{hf_1}{mc^2}\right)} \cong \frac{f_2}{\left(1+\frac{hf_2}{\gamma mc^2}\right)}\left[\frac{(\vec{v}_2-\vec{c})^2}{2c^2}\right] \cong \frac{1}{2}f_2\frac{(\vec{c}-\vec{v}_2)^2}{c^2} \end{array}\right\}$$

$$\alpha = \theta+\phi = \angle(\vec{v}_2,\vec{e}_2),\ \gamma = (1-v_2^2/c^2)^{-0.5},\ \gamma_c = (1-v_c^2/c^2)^{-0.5}.$$

Now, we are in the position to find relevant wave elements of the electron after its elastic scattering with a photon, and ***we see that only the electron's kinetic energy presents its wave-energy (meaning that the electron rest mass or state of rest energy is not a part of matter-wave energy)***. We shall also find that when electron and incident photon get close enough, then their local Center of Mass System becomes a dominant place where de Broglie matter waves would be "players of greater importance" for the final products or results of an interaction.

$$E_{k2} = hf_1 - hf_2 = h(f_1-f_2) = \frac{p_2 v_2}{1+\sqrt{1-v_2^2/c^2}} = (\gamma_2-1)mc^2 = hf_e = p_2 u_2 =$$

$$= hf_1\frac{(hf_1/mc^2)(1-\cos\theta)}{1+(hf_1/mc^2)(1-\cos\theta)} = hf_1\frac{x(1-\cos\theta)}{1+x(1-\cos\theta)} = E_{ke} \Rightarrow$$

$$\lambda_e = \frac{h}{p_2} = \frac{v_2}{(1+\sqrt{1-v_2^2/c^2})(f_1-f_2)} = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right],$$

$$f_e = \frac{E_{k2}}{h} = \frac{(\gamma_2-1)mc^2}{h} = \frac{p_2 v_2}{h(1+\sqrt{1-v_2^2/c^2})} = f_1 - f_2 = \Delta f, \quad (4.8\text{-}23)$$

$$u_2 = \lambda_e f_e = u_e = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{E_{k2}}{p_2} =$$

$$= \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}(f_1-f_2)\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right] \cong \frac{\overline{\lambda}^2}{\Delta\lambda}\Delta f\left[\frac{v_2/c}{1+\sqrt{1-v_2^2/c^2}}\right] \Rightarrow \overline{\lambda}^2\frac{\Delta f}{\Delta\lambda} \cong c.$$

$$E_{kr} = hf_1 - E_{kc} = hf_1 - hf_c = hf_2 + E_{k2} - E_{kc} = hf_2 + hf_e - hf_c = hf_r = mc^2 \frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}}$$

$$f_r = f_1 - f_c = f_2 + f_e - f_c = (\frac{mc^2}{h})\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} = \frac{f_1}{1+\frac{hf_1}{mc^2}},$$

$$f_e = f_r + f_c - f_2 = (\frac{mc^2}{h})\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} + f_c - f_2 = \frac{f_1}{1+\frac{hf_1}{mc^2}} + f_c - f_2 = f_1 - f_2, \tag{4.8-24}$$

$$(mc^2 >> hf_1) \Rightarrow f_r \cong f_1, \ (mc^2 < < hf_1) \Rightarrow f_r \cong \frac{mc^2}{h}$$

$$E_{kc} = hf_c = hf_1 - E_{kr} = hf_1 - hf_r = \left[1+\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}\right]mc^2 = \left[1+x-\sqrt{1+2x}\right]mc^2,$$

$$f_c = f_1 - f_r = f_1 - (\frac{mc^2}{h})\frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} = \frac{mc^2}{h}\left[1+\frac{hf_1}{mc^2} - \sqrt{1+2\frac{hf_1}{mc^2}}\right] = f_1 \frac{\frac{hf_1}{mc^2}}{1+\frac{hf_1}{mc^2}} = f_r \frac{hf_1}{mc^2},$$

$$(mc^2 >> hf_1) \Rightarrow f_c \cong f_1 \cong f_r \ ; (mc^2 << hf_1) \Rightarrow f_c \cong f_1 - \frac{mc^2}{h} \cong f_1 - f_r \ .$$

The virtual objects μ_r and m_c in the Center of Mass System, applying the same logic regarding motional energy of de Broglie matter waves, should have characteristic frequencies f_r and f_c. Since this is the case of an elastic collision, μ_r and m_c will eventually separate into a moving electron, γm, and a scattered photon, hf_2/c^2 (having characteristic frequencies f_e and f_2). Wavelengths, frequencies and phase and group velocities of such virtual objects (μ_r and m_c) vary (during a transitory phase of interaction), before they get stable and final frequency values f_e and f_2. Consequently, f_e and f_2 are causally related, or proportional to frequencies f_r and f_c (since what we know and calculate as f_e and f_2 are only their final values when interaction is completed). To continue the analysis of this situation, we can (mathematically) test several possibilities, as for instance:

$f_e = f_r, f_2 = f_c$, or $f_e = a \cdot f_r, f_2 = b \cdot f_c$, or $f_e = \alpha(f_r), f_2 = \beta(f_c)$,...

Practically, we assume that the Center of Mass (in a sufficiently close space-time vicinity of the impact) will become the “laboratory" where the electron's de Broglie matter wave frequency, f_e , and the frequency of the scattered photon, f_2, will be synthesized (from f_r and f_c), generating the following results:

$$\{f_e = f_r\,,\ f_2 = f_c\} \Rightarrow \begin{Bmatrix} \frac{f_c}{f_r} = \frac{hf_1}{mc^2} = \frac{f_c}{f_e} \\ \frac{f_e}{f_r} = 1 + \frac{f_c}{f_r} - \frac{f_2}{f_r} = \frac{f_1}{f_r} - \frac{f_2}{f_r} = 1 \end{Bmatrix},$$

$$f_r = f_e = f_1 - f_2 = \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right] = \frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}c = \frac{\lambda_c}{c}f_1 f_2 = \frac{hf_1 f_2}{mc^2},$$

$$f_c = f_2 = f_1 - f_r = f_1 - f_e = f_1 - \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right] = f_1(1 - \frac{hf_1}{mc^2}),$$

$$\lambda_c = \frac{h}{p_c} = \frac{c}{f_1} = \lambda_1 = \lambda_2 - \lambda_c(1 - \cos\theta),$$

$$u_c = f_c\lambda_c = c\left\{1 - \frac{mc^2}{hf_1}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right]\right\} = c(1 - \frac{hf_2}{mc^2}),$$

$$\lambda_r = \frac{h}{p_r} = \lambda_1 + \frac{h}{mc} = \lambda_1 + \lambda_c = \lambda_c + \lambda_c\,,\ (p_r = \frac{\frac{hf_1}{c}}{1 + \frac{hf_1}{mc^2}}),$$

$$u_r = f_r\lambda_r = \frac{mc^2}{h}\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right](\lambda_1 + \frac{h}{mc}) = c\left[\sqrt{1 + 2\frac{hf_1}{mc^2}} - 1\right](1 + \frac{mc^2}{hf_1}) =$$

$$= c\left[\sqrt{1 + 2x} - 1\right](1 + x) = c\frac{hf_2}{mc^2}(\frac{hf_1}{mc^2} - 1) = c\frac{hf_2}{mc^2}(x - 1),$$

$$\lambda_e = \frac{h}{p_2} = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2/c}{1 + \sqrt{1 - v_2^2/c^2}}\right] = \frac{u_e}{c}(\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}),$$

$$u_e = u_2 = \lambda_e f_e = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{v_2/c}{1 + \sqrt{1 - v_2^2/c^2}}\right](f_1 - f_2) = \frac{v_2}{1 + \sqrt{1 - v_2^2/c^2}}, \qquad (4.8\text{-}25)$$

$$(f_r = f_e) \Rightarrow \cos\theta = \frac{(\frac{hf_1}{mc^2} + 1)\left[\frac{hf_1}{mc^2} - \sqrt{1 + 2\frac{hf_1}{mc^2}}\right] + 1}{\frac{hf_1}{mc^2}\left[\frac{hf_1}{mc^2} - \sqrt{1 + 2\frac{hf_1}{mc^2}} + 1\right]} = \frac{(x + 1)\left[x - \sqrt{1 + 2x}\right] + 1}{x\left[x - \sqrt{1 + 2x} + 1\right]}.$$

Continuing the same (mathematical) testing, by the elimination of unacceptable results (where one of them is given above), we should be able to find the most significant and exact relations between all frequencies, wavelengths, and velocities of interaction participants. ***The objective in the above-elaborated analysis is to***

show that mutually interacting objects create transitory, (time and space dependent) variable and evolving phenomena, where dominant (interaction-decisive) frame is the local Center of Mass System.

One of the remaining possibilities to analyze such situations is also to use differential forms of energy and momentum conservation laws, such as, $d\tilde{E}_i = c^2 d(\gamma\tilde{m}_i) = hdf_i = v_i d\tilde{p}_i = d(\tilde{p}_i u_i) = -dE_{ki} = -c^2 d(\gamma m_i) = -v_i dp_i = -d(p_i u_i)$, and in the process of integration we should be able to take care about specific boundary conditions (extending the same procedure to the effects of rotation, electromagnetic fields etc.).

The most promising strategy in addressing such and similar problems is to understand that the incident photon changes its frequency (loses its initial energy), and the particle (or electron), which was in the state of rest, increasingly gets more of motional energy (in the transitional process when they approach each other). We can also conceptualize this situation as a kind of Doppler Effect, where the incident photon gradually reduces its frequency ($f_1 \rightarrow f_2$), or reduces its energy, and de Broglie electron's matter-wave gradually increases its frequency ($0 \rightarrow f_e$), in relation to the center of mass which has velocity v_c. Similar conclusions should also be valid for any other type of collision (see [6]).

In fact, the principal message here is to show that every collision event (elastic or plastic) creates certain (dynamic and transient) field perturbation around collision participants, this way producing de Broglie matter waves. The energy of matter waves is only a form of kinetic or motional energy composed of kinetic or other motional energies of interaction participants.

What we traditionally analyze as different collision types (found in all physics books) are mostly situations verifiable long before and long after the collision happens when we see and measure only steady or stationary states. For instance, the above analyzed Compton effect is traditionally explained based on energy and momentum conservation, considering only the initial situation long before the impact, and the situation long after the scattering happens (neglecting the transitory process between them), as for instance:

$$E_{tot.} = hf_1 + mc^2 = hf_2 + \gamma_2 mc^2 = hf_2 + E_{k2} + mc^2,$$
$$\frac{hf_2}{c}\vec{e}_2 + \vec{p}_2 = \frac{hf_1}{c}\vec{e}_1 ,$$

and here an attempt is made to show that such a strategy is insufficient to fully describe Compton's and familiar interactions. ***In addition, we still did not consider involvement of any electromagnetic interaction here, but as we know, both photon and moving electrons have electromagnetic nature and properties.*** ♣]

The process which is opposite (or inverse) to Compton Effect is the ***Continuous spectrum of x-rays*** *(or photons) emission, caused by impacts of electrons (accelerated in the electrical field between two electrodes) with the anode as the target. The emission of x-ray photons starts when the electrons are abruptly stopped* *on the anode surface. If the final, impact-electron speed is non-relativistic,* $v << c$*, kinetic energy of such electron is* $E_{ke} = mv^2/2$*, and the maximal frequency of the emitted x-rays is found from the relation:* $\tilde{E} = hf_{max.} = mv^2/2 = E_{ke}$*, and in cases of relativistic electron velocities, we have* $hf_{max.} = (\gamma - 1)mc^2$ (both of them being experimentally confirmed as correct). *If we now consider electrons (before an impact happen) as matter waves, where the* <u>*electron matter wave energy*</u> *corresponds only to a kinetic or motional electron energy, without a rest-mass energy content, we will be able to find de Broglie, matter wave frequency of such electrons (just before their impact with the anode). With impact realization, electrons are fully stopped, and the energy content of their matter waves is fully transformed and radiated in the form of X-ray photons (or into another form of waves), whose frequency corresponds to the matter wave frequency of electrons in the moment of the impact. This* ***equality of the frequencies of radiated X-ray photons and electron matter-waves (in the moment of impact) explain us the essential nature of electron matter-waves (eliminating the possibility that the rest mass of electrons belongs to any matter-wave energy content).***

Let us now analyze the ***<u>simplified case (of Compton's effect) that could also be placed between the Photoelectric and Compton Effect</u>***. We can imagine that an incident photon is "fully absorbed" by its target, the free electron that was in the state of rest before the impact with a photon (and, of course, the electron will get certain kinetic energy after *the* impact). This situation will be presented, as before, graphically and with the table of input-output energy and momentum states, as follows:

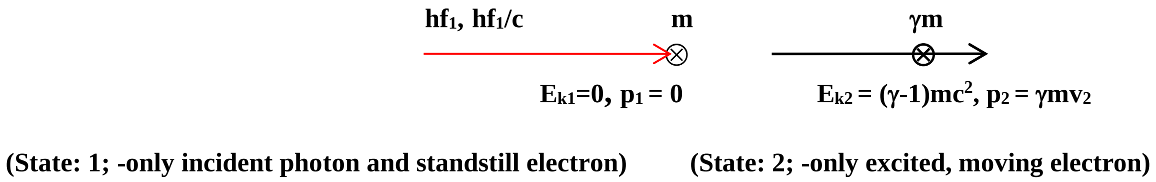


Fig.4.2.1 Direct Photon-Particle collision when the incident photon is fully absorbed by the electron. (Photon: $hf_1, hf_1/c$; Particle: mass m)

	States long before the collision (indexing: 1)	**States just after the collision (indexing: 2)**
Photon	$\tilde{E}_{f1} = hf_1, \tilde{p}_{f1} = hf_1/c$	$\tilde{E}_{f2} = 0, \tilde{p}_{f2} = 0$
Electron	$E_{k1} = 0, p_1 = 0,$ $v_1 = 0$	$E_{k2} = (\gamma - 1)mc^2 = p_2 u_2,$ $p_2 = \gamma mv_2 = p_e$

After the collision, the incident photon **(hf_1, hf_1/c)** disappears and its energy and momentum before impact are transformed into a moving particle (an electron) that was in a state of rest before the collision. When the particle (here an electron) was in a state of rest, we shall again assume that it did not have any wave energy or wave momentum (externally detectable). *The meaning of the particle-wave duality in this* *situation is that a particle (just) after the collision will get certain kinetic energy (*$E_{k2} = (\gamma-1)mc^2$*,* $p_2 = \gamma mv_2$*).* All particle and photon states before the collision will be marked using index “1” and after the collision using index “2”, as already presented in the Fig.4.2.1 and in the table, above.

Let us first apply the energy and momentum conservation laws,

$$hf_1 + mc^2 = \gamma mc^2, hf_1 = (\gamma - 1)mc^2 = E_{k2},$$
$$\frac{hf_1}{c} = \gamma mv_2 = \gamma mv_e = p_2 = p_e$$

From the energy and momentum conservation laws we can find all other matter wave characteristics of the excited particle (excited electron) after the collision (of course, using the already known **PWDC** relations between group and phase velocity from (4.1) - (4.3)),

$$p_2 = \gamma mv_2 = \frac{hf_1}{c} = p_e, \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1} = \frac{h}{(\gamma-1)mc}, f_2 = f_e = f_1 = \frac{(\gamma-1)mc^2}{h},$$
$$u_2 = \lambda_2 f_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{v_2}{1+1/\gamma} = c, \gamma = (1-\frac{v_2^2}{c^2})^{-\frac{1}{2}} \Rightarrow v_2 = c,$$
$$E_{k2} = (\gamma-1)mc^2 = p_2 u_2 = \frac{p_2 v_2}{1+\sqrt{1-v_2^2/c^2}} = \tilde{E}_2 = hf_2 = hf_1,$$

Obviously, it is not easy to imagine that any photon can immediately accelerate an electron from the state of rest to c, meaning that something is wrong in the above-analyzed example. Also, if we analyze the same case traditionally, when $hf_1 << mc^2$, we have $hf_1 \approx \frac{1}{2}mv_2^2, \frac{hf_1}{c} \approx mv_2, \gamma \approx 1 \Rightarrow v_2 \approx 2c$, which is again an impossible result since the particle velocity reaches $2c$. ***The only logical conclusion is that we should consider that the interaction between the photon and the electron starts long before the physical impact (or before unification between them) happens.***

Let us now analyze the same situation in the Center of Mass System, in the time-space domain before the photon and the electron become the united object. We could also say that in this first phase of interaction, analyses of the elastic and plastic impact are identical (if we can say that we have two interacting objects). From the earlier analysis of the Compton Effect, we already know,

$$E_{kr} = hf_r = \frac{hf_1}{1+\frac{hf_1}{mc^2}} = hf_1\frac{1}{1+x}, E_{kc} = hf_c = hf_r x = E_{kr}x = hf_1\frac{x}{1+x}, x = \frac{hf_1}{mc^2}.$$

We also know that the energy E_{kr}, after a plastic impact materializes, is injected (absorbed) in the total-system mass, making the excited electron after the impact has a (temporarily) higher rest mass (higher than m), and a lower kinetic energy (lower than hf_1). Consequently, after gradual evolving of this plastic impact situation, we have only one object (a moving and excited electron that has temporarily increased its rest mass, **m*;** -The electron stays excited until it radiates again the initially “absorbed photon”),

$$\gamma_c m_c = \frac{\sum E}{c^2} = \frac{hf_1 + mc^2}{c^2} = m + \frac{hf_1}{c^2} = m(1+x), \{= \gamma m \Rightarrow \gamma = 1+x, ?!\},$$

$$m_c = m^* = m\sqrt{1+2\frac{hf_1}{mc^2}} = m\sqrt{1+2x} = m\sqrt{\frac{1-v_c^2/c^2}{1-v_2^2/c^2}} = m + \Delta m, E_{oc} = m_c c^2$$

$$\gamma_c = \frac{1}{\sqrt{1-\frac{v_c^2}{c^2}}} = \frac{1+\frac{hf_1}{mc^2}}{\sqrt{1+2\frac{hf_1}{mc^2}}} = \frac{1+x}{\sqrt{1+2x}}, \gamma = \frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} = \sqrt{1+x^2}$$

$$\vec{p}_c = (\sum \vec{p}) = \frac{\sum E}{c^2}\vec{v}_c = (m + \frac{hf_1}{c^2})\vec{v}_c = \gamma_c m_c \vec{v}_c =$$

$$= \gamma m \vec{v}_c = \gamma m \vec{v}_2 = \vec{p}_2 = \frac{hf_1}{c}\vec{e}_1 = \tilde{p}_{f1}, \lambda_2 = \frac{h}{p_2} = \lambda_e = \frac{c}{f_1},$$

$$v_c = v_2 = \frac{hf_1}{\gamma mc} = c\frac{x}{\gamma} = cx\sqrt{1-v_2^2/c^2} \Rightarrow v_2 = \frac{cx}{\sqrt{1+x^2}},$$

$$u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{cx}{2} = \lambda_2 f_2 = c\frac{f_2}{f_1} = c\frac{f_e}{f_1},$$

$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2}$$

Now we can summarize the particle and wave properties of the excited electron, just after collision (before it radiates a photon).

$$v_2 = \frac{cx}{\sqrt{1+x^2}}, u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} = \frac{cx}{2} = \lambda_2 f_2 = \lambda_e f_e = c\frac{f_2}{f_1} = c\frac{f_e}{f_1},$$

$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$

$$p_2 = mcx = \frac{hf_1}{c} = p_e, \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1},$$

$$x = \frac{hf_1}{mc^2}, \gamma = \frac{1}{\sqrt{1-\frac{v_2^2}{c^2}}} = \sqrt{1+x^2}.$$

For instance, let us first analyze the case (1°) when the incident photon has an extremely low energy, meaning that the electron after the collision can be treated as a non-relativistic particle.

1°

$$(hf_1 = \text{Low} << mc^2) \Rightarrow (v_2 << c, \gamma \approx 1, x << 1) \Rightarrow$$
$$v_2 = \frac{cx}{\sqrt{1+x^2}} \cong cx,\ u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} \cong \frac{cx}{2} = \frac{v_2}{2}$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c},\ \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1}.$$

The second case (2°) of interest is when the incident photon has a remarkably high energy and when the electron after the collision can be treated as a relativistic particle:

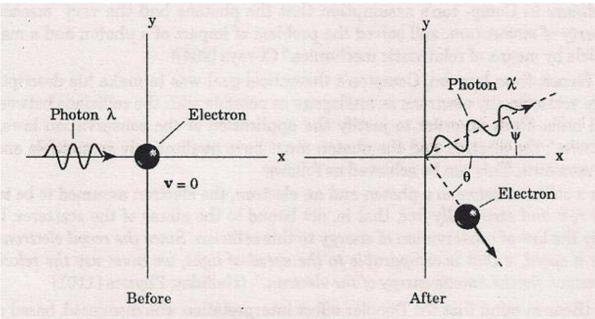
2°

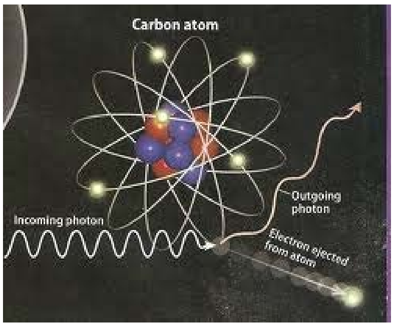
$$(hf_1 = \text{very high} >> mc^2) \Rightarrow (v_2 \approx c, \gamma \to \infty, x >> 1) \Rightarrow$$
$$v_2 = \frac{cx}{\sqrt{1+x^2}} \approx c,\ u_2 = \frac{v_2}{1+\sqrt{1-v_2^2/c^2}} \approx c,$$
$$E_{k2} = p_2 u_2 = \frac{hf_1}{c}\frac{cx}{2} = \frac{hf_1 x}{2} = hf_e \Rightarrow f_e = \frac{f_1 x}{2} = f_2$$
$$p_2 = mcx = \frac{hf_1}{c},\ \lambda_e = \lambda_2 = \frac{h}{p_2} = \frac{c}{f_1}.$$

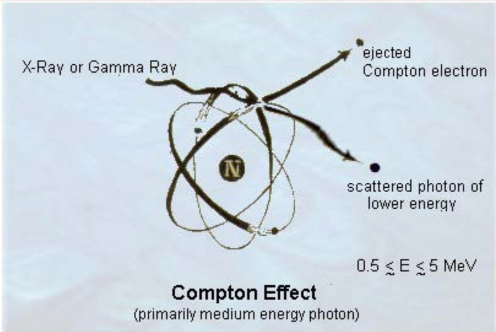
In both situations (non-relativistic 1°, and relativistic 2°), we can calculate all particle and wave characteristics of the excited electron (after collision), and the results look very realistic (or at least not directly contradictory to known conservation laws), contrary to the results of the traditional analysis of the same situation.

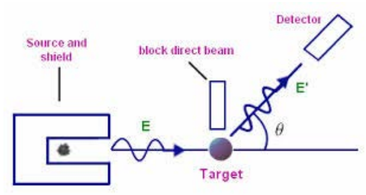
The situation under 2° is similar to conditions causing Cherenkov Effect: the accelerated (or excited) electron starts radiating photons behind (creating the back conus of its wave energy).

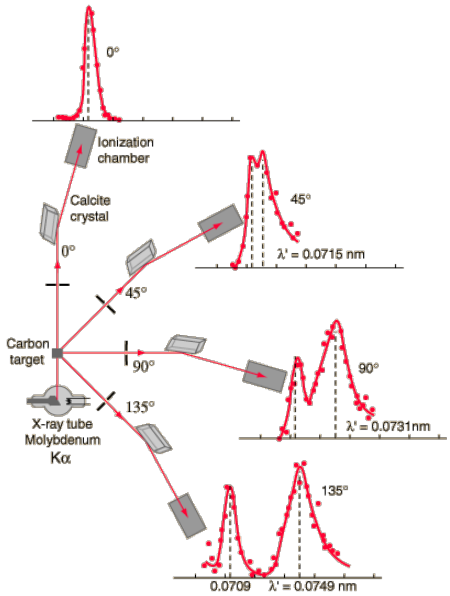
Compton Effect illustrations (found on the Internet)

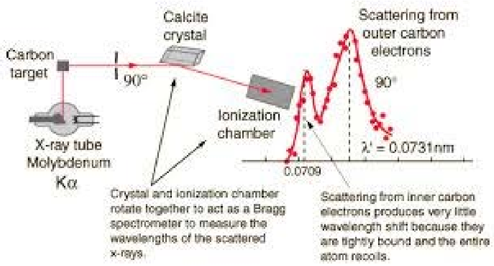




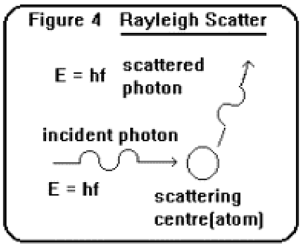


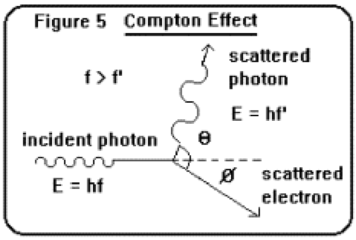


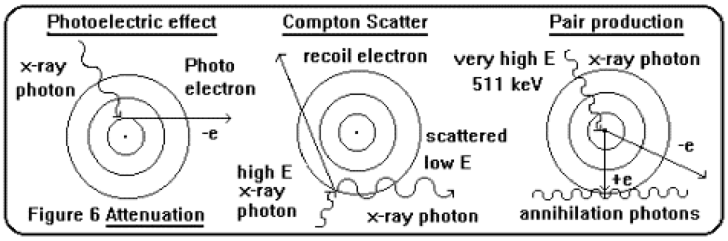


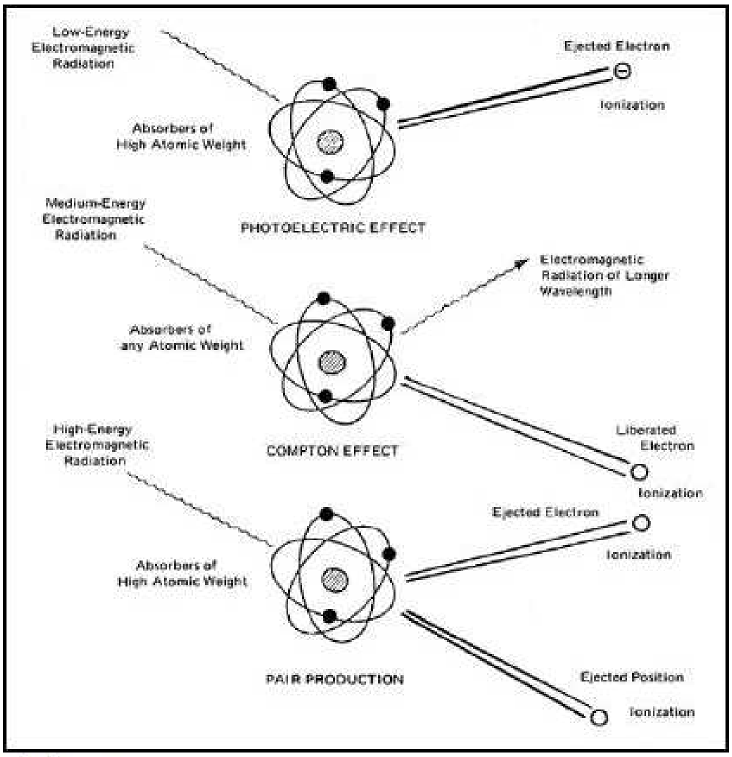


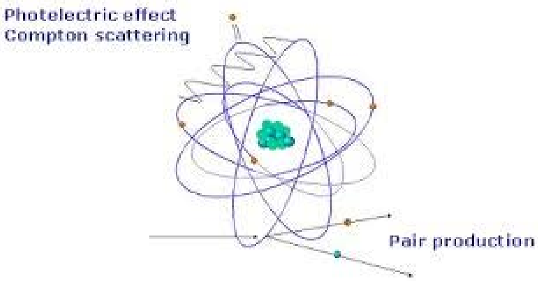
Compton Effect, Photoelectric Effect and Pair production pictures (from the Internet)

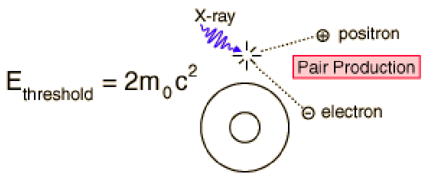


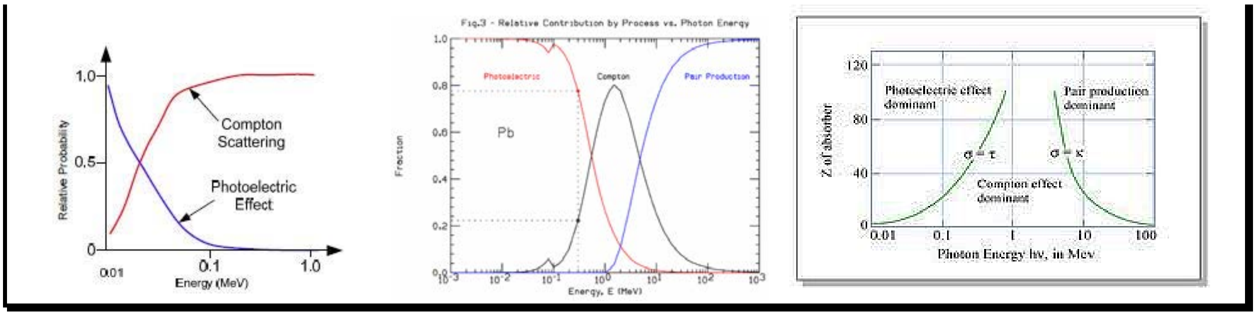












[♣ 4.2.3. Example 4: Doppler Effect (*still a draft; -needs significant modifications*)

Doppler Effect describes the frequency difference between the emitting source signal and received signal when the emitter and the receiver are mutually in relative motion.

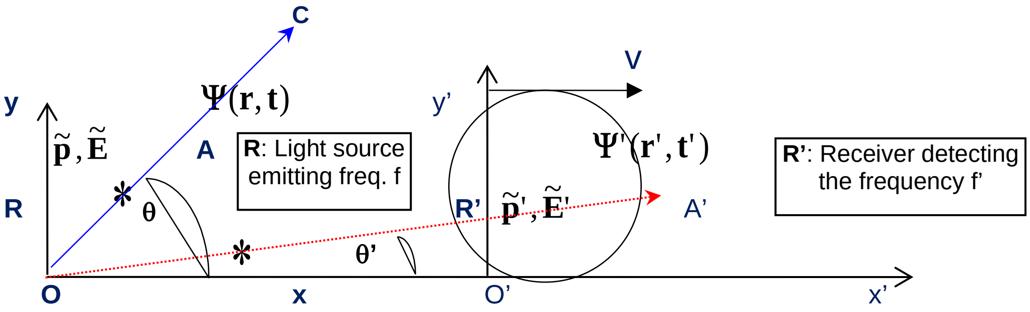
Let us imagine that the light source is in the referential system R: (Oxy), and signal receiver is in the system R': (O'x'y'), ant that relative speed between them is v. In the center O of the system R is the light signal source, and in the center O' of the system R' is the receiver of the same light signal. O emits a monochromatic plane wave that has the frequency f, directed along **OA** (angle θ against Ox axis). Referential receiver system R' moves by uniform speed v relative to R, along their common axis Ox - O'x', and the received signal is detected by along **OA'** (angle θ' against O'x' axis).

Monochromatic plane wave in complex notation in the system R can be characterized by its wave function $\overline{\Psi}(\mathbf{r},\mathbf{t})$:

$\overline{\Psi}(\mathrm{r},\mathbf{t}) = \mathbf{a}(\mathrm{r},\mathbf{t})\mathbf{e}^{-j(\omega t - kr)} = \mathbf{a}(\mathrm{r},\mathbf{t})\mathbf{e}^{j(k_x x + k_y y + k_z z - \omega t)}$, where $\mathbf{k_x}, \mathbf{k_y}$ **i** $\mathbf{k_z}$ are components of the wave vector $\vec{\mathbf{k}}$. In the 4-dimensional Minkowski space, photon wave vector $\overline{\mathbf{K}}_4$ and its radial position vector $\overline{\mathbf{R}}_4$ are known as, $\overline{\mathbf{K}}_4 = \overline{\mathbf{K}}_4(\mathbf{k_x}, \mathbf{k_y}, \mathbf{k_z}, \frac{\omega}{\mathbf{c}}) = \overline{\mathbf{K}}_4(\vec{\mathbf{k}}, \frac{\omega}{\mathbf{c}})$ and $\overline{\mathbf{R}}_4 = \overline{\mathbf{R}}_4(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{ct}) = \overline{\mathbf{R}}_4(\vec{\mathbf{r}}, \mathbf{ct})$, making it possible to express the photon wave function using the product between $\overline{\mathbf{K}}_4$ and $\overline{\mathbf{R}}_4$:

$$\overline{\Psi}(\mathbf{r},\mathbf{t}) = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{-j\overline{K}_4\overline{R}_4} = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{j(\vec{k}\vec{r} - \omega t)} = \mathbf{a}(\mathbf{r},\mathbf{t})\mathbf{e}^{j(k_x x + k_y y + k_z z - \omega t)}$$

$$(\mathbf{j}^2 = -1,\ \mathbf{k} = \frac{2\pi}{\lambda},\ \omega = 2\pi\ \mathbf{f})$$



Doppler Effect Illustration

After applying Lorenz transformations on the above-described case, creating relations between R and R', we will get:

$$\bar{K}'_4 = \bar{K}'(k'_x, k'_y, k'_z, \frac{\omega'}{c}),$$

$$k'_x = \gamma\ (k_x - \beta\frac{\omega}{c}),\quad k'_y = k_y,\quad k'_z = k_z,\quad \frac{\omega'}{c} = \gamma\ (\frac{\omega}{c} - \beta\ k_x),\quad (k_{x,y,z} = \frac{2\pi}{h}\tilde{p}_{x,y,z}),$$

$$c = \lambda f = \frac{\omega}{k} = \frac{\omega'}{k'},\quad \tilde{E} = \frac{h}{2\pi}\omega,\quad \tilde{E}' = \frac{h}{2\pi}\omega',\quad \gamma = (1-\beta^2)^{-\frac{1}{2}},\ \beta = \frac{v}{c}\ .$$

$$\mathbf{k_x = k\cos\theta = \frac{2\pi}{c} f\cos\theta},$$

The Doppler frequency difference between light signals in R and R' can be found from,

$$f' = f\frac{\tilde{E}'}{\tilde{E}} = \gamma\ f(1-\beta\ \cos\theta),\quad \lambda' = \frac{c}{f'} = \lambda\frac{\tilde{p}}{\tilde{p}'} = \frac{\lambda}{\gamma\ (1-\beta\ \cos\theta)}\ .$$

This situation looks like adding (or reducing) certain frequency shift Δf to the source frequency $\mathbf{f}$, or adding (or reducing) a certain amount of motional (wave) energy $\Delta\tilde{E}$ to the source photon energy. We can imagine that between R and R' certain intermediary wave-coupling state materializes, realizing the mentioned energy difference. We can also associate the same wave coupling state to certain wave moment $\tilde{\mathbf{p}}^*$ and mass $\tilde{\mathbf{m}}^*$. Since the source and the receiver photon frequency, and relative speed between R and R' are known, we can calculate all characteristics of the wave coupling state (see the table with all results, below).

The message of the above given Doppler Effect analysis is that this is not only an observation-related phenomenon but much more, it is the case of real wave interactions and energy and momentum conservation rules. In addition, the same case can be analogically applied to any mass movement, explaining the nature of the particle-wave duality from a larger perspective than presently known (or saying differently, every relative motion between minimum 2 particles, or quasiparticles should create similar wave coupling state/s).

	Source Photon in R	Differential Wave Coupling State Between R' and R	Detected Photon in R'
Wave energy	$\tilde{E} = hf$	$\tilde{E}^* = \Delta\tilde{E} = \tilde{E}' - \tilde{E} = h(f' - f) = h\Delta f = hf^* =$ $= (\gamma - 1)\tilde{m}^* c^2 =$ $= hf[\gamma(1 - \beta\cos\theta) - 1]$	$\tilde{E}' = hf'$
Moment	$\tilde{p} = \frac{hf}{c}$	$\vec{\tilde{p}}^* = \frac{\vec{hf'}}{c} - \frac{\vec{hf}}{c} = \gamma\,\tilde{m}^*\vec{v}$ $\tilde{p}^* = \gamma\,\tilde{m}^* v =$ $= \frac{hf}{c}\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]$	$\tilde{p}' = \frac{hf'}{c}$
Frequency	f	$f^* = \Delta f = f' - f = \frac{\Delta\tilde{E}}{h} = \frac{\tilde{E}^*}{h} =$ $= f[\gamma(1-\beta\cos\theta) - 1]$	$f' = f' = f\frac{\tilde{E}'}{\tilde{E}} =$ $= \gamma f(1 - \beta\cos\theta)$
Wavelength	$\lambda = \frac{c}{f}$	$\lambda^* = \frac{h}{\tilde{p}^*} =$ $= \frac{c}{f}\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]$	$\lambda' = \frac{c}{f'} =$ $\lambda\frac{\tilde{p}}{\tilde{p}'} = \frac{\lambda}{\gamma\,(1-\beta\cos\theta)}$
Group Velocity	**c**	$v = \frac{\partial\tilde{E}^*}{\partial\tilde{p}^*}$	**c**
Phase Velocity	**c**	$u = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \lambda^* f^* = \frac{\tilde{E}^*}{\tilde{p}^*}$	**c**
Effective Mass	$\tilde{m} = \frac{\tilde{E}}{c^2} = \frac{hf}{c^2}$	$\tilde{m}^* = \frac{\Delta\tilde{E}}{(\gamma-1)c^2} = \frac{h\Delta f}{(\gamma-1)c^2} = \frac{\tilde{p}^*}{\gamma v} =$ $= \frac{hf[\gamma(1-\beta\cos\theta)-1]}{(\gamma-1)c^2} = \frac{hf^*}{(\gamma-1)c^2} =$ $= \frac{hf\left[-\cos\theta \pm \sqrt{\gamma^2(1-\beta\cos\theta)^2 - \sin^2\theta}\right]}{\gamma vc}$	$\tilde{m}' = \frac{\tilde{E}'}{c^2} = \frac{hf'}{c^2}$

The principal objective in all above given examples was to show that only and exclusively kinetic or motional energy creates, or belongs to de Broglie matter waves, contrary to the mainstream of contemporary understanding in Physics, regarding the same situation (where in most cases the total energy, including rest mass energy is included in matter wave energy).

We can also notice that all the above-presented mathematical analysis has its limit since this way doing, we are only able to treat relatively simple situations concerning particle-wave duality. In fact, here we are dealing with mixed particle and field interactions, and we are faced with the necessity to introduce operations with wave functions.

After establishing the wave function concept, and after formulating generally valid wave equations (as follows), such and similar problems can be much more completely treated, but even at this level (previously presented) there is already enough (mathematical) substance to qualitatively understand the concept of particle-wave duality (of this paper) and its relation to inertial and reaction forces. ♣]

4.3. MATTER WAVES AND WAVE EQUATIONS

Wave motions, vibrations and oscillations are omnipresent in our universe. We know how to create and detect them, how to describe them mathematically, and how to make modeling of different Physics-related wave phenomenology (by connecting general wave concepts and wave equations with material properties, relevant geometry, and parameters of motional states). We know that any temporal-spatial function or signal can be decomposed on simple-harmonic sinusoidal components (or elementary waves), and that this really works in Physics producing measurable and useful spectral characteristics, and predictable interactions with matter (based on Fourier-Kotelnikov-Shannon-Nyquist-Whitaker analysis, and Denis Gabor Analytic signals concepts; -see [57], Michael Feldman, [109], Poularikas A. D., including [110], and [111]).

We also see that our universe has other mutually coupled matter states, such as different particles, gasses, plasma states, and fluids in mutually relative motions. From many experimental situations, we know that mutually interacting particles and/or waves can become sources or building elements of new waves and/or particles (in number of combinations), meaning that masses, atoms, and different forms of matter are compositions or superpositions of different wave-motion states including different forms of involved electromechanical and electromagnetic resonators and oscillators. Waves propagate with and without particles and particles tend to mutually synchronize and create wave-like movements when moving in large groups, etc.

We also know (directly or indirectly) that different wave formations and oscillations are present (on number of ways) inside and around atoms, molecules, and other particles. Consequently, we know that we need to master (conceptually, theoretically, and empirically) the wave motions and oscillations in their relations to particles, and this is a big part of what modern science and technology are busy with. By accepting the necessity and conveniences of operating with wave functions, we are touching the foundations of modern Quantum Mechanics (and Physics in general), since superposition and interference effects among matter-waves (or wave functions) is creating everything wat exist in our Universe.

All what we experience as different force effects is also linked to matter-waves, especially recognizable in cases of stationary, static or standing waves, resonant formations. For instance, what we consider as an attractive force (like gravitation) should exist around nodal, minimal oscillating amplitude (of matter and energy agglomerating) zones of standing matter-waves. Repulsive forces (opposite to Gravitation) should exist around antinode zones (with maximal oscillating amplitudes) of standing matter waves. Mentioned standing waves and associated attractive and repulsive forces can be of electromagnetic and/or any other physical nature.

We often tend to associate four of natural forces only to certain relevant natural charge entities (analog to electric charges), but natural forces are anyway and always detectable where non-zero gradients of energy and increased mass density are. Even electric charges are most probably standing-waves structural formations and manifestations of certain effective electromagnetic energy-mass agglomeration, like photons (meaning never being standstill).

Citation from [95], "A Student's Guide to Waves":
"A ***classical traveling wave is a self-sustaining disturbance of a medium, which moves through space transporting energy and momentum.***"

"What is required for a physical situation to be referred to as a wave is that ***its mathematical representation gives rise to a partial differential equation of a particular form, known as the wave equation***."

"[The essential feature of wave motion is that a ***condition of some kind transmitted from one place to another by means of a medium, but the medium itself is not transported.***"

"[A wave is] each of those ***rhythmic alternations of disturbance and recovery of configuration.***"

The most common defining characteristic is that ***a wave is a disturbance of some kind, that is, a change from the equilibrium (undisturbed) condition***. A string wave disturbs the position of segments of the string, a sound wave disturbs the ambient pressure, an electromagnetic wave disturbs the strengths of the electric and magnetic fields etc.

In propagating or traveling waves, the wave disturbance must move from place to place, carrying energy with it. But, you should be aware that ***combinations of propagating waves can produce non-propagating disturbances, sue as those of a standing wave***.

In periodic waves, the wave disturbance repeats itself in time and space. So, if you stay in one location and wait long enough, you're sure to see the same disturbance as you've seen previously. And if you take an instantaneous snapshot of the wave, you'll be able to find different locations with the same disturbance. But ***combinations of periodic waves can add up to non-periodic disturbances such as a wave pulse***.

Finally, ***in harmonic waves, the shape of the wave is sinusoidal***, meaning that it takes the form of a sine or cosine function.

So, ***waves are disturbances that may or may not be propagating, periodic and harmonic***.

In this book, we consider the existence of real and (directly or indirectly) measurable, momentum-energy carrying matter-waves or wavefunctions (being also de Broglie waves), presentable with power-related wavefunctions (as an example see (4.0.82) from Chapter 4.0).

This is contrary to the Orthodox Quantum Mechanics where wave-functions and de Broglie matter waves are treated exclusively as virtual mathematical objects, tools or artificial, abstract, non-dimensional "probability waves" (or using many of other continuously evolving exotic formulations like probability or possibility distributions in the form of waves). Later, it will be clarified how and why it was possible and convenient to associate the probability nature to matter-waves wavefunctions from a microphysics.

We shall see that probability concept for explaining the grounds of Orthodox Quantum Mechanics is ontologically and conceptually unclear and wrong, but it works sufficiently well in practice, in the frames, mathematical modeling and assumptions of Statistics, Probability theory, and Signal Analysis, including everything else mathematically assembled, as known in the contemporary Orthodox Quantum Mechanics, since Probability, Statistics, and Parseval theorems are on certain way (by taking into account all possible states of certain event, as a sum of probabilities that should be equal to one) accounting and representing laws of energy and moments conservation. It will be demonstrated why, how, and when such artificial theory works superficially well.

The question how to recognize what and where the material carrier of de Broglie matter waves is, remains often unanswered (especially in the frameworks of the contemporary Relativity and Quantum theory). In this book, we also assume that in every situation (of wave motions) certain matter-waves carrier-medium, or spatial-texture (presently known or unknown, or beyond the rich of our present state of the art of available technology) always exist, since if nothing like that exists is against common-sense logic, and against all our knowledge about natural and tangible wave motions (being like giving full legitimacy to magic, mysticism, and to an arbitrary metaphysics). We also know that even an idealized vacuum state still has measurable dielectric and magnetic permeability of something, what we could consider as a fine fluid, or an ether.

Quantum Mechanics anyway treats de Broglie matter waves and all interactions in the quantum world of microphysics using the imaginative and artificial concept of probability wavefunctions (meaning that de Broglie matter waves are considered as not directly measurable and being non-dimensional, or virtual, artificial, mathematical items).

Contrary, here favored (active power related) square of a wave function, represents motional or kinetic energy, or “power-flow”. Such (measurable wave function) concept is already introduced in (3.5) from Chapter 3.0, and in Chapter 4.0, with equations from (4.0.1) to (4.0.5), including “4.0.11. Generalized Wave Functions and Unified Field Theory”), and later summarized in Chapter 10. Power-related wave function (from this book) is a kind of realistic, deterministic, and natural, measurable wavefunction, compared to the probabilistic, non-dimensional Orthodox Quantum Mechanics’ wave function (meaning that, in this book, de Broglie matter-waves will be treated as any other known wave phenomena in electromagnetism, acoustics, fluid mechanics etc.).

Moreover, it will be shown that on the formal and mathematical level there is no big contradiction between probabilistic, and (normalized, dimensionless) power, or motional energy-carrying wave function, meaning that both could be on some essential way mutually isomorphic and (mathematically) respecting the same conservation laws known in Physics. Deterministic and power-related wave function $\Psi(\mathbf{t})$, if presented using the Analytic Signal model, perfectly describes associated and space-time evolving de Broglie matter waves or wave-packets, inside and around moving particles, following corresponding differential energy balance (4.7), as established in Chapter 4.1.

Mathematically (to satisfy universally valid Parseval's theorem, related to signal energy equivalence and conservation between time and frequency domains of the same signal; -see Chapter 4.0; -equations (4.0.4), (4.0.5)), it is very convenient to consider the square of a wave function, $\Psi^2(t)$, being an active-power function **S(t)**, or a matter-wave power (or wave "energy-current") since,

$$\left\{d\tilde{E} = hdf = dE_k = c^2 d(\gamma m) = vdp = d(pu) = -c^2 d\tilde{m} = -vd\tilde{p} = -d(\tilde{p}u)\right\} / dt$$

$$\Leftrightarrow \left\{\begin{array}{l} \Psi^2(t) = S(t) = \frac{d\tilde{E}}{dt} = h\frac{df}{dt} = c^2\frac{d(\gamma m)}{dt} = \frac{d(pu)}{dt} = v\frac{dp}{dt} = ... \quad (=)[W] \\ d\tilde{E} = \Psi^2(t)\cdot dt, dp = \frac{1}{v}d\tilde{E} = \frac{1}{v}\Psi^2(t)\cdot dt \end{array}\right\},$$

$$\Rightarrow \left(\begin{array}{l} \tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \\ = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \sum_{(i)}\tilde{E}_i = \sum_{(i)}\int_{-\infty}^{+\infty}\Psi_i^2(t)dt = \int_{-\infty}^{+\infty}\sum_{(i)}\left[\Psi_i^2(t)\right]dt = \\ = \sum_{(i)}\int_{-\infty}^{+\infty}\hat{\Psi}_i^2(t)dt = \int_{-\infty}^{+\infty}\sum_{(i)}\hat{\Psi}_i^2(t)dt = \sum_{(i)}\int_{-\infty}^{+\infty}\left[\frac{a_i(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\sum_{(i)}\left[\frac{a_i(t)}{\sqrt{2}}\right]^2 dt = \\ = \sum_{(i)}\int_{-\infty}^{+\infty}\left|\frac{\overline{U}_i(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{-\infty}^{+\infty}\sum_{(i)}\left|\frac{\overline{U}_i(\omega)}{\sqrt{2\pi}}\right|^2 dt = \sum_{(i)}\int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega = \int_{-\infty}^{+\infty}\sum_{(i)}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega \\ \Rightarrow \Psi^2(t) = \sum_{(i)}\left[\Psi_i^2(t)\right], \hat{\Psi}^2(t) = \sum_{(i)}\left[\hat{\Psi}_i^2(t)\right], \tilde{E} = \frac{1}{2}\int_{-\infty}^{+\infty}\left[\Psi^2(t)+\hat{\Psi}^2(t)\right]dt, \\ \Leftrightarrow \frac{1}{2}\left\{\Psi^2(t) + \hat{\Psi}^2(t)\right\} = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = \sum_{(i)}\frac{dE_i}{dt} = \sum_{(i)}\frac{dE_k}{dt} = \sum_{(i)}\frac{d\tilde{E}_i}{dt} = S(t). \end{array}\right), \quad (4.9\text{-}0)$$

where de Broglie or matter-wave energy is $\tilde{E} = \int S(t)dt = \int \Psi^2(t)dt = \tilde{p}u = hf = E_k$. All wave functions, Classical Wave equation, and *Schrödinger-like wave equations,* developed later in this book, will be in compliance with (4.9-0), *and also fully merged with **P**article-**W**ave **D**uality **C**ode, formulated in Chapter 4.1, around equations (4.1)-(4.3), and in Chapter 10., meaning that the Complex Analytic Signal wave function* $\overline{\Psi}(t)$ *created from* $\Psi(t)$ *will have all properties of de Broglie matter waves (such as matter-wave wavelength,* $\lambda = h/p = u/f$ *and belonging group and phase velocity* $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$*).*

For a wave function related to active power, it should be valid $\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 = \frac{1}{2}\left\{\Psi^2(t) + \hat{\Psi}^2(t)\right\} = \frac{d\tilde{E}}{dt} = \sum_{(i)}\frac{d\tilde{E}_i}{dt} = S(t)$ *(as in (4.9-0)). If we neglect or ignore such natural and mathematically correct links between Wave Function, Power, Energy, and universally valid Parseval's theorem, and consider a wave function to have only a probabilistic (non-dimensional and averaged) meaning, we should either accept to make some conceptual mistakes, or we need to attach something else to such probabilistic wave function processing, what mathematically compensates and corrects an initially artificial and weak conceptualization. Orthodox Quantum theory conveniently assembled set of useful mathematical rules and assumptions, making* *its probability related framework as an "in-average" and a particularly good and isomorphic replacement of real and dimensional wave functions for satisfying conservation laws of classical and deterministic Physics.*

To get sufficiently meaningful and practically applicable theory, founders of quantum mechanics postulated what was generally acceptable regarding wave-particle duality and effectively hybridized and merged the concepts of Probability, Statistics, Spectrum and Signal Analysis theory with Particle–Wave Duality concepts (see more in Chapter 10, under "10.00 DEEPER MEANING OF PWDC").

In addition, classical, differential, second-order wave equation and its wave function were (on certain unnatural, intuitive and by "divine-inspiration experienced way") mathematically upgraded (by Schrödinger) to comply with expected complex wave function solutions. Later, such wave function and wave equation were hybridized with additional rules, postulates, and definitions to become probabilistically and statistically operational, and effectively in compliance with conservation laws…, and after convenient mathematical shaping and fittings we got present Orthodox Quantum Theory, which is still abstract and unnatural, but working well (inside its own, self-defined terminology and boundaries). Future development of Quantum Theory and Wave-Particle Duality Theory will (most probably) reveal that what presently appears being only stochastic and by probability functions and concepts explicable, also has certain, deterministic nature (when appropriately clarified). Of course, statistics and probability can always be applied (in any of natural and other sciences) as an "in-average" mathematical modeling, whenever analyzed system has a sufficiently big number of identical elements (and not only in Quantum Theory).

The primary objective of this book is to modify and re-establish the foundations and most relevant step-stones of (renewed), more causal and not-essentially probabilistic Particle-Wave Duality, or better to say Particle-Wave Unity theory. In relation to here favored deterministic and more tangible (dimensional, power-related) wavefunction, we will also use well known mathematics from Spectrum or Signal Analysis, which is generally good for dealing with wavefunctions and wave equations (regardless of waves nature). In fact, the mathematical background and mathematical processing that is already well established in Orthodox Quantum Mechanics could stay very much the same in any new matter-waves theory, as an imaginative, particularly useful mathematical toolbox for all new or modified forms of wave equations, and wave analyzes that will be developed in the future, and not very much of that matter should be significantly changed.

Anyway, we can prove that deterministic, dimensional, and very tangible wave function in connection to Analytic Signal modeling, has number of natural, spatial, temporal, amplitude, phase and spectral characteristics, and significantly richer mathematical structure, when compared to an artificial probability assigned wavefunction from the Orthodox Quantum theory.

The message underlined here will be to show that for developing all wave functions and wave equations (known in modern physics), even for equations more general than presently known in quantum theory (and applicable to many of wave phenomena known in physics), we really do not need the Orthodox Quantum Mechanics' with probabilistic assumptions. We only need to respect

***all conservation laws of physics, merging them properly with Classical Wave Equation and Particle-Wave reality, described in this book as PWDC (see (4.1) - (4.3) in Chapter 4.1, and (10.1) from Chapter 10.). Eventually, we anyway must integrate all of that into a much more generally valid mathematical framework of Signals and Spectrum Analysis around Analytic Signal modeling** (based on Hilbert transformation: see about Analytic Signals in Chapter 4.0, Chapter 10., and later in this chapter, including literature references [57], [58], [59]).*

Regardless significant differences between treating the wave functions (in this book, and in Orthodox Quantum Mechanics), it is easy to show that the Schrödinger's wave equation, as one of the important differential equations in microphysics, mathematically always keeps the same form (properties, mathematical processing and results), either using a non-dimensional probability wavefunction, or "power-energy-carrying" wavefunction. In addition, conveniently modified form of Schrödinger equation could be equally well applied in describing cosmological situations, and satellites and planetary motions, since starting from Classical differential wave equation we can easily develop Schrödinger equation (when wavefunction is formulated as an Complex, Analytic Signal function); -see more in the second chapter, within equations under (2.11.20), and in Chapter 10.).

We also know that often we do not have the complete or good-enough answer about what kind of substance is the real carrier of de Broglie matter waves (or also a carrier of electromagnetic waves), <u>but this does not affect our mathematical modeling of such wave phenomena, since material waves carrier is anyway implicitly considered.</u> It is better to say that in this book we should (for the time being) forget or put aside questions about correct interpretation of the nature of wave function and its material carriers, and discuss the same items a bit later, when this situation naturally becomes clearer (and after some of present-days dilemmas and wrong concepts regarding wave motions and wave-particle duality will be considered as obsolete, since it is natural and logical that for any waves propagation should always exist a waves-carrier medium).

What is very impressive is that many of famous founders, experts, and followers of Quantum Theory teachings said honestly and proudly that Quantum theory works very well, but nobody understands (profoundly and essentially) why and how it works, meaning that conceptual and ontological picture of modern Quantum theory is still not sufficiently known, natural and completely clear. As we know, Orthodox Quantum theory is mathematically operational, but only in the frames of its own (artificial), self-defined and reduced scope of mathematical framework, which is very much postulated and artificially assembled, mostly for needs of such Quantum Mechanics.

We also know that Mathematics is the best language, logic, and processing tool for describing, explaining, and predicting phenomenology in Physics and Nature if mathematical conceptualization and modeling (of certain phenomenology) is well and naturally established, and smoothly connected with the remaining, tangible body of Mathematics and Physics.

For natural and favorable situations (where mathematical theory or modeling is not specifically and artificially constructed, or defined only for the purpose of supporting *some specific temporary and artificially designed needs), questions as conceptual, intellectual, philosophical, logical, deterministic, realistic and experimental understanding of certain problematic are almost self-explanatory, without internal, logical and structural contradictions. Such naturally conceptualized theories are fully integrated into remaining part of Physics and Mathematics, being analogically, and intuitively clear from different points of view (or being fully mathematically and empirically explicable and confirmable without artificial constructions and axiomatic assumptions).*

When we are constructing, extending, and applying, universally valid, natural Mathematics to Physics (what is always an interactive never-ending, iterative and extrapolating (self-adjusting) process, combined with inductive, deductive and analogical thinking and conclusions, since mathematics is also learning from Physics), we will always have clear conceptual pictures (of different Physics' related situations), being mutually and smoothly connected (what is still not the case with Orthodox Quantum theory). Here, we can characterize such mathematical approach as the natural, tangible body of generally valid and always applicable mathematics. Unfortunately, this is still not the case regarding contemporary Orthodox Quantum theory, where the applied mathematical framework is artificially constructed, postulated, or imaginatively defined, and very much disconnected from the remaining natural body of generally applicable mathematics and physics. This is producing that conceptual and natural understanding of such theory is full of complexity, assumptions, dilemmas, postulated definitions, and challenging contradictions in relation to Reality and remaining body of Physics.

In this book will be established a very much natural and analogical modeling of power or energy-flow related wavefunctions and wave equations, being smoothly connected with naturally and universally valid Mathematics and Physics (within the boundaries where such mutually non-contradictory Mathematics and Physics facts and practices are well known and universally applicable).

The same form of classical (second order, partial, differential) wave equation is established independently in many mutually different fields of Physics (much before Quantum Theory was formulated). ***It can be demonstrated that classical wave equation*** $\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=\frac{\partial^2\Psi}{\partial r^2}(=)\nabla^2\Psi$ ***is the most important partial differential wave equation, which (with certain modifications) generally describes all kinds of waves****, such as sound waves, electromagnetic waves, fluid dynamics waves…, and which is in the background of all wave equations familiar to Schrödinger equation, as known in modern Physics and Quantum theory. Variations of the same classical wave equation are also found in General Relativity theory. When operating with such wave equations (applied to Classical Physics), everything is tangible, clear, causal, explicable and without internal contradictions. Now, we will consider, and demonstrate in three steps (see later: -1-, -2-, -3-), that mentioned classical, second order differential wave equation is the best starting platform for constructing other wave equations, including quantum-mechanical ones.* ***Schrödinger equation will simply surface to be the Classical Wave equation, where involved wave function*** $\overline{\Psi}$ ***is a Complex Analytic Signal function*** *(merged with PWDC; -see more in (4.1) - (4.3) in Chapter 4.1, and (10.1) in Chapter 10.).*

In the following table (see below; -taken from ***[87]****, as an example) we can find several mutually analogical Classical Wave Equations, developed for different waving phenomena* *known in Physics. Starting from such wave equations, or from Classical Wave equation, we can easily develop several of famous wave equations known in modern Quantum Theory.*

Propagation d'onde non dispersive. Cordes vibrantes 101

Tableau des célérités de phénomènes classiques non dispersifs

	Exemple de propagation non dispersive	Équation de d'Alembert	Onde	Célérité	Caractéristique du milieu	Problème type
Ondes transversales	Corde vibrante	$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$	déplacement transversal $y(x,t)$	$c = \sqrt{\frac{F}{\mu}}$	F = tension de la corde; μ = masse linéique de la corde	ch.2 n°2
	Ondes de Marée	$\frac{\partial^2 z}{\partial x^2} = \frac{1}{gh}\frac{\partial^2 z}{\partial t^2}$	déplacement de la surface de l'océan $z(x,t)$	$c = \sqrt{g \cdot h}$	h = profondeur de l'océan ($h \ll \lambda$); g = accélération de la pesanteur	ch.4 n°6
	Ondes de torsion dans un cylindre	$\frac{\partial^2 \theta}{\partial z^2} = \frac{\rho}{G}\frac{\partial^2 \theta}{\partial t^2}$	rotation d'un élément $\theta(z,t)$	$c = \sqrt{\frac{G}{\rho}}$	G = module de rigidité; ρ = masse volumique	ch.2 n°16
	Ondes électromagnétiques dans le vide illimité	$\frac{\partial^2 B}{\partial x^2} = \sqrt{\varepsilon_0\mu_0}\frac{\partial^2 B}{\partial t^2}$	champ électrique ou magnétique $B(x,t)$	$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$	ε_0 = permittivité du vide = $\frac{1}{36\pi \cdot 10^9}$; μ_0 = perméabilité du vide = $4\pi \cdot 10^{-7}$	ouvrage « ondes électromagnétiques » (Dunod)
Ondes longitudinales	Ondes dans un ressort	$\frac{\partial^2 s}{\partial x^2} = \frac{\mu}{kL}\frac{\partial^2 s}{\partial t^2}$	déplacement longitudinal d'une spire du ressort $s(x,t)$	$c = \sqrt{\frac{kL}{\mu}}$	k = raideur du ressort; L = longueur du ressort à vide; μ = masse linéique du ressort	ch.2 n°17
	Ondes dans un barreau solide	$\frac{\partial^2 s}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 s}{\partial t^2}$	déplacement longitudinal de la tranche d'abscisse x $s(x,t)$	$c = \sqrt{\frac{E}{\rho}}$	E = module d'Young; ρ = masse volumique	ch.2 n°15
	Ondes dans une ligne électrique	$\frac{\partial^2 i}{\partial x^2} = \Lambda \cdot \Gamma \frac{\partial^2 i}{\partial t^2}$	courant électrique $i(x,t)$	$c = \frac{1}{\sqrt{\Lambda \cdot \Gamma}}$	Λ = inductance linéique; Γ = capacité linéique	ch.2 n°18

Classical differential wave equation is describing a wave that has the same wave-shape during motion (like elementary sinusoidal signals). If presented in one-dimension (traveling along the **x**, or **r**, or t axis), wave functions or solutions of relevant wave equation are composed of two similar waves (or wavelets), traveling with the same phase velocity, **u**, in mutually opposite directions (in the following elaborations marked as (+) and (-) directions).

- 1 -

The generalized wave function of such wave motions (or solution of Classical wave equation), applicable also to arbitrary function shapes can be formulated as $\Psi(t,r) = \Psi^{(+)}(r-ut) + \Psi^{(-)}(r+ut)$, where **u** is the *wave phase velocity*. It can be proven that for all of the particular wave functions such as $\underline{\Psi^{(+)}(r-ut)}$, or $\underline{\Psi^{(-)}(r+ut)}$, or $\underline{\Psi^{(+)}(r-ut) + \Psi^{(-)}(r+ut)}$, (when the wave propagates in linear media, without dispersion, with the same speed, being independent of wavelength, in both directions, and independent of amplitude) it is applicable the same Classical wave equation. **The most elementary solutions of such classical wave equations are also real, simple harmonic functions, since we know from Fourier and other Signal Analysis practices that arbitrary wave functions can be decomposed and presented as summations of elementary and simple harmonic wave components, such as,**

$$\Psi(r,t) = a \cdot \cos(kr - \omega t) + b \cdot \cos(kr + \omega t) =$$
$$= \Psi^{(+)}(r-ut) + \Psi^{(-)}(r+ut),\ u = \omega/k,\ (a, b) = \text{constants}.$$

Until here, we are still inside the framework of universally applicable mathematics (well and naturally connected to Physics).

- 2 -

What is even more important mathematical approach, which is more productive, is that real **components of Classical wave equation can be formally extended and generalized with superposition of similar complex signal forms** such as,

$$\Psi(r,t) \rightarrow \overline{\Psi}(r,t) = a \cdot e^{j(kr-\omega t)} + b \cdot e^{j(kr+\omega t)},\ (a, b = \text{constants},\ j^2 = -1),$$

what Schrödinger effectively applied when he created his wave equation and this was the significant step that facilitated the creation of Quantum theory Wave Equation, as presently known (of course merged with PWDC; -see about PWDC under (4.1) - (4.3) in chapter 4.1, and (10.1) in Chapter 10). Such mathematical transition from the world of Real numbers (or functions) to a world of Complex numbers (or functions) is already sufficiently revolutionary action step that demands certain mathematical and conceptual readjustments, previously not practiced (if we would like to stay inside boundaries of universally applicable mathematics). Using complex functions instead real functions is comparable to extending the basis of the four-dimensional world (meaning our world) to a higher number of dimensions (not to mention very promising, imaginative, and philosophical consequences of using even more advanced Analytic, Hyper-complex wavefunctions with several imaginary units for multidimensional worlds conceptualization). Schrödinger and other founders of Quantum theory did what they could (in the historical framework of their scientific environment), and formulated contemporary Quantum Theory while applying specifically constructed, and partially ad-hoc postulated mathematics, which is operational, but not naturally connected with a remaining body of Physics and Mathematics. Many of founders and followers of such Quantum Theory artificial foundations realized mentioned incoherence, but they have been satisfied, since anyway such artificial construction is mathematically working and producing satisfactory results. We only need to get familiar with several foggy and unnatural explanations, assumptions, definitions, and concepts (of by consensus-accepted Quantum Theory) and accept all of that as using necessary and prescribed mathematical tools, as a way for satisfying natural laws of Physics (at least in average). Why not to accept such original and well-working mathematical creation (as such Quantum theory is), many of us could ask? Founders and followers of Orthodox Quantum theory accepted such platform, but the author of this book is proposing better mathematical modelling ...

- 3 -

Obviously, the transition from real functions to complex functions is shown to be productive (regarding wave functions and wave equations). What is missing here (in contemporary Quantum theory) is to find a more natural, better connected, and smoother transition from real wavefunctions to complex wavefunctions (and to stay in the frames of universally applicable (natural) mathematics and conceptually clear Physics). In this book, we will simply consider that **solutions of any wave equation are best presentable as Complex, Analytic Signal functions**, created using Hilbert transform "**H**", as shown in (4.0.2) and (4.9), since such Analytic Signal functions are really, well and naturally, presenting wave phenomenology known in Physics, as follows,

$$\Psi(r,t) \rightarrow \overline{\Psi}(r,t) = a(r,t)\cdot e^{j(kr-\omega t)} + b(r,t)\cdot e^{j(kr+\omega t)} = \Psi(r,t) + j\hat{\Psi}(r,t) =$$
$$= A(r,t)e^{j\varphi(r,t)} = A(r,t)\cos\varphi(r,t) + j\cdot A(r,t)\sin\varphi(r,t),$$
$$\Psi(r,t) = A(r,t)\cos\varphi(r,t), \hat{\Psi}(r,t) = A(r,t)\sin\varphi(r,t) = H\left[\Psi(r,t)\right].$$

We will see later that such approach (combined with **PWDC**, formulated in Chapter 4.1, around equations (4.1) - (4.3), and in Chapter 10.), can easily produce all important wave equations known in Quantum Theory (and Physics), on a more natural, simpler, clearer, and more elementary level, compared to contemporary Orthodox Quantum theory mathematical processing. Even entanglement effects (as known in microphysics) are explicable as (mathematical) connections between described, mutually coupled wavefunction solutions of the same wave equation (for instance, like coupled photons, electrons and other micro particles). Of course, the wave velocity (here phase velocity, $\mathbf{u} = \omega/\mathbf{k}$) will depend on the medium properties through which the wave is propagating. All other, more complex waveforms and wave packets are presentable as an integral or discrete superposition of elementary, simple harmonic waves, as shown in (4.0.1), including a superposition of elementary waves (4.0.8). In addition, synchronous propagation of two, (or many in-pairs) coupled wave components in mutually opposed directions should always be considered as generally valid for all wave motions (having a much deeper meaning in the world of Physics, than presently seen).

Regardless of how complex, brilliant, vocal, imaginative, and magical explanations we are finding in Quantum Mechanics books regarding wave functions and wave equations, this problematic can also be very comprehensive on a quite simple level if initially addressed using Complex Analytic Signal modelling (what was not the case). The approach of Schrödinger and others was that Classical wave equation and wave functions were simply (analogically) replaced, hybridized, and generalized with convenient complex functions (not with complex analytic signals), and then, missing mathematical structure was additionally upgraded, or postulated according to the best intuitive estimations. Instead of searching for more natural and better mathematical modeling and explanations, Orthodox Quantum theory fans are still vocally and persistently glorifying, defending, and reinforcing such artificial and not very scientific approach (since, luckily, it works very well, and many of them are enthusiastically saying "what for to change or challenge something that works so well"). Here, it will be shown that it is worth and enormously beneficial to modify and rectify mentioned patchwork practices of contemporary Quantum theory (mostly related to Schrödinger equation and wave functions).

Let us now summarize foundations of wave equations and wave functions relevant for Physics, on a diffrent way (using Analytic Signals as wave functions). Since all ***natural and real-world matter waves***, pulses and different signals can be decomposed on periodical, elementary sinusoidal waves (using Fourier Analysis), we can initially analyze motion of such elementary waves in one spatial and one temporal dimension (x, t) to describe involved velocities, wavelength, frequency, signal energy, power… Let us use such elementary sinusoidal matter waves or signals to address matter-waves and wave-particle duality. This will later help us to develop Schrödinger and other familiar wave equations on a very elegant, clear, elementary, and exact way (of course, taking in consideration necessary elements and relations of particle-wave duality (in this book summarized as PWDC relations), and by using Analytic signal modelling for wave functions). This way we will avoid creating and repeating complicated historical and other imaginative references to what Schrödinger, Bohr, Heisenberg, P. Dirac, Luis de Broglie, and others had in mind when developing wave quantum mechanics. Here is the short, self-explanatory resume regarding wave motions of elementary sinusoidal waves:

$$\Psi(x,t) = A\sin\left(kx \mp \omega t\right) = A\sin\left[\frac{2\pi}{\lambda}(x \mp ut)\right] = A\sin\left[2\pi(\frac{x}{\lambda} \mp \frac{u}{\lambda}t)\right] = A\sin\left[2\pi(\frac{x}{\lambda} \mp f \cdot t)\right],\ \omega = 2\pi f,\ k = \frac{2\pi}{\lambda},$$

$$\psi(\omega t) = \underline{\psi(x,t)}\big|_{x=0} = \underline{\psi(kx,\omega t)}\big|_{x=0} = \psi(0,\omega t) = \psi\left(\frac{2\pi}{T}\cdot t\right),\ \text{sinusoidal wave only in a time domain } (x = 0)$$

$$\psi(kx) = \underline{\psi(x,t)}\big|_{t=0} = \underline{\psi(kx,\omega t)}\big|_{t=0} = \psi(kx,0) = \psi\left(\frac{2\pi}{\lambda}\cdot x\right),\ \text{sinusoidal wave only in a spatial domain } (t = 0)$$

$$\psi(kx - \omega t) = \psi(2\pi\frac{1}{\lambda}x - 2\pi\frac{1}{T}t) = \psi(2\pi\frac{1}{\lambda}\cdot x - 2\pi f\cdot t),\ \text{sinusoidal wave travels in a positive direction (right)}$$

$$\psi(kx + \omega t) = \psi(2\pi\frac{1}{\lambda}x + 2\pi\frac{1}{T}t) = \psi(2\pi\frac{1}{\lambda}\cdot x + 2\pi f\cdot t),\ \text{sinusoidal wave travels in a negative direction (left)}$$

$$\psi(kx,\omega t) = \psi(kx \mp \omega t) \Leftrightarrow \psi(kx + \omega t) + \psi(kx - \omega t),\ \text{sinusoidal waves travelling in a positive and negative}$$
$$\text{directions (right \& left)} \Leftrightarrow \psi\left[k(x \mp \frac{\omega}{k}t)\right] = \psi\left[k(x \mp ut)\right] \Leftrightarrow \psi(x - ut) + \psi(x + ut).$$

A more general form of a wave function , $\Psi(x,t)$ with an initial constant phase Φ, that shifts the same wave is, $\Psi(x,t) = A\sin\left(k\cdot x - \omega\cdot t + \Phi\right)$.

The wave number **k** and the angular frequency $\omega = 2\pi f = 2\pi\frac{u}{\lambda}$ are analogically defined as being directly dependent on involved spatial and temporal periodicities (or periods) such as wavelength λ and time period $T = \frac{1}{f}$, meaning $k = \boxed{2\pi\cdot\frac{1}{\lambda}},\ \omega = 2\pi f = \boxed{2\pi\cdot\frac{1}{T}}$.

The **<u>wave function phase velocity u</u>** is the velocity of a point on the wave $\Psi(x,t)$ that has constant phase (for example, its crest), and it is given by relations,

$$u = \lambda\cdot f = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\tilde{E}}{p},$$

and on a similar way we define and develop **<u>group wave-function velocity v</u>** as the velocity of the envelope of a wave $\Psi(x,t)$ (that has certain constant amplitude), and it is given by relations (see more in Chapters 4.0 and 4.1),

$$v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda}.$$

Now, we can briefly repeat and summarize how mathematical aproach to wave equations has been evolving from an ordinary classical wave equation (using real functions) towards using complex and hypercomplex functions.

<table>
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-1-

$$\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=\frac{\partial^2\Psi}{\partial r^2}(=)\nabla^2\Psi,\ u=\omega/k$$

Classical Wave Equation

This has been the basic and initial model for all other analogical extensions, of wave equations in Physics.
</td><td>

The wavefunction is a real function.

$$\Psi(r,t)=a\cdot\cos(kr-\omega t)+b\cdot\cos(kr+\omega t)=$$
$$=a\cdot\cos k(r-ut)+b\cdot\cos k(r+ut)=$$
$$=\Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut),\ (a,b)=\text{constants}$$

Solutions are also:

$$\Psi(r,t)=\Psi^{(+)}(r-ut)$$
$$\Psi(r,t)=\Psi^{(-)}(r+ut)$$
$$\Psi(r,t)=\Psi^{(+)}(r-ut)+\Psi^{(-)}(r+ut)$$

(including a convenient integral superposition of similar infinitesimal and elementary wave function components)
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<tr><td>

-2-

$$\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\frac{\partial^2\overline{\Psi}}{\partial r^2}(=)\nabla^2\overline{\Psi},\ u=\omega/k$$

Complex Function Wave Equation

Schrödinger Equation and Wave Quantum Mechanics evolved from here (combined with certain ad hoc mathematical attachments, and with **PWDC**, as formulated in chapter 4.1, around equations (4.2)).
</td><td>

The wavefunction is a simple complex function.

$$\Psi(r,t)\to\overline{\Psi}(r,t)=a\cdot e^{j(kr-\omega t)}+b\cdot e^{j(kr+\omega t)}=$$
$$=a\cdot e^{jk(r-ut)}+b\cdot e^{jk(r+ut)}=\left|\overline{\Psi}(r,t)\right|e^{j\varphi(r,t)}=$$
$$=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut),\ (a,b)=\text{constants}$$

Solutions are also:

$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(-)}(r+ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)$$

(including the convenient integral superposition of similar infinitesimal and elementary wave function components)
</td></tr>
<tr><td>

-3-

$$\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\frac{\partial^2\overline{\Psi}}{\partial r^2}(=)\nabla^2\overline{\Psi},\ u=\omega/k$$

Analytic Signal Wave Equation

In this book, considering wave functions as Complex, Analytic Signal Wave Functions, Schrödinger, and other wave equations of Wave Quantum Mechanics will be reestablished and generalized on a much simpler, natural and more elementary way compared to contemporary Quantum theory situation (of course, combined with **PWDC**, formulated in chapter 4.1, around equations (4.2)).
</td><td>

The wavefunction is an Analytic Signal, complex function.

$$\Psi(r,t)\to\overline{\Psi}(r,t)=a(r,t)\cdot e^{j(kr-\omega t)}+b(r,t)\cdot e^{j(kr+\omega t)}=$$
$$=a(r,t)\cdot e^{jk(r-ut)}+b(r,t)\cdot e^{jk(r+ut)}=\left|\overline{\Psi}(r,t)\right|e^{j\varphi(r,t)}=$$
$$=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)=\Psi(r,t)+j\hat{\Psi}(r,t)=$$
$$=\left[a(r,t)\cos(kr-\omega t)+b(r,t)\cos(kr+\omega t)\right]+$$
$$+j\left[a(r,t)\sin(kr-\omega t)+b(r,t)\sin(kr+\omega t)\right]$$

Solutions are also:

$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(-)}(r+ut)$$
$$\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)$$

(including the convenient integral superposition of similar infinitesimal and elementary wave function components; -see (4.10-12))
</td></tr>
</table>

There is another (fourth) option for future explorations of wave functions by imposing that a much more general wave function ***will be a Hypercomplex, Analytic Signal, function***, as summarized below, under -4- (see more about Hyper-Complex Analytic Signal functions in Chapter 4.0, under equations (6.10) and in Chapter 10). Such approach has a potential to address a variety of elementary and micro-world entities appearing in impact and scatterings interactions (see also (4.3-0)-p,q,r,s… in Chapter 4.1, where Minkowski 4-vector of momentum-energy is extended towards hyper-complex space with three imaginary units).

-4-	The wavefunction is an Analytic Signal, Hyper-complex function with 3 imaginary units.
$\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\frac{\partial^2\overline{\Psi}}{\partial r^2}(=)\nabla^2\overline{\Psi},\ u=\omega/k,$ $\overline{\Psi}=\overline{\Psi}(r,t)=\overline{\Psi}[r(x,y,z),t]=$ $=\overline{\Psi}\{r[x(t),y(t),z(t)],t\}=$ $=\overline{\Psi}[x(t),y(t),z(t),t],$ $\frac{\partial}{\partial t}=\frac{d}{dt}-(v\cdot\vec{\nabla}),$ $\vec{\nabla}=(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}),$ v(=) Frame velocity	$\Psi(r,t)\to\overline{\Psi}(r,t)=$ $=a(r,t)\cdot e^{i_1(kr-\omega t)}+b(r,t)\cdot e^{i_1(kr+\omega t)}+$ $+c(r,t)\cdot e^{i_2(kr-\omega t)}+d(r,t)\cdot e^{i_2(kr+\omega t)}+$ $+e(r,t)\cdot e^{i_3(kr-\omega t)}+f(r,t)\cdot e^{i_3(kr+\omega t)}=$ $=a(r,t)\cdot e^{i_1k(r-ut)}+b(r,t)\cdot e^{i_1k(r+ut)}+$ $+c(r,t)\cdot e^{i_2k(r-ut)}+d(r,t)\cdot e^{i_2k(r+ut)}+$ $+e(r,t)\cdot e^{i_3k(r-ut)}+f(r,t)\cdot e^{i_3k(r+ut)}=$ $=\left\|\overline{\Psi}(r,t)\right\|e^{I\varphi(r,t)}=$ $=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)$ $i_1^2=i_2^2=i_3^2=I^2=-1$ $i_1i_2=i_3, i_2i_3=i_1, i_3i_1=i_2, i_2i_1=-i_3, i_3i_2=-i_1...$
Hyper Complex Analytic Signal Wave Equation Will be left for future explorations. (Again, such wave equation should be combined with **PWDC**, formulated in chapter 4.1, around equations (4.1)-(4.3)).	Solutions are also: $\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)$ $\overline{\Psi}(r,t)=\overline{\Psi}^{(-)}(r+ut)$ $\overline{\Psi}(r,t)=\overline{\Psi}^{(+)}(r-ut)+\overline{\Psi}^{(-)}(r+ut)$ (including a convenient integral superposition of similar infinitesimal and elementary wave function components; -see (4.10-12))

In this book, Schrödinger and other wave equations of Wave Quantum Mechanics will evolve directly from Classical wave equation and a wave function that is an Analytic Signal function (here summarized under **-3-**). We will simply show that Nature or macro and micro-Physics are perfectly united, and that all wave equations are causally related to the old and universally applicable Classical Wave Equation. Schrödinger artificially assembled his equation by applying "*divine and magic assumptions, intuition and inspiration*" (under special life-motivating conditions), and here we will realize something similar, but being more general, using simple, smooth, provable, and deterministic mathematical steps.

Wavefunction, $\Psi(t)$*, as presently (and probabilistically) defined and interpreted in Orthodox Quantum theory, has no immediate phase information (where the phase is dependent on immediate spatial and temporal variables). To get certain relevant spatial or combined spatial-temporal distribution or spectrum, we need to rely on probabilistic and statistical events and modelling, until we start recognizing some wave shapes, or wave properties, as superposition, diffraction, and interference effects. Contrary, if we treat a wave function as a Complex, Analytic Signal function (as practiced in this book), we know from the very beginning that such wave function explicitly has rich spatial and temporal (immediate) phase and amplitude information, being causally related to, or connected with* ***PWDC*** *facts (see more about* ***PWDC*** *in the Chapters 4.1 and 10.).*

For instance, if certain solid body is assembled from number of atoms, molecules and/or other small particles, such ensemble of small items would realize certain level of internal and mutual coupling and synchronization between its constituents (thanks to wave-particle duality nature of involved entities). Conceptualizing this way, we can understand that every system of particles has certain specific, joint, or resulting phase function (in relation to de Broglie matter-waves concept). We also know that contemporary Quantum theory, based on probabilistic wavefunction, is unfortunately (by definition) neglecting mentioned phase-related properties.

Let us briefly (in three steps: **A**), **B**) and **C**)) summarize the starting points and results in the process of developing Schrödinger-like equations, **based on an Analytic Signal Wave function concept, as introduced in (4.9-0)**, almost apart (or independently) from any Quantum Theory background and assumptions:

A) By replacing (or extending) an arbitrary wave function $\Psi(t)$, (which could be applicable to represent any kind of waving process, wave-group, or motional particle, regardless if it would have probabilistic or deterministic nature), into its equivalent, temporal-spatial coordinates dependent complex form, $\Psi(t)\to\Psi(x,t)\to\overline{\Psi}(x,t)$, (considering that $\overline{\Psi}(x,t)$ will be treated as the **Complex Analytic Signal**, first time introduced by Dennis Gabor; -see [7] , [8], and Chapter 4.0.; -equations (4.0.1) - (4.0.5)), we will get:

$$\begin{aligned}
&\overline{\Psi}(x,t)=\Psi(x,t)+j\hat{\Psi}(x,t)=a(x,t)e^{j\varphi(x,t)}=\frac{1}{(2\pi)^2}\iint_{(-\infty,+\infty)}U(\omega,k)e^{-j(\omega t-kx)}dkd\omega=\\
&=\frac{1}{\pi^2}\iint_{(0,+\infty)}U(\omega,k)e^{-j(\omega t-kx)}dkd\omega=\frac{1}{\pi^2}\iint_{(0,+\infty)}A(\omega,k)e^{-j(\omega t-kx+\Phi(\omega,k))}dkd\omega\ ,\ j^2=-1,\\
&U(\omega,k)\ =\ U_c(\omega,k)\ -\ j\,U_s(\omega,k)\ =\ \iint_{(-\infty,+\infty)}\overline{\Psi}(x,t)\ e^{j(\omega t-kx)}dtdx=\ A(\omega,k)e^{-j\Phi(\omega,k)}\ ,
\end{aligned}\tag{4.9}$$

where $\hat{\Psi}(x,t)$ is the Hilbert transformation of $\Psi(x,t)=a(x,t)\cos\varphi(x,t)$, or $\hat{\Psi}(x,t)=H\left[\Psi(x,t)\right]=a(x,t)\sin\varphi(x,t)$.

More general wave function (instead of (4.9)) should be composed of at least two wave functions propagating in mutually opposite directions (see (4.10-12)), such as,

$$\begin{aligned}
&\Psi(x,t)\to\overline{\Psi}(x,t)=a(x,t)\cdot e^{j(kx-\omega t)}+b(x,t)\cdot e^{j(kx+\omega t)}=\\
&=a(x,t)\cdot e^{jk(x-ut)}+b(x,t)\cdot e^{jk(x+ut)}=\left|\overline{\Psi}(x,t)\right|e^{j\varphi(x,t)}=\\
&=\overline{\Psi}^{(+)}(x-ut)+\overline{\Psi}^{(-)}(x+ut)=\Psi(x,t)+j\hat{\Psi}(x,t)=\\
&=\left[a(x,t)\cos(kx-\omega t)+b(x,t)\cos(kx+\omega t)\right]+\\
&+j\left[a(x,t)\sin(kx-\omega t)+b(x,t)\sin(kx+\omega t)\right],
\end{aligned}$$

what will not affect later results regarding the development of different differential wave equations, since general solutions of mentioned wave equations are always complying with $\overline{\Psi}(x,t)=a(x,t)\cdot e^{j(kx-\omega t)}+b(x,t)\cdot e^{j(kx+\omega t)}$*, or more correct in three dimensional spatial and one temporal coordinate, such wave function will be presentable as* $\overline{\Psi}(x,t)=a(x,y,z,t)\cdot e^{j(kr-\omega t)}+b(x,y,z,t)\cdot e^{j(kr+\omega t)},\quad r=r(x,y,z)$.

Solutions of Classical differential (second order) wave equation are always presentable as two wave functions propagating in mutually opposite directions, or inwards and outwards in cases of more complicated spatial structures. This is kind of natural balance between always and coincidently present action and reaction forces, mutual inductions, mirror imaging effects, and it is closely related to respecting conservation laws and symmetries valid in Physics.

In fact, it will be demonstrated that Complex Analytic Signal (4.9) presents much more important (richer and more productive) generic framework for any wave function and wave equation formulation, than what we find in traditionally known Schrödinger and Quantum Mechanics wave equations practices. Starting from (4.9) we can easily develop all variants of Schrödinger, d 'Alembert, Classical, and number of other wave equations known in Physics.

Because of mathematical simplicity in developing Schrödinger equation, here we will consider $\overline{\Psi}(x,t)$ as a plane waves superposition (or wave group, or wave packet) function of one spatial coordinate and a time, but we know that the more general case is $\overline{\Psi}(r,t)=\overline{\Psi}(x,y,z,t)$. We also know that a wave and particle nature of electromagnetic radiation (as photons), and motional particles with non-zero rest masses, can be very well, mutually, and analogically, compared using the concept of wave packets (based on analyzes and explanations of Compton and Photoelectric effects, Brag diffraction etc.). We will simply, and analogically consider here, based on de Broglie matter wave's hypothesis, that the same can be applied to all matter waves and motional particles, as follows:

B) By applying multiple derivations to (4.9), (see the similar procedure in **[5],** pages: 175-179), we will be able to get (4.9-1):

$$\left[\begin{array}{l}
\left\{\begin{array}{l}
\overline{\Psi}(x,t)=\Psi(x,t)+j\hat{\Psi}(x,t)=a(x,t)e^{j\varphi(x,t)}=\frac{1}{(2\pi)^2}\iint\limits_{(-\infty,+\infty)}U(\omega,k)e^{-j(\omega t-kx)}dkd\omega= \\
=\frac{1}{\pi^2}\iint\limits_{(0,+\infty)}U(\omega,k)e^{-j(\omega t-kx)}dkd\omega=\frac{1}{\pi^2}\iint\limits_{(0,+\infty)}A(\omega,k)e^{-j(\omega t-kx+\Phi(\omega,k))}dkd\omega= \\
=\iint\overline{\Psi}''dkd\omega=\iint\overline{\Psi}''dxdt=\Psi+jH[\Psi]=\Psi+j\hat{\Psi}
\end{array}\right\}\Rightarrow \\
\Rightarrow\left\{\begin{array}{l}
\overline{\Psi}'_x=\frac{\partial\overline{\Psi}}{\partial x}=\frac{jk}{(2\pi)^2}U(\omega,k)e^{-j(\omega t-kx)}=jk\overline{\Psi},\ \overline{\Psi}''_x=\frac{\partial^2\overline{\Psi}}{\partial x^2}=jk\frac{\partial\overline{\Psi}}{\partial x}=-k^2\overline{\Psi}\Rightarrow\Delta\overline{\Psi}=-k^2\overline{\Psi}, \\
\overline{\Psi}'_t=\frac{\partial\overline{\Psi}}{\partial t}=\frac{-j\omega}{(2\pi)^2}U(\omega,k)e^{-j(\omega t-kx)}=-j\omega\overline{\Psi},\ \overline{\Psi}''_t=\frac{\partial^2\overline{\Psi}}{\partial t^2}=-j\omega\frac{\partial\overline{\Psi}}{\partial t}=-\omega^2\overline{\Psi}
\end{array}\right\}\Rightarrow \\
\Rightarrow\overline{\Psi}=-\frac{1}{k^2}\frac{\partial^2\overline{\Psi}}{\partial x^2}=-\frac{1}{\omega^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}\Leftrightarrow\frac{\partial^2\overline{\Psi}}{\partial x^2}-\frac{1}{\left(\frac{\omega}{k}\right)^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\Leftrightarrow\frac{\partial^2\overline{\Psi}}{\partial x^2}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\Rightarrow \\
\Rightarrow\boxed{\nabla^2\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=\Delta\overline{\Psi}-\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\Leftrightarrow\left\{\begin{array}{l}\Delta\Psi-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\\ \Delta\hat{\Psi}-\frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2}=0\end{array}\right\}}
\end{array}\right]\Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{c} \boxed{-\frac{\omega}{k^2}\Delta\overline{\Psi} = \omega\overline{\Psi} = \frac{-1}{\omega}\frac{\partial^2\overline{\Psi}}{\partial t^2} = -j\frac{\omega}{k}\nabla\overline{\Psi} = j\frac{\partial\overline{\Psi}}{\partial t}} \\ \Downarrow \\ -\frac{\omega}{k^2}\Delta\Psi = \omega\Psi = \frac{-1}{\omega}\frac{\partial^2\Psi}{\partial t^2} = \frac{\omega}{k}\nabla\hat{\Psi} = -\frac{\partial\hat{\Psi}}{\partial t} \\ -\frac{\omega}{k^2}\Delta\hat{\Psi} = \omega\hat{\Psi} = \frac{-1}{\omega}\frac{\partial^2\hat{\Psi}}{\partial t^2} = -\frac{\omega}{k}\nabla\Psi = \frac{\partial\Psi}{\partial t} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} \boxed{\Delta\overline{\Psi} = \frac{k^2}{\omega^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} = -k^2\overline{\Psi} = jk\nabla\overline{\Psi} = -j\frac{k^2}{\omega}\frac{\partial\overline{\Psi}}{\partial t}} \\ \Downarrow \\ \Delta\Psi = \frac{k^2}{\omega^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = -k^2\Psi = -k\nabla\hat{\Psi} = \frac{k^2}{\omega}\frac{\partial\hat{\Psi}}{\partial t} \\ \Delta\hat{\Psi} = \frac{k^2}{\omega^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = -k^2\hat{\Psi} = k\nabla\Psi = -\frac{k^2}{\omega}\frac{\partial\Psi}{\partial t} \end{array} \right\}.$$

(4.9-1)

Equivalent results to (4.9-1) can be, analogically and intuitively demonstrated, without the big need to "*invent*" and artificially attach any missing equation part, or to introduce imaginative discussions and concepts about waving probabilities and undulatory properties of involved events. Alternatively, if we simply start from the Classical Wave Equation, and analogically replace a real wave function $\Psi(x,t)$ with the corresponding complex Analytic Signal wavefunction $\overline{\Psi}(x,t)$, and develop and combine involved derivatives, we get again (4.9-1),

$$\begin{bmatrix} \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2}(=)\nabla^2\Psi,\ u = \omega/k \\ \\ \Psi(x,t) \to \overline{\Psi}(x,t) = \\ = a\cdot e^{j(kx-\omega t)} + b\cdot e^{j(kx+\omega t)} = \\ = [a\cdot\cos(kx-\omega t) + b\cdot\cos(kx+\omega t)] + \\ +j[a\cdot\sin(kx-\omega t) + b\cdot\sin(kx+\omega t)] \end{bmatrix} \to \begin{bmatrix} \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} = \frac{\partial^2\overline{\Psi}}{\partial x^2}(=)\nabla^2\overline{\Psi},\ u = \omega/k \\ \\ \overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = \\ = a(x,t)\cdot e^{j(kx-\omega t)} + b(x,t)\cdot e^{j(kx+\omega t)} = \\ = [a(x,t)\cos(kx-\omega t) + b(x,t)\cos(kx+\omega t)] + \\ +j[a(x,t)\sin(kx-\omega t) + b(x,t)\sin(kx+\omega t)] \end{bmatrix} \Rightarrow$$

$$\Rightarrow [\text{eventually resulting in partial differential wave equations (4.9-1)}].$$

C) Now, by implementing the following **P**article-**W**ave **D**uality **C**ode relations (as introduced in (4.1) - (4.3) and in Chapter 10.) into (4.9) and (4.9-1),

$$\boxed{m^* = \gamma m, p = \gamma m v = m^* v = \tilde{p},\ \tilde{E} = hf = \hbar\omega = m^* v u = pu,\ u = \frac{\omega}{k} = \lambda f, v = \frac{d\omega}{dk} = u - \lambda\frac{du}{d\lambda}, \lambda = \frac{h}{p}}$$
,

we will be able to formulate several matter wave equations belonging to a family of Schrödinger-like equations, as follows (see such equations from (4.9-2) to (4.10-6)):

(1)

Let us start with pure matter waves (without rest masses), propagating in a free space without presence of potential-field or central forces, with potential energy, which is here $\mathbf{U_p = 0}$. Of course, energy-momentum content of matter wave in question would have certain energy-mass equivalent (which is not a rest mass). Examples of such phenomena are electromagnetic waves and photons (propagating in a vacuum, far from any big masses agglomerations). In a few steps, from equations (4.9-1) we will get,

matter-waves equations in a free space, without rest mass

$$\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi}+\tilde{E}\,\overline{\Psi}=0,\ (U_p=0),$$

$$\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi}=-j\hbar\frac{\partial\overline{\Psi}}{\partial t}=-\tilde{E}\,\overline{\Psi}=\frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\hbar u\nabla\overline{\Psi}, \qquad (4.9\text{-}2)$$

$$(\frac{\tilde{E}}{\hbar})^2\cdot\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\ \frac{\partial\overline{\Psi}}{\partial t}+u\nabla\overline{\Psi}=0,$$

$$\Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0\,.$$

As the significant support to (4.9-1) and (4.9-2), we could draw relevant and equivalent conclusions by comparing well-known examples of similar classical, electromagnetic wave equations dealing with electric, $\mathbf{E}$, and magnetic, $\mathbf{B}$, field-vectors,

$$\Delta\mathbf{E}-\varepsilon_0\mu_0\frac{\partial^2\mathbf{E}}{\partial t^2}=\Delta\mathbf{E}-\frac{1}{u^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=0,$$

$$\Delta\mathbf{B}-\varepsilon_0\mu_0\frac{\partial^2\mathbf{B}}{\partial t^2}=\Delta\mathbf{B}-\frac{1}{u^2}\frac{\partial^2\mathbf{B}}{\partial t^2}=0,\ u=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=c. \qquad (4.9\text{-}2.1)$$

In [86] Victor Christianto (when being in his rational and scientific state of mind) elaborated correspondence, analogy, and mutual transformability between classical wave equations of electromagnetic waves (4.9-2.1) and quantum mechanics', Klein-Gordon, and Schrödinger wave equations. He did this by evoking and summarizing "Ward & Volkmer's derivation of Schrödinger equation from the Classical Wave equation", practically demonstrating how Electromagnetic Classical Wave Equation can evolve systematically, becoming Schrödinger equation. Equivalent elaborations about evolution of Schrödinger-family differential wave equations towards Classical wave equation, and vice-versa, when combined with correct wave-particle duality foundations (in this book addressed as **PWDC,** with (4.1) - (4.3)) is also presented in the significant reference [105], from Himanshu Chauhan, Swati Rawal and R K Sinha, -WAVE-PARTICLE DUALITY REVITALIZED; -CONSEQUENCES, APPLICATIONS, AND RELATIVISTIC QUANTUM MECHANICS.

The following resume (literature ref. under [86] and [39]) was summarized by Victor Christianto in his younger and still rational intellectual phase, before he started to work for J. C. He made his resume about Schrödinger wave equation based on publications and works of George Shpenkov (see literature references under [85], https://shpenkov.com/).

"A Review of Schrödinger Equation & Classical Wave Equation

Schrödinger equation

George Shpenkov points out that there are several weaknesses associated with (spherical solution of) Schrödinger's equation:

i. Its spherical solution is rarely discussed completely (especially in graduate or undergraduate quantum mechanics textbooks), perhaps because many physicists seem to feel obliged to hide from public that the spherical solution of Schrödinger's wave equation does not agree with any experiment.

ii. Schrödinger equation is able only to arrive at hydrogen energy levels, and it has to be modified and simplified for other atoms. For example, physicists are forced to use an approximate approach called Density Functional Theory (DFT) in order to deal with N-body system.1

iii. The introduction of variable wave number k in Schrödinger equation, depending on electron coordinates, and the omission of the azimuth part of the wave function, were erroneous [6]. Schrödinger's variable wave number should be questioned, because the potential function cannot influence the wave speed or consequently the wave number.

iv. Introduction of the potential function V in the wave equation, which results in dependence of the wave number *k* on the Coulomb potential, *generates divergences* that do not have a physical justification. They are eliminated in an artificial way. [6, p.27]

v. Modern physics erroneously interprets the meaning of polar-azimuthal functions in Schrödinger's equation, ascribing these functions to atomic "*electron orbitals*". [1, p.5]

vi. Schrödinger arrived at a "correct" result of hydrogen energy levels using only a radial solution of his wave equation, with one major assumption: the two quantum numbers found in the solution of his wave equation were assumed to be the same with Bohr's quantum number.

vii. Quantum mechanics solutions, in their modern form, contradict reality because on the basis of these solutions, the existence of crystal substances-spaces is not possible. [6, p.26]

viii. Schrödinger's approach yields abstract phenomenological constructions, which do not reflect the real picture of the micro-world.[2]

ix. Schrödinger himself in his 1926 paper apparently wanted to interpret his wave equation in terms of vibration of string [3][4]. This is why he did not accept Born's statistical interpretation of his wave equation until he died. Einstein and de Broglie also did not accept the statistical interpretation of quantum mechanics.

x. The interpretation and the physical meaning of the Schrödinger's wave function was a problem for physicists, and it still remains so, although many researchers understand its conditional character [6]."

Of course, we could also make attempts to explore the evolution of classical electromagnetic (and other) wave equations like (4.9-2.1), when electromagnetic vectors and associated waves will take **Complex, or Hyper-Complex Analytic Signal forms**, eventually transforming (4.9-2.1) to wave equations as found in (4.9-2). See also familiar elaborations (3.7-1) and (3.7-2) from the third chapter of this book.

Nevertheless, regarding all wave equations, starting from (4.9) until (4.9-2.1), we should also consider as extremely relevant and unavoidable critical comments, elaborations, and proposals (found in [35]) from Thomas E. Phipps, Jr., about Maxwell equations, that can be equally applicable to all quantum theory wave equations.

(2)

Next step will consider that matter waves phenomenology is also presentable (directly, indirectly, and analogically) as a motion of certain energy-momentum formation in an energy field of some bigger particle (which creates a potential field with energy $\mathbf{U_p}$), still without rest mass involvement. For such situation, new, generalized Schrödinger-like equation, developed from (4.9-1), will evolve to its final form considering certain process of “wave-energy translation”, as follows:

matter-waves equations in a negative potential energy field, without rest mass

$$\left[\tilde{E} \to \tilde{E}' = \tilde{E} - U_p,\ U_p \le 0\right] \Rightarrow \left[\bar{\Psi} \to \bar{\Psi}',\ m^* \to m^{*\prime}...\right] \Rightarrow$$

$$-\frac{\hbar^2}{m^{*\prime}}\left(\frac{u}{v}\right)'\Delta\bar{\Psi}' = j\hbar\frac{\partial\bar{\Psi}'}{\partial t} = (\tilde{E}-U_p)\bar{\Psi}' = \frac{-\hbar^2}{\tilde{E}-U_p}\frac{\partial^2\bar{\Psi}'}{\partial t^2} = -j\hbar u'\nabla\bar{\Psi}',$$

$$\begin{pmatrix} \Delta\bar{\Psi}' - \frac{1}{u'^2}\cdot\frac{\partial^2\bar{\Psi}'}{\partial t^2} = \left(\frac{\tilde{E}-U_p}{\hbar u'}\right)^2\bar{\Psi}' + \frac{1}{u'^2}\cdot\frac{\partial^2\bar{\Psi}'}{\partial t^2} = j\frac{\tilde{E}-U_p}{\hbar u'^2}\frac{\partial\bar{\Psi}'}{\partial t} + \frac{1}{u'^2}\cdot\frac{\partial^2\bar{\Psi}'}{\partial t^2} = 0, \\ \frac{\hbar^2}{m^{*\prime}}\left(\frac{u}{v}\right)'\Delta\bar{\Psi}' + (\tilde{E}-U_p)\bar{\Psi} = 0 \end{pmatrix} \quad (4.9\text{-}3)$$

Now, we can simply come back from (4.9-3) to usual wave functions notation, as,

$$\left[\bar{\Psi}' \to \bar{\Psi},\ m^{*\prime} \to m^*,\ \tilde{E}' = m^{*\prime}u'v' = \tilde{E}-U_p\right] \Rightarrow$$

$$\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\bar{\Psi} + (\tilde{E}-U_p)\bar{\Psi} = 0,$$

$$\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\bar{\Psi} - U_p\bar{\Psi} = -\tilde{E}\bar{\Psi} = -j\hbar\frac{\partial\bar{\Psi}}{\partial t} - U_p\bar{\Psi} = \frac{\hbar^2}{\tilde{E}-U_p}\cdot\frac{\partial^2\bar{\Psi}}{\partial t^2} - U_p\bar{\Psi}, \quad (4.10)$$

$$\left(\frac{\tilde{E}-U_p}{\hbar}\right)^2\cdot\bar{\Psi} + \frac{\partial^2\bar{\Psi}}{\partial t^2} = 0\ ,\ \frac{\partial\bar{\Psi}}{\partial t} + u\nabla\bar{\Psi} = 0,$$

$$\Delta\bar{\Psi} - \frac{1}{u^2}\cdot\frac{\partial^2\bar{\Psi}}{\partial t^2} = \left(\frac{\tilde{E}-U_p}{\hbar u}\right)^2\bar{\Psi} + \frac{1}{u^2}\cdot\frac{\partial^2\bar{\Psi}}{\partial t^2} = j\frac{\tilde{E}-U_p}{\hbar u^2}\frac{\partial\bar{\Psi}}{\partial t} + \frac{1}{u^2}\cdot\frac{\partial^2\bar{\Psi}}{\partial t^2} = 0.$$

The same equations from (4.10) can be transformed (or abbreviated) into operators' form:

$$\left(H = -\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta + U_p\ (=)\ \text{Hamiltonian}\right) \Rightarrow \left\{H\bar{\Psi} = \tilde{E}\bar{\Psi} = j\hbar\frac{\partial}{\partial t}\bar{\Psi} + U_p\bar{\Psi} = ...\right\}$$

$$\Rightarrow \tilde{E} \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} + U_p \Leftrightarrow -j\hbar u\nabla\ , \quad (4.11)$$

$$\left(\tilde{p}_i\bar{\Psi} = -j\hbar\nabla\bar{\Psi}\right) \Rightarrow \tilde{p}_i \Leftrightarrow -j\hbar\nabla \Leftrightarrow \frac{1}{u}\left(j\hbar\frac{\partial}{\partial t} + U_p\right) \Leftrightarrow \frac{1}{u}H.$$

In cases of non-relativistic velocities, after replacing relation between group and phase velocity with its approximate value $v \cong 2u$*, generalized Schrödinger's equation (4.10),* $\frac{\hbar^2}{m^*}\left(\frac{u}{v}\right)\Delta\bar{\Psi} + (\tilde{E}-U_p)\bar{\Psi} = 0$ *becomes equal to the traditionally known (original and non-relativistic) Schrödinger's wave equation,*

$$\mathbf{j}\overline{\Psi}\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} = -\frac{\hbar^2}{\mathbf{2m}^*}\Delta\overline{\Psi} + \mathbf{U}_{\mathbf{p}}\overline{\Psi} = \tilde{\mathbf{E}}\overline{\Psi} \Leftrightarrow \frac{\hbar^2}{\mathbf{2m}^*}\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathbf{U}_{\mathbf{p}})\overline{\Psi} = \mathbf{0}, \qquad (4.12)$$

which (according to contemporary Quantum Theory literature) has the universal applicability in a micro world of physics, and has an almost divine origin, created in an extraordinary period of Schrödinger's intellectual illumination (when he was especially and multidisciplinary motivated). Fortunately, the same situation is much different from being divine, and it is much more deterministic, and very well explicable in quite simple terms (when treating wavefunction as an Analytic Signal). This is also an example how obviously useful, practical, but only partially valid and oversimplified formulations, and by trial-&-error optimized results, could be successfully mystified, complicated and appropriately hybridized, and upgraded, eventually becoming celebrated foundations of our modern physics.

The above-presented forms of generalized Schrödinger's equation ((4.9-2) and (4.10)) consider only motional wave energy as a wave-group energy. This fact should be noticed as the biggest difference between classical interpretation of Schrödinger's equation from contemporary quantum theory (which usually takes a total, real particle with its rest-mass energy, into account), and all wave equation forms, (4.10) - (4.12), developed until here.

We can also notice (and demonstrate) that Dirac's relativistic modification of Schrödinger's equation is (or should be) automatically included in equations (4.10) - (4.12), since the energy $\tilde{\mathbf{E}}$ and ratio $\mathbf{u}/\mathbf{v}$ are already treated as relativistic, velocity-dependent functions (in relation to Lorentz transformations; -see (4.1) - (4.3)).

In (4.10) we made wave-energy level-translation for the potential field energy, $\tilde{E} \to \tilde{E}' = \tilde{E} - U_p, U_p \neq 0$, and we got the following, familiar-looking (classical) Schrödinger's equation (since when $\mathbf{v} << \mathbf{c} \Rightarrow \mathbf{u/v} = \mathbf{1/2}$),

$$\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} - \mathbf{U}_{\mathbf{p}})\overline{\Psi} = 0 \Rightarrow \frac{\hbar^2}{2m^*}\Delta\overline{\Psi} + (\tilde{E} - \mathbf{U}_{\mathbf{p}})\overline{\Psi} = 0.$$

Schrödinger most probably did not know for Analytic Signal functions and Hilbert transformation, when he intuitively, and by productive trial and error, and brainstorming (including divine enlightenment) created his famous equation by adding missing imaginary part (since Dennis Gabor theoretically established the Analytic Signal concept much later). Anyway, Schrödinger intuitively, effectively, and somewhat inexplicably created his equation that (non-intentionally or not-knowingly) complies with the Analytic Signal model, and this equation started to be particularly useful and productive in Quantum Physics. People simply accepted (after several verifications and mathematical optimizations) that such equation is essential, and important for describing the wave-particle duality world of microphysics. ***In this chapter, we will see, on a very elementary and simple way, that complex analytic signal based Classical Wave Equation (4.9-1) is smoothly, logically, and naturally generating Schrödinger and other familiar wave equations as presently known in Quantum Theory. There is no more need for divine and magic inspiration and involvement of brilliant minds, ad hock assumptions,***

and patchwork, as it was practiced in the early days of Quantum Theory creation. Consequently, in this book, micro-physics, and early days Quantum Theory concepts (regarding Particle-Wave Duality and Matter waves) are shown to be much better connected to Classical and Relativistic Physics (including Mechanics) than in the case of Orthodox Quantum theory. At the same time, we can profit and admire many of mathematical and scientifically adventurist, imaginative, fruitful, and productive concepts developed or postulated in Quantum Theory (that would never be developed in the framework of Classical Physics and Mechanics). What is still partially wrong, but important to be considered, is that modern Quantum Theory (including Standard Model of Particle Physics) is, as the model or theory, mathematically and conceptually incomplete, too much artificial, complicated, and not enough rich in necessary mathematical foundations and realistic options, to describe the real world of physics more naturally and correctly. For instance, here we can see that just by treating a wave function as an Analytic Signal, we can bring an enormous contribution to Wave mechanics, Quantum theory and to better, natural conceptualization of Particle-Wave Duality (see more in Chapter 10). Standard Model is still missing good and natural mathematical framework or modeling.

(3)

Let us now create another *wave-energy level translation* in a potential field, to “capture” the total (relativistic) energy of a moving particle (which has its rest mass $\mathbf{m_0 = E_0 / c^2 = const.}$), as follows. $\tilde{\mathbf{E}} \rightarrow \tilde{\mathbf{E}}' = \tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}, \mathbf{U_p} \neq \mathbf{0}, \mathbf{E_0} = \mathbf{const.}$, (understanding that relevant energy equals the sum of motional energy, $E_k = \tilde{E}$, and rest energy $\mathbf{E_0}$, reduced for the energy of surrounding potential field U_p (since we associate negative values to potential energy), this way creating relevant Lagrangian), and let us apply such “*energy translation*” on the wave equations (4.9-2), as for instance,

matter-waves equations in a negative potential energy field, with rest mass involvement

$$\left[\tilde{\mathbf{E}} \rightarrow \tilde{\mathbf{E}}' = \tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p} = \mathbf{E_{total}} - \mathbf{U_p}, (\mathbf{U_p} \leq \mathbf{0}, \mathbf{E_0} = \mathbf{const.})\right] \Rightarrow \left[\overline{\Psi} \rightarrow \overline{\Psi}', \mathbf{m}^* \rightarrow \mathbf{m}^{*'}...\right] \Rightarrow$$

$$-\frac{\hbar^2}{\mathbf{m}^{*'}}(\frac{\mathbf{u}}{\mathbf{v}})'\Delta\overline{\Psi}' = \mathbf{j}\hbar\frac{\partial\overline{\Psi}'}{\partial \mathbf{t}} = (\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p})\overline{\Psi}' = \frac{-\hbar^2}{\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}}\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = -\mathbf{j}\hbar\mathbf{u}'\nabla\overline{\Psi}',$$

$$\begin{pmatrix} \Delta\overline{\Psi}' - \frac{\mathbf{1}}{\mathbf{u}'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}}{\hbar\mathbf{u}'}\right)^2\overline{\Psi}' + \frac{\mathbf{1}}{\mathbf{u}'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p}}{\hbar\mathbf{u}'^2}\frac{\partial\overline{\Psi}'}{\partial \mathbf{t}} + \frac{\mathbf{1}}{\mathbf{u}'^2}\cdot\frac{\partial^2\overline{\Psi}'}{\partial \mathbf{t}^2} = \mathbf{0}, \\ \frac{\hbar^2}{\mathbf{m}^{*'}}(\frac{\mathbf{u}}{\mathbf{v}})'\Delta\overline{\Psi}' + (\tilde{\mathbf{E}} + \mathbf{E_0} - \mathbf{U_p})\overline{\Psi} = \mathbf{0} \end{pmatrix} \quad (4.9\text{-}4)$$

The next step is again to transform (4.9-4) to its basic energy-translated wave function,

$$\left[\overline{\Psi}' \to \overline{\Psi},\ \mathbf{m}^{*\prime} \to \mathbf{m}^{*},\ \tilde{\mathbf{E}}' = \mathbf{m}^{*\prime}\mathbf{u}'\mathbf{v}' = \tilde{\mathbf{E}} + \mathbf{E}_0 - \mathbf{U}_p = \mathbf{E}_{total} - \mathbf{U}_p\right] \Rightarrow$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0 - \mathbf{U}_p)\overline{\Psi} = \mathbf{0}$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} - \mathbf{U}_p\overline{\Psi} = -(\tilde{\mathbf{E}} + \mathbf{E}_0)\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} - \mathbf{U}_p\overline{\Psi} = \frac{\hbar^2}{\tilde{\mathbf{E}} + \mathbf{E}_0 - \mathbf{U}_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} - \mathbf{U}_p\overline{\Psi} = \qquad (4.10\text{-}1)$$

$$= \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi} - \mathbf{U}_p\overline{\Psi},\ (\frac{\mathbf{E}_{total} - \mathbf{U}_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0},\ \frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0}$$

$$\Delta\overline{\Psi} - \frac{1}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{E}_0 - \mathbf{U}_p}{\hbar\mathbf{u}}\right)^2\overline{\Psi} + \frac{1}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{E}_0 - \mathbf{U}_p}{\hbar\mathbf{u}^2}\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \frac{1}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}.$$

Practically, now we can symbolically transform the same equations (4.10-1) into another operators' form, as follows:

$$\left(\mathbf{H} = -\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta + \mathbf{U}_p\ (=)\ \text{Hamiltonian}\right) \Rightarrow \left\{\mathbf{H}\overline{\Psi} = \mathbf{E}_{total}\overline{\Psi} = \mathbf{j}\hbar\frac{\partial}{\partial \mathbf{t}}\overline{\Psi} + \mathbf{U}_p\overline{\Psi} = ...\right\}$$

$$\Rightarrow \mathbf{E}_{total} \Leftrightarrow \mathbf{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial \mathbf{t}} + \mathbf{U}_p \Leftrightarrow -\mathbf{j}\hbar\mathbf{u}\nabla, \qquad (4.11\text{-}1)$$

$$\tilde{\mathbf{p}}_i \Leftrightarrow -\mathbf{j}\hbar\nabla - \frac{\mathbf{E}_0 - \mathbf{U}_p}{\mathbf{u}} \Leftrightarrow \frac{1}{\mathbf{u}}(\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{t}} - \mathbf{E}_0) \Leftrightarrow \frac{1}{\mathbf{u}}(\mathbf{H} - \mathbf{E}_0 + \mathbf{U}_p).$$

(4)

We can also make similar energy translation, $\tilde{E} \to \tilde{E}' = E_{total} = \tilde{E} + E_0$, for a particle with rest mass in a force-free space (without potential energy field, $U_p = 0$) and apply it to wave equations from (4.10-1), as follows,

matter-waves equations in a free space, with rest mass involvement

$$\left[\overline{\Psi}' \to \overline{\Psi},\ \mathbf{m}^{*\prime} \to \mathbf{m}^{*},\ \tilde{\mathbf{E}}' = \mathbf{m}^{*\prime}\mathbf{u}'\mathbf{v}' = \tilde{\mathbf{E}} + \mathbf{E}_0 = \mathbf{E}_{total},\ \mathbf{U}_p = \mathbf{0}\right] \Rightarrow$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathbf{E}_{total}\overline{\Psi} = \mathbf{0},$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = -(\tilde{\mathbf{E}} + \mathbf{E}_0)\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} = \frac{\hbar^2}{\tilde{\mathbf{E}} + \mathbf{E}_0}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi}, \qquad (4.10\text{-}2)$$

$$(\frac{\mathbf{E}_{total}}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0},\ \frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0},$$

$$\Delta\overline{\Psi} - \frac{1}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{E}_0}{\hbar\mathbf{u}}\right)^2\overline{\Psi} + \frac{1}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{E}_0}{\hbar\mathbf{u}^2}\cdot\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \frac{1}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}.$$

or, again, the same equation can be presented with an operators' form:

$$\left(H = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta \ (=) \ \text{Hamiltonian}\right) \Rightarrow \left(H\overline{\Psi} = E_{total}\ \overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} = -j\hbar u\nabla\overline{\Psi}\right)$$

$$\Rightarrow E_{total} \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} \Leftrightarrow -j\hbar u\nabla\overline{\Psi}\ , \qquad (4.11\text{-}2)$$

$$\tilde{p}_i \Leftrightarrow -j\hbar\nabla - \frac{E_0}{u} \Leftrightarrow \frac{1}{u}(j\hbar\frac{\partial}{\partial t} - E_0) \Leftrightarrow \frac{1}{u}(H - E_0).$$

(5)

We can again exercise similar energy translation, $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathbf{E}_{total} + \mathbf{U}_p = \tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p$, for a particle with rest mass in a space with the positive potential energy field, $\mathbf{U}_p \geq \mathbf{0}$) and apply it to wave equations from (4.10-2),

matter-waves equations in a positive potential energy field, with rest mass involvement

$$\left[\overline{\Psi}' \to \overline{\Psi},\ \mathbf{m}^{*\prime} \to \mathbf{m}^*,\ \tilde{\mathbf{E}}' = \mathbf{m}^{*\prime}\mathbf{u}'\mathbf{v}' = \tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p = \mathbf{E}_{total} + \mathbf{U}_p,\ \mathbf{U}_p \geq \mathbf{0}\right] \Rightarrow$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\mathbf{E}_{total} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0},$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = -(\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p)\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} = \frac{\hbar^2}{\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi},$$

$$(\frac{\mathbf{E}_{total} + \mathbf{U}_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}\ ,\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0},$$

$$\Delta\overline{\Psi} - \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p}{\hbar\mathbf{u}}\right)^2\overline{\Psi} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p}{\hbar\mathbf{u}^2}\cdot\frac{\partial\overline{\Psi}}{\partial \mathbf{t}} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial \mathbf{t}^2} = \mathbf{0}. \qquad (4.10\text{-}3)$$

or, again, the same equation can be presented with operators' form:

$$\left(H = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta \ (=) \ \text{Hamiltonian}\right) \Rightarrow \left(H\overline{\Psi} = (E_{total} + U_p)\overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} = -j\hbar u\nabla\overline{\Psi}\right)$$

$$\Rightarrow (E_{total} + U_p) \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} \Leftrightarrow -j\hbar u\nabla\overline{\Psi}\ , \qquad (4.11\text{-}3)$$

$$\tilde{p}_i \Leftrightarrow -j\hbar\nabla - \frac{E_0}{u} \Leftrightarrow \frac{1}{u}(j\hbar\frac{\partial}{\partial t} - E_0) \Leftrightarrow \frac{1}{u}(H - E_0).$$

(6)

We can again make another energy translation, $\tilde{\mathbf{E}} \to \tilde{\mathbf{E}}' = \mathbf{E}_{total} - \mathbf{E}_0 = \tilde{\mathbf{E}} + \mathbf{U}_p$, for a particle with rest mass in a space with (a positive) potential energy field, $\mathbf{U}_p \neq \mathbf{0}$) and apply it to wave equations from (4.10-3),

matter-waves equations in a positive potential energy space, without rest mass involvement

$$\left[\overline{\Psi}' \to \overline{\Psi},\ \mathbf{m}^{*\prime} \to \mathbf{m}^{*},\ \tilde{\mathbf{E}}' = \mathbf{m}^{*}\mathbf{u}'\mathbf{v}' = \tilde{\mathbf{E}} + \mathbf{U}_p, \mathbf{U}_p \geq \mathbf{0}\right] \Rightarrow$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0},$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} = -(\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = -\mathbf{j}\hbar\frac{\partial\overline{\Psi}}{\partial\mathbf{t}} = \frac{\hbar^2}{\tilde{\mathbf{E}} + \mathbf{U}_p}\cdot\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{j}\hbar\mathbf{u}\nabla\overline{\Psi},$$

$$(\frac{\tilde{\mathbf{E}} + \mathbf{U}_p}{\hbar})^2\cdot\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{0}\ , \frac{\partial\overline{\Psi}}{\partial\mathbf{t}} + \mathbf{u}\nabla\overline{\Psi} = \mathbf{0},$$

$$\Delta\overline{\Psi} - \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \left(\frac{\tilde{\mathbf{E}} + \mathbf{U}_p}{\hbar\mathbf{u}}\right)^2\overline{\Psi} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{j}\frac{\tilde{\mathbf{E}} + \mathbf{U}_p}{\hbar\mathbf{u}^2}\cdot\frac{\partial\overline{\Psi}}{\partial\mathbf{t}} + \frac{\mathbf{1}}{\mathbf{u}^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial\mathbf{t}^2} = \mathbf{0}. \qquad (4.10\text{-}4)$$

or, again, the same equation can be presented with new operators' form:

$$\left(\mathrm{H} = -\frac{\hbar^2}{\mathrm{m}^*}(\frac{\mathrm{u}}{\mathrm{v}})\Delta\ (=)\ \text{Hamiltonian}\right) \Rightarrow \left(\mathrm{H}\overline{\Psi} = (\tilde{\mathrm{E}} + \mathrm{U}_p)\overline{\Psi} = \mathrm{j}\hbar\frac{\partial}{\partial \mathrm{t}}\overline{\Psi} = -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}\right)$$

$$\Rightarrow (\tilde{\mathrm{E}} + \mathrm{U}_p) \Leftrightarrow \mathrm{H} \Leftrightarrow \mathrm{j}\hbar\frac{\partial}{\partial \mathrm{t}} \Leftrightarrow -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}\ ,$$

$$\tilde{\mathrm{p}}_i \Leftrightarrow -\mathrm{j}\hbar\nabla - \frac{\mathrm{E}_0}{\mathrm{u}} \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{j}\hbar\frac{\partial}{\partial \mathrm{t}} - \mathrm{E}_0) \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{H} - \mathrm{E}_0).$$

(4.11-4)

Now we could summarize successive results regarding different energy-levels translations (what corresponds to different Lagrangians; -from (1) to 6)), as found in (4.9-2), (4.10), (4.10-1), (4.10-2), (4.10-3) and (4.10-4), and see that all of them produce mutually compatible and correct Schrödinger-like (or Dirac's) equations:

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} - \mathbf{U}_p)\overline{\Psi} = \mathbf{0}, (\mathbf{U}_p \neq \mathbf{0}),$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \tilde{\mathbf{E}}\overline{\Psi} = \mathbf{0},\ (\mathbf{U}_p = \mathbf{0}),$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0}, \qquad (4.10\text{-}5)$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0 - \mathbf{U}_p)\overline{\Psi} = \mathbf{0},$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + \mathbf{E}_{total}\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0)\overline{\Psi} = \mathbf{0},$$

$$\frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\tilde{\mathbf{E}} + \mathbf{E}_0 + \mathbf{U}_p)\overline{\Psi} = \frac{\hbar^2}{\mathbf{m}^*}(\frac{\mathbf{u}}{\mathbf{v}})\Delta\overline{\Psi} + (\mathbf{E}_{total} + \mathbf{U}_p)\overline{\Psi} = \mathbf{0}.$$

Equations like in (4.10-5) should also (and analogically) be applicable to electromagnetic waves vectors from (4.9-2.1), and this is the field where we could search for mutual, extended level of analogical predictions, based on Maxwell equations, conveniently applied to matter-waves equations.

It is worth to mention that based on (4.10-5) we could ask ourselves what Dirac's wave equation really predicted, and how much this prediction (matter-antimatter particles) was based (only) on Dirac's equation. In other words, Dirac's prediction was luckily successful (partially based on already known experimental results realized without and before Dirac's involvement), but not too much (and exclusively) related to what Dirac (and his faithful followers) afterward took as being the starting and strongest platform for making mentioned predictions. Much worse is that generations of scientists after Dirac, involved in similar fields of research, just continue, non-critically and non-creatively, repeating what Dirac (and his well-obeying followers) said in their early moments of brainstorming and divine inspiration.

We can also summarize previous results regarding different energy translations and corresponding operator forms of Schrödinger-like equations (found in (4.11), (4.11-1) - (4.11-4)), and see that all of them produce different operators,

$$\left\{\begin{array}{l} \tilde{\mathbf{E}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial t} + \mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar\mathrm{u}\nabla\ , \\ \tilde{\mathbf{p}}_\mathbf{i} \Leftrightarrow -\mathbf{j}\hbar\nabla \Leftrightarrow \frac{1}{\mathrm{u}}(\mathbf{j}\hbar\frac{\partial}{\partial t} + \mathbf{U_p}) \Leftrightarrow \frac{1}{\mathrm{u}}\mathrm{H} \end{array}\right\}, \qquad (4.11)$$

$$\left\{\begin{array}{l} \mathbf{E}_{\mathrm{total}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial t} + \mathrm{U_p} \Leftrightarrow -\mathbf{j}\hbar\mathrm{u}\nabla\ , \\ \tilde{\mathbf{p}}_\mathbf{i} \Leftrightarrow -\mathbf{j}\hbar\nabla - \frac{\mathrm{E_0} + \mathrm{U_p}}{\mathrm{u}} \Leftrightarrow \frac{1}{\mathrm{u}}(\mathbf{j}\hbar\frac{\partial}{\partial t} - \mathbf{E_0}) \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{H} - \mathrm{E_0} - \mathrm{U_p}) \end{array}\right\}, \qquad (4.11\text{-}1)$$

$$\left\{\begin{array}{l} \mathbf{E}_{\mathrm{total}} \Leftrightarrow \mathrm{H} \Leftrightarrow \mathbf{j}\hbar\frac{\partial}{\partial t} \Leftrightarrow -\mathbf{j}\hbar\mathrm{u}\nabla\ , \\ \tilde{\mathbf{p}}_\mathbf{i} \Leftrightarrow -\mathbf{j}\hbar\nabla - \frac{\mathrm{E_0} + \mathrm{U_p}}{\mathrm{u}} \Leftrightarrow \frac{1}{\mathrm{u}}(\mathbf{j}\hbar\frac{\partial}{\partial t} - \mathrm{E_0} - \mathbf{U_p}) \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{H} - \mathrm{E_0} - \mathbf{U_p}) \end{array}\right\}. \qquad (4.11\text{-}2)$$

$$\left(\mathrm{H} = -\frac{\hbar^2}{\mathrm{m}^*}(\frac{\mathrm{u}}{\mathrm{v}})\Delta\ (=)\ \mathrm{Hamiltonian}\right) \Rightarrow \left(\mathrm{H}\overline{\Psi} = (\mathrm{E_{total}} + \mathrm{U_p})\overline{\Psi} = \mathrm{j}\hbar\frac{\partial}{\partial t}\overline{\Psi} = -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}\right)$$
$$\Rightarrow (\mathrm{E_{total}} + \mathrm{U_p}) \Leftrightarrow \mathrm{H} \Leftrightarrow \mathrm{j}\hbar\frac{\partial}{\partial t} \Leftrightarrow -\mathrm{j}\hbar\mathrm{u}\nabla\overline{\Psi}\ , \qquad (4.11\text{-}3)$$
$$\tilde{\mathrm{p}}_\mathrm{i} \Leftrightarrow -\mathrm{j}\hbar\nabla - \frac{\mathrm{E_0}}{\mathrm{u}} \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{j}\hbar\frac{\partial}{\partial t} - \mathrm{E_0}) \Leftrightarrow \frac{1}{\mathrm{u}}(\mathrm{H} - \mathrm{E_0}).$$

$$\left(H = -\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta \ (=) \ \text{Hamiltonian} \right) \Rightarrow \left(H\overline{\Psi} = (\tilde{E} + U_p)\overline{\Psi} = j\hbar\frac{\partial}{\partial t}\overline{\Psi} = -j\hbar u\nabla\overline{\Psi} \right)$$

$$\Rightarrow (\tilde{E} + U_p) \Leftrightarrow H \Leftrightarrow j\hbar\frac{\partial}{\partial t} \Leftrightarrow -j\hbar u\nabla\overline{\Psi} \ , \tag{4.11-4}$$

$$\tilde{p}_i \Leftrightarrow -j\hbar\nabla - \frac{E_0}{u} \Leftrightarrow \frac{1}{u}(j\hbar\frac{\partial}{\partial t} - E_0) \Leftrightarrow \frac{1}{u}(H - E_0).$$

The most important conclusion from (4.10-3) and (4.11), (4.11-1) and (4.11-2) is that some of the considerably basic elements and step-stones of modern Quantum Mechanics and particle-wave dualism concept could and should be revised, as proposed in this book. *Obviously, different forms of Schrödinger equations can be simply developed without knowing that Orthodox Quantum Theory ever existed, and without stochastic and probabilistic patchwork and assumptions. None of the equations from (4.10-5) gives us the (generally valid) right to treat the total energy, or total particle mass (including the rest energy and rest mass) as a 1:1 equivalent to a total particle matter-wave-energy.* Only motional or kinetic energy creates de Broglie matter waves. Also, from (4.11), (4.11-1) - (4.11-4) it looks that there are no common sense and universally applicable operators that could uniquely present the whole family of Schrödinger's equations, since every time when we create an "energy level shift or translation", operators also change their form or content. Later, it will be shown that more universally valid operators for all Schrödinger-like equations can be formulated, but in a bit different form than presently oversimplified and cemented in the Orthodox Quantum Mechanics; -see (4.22) - (4.28).

(7)

Anyway, on a certain unnecessarily complicated way, but with similar results to (4.10-3) - (4.10-5), the official Quantum Mechanics (regarding Schrödinger's equation), effectively generated results, as shown in (4.10-6). The particle energy has been treated, either as kinetic energy, as known in Classical Mechanics, or using the relativistic expression for the total particle energy (Dirac), including potential energy, in both cases, and by approximating **u/v** = ½**,** for **v<<c**; - see **[9]** and (4.10-3)),

$$-\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + \mathbf{U_p}\,\overline{\Psi} = \frac{-\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2} = j\hbar\frac{\partial\overline{\Psi}}{\partial t} = -j\hbar u\nabla\overline{\Psi} = \tilde{E}\,\overline{\Psi} ,$$

$$\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} - \mathbf{U_p})\,\overline{\Psi} = \left[\cong \frac{\hbar^2}{2m^*}\Delta\overline{\Psi} + (\tilde{E} - \mathbf{U_p})\,\overline{\Psi} \Big|_{v<<c} \right] = 0 ,$$

$$(\frac{\tilde{E}}{\hbar})^2 \cdot \overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0 \ , \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0, \tag{4.10-6}$$

$$-\frac{(\hbar u)^2}{\tilde{E}}\left[\Delta\overline{\Psi} - \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} \right] + U_p\overline{\Psi} = 0,$$

$$j\hbar\frac{\partial\overline{\Psi}}{\partial t} + \frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0, -j\hbar u\nabla\overline{\Psi} + \frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,$$

$$\tilde{E}\,\overline{\Psi} + \frac{\hbar^2}{\tilde{E}}\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0, \ v \cong 2u.$$

From (4.10-6) we can extract the following operator forms of classical Schrödinger equation (most of them found in today's Quantum Mechanics):

$$\begin{array}{l}\left(H=-\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta + U_p\ (=)\ \text{Hamiltonian}\right)\Rightarrow\left\{H\bar{\Psi}=j\hbar\frac{\partial}{\partial t}\bar{\Psi}=-j\hbar u\nabla\right\}, \\ \tilde{E}\Leftrightarrow H\Leftrightarrow j\hbar\frac{\partial}{\partial t}\Leftrightarrow -j\hbar u\nabla\ ,\ \tilde{p}_i\Leftrightarrow -j\hbar\nabla\Leftrightarrow j\frac{\hbar}{u}\frac{\partial}{\partial t}\Leftrightarrow\frac{1}{u}H, \frac{u}{v}\cong\frac{1}{2}.\end{array} \quad (4.11\text{-}3)$$

However, we should not be too much fascinated with such simple, elementary, and non-essential operators' formulations for expressing wave equations, and we should not replace the real physics with a trivial and unnecessary complicated (only sporadically applicable) mathematical theory, playing with limited-applicability operators.

The differences between equations starting from (4.10-5), and (4.10-6), (4.11), (4.11-1), (4.11-2), and (4.11-3), are sufficiently small, that it could be difficult to find is it really everything in order there. Such superficially small differences have been sufficient to produce many challenges and waving in the Physics of the 20th century, (especially when a stable rest mass or rest energy is taken as a part of matter-waves energy, what is a generally wrong concept). Now we are in the position to explicitly formulate the principal differences between the wave energy concepts applied in the historically known Schrödinger equation (4.10-6), and in generalized Schrödinger-like equations from (4.10-5), as developed in this book:

Classical (contemporary and presently still official) particle-wave duality concept (applicable to equations (4.10-6)) considers rest mass as a static, constant, self-standing entity or parameter,

$$\left[\begin{array}{l}\tilde{E}_{classical}=E=hf=\hbar\omega\equiv\begin{cases}E_{total-Classic}=\frac{p^2}{2m}+U_p=E_k+U_p\ , & \text{or}\\ E_{total-Relativistic}=\sqrt{c^2p^2+(E_0)^2}+U_p=E_k+E_0+U_p\end{cases}\\ *!?\ \text{pay attention where the problem really is: } \left(\left.\left(u\cong v/2\right)\right|_{v<<c},\left(uv=c^2\ ;\ v\le c\right)\Rightarrow\left(u\ge c; v<<c\right)\right)*!?\end{array}\right]$$

Particle-wave duality in this book (applicable in equations (4.10-5)) in certain cases considers rest mass as potentially variable, being an evolving part or product of a specific interaction,

$$\left[\begin{array}{l}\tilde{\mathbf{E}}_{this-book}=\tilde{\mathbf{E}}=\mathbf{hf}=\hbar\omega=\tilde{\mathbf{p}}\mathbf{u}=(\text{particle}-\text{wave energy})=\mathbf{E}_k=(\gamma-1)\mathbf{mc}^2=\\ =\begin{cases}\left.\mathbf{E}_{total-Classic}-\mathbf{U}_p=\frac{\mathbf{p}^2}{2\mathbf{m}}=\mathbf{pu}=\mathbf{E}_k=\frac{\mathbf{mv}^2}{2}\right|_{v<<c}\ , & \text{or}\\ \mathbf{E}_{total-Relativistic}-\mathbf{E}_0-\mathbf{U}_p=\sqrt{\mathbf{c}^2\mathbf{p}^2+(\mathbf{E}_0)^2}-\mathbf{E}_0=\mathbf{E}_k=\mathbf{pu}=\frac{\mathbf{pv}}{1+\sqrt{1-\mathbf{v}^2/\mathbf{c}^2}}\end{cases}\\ \mathbf{d}\tilde{\mathbf{E}}=\mathbf{hdf}=\mathbf{v}\mathbf{d}\tilde{\mathbf{p}}=\mathbf{d}(\tilde{\mathbf{p}}\mathbf{u})=-\mathbf{dE}_k=-\mathbf{vdp}=-\mathbf{d}(\mathbf{pu}),\mathbf{u}=\lambda\mathbf{f}=\frac{\mathbf{v}}{1+\sqrt{1-\mathbf{v}^2/\mathbf{c}^2}}\\ \mathbf{v}=\mathbf{u}-\lambda(\mathbf{du}/\mathbf{d}\lambda)=-\lambda^2(\mathbf{df}/\mathbf{d}\lambda),\ (\mathbf{v}/\mathbf{u})=1+\sqrt{1-\mathbf{v}^2/\mathbf{c}^2}\ \Rightarrow\ 0\le 2\mathbf{u}\le\sqrt{\mathbf{uv}}\le\mathbf{v}\le\mathbf{c}.\end{array}\right] \quad (4.10\text{-}5)$$

Finally, after correlating different concepts, sufficiently acceptable and operational mathematical modeling was established in the form of today's Orthodox Quantum Mechanics, see **[9]**). We could say that now we have well-operating (and somewhat artificial) mathematical theory (*which mathematically united all common areas and similarities (between particles and waves presentations), and bridged, or compensated all differences mentioned above, in (4.10-5), even without noticing them from the same point of view, as elaborated in this book*). Sporadically, in the absence of enough conceptual and factual argumentation, in many books regarding modern Quantum Mechanics, we can find too optimistic and too affirmative statements about brilliant results of the contemporary Quantum theory (like creating preventive and advanced warnings against all possible critical voices). Mentioned statements are often based on pointing to correct experimental and theoretical background, saying that countless number of predictions and applications (or explanations based on using Schrödinger's equation in the frames of Orthodox Quantum Mechanics) have been justifying involved ad-hock-postulated, mathematical steps, or equations and models that are not fully and systematically developed starting from some basic grounds. *This way, indirectly and deductively, Orthodox Quantum Mechanics established its strongest supporting arguments and step-stones, which are also supporting systematic and much more naturally developed equations in this book. Orthodox Quantum Mechanics is mostly addressing already known experimental results and databases, and conveniently connecting all involved participants, making an impression that it can always explain and predict obtained results. Anyway, in most of cases, mentioned supportive, positive experimental results are primarily and independently found almost by chance, without significant theoretical guiding within Quantum theory, and later being alternatively and successfully explained as glorious consequences of the probabilistic Quantum theory. After many of such patching, fitting, and modeling steps, a big number of already obvious and trivial predictions can really be made, including predictions that are anyway based on something that is repeating experimentally, and naturally respecting, known, deterministic conservation laws of Physics, regardless of other Quantum Theory, mathematical options.*

The bottom-line reality regarding Schrödinger equation is that it was initially formulated by analogical generalization and simple hybridization of Classical Wave Equation, which is related to standing waves oscillations of a string. *In fact, the same, classical wave equation (being certain analogical form of d'Alembert equation) has been known in different fields of Classical Mechanics, Fluid Mechanics, Acoustics and Maxwell Electromagnetic Theory, long before Schrödinger. Quantum Theory effectively and luckily "renamed and analogically applied" such ordinary wave equations to micro-world matter-waves phenomenology in Physics (where an ordinary displacement or amplitude of a string in a state of standing waves oscillating,* $\Psi(x,t) \rightarrow \bar{\Psi}(x,t)$*, is getting an associated meaning of certain generally valid non-dimensional, and probabilistic wave function …).* ***Since our universe is anyway naturally united, without taking care of our ignorance regarding certain uniting field theory, natural wave or matter-wave phenomena should have the same nature, or mutually analogical basic mathematical skeleton.*** *We have seen that to develop Schrödinger equation it was enough to use the Complex Analytic Signal model, and to hybridize Classical Wave Equation with the* ***PWDC*** *facts, as formulated in this book in Chapters 4.1 and 10. (but this is not an explanation, we usually find in Quantum Theory books).* The next important situation is to clarify the meaning of the complex Analytic Signal wave function (4.9), favored here as the best model for all matter-waves (and all signals), that is not equivalent to the usual complex wave functions used in Quantum Mechanics. *In fact, we could also use only a real wave function,* $\Psi(x,t)$*, to represent de Broglie matter waves*. Because of number of mathematical conveniences, we transform a real wave function into its complex replacement $\Psi(x,t) \to \overline{\Psi}(x,t)$, known in mathematics as an Analytic Signal, where we can easily find and represent all signal characteristics (such as amplitude, phase, and frequency, both in time and frequency domains) that are equivalent (or similar) to any simple harmonic (sinusoidal) function $\Psi = a \cdot \sin\omega t$, as for instance (see chapter 4.0):

$$\overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)} = \frac{1}{\pi^2}\iint_{(0,+\infty)} A(\omega,k)e^{-j(\omega t - kx + \Phi(\omega,k))}dkd\omega\,,$$

$$\hat{\Psi}(x,t) = H\left[\Psi(x,t)\right], a(x,t) = \sqrt{\Psi^2 + \hat{\Psi}^2}\,,$$

$$\varphi(x,t) = \text{arctang}\left[\hat{\Psi}(x,t)/\Psi(x,t)\right], \omega(t) = 2\pi f(t) = \partial\varphi/\partial t\,,\ \omega(x) = 2\pi f(x) = \partial\varphi/\partial x\,, \qquad (4.10\text{-}6)$$

$$U(\omega,k) = U_c(\omega,k) - j\,U_s(\omega,k) = \iint_{(-\infty,+\infty)} \overline{\Psi}(x,t)\, e^{j(\omega t - kx)}dtdx = A(\omega,k)e^{-j\Phi(\omega,k)}\,.$$

Of course, more general wave function always has two coupled wave elements propagating in mutually opposite directions (or inwards and outwards), but mathematical processing, resulting with differential wave equations, and Analytic Signal structure will stay similar or the same as in (4.9-1) - (4.10-6),

$$\begin{aligned}\Psi(r,t) \to \overline{\Psi}(r,t) &= a(r,t)\cdot e^{j(kr-\omega t)} + b(r,t)\cdot e^{j(kr+\omega t)} = \\ &= a(r,t)\cdot e^{jk(r-ut)} + b(r,t)\cdot e^{jk(r+ut)} = \left|\overline{\Psi}(r,t)\right| e^{j\varphi(r,t)} = \\ &= \overline{\Psi}^{(+)}(r-ut) + \overline{\Psi}^{(-)}(r+ut).\end{aligned}$$

To fully understand all advantages of Analytic Signal forms, it is useful to refer to relevant chapters addressing Signal Analysis (see **[7]** and **[8]**). Here we can briefly say that Analytic Signal model (besides many other advantages) produces explicit forms of immediate and time-evolving amplitude, phase, and frequency functions of any signal (or arbitrary wave function), both in time and frequency domains (even in a joint time-frequency domain). This is not possible to have when using Orthodox Quantum Mechanics complex (and probabilistic) wave function (since phase functions have no use, or meaning in today's Quantum Mechanics, because "probability philosophy" takes care only about stochastic amplitudes distributions and resulting mean effects of certain process). *After operating with complex wave functions in the form of Analytic Signals, the final result can easily be transformed (back) into real wave function,* $\overline{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)} \to \Psi(x,t) = a(x,t)\cos\varphi(x,t)$*, as the solution relevant for wave functions and equations (being dimensionally the square root of relevant active power, and giving information regarding field distribution of certain matter-wave phenomena).* Contrary to Analytic Signal wave function (which could be either harmonic or an arbitrary function), in the contemporary Quantum Mechanics, the wave function is very much artificially formulated (from a very beginning and later; -see **[9]**) as a simple complex and harmonic function (and not as an Analytic Signal function). Later, results can be again transformed into real functions, finding their absolute values (also using specific complex operators for every specific case. In many aspects, comparing, (**a**) -typical Fourier signal analysis (applied on wave functions), (**b**) -Quantum Mechanics operations with wave functions, and (**c**) -Analytic Signal wave functions, we can notice many (mathematical) similarities between them. One should have a passionate and profound attention to discover number of small details, to extract all finesses, differences, and advantages of the Analytic Signal model (see **[7]** and **[8]**). *When we come back to physics, we can distinguish two different, but mutually not-contradictory understandings of the wave function.*

a) In the Orthodox Quantum Theory, we say that probability (distribution) of finding the particle (including its rest mass and its total energy constituents) in a certain space-element is presented by the square of (the Schrödinger's) wave function. This is accepted axiomatically and by consensus of involved authors of Quantum theory.

b) And here (in this book), regarding the same moving particle, for its Analytic Signal Wave Function, we consider the square of this wave function to be an active power (of the involved matter-wave phenomena). Here, for a moving particle, we address spatial-temporal distribution of de Broglie, or Matter-Wave field that has amplitude, phase, group and phase velocity, power, and energy... ($P = \Psi^2 = d\tilde{E}/dt = S$*), (but not taking into an account the particle's rest mass, or rest energy). Of course, later, eventually we can also apply different averaging, smoothing, and statistical mathematical practices, and make comparisons with Orthodox Quantum Theory predictions.*

*[♣ **COMMENTS & FREE-THINKING CORNER:** There are more mathematical possibilities (here only briefly mentioned) to be exploited in relation to Analytic Signal and wave equations, as for instance:*

1° Schrödinger equation (in this book) is created taking into account the complex analytic signal, wave function $\overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)}$*, and we can also formulate similar equations dependent only on* $\Psi(x,t)$ *or* $\hat{\Psi}(x,t)$*.*

2° The next interesting possibility would be to separate (or develop) wave equations dependent only on amplitude, $a(\mathbf{x},\mathbf{t})$ *and/or phase function,* $\boldsymbol{\varphi}(\mathbf{x},\mathbf{t})$ *to present all aspects of phase and group velocities, wave power and and associated energy transfer.*

3° Since the wave function Ψ *and active power* $\mathbf{P}$ *are closely related we can also formulate all wave equations to be dependent only on active power function (*$P(t) = \Psi^2(t) = d\tilde{E}/dt$*). It is just a matter of intellectual rigidity and inertia that we are used to treating wave functions as* $\Psi(t)$*, or in classical mechanics as certain amplitude function. Any wave motion is a form of energy transfer, or certain energy state with wave-motion attributes, or simply power in motion. Much more general case would be to establish (or transform) all wave equations to be related only to involved power* $P(t)$*, or that we start getting familiar with a new wave function which is only a power function. Creating new differential equations where only power function is used will be relatively easy, as for instance,*

$$\Psi^2(t) = P(t) \Rightarrow \frac{\partial P}{\partial t} = 2\Psi\frac{\partial \Psi}{\partial t} \Rightarrow \left\{ \begin{array}{l} \frac{\partial \Psi}{\partial t} = \frac{1}{2P^{0.5}}\frac{\partial P}{\partial t} \\ \frac{\partial^2 \Psi}{\partial t^2} = \frac{1}{2P^{0.5}}\frac{\partial^2 P}{\partial t^2} - \frac{1}{4P^{1.5}}(\frac{\partial P}{\partial t})^2 \\ .... \end{array} \right\}. \quad (4.10\text{-}7)$$

Later, by replacing $\frac{\partial \Psi}{\partial t}$ *and* $\frac{\partial^2 \Psi}{\partial t^2}$ *(in all already known wave equations) with new members where only power* $P(t)$ *is figuring, we will get new set of (relatively simple) equivalent wave equations, without the need to use* $\Psi(t)$*. This is a small mathematical step, but consequences related to physics (and to the understanding of wave motions) would be significant.*

4° The Analytic Signal modeling of the wave function can easily be installed in the framework of the Fourier Integral Transform, which is based on summation of simple harmonic functions such as cosωt,

$$\Psi(t) = a(t)\cos\phi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})e^{j2\pi ft}df = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\bar{H}\left[\cos 2\pi ft\right]\right\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi}) = A(\frac{\omega}{2\pi})e^{j\Phi(\frac{\omega}{2\pi})} = \int_{-\infty}^{\infty}\Psi(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty}\Psi(t)\left\{\bar{H}^*\left[\cos 2\pi ft\right]\right\}dt = F\left[\Psi(t)\right], \quad \omega = 2\pi \mathbf{f}, \quad (4.10\text{-}8)$$

where the meaning of symbols is:

F *(=) Direct Fourier transform,*

F^{-1} *(=) Inverse Fourier transform,*

$\bar{H} = 1 + jH$ *(=) Complex Hilbert transform,* $j^2 = -1$,

$\bar{H}^* = 1 - jH$ *(=) Conjugate complex Hilbert transform.*

$$\begin{array}{l} \bar{H}\left[\cos\omega t\right] = e^{j\omega t}, \quad H\left[\cos\omega t\right] = \sin\omega t, \\ \bar{H}^*\left[\cos\omega t\right] = e^{-j\omega t}, \quad H\left[\sin\omega t\right] = -\cos\omega t, \\ e^{\pm j\omega t} = (1 \pm jH)\left[\cos\omega t\right], \\ \bar{H}\left[\Psi(t)\right] = \bar{\Psi}(t) = \Psi(t) + jH\left[\Psi(t)\right] = \Psi(t) + j\hat{\Psi}(t), \\ \bar{H}^*\left[\Psi(t)\right] = \bar{\Psi}^*(t) = \Psi(t) - jH\left[\Psi(t)\right] = \Psi(t) - j\hat{\Psi}(t). \end{array} \quad (4.10\text{-}9)$$

The further generalization of the Fourier integral transformation can be realized by convenient replacement of its simple harmonic functions basis cosωt by some other (compatible) signal basis $\alpha(\omega,t)$. Now, the general wave function (in the generalized framework of Fourier transform) can be represented as,

$$\Psi(t) = \int_{-\infty}^{\infty} U(\frac{\omega}{2\pi})\left\{\bar{H}\left[\alpha(\omega,t)\right]\right\}df = F^{-1}\left[U(\frac{\omega}{2\pi})\right],$$

$$U(\frac{\omega}{2\pi}) = \int_{-\infty}^{\infty}\Psi(t)\left\{\bar{H}^*\left[\alpha(\omega,t)\right]\right\}dt = F\left[\Psi(t)\right]. \quad (4.10\text{-}10)$$

Let us imagine that $\alpha(t)$ presents the finite-energy elementary signal that carries the energy of a single energy quantum. Now Planck's wave energy expression should correspond to,

$$\int_{[t]}[\alpha(t)]^2\,dt = \frac{1}{2\pi}\int_{[0,\infty]}[B(\omega)]^2\,d\omega = h\,\bar{f} = hf,\quad f=\frac{\omega}{2\pi}\ ,\ h = const.. \tag{4.10-11}$$

5° Another possibility is to treat the wave function (4.9) as the composition of two (mutually coupled) waves, propagating in mutually opposed directions,

$$\begin{aligned}
&\overline{\Psi}(x,t) = \Psi^{+}(x,t) + j\hat{\Psi}^{+}(x,t) + \Psi^{-}(x,t) + j\hat{\Psi}^{-}(x,t) = \\
&= [\Psi^{+}(x,t) + \Psi^{-}(x,t)] + j[\hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t)] = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)} = \\
&= \frac{1}{(2\pi)^2}\iint_{(-\infty,+\infty)} U^{+}(\omega,k)e^{-j(\omega t-kx)}dkd\omega + \frac{1}{(2\pi)^2}\iint_{(-\infty,+\infty)} U^{-}(\omega,k)e^{-j(\omega t+kx)}dkd\omega = \\
&= \frac{1}{\pi^2}\iint_{(0,+\infty)} U^{+}(\omega,k)e^{-j(\omega t-kx)}dkd\omega + \frac{1}{\pi^2}\iint_{(0,+\infty)} U^{-}(\omega,k)e^{-j(\omega t+kx)}dkd\omega = \\
&= \frac{1}{\pi^2}\iint_{(0,+\infty)} A^{+}(\omega,k)e^{-j(\omega t-kx+\Phi(\omega,k))}dkd\omega + \frac{1}{\pi^2}\iint_{(0,+\infty)} A^{-}(\omega,k)e^{-j(\omega t+kx+\Phi(\omega,k))}dkd\omega,\ j^2=-1, \\
&U^{+/-}(\omega,k) = U_c^{+/-}(\omega,k) - j\,U_s^{+/-}(\omega,k) = A^{+/-}(\omega,k)e^{-j\Phi(\omega,k)}\ , \\
&a(x,t) = \sqrt{[\Psi^{+}(x,t) + \Psi^{-}(x,t)]^2 + [\hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t)]^2} = \sqrt{[\Psi(x,t)]^2 + [\hat{\Psi}(x,t)]^2}, \\
&\Psi(x,t) = \Psi^{+}(x,t) + \Psi^{-}(x,t),\ \hat{\Psi}(x,t) = \hat{\Psi}^{+}(x,t) + \hat{\Psi}^{-}(x,t), \\
&\tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = \int_{0}^{\infty}\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2 d\omega, \\
&\frac{1}{2}\left\{\Psi^2(t) + \hat{\Psi}^2(t)\right\} = \frac{dE}{dt} = \frac{dE_k}{dt} = \frac{d\tilde{E}}{dt} = \sum_{(i)}\frac{dE_i}{dt} = \sum_{(i)}\frac{dE_k}{dt} = \sum_{(i)}\frac{d\tilde{E}_i}{dt} = S(t).
\end{aligned} \tag{4.10-12}$$

From the signal energy (4.10-12), it is almost obvious that Ψ and its Hilbert couple $\hat{\Psi}$ are in a mutual coupling and synchronously communicating. This property presents ***Quantum Entanglement*** *because wave parts (or elements) of involved signals traveling in mutually opposite directions ($\Psi^{+}(x,t), \Psi^{-}(x,t), \hat{\Psi}^{+}(x,t), \hat{\Psi}^{-}(x,t)$) are equally and coincidently present on both sides or directions of waves propagation (in the expressions for signal amplitude, power, and energy). We know for sure that Entanglement is an experimentally verifiable reality of specifically coupled particle-wave states like photons, electrons, protons, atoms, and small atomic clouds (see more about entanglement in [56]). Obviously, explanations based on probabilities and stochastics' grounds of Entanglement are on some way useful, but still not good enough explanation of the entanglement nature. For instance, there are publications, like in [36], with innovative thinking, proposals, arguments, and indications that all orbital and spinning moments (in micro and macro physics) could be mutually connected (universally, intrinsically, holistically, and synchronously, allover our Universe), with mentioned entanglement couplings, or to have immediate communications (without delays).*

There are also numerous interesting possibilities to continue representing and analyzing wave functions using different signal basis functions (to be analyzed some other time).

For instance, originally, the Schrödinger wave equation was analogically established based on the standing-waves equation of an oscillating string. The string is a line that has limited (fixed) length between two points of fixation. Then we imagine how we transform such oscillating string into a similar, but circular (closed line) oscillating string to get self-closed and self-stabilized resonant states.

One of the options for realizing such transformation is that we find the imaging or functional (bidirectional) transformation (or mapping) that is converting the line segment to the circle, and vice versa under the following initial conditions:

$$\left\{\begin{array}{l} y-y_1=k(x-x_1)\Leftrightarrow y=kx+b, \\ k=\dfrac{y-y_1}{x-x_1}=\dfrac{y_2-y_1}{x_2-x_1}, \\ b=y_1-kx_1=\dfrac{x_2y_1-x_1y_2}{x_2-x_1}, \\ x\in[x_1,x_2]\,,y\in[y_1,y_2]. \end{array}\right\} \begin{array}{c}\leftrightarrow \\ \text{(bidirectional} \\ \text{transformation)}\end{array} \left\{\begin{array}{l} (x-x_0)^2+(y-y_0)^2=r^2, \\ 2\pi r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}, \\ x_0=\dfrac{1}{2}(x_1+x_2),\ y_0=\dfrac{1}{2}(y_1+y_2), \\ r=r(x_0,y_0). \end{array}\right\}. \quad (4.10\text{-}13)$$

The same process can be additionally upgraded with the objective to find another functional transformation that is converting the circle into a sphere:

$$\left\{\begin{array}{l} (x-x_0)^2+(y-y_0)^2=r^2, \\ 2\pi r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}, \\ x_0=\dfrac{1}{2}(x_1+x_2),\ y_0=\dfrac{1}{2}(y_1+y_2), \\ r=r(x_0,y_0). \end{array}\right\} \leftrightarrow \left\{\begin{array}{l} (x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2, \\ r=r(x_0,y_0,z_0). \end{array}\right\}. \quad (4.10\text{-}14)$$

Of course, for the sake of higher generality, we could extend the same task towards Line to Ellipse and Ellipse to Ellipsoid transformations (in both directions).

Topology or geometry platform in the background of here conceptualized links between linear, planar, and spatial objects, could belong to Möbius transformations and Riemann Sphere concepts, where such transformations are something obvious and natural.

After establishing mentioned imaging (or transformations) we will be able to make the equivalency and correspondences (in both directions) between standing waves equations on a line segment (or string) and standing waves on a corresponding circle and ellipse, as well as to extend such standing waves on a Sphere and Ellipsoid. ♣]

4.3.1. Wave Energy and Mean Values

Up to present we only demonstrated applicability and compatibility of Planck's wave energy relation, $\tilde{E}_i = hf_i$, with Energy and Momentum conservation laws, as well as with de Broglie matter wavelength, $\lambda_i = h/p_i$, without precisely showing what really makes those relations correct. The answer on the simplest question how and why only one specific frequency (multiplied by Planck's constant) can represent the motional energy of a quantized wave group (or what means that frequency) should be found. The first intuitive and logical starting point is the mean frequency, $\bar{\mathbf{f}}$, of the corresponding narrow-banded matter-wave group calculated concerning its energy (since wave group, or wave packet, or de Broglie matter wave is composed of an infinity of elementary simple-harmonic waves, covering certain, not too wide, frequency interval: $0 \leq f_{min.} \leq \overline{f} \leq f_{max.} <<< \infty$).

$$\tilde{E} = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{+\infty} \hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty} \left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \tilde{p}u = h\overline{f} . \quad (4.13)$$

As known from Signal and Spectrum analysis, time, and frequency domains of a wave function (4.9) can be connected using Parseval's theorem, thus the energy of de Broglie matter wave can also be presented as,

$$\tilde{E} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left|U(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_0^{\infty} [A(\omega)]^2 d\omega = \tilde{p}u = h\overline{f} . \quad (4.14)$$

Now (by definition) we can find the mean frequency as,

$$\overline{f} = \frac{\frac{1}{\pi}\int_0^{\infty} f \cdot [A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty} [A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} f \cdot [A(\omega)]^2 d\omega}{\tilde{E}}, \quad (4.15)$$

and replace it into the wave energy expression, (4.14),

$$\tilde{E} = \frac{1}{\pi}\int_0^{\infty} [A(\omega)]^2 d\omega = h\overline{f} = h\frac{\frac{1}{\pi}\int_0^{\infty} f \cdot [A(\omega)]^2 d\omega}{\tilde{E}} \Rightarrow \tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f \cdot [A(\omega)]^2 d\omega . \quad (4.16)$$

Using one of the most general formulas valid for all definite integrals (and applying it to (4.16)), we can prove that the wave energy (of a de Broglie wave group) should be equal to the product between Planck's constant and mean frequency of the wave group in question, as follows:

$$\left\{ \begin{array}{l} \int_a^b f(x) \cdot g(x)dx = f(c)\int_a^b g(x)dx , a < c < b, \ g(x) \geq 0, \\ f(x) \text{ and } g(x) - \text{continuous in } [a \leq x \leq b], \\ f(x) = f , g(x) = [A(\omega)]^2 > 0 , x = \omega \in (0, \infty) \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left[\tilde{E}^2 = \frac{h}{\pi}\int_0^{\infty} f \cdot [A(\omega)]^2 d\omega = \frac{h}{\pi} \cdot \overline{f} \cdot \int_0^{\infty} [A(\omega)]^2 d\omega = h\overline{f} \cdot \tilde{E} \right] \Rightarrow \tilde{E} = h\overline{f} . \quad (4.17)$$

If Planck's energy of a quantized wave group, effectively and essentially deals with mean frequency (of that narrow-band wave group), the same should be valid for de Broglie wavelength, as well as for its phase and group velocities. All the mentioned items should be treated as mean values describing the motion of an effective center of inertia, or center of gravity of that wave group). Consequently, we do not need to mark them, as it was the case with a mean frequency (since we know that all of them should anyway be mean values $\overline{f} = f$, $\overline{\lambda} = \lambda$, $\overline{u} = u$, $\overline{v} = v$). *The next important conclusion is that* *__all forms of wave equations, (4.10-3), effectively deal only with mean values (regarding energy, momentum, frequency, wavelength, velocities...).__* For instance, group and phase velocity can also be found as mean values in the following way:

$$\overline{v}_g = \overline{v} = \frac{\frac{1}{\pi}\int_0^{\infty} v\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{d\tilde{E}}{d\tilde{p}} \Rightarrow \tag{4.17-2}$$

$$\Rightarrow \tilde{E}\frac{d\tilde{E}}{d\tilde{p}} = \frac{1}{\pi}\int_0^{\infty}\frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega = \frac{1}{\pi}\int_0^{\infty}\frac{d\tilde{E}}{d\tilde{p}}\cdot[A(\omega)]^2 d\omega \Rightarrow \tilde{E} = \frac{\frac{1}{\pi}\int_0^{\infty}\frac{d\tilde{E}}{d\tilde{p}}\cdot[A(\omega)]^2 d\omega}{\frac{d\tilde{E}}{d\tilde{p}}} = h\overline{f},$$

$$\overline{v}_f = \overline{u} = \frac{\frac{1}{\pi}\int_0^{\infty} u\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\frac{1}{\pi}\int_0^{\infty}[A(\omega)]^2 d\omega} = \frac{\frac{1}{\pi}\int_0^{\infty} \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega}{\tilde{E}} = \frac{\tilde{E}}{\tilde{p}} \Rightarrow \tag{4.17-1}$$

$$\Rightarrow \tilde{E}\frac{\tilde{E}}{\tilde{p}} = \frac{1}{\pi}\int_0^{\infty}\frac{\omega}{k}\cdot[A(\omega)]^2 d\omega,\ \tilde{E}\frac{\omega}{k} = \frac{1}{\pi}\int_0^{\infty}\frac{\omega}{k}\cdot[A(\omega)]^2 d\omega \Rightarrow \tilde{E} = h\overline{f}.$$

[♣ ***__COMMENTS & FREE-THINKING CORNER:__*** *We could additionally test the Planck's radiation law, regarding photon energy* $\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega$*. It is well proven that a photon has the wave energy equal to the product between Planck's constant h and photon's frequency f. Photon is a wave phenomenon, and it should be presentable using certain time-domain wave function* $\psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$*, expressed in the form of an Analytic signal. Since the Analytic signal gives the chance to extract immediate signal amplitude* $\mathbf{a}(\mathbf{t})$*, phase* $\varphi(\mathbf{t})$*, and frequency* $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial\mathbf{t}} = 2\pi\mathbf{f}(\mathbf{t})$*, let us explore the meaning of Planck's energy when: instead of constant photon frequency (valid for a single photon), we take the mean wave-group frequency,* $\Omega = \langle\omega(\mathbf{t})\rangle$*, of the time-domain wave function* $\psi(\mathbf{t})$*.*

$$\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega$$

$$\mathbf{E} = \int_{[T]} \psi^2(\mathbf{t})\mathbf{dt} = \int_{[T]} [\mathbf{a(t)}\cos\varphi(\mathbf{t})]^2\mathbf{dt} = \int_{[T]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}\ (=)\ \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\langle\omega\rangle$$

$$\psi(\mathbf{t}) = \mathbf{a(t)}\cos\varphi(\mathbf{t}) = -\mathbf{H}[\hat{\psi}(\mathbf{t})],\ \hat{\psi}(\mathbf{t}) = \mathbf{a(t)}\sin\varphi(\mathbf{t}) = \mathbf{H}[\psi(\mathbf{t})]$$

$$\mathbf{a(t)} = \sqrt{\psi^2(\mathbf{t}) + \hat{\psi}^2(\mathbf{t})}$$

$$\varphi(\mathbf{t}) = \mathbf{arctg}\frac{\hat{\psi}(\mathbf{t})}{\psi(\mathbf{t})}$$

$$\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial\mathbf{t}} = 2\pi\mathbf{f(t)} \Rightarrow \quad \Omega = \langle\omega(\mathbf{t})\rangle = \frac{\mathbf{1}}{\mathbf{T}}\int_{[T]} \omega(\mathbf{t})\mathbf{dt}\ (=)\ \frac{\frac{\mathbf{1}}{\mathbf{T}}\int_{[T]} \mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}{\int_{[T]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}}$$

$$\mathbf{E} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega \Rightarrow \mathbf{E} = \frac{\mathbf{h}}{2\pi}\Omega \Rightarrow$$

$$\mathbf{E} = \frac{\mathbf{h}}{2\pi}\frac{\mathbf{1}}{\mathbf{T}}\frac{\int_{[T]} \mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}{\int_{[T]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}} (=) \frac{\mathbf{h}}{2\pi}\frac{\mathbf{1}}{\mathbf{T}}\int_{[T]} \omega(\mathbf{t})\mathbf{dt} = \int_{[T]} \mathbf{a}^2(\mathbf{t})\mathbf{dt} \Rightarrow$$

$$\Rightarrow \frac{\left[\int_{[T]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}\right]^2}{\frac{\mathbf{1}}{\mathbf{T}}\int_{[T]} \mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}} = \frac{\mathbf{h}}{2\pi} = \mathbf{Const.}, \quad \mathbf{or} \quad \frac{\int_{[T]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}}{\frac{\mathbf{1}}{\mathbf{T}}\int_{[T]} \omega(\mathbf{t})\mathbf{dt}} = \frac{\mathbf{h}}{2\pi} = \mathbf{Const.}$$

Depending on how we calculate the mean frequency, we should be able to prove above given relations, or to find a family of wave functions which describe photon in a time domain, or in any case, to see how universal Planck's energy law could evolve regarding the energy of arbitrary wave functions. ♣]

4.3.2. Inertial and Reaction Forces

Now we can formulate the starting platform for establishing unified force/fields theory (valid for dynamic interactions between different motional states of particles, quasi-particles, and similar matter-waves objects). De Broglie waves address waiving or oscillating field structure inside and around interacting objects, this way naturally manifesting specific forces and field formations between them. If de Broglie, or matter-wave function $\Psi(r,t)$ represents an active power, we should be able to determine a force and field distribution in its space of definition, based on the idea first introduced in the second chapter of this book (see equations from (2.5.1) to (2.9) and later (4.3)), regarding the unity of linear and rotational motions:

$$\begin{aligned} &dE = c^2 d(\gamma m) = mc^2 d\gamma = vdp + \omega dL = \Psi^2 dt, \quad p = \gamma mv, \quad \gamma = (1 - v^2/c^2)^{-0.5} \\ &\Psi^2 = \frac{dE}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = \frac{dx}{dt}\frac{dp}{dt} + \frac{d\alpha}{dt}\frac{dL}{dt} = vF + \omega\tau = mc^2\frac{d\gamma}{dt} = \text{Power} = [W] \end{aligned} \tag{2.5.1}$$

The objective behind (2.5.1) is to establish the concept that any particle (regarding its total energy content) is composed of linear motion components $\mathbf{vdp}$ and spinning motion components, $\omega d\mathbf{L}$, and that both of them are intrinsically involved in creating total particle energy ($dE = mc^2 d\gamma = vdp + \omega d\mathbf{L}$). The relation between linear and rotational motion components (related to the same moving particle) could be "analogically visualized" as a relation between current and voltage components inside of a closed, oscillating R-L-C circuit (where electric and magnetic field mutually communicate by currents and voltages through inductive and capacitive elements). The wave function behind such modeling should have two wave components (one from linear motion and the other from spinning),

$$\begin{aligned} &\Psi^2 = \frac{d\tilde{E}}{dt} = v\frac{dp}{dt} + \omega\frac{dL}{dt} = vF + \omega\tau = -v\frac{d\tilde{p}}{dt} - \omega\frac{d\tilde{L}}{dt} = -v\tilde{F} - \omega\tilde{\tau} = \\ &= vF_{linear} + \omega F_{angular} = \Psi^2_{linear} + \Psi^2_{angular} = mc^2\frac{d\gamma}{dt} \\ &\left.\begin{cases} \Psi^2_{linear} = vF_{linear} \Leftrightarrow F_{linear} = \frac{dp}{dt} = -\frac{d\tilde{p}}{dt} = \frac{1}{v}\Psi^2_{linear} \\ \Psi^2_{angular} = \omega F_{angular} \Leftrightarrow F_{angular} = \tau = \frac{dL}{dt} = -\frac{d\tilde{L}}{dt} = \frac{1}{\omega}\Psi^2_{angular} \end{cases}\right\} \Rightarrow F_q = \frac{1}{V_q}\Psi^2_q \\ &v = \frac{d\tilde{E}}{d\tilde{p}} = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = -\frac{\lambda^2}{2\pi}\frac{d\omega}{d\lambda}, \; u = \frac{\tilde{E}}{\tilde{p}} = \lambda f = \lambda\frac{\omega}{2\pi}, \; \tilde{E} = hf, \end{aligned} \tag{4.18}$$

where $\mathbf{v}$ and $\mathbf{u}$ are group and phase speed of de Broglie wave packet. Practically, for any specific situation, it would be necessary to find the wave function, $\Psi(r,t)$, by solving relevant Schrödinger's equation (4.10) - (4.12), and then to determine the linear and radial force-field structure (see also some attempts to generalize force laws formulated by (4.30), (4.31) and (4.37)).

Nuclear (and other known or still undiscovered) forces, because of specific force field distributions, should also be presentable like (4.18) or (4.37).

In addition to (4.18), we could evoke the ideas of the philosopher R. Boskovic, who was the first to explain the (need of) existence of certain alternating force field (with attractive and repulsive forces) in a narrow zone of impact between two objects (see [6]).

If the above-mentioned concept proves logical, or at least introducing fruitful brainstorming ideas, the road to a unified field theory will be largely paved (see also (4.26), (4.29), (4.30), (4.31), (5.15) and (5.16) to get an idea how the particle-wave duality, universal force law, and Schrödinger-like equations can additionally be upgraded).

[♣ COMMENTS & FREE-THINKING CORNER:

*R. Boskovic (with an obviously south-Slavic origin, and with typical Serbian family name (coming from his Serbian father), including Italian genetic ingredients from his mother) published the **universal-natural-force** concept and about three centuries before Quantum Theory was established. This happened in a historical period when Balkan, dominantly Slavic and Serbian population, still did not know who and how much of them are or will be future Croats, Bosnians, Slovenians, Macedonians etc. Much later, shady, and ideologically colored forces, that are still alive and active in the same area, always using the same agendum, united and helped to some parts of religiously, geographically, nationally, and geopolitically confused Balkan population to learn and recognize who they really are (creating new national brands and flavours of an originally and dominantly Serbian and old Slavic population). This was realized by transforming and magnifying their geographical or regional identities, and negligible differences regarding many of existing or newly composed political, national, ideological and cultural identities, except of their common Serbian language that was not easy to be transformed or replaced in a short time. It is also interesting to know that such Balkan, forcefully and imaginatively created, quasi multicultural and multinational mixture flourished in the center of the decaying Roman Empire, between Byzantine, Roman-catholic, Slavic, and European, mutually conflicting interests and peripetia-like activities.*

More than fifteen of Roman Imperators, from the period when such big Roman conglomeration was in power, geographically overlapping with areas populated with Serbs and other old Slavic variants, were generated directly from Balkan (or dominantly Serbian) population (since Croats, Bosnians etc. were discovered, or only being like software modified, much later). Similar trend and practice of domesticating and absorbing Slavic population is also known in Ottoman Empire, where many of military and high-ranking administration leaders were Serbs. Even more indicative is that in Ottoman Empire (during five centuries) one of two official languages was the Serbian language (not at all Croatian or Bosnian). With such common language it was possible to communicate and administrate dominant part of Ottoman Empire including all Balkan varieties (nobody in that time even slightly mentioned Croatian and other of recently composed languages, which are presently surfacing in the same area as some weak, often comical, and intentional modifications of the core Serbian language). Tragicomically, we could also find that original (common) Christianity (as an official and state-supported religion, initially without Orthodox and Catholic brands), was established by Serbs, in Serbia (within the old Roman Empire, in the town Naisus, in the middle of Serbia, by the Roman Imperator Constantin, who was born from a Serbian mother). Later, and sarcastically (in our modern and contemporary time), newly created catholic brand of Christianity, gradually and forcefully indoctrinated one part of confused and ignorant Serbs to become Croats, ideologically pushing them "by Vatican god's supported activities of genocide and extermination" against other Serbs that were not susceptible and willing to pass to Catholic religion brand.

Apart from religion, military, and administrative leadership examples, many of more recent, Balkan-originated science flowers (mostly Serbs) are also known worldwide, such as Nikola Tesla, Milutin Milankovic, Mihajlo Pupin, Mileva Maric (A. Einstein's first wife), Pavle Savic… Modern world industry, technology, and science would never exist in present form, without inventions of Nikola Tesla. Anyway, number of family members holding the family name Tesla where exterminated by Croats during the Second World War. Of course, it is good and honest to say, that this happened only by actions of some too much indoctrinated Croats belonging to extreme fascist and catholic brands, to avoid accusing the whole Croat's population that is anyway, still and overwhelmingly supporting mentioned historical transitions and actions, and still being warmly backed by Vatican and familiar forces from western world to continue acting against their Serbian origins.

The author needs to express excuses to the Asian, Arab, and Indian population for not mentioning similar examples (of real scientific creations) radiating from their part of history that have been borrowed or copied by European and Western-world innovators (since Europeans forgot to mention the real authors and references, they exploited and copied). Of course, these are irrelevant comments (at least for Wave-Particle Duality), but useful to challenge our history knowledge, objectivity, and independence regarding multidisciplinary and multicultural items.

It is especially important to notice that (based on (4.2) and (4.18)) the particle-wave duality concept is extended to any situation where motional energy (regardless of its origin) is involved. This is indicating that motional energy is intrinsically coupled with de Broglie matter waves, creating inertia-like reaction forces in the form of waves (where reaction and inertial waving forces can have gravitational, mechanical, rotational, electromagnetic... or some other nature). The most significant relation, found in (4.2), leading to such conclusion is the connection between wave and kinetic energy, $d\tilde{E} = h \cdot df = vd\tilde{p} = h \cdot vdf_s = d(\tilde{p}u) = dE_k = vdp = d(pu)$*, which is in agreement with de Broglie wavelength* $\lambda = h / \tilde{p}$*, and with relations expressing group and phase velocities* $v = d\omega / dk = dE / dp$, $u = \omega / k = \tilde{E} / \tilde{p}$, $v = u - \lambda du / d\lambda$*. Using a similar conclusion process (based on analogies), we could also develop the following particle-wave duality relations* $\omega d\tilde{L} = -\omega dL$, $ud\tilde{q}_{electric} = -udq_{electric}$, $id\tilde{q}_{magnetic} = -\mathbf{idq_{magnetic}}$ *, (where* $\tilde{q}_{electric}$, $q_{electric}$ *and* $\tilde{q}_{magnetic}$, $q_{magnetic}$ *are electric and magnetic charges). From the point of view of cosmology and "expanding universe" we should also conclude that what we see as an expansion (driving force, or positive energy: characterized by* dE_k, dp, dL,... *) should always be balanced with something what we, most probably, do not see (named here as reaction energy or inertial forces,* $-\mathbf{d\tilde{E}}, -\mathbf{d\tilde{p}}, -\mathbf{d\tilde{L}}, ...$ *). It looks that here we are dealing with forces described by Newton action-reaction, or inertia law (extended to a rotation, electromagnetism etc.). In (4.2) we also find that after integration of differential relations connecting particle and wave aspect of motion, we can get very useful finite differences relations, such as:* $\Delta E_k = \Delta\tilde{E}$, $\Delta p = -\Delta\tilde{p}$, $\Delta L = -\Delta\tilde{L}$, $\Delta \mathbf{q} = -\Delta\tilde{\mathbf{q}}$, $\Delta\dot{p} = -\Delta\dot{\tilde{p}}$, $\Delta\dot{L} = -\Delta\dot{\tilde{L}}$, $\Delta\dot{q} = -\Delta\dot{\tilde{q}}$*,...(see the end of chapter 5 of this paper, where advantages of using Central Differences are presented). Modern physics also addresses the same problem, non-systematically and often implicitly (when analyzing origins of Inertia), using the terminology of "transient mass fluctuations, electromagnetic radiation reaction forces, induction laws, inertial reaction forces" etc.*

This book initially started by establishing the wide analogy platform between different physical entities. For instance (see T.1.1 to T.1.6 and T.3.1 to T.3.3), there is a multilevel analogy between speed and voltage (or potential difference), and between electric charge and linear and angular momentum. This could be directly and imaginatively applied (in fact tested) in the equation (4.2) that connects group and phase speed $v_g = v, v_p = u$ *(producing that group velocity,* $v_g = v$*, can be analog to "**group voltage**" (=)* u_g*, or* $v_g = v \Leftrightarrow u_g$*, and phase velocity,* $v_p = u$*, analog to "**phase voltage**" (=)* u_p*", or* $v_p = u \Leftrightarrow u_p$*,*

$$\left\{ \begin{array}{l} (v = v_{group} = v_g \Leftrightarrow u_g),\ (u = v_{phase} = v_p \Leftrightarrow u_p), (q \Leftrightarrow p, \tilde{q} \Leftrightarrow \tilde{p}), \\ v_g = v_p - \lambda \dfrac{dv_p}{d\lambda} = v_p + \tilde{p}\dfrac{dv_p}{d\tilde{p}} = \dfrac{d\tilde{E}}{d\tilde{p}},\ v_p = \dfrac{\tilde{E}}{\tilde{p}} \end{array} \right\} \Rightarrow$$

$$\Rightarrow \quad u_g = u_p - \lambda' \frac{du_p}{d\lambda'} = u_p + \tilde{q}\frac{du_p}{d\tilde{q}} = \frac{d\tilde{E}}{d\tilde{q}}, u_p = \frac{\tilde{E}}{\tilde{q}}. \qquad (4.19)$$

*Using analogies (see T.1.8, Generic Symmetries and Analogies of the Laws of Physics) in the similar way as in (4.19) we can (hypothetically) extend "**group phase**" concept to magnetic field and rotational motion (connecting "**magnetic group-voltage**" and "**magnetic phase-voltage**" as well as "**angular group**" and "**angular phase**" velocity) etc., (see also (3.4), (3.5), T.3.1 - T.3.3 for understanding the meaning of magnetic voltages and currents). It is another question (still not answered here) to prove if the "**group phase**" concept (4.19), based only on analogies, is universally applicable, and how to integrate it into positive knowledge of today's relevant physics (see also (5.15) and (5.16)). One of the possibilities that should be analyzed is to connect "**group phase**" concept to Retarded Lorentz Potentials (known in Maxwell Electromagnetic Theory). ♣]*

4.3.3. Probability and Conservation Laws

Obviously, it looks possible to develop many wave equations (known in Quantum Mechanics and wider, such as (4.10) - (4.10-3) ... (4.12)), without associating the probability nature to wave functions. It is also clear that we shall not sacrifice the flexibility and positive results of Quantum Mechanics by conveniently modifying (or correcting) the meaning of non-dimensional wave function, from probability to natural and dimensional "active power" wave function. By simple normalization, we can always make "active power" wave function, (4.9), to be dimensionless, and use it almost in the same way as it is currently used in Quantum Mechanics, but with much more of rich and naturally suitable mathematics related to Signal Analysis and Analytic Signal concepts. Wave functions concept presented in this book is widely extended to everything what presents temporal-spatial signals, voltages, currents, velocities, forces, and different moments (see Chapter 4.0, and (4.0.82)). In electric sciences we already have very well developed and particularly useful concept of complex Phasors. There, we know how currents and voltages are being phase-shifted or transformed by different electric loads (or impedances), we operate with different meanings of power (such as active, reactive, apparent, complex, reflected...), we define different electric impedances, and we can analyze coherence or correlation relations between different signals or Phasors. We also know a lot about different and natural (multilevel) analogies between electrical and mechanical entities, and we should be able to transfer all such concepts and methodology of electric Phasors to a world of Mechanics and Physics (see the first chapter of this book). This way we will enormously enrich understanding of interactions in mechanics and Wave-Particle Duality field, but we are still not taking such steps, and we are diving into insufficiently clear and very incomplete and limited probability-related strategies. Of course, Probability and Statistics will always have their significant place in all mass data, and associated processes descriptions, quantifications, and modeling, in any field of science and life.

Here we modeled (or explained) de Broglie matter waves as the phenomena that (besides other aspects) unites linear motion with rotation, (see (4.3) - (4.4), Fig.4.1 and chapter 10). The generalized Schrödinger equation, (4.10) - (4.12), (4.10-3), presents an additional (deductive) support to the hypothesis of this book stating that fields and forces related to rotation, associated to linear motions, should be much better accounted when explaining Wave-Particle duality.

To understand the profound background regarding how Orthodox Quantum Mechanics successfully established a wave function as a probability function, let us start from natural conservation laws as the strongest platform in physics. Most conservation laws in physics are covered by the law of energy conservation, and several laws of different vector-values conservations (such as moments conservation laws). Since a total energy input, $\mathbf{E}_{\mathbf{inp.}}$, of one isolated system (passing through certain transformation) will always stay equal to its total energy output, $\mathbf{E}_{\mathbf{outp.}}$, and since the same is valid for all other important vector properties of that system, $\vec{\mathbf{A}}_{\mathbf{inp.}} = \vec{\mathbf{A}}_{\mathbf{outp.}}$ (its moments, for instance), we can easily formulate the following generalized and normalized forms of such conservation laws:

$$\left\{\begin{array}{l} E_{tot.}=E_{inp.}=\sum_{(i)}E_i=E_{outp.}=\sum_{(j)}E_j \\ \vec{A}_{tot.}=\vec{A}_{inp.}=\sum_{(i)}\vec{A}_i=\vec{A}_{outp.}=\sum_{(j)}\vec{A}_j=\left|\vec{A}_{tot.}\right|\cdot\vec{a}_0=A\cdot\vec{a}_0 \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} 1=\frac{E_{inp.}}{E_{tot.}}=\sum_{(i)}\frac{E_i}{E_{tot.}}=\frac{E_{outp.}}{E_{tot.}}=\sum_{(j)}\frac{E_j}{E_{tot.}} \\ 1=\frac{\left|\vec{A}_{inp.}\right|}{A}=\left|\sum_{(i)}\frac{\vec{A}_i}{A}\right|=\frac{\left|\vec{A}_{outp.}\right|}{A}=\left|\sum_{(j)}\frac{\vec{A}_j}{A}\right|=\left|\vec{a}_0\right| \end{array}\right\}\Leftrightarrow\left\{\begin{array}{l} 1=\sum_{(i)}e_i=\sum_{(j)}e_j \\ 1=\left|\sum_{(i)}\vec{a}_i\right|=\left|\sum_{(j)}\vec{a}_j\right| \end{array}\right\}. \quad (4.20)$$

For situations where average motional or wave energy $\langle\tilde{E}\rangle$ of certain system is stable (meaning conserved) we can also apply similar normalization like in (4.20), for instance using motional or wave energy relations from (4.9-0) and (4.13),

$$\left\{\begin{array}{l} \langle\tilde{E}\rangle=\int_{-\infty}^{+\infty}P(t)dt=\int_{-\infty}^{+\infty}\Psi^2(t)dt=\int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt=\frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2dt=\frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt=\int_{-\infty}^{+\infty}d\tilde{E} \\ d\tilde{E}=hdf=dE_k=c^2d(\gamma m)=vdp=d(pu) \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} \frac{\tilde{E}}{\langle\tilde{E}\rangle}=\frac{1}{\langle\tilde{E}\rangle}\int_{-\infty}^{+\infty}P(t)dt=\frac{1}{\langle\tilde{E}\rangle}\int_{-\infty}^{+\infty}d\tilde{E}= \\ =\frac{1}{\langle\tilde{E}\rangle}\int_{-\infty}^{+\infty}\Psi^2(t)dt=\frac{1}{\langle\tilde{E}\rangle}\int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt=\frac{1}{2\langle\tilde{E}\rangle}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2dt=\frac{1}{2\langle\tilde{E}\rangle}\int_{-\infty}^{+\infty}a^2(t)dt=1 \end{array}\right\}\Leftrightarrow \quad (4.20\text{-}1)$$

$$\left\{\frac{\tilde{E}}{\langle\tilde{E}\rangle}=\int_{-\infty}^{+\infty}\frac{P(t)}{\langle\tilde{E}\rangle}dt=\int_{-\infty}^{+\infty}\frac{d\tilde{E}}{\langle\tilde{E}\rangle}=\int_{-\infty}^{+\infty}\frac{\Psi^2(t)}{\langle\tilde{E}\rangle}dt=\int_{-\infty}^{+\infty}\frac{\hat{\Psi}^2(t)}{\langle\tilde{E}\rangle}dt=\int_{-\infty}^{+\infty}\frac{\left|\overline{\Psi}(t)\right|^2}{2\langle\tilde{E}\rangle}dt=\int_{-\infty}^{+\infty}\frac{a^2(t)}{2\langle\tilde{E}\rangle}dt=1\right\}\Leftrightarrow$$

$$\Leftrightarrow\int_{-\infty}^{+\infty}de=1,\ de=\frac{P(t)}{\langle\tilde{E}\rangle}dt=\frac{d\tilde{E}}{\langle\tilde{E}\rangle}=\frac{\Psi^2(t)}{\langle\tilde{E}\rangle}dt=\frac{\hat{\Psi}^2(t)}{\langle\tilde{E}\rangle}dt=\frac{\left|\overline{\Psi}(t)\right|^2}{2\langle\tilde{E}\rangle}dt=\frac{a^2(t)}{2\langle\tilde{E}\rangle}dt$$

Based on (4.20-1) we can conclude that the square of a normalized wave function $\psi^2(t)$ (originating from power function) will be,

$$\psi^2(t)=\frac{P(t)}{\langle\tilde{E}\rangle}=\frac{\Psi^2(t)}{\langle\tilde{E}\rangle}=\frac{\hat{\Psi}^2(t)}{\langle\tilde{E}\rangle}=\frac{\left|\overline{\Psi}(t)\right|^2}{2\langle\tilde{E}\rangle}=\frac{a^2(t)}{2\langle\tilde{E}\rangle}\Rightarrow\int_{-\infty}^{+\infty}\psi^2(t)dt=1. \quad (4.20\text{-}2)$$

Similar and more tangible superposition and normalization should also be applicable on a 4-vector of momentum-energy, from Relativity theory, as for instance (just to give some brainstorming ideas to start with),

$$\left\{\begin{array}{l} P_4(\vec{P},\frac{E}{c}) = (\sum_{(i)}\vec{p}_i, \frac{\sum_{(i)} E_i}{c}) \Rightarrow \left(\sum_{(i)}\vec{p}_i\right)^2 - \frac{\left(\sum_{(i)} E_i\right)^2}{c^2} = -\frac{-\left(\sum_{(i)} E_{0i}\right)^2}{c^2} \\ E = E_{tot.} = E_{inp.} = \sum_{(i)} E_i = E_{outp.} = \sum_{(j)} E_j \\ \vec{P} = \vec{P}_{tot.} = \vec{P}_{inp.} = \sum_{(i)}\vec{p}_i = \vec{P}_{outp.} = \sum_{(j)}\vec{p}_j = \left|\vec{P}_{tot.}\right| \cdot \vec{a}_0 = P \cdot \vec{a}_0,\ P = \left|\vec{P}\right| \\ (\vec{p}_i, \frac{E_i}{c}) \Rightarrow \vec{p}_i^2 - \frac{E_i^2}{c^2} = -\frac{E_{0i}^2}{c^2},\ \vec{P}^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} E^2 = E_0^2 + c^2\vec{P}^2 \\ E_i^2 = E_{0i}^2 + c^2\vec{p}_i^2 \\ \left(\sum_{(i)} E_i\right)^2 = \left(\sum_{(i)} E_{0i}\right)^2 + c^2\left(\sum_{(i)}\vec{p}_i\right)^2 \\ \sum_{(i)} E_i^2 = \sum_{(i)} E_{0i}^2 + c^2\sum_{(i)}\vec{p}_i^2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\left[\left(\sum_{(i)} E_i\right)^2 - \sum_{(i)} E_i^2\right] = \left[\left(\sum_{(i)} E_{0i}\right)^2 - \sum_{(i)} E_{0i}^2\right] + c^2\left[\left(\sum_{(i)}\vec{p}_i\right)^2 - \sum_{(i)}\vec{p}_i^2\right]\right\} \Rightarrow \sum_{(i\neq j)} E_i E_j = \sum_{(i\neq j)} E_{0i}E_{0j} + c^2\sum_{(i\neq j)}\vec{p}_i\vec{p}_j$$

$$\Rightarrow \left\{\begin{array}{l} \frac{\left(\sum_{(i)} E_i\right)^2}{E^2} = \frac{\left(\sum_{(i)} E_{0i}\right)^2}{E^2} + \frac{c^2}{E^2}\left(\sum_{(i)}\vec{p}_i\right)^2 = 1 \\ \frac{\sum_{(i)} E_i^2}{E^2} = \frac{\sum_{(i)} E_{0i}^2}{E^2} + \frac{c^2}{E^2}\sum_{(i)}\vec{p}_i^2 = 1...\ (?!) \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \left(\sum_{(i)}\frac{E_i}{E}\right)^2 = \left(\sum_{(i)}\frac{E_{0i}}{E}\right)^2 + \left(\sum_{(i)}\frac{c\vec{p}_i}{E}\right)^2 = 1 \\ \sum_{(i)}\frac{E_i^2}{E^2} = \sum_{(i)}\frac{E_i^2}{E^2} + \sum_{(i)}\frac{c^2\vec{p}_i^2}{E^2} = 1...\ (?!) \end{array}\right\}.$$

(4.20-3)

Or, if we exploit similar 4-vector relations (4.0.5-4) from chapter 4.0, we will be able to get a normalized sum of all energy-momentum members of certain system as,

$$\left\{\begin{array}{l} (p, \frac{E}{c}) \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} \Leftrightarrow E^2 = E_0^2 + c^2p^2 = E_0^2 + E_p^2 \Rightarrow E = \left|\overline{E}\right| = \gamma mc^2 \\ E = \left|\overline{E}\right| = \gamma mc^2, E_0 = mc^2, E_p = cp = \gamma mcv,\ \gamma = (1-\frac{v^2}{c^2})^{-0.5}, E_k = E - E_0 = (\gamma-1)E_0 = \frac{\gamma-1}{\gamma}E \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left(\begin{array}{l} \overline{E} = \sqrt{E_0^2 + E_p^2} \cdot e^{j\cdot arctg(\alpha)} = \left|\overline{E}\right| \cdot e^{j\cdot arctg(\alpha)} = E \cdot e^{j\cdot\theta} \\ \overline{E}_k = \frac{\gamma-1}{\gamma}\overline{E} = E_k \cdot e^{j\cdot arctg(\alpha)} = E_k \cdot e^{j\cdot\theta} = \tilde{E} \cdot e^{j\cdot\theta} \end{array}\right) \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \overline{E}_{tot.} = \sum_{(i)}\overline{E}_i = \sum_{(i)}\sqrt{E_{0i}^2 + E_{pi}^2} \cdot e^{j\cdot arctg(\alpha_i)} = \sqrt{\sum_{(i)}(E_{0i}^2 + E_{pi}^2)} \cdot e^{j\cdot\theta_\Sigma} = \left|\overline{E}_{tot.}\right| \cdot e^{j\cdot\theta_\Sigma} = E_{tot.} \cdot e^{j\cdot\theta_\Sigma}, \\ \left|\overline{E}_{tot.}\right|^2 = E_{tot.}^2 = \left|\sum_{(i)}\overline{E}_i\right|^2 = \sum_{(i)}(E_{0i}^2 + E_{pi}^2),\ \left|\sum_{(i)}\overline{E}_{ki}\right| = \sqrt{\sum_{(i)}\overline{E}_{ki}^2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{1 = \frac{E_{tot.}^2}{E_{tot.}^2} = \left|\sum_{(i)}\frac{\overline{E}_i}{E_{tot.}}\right|^2 = \sum_{(i)}(\frac{E_{0i}^2 + E_{pi}^2}{E_{tot.}^2}) = \sum_{(i)} e_i^2\right\}.$$

(4.20-4)

Founders of Quantum Mechanics succeeded to model the wave function, that can mimic, represent, or (analogically, and in-average, or approximately) replace and generalize equations of normalized energy and vectors conservation laws, (4.20), (4.20-1), (4.20-3), and (2.20-4) of a system in certain transformation, $\left(\sum_{(i)} e_i, \sum_{(j)} e_j\right) = \frac{E_{i,j}}{E_{tot.}}$, $\sum_{(i)} e_i^2 = \sum_{(j)} e_j^2 = 1$, $\left(\left|\sum_{(i)}\vec{a}_i\right|, \left|\sum_{(j)}\vec{a}_j\right|\right) = \left|\frac{\vec{A}_{i,j}}{A}\right|$, by convenient finite or infinite summations (for instance using Fourier Integral Transformation, and framework of Signal Analysis, Probability theory and Statistics). This practically replaces finite (or discrete) summation elements $\sum_{(i)} e_i^2, \sum_{(j)} e_j^2, \left|\sum_{(i)} \vec{a}_i\right|^2, \left|\sum_{(j)} \vec{a}_j\right|^2$ by an equivalent integral summation form of certain wave function that could have a finite or infinite number of elements (which we can modify and fit to represent spectral and/or probability components of such wave function, such as $\int_{-\infty}^{+\infty} \psi^2(t)dt = 1$). *When we have sufficiently big number of elements of certain system (even mathematically constructed by applying certain abstractions, approximations, and generalizations, or integral transformations and Signal Analysis decomposition methods), this could be workable background to (conveniently and approximately), apply or mimic laws of Probability and Statistics to such system. Probability and Statistics can also be applied to any elementary signal-functions-basis of the same system (while respecting, and/or constructing necessary conditions for such mathematical fitting, merging and makeup), to get applicable and seemingly or sufficiently correct results.* Conveniently formulating or modeling the wave function as a probability-behaving function ($|\psi|^2$), Quantum Mechanics **"invented"**, accepted, or better to say postulated the rules of the following replacements (what is isomorphic, equivalent or analog to energy conservation, averaging and normalization formalism, as presented in (4.20) - (4.20-4)):

$$\left\{\sum_{(i)} e_i^2 = \sum_{(j)} e_j^2 = 1\right\} \Leftrightarrow \left\{\begin{array}{l} \int |\psi|^2 dq = 1 \, , dq = dx_1 \cdot dx_2 \cdot ... \cdot dx_n \\ \psi = \psi(x_1, x_2, ... x_n) = \sum_m \alpha_m \psi_m \, , \, \alpha_m = const. \end{array}\right\} \quad (4.21)$$

The understanding of matter-waves duality with associated energy components is supporting such approach, since interacting particles and waves are surrounded and coupled with many of energy-momentum entities, in forms of particles and waves. This is giving us a chance to extend such concepts to spatial and temporal distributions of relevant states. Probability and statistics skeleton were just a possible, fruitful, universally, and naturally applicable mathematical toolbox "to domesticate" such new visions of energy-momentum conservation (since the sum of all probabilities of certain system or an ensemble is always equal to 1, and this is very much like a sum of normalized energy members of the same system). Nobody can make a reliable and clear conceptual and mathematical difference (between *dimensional power-related* or *non-dimensional probabilistic-interpretations*, wave functions in cases with big (or infinite) number of particle-wave components, since anyway sum of probabilities corresponds (in average) to a sum of normalized energies of wave packets (involved in the same process). To get very safe (and almost always valid) theoretical approach, it was sufficient to maximally unify or merge basic signal analysis and probability theory concepts, especially by respecting different forms of Parseval's theorems and identity relations (both regarding discrete and infinite summations, or waves superposition, what is the case of the contemporary, orthodox Quantum theory). It is correct to say that such methodology could be successfully applied to any set or ensemble with big number of similar or identical participants (and not only to the Quantum theory world).

Another very strong support here is coming from the fact that temporal-spatial dependent wave functions or signals (including power functions, motional energy functions, currents, voltages, forces, velocities and moments), can be naturally and exactly decomposed on simple harmonic members, or on similar elementary wave groups like Gaussian-Gabor signals, and again synthetized based on discrete and integral forms of Fourier and Analytic Signals analysis (supported by "Kotelnikov-Shannon, Whittaker-Nyquist Sampling Theory". See more in [57, 58, and 59]). The fact (in deep and always existing relation with Wave-Particle Duality theory) is that when we decompose certain wave function on many of elementary waveforms, such wave elements are still real matter-waves, being experimentally detectable or verifiable signals (see more in chapter 10). Solutions of Classical or Schrödinger wave equations (whatever the wave function is) are sets of wave function components propagating in many of wave-pairs with mutually opposed directions (inwards and outwards, or on a similar way in all directions). This is giving chances to speculate (or imagine) that wave function could be everywhere (around) with certain probability. The fact is that a probability here is not the unique, dominant and an essential or ontological property of any wave function. Probability modelling of wavefunction is only very much useful, similar, and approximately equal, to mimic and/or replace power-related, dimensional wave function).

The remaining part of the process (in creating Orthodox Quantum Theory) was to establish and postulate all necessary mathematical framework as normalizations, transformations, and definitions (belonging to the content of modern Quantum Theory) that will satisfy, support and defend above-presented steps ((4.20), (4.20.1)-(4.20-4) and (4.21)). Even the statement that $|\psi|^2$ represents the probability distribution of a certain energy state could be considered only as an isomorphic mathematical equivalent, and behaving like a **probability function** because its intrinsic and hidden origin is anyway and dominantly related to satisfying basic conservation laws of physics, while being fully in agreement with Spectral and Signal Analysis, and Parseval's theorems (at least in average). *Since the probability of certain multi-component event also obeys the law of total probability conservation (sum over all single probabilities is equal to one), this can be considered as an equivalent formulation of (4.21). Another extremely important aspect of wave-function normalization in relation to satisfying conservation laws will be addressed later in the same chapter (see (4.34)). The rest of the modeling work (of modern Orthodox Quantum Theory) has been to make its wave function fully mathematically compatible (complying, or not being contradictory) to all other physics related laws, principles, and good practices, initially not considered (such as Hamilton-Lagrange-Euler mechanics etc.).* ***This way, the simple mathematical structure of several of conservation laws (known in Physics before Quantum Theory was established) was specifically unified, normalized, generalized, and replaced or hybridized by another isomorphic mathematical structure that is taken from Statistics, Probability and Signal Analysis, by conveniently merging all of them. Where merging was not very much evident, missing and somewhat unnatural, new rules and mathematical bridges have been postulater or invented, and "by consensus legalized", as the rules of, the "self-proclaimed and forever valid", and victorious, contemporary Quantum Theory.***

Significant contribution and mathematical framework in such legalizing, merging, and modeling process, Quantum Theory could also take from the following, Amalie Nether theorem (formulated 1905):

-For every continuous symmetry of the laws of physics, there must be a conservation law.
-For every conservation law, there should exist a continuous symmetry.

Consequently, the most logical (and naturally applicable), has been to construct or complete the missing positions of the Quantum Theory by completing already recognizable symmetries, knowing that this way we are also in agreement with all known or not well-known conservation laws that should be in the background of symmetries in question. Unfortunately, A. Nether theorem and consciousness that it can be used for here described purpose (which is a final modeling of Quantum Theory) has been taken very seriously much later, after it was first time published, but still, it has been useful to finalize, generalize and test the building of already established Orthodox Quantum Theory.

The next "*multi-level of conceptual confusion* and mystification" in the same field has been unintentionally and non-consciously introduced by people involved in popularization and simplified explanations of what modern Quantum Theory should mean from different aspects of Reality. Eventually a "*mass of uncritical and well-obeying, good-will followers took everything of such Quantum Theory as being the final, literal and eternal truth of divine messages and ultimate orders*". By the way, it is also important to say, that contemporary Quantum Theory really works very well, and it is honest and sympathetic what many of the founders and followers of such Quantum Theory are proudly saying, "***nobody understands why it works so well***". Obviously, there is really a need, and it is already a time to clarify and rectify this situation. As we can see from (4.20) and (4.20.1)-(4.20-4), the reality behind all of Quantum Theory foundation is very tangible, clear, stable and bottom-line simple. It is still respecting, on some ways modified and generalized, but unnecessarily complicated functional mapping, imaging and equivalent reformulation of basic conservation laws.

In fact, (the opinion of the author of this book is that) only correct mathematics belonging to Signal and Spectrum Analysis was largely sufficient to formulate majority of rules, theorems, definitions, and models belonging to modern Quantum Mechanics (of course when properly merged with **PWDC**, from (4.1) - (4.3), and with other conservation laws; -see more in Chapter 10.). Probability and Statistics have been, in addition, a complementary and very convenient modeling framework and skeleton structure that gave the final form to the book of Orthodox Quantum Mechanics because (4.21) looks like a summation of probabilities. All rules of Probability and Statistics should anyway be satisfied in all events where a big number of identical or similar items are involved, what is often the case in a micro cosmos of real matter, or physics-related systems (in too many of different domains). *Since all forms of wave equations, (4.10-3), operate only with mean values (regarding energy, momentum, frequency, wavelength, velocities...-see (4.17)), there is only a small theoretical and modeling step to be made to formulate or tune the complete Quantum Mechanics using the language and rules of Probability Theory* *and Statistics. Statistics and Probability approach is especially convenient modeling concept when we neglect immediate spatial-temporal phase information, neglect existence and intrinsic synchronizations and coupling (and entanglement effects) between Analytic Signal pairs* $\psi(t)$ and $\hat{\psi}(t)$ *, and when we forget that every wave equation describes at least two wave groups propagating in mutually opposite directions. It should also be clear that operating with mean values is useful, but not sufficient argument to say that Microphysics ontologically and intrinsically deals only with Probability and Statistics manageable items.*

One of the problems in (4.21) to be solved is in the fact that normalized energy members $\sum_{(i)} \mathbf{e_i} = \sum_{(j)} \mathbf{e_j} = \mathbf{1}$, $\sum_{(i)} e_i^2 = \sum_{(j)} e_j^2 = 1$ present summations of state of rest (static, or constant) energy members, $\mathbf{e_{oi}}$ and $\mathbf{e_{oj}}$, and their remaining motional energies, $\mathbf{e_{mi}}$ and $\mathbf{e_{mj}} \Rightarrow \sum_{(i)} \mathbf{e_i} = \sum_{(i)} (\mathbf{e_{oi}} + \mathbf{e_{mi}}) = \sum_{(j)} \mathbf{e_j} = \sum_{(j)} (\mathbf{e_{oj}} + \mathbf{e_{mj}}) = \mathbf{1}$.
Quantum Mechanics pretends to represent all such energy members (static = rest energy, and dynamic = motional energy members) in the same way (using wave functions, as wave groups or wave packets). ***In this book, we support the platform that proper wave function can exclusively represent only motional (dynamic) energy (or power) content and that states of rest and motional energy members should be separately treated.*** Also, after normalizing vector components, (4.21); $\left|\sum_{(i)} \vec{\mathbf{a}}_i\right| = \left|\sum_{(j)} \vec{\mathbf{a}}_j\right| = \mathbf{1}$, we additionally sacrifice their motional, or immediate, time-space evolving phase information (losing the chance to deal with the concept of active and reactive power and energy, and to analyze optimal active power transfer, like in electronics). To find a solution to such and similar problems, Quantum Mechanics simply introduced different (and mathematically manageable) artificial complex functions, normalization or renormalization rules, Feynman diagrams, Operators' algebra etc., without modifying the foundations of Orthodox Quantum Mechanics, making this situation more complicated and "conceptually fogy" (but still mathematically operational and applicable to Physics).

*[♣ **COMMENTS & FREE-THINKING CORNER:** The official history of modern Quantum Theory is not presented in the same way as here, neither its historical development was following mathematical processing presented with (4.20)-(4.21). The opinion of the author of this book is that the answer to the question why Quantum Theory has been so successful should be something very much obvious. This is just the consequence of correct merging of conservation laws of Physics with mathematical rules and models of Signal Analysis, Probability and Statistics, also effectively merged with effects of rotation complemented to linear motion, here formulated as **PWDC**. Probability framework associated to Quantum Mechanics is only a useful, neither particularly good, nor essential, nor wrong theoretical and mathematical modeling tool. Such stochastic approach is made fully compatible with the meaning of normalized and dimensionless wavefunction on a way to respect conservation laws of Physics (at least in average). Such methodology is relatively easy applicable whenever possible and logical (and, of course, we should agree that the nature of certain phenomena does not become exclusively probabilistic just because we use Statistics and Probability to make mathematical modeling of that phenomena. Maybe we will discover some of strange behaviors of de Broglie matter waves and light (or electromagnetic waves) what may be indirectly telling us that we are possibly dealing with something that has some hidden (still unknown, or not considered) parameters, or it has certain missing part of still not conceptualized reality. Alternatively, we maybe have oscillating and wave phenomena of some strange and for us still undetectable media. Anyway, sometimes it looks that we measure effects and results of particles and/or waves interactions within certain undetectable media, but it also could be that we do not see what is really oscillating, regarding de Broglie matter*

waves. We could also say (with big probability and confidence) that some revitalized ether, or carrier fluid should exist (where matter forms and motions from our detectable world are experiencing interactions, propagation and waving, such as light or electromagnetic waves propagation in an "empty space"), still without detecting such carrier fluid. Effectively, we should conclude that there is no empty space in our universe, but maybe we do not know what exactly exist in the space where we "see or detect nothing". Regardless of our inability to (directly) see and detect such strange media, or fluid, we can make mathematical modeling that is exactly predicting results of particle-wave interactions in it. From an observer's position, it could look to us that we are dealing with probabilities that something could happen. For instance, often waves from micro-world-physics really perform as predictable using Schrödinger's equation, confirming only that we have practical and sufficiently well operating mathematical tools and models for handling matter waves phenomenology, but at the same time, we still do not have a complete conceptual picture of the same reality. Being more imaginative, we could assume the existence of this (invisible and strange) ideal fluid-like media, where we can make (and mathematically and mentally visualize) wave propagation and perturbations, vortices, diffraction, and interference effects... Any particle moving inside such strange, or any other fluid will always create associated (de Broglie) waves around itself, like a stone thrown in a water.

The words or names like Quantum, Quant, Quantization, Quantum Theory, Quantum Mechanics… are so fashionable and popular since a long time (almost for one century and the same trend is still getting momentum). Many of us could be by intellectual inertia, emotionally, ideologically, or just linguistically in love, or sympathizing with such terms, thanks to long and almost hypnotizing or indoctrinating publicity of such mathematical elaborations in relation to Quantum theory. Terms like Quantum or Quant present certain fixed, constant (elementary, or minimal) package or unit of something ***that can be quantified, measured and counted with integers****, and it is nothing more than such bottom-line, packing and counting situation. There is nothing very much universal and significant in names related to "quantum", except the fact that certain (but not all) matter and energy-momentum states tend to create atomized units and elementary packets (including standing waves), where we can count them using integers. Quantizing is a synonym for counting (when using integers), and phenomenology when and where matter (in motion) is creating some elementary and countable units. We consider such situations, formations, and events as describing specific "quantum nature of matter". We should not forget that Quant or Quantum presents a synonym for fixed (or discretized) quantity of something what is countable and measurable, and nothing else.*

In this book, we have also seen that quantum mostly corresponds to one minimal, elementary, or resonant and/or standing wave formation of certain matter-wave, ***usually being equal to its half-wavelength domain*** *(or size). These (half-wavelength standing waves elements) are building blocks or quantifying units of matter and its energy-momentum states (of course, being naturally countable using integers). Such quantizing is the familiar property of all relatively stable material structures. Energy-momentum exchanges (or communications) between mentioned structures present (integers countable) exchanges in units of their elementary quantum states (meaning that really nothing more magic and symbolic, or illuminating is a part of Quantum nature of matter). Synonyms for bottom-line, ordinary, explicable events and elementary mathematics terms should not become foundations and excuses for creating some mystic world of strange and artificial Physics. Anyway, modern science and technology are often obliged to clarify, describe, justify, and explain many of new experiments and technological situations, and this way, certain still unclear or artificial, or unnatural theory would gradually evolve towards more natural structures.*

What is unfortunate and critical here is that any humans organized mainstream establishment is, usually (or often), suffering from a Ptolemy syndrome. We know that Ptolemy geocentric system was kept alive too long time (just because of religious and ideological dogmas), and Ptolemy (and his followers) explained planetary syszems concepts very well mathematically, showing (with mathematical backing) that our planet Earth is the center of universe… and many brilliant, open-minded thinkers were sacrificed to keep this theory alive as long as possible. We should avoid repeating such situations within modern Physics and Quantum Theory. ♣]

4.3.4. Matter Waves and Unified Field Theory

By simple algebraic transformations of different forms of Schrödinger equations (4.9-2), (4.10), (4.10-1) - (4.10-5), valid for different levels of energy translation, such as,

$$\left\{\begin{array}{l} \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0. \\ \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}-U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}-U_p}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0. \\ \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+E_0-U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+E_0-U_p}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0. \\ \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+E_0}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+E_0}{\hbar u^2}\cdot\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0. \\ \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+E_0+U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+E_0+U_p}{\hbar u^2}\cdot\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0. \\ \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{\tilde{E}+U_p}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{\tilde{E}+U_p}{\hbar u^2}\cdot\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0. \end{array}\right\}\Rightarrow \quad (4.22)$$

$$\Rightarrow \boxed{\begin{array}{l} \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{L}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0, \\ L\in\left[(\tilde{E}),\text{ or }(\tilde{E}-U_p),\text{ or }(\tilde{E}+E_0-U_p),\text{ or }(\tilde{E}+E_0),\text{ or }(\tilde{E}+E_0+U_p),\text{ or }(\tilde{E}+U_p)...\right] \end{array}},$$

we can easily obtain generalized (or universal) wave equation with different Lagrangian, or (for instance) a wave equation that has the same form as Classical or electromagnetic waves equation $\underline{\Delta\Psi=\Delta(\mathcal{E},\mathcal{H})=\frac{1}{c^2}\frac{\partial^2(\mathcal{E},\mathcal{H})}{\partial t^2}}$, {$(\mathcal{E},\mathcal{H})$ = (*electric and magnetic fields*)}.

From (4.22), especially from $\Delta\overline{\Psi}=\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} \Leftrightarrow \Delta(\mathcal{E},\mathcal{H})=\frac{1}{c^2}\frac{\partial^2(\mathcal{E},\mathcal{H})}{\partial t^2}$, *it is obvious that electromagnetic waves (in a free space,* $U_p = 0$*) are just one of measurable (and non-probabilistic) manifestations of de Broglie matter waves (when stable rest mass does not exist* $\Rightarrow \tilde{E}=E_{total}=E_k$*).* See also familiar elaborations (3.7-1) and (3.7-2) from the third chapter of this book.

There are also forms of (4.22) where rest mass should be involved, and where we can recognize other forms of matter waves. We also see that the famous Schrödinger equation is nothing else but just another analogically modified form of well-known (classical, D'Alembert) wave equation, valid in electromagnetic theory, mechanics, acoustics, fluid dynamics, simply formulated in the form of a Complex, Analytic Signal... Going backwards to some of the earlier chapters of this book, we shall be able to defend initial hypothetical statements (of this book) that New Theory of Gravitation could be constructed following analogy with Faraday-Maxwell Electromagnetic Theory (of course, first by upgrading both to become more generally valid and more compatible for unification).

The way to establish a Unified Field Theory (suggested in this paper) will go back to the presentation of analogies found at the beginning of this paper. To give an idea how to relate Classical and Schrödinger equations (4.22) to Gravitation (and to any other field), we should remember that the square of the wave function in this paper presents the active power function $\Psi^2(\mathbf{t},\mathbf{r}) = \mathbf{P}(\mathbf{t},\mathbf{r}) = \mathbf{d}\tilde{\mathbf{E}}/\mathbf{dt} = -\mathbf{dE_k}/\mathbf{dt}$. Since we are already used to express power functions as products between corresponding current and voltage (in electro technique), or force and velocity (in mechanics) etc. (*see the first chapter of this book dealing with analogies*), this will directly enable us to develop new forms of wave equations (valid for Gravitation and other fields), formally like Schrödinger and Classical wave equations (4.22), but dealing with velocities, forces, currents, voltages ..., as for instance,

$$\Psi^2(t,r) = P(t,r) = \frac{d\tilde{E}}{dt} (=) \text{Active Power } (=)$$
$$(=)\begin{Bmatrix} i(t)\cdot u(t) & (=) & [\text{Current}\cdot\text{Voltage}], \text{ or} \\ f(t)\cdot v(t) & (=) & [\text{Force}\cdot\text{Velocity}], \text{ or} \\ \tau(t)\cdot\omega(t) & (=) & [\text{Orb.-moment}\cdot\text{Angular velocity}], \text{or} \\ (\vec{E}\times\vec{H})\cdot\vec{S} & (=) & [\overline{\text{Pointyng Vector}}]\cdot\overline{\text{Surface}} \\ ----- & (=) & --------- \\ s_1(t)\cdot s_2(t) & (=) & [(\text{signal}-1)\cdot(\text{signal}-2)] \end{Bmatrix} \quad (4.23)$$

(see also (4.0.82), chapter 4.0).

*[♣ **COMMENTS & FREE-THINKING CORNER:** We can also profit from the well-developed methodology (in electronics) related to an optimal active power transfer, to real, imaginary, and complex power, to real and complex impedances etc., applying by analogy, models, structures, and conclusions (already developed in Electronics), to Classical Mechanics, Gravitation, Quantum Mechanics...*

Here we try to connect arbitrary Power Function (product between current and voltage, or product between force and velocity, or product between any other relevant, mutually conjugate functions creating power) to a Wavefunction as it is known in Quantum Mechanics. Energetically analyzed, any wave propagation in time and frequency domain can be mutually (space-time-frequency) correlated using Parseval's theorem. Consequently, the immediate (time domain) Power signal can be presented as the square of the wave function $\Psi^2(\mathbf{t})$*. Analysis of the optimal power transfer can be extended to any wave propagation field (and to arbitrarily shaped signals). Thus, we profit enormously (in booth, directions) after unifying traditional concepts of Active, Reactive and Apparent power with the* $\Psi^2(\mathbf{t})$ *wave function mathematics, based on Analytic Signal methodology.*

In Quantum Mechanics the wave function $\Psi^2(\mathbf{t})$ *is conveniently modeled as a probability function, but effectively it behaves like normalized and dimensionless Power function, and here it will be closely related to Active Power or power delivered to a load expressed in Watts as its units:*

$$\Psi^2(\mathbf{t}) = \mathbf{P}(\mathbf{t}) = \mathbf{S}(\mathbf{t})\cos\theta(\mathbf{t}) = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{ui}+\hat{\mathbf{u}}\hat{\mathbf{i}}) = \mathbf{Q}(\mathbf{t})\cdot\mathbf{cotan}\,\theta(\mathbf{t})\ (=)[\mathbf{W}].$$

The power reflected from a load, or Reactive Power, can be formulated as:

$$\mathbf{Q}(\mathbf{t}) = \mathbf{S}(\mathbf{t})\sin\theta(\mathbf{t}) = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{u}\hat{\mathbf{i}}-\hat{\mathbf{u}}\mathbf{i}) = \Psi^2(\mathbf{t})\cdot\tan\theta(\mathbf{t}) = \mathbf{P}(\mathbf{t})\cdot\tan\theta(\mathbf{t})\,(=)\,[\mathbf{VAR}]$$

Electric Power and Energy transfer analysis (especially for arbitrary voltage and current signal forms) can be related to Wave function analysis if we establish the Wave function (or more precisely, the square of the wave function) in the following way:

$$P(t) = \Psi^2(t) = \left[a(t)\cos\varphi(t)\right]^2 = \text{Wave function} \ , \ t \in [T],$$

$$\Psi(t) = a(t)\cos\varphi(t), \ \hat{\Psi}(t) = a(t)\sin\varphi(t),$$

$$\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = \Psi(t) + jH[\Psi(t)] = a(t)e^{j\varphi(t)} = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} U(\omega)e^{-j\omega t}d\omega =$$

$$= \frac{1}{\pi^2}\int_{0}^{+\infty} U(\omega)e^{-j\omega t}d\omega = \frac{1}{\pi}\int_{(0,+\infty)} A(\omega)e^{-j\omega t}d\omega,$$

$$U(\omega) \ = \ U_c(\omega) \ - j\, U_s(\omega) \ = \int_{(-\infty,+\infty)} \overline{\Psi}(t)\ e^{j\omega t}dt = A(\omega)e^{-j\Phi(\omega)} \ ,$$

$$U_c(\omega) = A(\omega)\cos\Phi(\omega), \ U_s(\omega) = A(\omega)\sin\Phi(\omega),$$

$$a(t) = \sqrt{\Psi(t)^2 + \hat{\Psi}(t)^2} \ , \ \ \varphi(t) = \operatorname{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)} \ ,$$

$$A^2(\omega) = U^2{}_c(\omega) + U^2{}_s(\omega), \ \ \Phi(\omega) = \operatorname{arctg}\frac{U_s(\omega)}{U_c(\omega)} \ ,$$

$$T\cdot\langle P(t)\rangle = \int_{-\infty}^{+\infty} P(t)dt = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty}\left|\Psi(t) + j\hat{\Psi}(t)\right|^2 dt = T\cdot\langle\hat{P}(t)\rangle = \int_{-\infty}^{+\infty}\hat{P}(t)dt =$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} a^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|U(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{\infty}\left[A(\omega)\right]^2 d\omega,$$

As we can see, every single wave function has at least two wave components (since to create power function it is essential to make the product of two relevant, mutually conjugate signals, like current and voltage, velocity and force, or some other equally important couple of signals: see (4.18)):

$$\Psi^2(t) = P(t) = S(t)\cos\theta(t) = \frac{1}{2}(ui + \hat{u}\hat{i}) = \Psi_1{}^2(t) + \Psi_2{}^2(t), \ \Psi_1^2 = \frac{ui}{\sqrt{2}}, \ \Psi_2^2 = \frac{\hat{u}\hat{i}}{\sqrt{2}},$$

and it shouldn't be too big success to causally explain quantum mechanical diffraction, superposition and interference effects, when a "single wave object" and/or a single particle (like an electron, or photon) passes the plate with (at least) two small, diffraction holes, because in reality (in this situation) there isn't a single object (there are always minimum 2 mutually conjugate wave elements and their mixed products, somehow energetically coupled with their environment, extending the number of interaction participants). What looks to us like strange quantum interaction, or interference of a single wave or particle object with itself, in fact, presents an interaction of at least 2 wave entities with some other, third object ($\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$ *). Somehow, Nature always creates complementary and conjugate couples of important elements (signals, particles, energy states…) belonging to every kind of matter motions. We can also say that every object (or energy state) in our universe has its non-separable and conjugate image (defined by Analytic Signal concept). Consequently, the quantum mechanical wave function and wave energy should represent only a motional energy (or power) composed of a minimum of two mutually coupled wave functions (* $\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$ *).*

Here applied mathematics, regarding wave functions $\Psi^2(t) = \Psi_1{}^2(t) + \Psi_2{}^2(t)$*, after making appropriate normalizations and generalizations, would start (only superficially and approximately) looking as applying Probability Theory laws, like in the contemporary Quantum Theory. Consequently, modern Quantum Theory could also be treated as the generalized mathematical modeling of the micro world phenomenology, by conveniently unifying all conservation laws of physics in a joint, dimensionless, mutually well-correlated* *theoretical platform, creating a new mathematical theory that is different by appearance, but, isomorphic to remaining Physics. Also, a kind of generalized analogy with Norton and Thevenin's theorems (known in Electric Circuit Theory) should also exist (conveniently formulated) in all other fields of Physics and Quantum Theory, since the cause or source of certain action produces a certain effect, and vice versa and such events are always mutually coupled. ♣]*

4.3.5. Wave Function and Euler-Lagrange-Hamilton Formalism

One of the biggest achievements of Classical Mechanics is Euler-Lagrange-Hamilton formalism derived from Calculus of Variations. In this methodology, we usually formulate the most appropriate Lagrange function, or **Lagrangian (=) L**, and apply Euler-Lagrange equations on it, to find all relevant elements of certain complex motion. Without going too far in discussing Euler-Lagrange formalism, just analyzing different forms of Schrödinger wave equations (4.22), we can conclude that **Lagrangian** can also have different (floating energy level) forms, as for instance,

$$\left[\begin{array}{l}
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} - U_p)\overline{\Psi} = 0, (U_p \neq 0) \Rightarrow \left\{\underline{L = \tilde{E} - U_p} = -\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} \Leftrightarrow E_k - U_p, \frac{1}{c^2} \leq (\frac{\tilde{E}}{L})\frac{1}{u^2} < \infty\right\} \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + \tilde{E}\,\overline{\Psi} = 0, (U_p = 0) \Rightarrow \left\{\underline{L = \tilde{E}} = -\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} \Leftrightarrow E_k, \frac{1}{c^2} \leq \frac{1}{u^2} < \infty\right\}, \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} + U_p)\overline{\Psi} = 0 \Rightarrow \left\{\underline{L = \tilde{E} + U_p} = -\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} \Leftrightarrow E_k + U_p, \frac{1}{c^2} \leq (\frac{\tilde{E}}{L})\frac{1}{u^2} < \infty\right\} \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} + E_0 - U_p)\overline{\Psi} = 0 \Rightarrow \left\{\underline{L = E_{total} - U_p} = -\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}}, \frac{1}{c^2} \leq (\frac{\tilde{E}}{L})\frac{1}{u^2} < \infty\right\}, \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + E_{total}\overline{\Psi} = \frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} + E_0)\overline{\Psi} = 0 \Rightarrow \left\{\underline{L = E_{total}} = -\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}}, \frac{1}{c^2} \leq (\frac{\tilde{E}}{L})\frac{1}{u^2} < \infty\right\}, \\
\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (\tilde{E} + E_0 + U_p)\overline{\Psi} = \frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi} + (E_{total} + U_p)\overline{\Psi} = 0 \Rightarrow \\
\Rightarrow \left\{\underline{L = \tilde{E} + E_0 + U_p} = -\frac{\hbar^2}{m^*}(\frac{u}{v})\frac{\Delta\overline{\Psi}}{\overline{\Psi}} \Leftrightarrow E_k + E_0 + U_p, \frac{1}{c^2} \leq (\frac{\tilde{E}}{L})\frac{1}{u^2} < \infty\right\}.
\end{array}\right] \Rightarrow$$

$$\Rightarrow \boxed{\begin{array}{l}
\Delta\overline{\Psi} - \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = \left(\frac{L}{\hbar u}\right)^2\overline{\Psi} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0, \\
L \in \left[\tilde{E} \text{ or } \tilde{E} - U_p \text{ or } \tilde{E} + E_0 - U_p \text{ or } \tilde{E} + E_0 \text{ or } \tilde{E} + E_0 + U_p \text{ or } \tilde{E} + U_p \ldots\right]
\end{array}}. \tag{4.24}$$

*[♣ **COMMENTS & FREE-THINKING CORNER:** Based on (4.24) and (4.10-3) we can formulate the unique and even more general forms of relativistic Schrödinger (or Dirac's) wave equations, which replace all previously formulated wave equations,*

$$\left\{\begin{array}{l} \Delta\overline{\Psi}-\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=\left(\frac{L}{\hbar u}\right)^2\overline{\Psi}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t}+\frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2}=0, \\ \frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\overline{\Psi}+L\overline{\Psi}=0\ ;\ \Delta\overline{\Psi}=\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2}, \\ (\frac{L}{\hbar})^2\overline{\Psi}+\frac{\partial^2\overline{\Psi}}{\partial t^2}=0,\ \frac{L}{\hbar}\overline{\Psi}=j\frac{\partial\overline{\Psi}}{\partial t}, \\ L\in\left[\tilde{E}\text{ or }\tilde{E}-U_p\text{ or }\tilde{E}+E_0-U_p\text{ or }\tilde{E}+E_0\text{ or }\tilde{E}+E_0+U_p\text{ or }\tilde{E}+U_p...\right] \\ S=\int_{t_1}^{t_2}L(q_i,\dot{q}_i...,t)dt=\text{extremum}, \end{array}\right\}\Rightarrow$$

$$\left(H=-\frac{\hbar^2}{m^*}(\frac{u}{v})\Delta\ (=)\ \text{Hamiltonian}\right)\Rightarrow\left(H\overline{\Psi}=L\overline{\Psi}=j\hbar\frac{\partial}{\partial t}\overline{\Psi}=-j\hbar u\nabla\overline{\Psi}\right)\Rightarrow$$

$$\Rightarrow L\Leftrightarrow H\Leftrightarrow j\hbar\frac{\partial}{\partial t}\Leftrightarrow -j\hbar u\nabla\overline{\Psi},\ \tilde{E}-U_p\le L<\infty,\tilde{E}\Leftrightarrow E_k,\overline{\Psi}=\overline{\Psi}(t,r),\ j^2=-1, \quad (4.25)$$

*where Lagrangian **L** should be considered as the floating (and variable:* $\mathbf{L}=\mathbf{E}_{tot.}-\mathbf{U_p}$, $\mathbf{0}\le\tilde{\mathbf{E}}\le\mathbf{E}_{tot.},\ \mathbf{U_p}\ge 0$*) energy level, and* $\tilde{\mathbf{E}}-\mathbf{U_p}$ *as its lowest or minimal energy level. For instance, in the case of electromagnetic waves in a free space (*$U_p=0$*), floating energy level (Lagrangian) is equal to the wave energy* $\mathbf{L}=\tilde{\mathbf{E}}$ *. This is (most probably) related to the fact that photons do not have a rest mass (but when strong* γ *photon penetrates sufficiently close to an atom, it can transform its energy-momentum state into an electron-positron pair, because there is a certain potential field involved in the reaction). By performing an energy translation on the scale (or axis) of a floating energy level* $\tilde{\mathbf{E}}-\mathbf{U_p}\le\mathbf{L}<\infty$*, we should be able "to materialize" all elementary particles and other energy forms of our universe. It should also be highlighted that all of the wave equations in (4.25) are mutually compatible or describe the same wave phenomena. The solutions of such equations are essentially dependent on the solutions of Euler-Lagrange equations applied on the relevant Lagrangian **L**.* It is also interesting to notice that in (4.25) the factor $(\frac{\tilde{E}}{L})\frac{1}{u^2}$ should determine a kind of wave (phase) velocity. If we just limit this speed to be $\frac{1}{c^2}\le(\frac{\tilde{\mathbf{E}}}{L})\frac{1}{u^2}<\infty$, we shall get $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\ge(\frac{\mathbf{u}}{\mathbf{c}})^2$ and $-\mathbf{L}\le\frac{\tilde{\mathbf{E}}-\mathbf{L}}{1-\frac{\mathbf{u}^2}{\mathbf{c}^2}}\le\frac{\mathbf{U_p}}{1-\frac{\mathbf{u}^2}{\mathbf{c}^2}}$.

If we imagine that $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\frac{\mathbf{1}}{\mathbf{u}^2} = \frac{\mathbf{1}}{\mathbf{v}^2}$ determines the group speed of the same wave group (because there is not a big choice of possible velocities of a wave packet, since $\mathbf{u}$ is already its phase speed), we get $(\frac{u}{v})^2 = (\frac{\tilde{\mathbf{E}}}{L})$, and since from (4.2) we already know the relation between group and phase velocity, we shall easily obtain the following relations: $\{\mathbf{0} \leq (\frac{\mathbf{u}}{\mathbf{c}})^2 \leq (\frac{u}{v})^2 = (\frac{\tilde{\mathbf{E}}}{\mathbf{L}}) = \left\{\frac{1}{2} \leq 1 / \left[1 + \sqrt{1 - \frac{v^2}{c^2}}\right]^2 \leq 1\right\}\}$. This mathematical exercise (regarding group and phase velocity) at present, is a brainstorming to initiate the search for the real meaning of the velocity involved in $(\frac{\tilde{\mathbf{E}}}{\mathbf{L}})\frac{\mathbf{1}}{\mathbf{u}^2} = \frac{\mathbf{1}}{\mathbf{v}^2}$).

We can also use the following expressions (from (4.1) - (4.3) and T.4.1.):

$$\left\{\begin{array}{l} L = \frac{\tilde{p}c^2}{u} = \tilde{E}(\frac{c}{u})^2 = hf(\frac{c}{u})^2 = E_{tot.} - U_p = \sqrt{c^2p^2 + (E_0)^2} = \frac{pv}{1+\sqrt{1-v^2/c^2}} + E_0 = pu + E_0, \\ E_{tot.} = \sqrt{c^2p^2 + (E_0)^2} + U_p = E_k + E_0 + U_p = pu + E_0 + U_p = \frac{pv}{1+\sqrt{1-v^2/c^2}} + E_0 + U_p, \\ \tilde{p} = (\frac{u}{c})\sqrt{p^2 + (\frac{E_0}{c})^2} = \frac{\tilde{E}}{u} = mu\sqrt{(\frac{\gamma v}{c})^2 + 1} = \frac{mv}{(1+\sqrt{1-v^2/c^2})\sqrt{1-v^2/c^2}} = \frac{mu}{\sqrt{1-v^2/c^2}} \\ \tilde{p} = \frac{\tilde{E}}{u} = p = \gamma mv = \frac{E_k}{u}, \ E_0 = mc^2 \end{array}\right\}. \tag{4.25-2}$$

Of course, if once somebody would find and prove that **SRT** is wrong in certain of its segments, we should be ready for new refinements of wave equations (and this is, most probably, already the case based on recent publications from Thomas E. Phipps, Jr.; -see literature under [35]).

From (4.25) we can find Lagrangian $\mathbf{L}$ and apply Euler-Lagrange-Hamilton equations on it, considering all possible coordinates and relevant field parameters of certain wave function (4.23), as for instance:

$$\begin{array}{l} \overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i, ..., t), \ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, ..., t)dt = \text{extremum}, \ H\overline{\Psi} = 0, \\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}\tilde{p}, \ \tilde{E}\text{-}U_p \leq L < \infty, \ L \in \left\{ (\tilde{E} - U_p), ... \ \tilde{E}, ... \ (E_{total} - U_p), ... \ E_{total} ... \right\}, \\ \tilde{E} = \int_{-\infty}^{+\infty} \Psi^2(\omega t - kx)dt = \int_{-\infty}^{+\infty} \left|\overline{\Psi}(\omega t - kx)\right|^2 dt \Rightarrow \\ \frac{d\tilde{E}}{dt} = 2(\omega - k\frac{dx}{dt})\int_{-\infty}^{+\infty} \Psi(\omega t - kx)dt, \frac{\partial \tilde{E}}{\partial x} = -2k\int_{-\infty}^{+\infty} \Psi(\omega t - kx)dt \Rightarrow \\ \frac{d\tilde{E}}{dt} = \frac{\omega - k\frac{dx}{dt}}{-k} \cdot \frac{\partial \tilde{E}}{\partial x} = (v - u) \cdot \frac{\partial \tilde{E}}{\partial x} \Rightarrow \frac{d\tilde{p}}{dt} = (1 - \frac{u}{v}) \cdot \frac{\partial \tilde{E}}{\partial x} = \frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \cdot \frac{\partial \tilde{E}}{\partial x}. \end{array} \tag{4.26}$$

In simpler situations when Lagrangian presents a function of certain coordinates and their first derivatives, $L = L(q_i, \dot{q}_i, t)$, Euler-Lagrange equations will generate,

$$\left\{\begin{array}{l} L = L(q_i, \dot{q}_i, t), \overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i ..., t)\ ,\ \tilde{E} = \int \left\| \overline{\Psi} \right\|^2 \cdot dt\,, \\ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t) dt = \text{extremum}\,,\ H\overline{\Psi} = 0, \end{array}\right\} \Rightarrow \quad (4.27)$$
$$\Rightarrow \frac{d}{dt}\left[\frac{\partial L(q_i, \dot{q}_i, t)}{\partial \dot{q}_i}\right] - \frac{\partial L(q_i, \dot{q}_i, t)}{\partial q_i} = 0.$$

In all other situations when Lagrangian presents more complex function, Euler-Lagrange equations will be,

$$\left\{\begin{array}{l} L = L(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t), \overline{\Psi} = \overline{\Psi}(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)\ ,\ \tilde{E} = \int \left\| \overline{\Psi} \right\|^2 \cdot dt\,, \\ S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) dt = \text{extremum}\,,\ H\overline{\Psi} = 0 \end{array}\right\} \Rightarrow \quad (4.28)$$
$$\Rightarrow \frac{\partial \int L dt}{\partial q_i} - \frac{d}{dt}\left[\frac{\partial \int L dt}{\partial \dot{q}_i}\right] + \frac{d^2}{dt^2}\left[\frac{\partial \int L dt}{\partial \ddot{q}_i}\right] - ... + (-1)^n \cdot \frac{d^n}{dt^n}\left[\frac{\partial \int L dt}{\partial q_i^{(n)}}\right] = 0.$$

The Orthodox Quantum Mechanics developed a big part of its mathematical apparatus exploiting the achievements of Euler-Lagrange and Hamilton Theory, basically accepting restraints of Classical Mechanics. Euler-Lagrange equations have been extended to Hamilton equations, producing the rules of Poisson brackets, leading to “Evolution Equation”, to Schrödinger’s Equation, to Operators Mechanics etc. The simple and seducing mathematical symmetries and elegant forms of Euler-Lagrange and Hamilton’s equations, discovered more than a century ago, fully modeled at that time to describe Classical, Newton Mechanics, still influence the physics in its foundations (but often their applicability is limited only to non-relativistic motions, $v << c$). It is really the time to transform and upgrade this excellent concept and methodology (based on Calculus of Variations) to the new level of a more general physics (without blindly and exclusively accepting the restrictions and prescriptions of Classical, Relativistic and Quantum Mechanics, as somebody by game of best chances and creative patchwork established a long time ago). By using the relativistic form of Lagrangian, here we propose how it would be possible to extend the applicability of Euler-Lagrange equations, (4.26)-(4.28). This way, a new theory would replace traditionally known Euler-Lagrange and Hamilton's mathematics (and, of course, there is a lot of work to be done in the same direction before we create new theory).

In all of the above given equations, (4.25)-(4.28), we do not have the real restriction to treat (motional and wave) energy and Lagrange functions only from the point of view of Classical Mechanics, as usual in Classical and Quantum Mechanics (since here we already use relativistic-compatible expressions for energy). Of course, when using only Classical Mechanics formulas for mass, momentum, and energy, the system of Euler-Lagrange equations generates attractive and symmetrical mathematical forms, but this should not be the only reason to sacrifice domain of applicability of Euler-Lagrange concept (which is universally valid, since it is the result of Hamilton's Principle derived from Calculus of Variations, applied on Lagrangian). If once physics finds a better theory to replace Einstein's Relativity Theory, the Euler-Lagrange concept will still hold valid, except that mass, momentum, and energy would be differently described. ♣]

Obviously, equations (4.25) - (4.28) are the demonstration of connections between Classical, Relativistic and Quantum Mechanics with Particle-Wave Duality Theory as presented in this book. Such equations are universally valid for any particle-wave duality aspect of motion, and applicable to Gravitation, Electromagnetism, Linear and Rotational motions, coupled Action and Reaction forces etc. (see also T4.1, T5.2, (4.29) - (4.31), Uncertainty Relations (5.15), and (5.16) that complement the same statement). In the background of (4.25) - (4.28), summarizing the results, we can find:

1° "Particle-Wave Duality Code" or PWDC, and coupled Action-Reaction forces, given by (Relativity Theory compatible) relations found in (4.1), (4.2) and (4.3),

2° with generally valid wave function (and its differential equation: Schrödinger equation) given in the form of an Analytic Signal (4.9),

3° respecting the fact that only motional energy creates de Broglie matter waves, as summarized by the second part of (4.10-5),

4° also respecting Energy and Momentum conservation laws,

5° and placing all of them (already-mentioned) inside the frames of universally valid Calculus-of-Variation Principles (from Euler-Lagrange-Hamilton mechanics).

Since the Orthodox Quantum Mechanics is still the leading and only officially accepted theory of micro world, it would be useful to underline the most important differences and similarities between here presented particle-wave duality concept and particle-wave duality in Orthodox Quantum Mechanics (or in Schrödinger's wave mechanics), following above-listed step-stones (from 1° to 5°).

1° Orthodox Quantum Mechanics (OQM) also made necessary integration of "Particle-Wave Duality Code" (or PWDC) with its probabilistic wave function and Schrödinger (or almost Classical) wave equation, but not using all options found in (4.1) - (4.3), and staying on the grounds of Classical Mechanics (regarding particle speed, energy etc.). Especially significant negative aspect of OQM can be found in its imprecise phase and group velocity treatment, and in a missing spatial-temporal phase information. Also, coupled Action-Reaction forces and effects of intrinsic rotation (or torsion fields) are not addressed in OQM.

2° The wave function and wave equation in OQM is very much artificially postulated and assembled. It is not created following the general model that is very good for representing almost any arbitrary wavefunction by an Analytic Signal (in comparison with the wave function (4.9)). In addition, such OQM wave function is not creating or using the immediate temporal-spatial-frequency and signal phase information. Simply, and luckily, it is eventually and correctly completed and modeled to serve its function properly. In many other aspects (except missing phase function), OQM wave function behaves almost like a complex Analytic Signal wave function, and produces the same type of Schrödinger equation (as the wave function in (4.9)).

3° In OQM, the wave function represents the totality of a particle, including its rest energy and rest mass states, summarized by the first part of (4.10-5). In many cases, this does not present a serious problem (except that analysis of relativistic particles is not well supported), since after every differentiation (when creating wave equations) we lose the freestanding constant members (rest mass and rest energy, for instance).

4° Instead of directly respecting Energy and Momentum conservation laws, OQM creates an equivalent platform, operating with dimensionless probability functions, which are (or can be) effectively created by simple normalization of relevant energy and momentum conservation functions. Since OQM wave function does not have the information about real (immediate, time-space-frequency dependent) signal phase of de Broglie matter wave which it should represent, in order to avoid possible mistakes, it was very convenient (for OQM) to accept the modeling frame borrowed from Statistics and Probability theory (mixed with appropriate Signal and Spectrum Analysis practices). This approach is intrinsically compatible with all conservation laws known in Physics. This way, OQM dimensionless wave function effectively compensated the part of "its missing body", and became very operational, but only in averaged statistical terms, and in the frames of (re)invented or postulated mathematics of OQM.

5° Euler-Lagrange-Hamilton mechanics and variation principles cannot be directly or literally represented and satisfied in the structure of OQM, because in OQM we operate with dimensionless probability-compatible wave functions. However, indirectly OQM naturalized a big part of mathematical apparatus exploiting the achievements and models of non-relativistic Euler-Lagrange and Hamilton Theory, basically accepting restraints and conservation laws of Classical Mechanics.

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*[♣ **COMMENTS & FREE-THINKING CORNER:** The opinion of the author of this paper regarding Quantum Mechanical "probabilistic philosophy" and probabilistic wave function (including a lot of assumptions) is that this has been just one of the acceptable mathematical modeling possibilities (not the best and not the most general). It obviously has a lot of supporting background, because of its **"in-average and implicit compatibility"** with Energy and Momentum (and other) conservation laws, as well as compatibility with generally valid mathematics (from Signal and Spectrum Analysis and Probability theory). Thanks to all associated "mathematical cosmetics", assumptions and proper fitting into positive knowledge of Physics (based on de Broglie, Heisenberg, and Schrödinger contributions regarding particle-wave duality), a sufficiently well and still one of the best-known operating structure of Orthodox Quantum Mechanic was established. **Effectively, this has been a long-lasting triumph, and at the same time, the biggest conceptual, common sense and philosophical obstacle (almost dead-end street) of modern physics regarding future (more significant) advances in the same field. We should be ready to start general and large-scale refinement of modern physics, especially of Orthodox Quantum Mechanics (what is one of the messages of this paper). In other words, we could say that existing Quantum Mechanics effectively blocks, masks (or misleads) creation of new and more advanced concepts in Physics (regardless of its very important contribution to modern physics). Under more significant advances in Physics, here we understand a creation of Unified and Open Architecture Field-Theory (which unites all known natural forces and creates open links for integration of forces to be discovered; -which is probably a kind of Superstring theory).** ♣]*

4.3.6. Extensions of Wave Function and Universal Field Theory

It is especially important to notice that the wave function presented in this book is different from Quantum Mechanic's wave function. Generally valid in this book is that we can associate the wave function to any form of motional energy (or particularly to linear motion, to Rotation, to motion in Electric field and to motion in the Magnetic field, or to any of their combinations). If there are some other forms of motional energy presently unknown or not mentioned, for each one of them it should exist one associated wave function. However, what looks probable regarding a plurality of motional energy components is that the real motion energy nature could have its profound origins only in some of the different manifestations of an electromagnetic field.

To develop a more general (and Relativity compatible) forms of wave and Euler-Lagrange equations we shall use the expression for kinetic energy in a differential form, that is (mathematically) the same in Classical and Relativistic Mechanics (instead of using the expression for non-relativistic kinetic energy), as for instance:

$$dE_k = vdp = d\tilde{E} = hdf \text{ ,}$$

since in Classical Mechanics for kinetic energy we have,

$$\{E_k = mv^2/2 = pv/2, p = mv, m = \text{const.}\} \Rightarrow \underline{dE_k} = mvdv = vd(mv) = \underline{vdp} \text{ ,}$$

and we can get the same result in Relativistic Mechanics,

$$\left\{E_k = \frac{\gamma mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = (\gamma-1)mc^2, \gamma = 1/\sqrt{1-\frac{v^2}{c^2}}, p = \gamma mv\right\} \Rightarrow \underline{dE_k = vdp}.$$

It is also obvious that the differential of kinetic energy, from the point of view of Classical Mechanics, can be found as $dE_k = pdv \; (= vdp)$, but we eliminate this possibility since in Relativistic Mechanics we can get only $\mathbf{dE_k = vdp}$. This way we indirectly implement compatibility of any form of kinetic energy (and wave functions developed based on motional energy) with Lorentz transformations and Euler-Lagrange equations.

In a situation when particle only performs the rotation, its kinetic (rotational and relativistic) energy, by analogy with the above-given example (see T.3.1-T.3.3 and T.4.3.1), can be expressed as a function of its angular momentum and angular speed, $dE_k = \omega d(J\omega) = \omega dL$.

The square of the wave function in this paper is just the active (motional) power, and using analogies developed before (see T.1.8, Generic Symmetries, and Analogies of the Laws of Physics), we can summarize several forms of possible wave functions, presented in the following table:

T.4.3.1

Linear motion	$dE_k = vdp = -\Psi^2 \cdot dt$, $\Psi^2 = v \cdot \tilde{F}$	**v = velocity, $\tilde{F}$ = force**	**p = momentum,** $\mathbf{dp = -d\tilde{p}}$, $\tilde{F} = d\tilde{p}/dt$
Rotation	$\mathbf{dE_{kr} = \omega dL = -\Psi^2 \cdot dt}$, $\Psi^2 = \omega \cdot \tilde{\tau}$	**ω = angular velocity, $\tilde{\tau}$ = torque**	**L = angular momentum,** $dL = -d\tilde{L}$, $\tilde{\tau} = \mathbf{d\tilde{L}/dt}$
Electric field	$dE_{ke} = udq_e = -\Psi^2 \cdot dt$, $\Psi^2 = u \cdot \tilde{i} = i_{mag.} \cdot \tilde{u}_{mag.}$	**u = electric voltage, $\tilde{i}$ = electric current**	**q_e = electric charge,** $dq_e = -d\tilde{q}_e$, $\tilde{i} = d\tilde{q}_e / dt$
Magnetic field	$dE_{km} = id\Phi = -\Psi^2 \cdot dt$, $\Psi^2 = u_{mag.} \cdot \tilde{i}_{mag.} = i_{el.} \cdot \tilde{u}_{el.}$	**$u_{mag.}$ = magn. voltage, $\tilde{i}_{mag.}$ = magn. current**	**Φ = magnetic flux,** $d\Phi = -d\tilde{\Phi}$, $\tilde{i}_{mag.} = d\tilde{\Phi}/dt$

where: $(u = u_{el.}, \tilde{u} = \tilde{u}_{el.}) \equiv (i_{mag.}, \tilde{i}_{mag.})$; $(i = i_{el.}, \tilde{i} = \tilde{i}_{el.}) \equiv (u_{mag.}, \tilde{u}_{mag.})$;
$q = q_{el.} = \Phi_{el.}, \tilde{q} = \tilde{q}_{el.} = \tilde{\Phi}_{el.}, q_{mag.} = \Phi_{mag.} = \Phi, \tilde{q}_{mag.} = \tilde{\Phi}_{mag.} = \tilde{\Phi}$.

In a case when all of the above energy elements are present in the same motional situation, we shall have:

$$dE_{k-total} = \sum_i dE_k(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) = vdp + \omega dL + udq_e + id\Phi + ... =$$
$$= -vd\tilde{p} - \omega d\tilde{L} - ud\tilde{q}_e - id\tilde{F} - ... = \Psi^2_{total} dt = \sum_i \Psi^2(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)dt = \tag{4.29}$$
$$= \sum_i \Psi^2(q_i, v, \omega, u, i, p, L, q_e, \Phi, ..., t)dt = d\tilde{E}_{total} .$$

We could again apply the Euler-Lagrange formalism, (4.26) - (4.28) on (4.29) and analyze much more complex field situations, this way approaching the fields' unification objective from a more general platform.

For instance, we could now generalize the meaning of (multi-component, linear) force, extending by analogy the force expression (4.18), (respecting also (4.19) and (4.26)-(4.29)), to the following form:

$$\tilde{F}(t,r) = \frac{1}{v}\Psi^2(t,r) \Rightarrow \tilde{F}_i(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) = \frac{1}{\dot{q}_i}\Psi^2(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t),$$
$$\tilde{F} = \sum_{(i)} \alpha_i \tilde{F}_i = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i}\Psi_i^2 = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \frac{d\tilde{E}(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)}{dt} =$$
$$= \sum_{(i)} \alpha_i \frac{E_i}{q_i} = \sum_{(i)} \alpha_i \frac{E_{ki}}{q_i}, \quad \alpha_i \in \{Const...\} \forall_i , \tag{4.30}$$

$$\tilde{E} = -k^2 L \frac{\overline{\Psi}}{\Delta\overline{\Psi}} = Lu^2 \Delta\overline{\Psi} / \frac{\partial^2 \overline{\Psi}}{\partial t^2} = j\frac{\hbar k^2}{\Delta\overline{\Psi}} \frac{\partial \overline{\Psi}}{\partial t} = j\hbar u^2 \frac{\Delta\overline{\Psi}}{\overline{\Psi}} \frac{\partial \overline{\Psi}}{\partial t} .$$

It looks that generalized force law (4.30) represents only dynamical (transitory, or time-dependent) forces, but we can easily see that this is not correct, since every component of motional (or kinetic) energy E_{ki} plus certain constant energy $\mathbf{E_{0i}}$ can create the total energy $E_{total-i}$,

$$E_{total-i} = E_{ki} + E_{0i}\,,\; E_{0i} = \text{Constant.} \Rightarrow \frac{dE_{ki}}{dt} = \frac{dE_{total-i}}{dt} \Rightarrow$$
$$\Rightarrow \tilde{\mathbf{F}} = \sum_{(i)} \alpha_i \tilde{\mathbf{F}}_i = \sum_{(i)} \alpha_i \frac{\dot{\tilde{E}}_i}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{E}_{ki}}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{E}_{total-i}}{\dot{q}_i}, \alpha_i = \text{const..} \tag{4.31}$$

The generalized force law (4.31) should be considered (presently) more as a an example for discussion in a direction of creating more practical force formula, since the force is a vector. Constants $\boldsymbol{\alpha}_\mathbf{i}$ should take care of dimensional agreements and generalized coordinates $\dot{\mathbf{q}}_\mathbf{i}$ should belong to the set of generalized velocities, $q_i \in \{v_i, \omega_i, ...\}$.

The most important "secret" (hidden) in the force law (4.31) is that certain motional (time-dependent and stationary) energy and field components are somehow trapped (blocked or permanently captured) inside every constant energy level $\mathbf{E_{0i}}$*. For instance, every magnet (electromagnet or permanent magnet) is created as the result of certain current flow, but in the case of permanent magnets, we do not see such current flow. This is because it is permanently captured inside small, atomic-size magnet domains, created by circulation of electrons around their cores. Consequently, the ultimate structure of matter (or of our universe) is that we have frozen (permanently captured, stationary and stable) energy states (or levels)* $\mathbf{E_{0i}}$*, and free energy levels* $\mathbf{E_{ki}}$*. Whenever we can penetrate (experimentally or theoretically) into the internal structure of such constant energy states* $\mathbf{E_{0i}}$*, we find different motional energy states, and sometimes other relatively stable energy states, or particles. Again, it is obvious that the most important forces creating such (internal) energy structure should be the forces in some close relation to the rotation. Now it becomes clearer that the real sources of Gravity should be hidden in a certain more sophisticated entity than in only what is related to a particle rest mass.*

Almost a perfect symmetry between electric and magnetic voltages and currents has been established as the result of presentations based on analogies (see T.3.1-T.3.3 and T.4.3.1). Of course, this symmetry exists as an achievement of Maxwell-Faraday Electromagnetic Theory, and here it becomes evident just because of the way it is presented. *It is also clear (after reading the entire contents of this book) that presently we do not have the same level of symmetry regarding Linear motion and Rotation, and that the missing link (between them) should be related to de Broglie matter waves. Therefore, every rectilinear motion should have some associated effects of rotation and waving (and vice versa, since every form of rotation is also a source of harmonic oscillations). Consequently, a new theory must be established to cover all aspects of here presented analogies and particle-wave duality phenomenology.*

4.3.7. Matter-Wave 4-Vectors in Minkowski Space and Elements for New Topology

The principal idea here is that when creating New Topology Basis, we should consider only coordinates, motional elements and/or degrees of freedom which are really (and always) contributing to the total energy of a certain system (see also (5.16)). Based on (4.30) and (4.31) we can extract the elements for a New (Universal Field) Topology if we determine the resulting velocity $\mathbf{V}_\Sigma$ *and its path element* $\mathbf{dX}_\Sigma$ *caused by complex (multi-component) force* $\mathbf{F}_\Sigma$*, or by complex momentum* $\mathbf{P}_\Sigma$*, as for instance:*

$$\begin{aligned}
&F_\Sigma = \sum_{(i)} \alpha_i \frac{\dot{E}_{ki}}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{E}_{total-i}}{\dot{q}_i} = \\
&= \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \frac{dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)}{dt} = \frac{dP_\Sigma}{dt} = \dot{P}_\Sigma = \frac{1}{V_\Sigma}\frac{dE_\Sigma}{dt} \Rightarrow \\
&\tilde{F}_\Sigma = -F_\Sigma = -\sum_{(i)} \alpha_i \frac{\dot{E}_{ki}}{\dot{q}_i} = -\sum_{(i)} \alpha_i \frac{\dot{E}_{total-i}}{\dot{q}_i} = \sum_{(i)} \alpha_i \frac{\dot{\tilde{E}}_i}{\dot{q}_i} = \\
&= -\sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \frac{dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t)}{dt} = -\frac{dP_\Sigma}{dt} = -\dot{P}_\Sigma = \dot{\tilde{P}}_\Sigma \Rightarrow \\
&V_\Sigma = \frac{dE_\Sigma}{dP_\Sigma} = \frac{d\left[\sum_{(i)} E_i\right]}{dP_\Sigma} = \frac{\dot{E}_\Sigma}{F_\Sigma} = \frac{\sum_{(i)} \dot{E}_i}{F_\Sigma} = \frac{dX_\Sigma}{dt}, \qquad (4.32) \\
&dP_\Sigma = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t), \\
&E_i = E(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t), \\
&dX_\Sigma = V_\Sigma dt = \frac{\sum_{(i)} dE_i}{dP_\Sigma} dt = \frac{\sum_{(i)} \dot{E}_i}{F_\Sigma} dt = \frac{\sum_{(i)} dE_i}{F_\Sigma} = \frac{\sum_{(i)} dE_i}{\sum_{(i)} \frac{\alpha_i}{\dot{q}_i} dE_i} dt = \frac{\sum_{(i)} dE_i}{\sum_{(i)} \alpha_i \frac{dE_i}{dq_i}}.
\end{aligned}$$

With (4.32) only the principal idea and starting platform for creating New Topology is formulated (but to finalize this task will request much more efforts; -see also (5.15)-(5.17)).

In a contemporary physics (especially in Relativity Theory) we use (under Lorentz transformations) covariant forms of 4-vectors in Minkowski space, as the most elegant and most general synthesis of Energy and Momentum conservation laws. For instance, covariant 4-vectors of particle velocity (here we can also say group velocity when we are using wave packet or wave group model) and momentum are known as:

$$\overline{V}_4 = \overline{V}[\gamma\vec{v}, \gamma c], \overline{V}_4^2 = \text{invariant}, \ \overline{P}_4 = m\overline{V}_4 = \overline{P}\left[\vec{p} = \gamma m\vec{v}, \frac{E}{c} = \gamma mc\right] =$$
$$= \overline{P}(\vec{p}, \frac{E}{c}) = \overline{P}\left[\vec{p}, (\frac{E_0}{c} + \frac{\tilde{E}}{c})\right], \overline{P}_4^2 = \text{invariant}., \tilde{E} = E_k = (\gamma - 1)mc^2 = pu, \ E_0 = mc^2. \quad (4.33)$$

The wave energy and wave momentum can be connected in a similar way using differential forms of 4-vector wave momentum,

$$\overline{P}_4 = \overline{P}(\vec{p}, \frac{E}{c}) = \overline{P}\left[\vec{p}, (\frac{E_0}{c} + \frac{\tilde{E}}{c})\right] = \overline{P}_{4,0}(0, \frac{E_0}{c}) + \tilde{P}_4(\vec{p}, \frac{\tilde{E}}{c}), \vec{p} = \tilde{p},$$
$$d\overline{P}_4 = d\tilde{P}_4 = d\tilde{P}(\tilde{p}, \frac{\tilde{E}}{c}) = \tilde{P}(d\tilde{p}, \frac{d\tilde{E}}{c}) = \tilde{P}(\frac{d\tilde{E}}{v}, \frac{d\tilde{E}}{c}) = \tilde{P}(\frac{\Psi^2}{v}dt, \frac{\Psi^2}{c}dt) = \frac{1}{v}\overline{\Psi}_4^2 dt \Rightarrow$$
$$\overline{\Psi}_4^2 = \frac{v d\tilde{P}_4}{dt} = \overline{\Psi}_4^2(\frac{v d\tilde{p}}{dt}, \frac{v}{c}\frac{d\tilde{E}}{dt}) = \overline{\Psi}_4^2(\frac{d\tilde{E}}{dt}, \frac{v}{c}\frac{d\tilde{E}}{dt}) = \overline{\Psi}_4^2(\Psi^2, \frac{v}{c}\Psi^2) \Rightarrow \quad (4.34)$$
$$\overline{P}_4 = \overline{P}_4(\int \frac{1}{v}\overline{\Psi}_4^2 dt) = \overline{P}_4(\int \frac{1}{v}\Psi^2 dt, \ \frac{1}{c}\int \Psi^2 dt) = \overline{P}(\vec{p}, \frac{E}{c}),$$
$$\overline{P}_4^2 = (\int \frac{1}{v}\Psi^2 dt)^2 - (\frac{1}{c}\int \Psi^2 dt)^2 = \text{invariant } (= \ mc^2),$$
$$d\tilde{E} = hdf = vd\tilde{p} = d(\tilde{p}u) = dE_k = dE = vdp = d(pu) = \Psi^2 dt.$$

A 4-wave vector and 4-phase velocity vector could also be formulated as,

$$\overline{V}_4 = \overline{V}(\gamma v, \gamma c) = \overline{V}(\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}}, \frac{c}{\sqrt{1 - \frac{v^2}{c^2}}}) = \text{ 4-group velocity} \Rightarrow$$
$$\Rightarrow \overline{U}_4 = \overline{U}(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}}, \frac{c}{\sqrt{1 - \frac{u^2}{c^2}}}) = \text{ 4-phase velocity},$$
$$\overline{K}_4 = \frac{2\pi}{h}\tilde{P}_4 = \overline{K}(\vec{k}, \frac{\omega}{c}) \quad (= \text{ 4-wave vector}), \quad (4.35)$$
$$\overline{V}_4^2 = \text{inv.}, \ \overline{U}_4^2 = \text{inv.}, \ \overline{K}_4^2 = \text{inv.}, \ \overline{K}_4\overline{U}_4 = \text{inv.} = 0,$$
$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}\tilde{p}, \omega = 2\pi f, \ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}},$$
$$v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}}.$$

From (4.34) we can also develop the force 4-vectors which are Lorentz-covariant in Minkowski space,

$$\tilde{F}_4 = \gamma\frac{d\tilde{P}_4}{dt} = \left[\gamma\frac{d\tilde{p}}{dt}, \frac{\gamma}{c}\frac{d\tilde{E}}{dt}\right] = \left[\gamma\tilde{F}, \gamma\frac{\bar{v}}{c}\tilde{F}\right] =$$
$$= -\gamma\frac{d\overline{P}_4}{dt} = -\overline{F}_4 = -\left[\gamma\frac{d\vec{p}}{dt}, \frac{\gamma}{c}\frac{dE}{dt}\right] = -\left[\gamma\vec{F}, \gamma\frac{\bar{v}}{c}\vec{F}\right], \qquad (4.36)$$
$$\underline{\tilde{F}_4^{\,2} = \overline{F}_4^{\,2} = \text{inv.}, \tilde{F}_4\cdot\overline{V}_4 = -\overline{F}_4\cdot\overline{V}_4 = \tilde{F}'_4\cdot\overline{V}'_4 = -\overline{F}'_4\cdot\overline{V}'_4 = 0.}$$

All binary or two-states interactions can be treated in the most general way by applying (4.33) - (4.36). (4.37) also express Newton law of action and reaction (regarding inertial forces), $\overline{F}_4 = -\tilde{F}_4$. To expose a more general view of force law (4.36), we should treat such force as being multi-component (complex) force, as presented in (4.32), modeling it towards creating Lorentz-covariant 4-vectors in the Minkowski space.

Implicitly, from 4-vectors found in (4.34) and (4.36) we can also determine what should mean Lorentz-covariant wave function Ψ^2 in the Minkowski space. This way, we have a chance to establish higher compatibility between Quantum Mechanics and Relativity Theory. ***The full power and advantages of Minkowski 4-vectors will be exposed if we properly and creatively unite the Analytic Signal concept and 4-vectors, what is resulting in relevant complex (Analytic Signal) Phasors, analog to Phasors in electrical sciences (see more in chapter 10).***

*[♣ **COMMENTS & FREE-THINKING CORNER** (only brainstorming ideas based on analogies):*
Using analogies (see T.3.1-T.3.3) we could "invent" possible (and hypothetical) directions of further extensions of 4-space vectors (in Minkowski space) towards rotation, electromagnetism etc. Let us reformulate the 4-vector of velocity (4.33) to become a direct consequence of 4-vector of momentum,

$$\overline{V}_4 = \frac{1}{m}\overline{P}_4 = (\frac{\vec{p}}{m}, \frac{E}{mc}) = (\frac{\vec{p}}{m}, \frac{pc}{mv}) = (\frac{\gamma m\vec{v}}{m}, \frac{\gamma mc^2}{mc}) = \overline{V}(\gamma\vec{v}, \gamma c) \Rightarrow \qquad (4.38)$$
$$\Rightarrow V^2(\gamma\vec{v}, \gamma c) = \text{invariant}.$$

*In terms of rotation (based on analogies), instead of linear speed **v**, we have angular speed ω, and instead of speed **c** we should have maximal angular speed ω_c. By replacing all values from (4.38) with their analogies in rotational motion we will get:*

$$\begin{Bmatrix} \overline{\Omega}_4 = \frac{1}{\mathbf{J}}\overline{L}_4 = \overline{\Omega}(\gamma\vec{\omega}, \gamma\omega_c), \vec{L} = \gamma\mathbf{J}\vec{\omega} \\ \gamma = (1 - v^2/c^2)^{-1/2} = (1-\omega^2/\omega_c^{\,2})^{-1/2} \end{Bmatrix} \Rightarrow \overline{\Omega}^2(\gamma\vec{\omega}, \gamma\omega_c) = \text{inv.}, \qquad (4.39)$$

*In terms of the magnetic field (using conclusions based only on analogies), instead of **v** we have electric current (or "magnetic voltage") $i_{el.}$. Instead of speed **c** we would have a certain maximal electric current $i_{el\text{-}c.}$. By replacing all values from (4.38) with their analogies in the magnetic field we would formally or analogically get:*

$$\overline{I}_{el.-4} = \overline{I}_{el.}(\gamma i_{el.}, \gamma i_{el-c.}) = \overline{I}_{el.}(\gamma i_{el.}, \gamma i_{el.} \cdot \frac{v}{c}),\ i_{el-c.} = i_{el.} \cdot \frac{v}{c},$$
$$(\overline{I}_{el.-4})^2 = -(i_{el-c.})^2,\ \gamma = (1-v^2/c^2)^{-1/2} = \left[1-(i_{el.}/i_{el-c.})^2\right]^{-1/2}. \qquad (4.40)$$

*In terms of the electric field (using analogies), instead of **v** we have electric voltage (or "magnetic current") $u_{el.}$, and instead of speed **c** we would have maximal electric voltage $u_{el-c.}$. By replacing all values from (4.38) with their analogies in the electric field we will get:*

$$\overline{U}_{el.-4} = \overline{U}_{el.}(\gamma u_{el.}, \gamma u_{el-c.}) = \overline{U}_{el.}(\gamma u_{el.}, \gamma u_{el.} \cdot \frac{v}{c}),\ u_{el-c.} = u_{el.} \cdot \frac{v}{c},$$
$$(\overline{U}_{el.-4})^2 = -(u_{el-c.})^2,\ \gamma = (1-v^2/c^2)^{-1/2} = \left[1-(u_{el.}/u_{el-c.})^2\right]^{-1/2}. \qquad (4.41)$$

We could continue similar mathematical experiments (creating analogies of possible 4-space vectors) towards many other values found in T.3.1-T.3.3. The principal question, whether something like that really produces useful, logical, and correct results (or results that later can be transformed into more realistic formulas), could be analyzed some other time.

Another important conceptual difference between our usual modeling regarding mechanical movements and electromagnetic family of phenomena (where currents and voltages are involved) is that in most of the situations regarding electromagnetic phenomenology we know or understand that electric circuits are by their nature kind of fully closed circuits (Kirchoff's and Ohm's laws etc.). Regarding (mechanical) particles motions we are often talking about (somehow free hanging) linear and/or rotational motions without defining what should be their closed mechanical circuits. Both electrical and mechanical phenomena, models and motions should have similar, closed circuits nature. This is certainly in relation to Lorentz transformations and 4-vector rules, as presently formulated, and a reason for extending Lorentz and 4-vectors framework, towards unified models that would take care about the unity of linear and rotational motions (what is basically proposed here).

In the first three chapters of this book, we already established analogies between different "field charges" or values equivalent to linear and orbital moments. Let us try "analogically" to extend all such values to 4-vectors in Minkowski space. By analogy with the linear momentum 4-vector (see (2.5.1-7)) we would be able to formulate a similar relation for an orbital moment, as for instance,

$$\overline{P}_4 = m\overline{V}_4 = \overline{P}(\vec{p}, \frac{E}{c}) = \overline{P}(\gamma m\vec{v}, \gamma mc) \Leftrightarrow \overline{L}_4 = \overline{L}(\vec{L}, \frac{E}{\omega_c}) = J\overline{\Omega}_4 = (\vec{L}, \frac{\gamma J\omega_c^2}{\omega_c}) \Rightarrow$$
$$\Rightarrow \left\{\overline{P}_4^2 = \overline{P}^2(\vec{p}, \frac{E}{c}) = \vec{p}^2 - \frac{E^2}{c^2} = \vec{p}'^2 - \frac{E'^2}{c^2} = \vec{p}''^2 - \frac{E''^2}{c^2} = \text{invariant} = -\frac{E_0^2}{c^2} = -M^2c^2,\right\} \Rightarrow$$
$$\Rightarrow \left\{\overline{L}_4^2 = \overline{L}^2(\vec{L}, \frac{E}{\omega_c}) = \vec{L}^2 - \frac{E^2}{\omega_c^2} = \vec{L}'^2 - \frac{E'^2}{\omega_c^2} = \vec{L}''^2 - \frac{E''^2}{\omega_c^2} = \text{invariant} = -\frac{E_0^2}{\omega_c^2} = -J^2\omega_c^2\right\},$$
$$E^2 = E_0^2 + \vec{p}^2c^2 = E_0^2 + \vec{L}^2\omega_c^2,\ E_0 = mc^2 + E_s = Mc^2 = J\omega_c^2,$$
$$\vec{p} = \gamma M\vec{v},\ \vec{L} = \gamma J\vec{\omega} = \vec{p} \times \vec{R},$$
$$dE = dE_k = vdp = \omega dL,\ pdv = Ld\omega,\ vdp + pdv = \omega dL + Ld\omega,$$
$$E_k = E - E_0 = (\gamma-1)Mc^2 = \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} = (\gamma-1)J\omega_c^2 = \frac{L\omega}{1+\sqrt{1-\frac{v^2}{c^2}}},$$
$$\vec{v} = \vec{v}_t = \vec{\omega}R,\ pv = L\omega,\ pc = L\omega_c,\ \frac{c}{\omega_c} = \frac{v}{\omega} = \sqrt{\frac{J}{M}},\ \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{\omega^2}{\omega_c^2}}}. \qquad (4.33\text{-}1)$$

In reality, the same particle could have linear, orbital and electromagnetic moments (at the same time), and we would need to find a way to address such complex moments reality, for instance, by proposing united 4-vectors and scalar fields formulation. Based on analogies, we could propose the following table, T.4.3.2, just to stimulate creative curiosity.

T.4.3.2	**[Q] = CHARGES / MOMENTS**	**Meaning**	**4-vectors: Moment / Force** (just ideas, proposals)
Gravitation & Linear Motion	**p = mv**	**Linear moment**	$\overline{P}_4 = \overline{P}(\vec{p}, \frac{E}{c}) = m\overline{V}_4,$ $\overline{F}_4 = \gamma \frac{d\overline{P}_4}{dt} = (\gamma\vec{F}, \frac{\gamma}{c}\frac{dE}{dt}) = (\gamma\vec{F}, \gamma\vec{F}\cdot\frac{\vec{v}}{c})$
Spinning	**L = Jω**	**Orbital moment**	$\overline{L}_4 = \overline{L}(\vec{L}, \frac{E}{\omega_c}) = J\overline{\Omega}_4,$ $\overline{\tau}_4 = \gamma \frac{d\overline{L}_4}{dt} = (\gamma\vec{\tau}, \frac{\gamma}{\omega_c}\frac{dE}{dt}) =$ $= (\gamma\vec{\tau}, \frac{\gamma}{\omega_c}\vec{\tau}\cdot\vec{\omega}) = (\gamma\vec{\tau}, \gamma\vec{\tau}\cdot\frac{\vec{v}_t}{c})$
Electric Field	**$\Phi_{el} = q_{el.}$**	**Electric charge**	**? ! (to think about)** $\overline{Q}_{el.-4} = \int \overline{I}_{el.-4} dt = \overline{Q}(q_{el.}, q_{el.-c}) =$ $= \overline{Q}(\int \gamma i_{el.} dt, \int \gamma i_{el.} \cdot \frac{v}{c} dt),$ $\overline{I}_{el.-4} = \overline{I}_{el.}(\gamma i_{el.}, \gamma i_{el.} \cdot \frac{v}{c})$
Magnetic Field	**$\Phi_{mag} = q_{mag.}$**	**Magnetic flux**	**? ! (to think about)** $\overline{\Phi}_{4\text{-}mag.} = \int \overline{U}_{el.-4} dt = \overline{\Phi}(q_{mag.}, q_{mag\text{-}c}) =$ $= \overline{\Phi}(\int \gamma u_{el.} dt, \int \gamma u_{el.} \cdot \frac{v}{c} dt),$ $\overline{U}_{el.-4} = \overline{U}_{el.}(\gamma u_{el.}, \gamma u_{el.} \cdot \frac{v}{c})$

Obviously, for scalar values of electric charge and magnetic flux (in T.4.3.2) it would not be analogically applicable to place them directly in a format of the 4-vectors of Minkowski-space (as here presented), but just for the purpose of brainstorming (and maybe in relation to some future redefinition of such entities, which would give them more of vectors meanings), here is initiated the first step. ♣]

4.3.8. Mass, Particle-Wave Duality and Real Sources of Gravitation

By applying different analogies all over this paper we see that mass and gravitation somehow "avoid" being simply presentable, following similar patterns of other natural forces and their sources (or charges). It looks (based upon here established analogies in earlier chapters) that real sources of gravity (between particles with non-zero rest masses) should be mutually coupled, relevant linear and orbital moments, also coupled with other electrodynamic parameters such as involved electric charges and magnetic fluxes. This is contrary to the common opinion that pure, static or standstill rest masses are proper gravity charges, or primary sources of Gravitation (based on a direct analogy with Coulomb laws for electric and magnetic charges. See also Chapter 2: GRAVITATION, starting from equations (2.3) to (2.4-3)).

In order to understand better what means mass, let us summarize the most important expressions concerning the energy of a particle (moving in a free space):

$$\begin{aligned} &E = E_{total} = E_0 + E_k = \gamma mc^2 = \sqrt{E_0^2 + p^2c^2} = m_{total}c^2 = m_t c^2, \\ &E_k = E - E_0 = (\gamma - 1)mc^2 = E(1 \pm \sqrt{1 - \frac{p^2c^2}{E^2}}) = m_{motional}c^2 = m_m c^2, \\ &E_0 = mc^2 = m_0 c^2, \end{aligned} \tag{4.41-1}$$

The most general understanding of the mass concept is to present mass just as another form of *"particle energy storage (devised by the constant c^2)"*, where $\mathbf{m_t}$ is the total mass, $\mathbf{m_m}$ is the motional mass and $\mathbf{m} = \mathbf{m_0}$ is the initial rest mass:

$$\begin{aligned} &m_t = \frac{E}{c^2} = \gamma m = m^* = m + m_m = m + \tilde{m} = \frac{1}{c^2}\sqrt{E_0^2 + p^2c^2} = \frac{E_0}{c^2}\sqrt{1 - \frac{p^2c^2}{E_0^2}}, \\ &m_m = \tilde{m} = \frac{E_k}{c^2} = (\gamma - 1)m = m_t - m = \frac{E}{c^2}(1 \pm \sqrt{1 - \frac{p^2c^2}{E^2}}), \\ &m = m_0 = \frac{E_0}{c^2}. \end{aligned} \tag{4.41-2}$$

This way, the same mass concept can be extended to all constituents of our universe, such as elementary particles and quasi-particles that do not have a rest mass (like photons), as well as to energy of fields and waves around particles. We could also describe the mass like a "*space-time-spread*" entity. Here, minimal internal mass content captured by internal particle space (by a particle geometry, or by its mechanical or solid boundaries) equals $\mathbf{m}$, and external mass content, spread in the space around the particle, equals $(\gamma - \mathbf{1})\mathbf{m}$, what effectively (for moving particle) makes the total (relativistic) particle mass equal $m_t = \gamma m = m^* = m + (\gamma - 1)m = m + \tilde{m}$. This kind of conceptualizing is directly linked to a particle-wave duality of matter, as imaginatively described in Chapter 10. under "*10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality*". Of course, here are only introduced ideas and indicative directions how to understand mass, energy, and wave-particle duality.

Let us now transform relations for energy and mass into relations applicable to photons,

$$\tilde{E} = E_p = E = E_{total} = E_0 + E_k = E_k = \tilde{p}c = m_t c^2 = \tilde{m}c^2,\ \tilde{m} = m_t = \frac{\tilde{E}}{c^2} = \frac{\tilde{p}}{c} = m_p,$$
$$\tilde{E} = E_p = E_k = E - E_0 = E = E(1 \pm \sqrt{1 - \frac{p^2c^2}{E^2}}) = m_m c^2 = \tilde{m}c^2, m_m = \tilde{m} = \frac{\tilde{E}}{c^2} = \frac{\tilde{p}}{c}, \quad (4.41\text{-}3)$$
$$E_0 = 0,\ \ m = m_0 = m_{p0} = \frac{E_0}{c^2} = 0.$$

Now, it is possible to introduce a new understanding of a stable rest mass as the form of motional (wave) energy, conveniently stabilized and (for instance) captured and packed into a big number of more elementary vortex or rotating formations of self-sustaining, self-closed stationary and standing waves (see similar and complementary conceptualization in Chapter 2., under "*2.3. How to account Rotation concerning Gravitation*").

All stable-mass particles also have natural, external electromagnetic and matter-waves field couplings, and different energy-mass-momentum communications with a surrounding Universe.

If the total *internal wave energy content* of a certain particle is, $\mathbf{E}_{\mathbf{k}-\text{int.}} = \tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{\text{int}} = \tilde{\mathbf{m}}_{\text{int.}}\mathbf{c}^2$, then the particle rest mass should be $\mathbf{m}_0 = \tilde{\mathbf{m}}_{\text{int.}} = \frac{\tilde{\mathbf{E}}_{\text{int}}}{\mathbf{c}^2}$. This way, we can equally treat "internal rest mass", and "externally spread (moving) mass", since externally spread mass is already defined as an equivalent to a kinetic particle energy (devised by $\mathbf{c}^2$). The equality of mass treatments is again valid, since in both cases (when we calculate mass), we are talking about kinetic or motional (or wave) energy, but:

1° For a rest mass we are considering or using as relevant only an internal wave-formation, or internal motional energy content, and

2° For a mass equivalent corresponding to an "external" particle motion we will consider its "external" particle kinetic energy $\mathbf{m}_\mathbf{m} = \frac{\mathbf{E}_\mathbf{k}}{\mathbf{c}^2} = (\gamma - \mathbf{1})\mathbf{m}$.

Of course, this situation (regarding mass concept) becomes much more general and better unified if we treat the particle kinetic energy also as a form of wave energy (what presents the proper meaning of the **P**article-**W**ave **D**uality in this book). In case of an elementary particle, we will have,

$$\begin{aligned}
&m_0 = \tilde{m}_{int.} = \frac{\tilde{E}_{int}}{c^2} = \frac{h\langle f_{int.}\rangle}{c^2}, \\
&m_m = m_{ext.} = \frac{E_k}{c^2} = (\gamma - 1)m = \frac{\tilde{E}_{ext.}}{c^2} = \frac{h\langle f_{ext.}\rangle}{c^2} \\
&E_k = (\gamma - 1)mc^2 = \tilde{E}_{ext.} = \tilde{E} = h\langle f_{ext.}\rangle = hf \\
&m_t = \tilde{m}_{int.} + m_{ext.} = \frac{E}{c^2} = \frac{\tilde{E}_{int} + \tilde{E}_{ext.}}{c^2} = \frac{E_0 + E_k}{c^2} = \gamma m = m^*
\end{aligned} \tag{4.41-4}$$

Here used terminology “internal and external” motional (or wave) energy is maybe not the best choice of names, but here it serves the need for simplified and faster (or an indicative) explanation of the fact that what we consider as particles (with our instruments), “externally” only looks like as particles, but “internally” such entities have their proper wave nature. Also, similar wave nature should exist "externally", as a motional energy of particles (being fully connected or coupled with internal wave structure of a relevant particle in motion). Rotation, spinning, and wave packing is what makes the difference between particles with rest masses and freely propagating waves.

Finally, the mass concept (as here introduced) tells us that in the background of what makes our universe should only be a certain kind of kinetic, motional or wave energy, “**assembled in different packing, stabilized or free propagating, and/or resonant mass-energy-momentum formats**”.

Particularly interesting consequences of previous assumptions and analyses are extending **the PWDC** concept to the internal macro-particles (or elementary mass) domains, meaning that:

A) A single particle has its "external" kinetic or waves energy content, $\mathbf{E_k = (\gamma - 1)mc^2 = \tilde{E}_{ext.} = h\langle f_{ext.}\rangle = \tilde{E} = hf}$ *, and*

B) The same particle can be at the same time "internally excited" on many ways (for instance, by heating, or passing electrical currents and/or mechanical vibrations through it), this way getting an elevated content of internal wave energy, $\tilde{\mathbf{E}}_{int.} = \mathbf{hf}_{int.}$*. Consequently, the internally excited particle would once become able to radiate the surplus of its "internal" wave energy (when necessary conditions will be satisfied), what really happens in many of well-known experimental situations. Sometimes, the surplus of internally excited wave states is so high that a macroparticle becomes a spontaneous radiator of real (smaller) particles, such as electrons,* α*-particles etc., besides radiating photons (all of that leading to the conclusion that differences between real particles that have stable rest masses, and pure waves without rest mass content, is not in their essential nature, but only in their appearance, or in a way of “**energy packing**”).*

Until here, we presented only a kind of simplified and approximate picture about real masses. It is already empirically known that relativistic mass descriptions have some not completely explained nature, exceptions, and deviations. For instance, by heating certain mass we have both negative and positive contributions of the total mass (see more in [101]), but Relativity Theory is confirming only the positive contribution of added heat energy on a total mass (see more in the Chapter 9. of this book).

Quote (The Negative Temperature Dependence of a Gravity is a Reality, Professor Alexander L. Dmitriev and Sophia A. Bulgakova, [101]): "Temperature dependence of force of gravitation - one of the fundamental problems of physics. The negative temperature dependence of weight of bodies is confirmed by laboratory experiments and like Faraday phenomenon in electrodynamics is a consequence of natural "conservatism" of a physical system, its tendency to preserve a stable condition. Realization of experimental research of the influence of the temperature of bodies on their gravitational interaction is timely and, undoubtedly, will promote the progress of development of physics of gravitation and its applications".

Mass is also a property dependent on internal packing, formatting, and spatial configuration, of involved fields and binding energy of its constituents, meaning that stable, static, and constant mass only conditionally exist (within a certain set of conditions). For instance, mass is composed of atoms (and molecules), and atoms are composed of neutrons, protons, and electrons. Usually (in most of the cases) an atom has less of its total mass content than if we calculate a sum of all masses of its neutrons + protons + electrons, considering them as independent (or isolated) elements, and in physics, this is specified as the *atomic or nuclear mass defect*. However, there are also minor exceptions for a certain number of stable atoms, when the situation with mass defect has the opposite sign, meaning that the rest and stable mass content of specific particles only conditionally exist (influenced with specific internal fields and waves packings and configurations). ***We probably need to consider that real mass-energy-momentum boundaries (including associated electromagnetic items) are larger than what we consider only as a strict mechanical, geometric, static mass boundaries). This will also affect our understanding of Uncertainty Relations.***

We should not forget that the ontological background of all discussions regarding different forms of energy, mass and matter waves should have a relatively simple conceptual framework, as already presented on the Fig. 4.1.2, and with energy conservation (4.5-3). All higher-level and more sophisticated concepts (related to mass-energy states) should have their roots there. *See Chapter 10; - (PARTICLES AND SELF-CLOSED STANDING MATTER WAVES) addressing similar problematic, where it is clearly underlined that mass should have Complex (or Hypercomplex) nature, well integrated with matter-waves and wave-particle duality properties (as shown around equations: (10.1.1) to (10.1.5)).*

*[♣ **COMMENTS & FREE-THINKING CORNER:** Let us now try to show how linear and orbital moments could assist the creation of rest masses. We can analyze a stable and neutral particle (consisting of one or many atoms, or a big number of different elementary entities) in the state of relative rest, which has a mass* $\mathbf{m}$*. We can also say that such particle consists of different molecules, atoms, electrons, nuclear and sub-nuclear particles, and fields, all of them moving, circulating, spinning, or oscillating inside the "particle shell". Externally, it appears that a particle with rest mass is stable, electrically neutral, and in the state of rest. Consequently, total linear and orbital (internal) particle moments,* $\vec{P}_{int.-total}, \vec{L}_{int.-total}$ *(looking externally, outside of the "particle shell"), should be close to zero, or should have negligibly small values (if we make a vector sum of all internal particle moments), as for instance,*

$$\vec{P}_{int.-total} = \sum_{(i)} \vec{p}_i = \sum_{(i)} m_i \vec{v}_i \to 0 \ , \ \vec{L}_{int.-total} = \sum_{(i)} \vec{L}_i = \sum_{(i)} J_i \vec{\omega}_i \to 0 \ . \qquad (4.42)$$

For the particle in the state of rest we could also estimate its center-of-mass linear speed, and center-of-inertia angular speed, $\vec{v}_c$, $\vec{\omega}_c$ *that should again be negligibly small or close to zero values (when the particle is globally in its rest state),*

$$\vec{v}_c = \frac{\vec{P}_{int.-total}}{\sum_{(i)} m_i} = \frac{\sum_{(i)} m_i \vec{v}_i}{\sum_{(i)} m_i} \to 0 \ , \ \vec{\omega}_c = \frac{\vec{L}_{int.-total}}{\sum_{(i)} J_i} = \frac{\sum_{(i)} J_i \vec{\omega}_i}{\sum_{(i)} J_i} \to 0 \ . \qquad (4.43)$$

Now, combining (4.42) and (4.43) we can find the total mass and total moment of inertia of the particle in the state of rest as the result of an internal superposition and mutual interferences of all its internal, linear and orbital moments,

$$m = \sum_{(i)} m_i = \frac{\left|\vec{P}_{int.-total}\right|}{\left|\vec{v}_c\right|} = \frac{\left|\sum_{(i)} m_i \vec{v}_i\right|}{\left|\vec{v}_c\right|} = \frac{E_0}{c^2} = \frac{1}{c^2}\int_{[P]} \vec{v}d\vec{p} = const. \ ,$$

$$J = \sum_{(i)} J_i = \frac{\left|\vec{L}_{int.-total}\right|}{\left|\vec{\omega}_c\right|} = \frac{\left|\sum_{(i)} J_i \vec{\omega}_i\right|}{\left|\vec{\omega}_c\right|} = \sum_{(i)} m_i r_i^2 = \frac{E_0}{\omega_c^2} = \frac{1}{\omega_c^2}\int_{[L]} \vec{\omega} d\vec{L} = Const. \qquad (4.44)$$

Effectively, in (4.44) we have divisions between two values; both negligibly small, but the results of such divisions are constant and realistically high numbers, meaning that matter or mass should always be in some motional state.

Just for the purpose of creating certain quantifiable and simple mathematical forms (at least dimensionally correct), we could "invent" the following indicative relations (that would be later, most probably modified, but presently are good enough to show that stable particle in a state of rest is intrinsically composed of permanently moving and rotating internal wave-energy states):

$$E_o = J\omega_c^2 = mc^2 = \int_{[P-int.]} \vec{v}d\vec{p} = \int_{[L-int.]} \vec{\omega}d\vec{L} = hf_c \Rightarrow \omega_c = c\sqrt{\frac{m}{J}} = \frac{c}{r^*} = 2\pi f_c \Rightarrow$$

$$\Rightarrow f_c = \frac{c}{2\pi}\sqrt{\frac{m}{J}} , \ c = \lambda_c f_c \Rightarrow \lambda_c = 2\pi r^* . \qquad (4.44\text{-}1)$$

If we go back to the Newton law of gravitation between two masses (see (2.3), (2.4) - (2.4-3), in Chapter 2., we can conclude that there we explicitly deal with masses, but implicitly also with their linear and orbital moments, like in (4.44), and with many other dynamic properties of internal (and external) mass constituents. There are some other, more profound (macrocosmic and microcosmic) consequences of such active mass modeling, especially if in (4.44) we apply Minkowski space 4-vectors, (4.32) - (4.37), in order to establish a more complex active mass modeling, as speculated in Chapter 10., under "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality" (see also (2.3) - (2.4-3), (4.5-1) and (5.15) - (5.17)).

Under certain conditions, we know that rest mass can be created combining the energy states (matter waves) that do not have their own rest masses, as for instance,

$$m = m_0 = \frac{1}{c^2}\sqrt{E_t^2 - P_t^2 c^2} \ , \ E_t = \sum_{(i)} E_{t-i} \ , \ \vec{P}_t = \sum_{(i)} \vec{P}_{t-i} \ . \qquad (4.44\text{-}2)$$

By analogy (presently without claiming that such result would be correct) we could create similar relation for a corresponding static moment of inertia,

$$J = J_0 = \frac{1}{\omega_c^2}\sqrt{E_t^2 - L_t^2\omega_c^2}\ ,\ E_t = \sum_{(i)} E_{t-i}\ ,\ \vec{L}_t = \vec{r}^* \times \vec{P}_t = \sum_{(i)} \vec{L}_{t-i}\ ,\left(\left|\vec{L}_{t-i}\right| = \frac{c^2}{r^* \omega_c^2}\right). \qquad (4.44\text{-}3)$$

By combining (4.44-2) and (4.44-3) we get,

$$mc^2 + P_t^2 c^2 = J\omega_c^2 + L_t^2\omega_c^2 = E_t^2 = \gamma\, mc^2 \quad . \qquad (4.44\text{-}4)$$

Most probably that combining (4.44-4) and (4.33-1) we would be able to find more appropriate relations that show that linear and rotational motion elements are well synchronized and united (or coupled like mutually conjugate functions, producing what we consider as a wave-particle duality).

Like in (4.44), using analogies given in T.3.1-T.3.3, $\mathbf{m} \leftrightarrow \mathbf{C},\ \mathbf{J} \leftrightarrow \mathbf{L}$*, we could later create new formulations of electric capacitance and magnetic inductance. This situation can also be compared to an atom structure: Externally, far-enough, in a space around an atom, we could measure that atom is electrically neutral and in the state of relative rest, although inside the atom structure everything is waving and moving (and that there is no electromagnetic neutrality there). The next step is to conclude that a total energy of active (or moving) atom constituents, conveniently integrated into a stable and (externally) neutral atom, creates atom mass (where rest mass can be presented using Einstein relation* $E_{tot.} = mc^2$*). It is also clear that very particular dynamic "gearing and fitting" between atom constituents should be realized to create an atom (and this gearing and resonant fitting we are presently explaining as a quantum or discretized nature of matter; -see also familiar Wilson-Sommerfeld rules, (5.4.1)).*

The same situation could also be analyzed using generalized Schrödinger equation, and equations of Relativistic Electrodynamics (producing more complex mathematical picture, than here presented), but important qualitative, phenomenological, and conceptual aspects of particle-wave duality and action-reaction forces would not be clearly and simply identified as here presented.

We could also conclude that de Broglie matter waves (belonging to moving particles) should be very much present inside an atom structure, complementing the above-mentioned dynamic gearing and fitting among atom constituents. A similar concept can be extended to any other stable particle. The same statement differently formulated is that sources of de Broglie matter waves should be found inside an internal structure of particles, in the form of standing wave formation of internal particle structure (externally manifesting as orbital moments and spin attributes). When a particle changes its state of motion, intrinsic, internal wave formation (previously being in the state of stationary and standing waves structures) is becoming an active, de Broglie matter wave, producing externally measurable consequences, as for instance: Photoelectric and Compton Effect, particles diffraction…

Every wave motion has its equivalent mass that can be found (combining (4.17-1) and (4.44)) as,

$$m^* = \frac{\tilde{E}}{uv} = \frac{\left[\int_0^\infty [A(\omega)]^2 d\omega\right]^3}{\pi\left[\int_0^\infty \frac{\omega}{k}\cdot[A(\omega)]^2 d\omega\right]\left[\int_0^\infty \frac{d\omega}{dk}\cdot[A(\omega)]^2 d\omega\right]}$$ *, or in case of electromagnetic waves,*

$$m^* = \frac{\tilde{E}}{uv} = \frac{\tilde{E}}{c^2} \qquad (4.45)$$

Intuitively, we see that a stable particle (which has stable rest mass) should have certain stationary waving structure (internally organized and balanced), and we also know that in many interactions' stable particles manifest particle-wave duality properties. Moreover, particles could be disintegrated into pure wave-energy constituents. It is also known that a convenient superposition of pure wave elements could produce a stable particle (electron-positron creation, for instance). Consequently, the general case of a stable particle should be that its internal constituents are composed of wave-mass *elements in a form of (4.45). The proper internal and dynamic "gearing and fitting" of all particle constituents should produce a stable particle, which has stable rest mass (found by applying the rules of Relativity Theory: -using known connections between total energy, rest energy and particle momentum). Of course, there are many intermediary and mixed particle-wave states and objects, which sometimes behave more as particles or as waves.*

*Obviously, particle-wave duality concept favored in this book creates sufficiently clear frontier between stable particles (which have constant rest mass in their center of mass reference system), and wave or particle-wave phenomena that belong only to different states of motion. Contrary to this position, we also know that internal structure of a stable particle is composed of wave and particle-wave constituents, which are properly "geared and fitted" (looking only externally) producing a stable particle (**based on standing waves, resonant field structures**). What is missing in such conceptual picture of particle-wave duality is to explain conditions when certain dynamic combination (superposition) of waves transforms into a stable particle; -for instance, when wave mass, given by (4.45), will create a stable particle (**by closing an open and relatively free propagating waveform into itself, internally structured as standing waves, or self-stabilized field in resonance**). The cornerstones and frames for wave-to-particle transformation should be found in the following ideas:*

1. ***A stable particle will be created when a wave-mass (4.45) is transformed into a constant (externally measurable and stable) rest mass, $\mathbf{m} \rightarrow \mathbf{m_0}$, that is time independent and localized in certain limited space (satisfying also (4.44)).***
2. ***Stable particle should have (in its Center of Mass System) non-zero rest mass.***
3. ***The center-of-mass velocity of all internal wave-particle constituents (in its Laboratory System) should be equal zero.***
4. ***Rest mass (created from a wave group as kind of superposition and interaction between involved wave groups) can be formulated based on Minkowski-space 4-vector relation between total energy and momentum, as:** $m = m_0 = (\sqrt{E^2_{tot.} - P^2_{tot.}c^2})/c^2$**...** Here we can make another conceptual step regarding wave-packing nature of stable particles that have non-zero rest masses. Let us again start from the general relation between mass, total energy and total momentum presented as:*

$$mc^2 = E_0 = \sqrt{E^2_{tot.} - P^2_{tot.}c^2} = \sqrt{E^2 - p^2c^2} = const. \Rightarrow$$
$$\Rightarrow E_0^2 = E^2 - p^2c^2 = (E - pc)\cdot(E + pc) = \gamma mc(c - v)(c + v) = Const.$$

In other words, we could state that a rest mass $\mathbf{m}$ is created as superposition (cross-correlation, or interaction) between two wave groups propagating in mutually opposed directions, that have corresponding time-space domain functions $\overline{\psi}_1(t,x) = \overline{\psi}(\omega t - kx)$ and $\overline{\psi}_2(t,x) = \overline{\psi}(\omega t + kx)$.

Now relations between mutually corresponding time and frequency domains can be conceptually presented (carrying an over-simplified brainstorming message) as,

$$E_0^2 (\Leftrightarrow) \begin{Bmatrix} E_0 \cdot E_0 \\ \overline{\psi}(t); \overline{\psi}(t) \end{Bmatrix} (\Leftrightarrow) \begin{Bmatrix} (E - pc)\cdot(E + pc) \\ \overline{\psi}_1(t,x); \overline{\psi}_2(t,x) \end{Bmatrix} (\Leftrightarrow) \begin{Bmatrix} \text{Energy} \\ \text{time - space} \end{Bmatrix}$$
$$E = E(\omega, k)\ ,\ p = \frac{h}{2\pi}k = \frac{h}{\lambda}$$

$$\begin{aligned}
&\overline{\psi}_1(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U_1(\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U(E-pc)e^{j\omega t}d\omega=a(t)e^{j\Phi(E-pc)}\\
&\overline{\psi}_2(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U_2(\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U(E+pc)e^{j\omega t}d\omega=a(t)e^{j\Phi(E+pc)}\\
&\overline{\psi}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U_1\cdot U_2e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U(E,pc)e^{j\omega t}d\omega=a(t)e^{j\Phi(E,pc)}=\\
&=\int_{-\infty}^{+\infty}\psi_1(t)\cdot\psi_2(t-\tau)e^{j\omega t}d\omega=\int_{-\infty}^{+\infty}\psi_1(t-\tau)\cdot\psi_2(t)e^{j\omega t}d\omega.
\end{aligned} \tag{4.46}$$

It is almost obvious that a stable rest mass is created as the space-time cross-section (or convolution) of the mutually phase-shifted wave functions $\psi_1(t,x)$ and $\psi_2(t,x)$. In this way, the common space-time variable wave content is captured into a time stable wave packing format. In other words, here we are on a way to conceptualize how waves of matter could create stable particles, and later (using similar methodology) we could also start thinking about how particles could be transformed into waves.

*Most probably that by continuing this process we can find other important conditions for wave-to-particle transformation, related to solutions of Schrödinger-type wave equations. As we know, the Orthodox Quantum Mechanics does not respect too much the frontiers between waves and particles, treating all particles and particle-wave objects as waves (including a rest mass), and associating the proper wave function to every "quantum entity". This **is basically correct in the framework of Quantum Theory, but still, the difference should exist between waves that are unbounded and propagating relatively free, and when the same wave structure is internally self-closed, creating standing waves resonant field structure, spatially localized**). This is one of the differences between the particle-wave duality explained here and official particle-wave duality found in today's Quantum Theory (since in this book the rest mass and rest energy are excluded from the free propagating wave energy). Luckily (for Quantum Theory), in many practical situations, the mentioned difference is even not noticed (or not too much relevant) since when creating differential wave equations (as Schrödinger equation) we are simply eliminating constant mass or constant energy members (since differentiation of constants is zero).*

Here, the main intention has been to introduce sufficiently clear intuitive and conceptual message about real nature of mass on a simple way, indirectly underlying that Gravitation, as presently supported in Physics, could be (or should be) differently established, since the meaning of mass needs to be significantly updated. It is also clear that one should invest much more work to make the mentioned mass concept more operational.

As we can see, the differentiation between mass and energy related to a particle (or wave group) in different states of motion should be more systematically and profoundly analyzed if we would like to capture by the same modeling the unity of linear and rotational motions. To start realizing such project, an initial step could be to encounter all possible, mutually distinctive states of motion, as for example, listed in the T.4.3.3, and illustrated on the Fig.4.3.1. Eventually, we will be able to specify at least 16 of energy-moments mass states that are mutually different and mutually related (16 states in the T.4.3.3, from 1.0.1 to 4.3.2), showing the basic relations and connections (or complementary nature) between linear motion and rotation. See in Chapter 10., "Particles and self-closed standing matter waves", specifically dedicated to similar problematic.

T.4.3.3. The chart of possible particle states (only a brainstorming)

<table>
<tr><th>Different motional states of a stable particle</th><th>1. Standstill state</th><th>2. Linear Motion</th><th>3. Rotational Motion</th><th>4. Combined Linear Motion and Rotation</th></tr>
<tr><td rowspan="3">From the point of view of different observers (see 1. and 2., below):</td><td rowspan="3">1.0. No external motion. The particle is in the state of rest, looking from the outside space.</td><td rowspan="3">2.0. A particle as a whole is only in linear motion relative to a certain external system of reference.</td><td>3.1. A particle as a whole is only rotating around a certain point, relative to the certain external system of reference (or center, which is in external space, not captured by the particle). No linear motion.</td><td>4.1. A particle as a whole is in linear motion and at the same time rotating relative to a certain external system of reference (or center, which is in external space, not captured by the particle).</td></tr>
<tr><td>3.2. A particle as a whole is only rotating around itself (or around its center of gravity, or around one of its axes). No linear motion.</td><td>4.2. A particle as a whole is in linear motion and at the same time rotating around itself, relative to a certain (moving) system of reference that is inside the domain captured by the particle.</td></tr>
<tr><td>3.3. A particle as a whole is rotating around itself (or around its center of gravity, or around one of its axes), and at the same time also rotating around a certain point which is in external space, not captured by the particle (performing a multi-component rotational motion). No linear motion.</td><td>4.3. A particle as a whole is in linear motion and at the same time rotating relative to a certain external system of reference, and also rotating around itself, relative to a certain (moving) system of reference that is inside the domain captured by the particle.</td></tr>
<tr><td>Comments:</td><td>Only internal particle constituents or matter waves are in complex motion (united rotation and linear motion inside the particle structure; -nothing of that being visible externally).</td><td colspan="3">No externally visible rotation exists; -internal particle structure and internal matter-wave motions are not visible externally.</td></tr>
<tr><td rowspan="3">1. The (virtual) observer who is placed inside the particle structure</td><td rowspan="3">1.0.1.</td><td rowspan="3">2.0.1.</td><td>3.1.1.</td><td>4.1.1.</td></tr>
<tr><td>3.2.1.</td><td>4.2.1.</td></tr>
<tr><td>3.3.1.</td><td>4.3.1.</td></tr>
<tr><td rowspan="3">2. An observer who is placed in the external particle space (external, independent system of reference, not captured by the particle domain).</td><td rowspan="3">1.0.2.</td><td rowspan="3">2.0.2.</td><td>3.1.2.</td><td>4.1.2.</td></tr>
<tr><td>3.2.2.</td><td>4.2.2.</td></tr>
<tr><td>3.3.2.</td><td>4.3.2.</td></tr>
</table>

The same energy state (particle, wave packet…) could have many mutually coupled levels of its (internally and externally), energetically atomized structure, where each level would have its own linear and a rotational couple of motional components, symbolically visualized on the Fig.4.3.1 with 4 of such levels (see also chapter 6., MULTIDIMENSIONALITY, where an attempt is made to formulate similar concepts mathematically).

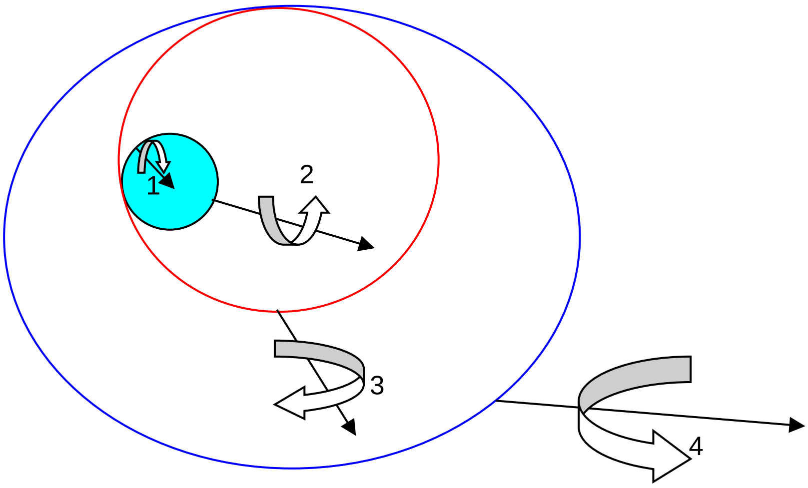

Fig.4.3.1. Symbolic visualization of multilevel linear-rotational motional couples ♣]

5. UNCERTAINTY RELATIONS AND ELEMENTARY MATTER DOMAINS

Until present, in this book we have been effectively exploiting different analogies, including interactions, couplings and complementary nature between Mechanics, Electromagnetism, Waves and Particles, while respecting the most essential Symmetries of Physics (see Chapter 1, T.1.8, Generic Symmetries and Analogies of the Laws of Physics).

Before we start discussing and analyzing *universally valid Uncertainty Relations in Mathematics and in all domains of micro and macro world of Physics*, let us first explain what kind of entities in our Universe, or in mathematics, Uncertainty Relations are addressing, as follows.

Every signal, wave-group, particle, or energy-momentum state has its spatial, temporal, and spectral durations. Briefly saying, Uncertainty Relations in this book, as well as in Mathematics and Physics, deal with quantifiable relations between signal (or wave function) absolute (or total) intervals-durations (or lengths) belonging to couples of mutually Original and Spectral domains, regardless of this is the world of atoms and micro-particles, or everything else that belongs to a macroworld of planets and galaxies. In this book we assign more importance to relations between mentioned total signal durations (in all domains) than to statistically presentable durations based on standard deviations.

We also know (from Signal Analysis) that signals with Gaussian, or *like bell-curve envelopes* (both in the original and spectral domain), have finite lengths or (temporal-spatial and spectral) durations in all mentioned domains. This is kind of stability and structural compactness condition producing that any stable particle in motion, elementary particles, and other relatively stable energy-momentum states, including their matter-wave packets, are well-localized, energy-finite, non-dispersive, and precisely defined in all relevant domains. In such cases **Uncertainty** or Inequality relations can be considered as **Certainty** relations.

There are different platforms for understanding Uncertainty Relations. Contemporary Physics is starting with Heisenberg Uncertainty concepts (anyway, initially borrowed from mathematics, then imaginatively modified to address only Quantum Theory items in microphysics, and then retroactively, too often and falsely, presented as an original, only microphysics-related concept, wrongly implicating those mathematical relations of uncertainties (or inequalities) as almost exclusive consequences of Heisenberg Uncertainty assumptions.

Here, we will start from the generally valid, universally applicable, mathematical grounds of Uncertainty or Inequality relations, regarding mutually *Conjugate Domain Duration Relations*, since starting from there we can develop and understand all other physics-related Uncertainty Relations applicable in our Universe (see an introduction about Uncertainty relations in Chapter 4.0, and later in this chapter, under: “5.1. Uncertainty in Physics and Mathematics”, under (5.5)).

In addition, it is useful to mention that Quantum Theory (**QT**) Uncertainty Relations, as formulated in the early days of **QT** foundations, are slightly different, when compared

All over this paper are scattered small comments placed inside the squared brackets, such as:
[♣ COMMENTS & FREE-THINKING CORNER… ♣]. The idea here has been to establish intuitive and brainstorming, not-confirmed and freethinking corners for making quick comments, and presenting challenging ideas that could be some other time developed towards something much more meaningful and more appropriately integrated into Physics.

to what we find in the contemporary Quantum Theory publications (since the same concept evolved too much towards statistics-theory interpretations). The reasons are that priority has been gradually given only to Orthodox Quantum theory stochastics concepts (and not given to the much more generally valid, clearer, and forever stable mathematical concept of universally valid Uncertainty relations dealing with total domains durations). In mathematics, Uncertainty (or inequality) relations are originally related to absolute (or total) durations of certain signal in its time and frequency or spectral domains (see (5.5)). After we apply such relations to an elementary (narrow-band shaped) wave-packet, or *matter-wave energy quant*, or to corresponding wave-group (which has wave energy equal to $\tilde{E} = hf$ or $\tilde{E} = Hf$; $[h , H]$ (=) constants, and respects other **PWDC** relations, as summarized in Chapter 10.), we are directly creating Physics-related Uncertainty Relations (without a need to rely on Heisenberg's and contemporary Quantum Theory foundations and associated, statistics and probability related assumptions).

Mathematically, using generally applicable knowledge from Statistics, Probability theory, and Signal-Spectrum analysis, we can define Uncertainty relations or inequalities concerning average and mean values, standard deviations, and signal spectral distributions. All of that should be equally (mathematically) applicable to a micro and macro-Universe around us. Nothing essential (except an erroneous conceptualization) is supporting "*Uncertainty exclusivity*" to belong only to a microworld of Quantum Theory. ***It will be enough to study modern Informatics, Signal Analysis, Telecommunication theories, signals sampling, and signals reconstruction rules, based on Kotelnikov-Shannon-Nyquist-Whittaker theory to understand the nature of Uncertainty relations between durations of different signal domains. This (about signals sampling and reconstruction) has bigger significance than, both mathematical and Quantum-Theory Uncertainty relations. Of course, in a time of Heisenberg, not many facts about Kotelnikov-Shannon-Nyquist-Whittaker theorems and concepts were known or being completely formulated and clear. Anyway, Heisenberg had a chance to formulate his best and successful, imaginative guesswork based either on mathematical inequality or uncertainty relations between signal-durations of mutually conjugate spectral domains, or based on his understanding of commutation properties of matrices in linear algebra. Heisenberg assigned mentioned relations only to a micro-world of Physics. Later, nobody updated or modified such incomplete initial assumption.***

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Citation from: Mark John Fernee. Works at University of Queensland May 3: Quora
How does the Fourier transform tell us the uncertainty principle?

The Fourier transform deals with conjugate variables, such as time and frequency, or position and momentum. So, let's start with the underlying principle, and that is the superposition of sine waves can be used to build any waveform. Now consider a sine wave, it has no beginning or end, but it has a perfectly defined frequency. Now consider a spike that is zero everywhere except at a single point. How many sine waves would we need to construct such a spike? It turns out that we'd need infinitely many covering all frequencies. Now the spike has a perfectly defined time or position (depending on your axis) but a completely undefined frequency or wave-vector (i.e., momentum). So, we have two extremes: A perfect sinusoid that has an undefined position in time or *space; or a spike of perfectly defined time and space, but undefined frequency and momentum. This is purely a mathematical property that arises because we wanted to understand the superposition of sine waves. Because there are two extremes, the question arises about what would be the optimum midpoint where both time and frequency, or position and momentum, were sufficiently well-defined? Clearly, there must be some uncertainty in both measurements at such a point, as perfect certainty can only exist in one property if the other is completely undefined. The minimum product of the uncertainties is the basis of the Heisenberg uncertainty principle.*

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Citation: **Heisenberg Uncertainty Principle,** David Kahana, physicist unhinged May 16
Did a scientist find a loophole in Heisenberg's uncertainty principle?
https://www.quora.com/Did-a-scientist-find-a-loophole-in-Heisenbergs-uncertainty-principle

First, let's drop the vague term Heisenberg's Uncertainty Principle - since Heisenberg never actually proved an uncertainty principle for position and momentum, and he certainly didn't prove one for time and energy, but rather he strongly argued and suggested that such a relation should exist for both pairs of variables, proceeding heuristically since he understood that position and momentum were a canonically conjugate pair of "matrices" in the terminology that Born and Jordan made up for Heisenberg's fundamental theoretical objects. Note very carefully they were not simply finite dimensional matrices such as we are all familiar with or at least imagine we are familiar with. It's a common human failing or hope that because you can understand the simplest possible sentences in a language, you also can understand the most general sentences, but in fact you can't. It was Born and Jordan who first formalized Heisenberg's breakthrough paper of July 1925 into what became known as matrix mechanics. There then followed a very intense argument between Schrödinger and supporters of his wave mechanics, and Heisenberg and the supporters of his matrix mechanics. After all, jobs were at stake, reputations had to be made. Schrödinger argued, after Max Born once again intervened and gave Schrödinger the proper interpretation of Schrödinger's own wavefunction, that wave mechanics and matrix mechanics were equivalent. Heisenberg argued that wave mechanics was "crap". Heisenberg insisted that discontinuous quantum jumps were absolutely basic and that any continuous and visualizable theory such as Schrödinger's was abhorrent and must be beside the point.

But there existed the Born rule in addition to Schrödinger's deterministic evolution equation for the wavefunction, without which any quantum theory was incomplete, and the measurement process was then largely a mystery. The measurement problem is still in my view a black hole from which very little has so far emerged from those who travelled inside the event horizon of that sub-area of theoretical physics and reside and work there, or maybe even it is an area of philosophy. Heisenberg and Schrödinger argued endlessly and violently in Copenhagen, with Niels Bohr I am sure, smoking his pipe and enjoying the whole thing immensely, listening to the youngsters and no doubt intervening to hold forth endlessly on some point or another at least once a day - and very little light was actually generated for all of that heat.

Meanwhile, PAM Dirac was looking at both approaches, and he finally settled the matter, they were indeed exactly equivalent formulations of the same theory. But this all took time to come out. So, the history is very confusing.

Start with Heisenberg's first paper on the idea of an uncertainty principle: I am not saying Heisenberg doesn't deserve credit for it, but I am saying that his original argument was not convincing.

Heisenberg argued that because position and momentum matrices obeyed a canonical commutation relation of the form: $[X, P] = -i1$, *in his theory as formulated by Born and Jordan, the disturbance that any simultaneous measurement of such canonically conjugate variables that was attempted, would be defeated by the disturbance inherent in the measurement effect. The measuring device he said, would disturb the measurement inevitably since something must interact with the particle that was observed (he considered the case of a single massive particle), and that would always result in an inherent irreducible uncertainty in the measurement of both position and momentum.*

Heisenberg ***did not state*** *in his original work on the subject any actual inequality, but merely argued that due to the existence of this commutator, the uncertainty in any simultaneous measurement of both position and momentum for a single particle must be on the order of* $\hbar$, *that is -* $\Delta x \Delta p \sim \hbar$ *(sic). The position momentum uncertainty relation, let's be quite clear, is stated as an inequality, and it was only proved sometime later, after Heisenberg first published his thinking on the matter. Opinions differ on who first proved the inequality. You can take your choice, I would go with Kennard, since he was first into print, but Hermann Weyl also proved it, and he did so within the same year, I think. So, who knows which one was actually first? It takes time to write up a result …*

However, let this be said - if this inequality is false, then quantum mechanics is false, or else John von Neumann's axiomatization of quantum mechanics is incorrect. Because the position momentum uncertainty relation: $\Delta x \Delta p \geq \hbar 2$

is a trivial corollary of the axioms - it follows because the Cauchy-Schwartz inequality is valid in a Hilbert space and all the quantum states for a single particle, which is all that Heisenberg was considering, lie in a Hilbert space.

So, the inequality is automatically satisfied by any quantum state.

Notice the factor of 2 that appears in the inequality relative to what Heisenberg heuristically argues for in his original work on the uncertainty principle. So, we can, first of all, violate "Heisenberg's order of magnitude argument" without violating the actual position momentum uncertainty principle, which is a weaker uncertainty, by a factor 2, than what Heisenberg argues for.

I suggest that while it is very interesting to demonstrate "entanglement", meaning to show nothing more than that a highly correlated quantum state for a two or more particle system exists, in this case what one might by a small stretch of the imagination call a pair of macroscopic systems even if at very low temperature, and to do this for the coupled oscillations of two such systems is a very, very nice achievement, and may well have implications for better measurement devices for various purposes - or at least we can hope so, there is actually no violation of quantum mechanics implied here at all, much less a violation of the uncertainty principle as I stated it above.

On reading the arxiv preprints of these works no such claim is made, and "entangled states", I actually hate the word entangled since entangled states are just correlated many particle quantum states in which a conservation law generally exists and which entail stronger correlations than are possible classically, that's all they are, are also quantum states, they don't violate quantum mechanics, and so, they are subject to all the uncertainty principles that quantum mechanics implies about quantum states.

Now as to whether this violates "the Heisenberg Uncertainty Principle" and whether that matters to you or not, I leave it up to you to decide. I hope I have made it clear it doesn't matter to me because "the Heisenberg Uncertainty Principle" is not actually a well-defined notion, but instead is a phrase which is tossed around all the time by people who half the time don't understand what Heisenberg actually argued for and what he actually proved and did accomplish, and who are promoting quantum woo half the time. What Heisenberg did accomplish was the singlehanded CREATION of matrix mechanics even if he didn't formulate it completely on his own and he made the first SUGGESTION - and it's a brilliant suggestion at that - that an uncertainty relation should exist. isn't that enough?!

……………………………

Officially, the Fourier transform is not the historical path that led Heisenberg to the uncertainty principle. *In fact, Heisenberg initiated the linear algebra approach to Quantum Mechanics, where his famous uncertainty principle is based on the commutation property of matrices. In that time Heisenberg formulated his intuitive and basically correct assumptions about Uncertainty, without offering any strict and strong mathematical proof.*

Whatever the real life (or historical) case about original formulation of Heisenberg Uncertainty Relations is, the fact is that only logical, conceptually clear, and mathematically well supported explanations and development of generally valid and generally applicable Uncertainty Relations is coming from Fourier Signal Analysis (meaning from Mathematics and not from Quantum Theory). Such relations are mathematically or originally dealing with absolute or total signal (or wave function) durations in mutually conjugate, or original and spectral domains. The fact is that Quantum Mechanics on certain stochastic way transformed or interpreted such relations (between durations of conjugate domains) to be presentable only statistically, as relations between relevant standard deviations, what is possible, but has its limits. In this book, we will connect all aspects of uncertainty relations, and present them on all possible ways, since the natural and mathematical background is the same for all of them (in a micro and macro world of Physics). What Heisenberg really had in mind and how he really developed and presented his visions about Uncertainty is not so *much essential and important in comparison to generally valid mathematical approach based on Fourier signal analysis.*

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In 1905, a mathematician Amalie Nether proved the following theorem (regarding universal laws of Symmetries):

-For every continuous symmetry of the laws of physics, there must exist a conservation law.

-For every conservation law, there should exist a continuous symmetry.

Since Uncertainty Relations are addressing signal (or wave-packet) durations in its Original and Spectral domains, let us first summarize the already known conservation laws and symmetries between Original and Spectral domains (in this paper initially introduced in the first chapter, regarding electromechanical Analogies), by creating the table T.5.1.

T.5.1. Symmetries of the Laws of Physics

Original Domains ↔	↔ Spectral Domains
Time = t	Energy = $\tilde{E}$ (or frequency = f, or mass = m)
Time Translational Symmetry	Law of Energy Conservation
Displacement = $x = S\dot{p} = S\tilde{F}$, ($\tilde{F}$ = force)	Momentum = $\tilde{p} = \tilde{m}\dot{x} = \tilde{m}v = p$
Space Translational Symmetry	Law of Momentum Conservation
Angle = $\alpha = S_R \dot{L} = S_R \tau$	Angular momentum = $L = J\dot{\alpha} = J\omega$
Rotational Symmetry	Law of Angular Momentum Conservation
Electric Charge = $q_{el.} = \Phi_{el.} = C\dot{q}_{mag.} = Ci_{mag.}$	Magn. Charge = $q_{mag.} = \Phi_{mag.} = L\dot{q}_{el.} = Li_{el.}$
Law of Total Electric Charge Conservation	The Electric Charge-reversal Symmetry
The Magnetic Charge-reversal Symmetry	"Total Magnetic Charge" Conservation

(Mono-magnetic charge does not exist as a free and self-standing, natural entity)

Apparently (as we can see from T.5.1), couples of the conjugate, mutually Original and Spectral domains, created using simple analogies, are also in formal agreement with the most essential symmetries and conservation laws of our universe (see T.1.8 from the first chapter of this book).

There are many mathematical forms of uncertainty relations between mutually coupled or conjugate domains-durations, or interval-lengths (in connection to entities presented in table T.5.1), belonging to a specific signal, wavefunction, or energy state, and related to Fourier-type transformation domains of that signal. Let us start from the simplest, already known (most generally valid, both for micro and macro world of Physics) mathematical forms of Uncertainty relations that can be applied to any waveform, or wave packet, as often found in Signal (or Spectrum) Analysis, Telecommunications Theory and in certain earlier works regarding Quantum theory (see (5.5)). We will consider in this analysis, that there is a wave function or wave packet model, which is by its definition or formation equivalent (only in a couple of most important kinematic aspects) to certain moving particle, or to a specific moving "energy-moments" state. The wave-energy of the wave-packet in question ($\tilde{E} = hf$ or $\tilde{E} = Hf$, $[h , H]$ (=) constants) should be equal to the total motional (or kinetic) energy of its equivalent particle couple. The wave momentum of the wave-packet will be equal to the particle momentum, and the relevant *group wave-velocity* will be equal to the particle velocity (see more in Chapters 4.1 and 10.).

Let us take into account only the ***absolute or total wave-packet durations (or lengths)*** *in all its domains* (in time, frequency, and space, as it is already introduced in Chapter 4.0 – "Wave functions wave velocities and uncertainty relations", starting from the equations under (4.0.55) and (5.5)). Then, "***non-statistical***" uncertainty relations, addressing a world of microphysics, molecules, atoms, and subatomic entities, are given as:

$$\Delta x \cdot \Delta p = \Delta t \cdot \Delta E = h \cdot \Delta t \cdot \Delta f \geq h/2 \Leftrightarrow \Delta x \cdot \Delta \tilde{p} = \Delta t \cdot \Delta \tilde{E} = h \cdot \Delta t \cdot \Delta f \geq h/2 , \quad (5.1)$$

where meaning of the symbols is the same as introduced all over this book ($\mathbf{x}$ - displacement, $p = \tilde{p}$ -particle and/or wave momentum, $\mathbf{E_k} = \tilde{\mathbf{E}}$, $\Delta\mathbf{E} = \Delta\mathbf{E_k}$, -particle and/or wave-packet motional energy, $\mathbf{f}$ -mean frequency, $\mathbf{t}$ -time and $\mathbf{h}$ -Planck's constant, see expressions (4.1) - (4.3)). Of course, similar domains relation can be formulated for macro-world objects since there are Planck analog constants $H_i >>> h$ applicable in a world of solar systems and galaxies (see more in Chapter 2).

Since here we are still addressing an integral wave packet or associated motional particle (which should be mutually and only kinematically equivalent on a described way), it is clear that *in (5.1) momentum and energy values are the total momentum and motional energy amounts, regardless of using delta-difference symbols, effectively meaning* $\Delta p = \Delta \tilde{p} = \tilde{p} = p$ *and* $\Delta \tilde{E} = \Delta E = \tilde{E} = E_k = c^2 \Delta m$ *.* To avoid repeating lengthy introductions and explanations regarding relations (5.1), it is very much recommendable first to review most of the necessary background (related to wave packets and introductory aspects of uncertainty relations) from Chapter 4.0; - "Wave functions wave velocities and uncertainty relations".

Here is also appropriate to mention that modern Quantum Theory modified a little bit the original and unambiguous mathematical meaning of Uncertainty Relations (5.1) by considering delta differences (Δ), or essential and total signal durations, as statistical, standard-deviation intervals, because this has been more appropriate to support probabilistic and statistical thinking, modeling and conceptual environment forged by the Orthodox Quantum Theory establishment. ***The position of the author of this book is that it should exists only one basic, fundamental set of Uncertainty Relations or inequalities (universally applicable in Mathematics and Physics), which is ontologically purely mathematical, and all other kind of Uncertainty relations surfacing in Quantum Physics should be equivalent or identical to general mathematical foundations of Uncertainty (of course, properly assigned to physics-related items).***

The principal objective in this book is to establish a much broader conceptual framework regarding universally valid Uncertainty Relations than presently found in Physics (condensing and simplifying, as much as possible, associated mathematical complexity). *What we can overwhelmingly find in Physics literature is that in many ways, all Uncertainty relations are intrinsically linked to Planck's constant **h**. This is traditional, old, and still contemporary platform, but we can also find that the Planck's constant applicable in the world of atoms, photons, and elementary particles, has its macro-world equivalent in a much bigger constant **H** >> **h**, which is relevant for planets, solar systems and galaxies with intrinsic periodicities related structures (see Chapter 2; -equations (2.11.12) - (2.11.21)). The fact is that traditional, micro-world Planck's constant **h** cannot be the unique and exclusive attribute of our macro-Universe. This fact brings a new light on future analyses and presentations regarding generalized Uncertainty relations (since we will be able to create at least two sets of micro and macro worlds Uncertainty relations, each of them with its own Planck-like constants, most probably on some way connected). To comply with simplicity and continuity with already existing knowledge about Uncertainty relations in Physics, in this chapter, we will (only initially) pay dominant attention to the micro-world environment, where Planck constant **h** has its significant place. Then, we will realize that most of the uncertainty relations developed for micro-world events are analogically applicable to similar relations valid for planetary systems, stars, and galaxies (using new and much bigger constant **H**, or number of such constants, like shown in Chapter 2., but this time devoted to planetary and other periodical motions). Signal analysis (or mathematics) is not making any difference between signals from micro or macro world situations; -it is universally valid and applicable on every physics-related, mathematically formulated wave function.*

When using electromechanical analogies (established in the earlier chapters of this book), it is possible to address and additionally extend all Uncertainty Relations (5.1). Taking data from T.1.2, T.3.1, T.3.2, T.3.3, and T.5.1, let us analogically create T.5.2, producing the results that will **dimensionally** give a mutually analog and complementing energy-time product $\Delta t \Delta E$ for situations regarding electromagnetic fields and linear and rotational motions (always applied on the same object or wave-group). Here, we will consider only total amounts, or total (non-statistical) domain durations of a relatively isolated and compact, self-standing energy states, such as a motional particle, or an equivalent wave-packet.

T.5.2. Analogies between electromagnetic fields and mechanical motions

Electro-Magnetic Field	Linear Motion	Rotation
$u_{el.} = i_{mag.} = -\frac{d\Phi_{mag.}}{dt} = -\frac{dq_{mag.}}{dt} \downarrow$ $(=)$ El.volt.$(=)$ Mag.current	$v = \frac{dx}{dt}$ $(=)$ Velocity $\downarrow$	$\omega = \frac{d\alpha}{dt}$ $(=)$ Angular Velocity $\downarrow$
$i_{el.} = u_{mag.} = +\frac{d\Phi_{el.}}{dt} = +\frac{dq_{el.}}{dt}$ (=) El.current (=) Mag.voltage	$F = \frac{dp}{dt}$ $(=)$ Force	$\tau = \frac{dL}{dt}$ $(=)$ Torque
$P = u_{el.} i_{el.} = i_{mag.} u_{mag.} = \frac{dE}{dt} = -\frac{dq_{mag.}}{dt} \cdot \frac{dq_{el.}}{dt}$ $(=)$ Power $(=)$ Ψ^2	$P = vF = \frac{dx}{dt} \cdot \frac{dp}{dt} = \frac{dE}{dt}$ $(=)$ Power $(=)$ Ψ^2	$P = \omega\tau = \frac{d\alpha}{dt} \cdot \frac{dL}{dt} = \frac{dE}{dt}$ $(=)$ Power $(=)$ Ψ^2
$\Phi_{mag.} = q_{mag.}$ (=) **Magn. Flux / Charge**	x $(=)$ Displacement	α $(=)$ Angle
$\Phi_{el.} = q_{el.}$ **(=) El. Flux / Charge**	**p (=) Momentum**	**L (=) Angular Momentum**
$\Delta E = P\Delta t$ **(=) Energy**	$\Delta E = P\Delta t$ **(=) Energy**	$\Delta E = P\Delta t$ **(=) Energy**
$\Delta\Phi_{mag.}\Delta q_{el.} = \Delta q_{mag.}\Delta\Phi_{el.} = P(\Delta t)^2 = \Delta E\, \Delta t$	$\Delta x \Delta p = P(\Delta t)^2 = \Delta E\, \Delta t$	$\Delta\alpha\Delta L = P(\Delta t)^2 = \Delta E\, \Delta t$

Combining (5.1) and (4.2) with relations found in the bottom line of T.5.2, it would be possible (by analogy, taking a complete and finite, minimal stable-size, or energy amount of the wave-packet of certain kind) to extended uncertainty relations (5.1) for two more members, as follows,

$$\begin{aligned}
&\Delta q_{mag.} \cdot \Delta q_{el.} = \Delta\alpha \cdot \Delta L = h \cdot \Delta t \cdot \Delta f = \Delta x \cdot \Delta p = \Delta t \cdot \Delta\tilde{E} = c^2 \Delta t \cdot \Delta m = \Delta s_1 \cdot \Delta s_2 \geq h/2, \\
&\Delta E = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta t} = \frac{\Delta\alpha \cdot \Delta L}{\Delta t} = \frac{\Delta x \cdot \Delta p}{\Delta t} = \overline{v} \cdot \Delta p = h\Delta f = c^2 \Delta m = P\Delta t = \Delta\tilde{E} \geq \frac{h}{2\Delta t}, i_{el.} = \frac{\Delta q_{el.}}{\Delta t}, \\
&\Delta m = \frac{\Delta E}{c^2} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 \Delta t} = \frac{\Delta q_{mag.}}{c^2} i_{el.} = \frac{\Delta\alpha \cdot \Delta L}{c^2 \Delta t} = \frac{\Delta x \cdot \Delta p}{c^2 \Delta t} = \frac{\overline{v} \cdot \Delta p}{c^2} = \frac{h\Delta f}{c^2} = \frac{P\Delta t}{c^2} \geq \frac{h}{2c^2 \Delta t}.
\end{aligned} \quad (5.2)$$

With (5.2) we are again addressing the same energy-momentum entity, object, or matter-wave state, meaning that such state should have (on some way) mutually coupled and dependent linear and angular moments, linear and angular displacements, and associated electromagnetic complexity. Later, we will see that in addition to uncertainty relations (or inequalities), such mutually connected properties should also comply with Kotelnikov-Shannon-Nyquist-Whitaker theorems regarding signals' sampling and reconstructing (concerning elementary sampling intervals and total signal durations in different, mutually conjugate signal definition domains). Important background here is the universal tendency of matter constituents to be mutually synchronized (or to evolve towards mutual spectral and resonant synchronizations).

The above-mentioned mathematical options seem a bit oversimplified, but we will see that the relations (5.2) and their consequences are correct. For instance, when we take $\mathbf{\Delta q_{el.}} = \mathbf{e}$, as an elementary (stable and minimal possible) electric charge of an electron, we will get $(\Delta q_{el.} = \Delta\Phi_{el.})_{min.} = e \Rightarrow \Delta\Phi_{mag.} = \Delta q_{mag.} \geq h/2e$. It is already known (in a micro world physics) that **h/2e** presents an elementary charge of a magnetic flux, being an elementary magnetic charge, $(\Delta q_{mag.})_{min.} = h/2e$ (of course, natural, free-standing, isolated and static magnetic monopole does not exist, but certain kind of (mathematically equivalent and effective) one-side closed electric current-loop should be in the background of such coupled magnetic charges. Since magnetic flux is anyway related to a certain magnetic dipole or closed electric current loop (with two magnetic poles), electric charge or delta-interval in (5.2) should also be one part of the corresponding electric dipole with positive and negative electric charges. Similarly, we can also support the resonant-like quantization of orbiting motions ($\mathbf{\Delta\alpha, \Delta L}$) and connect them to spin (and gyromagnetic) properties of elementary particles (or possibly find other couples of conjugate variables, $\mathbf{s_1}$, $\mathbf{s_2}$ satisfying similar relation). To simplify further elaborations of Uncertainty inequalities (or relations), in (5.2) is introduced formal and symbolic generalization of all mutually conjugate "delta segments" with $\Delta s_1 \cdot \Delta s_2 \geq h/2$, when the following Uncertainty inequalities are equally applicable (see [62]),

$$\left.\begin{cases}\Delta s_1 \cdot \Delta s_2 = (\Delta s_1)_{min.} \cdot (\Delta s_2)_{max.} = (\Delta s_1)_{max.} \cdot (\Delta s_2)_{min.} \geq h/2 \\ \dfrac{(\Delta s_2)_{min.}}{(\Delta s_1)_{min.}} = \dfrac{(\Delta s_2)_{max.}}{(\Delta s_1)_{max.}} = \dfrac{\Delta s_2}{\Delta s_1}\end{cases}\right\} \Rightarrow$$

$$(\Delta s_1 \cdot \Delta s_2)_{min.} = h/2,\ (\Delta s_1 \cdot \Delta s_2)_{max.} > h/2, \tag{5.2.1}$$

$$\left(\text{or } (\Delta s_1 \cdot \Delta s_2)_{max.} = n \cdot \frac{h}{2},\ n = 2, 3, 4, \ldots\ ?!\right).$$

What is significant here is that both particles and waves are treated equally (as being kinematically mutually equivalent), applying the same Uncertainty Relations (and the same PWDC relations, as described in Chapters 4.1 and 10.). The only difference is that real (or ordinary) particles (with non-zero rest masses) should be considered as “frozen” or well-packed, self-stabilized and self-closed standing waves structures (this way creating non-zero rest masses). Here, we have one challenging situation related to how we understand and specify total durations or lengths of certain (solid body) real particle and its kinematic properties (in mutually conjugate original and spectral domains), compared to similar total durations, lengths and/or dimensions of the kinematically and mathematically equivalent wave group. Real (solid and rigid) particle dimensions or absolute, total lengths and durations are not directly and fully equal to similar properties of the corresponding and kinematically equivalent wave group, meaning that certain compromising and realistic, mathematically correct solution should be established here, regarding particle durations in all domains. ***Furthermore, it looks logical to conclude that in cases of stable elementary particles from the world of micro-physics, meaning of UNCERTAINTY is transformed into a CERTAINTY, where an inequality sign, “≥ or ≤” is transformed into an equality sign, “=”.*** Obviously, (based on (5.2) and (5.2.1)), we can conclude (maybe at this time still a bit prematurely) that metrics and energy formatting of nature, regarding its elementary parts (such as atoms and elementary particles), has come to certain, conditionally non-divisible (and minimal) atomized or discretized units, or building blocks. This way Nature realizes optimal matter packing (or formatting) with certain minimal and finite elementary domain intervals. Such elementary and minimal matter building blocks should satisfy the following “resonant gearing, fitting, packing, or *elementary-certainty-relations*”,

$$\left(\Delta q_{mag.} \cdot \Delta q_{el.}\right)_{min.} = \left(\Delta\alpha \cdot \Delta L\right)_{min.} = h \cdot \left(\Delta t \cdot \Delta f\right)_{min.} = \left(\Delta x \cdot \Delta p\right)_{min.} =$$

$$= \left(\Delta t \cdot \Delta\tilde{E}\right)_{min.} = c^2\left(\Delta t \cdot \Delta m\right)_{min.} = \left(\Delta s_1 \cdot \Delta s_2\right)_{min.} = h/2,\ (\Delta x)_{min.} = \frac{\lambda}{2} = \frac{h}{2p}, \tag{5.3}$$

$$\left\{\begin{matrix}\tilde{E} = \tilde{E}(f),\ f = \omega/2\pi\left(\approx \dfrac{1}{\Delta t}\right),\ p = p(k),\ k = 2\pi/\lambda = \dfrac{2\pi}{h}p\left(\approx \dfrac{\pi}{\Delta x}\right), \\ v = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta E}{\Delta p} = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta s_2}{\Delta s_1} = \overline{v} \leq c\ (\cong \text{constant})\end{matrix}\right\},$$

where $(\Delta s_1)_{min.}$ and $(\Delta s_2)_{min.}$ symbolize all other, minimal, and elementary, quantifiable, finite interval lengths, which are also mutually conjugate variables.

*Analogically concluding (or just hypothetically exploring), for macrocosmic systems, like stable solar systems and galaxies, we need to apply macrocosmic “Planck-analog” constant H, and we will have similar **CERTAINTY** relations applicable on relevant macro participants,*

$$\left(\Delta q_{mag.}\cdot\Delta q_{el.}\right)_{min.}=\left(\Delta\alpha\cdot\Delta L\right)_{min.}=h\cdot\left(\Delta t\cdot\Delta f\right)_{min.}=\left(\Delta x\cdot\Delta p\right)_{min.}=$$
$$=\left(\Delta t\cdot\Delta\tilde{E}\right)_{min.}=c^2\left(\Delta t\cdot\Delta m\right)_{min.}=\left(\Delta s_1\cdot\Delta s_2\right)_{min.}=H/2,\ (\Delta x)_{min.}=\frac{\lambda}{2}=\frac{H}{2p}.$$

Here, in (5.2) and (5.3), we are dealing with mutually proportional and coupled, finite, absolute, spatial, frequency and time durations of matter-wave packets (in different domains of their existence). From mathematics (or signal analysis), we know that if a time domain duration of certain signal is very short, its spectral function duration is very long or tends to become infinite, and vice versa (valid equally for spatial distances and spatial frequencies; -here valid for Δx and Δp). ***Anyway, Nature, or our Universe, is on some convenient and natural way always creating durations of both, time, and frequency domains, of relatively stable particles and wave-packets, to be finite and energy limited (like in cases of Gaussian and Gabor, bell-curve-envelope wavelets, or wave-packets, since Gaussian waveforms are well defined and finite, both in a temporal and frequency domain).***

We could also make more far-reaching conclusions, based on relations (5.2). For instance, if we would like to realize "mass-radiator", or "mass-emitter", practically meaning to create and/or radiate a mass surplus as an equivalent to specific dynamic, and/or oscillatory motional energy-momentum situation, we just need to address (creatively and innovatively) the following relations, developed from (5.2),

$$\Delta m=\frac{\Delta x\cdot\Delta p}{c^2\Delta t}=\boxed{F\frac{\Delta x}{c^2}}=\frac{\bar{v}\cdot\Delta p}{c^2}=\frac{\Delta\alpha\cdot\Delta L}{c^2\Delta t}=\boxed{\tau\frac{\Delta\alpha}{c^2}}=\frac{\bar{\omega}\cdot\Delta L}{c^2}=\frac{h\Delta f}{c^2}=\frac{P\Delta t}{c^2}=\frac{\Delta E}{c^2}=$$
$$=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{c^2\Delta t}=\frac{\Delta q_{mag.}}{c^2}i_{el.}\geq\frac{constant}{\Delta t},\ \Delta m=\frac{\Delta x}{c^2}F=\frac{\Delta\alpha}{c^2}\tau\rightarrow F=\frac{\Delta m}{\Delta x}c^2=\frac{\Delta\alpha}{\Delta x}\tau$$

Mass defect or surplus Δm (or this can also be a radiated kinetic energy, matter waves, Tesla's Radiant energy, or some evaporation) will be directly proportional to involved linear and angular displacements, $\Delta x, \Delta\alpha$, or linear and angular forces F, τ, created by certain oscillatory process, $\Delta m=\frac{\Delta x}{c^2}F=\frac{\Delta\alpha}{c^2}\tau$, ($\tau(=)$ torque). Alternatively, the same situation and conclusions can be extended to pulsed and/or oscillatory, linear, and angular motions, electromagnetic excitations, etc. (see more in Chapter 10.). Gravitation (including antigravitation) and flying objects thrust related technologies (or forces) could be developed starting from such relations. The simplified understanding of natural forces can also be generalized based on "Uncertainty and Certainty Relations", as for example,

$$\Delta x\cdot\Delta p=\Delta t\cdot\Delta\tilde{E}=\Delta\alpha\cdot\Delta L=h\cdot\Delta t\cdot\Delta f=c^2\Delta t\cdot\Delta m=\Delta q_{mag.}\cdot\Delta q_{el.}=\Delta s_1\cdot\Delta s_2\ \left(=h/2,\text{ or }H/2\right)\Rightarrow$$
$$\Rightarrow F\ (=)\ \frac{\Delta p}{\Delta t}=\frac{\Delta E}{\Delta x}=\frac{\Delta\tilde{E}}{\Delta x}=\frac{h\cdot\Delta f}{\Delta x}=\frac{c^2\cdot\Delta m}{\Delta x}=\frac{\Delta\alpha}{\Delta t}\cdot\frac{\Delta L}{\Delta x}=\frac{\omega\cdot\Delta L}{\Delta x}=\frac{v\cdot\Delta p}{\Delta x}=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta x\cdot\Delta t}=\frac{i_{el.}\cdot\Delta q_{mag.}}{\Delta x}\Rightarrow$$
$$\Rightarrow F=grad(E)=\nabla E=\nabla(\omega\cdot\Delta L)=\nabla(v\cdot\Delta p)=\nabla(i_{el.}\cdot\Delta q_{mag.})=...\ ,$$

meaning (at least dimensionally) that every spatial, energy-related gradient (or mass agglomeration) is producing effects we identify as different forces. Alternatively, we could also creatively say that broader understanding of Uncertainty relations is directly influencing or *explaining effects of natural forces (see more in Chapter 10. under (10.2-2.2) and "10.02 MEANING OF NATURAL FORCES"). Obviously, that contemporary understanding of nuclear forces should also significantly evolve from a present state of oversimplified and unclear, or simple statements, and from indirect, fuzzy and naive verbal descriptions without tangible mathematical modeling (saying essentially that such forces should exist on some way, what is a trivial statement, and not an explanation), towards much more mathematically sophisticated, resonant, nodal-zones related force-effects within matter-wave states with specific standing-waves stabilized structures (within an atom nucleus zone). See more about generalized natural forces in Chapter 10. under "10.02 MEANING OF NATURAL FORCES ".*

The intrinsic structure of our universe is most-probably organized on a way that all relevant, mutually coupled, or conjugate signal durations are finite in both domains (explicable while using reasonable approximations, and convenient mathematical modeling such as Gaussian-Gabor Analytic signals). Mutual domains proportionality and coupling is closely related to relevant signal or matter waves speed of propagation, $v = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t} = \frac{\Delta s_2}{\Delta s_1} = \overline{v} \leq c\ (\cong \text{constant})$; -see relation (4.0.66) from chapter 4.0. ***The equality symbol in (5.3) is also mathematically defendable in cases when relevant signals or wave-packets (or their wave functions) are presentable in the form of Gaussian-envelope window functions (in both of conjugate domains) because the Gaussian function is optimally concentrated (finite or limited) in its joint time-frequency domain.*** This is indirectly telling us how we should make modeling of elementary wave functions, or wave packets including photons, that have a relevance in physics (or to be useful for creating or representing moving particles).

Since all energy states, masses, particles, and waves in our universe are always in different states of mutually related or relative motions, we can define the unique energy-propagation speed of such energy states or matter waves, which is naturally connecting all matter waves durations in relevant space-time-frequency domains as in (5.3), $v = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t}$.
Depending on properties of specific (non-dispersive) media, where matter waves are being observed or analyzed, we know that relevant wave-energy propagation-speed has its relatively stable value. For instance, sound-waves velocities in different gas, liquid or solid states are mutually different. Eelectromagnetic waves velocities in different materials are also propagation-media parameters-dependent. All of them are also relatively stable and limited if propagation media parameters are stable… This is at the same time the very important condition describing the stability and integrity of a specific energy state (also presenting stability condition of all micro and macro particles in our universe, since $v = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t} = \text{const.} \Rightarrow \ \Delta\tilde{E} = \Delta E = \text{const.}\cdot\Delta p,\ \ \Delta\omega = \text{const.}\cdot\Delta k,\ \ \Delta x = \text{const.}\cdot\Delta t$
…). <u>In other words, time and space (or spatial and temporal lengths) are mutually directly proportional, where the constant of proportionality has dimensions of certain speed (see more in Chapter 10.).</u>

....

Here is convenient place to clarify the meaning of associated forces and velocities that are linked to Uncertainty relations. Let us take mathematical Uncertainty relations between total or absolute durations of certain wave-packet (or a moving particle), as given in (5.1).

$$\Delta x \cdot \Delta p = \Delta t \cdot \Delta E = h \cdot \Delta t \cdot \Delta f \geq h/2 \Leftrightarrow \Delta x \cdot \Delta\tilde{p} = \Delta t \cdot \Delta\tilde{E} = h \cdot \Delta t \cdot \Delta f \geq h/2 . \quad (5.1)$$

Let us now rename relevant delta (Δ) values, since here we are talking about absolute, total, and finite domain durations, as follows,

$$\left(\Delta x = X,\ \Delta p = P,\ \Delta t = T,\ \Delta E = \Delta\tilde{E} = E_k = \tilde{E}\right) \Leftrightarrow X \cdot P = T \cdot \tilde{E} \geq h/2 .$$

Since here we are addressing moving and stable object, particle, or an equivalent wave packet, we could say (assuming that wave phenomena or wave-packet in question is not dispersive), that initial absolute amounts $X, P, T, \tilde{E}$ will, after certain minor time interval δt, evolve as follows,

$$X \cdot P = T \cdot \tilde{E} \geq h/2 \Rightarrow \begin{Bmatrix} (X + \delta x) \cdot (P + \delta p) = (T + \delta t) \cdot (\tilde{E} + \delta\tilde{E}) \geq h/2, \\ X \cdot P = T \cdot \tilde{E} \geq h/2, \\ \delta x \cdot \delta p = \delta t \cdot \delta\tilde{E} \geq h/2,\ v \cdot \delta p = \delta E = d\tilde{E} = dE = dE_k = vdp,\ v = \delta x/\delta t = dx/dt \end{Bmatrix} \Rightarrow$$

$$(X + \delta x) \cdot (P + \delta p) = (T + \delta t) \cdot (\tilde{E} + \delta\tilde{E}) =$$

$$= \boxed{X \cdot P} + X \cdot \delta p + \delta x \cdot P + \boxed{\delta x \cdot \delta p} = \boxed{T \cdot \tilde{E}} + T \cdot \delta\tilde{E} + \delta t \cdot \tilde{E} + \boxed{\delta t \cdot \delta\tilde{E}} \Rightarrow$$

$$\Rightarrow \left\{X \cdot \delta p + \delta x \cdot P = T \cdot \delta\tilde{E} + \delta t \cdot \tilde{E}\right\} \Rightarrow$$

$$\Rightarrow \begin{Bmatrix} \text{Force} = F\ (=)\ \dfrac{\delta p}{\delta t} = \dfrac{1}{X}\left[\tilde{E} + T \cdot \left(\dfrac{\delta\tilde{E}}{\delta t}\right) - P \cdot v\right] \\ \text{Force} = F\ (=)\ \dfrac{\delta\tilde{E}}{\delta x} = \dfrac{1}{T}\left[P + X \cdot \left(\dfrac{\delta p}{\delta x}\right) - \dfrac{\tilde{E}}{v}\right] \end{Bmatrix} \Rightarrow$$

$$\Rightarrow \tilde{E} = P \cdot v - T \cdot \left(\frac{\delta\tilde{E}}{\delta t}\right) + \frac{X}{T}\left[P + X \cdot \left(\frac{\delta p}{\delta x}\right) - \frac{\tilde{E}}{v}\right] = \int v \cdot dp .$$

$$\Rightarrow X = T \cdot v + \frac{\tilde{E}}{F} - v\frac{P}{v(\delta p/\delta x)} = T \cdot v + \frac{1}{F}\left(\tilde{E} - v \cdot P\right) = \int_{(T)} vdt$$

In conclusion, we can say that a meaning of force is related to spatial and temporal energy distribution, or to relevant (mass-energy) gradients, and to absolute durations (size, lengths, or dimensions) of associated wave-packets in all their (original and spectral) domains, also implicating that Uncertainty relations and associated Forces phenomenology are mutually related.

The next conclusion is that vibrational or oscillatory phenomenology can also create forces, since in force formulations we have members like, $\frac{\delta p}{\delta t} = \frac{dp}{dt}, \frac{\delta\tilde{E}}{\delta t} = \frac{d\tilde{E}}{dt}$, and we already know that spatial and temporal durations are mutually related. Here, we also see that any solid body or stable, moving particle with well-defined geometry has different effective size, when analyzed as an equivalent mater-wave packet. Relativity theory concepts about Gravitation, related to spatial-temporal deformations and transformations, should also have certain grounds in "Uncertainty or Certainty" relations and mass-energy gradients, as initiated here (see more about natural forces in Chapters 2., and 10.).

The message here is again that dominant and primary Uncertainty relations are mathematical relations (between absolute durations of mutually conjugate domains), and that Heisenberg Uncertainty relations (including statistically formulated uncertainty relations) should be strictly and entirely developed only from basic, generally applicable, and always valid mathematical Uncertainty relations between absolute and total signal durations. Later formulated statistical makeup of Uncertainty in Physics could also be appropriately updated.

In fact, no one field theory, like Maxwell electromagnetic theory, and A. Einstein Relativistic theories will be completed, more universally valid, and better mutually connected, if questions regarding satisfying Uncertainty relations (and Parseval's theorems) are not appropriately addressed there.

We should not forget that statistical processing of sets with sufficiently big number of similar or identical items, when applied in all natural and other sciences is especially useful, practical, very correct, universal, and powerful mathematical tool for mass data modeling, mainstream curve fittings, events spreading or distributions estimations, trends determination etc. We should not misunderstand the non-doubtful and universally applicable power of mathematical theories like Statistics and Probability, and wrongly consider them as being presentable as fundamental ontological basis of certain specific discipline of Natural sciences (like promoted, and by consensus accepted in Orthodox Quantum community). Powerful, sophisticated, and particularly good mathematical tools are only useful tools (for good operators), and not foundations and stepstones of Physics).

Mathematics was initially advancing by learning from Nature, explaining, measuring, and mastering the Nature, and formulating its laws, gradually becoming the best language, calculating toolbox, and logic of our Universe. Now, we see that contemporary mathematics advanced enormously, almost independently, generalizing what it was initially learned from Nature, and we now have chances to make deductive "mining of mathematical treasures", going backwards to basic principles, or foundations of Physics. For instance, we find that Uncertainty relations (and Parseval's theorems) should always be satisfied or respected in Physics and in all definition domains of certain energy-moments state (essentially presenting energy conservation laws). We can also find that number of other mathematically formulated concepts and conclusions are universally applicable, and we need to implement such universal mathematical foundations, critically and creatively, within theories of present Physics. For example, Signal Analysis based on Kotelnikov-Nyquist-Shannon-Whittaker sampling and signals reconstruction techniques, and Complex Analytic Signals, established by Dennis Gabor, are universally applicable.

If we consider that all elementary, (relatively stable) matter building blocks are self-closed standing-waves formations, we can transform (5.3) into an equivalent matter-wave half-wavelength $\frac{\lambda}{2}$ as,

$$\left\{\lambda = h/p = h/\Delta p,\ \Delta p = \frac{\Delta\tilde{E}}{(\frac{\Delta x}{\Delta t})} = \frac{\Delta\tilde{E}}{\bar{v}} = \frac{c^2\Delta m}{\bar{v}},\ \bar{v} \leq c\right\} \Rightarrow$$

$$\frac{\lambda}{2} = \frac{h}{2\Delta p} \leq \frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\tilde{E}}\cdot\bar{v} = \frac{\Delta\alpha\cdot\Delta L}{\Delta\tilde{E}}\cdot\bar{v} = \frac{h\cdot\Delta t\cdot\Delta f}{\Delta\tilde{E}}\cdot\bar{v} = \frac{\Delta s_1\Delta s_2}{\Delta\tilde{E}}\bar{v} = \frac{\Delta x\cdot\Delta p}{\Delta\tilde{E}}\cdot\bar{v} = \bar{v}\cdot\Delta t = \Delta x \Rightarrow \quad (5.4)$$

$$\frac{\lambda_{min.}}{2} = \left(\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\tilde{E}}\cdot\bar{v}\right)_{min.} = \left(\frac{\Delta\alpha\cdot\Delta L}{\Delta\tilde{E}}\cdot\bar{v}\right)_{min.} = \left(\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\tilde{E}}\cdot\bar{v}\right)_{min.} =$$

$$= \left(\frac{\Delta s_1\Delta s_2}{\Delta\tilde{E}}\bar{v}\right)_{min.} = \left(\frac{\Delta x\cdot\Delta p}{\Delta\tilde{E}}\cdot\bar{v}\right)_{min.} = (\bar{v}\cdot\Delta t)_{min.} = (\Delta x)_{min.} = \left(\frac{h}{2\Delta p}\right)_{min.} = \frac{h}{2mc},$$

what in reality presents kind of resonant states and standing-waves relations between each couple of mutually conjugate variables (like acoustic resonators, antenna dipoles, etc.). Here we should also notice that $\frac{\lambda_{min.}}{2} = \frac{\lambda_c}{2} = \frac{h}{2mc}$ is the Compton wavelength $\lambda_c = h/mc$ (just to initiate an indicative and challenging brainstorming).

Intentionally choosing the matter-wave half-wavelength $\frac{\lambda}{2}$, as the most crucial elementary and resonant energy-formatting and packing unit (or size) of anything that is quantifiable in our universe, (see (5.2) - (5.4)), we are merely stating that quantization and standing-waves *relations are phenomenological and conceptual synonyms (regarding elementary matter-domains and different matter packing or formatting). Clear manifestations of such quantizing events are significant only in cases of self-closed, atomized domains, and energy exchanges between them, where the smallest space-domain unit is equal to a half-wavelength* $\lambda / 2$*, or to certain of familiar (analogical and more applicable) space-periodicity related values from T.5.3 and T.5.4. Bottom line explanation of the quantum nature of our Universe is that all stable energy-moments states (for instance masses) are kind of quasi-resonant, standing-matter wave's structures. Such states are also mutually communicating and interacting by exchanging similar (finite, discretized and countable) packets or quants of energy-moments standing matter-wave states. Quantizing of matter has two phenomenological aspects, such as, Energy-communications between matter-states are not continuous but more like sending or exchanging small, finite, or discrete packets of energy, and if we associate some integers to such energy quants, this is closely related to standing-waves wavelengths counting. The real and mathematically valid theory about matter quantizing is much closer to Kotelnikov-Nyquist-Shannon-Whitaker sampling, signal analysis and signals synthesis (or reconstruction), than to what modern Quantum theory is promoting regarding quantization. Of course, associated Euler-Lagrange-Hamilton equations and concepts (taken from analytical Mechanics) are additionally arranging realistic situations in the same field. More of common supporting arguments regarding standing-waves matter structure can be found in the Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.*

Schrödinger equation is among the most famous equations closely related to matter waves. We should not forget that it can be formulated by an analogical transformation of the Classical wave equation of standing waves oscillations of a string, where an integer number of <u>*half-wavelength*</u> *segments is a natural stability condition (and where a relevant wave function is presented as an Analytic signal). Of course, the meaning of matter wavelength should be analogically extended to all other aspects of spatially self-structured waves and fields, as for instance to rotational or angular motions and to corresponding electromagnetic values (as we can find in T.5.3 and T.5.4). By continuing developing the same concept, we can find that Euler-Lagrange-Hamilton formalism presents another framework to express a* tendency to optimal and dynamic energy packing and synchronization of interacting matter in motion.

Let us now address the (resonant) periodicity of motional states, fields and wave packets using the framework of analogies (established in the first and third chapter of this book) and Wilson-Sommerfeld action integrals (see [9]), known from early days of Quantum Mechanics. By "playing with analogies", based on data from T.3.1 and T.3.3, (Chapter 3.), combining them with "*elementary particle metrics"* (5.3) and (5.4), we could introduce even broader meaning of (new) de Broglie-like wavelengths. Such wavelengths can also be considered as periodical and quantifiable, resonant matter wave intervals, being discrete <u>matter building blocks</u>, or elementary "Certainty Intervals" (as already elaborated in this chapter, in Chapter 4.1, by T.4.2, and in Chapter 10.). For instance, we will be able to formulate analogically "*de Broglie-like, matter waves angular wavelength*", $\vec{\theta} = \frac{h}{L}$ and "*de Broglie-like electromagnetic charges and wavelengths*", $\vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}} = \frac{h}{q}$, $\vec{\Phi}_{el.} = q = \frac{h}{\Phi_{mag.}}$, as given in the T.5.3.

T.5.3 Analogical Parallelism between Different Aspects of Matter Waves

Waves Periodicity of Fields and Motional States	[DISPLACEMENTS] $[X]=[q_{mag.}, q_{el.}, x, \alpha]$	[CHARGES] $[Q]=[q_{el.}, q_{mag.}, p, L]$	De Broglie, or Periodicity Wave Intervals $[\overset{\leftrightarrow}{X}][Q]=[h]$
Electric Field (and total electric charge conservation)	$\Phi_{mag} = L_{mag} i_{el} = q_{mag}$	$[C_{el.}u_{el.}^{1}]$ (=) $[q_{el.}u_{el.}^{0}]$ (=) $[q_{el.}]$ (=) $[\Phi_{el}]$	$\overset{\leftrightarrow}{\Phi}_{mag.} = \frac{h}{\Phi_{el.}} = \frac{h}{q} = \lambda_e = \frac{h}{q_{el.}} = q_{mag.}$ (new)
Magnetic Field (and total, or dynamic magnetic charge conservation)	$\Phi_{el.} = L_{el.} i_{mag.} = q_{el.}$	$[C_{mag.}u_{mag.}^{1}]$ (=) $[q_{mag.}u_{mag.}^{0}]$ (=) $[q_{mag.}]$ (=) $[\Phi_{mag}]$	$\overset{\leftrightarrow}{\Phi}_{el.} = q = \frac{h}{\Phi_{mag.}} = \lambda_m = \frac{h}{q_{mag.}} = q_{el.}$ (new)
Gravitation & Linear Motion (and linear momentum conservation)	$x = Sf$	$[mv^{1}]$ (=) $[pv^{0}]$ (=) $[p]$	$\overset{\leftrightarrow}{\lambda} = \frac{h}{p} = \lambda$ (already known)
Rotation (and angular momentum conservation)	$\alpha = S_R M$	$[J\omega^{1}]$ (=) $[L\omega^{0}]$ (=) $[L]$	$\overset{\leftrightarrow}{\theta} = \frac{h}{L}$ (new)

(Periodicity – here invented, unifying formulation, $q_{mag.} = \Phi_{mag.}$ ***is not a free and independent magnetic charge)***

The idea here is to show that micro-world quantization is not too far from standing matter waves, resonant energy packing (of self-closed or self-stabilized oscillatory formations), since the general meaning of quantization is that elementary and stable matter domains (elementary particles) are composed or structured from elementary matter-waves intervals such as $\lambda = \frac{h}{p}$, $\overset{\leftrightarrow}{T} = \frac{h}{\tilde{E}}$, $\overset{\leftrightarrow}{\theta} = \frac{h}{L}$, $\overset{\leftrightarrow}{\Phi}_{el.} = \frac{h}{\Phi_{mag.}}$, $\overset{\leftrightarrow}{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}$, $\overset{\leftrightarrow}{s}_{1,2} = \frac{h}{s_{2,1}}$, multiplied by integers (see later (5.2.1) and (5.4.1)). The conceptual grounds of Strings Theory are not too far from what is elaborated here (see *Appendix, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES*).

In fact, what T.5.3, equations (5.2) - (5.4) and Wilson-Sommerfeld action integrals really describe (and predict) should be a kind of universal (resonant) matter waves periodicity, or kind of fields, waves and charges, energy, and space-related atomization (based on standing waves formations, where half-wavelength, $\lambda/2$, is the minimal building-block size). This way we can formulate "fitting and gearing rules", expressing some of the fundamental Symmetries of our universe (or simply saying, we are expressing "**optimal matter waves and energy formatting and packing rules**"). Here we can also formulate an extension of de Broglie particle-wave hypothesis. *Since kind of Particle-Wave Duality, periodicity, waving, and resonant quantization is associated to all linear motions (already known in the form of* $\lambda = h/p$, $\tilde{E} = hf$ *), the same or analogical idea (under similar standing-waves conditions, such as,*

$\lambda = \frac{h}{p}, \vec{T} = \frac{h}{\tilde{E}}, \vec{\theta} = \frac{h}{L}, \vec{\Phi}_{el.} = \frac{h}{\Phi_{mag.}}, \vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}, \vec{s}_{1,2} = \frac{h}{s_{2,1}}$ *) should apply to rotational motions, to electromagnetic phenomena, to any kind of motional energy, and to all other fields and movements of Nature (including the macro world of astronomy). Also, all such manifestations should exist coincidently being mutually well synchronized (often united, coupled inside of the same object, or belonging to the same energy state).*

*All of that is also and essentially based on Translational and Rotational Symmetry, resulting in (or from) laws of linear, angular moments, and energy conservation. All stable, linear, uniform, and inertial motions are mutually relative motions, but similar (relative motions) concept is also applicable to stable rotational or orbital motions that are accelerated motions. Wave-Particle Matter Duality phenomenology also presents a "bridge" between the **linear motions with spatial translational symmetry**, and always and naturally associated **rotational motions with spatial rotational symmetry**. In addition, under linear motions of masses, we can analogically and phenomenologically associate linear motions of electrically charged particles being in certain electric field. The same way, under rotational motions of masses, we can analogically and phenomenologically associate rotational (spinning and helix) motions of electrically charged particles being in certain magnetic field.*

By analogy with (5.2), and using the newly formulated "*de Broglie-like matter wavelengths*" from T.5.3, we can express the following, extended matter-waves periodicity relations:

$$\left\{\begin{array}{l} 2\Delta q_{mag.}\cdot\Delta q_{el.} = 2\Delta\alpha\cdot\Delta L = 2h\cdot\Delta t\cdot\Delta f = 2\Delta x\cdot\Delta p = 2\Delta t\cdot\Delta\tilde{E} = 2c^2\Delta m\cdot\Delta t = 2\Delta s_1\cdot\Delta s_2 \geq h, \\ \lambda = \frac{h}{p}, \vec{T} = \frac{h}{\tilde{E}} = \frac{h}{\Delta\tilde{E}} = \frac{h}{\Delta E}, \vec{\theta} = \frac{h}{L} = \frac{h}{\Delta\tilde{L}} = \frac{h}{\Delta L}, \frac{\lambda}{\vec{\theta}} = \frac{L}{p}, p = \Delta p, L = \Delta L \; ..., \\ \vec{\Phi}_{el.} = \frac{h}{\Phi_{mag.}} = \frac{h}{\Delta\Phi_{mag.}}, \vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}} = \frac{h}{\Delta\Phi_{el.}}, \vec{s}_{1,2} = \frac{h}{s_{2,1}} = \frac{h}{\Delta s_{2,1}}, \frac{\vec{\Phi}_{el.}}{\vec{\Phi}_{mag.}} = \frac{\Phi_{el.}}{\Phi_{mag.}}, i_{el.} = \frac{\Delta q_{el.}}{\Delta t}. \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} \lambda \leq 2\frac{\Delta s_1\Delta s_2}{\Delta\tilde{E}}\cdot\overline{v} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\tilde{E}}\cdot\overline{v} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta\tilde{E}}\cdot\overline{v} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\tilde{E}}\cdot\overline{v} = 2\frac{\Delta x\cdot\Delta p}{\Delta\tilde{E}}\cdot\overline{v} = 2\overline{v}\cdot\Delta t = 2\Delta x, \\ \vec{T} \leq 2\frac{\Delta s_1\cdot\Delta s_2}{\Delta\tilde{E}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\tilde{E}} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta\tilde{E}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\tilde{E}} = 2\frac{\Delta x\cdot\Delta p}{\Delta\tilde{E}} = 2\frac{c^2\Delta m\cdot\Delta t}{\Delta\tilde{E}} = 2\Delta t, \\ \vec{\theta} \leq 2\frac{\Delta s_1\Delta s_2}{\Delta\tilde{L}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\tilde{L}} = 2\frac{\Delta\alpha\cdot\Delta\tilde{L}}{\Delta\tilde{L}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\tilde{L}} = 2\frac{\Delta x\cdot\Delta p}{\Delta\tilde{L}} = 2\frac{\Delta t\cdot\Delta\tilde{E}}{\Delta\tilde{L}} = 2\overline{\omega}\cdot\Delta t = 2\Delta\alpha, \\ \vec{\Phi}_{el.} \leq 2\frac{\Delta s_1\cdot\Delta s_2}{\Delta\Phi_{mag.}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\Phi_{mag.}} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta\Phi_{mag.}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\Phi_{mag.}} = 2\frac{\Delta x\cdot\Delta p}{\Delta\Phi_{mag.}} = 2\frac{\Delta t\cdot\Delta\tilde{E}}{\Delta\Phi_{mag.}} = 2\frac{\Delta\tilde{E}\cdot\Delta t}{\Delta\Phi_{mag.}} = 2\Delta q_{el.}, \\ \vec{\Phi}_{mag.} \leq 2\frac{\Delta s_1\cdot\Delta s_2}{\Delta\Phi_{el.}} = 2\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta\Phi_{el.}} = 2\frac{\Delta\alpha\cdot\Delta L}{\Delta\Phi_{el.}} = 2\frac{h\cdot\Delta t\cdot\Delta f}{\Delta\Phi_{el.}} = 2\frac{\Delta x\cdot\Delta p}{\Delta\Phi_{el.}} = 2\frac{\Delta t\cdot\Delta\tilde{E}}{\Delta\Phi_{el.}} = 2\frac{\Delta\tilde{E}\cdot\Delta t}{\Delta\Phi_{el.}} = 2\Delta q_{mag.}. \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \lambda_{min.} = \left(\frac{h}{p}\right)_{min.} = 2(\Delta x)_{min.} = \frac{h}{mc}, \\ \vec{T}_{min.} = \left(\frac{h}{\tilde{E}}\right)_{min.} = 2(\Delta t)_{min.} = \left(\frac{h}{c^2\Delta m}\right)_{min} = \frac{h}{mc^2}, \\ \vec{\theta}_{min.} = \left(\frac{h}{L}\right)_{min.} = 2(\Delta\alpha)_{min.} = \frac{2\pi}{n}, \; n = 1,2,3,..., \\ (\vec{\Phi}_{el.})_{min.} = \left(\frac{h}{\Phi_{mag.}}\right)_{min.} = 2(\Delta q_{el.})_{min.} = 2e, \\ (\vec{\Phi}_{mag.})_{min.} = \left(\frac{h}{\Phi_{el.}}\right)_{min.} = 2(\Delta q_{mag.})_{min.} = \frac{h}{2e}. \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} \frac{\lambda_{min.}}{2} = \frac{1}{2}\left(\frac{h}{p}\right)_{min.} = (\Delta x)_{min.} = \frac{h}{2mc}, \\ \frac{\vec{T}_{min.}}{2} = \frac{1}{2}\left(\frac{h}{\tilde{E}}\right)_{min.} = (\Delta t)_{min.} = \left(\frac{h}{2c^2\Delta m}\right)_{min} = \frac{h}{2mc^2}, \\ \vec{\theta}_{min.} = \left(\frac{h}{L}\right)_{min.} = 2(\Delta\alpha)_{min.} = \frac{2\pi}{n}, \; n = 1,2,3,..., \\ \frac{(\vec{\Phi}_{el.})_{min.}}{2} = \frac{1}{2}\left(\frac{h}{\Phi_{mag.}}\right)_{min.} = (\Delta q_{el.})_{min.} = e, \\ (\vec{\Phi}_{mag.})_{min.} = \left(\frac{h}{\Phi_{el.}}\right)_{min.} = 2(\Delta q_{mag.})_{min.} = \frac{h}{2e}. \end{array}\right\}$$

Magnetic flux quantum (From Wikipedia, the free encyclopedia); *-Motivating to think about and make associations and initiate brainstorming... Pay attention on Φ0 = h/(2e) = 2.067 833 636×10−15 Wb.*

*The **magnetic flux quantum** Φ_0 is the quantum of magnetic flux passing through a superconductor. The phenomenon of flux quantization was discovered B. S. Deaver and W. M. Fairbank[2] and, independently, by R. Doll and M. Nabauer,[3] in 1961. The quantization of magnetic flux is closely related to the Little-Parks effect but was predicted earlier by Fritz London in 1948 using a phenomenological model.*

*The inverse of the flux quantum, $1/\Phi_0$, is called the **Josephson constant** and is denoted K_J. It is the constant of proportionality of the Josephson Effect, relating the potential difference across a Josephson junction to the frequency of the irradiation. The Josephson Effect is very widely used to provide a standard for high precision measurements of potential difference, which (since 1990) have been related to a fixed, "conventional" value of the Josephson constant, denoted K_{J-90}.*

Values (CODATA 2006)[1]
Units
$\Phi_0 = 2.067\,833\,667(52)\times10^{-15}$ Wb
$K_J = 483\,597.891(12)\times10^{9}$ Hz/V
$K_{J-90} = 483\,597.9\times10^{9}$ Hz/V

***Magnetic flux quantum** Φ_0 is a property of a supercurrent (superconducting electrical current) that the magnetic flux passing through an area bounded by such a current is quantized. The quantum of magnetic flux is a physical constant, as it is independent of the underlying material as long as it is a superconductor. Its value is $\Phi_0 = h/(2e) = 2.067\,833\,636\times10^{-15}$ Wb.*

If the area under consideration consists entirely of superconducting material, the magnetic flux through it will be zero, for supercurrents always flow in such a way as to expel magnetic fields from the interior of a superconductor, a phenomenon known as the Meissner effect. A non-zero magnetic flux may be obtained by embedding a ring of superconducting material in a standard (non-superconducting) medium. There are no supercurrents present at the center of the ring, so magnetic fields can pass through. However, the supercurrents at the boundary will arrange themselves so that the total magnetic flux through the ring is quantized in units of Φ_0. This is the idea behind SQUIDs, which are the most accurate type of magnetometer available.

*A similar effect occurs when a type II superconductor is placed in a magnetic field. At sufficiently high field strengths, some of the magnetic field may penetrate the superconductor in the form of thin threads of material that have turned normal. These threads, which are sometimes called **fluxons** because they carry magnetic flux, are in fact the central regions ("cores") of vortices in the supercurrent. Each fluxon carries an integer number of magnetic flux quanta.*

Quantization of magnetic flux is determined by a unified geometric theory [4] of electromagnetism and gravitation. The elementary electric charge is simultaneously determined by the theory. Furthermore, these charge and flux quanta directly give the fractions which appear in the fractional quantum Hall effect.

Measuring the magnetic flux

The magnetic flux quantum may be measured with high precision by exploiting the Josephson effect. In fact, when coupled with the measurement of the von Klitzing constant $R_K = h/e^2$, this provides the most precise values of Planck's constant h obtained to date. This is remarkable since h is generally associated with the behavior of microscopically small systems, whereas the quantization of magnetic flux in a superconductor and the quantum Hall effect are both collective phenomena associated with thermodynamically large numbers of particles.

Wilson-Sommerfeld action integrals (see [9] and (5.4.1)), related to any periodical or wave motion on a self-closed stationary orbit, applied over one period of a motion (in question), present the kind of general quantifying rule (for all closed standing waves, which are energy carrying structures) what was successfully used in supporting N. Bohr's Planetary Atom Model. By analogical extension of Wilson-Sommerfeld action integrals to all "**CHARGE**" elements found in T.5.3 and (5.2.1), we can formulate the following quantifying expressions (again between mutually conjugate variables) that are also in agreement with "Periodicity relations, or de Broglie Wave Intervals" from T.5.3, presenting important metrics of different elementary energy-momentum states:

Metrics of Elementary Particles:

$$\left\{\begin{array}{l} 2\left|\Delta q_{mag.}\cdot\Delta q_{el.}\right|_{min.}=2\left|\Delta\alpha\cdot\Delta L\right|_{min.}=2h\cdot\left|\Delta t\cdot\Delta f\right|_{min}=2\left|\Delta x\cdot\Delta p\right|_{min.}=2\left|\Delta t\cdot\Delta\tilde{E}\right|_{min.}= \\ =2c^2\left|\Delta t\cdot\Delta m\right|_{min.}=2\left|\Delta s_1\cdot\Delta s_2\right|_{min.}=h, \\ \left\{\lambda=\frac{h}{p}=\overleftrightarrow{\lambda},\ \tilde{E}=hf=h\frac{1}{\overleftrightarrow{T}}\Leftrightarrow\overleftrightarrow{T}=\frac{h}{\tilde{E}}\right\}\Rightarrow\left\{\overleftrightarrow{\theta}=\frac{h}{L},\ \overleftrightarrow{\Phi}_{el.}=\frac{h}{\Phi_{mag.}},\ \overleftrightarrow{\Phi}_{mag.}=\frac{h}{\Phi_{el.}},\overleftrightarrow{s}_1=\frac{h}{s_2}\right\}, \\ [X]=\left[q_{mag.},q_{el.},x,\alpha\right],\ [Q]=\left[q_{el.},q_{mag.},p,L\right],\ \Phi_{el.}=q_{el.},\ \Phi_{mag.}=q_{mag.}. \end{array}\right\}\Leftrightarrow$$

$$\Leftrightarrow\left\{\begin{array}{l} \overleftrightarrow{\Phi}_{mag.}\cdot\Phi_{el.}=\overleftrightarrow{\Phi}_{el.}\cdot\Phi_{mag.}=\overleftrightarrow{\lambda}\cdot p=\overleftrightarrow{\theta}\cdot L=\overleftrightarrow{T}\cdot\tilde{E}=...=\overleftrightarrow{s}_1\cdot s_2=h, \\ (\overleftrightarrow{\Phi}_{mag.}\cdot\Phi_{el.})_1+(\overleftrightarrow{\lambda}\cdot p)_2+(\overleftrightarrow{\theta}\cdot L)_3+(\overleftrightarrow{T}\cdot\tilde{E})_4+...+(\overleftrightarrow{s}_1\cdot s_2)_m=mh,\ m=1,2,3... \end{array}\right\}\Rightarrow$$

$$\Rightarrow [\overleftrightarrow{\mathbf{X}}][\mathbf{Q}]=[h]\Rightarrow$$

Wilson-Sommerfeld action integrals

$$\left\{\begin{array}{l} \oint_{C_n} p_\lambda d\lambda = n_\lambda h,\ \oint_{C_n} L_\theta d\theta = n_\theta h, \\ (n_\lambda, n_\theta) = \text{integers } (=1,2,3,...) \end{array}\right\}\wedge\left\{\begin{array}{l} \oint_{C_n}\Phi_{el.}d\Phi_{magn.}=n_{el.}h,\ \oint_{C_n}\Phi_{mag.}d\Phi_{el.}=n_{mag.}h, \\ \oint_{C_n}\tilde{E}_n dt = nh,\ (n_{el.},n_{mag.},n)=\text{integers} \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} \oint_{C_n}[X]d[Q]^T=\oint_{C_n}[Q]d[X]^T=\oint_{C_n}\tilde{E}dt=n\cdot[X]\cdot[Q]^T=[n]\cdot[Q]\cdot[X]^T=[n]h,\ n=1,2,3,... \\ [X]^T=[X(t)]^T=\begin{bmatrix} q_{mag.} \\ q_{el.} \\ x \\ \alpha \end{bmatrix},\ [Q]^T=[Q(t)]^T=\begin{bmatrix} q_{el.} \\ q_{mag.} \\ p \\ L \end{bmatrix},\ [n]=\begin{bmatrix} n_{el.} \\ n_{mag.} \\ n_\lambda \\ n_\theta \end{bmatrix} \end{array}\right. \quad (5.4.1)$$

What we could additionally conclude from T.5.3 and Wilson-Sommerfeld action integrals (5.4.1) is that Uncertainty relations (recognized by using the symbol “≤”) in the form (5.1) - (5.2), have certain mathematical power only for cases of relatively unbounded, open-space and sufficiently freely propagating and spatial-temporal evolving wave motions. On the contrary, stable, and self-closed spatial structures, (5.3) - (5.4), like stationary and resonant energy states, and standing waves formations inside atoms and elementary particles (where rest masses could be involved), effectively and “in average” alter this form of Uncertainty, *making it much more certain than uncertain. Something remarkably similar can be shown valid for closed-paths periodical motions of planetary or solar systems like elaborated in the second chapter (except that Planck-like constant will be something else, H >>h). In other words, instead of “≤”, or “≥” in cases related to localized and stable energy states, like atoms and certain elementary particles are, we should have an equality sign “=”, since only an integer number of half or full wavelengths of any relevant matter-wave entity that creates standing waves there (as in (5.2.1)), could exist in such structures with number of periodicities (see T.5.4).* ***The minimal and basic set of such elementary building entities of our universe is captured by (5.4.1).*** *Quantization in Physics or in our Universe is related only to such self-closed, stabilized standing-waves systems (but there are many other transient and not quantized, evolving states of matter). Quantum World and Quantum Physics are dealing with optimal packing, interactions, and energy-momentum exchanges between here described elementary and resonantly self-closed building blocks of matter. Later in this chapter (see (5.14-1)), we will formulate more general conditions for signal discretization or atomization, explaining the meaning of* ***elementary matter-waves domains*** *as being energy finite Gaussian-Gabor signals, optimally concentrated in all mutually conjugate domains. See more of supporting information in the Appendix of this book, Chapter 10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES.*

We can, also say, regarding contemporary Physics and Electromagnetism, that everything related to unity, connections and coupling between electric and magnetic charges and fields is mathematically still better described than analogous relations between linear and rotational motions that should also exist in a similar way in Mechanics. In other words, essential definitions, and parameters of linear and rotational motions (see T.5.4) should be appropriately reformulated (or slightly modified), to get the same elegance simplicity and symmetry as in the case of symmetry between electric and magnetic fields and charges. We probably need to pay more attention to the neglected works of founders of electromagnetism in the period before Maxwell. For instance, on Ampère's force law between electric-current elements, and on Wilhelm Eduard Weber force law between electric charges, where conceptualization gap between electromagnetism and Newtonian mechanics was almost inexistent, compared to what we presently have with self-proclaimed dominance of Maxwell field equations, Lorentz transformations, Relativity, and Orthodox Quantum theory… With sufficient dose of open-minded and intellectually flexible thinking, and with specific remodeling and updating of old Ampère and Weber concepts (not neglecting works of others, like Faraday, Lenz, Biot and Savart…), we could establish much stronger unity between Mechanics and Electrodynamics (compared to what we practice presently), and we can better expose (and later improve) weak areas of Maxwell, Lorentz, Relativity and Quantum theory.

To extend mentioned analogies and symmetry between electromagnetic and mechanical motions, let us summarize already introduced concept of wavelength analogies and symmetries between mechanical and electromagnetic characteristics of matter waves (T.4.2 and T.5.3), by creating the table T.5.4. The terminology in T.5.4 is in some cases slightly modified (compared to what we could find in Physics literature) to additionally expose previously mentioned analogies.

T.5.4. Wavelength analogies in different frameworks

Matter Wave Analogies	Linear Motion	Rotation	Electric Field	Magnetic Field
Characteristic Charge	Linear Momentum p	Orbital Momentum $L = pR$	Electric Charge $q_e = q$	"Magnetic Charge" $q_m = \Phi$
Matter Wave *Periodicity*	*Linear Path Periodicity* $\lambda = \frac{h}{p}$ (Linear motion Wavelength)	*Angular Motion Periodicity* $\theta = \frac{h}{L}$ (Angular motion Wavelength)	*"Electric Periodicity"* $\lambda_e = \frac{h}{q_e} = q_m$	*"Magnetic Periodicity"* $\lambda_m = \frac{h}{q_m} = q_e$
Standing Waves on a circular self-closed path	$n\lambda = 2\pi R$ $p = n\frac{h}{2\pi}\cdot\frac{1}{R}$	$n\theta = 2\pi$ $L = n\frac{h}{2\pi}$	$\lambda_e\lambda_m = q_e q_m = h$	
	$\theta L = \lambda p = h$, $\theta = \frac{\lambda}{R} = \frac{2\pi}{n}$			

(Periodicity – here adopted the term for unifying all de Broglie wavelengths, $q_m = \Phi$ is not a free and independent, self-standing magnetic charge)?

The next unifying (still challenging) step in this process will be to show (macroscopically and microscopically) the existence of intrinsic coupling between magnetic phenomena and rotation, and electric events and linear motions, starting, for instance, from the well-known relations between orbital and magnetic moments valid in atom world (for electrons and protons).

In fact, understanding structures of elementary particles, based on (5.2.1), (5.4.1), and wavelengths analogies T.5.3 and T.5.4 can reveal a straightforward clear picture about how our universe is structured (see also the second chapter equations from (2.5.1) to (2.11)). A very much analogical concept (regarding standing-waves structural stability; -see equations from (2.11.10) to (2.11.20) in the second chapter), could also be established for solar and galactic systems of our universe (based on generalized orbital and spin conservation laws. Consequently, for planetary and some of galactic systems, we need to have an analogical macro-world Planck-like **H** constant, which is much bigger, compared to micro-world Planck constant **h** (and being generally applicable whenever we have stable orbiting and spinning planetary and galactic systems with periodical-motion elements, potentially hosting standing-waves formations, and conserving total angular moment).

In other words, whenever we can successfully apply Planck's constant h (or specific macrocosmic constant H), for process modeling (or for certain interaction description), this is a direct sign that we are dealing with some spatially closed and resonant (circular, elliptic, toroidal, spherical ...) objects, particles, fields, and waves formations where matter-waves are creating standing waves (like what Wilson-Sommerfeld integrals are supporting). To understand better how and why the same Uncertainty Relations are applicable to micro and macro systems, we should remember that real mass-energy-momentum (including electromagnetic items) boundaries of a certain macro-object (or macro mass) are different and more substantial than strict spatial or geometric limits of its static (or rest) mass. Also, Planck constant h is primarily applicable or linked to photons, interatomic entities with number of periodicities, and to belonging matter-waves. For macroparticles, solar systems, and astronomic configurations we should analogically apply different and much bigger Planck-like constants H_i (see the second chapter of this book about Gravitation). This way thinking, we can understand why and how Uncertainty Relations can apply to macro masses. Stable, self-closed standing matter-waves systems are also presenting inertial states (meaning that Newtonian definition of inertia should be appropriately extended). Multilevel and structural, multi-parameter standing-waves quantization, as presented in T.5.4 and (5.2.1) - (5.4.1), in real physics can be analogically compared with Finite Elements Analysis results when we are searching for modal and natural resonant (or standing waves) states of solid bodies. Macroparticles and solid objects usually have many resonant frequencies and harmonics (meaning many standing-waves states) and have many of frequency-zones without resonant properties, but if we go deeper into smaller spatial matter domains, until atoms and inside of atoms, we will again find only self-stabilized, self-closed, and standing-waves, resonant or oscillatory structures with intrinsic periodicities.

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We usually think about time domain vibrations as something what has temporal periodicity and producing vibrational or wave energy effects. On the same way (or analogically), different objects or static, ***geometry related shapes with spatial periodicities (like crystalline structures), when on some way agitated, can produce dynamic effects****, affecting surrounding masses (giving them, or radiating waves with linear and angular moments).* To demonstrate such unusual behaviors of geometry-related periodicities, let us start with certain finite-energy wave packet, which has the following wavefunction:

$$\begin{bmatrix} \Psi(t) \to A(\omega) \\ \Psi(x) \to A(k) \end{bmatrix} \Rightarrow \begin{bmatrix} \Psi(x,t) \to A(k,\omega), \\ \varphi(=)\text{phase}(=)\omega t \pm kx \end{bmatrix} \Rightarrow \overline{\Psi}(x,t) = \left|\overline{\Psi}\right| \cdot e^{I\varphi}$$

Extended, de Broglie, or matter wave properties and wave-particle-duality relations of such wave-packet are:

$$\left.\begin{matrix} \omega = \omega_t = 2\pi f_t = 2\pi f = k_t = \left|\frac{\partial\varphi}{\partial t}\right| = \frac{2\pi}{T} = 2\pi\frac{\tilde{E}}{h}, \\ k = k_x = \omega_x = 2\pi f_x = \left|\frac{\partial\varphi}{\partial x}\right| = \frac{2\pi}{\lambda} = 2\pi\frac{p}{h} \\ k_\alpha = \omega_\alpha = 2\pi f_\alpha = \left|\frac{\partial\varphi}{\partial\alpha}\right| = \frac{2\pi}{\theta} = 2\pi\frac{L}{h} \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} u = \frac{\omega}{k} = \frac{f_t}{f_x} = \frac{\lambda}{T} = \lambda f = \frac{\tilde{E}}{p} = \frac{\partial x}{\partial t}(=) \\ \text{phase velocity} \\ v = \frac{d\omega}{dk} = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{dx}{dt}(=) \\ \text{group velocity} \end{matrix}\right\}$$

Spatial, temporal and angular periodicity periods of certain energy-moments characterized structure, and their mutual relations are (see more in Chapter 10.):

$$\begin{bmatrix} \lambda = \frac{h}{p} = \frac{2\pi}{k_x} \\ p = \frac{h}{2\pi}k_x = hf_x = \frac{h}{\lambda} \\ \Delta p = \frac{h}{2\pi}\Delta k = h\Delta f_x = \frac{h}{2\Delta x} \\ (\Delta x \cdot \Delta p)_{min.} = h/2 \end{bmatrix}, \begin{bmatrix} T = \frac{h}{\tilde{E}} = \frac{2\pi}{k_t} \\ \tilde{E} = \frac{h}{2\pi}\omega_t = hf_t = \frac{h}{T} \\ \Delta\tilde{E} = \frac{h}{2\pi}\Delta\omega = h\Delta f_t = \frac{h}{2\Delta t} \\ (\Delta t \cdot \Delta\tilde{E})_{min.} = h/2 \end{bmatrix}, \begin{bmatrix} \theta = \frac{h}{L} = \frac{2\pi}{k_\alpha} \\ L = \frac{h}{2\pi}k_\alpha = hf_\alpha = \frac{h}{\theta} \\ \Delta L = \frac{h}{2\pi}\Delta k_\alpha = h\Delta f_\alpha = \frac{h}{2\Delta\alpha} \\ (\Delta\alpha \cdot \Delta L)_{min.} = h/2 \end{bmatrix}$$

Certainty Relations between relevant, dimensional durations or wave packet lengths (when wavefunction is well localized and limited in all its domains) are:

$$(\Delta t \cdot \Delta\tilde{E})_{min.} = h \cdot \left(\Delta t \cdot \frac{\Delta\omega_t}{2\pi}\right)_{min.} = h \cdot (\Delta t \cdot \Delta f_t)_{min.} = (\Delta x \cdot \Delta p)_{min.} = h\left(\Delta x \cdot \frac{\Delta k_x}{2\pi}\right)_{min.} = h(\Delta x \cdot \Delta f_x)_{min.} =$$
$$= (\Delta\alpha \cdot \Delta L)_{min.} = h\left(\Delta\alpha \cdot \frac{\Delta k_\alpha}{2\pi}\right)_{min.} = h(\Delta\alpha \cdot \Delta f_\alpha)_{min.} = (\Delta q_{mag.} \cdot \Delta q_{el.})_{min.} = c^2(\Delta t \cdot \Delta m)_{min.} = h/2$$
$$(\Delta t \cdot \Delta\omega_t)_{min.} = (\Delta x \cdot \Delta k_x)_{min.} = (\Delta\alpha \cdot \Delta k_\alpha)_{min.} = \pi, \quad (\Delta t \cdot \Delta f_t)_{min.} = (\Delta x \cdot \Delta f_x)_{min.} = (\Delta\alpha \cdot \Delta f_\alpha)_{min.} = 1/2$$

Consequently, spatial and temporal periodicities of certain energy-moments state or object are (potentially and still hypothetically) producing mutually related dynamic and motional effects, such as:

$$\left\{\begin{bmatrix} \begin{pmatrix} \text{spatial periodicity} \\ \text{linear cristalline structures} \end{pmatrix} \\ f_x = \frac{1}{\lambda} = \frac{p}{h} = \frac{k_x}{2\pi} = \frac{\omega_x}{2\pi} \\ \text{generating linear momentum} \\ p = hf_x, \Delta p = h\Delta f_x \end{bmatrix}, \begin{bmatrix} \begin{pmatrix} \text{temporal periodicity} \\ \text{oscillatory circuits} \end{pmatrix} \\ f_t = \frac{1}{T} = \frac{\tilde{E}}{h} = \frac{k_t}{2\pi} = \frac{\omega_t}{2\pi} \\ \text{generating wave energy} \\ \tilde{E} = hf_t, \Delta\tilde{E} = h\Delta f_t \end{bmatrix}, \begin{bmatrix} \begin{pmatrix} \text{angular periodicity} \\ \text{angular cristalline structures} \end{pmatrix} \\ f_\alpha = \frac{1}{\theta} = \frac{L}{h} = \frac{k_\alpha}{2\pi} = \frac{\omega_\alpha}{2\pi} \\ \text{generating angular momentum} \\ L = hf_\alpha, \Delta L = h\Delta f_\alpha \end{bmatrix}\right\}$$

If our Universe is globally and structurally oscillating, rotating and performing number of relative motions, generating and receiving radiant and heat energy (as Nikola Tesla conceptualized regarding his Dynamic Gravity theory), then stable and solid structures, for instance like big solid masses of granite and other polycrystalline materials in different geometric shapes (or like big pyramids), could be affected by mentioned (radiant energy) perturbations, start resonating on different modal and natural frequencies and producing specific radiations, streaming of fluids, acoustic and electromagnetic waves emissions, explicable based on Certainty and Uncertainty relations between different durations or lengths of relevant original and spectral domains of mentioned objects. Big and very big masses are simply presenting big receivers and emitters of matter waves (or producing and receiving radiant energy, dominantly in a very low frequency range). Also, other complex (by geometry or shape) objects could serve as resonators and specific frequency, amplitude and phase modulators of surrounding and natural matter waves, this way explaining unusual or strange electromagnetic and acoustic effects measured inside and around pyramids and high mountains.

For instance, certain big mass composed of hard stones, granite and minerals has countless number of small crystals (on a molecular level). Such crystals (presumably oscillating because of different environmental and thermal excitations) have certain spatial periodicity with a mean spatial frequency, $f_x = \frac{1}{\lambda} = \frac{p}{h} = \frac{k_x}{2\pi} = \frac{\omega_x}{2\pi}$. Every (excited and oscillating) spatial periodicity structure is also generating (oscillating) linear moments, $p = hf_x, \Delta p = h\Delta f_x$, and such linear moments should produce certain streaming and wave-like manifestations within surrounding fluids, and on electromagnetically charged particles. Such almost inexplicable or unexpected effects are measurable and already detected inside and around big pyramids. Based on spontaneous linear moments generating, and based on surrounding spatial periodicity, internal crystalline and atomic resonators could be exited and start producing resonant oscillations with certain mean frequency of temporal periodicity $f_t = \frac{1}{T} = \frac{\tilde{E}}{h} = \frac{k_t}{2\pi} = \frac{\omega_t}{2\pi}$, this way creating matter waves, having mean energy $\tilde{E} = hf_t, \Delta\tilde{E} = h\Delta f_t$. If the same pyramid internally has another type of spatial and torsional or angular crystalline structure, this could generate an oscillating angular periodicity $f_\alpha = \frac{1}{\theta} = \frac{L}{h} = \frac{k_\alpha}{2\pi} = \frac{\omega_\alpha}{2\pi}$ that would radiate spinning and spiraling particles with angular moments $L = hf_\alpha, \Delta L = h\Delta f_\alpha$.

....

As we can find in [36], Anthony D. Osborne, & N. Vivian Pope effectively and analogically (could be also unintentionally) made a significant extension of Wilson-Sommerfeld standing-waves concept (5.4.1) to the macro-world of planets, stars, and galaxies, but apparently, here we are starting to realize that there is a lot more to do around similar concepts. Before addressing the Universe and galaxies, let us first analyze stability conditions of micro-world of atoms and elementary particles, as follows.

A) Because we already know that all elementary particles (and quasiparticles) have certain intrinsic and quantized spin and/or orbital moments (expressed in units $\frac{h}{2\pi} = \hbar$ multiplied by integers), this clearly tells us that kind of stable, periodical rotation and/or spinning is present there. Every elementary particle exists as a measurable entity (meaning that it can be localized with a certain precision in specific time and space). *Spatial domain of such elementary particle-wave object is hosting an effective spinning mass m (or rotating matter wave with an energy content of mc^2), which could be geometrically conceptualized as a thin and narrow toroid or ring form, having spatially or radially distributed mass m. The condition, which makes such conceptualization defendable, is that when the average radius of rotation R is much bigger than any other rotating particle or rotating ring dimensions, a moment of inertia of such object is in all cases approximately equal to $J = mR^2$.* Angular periodicity belonging to such object, that is analog to (here invented) angular matter wavelength (see T.5.4), should be

$\theta = \frac{h}{L}$. Since we are describing a spatially localized and relatively stable object, only integer number of such angular periodicity sectors captures the entire particle domain, $n\theta = 2\pi = n\frac{h}{L}$ with a consequence that the orbital moment of the intrinsic rotation in question would be $L = n\frac{h}{2\pi} = n\hbar$, $\mathbf{n = 1, 2, 3}$....

B) Now, the idea about self-closed, orbiting circular zone (of an elementary particle structure) is already getting stronger legitimacy, and we could (mathematically) imagine the same rotating object (or an elementary particle) as being an equivalent rotating mass or wave group that revolves around its center of mass. Such rotating mass on its closed-line orbit performs a kind of linear motion, which should be presentable as a matter wave that has its de Broglie matter wavelength equal to $\lambda = \frac{h}{p}$. Since we are describing the same rotating object as before, only an integer number of such wavelengths should cower the circular zone perimeter (like in Wilson-Sommerfeld standing-waves concept (5.4.1)),

$$n\lambda = n\frac{h}{p} = 2\pi R \Leftrightarrow p = n\frac{h}{2\pi}\cdot\frac{1}{R} = n\hbar\cdot\frac{1}{R},\ L = pR = J\omega = J\frac{v}{R} . \qquad (5.4.2)$$

The "rotating motional energy" associated to an orbital moment $L = \mathbf{n}\hbar$, should be equal to a particle linear-motion energy (on a self-closed circular path, where an equivalent mass content moves with its tangential velocity $v = \omega R$), which is associated to linear particle momentum $p = n\hbar\cdot\frac{1}{R}$. Here, we are describing two mathematical aspects of the same motion, belonging to the same, stable and space localized object. Such motional energy equivalence means that the following identities are automatically satisfied (when for linear and orbital moments we take quantized values, $\mathbf{p} = \mathbf{n}\hbar\cdot\frac{\mathbf{1}}{\mathbf{R}}$, $L = \mathbf{n}\hbar$):

$$\left\{pv = L\omega, \frac{mv^2}{2} = \frac{J\omega^2}{2}, vdp = \omega dL, \frac{dp}{p} = \frac{dL}{L}\right\} \text{ and } \left\{p = n\hbar\cdot\frac{1}{R} \ ,\ L = n\hbar\right\} \qquad (5.4.3)$$

$$\Leftrightarrow \{n\hbar\cdot\frac{v}{R} = n\ \hbar\omega,\ v = \omega R\ , \theta L = \lambda p = h,\ \theta = \frac{\lambda}{R} = \frac{2\pi}{n}\},$$

what is obviously and indicatively correct.

C) The conceptual picture regarding the basic structure of elementary particles, as just presented, is oversimplified, but being clear and elegant. Most probably, mentioned elementary and rotating or spinning matter domains are internally composed of electromagnetic waves in a specific form of stationary and standing waves structures, since most of elementary particles have their magnetic moments, and most of them are in some ways being sensitive to external electric and magnetic fields. What is characteristic for many elementary particles is that they have stable gyromagnetic ratios, meaning that both magnetic and orbital moments in question are causally and strongly coupled and coincidently present (having the same origin). Here, we also come close to understanding how nature creates elementary matter domains with rest masses, using (electromagnetic) waves and fields as a primary building substance. Also, spin numbers of bosons and fermions could be addressed inside the same picture given here, since involved quantizing integers regarding linear and orbital motion-periodicity can be mutually different, as for instance, $p = n\hbar \cdot \frac{1}{R}$, $\mathbf{L} = m \cdot \hbar$, $n, m \in [1,2,3...]$. By the nature of here described self-closed circular zone (captured by an elementary particle), it is clear that in some cases integer **m** = **n**, and in other examples could be m = **n/2**, or m = **n/2k,** $n, m, k \in [1,2,3...]$ this way respecting $n\hbar \cdot \frac{v}{R} = m\hbar\omega$, $v = \frac{m}{n}\omega R = \omega R$ ... (see also Chapter 4.1, Fig.4.1, and equations under (4.3)).

In any case, specific and relatively stable elementary domain, or its equivalent wave-packet (or corresponding moving particle) will always have its minimal and elementary building blocks (as standing waves formations) mutually related as found in (5.4),

$$2\left|\Delta q_{mag.} \cdot \Delta q_{el.}\right|_{min.} = 2\left|\Delta\alpha \cdot \Delta\mathbf{L}\right|_{min.} = 2h \cdot \left|\Delta t \cdot \Delta f\right|_{min} = 2\left|\Delta x \cdot \Delta p\right|_{min.} = 2\left|\Delta t \cdot \Delta\tilde{E}\right|_{min.} =$$
$$= 2c^2\left|\Delta t \cdot \Delta m\right|_{min.} = 2\left|\Delta s_1 \cdot \Delta s_2\right|_{min.} = h.$$

D) Also, based on (5.2), and later elaborations (from (5.2.1) until (5.4.1)) it is clear that everything that could be relatively stable, space-time isolated, or qualified as an energy-finite particle, or an equivalent wave packet, should have in its structure coupled elements of linear and rotational motions ($\Delta x, \Delta p$ and $\Delta\alpha, \Delta\mathbf{L}$). This is also equivalent to coupling of mutually dependent electrical and magnetic charges ($\Delta q_{mag.}, \Delta q_{el.}$), also equal to the coupling of (associated) elementary spatial and temporal blocs ($\Delta x, \Delta t$), and causally related to the meaning of energy, mass, and time, since,

$$\Delta E = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\Delta t} = \frac{\Delta\alpha \cdot \Delta\mathbf{L}}{\Delta t} = \frac{\Delta x \cdot \Delta p}{\Delta t} = \overline{v} \cdot \Delta p = h\Delta f = c^2\Delta m = P\Delta t \geq \frac{h}{2\Delta t}.$$

Sometimes, like in the case of Gravitation, we do not see directly or explicitly where the vital place of electric and magnetic charges is, but it should indeed exist (within an internal structure of interacting participants or masses). Regarding Gravitation, presently we are wrongly considering masses as the only relevant gravitational charges (or sources), but (based on (5.2)), every mass is composed of coupled electric and magnetic charges, dipoles, multipoles, or combined linear and spinning motional states, as for instance,

$$\Delta m = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2\Delta t} = \frac{\Delta q_{mag.}}{c^2} i_{el.} = \frac{\Delta\alpha \cdot \Delta\mathbf{L}}{c^2\Delta t} = \frac{\Delta x \cdot \Delta p}{c^2\Delta t} = \frac{\overline{v} \cdot \Delta p}{c^2} = \frac{H\Delta f}{c^2} = \frac{P\Delta t}{c^2} = \frac{\Delta E}{c^2} \geq \frac{H}{2c^2\Delta t}. \quad (5.2.2)$$

Consequently, we are close to the conclusion that in the background of Gravitation we should experience an electromagnetic attraction, because even electrically and magnetically neutral, moving macro masses (when being in a mutual vicinity) have a tendency for creating internal electric and magnetic dipoles or polarizations (because of having elements of angular and accelerated motions). Here it is beneficial to exploit an analogical mass conceptualization, if we treat a photon on a usual and well-proven way, as known in cases of analyses of Compton effect, Photoelectric effect, electron-positron creation, or annihilation, etc. (see in more details in Chapter 4.1. under “4.1.1.1. Photons and Particle-Wave Dualism”). Photon is known to have an equivalent mass that is equal to $m_{ph.} = \frac{\Delta E}{c^2} = \frac{\tilde{E}}{c^2} = \frac{hf}{c^2}$, and we also know that big masses are making gravitational bending of photon beams (meaning attracting photons). It is also known for a photon to be an electromagnetic wave with alternating and mutually coupled (also mutually orthogonal) electric and magnetic field vectors, what could be coupling between specific electric and magnetic dynamic charges, $m_{ph.} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 \Delta t}$. It is also commonly accepted that we can associate an orbital or spinning moment to a photon equal to $L_{ph.} = \frac{h}{2\pi} \Rightarrow m_{ph.} = \frac{\Delta\alpha \cdot \Delta L}{c^2 \Delta t} = \frac{\omega \cdot L_{ph.}}{c^2} = \frac{hf}{c^2}$. As we can see, all equivalent qualifications and quantifications of a photon are again confirming (by analogy) that every finite and space-time localized mass or particle, or energy state, should be in its natural structure kind of coupling of mutually-conjugate states, like we see in (5.2) – (5.2.2).

Another essential meaning of Uncertainty-relations presented here is that we are also this way defining mutual relations, sizes, limits, and dimensions of elementary matter-building blocks, and that specific time interval (or temporal duration) is involved in all of them. For instance, for a photon, relevant time interval, period, or duration will be,

$$\left(m_{ph.} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 \Delta t} = \frac{\Delta\alpha \cdot \Delta L}{c^2 \Delta t} \right) \Rightarrow \Delta t = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{m_{ph.} c^2} = \frac{\Delta\alpha \cdot \Delta L}{m_{ph.} c^2} = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{\tilde{E}_{ph.}} = \frac{\Delta\alpha \cdot \Delta L}{\tilde{E}_{ph.}}. \qquad (5.2.3)$$

If we like to get an idea what could happen with such elementary matter blocks in motion, we can simply apply (on (5.2.2)) the known relation between a rest mass and its equivalent mass in motion, as for instance,

$$\gamma\Delta m = \frac{\Delta q_{mag.} \cdot \Delta q_{el.}}{c^2 (\frac{\Delta t}{\gamma})} = \frac{\Delta\alpha \cdot \Delta L}{c^2 (\frac{\Delta t}{\gamma})} = \frac{\Delta x \cdot \Delta p}{c^2 (\frac{\Delta t}{\gamma})} = \gamma \frac{\bar{v} \cdot \Delta p}{c^2} = \frac{h(\gamma \Delta f)}{c^2} = \gamma \frac{\Delta E}{c^2} \geq \frac{h}{2c^2 (\frac{\Delta t}{\gamma})}, \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \qquad (5.2.4)$$

The conclusion from (5.2.4) is that specific dimensions (sizes, or intervals) of elementary building blocks of matter should also be (mutually) dependent involving Lorentz factor $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$. For instance, if motional (elementary domain) mass is equal to $\gamma\Delta m$, the direct consequence of such mass evolution will be that relevant elementary time domain will evolve to $\frac{\Delta t}{\gamma}$, to keep the validity of (5.2.2). Something similar we should be able to conclude related to Δx and Δp, or to $\Delta\alpha$ and ΔL …

Of course, similar results (regarding actual and proper time) are known in Relativity Theory, but here we are introducing the possibility to extend, correct and revise such insights and practices on a simpler, straightforward, and more generally valid way.

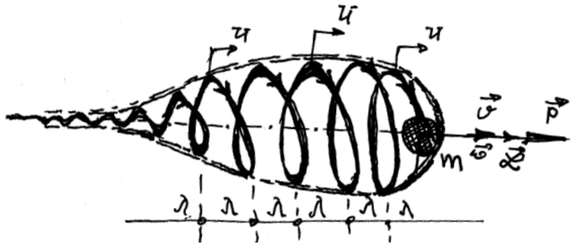
E) We are usually addressing matter waves in relation to de Broglie wavelength, M. Planck and M. Einstein energy of an elementary wave packet or quant, relationships between group and phase velocity, and time-space dependent, waving and oscillatory signals. Judging from time-domain wavefunctions, and related to our intuitive perception, waves for us are moving, oscillating and have certain specific temporal-spatial periodicity and dynamics. To better qualify time and frequency domains of wavefunctions, we are using spectral analysis (based on Fourier integral transformation, on Analytic Signal methods, Hilbert transform, Kotelnikov-Shannon-Nyquist-Whittaker signals sampling and reconstructing methods etc.), practically generating different amplitude, phase, power and energy, time and frequency dependent spectral distributions. Based on the nature of our existence and perception, time-domain functions are something what we see as motional and alive, or active entities. Opposite to that, a frequency or spectral domain functions associated to such time-domain functions are looking as something static, relatively stable. We can also make spectral functions of typical spatial-dimensions-dependent functions (which could be related to standing waves, cristaline structures, and differently shaped objects). For instance, if we have specific thin plate with one or several vertical slits (like in cases of diffraction plates used in laboratories for visualizing light and/or particles diffraction), we can equally well make spectral pictures (applying Fourier transformation) of such spatial-domain functions (or pictures) and get amplitude distributions that are by shape associating on waves (see more in Chapter 10. under "10.00 DEEPER MEANING OF PWDC". It is a matter of habits (and because of the nature of our biological existence) that we do not consider such geometrical and spectral situations as being familiar with matter waves. Here is the place where Uncertainty or "Certainty" relations, like (5.3) and (5.4), are offering additional clarification about the plausible reality of matter waves. Briefly to conclude, matter waves (and other waves) are not something uniquely related or linked to time-domain wave functions. Coincidently with a time-domain, we also have space-domain wave functions, and all of them are mutually dependent respecting (5.3) and (5.4),

$$\left(\Delta q_{mag.} \cdot \Delta q_{el.}\right)_{min.} = \left(\Delta\alpha \cdot \Delta L\right)_{min.} = h \cdot \left(\Delta t \cdot \Delta f\right)_{min.} = \left(\Delta x \cdot \Delta p\right)_{min.} =$$
$$= \left(\Delta t \cdot \Delta\tilde{E}\right)_{min.} = c^2\left(\Delta t \cdot \Delta m\right)_{min.} = \left(\Delta s_1 \cdot \Delta s_2\right)_{min.} = h/2,\ (\Delta x)_{min.} = \frac{\lambda}{2} = \frac{h}{2p}.$$

This is producing that energy-related spectral function will be temporal-frequency dependent, $\tilde{E} = \tilde{E}(f),\ f = \omega / 2\pi\left(\approx \frac{1}{\Delta t}\right)$, that momentum will be spatial-frequency dependent, $p = p(k),\ k = 2\pi / \lambda = \frac{2\pi}{h}p\left(\approx \frac{\pi}{\Delta x}\right)$, and that the signal or group velocity (of energy propagation) will be space-time-frequency dependent, $v = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t} = \overline{v}.$

[♣ COMMENTS & FREE-THINKING CORNER:

***WHAT UNCERTAINTY RELATIONS REALLY PRESENT**: Let us create an oversimplified model for the visual and intuitive understanding of the relation between real and stable particle (or constant mass) size and its matter-wave size in its spatial, temporal and frequency domains (see the picture below). We could realistically assume that mass in question is (geometrically or spatially) smaller than space (or volume) captured by its* *kinematically equivalent matter-wave packet. We can also consider that if the particle is stable, the spatial length* Δx *of its matter-wave equivalent should be on some way* *__standing waves__ structured and stabilized, meaning that* $\Delta x = n \cdot \frac{\lambda}{2},\ n = 1, 2, 3....$ *Now, we assume that mass in question,* m*, has specific speed v and linear moment,* $p = mv$*, and that it started to move from the state of relative rest, meaning that mass gained the linear momentum of,* $\Delta p = p - p_0 = mv,\ (p_0 = 0, p = mv)$.



Now we can create the self-explanatory Uncertainty Relations product, without involving any probability or statistics related thinking,

$$\Delta x \cdot \Delta p = n \cdot \frac{1}{2}\lambda \cdot mv = n \cdot \frac{1}{2}\frac{H}{mv} \cdot mv = n \cdot \frac{H}{2} \geq \frac{H}{2},\ n \geq 1$$

$$\left(\tilde{E} = E_k = Hf = \frac{pv}{1 + \sqrt{1 - \frac{v^2}{c^2}}},\ \lambda = \frac{H}{p} = \frac{H}{mv},\ u = \lambda v, v = u - \lambda \frac{du}{d\lambda} \right).$$

Let us observe mass m in linear motion from its initial position (when x = 0) arriving after certain time Δt *to its new location, passing the length interval equal to* Δx*, which is precisely equal to the effective total length of the same matter wave packet* Δx *that belongs to mass m.*

Under such conditions, we could say that particle with mass m *has an average linear speed equal to,* $v = \frac{\Delta x}{\Delta t}$*, where* Δt *is its temporal duration. We also know that, both in classical and relativistic mechanics, the differential of kinetic (or total) energy is equal to* $\left(d\tilde{E} = dE_k = vdp = Hdf\right) \Leftrightarrow \left(\Delta\tilde{E} = \Delta E_k = v\Delta p = H\Delta f\right)$*, meaning that the same mass* m *has its total motional energy content equal to* $\Delta\tilde{E} = \Delta E_k$*, and it captures certain frequency interval equal to* Δf*. We already know, or we can again verify here, that the product between motional energy content and corresponding temporal duration (of the same matter wave packet) is,*

$$\Delta E \cdot \Delta t = v\Delta p \cdot \Delta t = \frac{\Delta x}{\Delta t}\Delta p \cdot \Delta t = \Delta x \cdot \Delta p \geq \frac{H}{2},\ (\Delta E = \Delta\tilde{E} = \Delta E_k = v\Delta p = H\Delta f).$$

Now, we can just rewrite the same relations as,

$$\frac{\Delta x \cdot \Delta p}{\Delta t \cdot \Delta E} = v\frac{\Delta p}{\Delta E} = v\frac{\Delta p}{H\Delta f} = 1.$$

In cases when our matter wave packet is a photon, we will have, $v\frac{\Delta p}{\Delta E} \to c\frac{\Delta p}{\Delta E} = c\frac{\Delta\tilde{p}}{\Delta\tilde{E}}$,

$$\left(c\frac{\Delta p}{\Delta E} = c\frac{\Delta p}{h\Delta f} = 1 \right) \Leftrightarrow \left(c\frac{dp}{dE} = c\frac{dp}{hdf} = 1 \right).$$

Also, we can extend the same relationship using relativistic energy form $dE = c^2 dm$, *getting already well-known relations,*

$$\left(v\frac{dp}{dE} = v\frac{dp}{hdf} = v\frac{dp}{c^2 dm} = 1 \right) \Leftrightarrow \left(dE = c^2 dm = vdp = hdf \right).$$

Now, we can realize what the real and essential meaning of Uncertainty relations is explained entirely without probabilities and statistics related items. We operated only with total signal (or wave packet) durations, or lengths in different spectral and original domains, and we assumed that the wave packet in question creates standing waves where $\Delta x = n \cdot \frac{\lambda}{2},\ n = 1, 2, 3...$ *Similar situations are extendable on the same way to standing waves formatting of* $\Delta E, \Delta p, \Delta f$ *.*

With such conceptualizing, we can realize that Uncertainty relations are equally applicable (on the same way) to the world of micro and macro-Physics, without any limits, and that statistical interpretations are not ontologically and mathematically necessary here. Of course, the statistical approach is applicable whenever conditions for proper applications of statistics are met. Also, in Classical or macro-world mechanics, we have a false impression that we can fully and precisely define all mass parameters and properties without Uncertainty relations (like position, moments, and kinetic energy), just because spatial macro mass geometry, position, volume, or domain is not considered correctly or on the same way as in micro world physics. For applying Uncertainty relations in macro-mechanics, we should consider relevant spatial-temporal and matter-waves (or wavefunctions) dimensions and kinematic parameters on a similar way as in microphysics world. (see more in Chapter 10). ♣]

There should be no wonder or mystery about the real background of diffraction and interference patterns behind single or multiple vertical slits in experiments with photons, electrons, and other microparticles. Space (or by geometry) defined patterns of vertical slits have certain spatial-frequency dependent spectral distributions, and such spectral (or spatial) distributions are channeling and guiding diffracting particles (since the total diffracting system before and after diffraction should be considered as, at least two-body coupled system where temporal and spatial dimensions or durations are mutually related). All other explanations related to probability and possibility distributions of involved matter waves are very imaginative, operational in cases when statistics is well applicable, but not entirely correct and always valid concepts.

We could also ask ourselves how to understand and apply Uncertainty relations to the hypothetic “Big bang” event that (maybe) happened during an extremely short time interval, meaning that its spectral domain should be extremely large. Also, hypothetical cosmic “Dark matter” or “Dark mass and dark energy” or missing and undetectable mass and energy, should comply with uncertainty relations (5.2.2) - (5.2.4), meaning that we should not simply invent such virtual and experimentally missing entities (by creating some strange names or labels), just to save Relativity theory, without satisfying generally valid mathematics.

*[♣ **COMMENTS & FREE-THINKING CORNER**:*

Quantizing and Kotelnikov-Shannon-Whittaker-Nyquist Sampling Theorem

Here we will show mathematically that signal (or wave function) quantizing and standing waves structural packing are mutually related and important when creating particles and other matter structures. (see more in chapter 10. of this book).

Let us take a time and frequency band-limited, duration-limited, energy finite, signal $\psi(t)$, *where* T *(=) total time duration of the signal* $\psi(t)$, *and* $F = F_{max.}$ *(=) total frequency spectrum duration (or width) of* $\psi(t)$ *in its frequency domain* $A(\omega)$. *If we cannot say that* $\psi(t)$ *is time and frequency durations limited in both domains (in* $\psi(t)$ *and in* $A(\omega)$*), we could just say (for instance) that* T *and* F *are signal durations where 99% or 99.99%... of the signal energy is captured (to be sure that approximately we took almost the whole object or wave packet). Of course, the meaning of signal energy here is closely related to the application of Parseval's theorem (the same energy should be taken into account both in a time and frequency domains). From Uncertainty relations (between total time and frequency durations* T *and* F *of the same signal) we also know that it is generally valid* $TF \geq \frac{1}{2}$.

A band-limited signal $\psi(t)$ *with bandwidth* $\Omega_{max} = 2\pi F_{max.}$ *is entirely determined by the countable set of samples* $\psi(n \cdot \Delta t)$ *of the signal* $\psi(t)$ *if the time sampling interval satisfies* $\Delta t \leq \pi/\Omega_{max}$. *Furthermore,* $\psi(t)$ *may be obtained from these values using the following sampling and reconstruction relations:*

$$\psi(t) = \sum_{n=-\infty}^{\infty} \psi(n \cdot \Delta t) \cdot \frac{\sin\left(\frac{\omega_s (t - n\Delta t)}{2}\right)}{\frac{\omega_s (t-n\Delta t)}{2}} = \sum_{k=-\infty}^{\infty} \psi(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)}$$

$$\omega_s = 2\pi f_s = \frac{4\pi}{\Delta t} = 2\omega^* > 2\Omega_{max} = 4\pi F,\ f_s > 2F,$$

$$\Delta t \leq \frac{1}{2F} = \frac{\pi}{\Omega_{max}},\ \omega^* = \frac{\omega_s}{2},\ \omega_n = n\frac{\omega_s}{2}, F = \frac{\Omega_{max}}{2\pi} = F_{max.}, n = 1, 2, 3...$$

If we now start (backwards) from a known spectral function $A(\omega)$, *(*$\psi(t) \rightarrow A(\omega)$*), we could again analogically apply the same sampling theorem*

$$(t \rightarrow \omega,\ \Delta t \rightarrow \Delta\omega,\ \omega_s \rightarrow t_s,\ \omega^* \rightarrow t^*, F \rightarrow T) \Rightarrow$$

$$t_s \frac{\omega}{2} = t^*\omega,\ t_s > 2T, t_n = n\frac{t_s}{2},\ \Delta\omega = 2\pi\Delta f \leq \frac{\pi}{T},\ \Delta f \leq \frac{1}{2T}, n = 1, 2, 3...$$

$$A(\omega) = \sum_{n=-\infty}^{\infty} A(n \cdot \Delta\omega) \cdot \frac{\sin\left(\frac{t_s (\omega - n\Delta\omega)}{2}\right)}{\frac{t_s (\omega - n\Delta\omega)}{2}} = \sum_{n=-\infty}^{\infty} A(n \cdot \Delta\omega) \cdot \frac{\sin(t^*\omega - t_n \Delta\omega)}{(t^*\omega - t_n \Delta\omega)}.$$

As we can see, for energy finite signals (or wave functions) the same signal can be discretized or quantized (on similar ways) both in its time and frequency domain. Taking into account known uncertainty relation (between total signal durations in a time and frequency domain) and known or necessary sampling intervals, we can create the following, extended uncertainty relations between minimal or sufficient sampling intervals,

$$\left\{ TF \geq \frac{1}{2},\ (\Delta t)_{max.} = \Delta t \leq \frac{1}{2F},\ (\Delta f)_{max.} = \Delta f \leq \frac{1}{2T} \right\} \Rightarrow$$
$$\left\{ 0 < \Delta t \cdot \Delta f \leq \frac{1}{2} \leq TF \leq \frac{1}{4\Delta t \cdot \Delta f} \right\} \Rightarrow (\Delta t \cdot \Delta f)_{min.} = \frac{1}{2}.$$

An optimal signal discretization (or quantization) is implicating that minimal sampling signal intervals or durations in all signal domains may also define standing waves frequency and wavelength. Quantum Theory Uncertainty relations are also related to sampling intervals, but this is now much clearer and more deterministic concept than Uncertainty presented in a contemporary Quantum Theory.

Now we can generalize wave function, meaning to make it presentable as an analytic complex signal function,

$$\psi(t) = \sum_{n=-\infty}^{\infty} \psi(n\cdot\Delta t)\cdot\frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)} = a(t)\cdot\cos\varphi(t),$$
$$\hat{\psi}(t) = H[\psi(t)] = a(t)\cdot\sin\varphi(t) = \sum_{n=-\infty}^{\infty} \hat{\psi}(n\cdot\Delta t)\cdot\frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)},$$
$$\overline{\Psi}(t) = \psi(t) + j\hat{\psi}(t) = a(t)\cdot e^{j\varphi(t)},\ a(t) = |\psi(t)| = |\hat{\psi}(t)| = |\overline{\Psi}(t)| = \sqrt{\psi^2(t) + \hat{\psi}^2(t)},\ j^2 = -1,$$

We can again apply Whittaker-Nyquist-Kotelnikov-Shannon Sampling Theorem and realize wave function quantizing or discretization as,

$$\overline{\Psi}(t) = \sum_{n=-\infty}^{\infty} \overline{\Psi}(n\cdot\Delta t)\cdot\frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)} = e^{j\varphi(t)}\cdot\sum_{n=-\infty}^{\infty} a(n\cdot\Delta t)\cdot\frac{\sin(\omega^* t - \omega_n \Delta t)}{(\omega^* t - \omega_n \Delta t)}$$

In cases of planar unidirectional elementary waves (propagating in a positive x*-direction), the wave function could be something as,*

$$\overline{\Psi}(t) \Rightarrow \overline{\Psi}(\omega t - kx) = \sum_{n=-\infty}^{\infty} \overline{\Psi}(n\cdot\Delta t)\cdot\frac{\sin(\omega^* t - \omega_n \Delta t - kx)}{(\omega^* t - \omega_n \Delta t - kx)} =$$
$$= e^{j(\omega t - kx)}\cdot\sum_{n=-\infty}^{\infty} a(n\cdot\Delta t)\cdot\frac{\sin(\omega^* t - \omega_n \Delta t - kx)}{(\omega^* t - \omega_n \Delta t - kx)} = e^{j(\omega t\ -\ 2n\pi - kx)}\cdot\sum_{n=-\infty}^{\infty} a(n\cdot\Delta t)\cdot\frac{\sin(\omega^* t - \omega_n \Delta t - kx)}{(\omega^* t - \omega_n \Delta t - kx)}$$

intended to produce synchronized, uniform, and linear superposition of elementary (or quantized) waves, with the unique group and phase velocity, as follows,

$$(\omega t\ -\ 2n\pi - kx = \omega^* t - \omega_n \Delta t - kx) \Rightarrow \omega t\ -\ 2n\pi = \omega^* t - \omega_n \Delta t \Rightarrow$$
$$\omega = \omega^*,\ 2n\pi = \omega_n \Delta t = n\frac{\omega_s}{2}\Delta t \Leftrightarrow 4\pi = \omega_s \Delta t,\ \omega_s = \frac{4\pi}{\Delta t}$$

And finally, we will have,

$$\overline{\Psi}(\omega t - kx) = \sum_{n=-\infty}^{\infty} \overline{\Psi}(n \cdot \Delta t) \cdot \frac{\sin(\omega^* t - \omega_n \Delta t - kx)}{(\omega^* t - \omega_n \Delta t - kx)} =$$

$$= e^{j(\omega t - kx)} \cdot \sum_{n=-\infty}^{\infty} a(n \cdot \Delta t) \cdot \frac{\sin(\omega t - \omega_n \Delta t - kx)}{(\omega t - \omega_n \Delta t - kx)}, \ \omega = \omega^* = \frac{1}{2}\omega_s .$$

F) As we know (empirically) every wave phenomenon (or matter wave, wave-packet, photon, quasi-particle…) in a certain environment with fixed and stable properties has its relatively stable and fixed energy propagation speed (or group velocity), which will be denoted here as $v = v_c = \text{const.}$. In all such cases, the same matter wave would also have its phase velocity $u = \lambda f$. It is also possible to verify the inequality relation between any group and phase velocity (valid for any matter wave propagation) as $0 \leq u = \lambda f < 2u \leq \sqrt{uv} \leq v \leq v_c \leq c$. Consequently, if the speed of energy-propagation of specific matter wave is stable and constant $v = v_c = \text{const.}$, under different motional conditions relative to a particular observer, its phase velocity would be variable (dependent on the relative speed between the observer and matter wave generator). Analyzing matter waves this way, we will also be able to address *Doppler Effect relations*.

Let us start from the (always-valid) relation between any group and phase velocity, making the following chain of conclusions (under $v = \overline{v} = v_c = \text{const.}$),

$$\left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{2u}{1+\frac{uv}{c^2}} = (1+\frac{1}{\gamma})u \ , \\ \lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{h}{mc}\cdot\frac{c}{\gamma v} = \lambda_0 \cdot \left(\frac{c}{\gamma v}\right), \ \lambda_0 = \frac{h}{mc} = \text{const.}, \gamma = 1/\sqrt{1-\frac{v^2}{c^2}}, \\ f = \frac{1}{\Delta t} = \frac{E_k}{h} = \frac{mc^2}{h}(\gamma - 1) = f_0 \cdot (\gamma - 1) = \frac{1}{\Delta t_0}\cdot(\gamma-1), \ f_0 = \frac{mc^2}{h} = \text{Const.} \Rightarrow \Delta t = \frac{\Delta t_0}{(\gamma-1)}, \\ u = \lambda f = \lambda_0 f_0 \cdot \frac{\frac{\gamma v}{c}}{1+\gamma} = \frac{\gamma}{1+\gamma}\cdot v = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p}, \ \lambda_0 f_0 = u_0 = c \\ \overline{v} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta E}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\Delta x}{\Delta t}, \ \Rightarrow 0 \leq u < 2u \leq \sqrt{uv} \leq v \leq c. \end{array}\right\} \Rightarrow$$

$$v = \overline{v} = v_c = \text{const.} \Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} v = v_c = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} \Rightarrow \\ v_c d\lambda = u d\lambda - \lambda du \Rightarrow v_c\frac{d\lambda}{\lambda^2} = d(\frac{u}{\lambda}), \\ v_c\frac{d\lambda}{\lambda^2} = -df \Rightarrow \frac{v_c}{\lambda} = f + F_0 \Leftrightarrow \lambda = \frac{v_c}{f+F_0},\ u = v_c\frac{f}{f+F_0},\ F_0 = CONST. \\ v_c dk = d\omega \Rightarrow v_c k = \omega + \omega_0 \Leftrightarrow k = \frac{2\pi}{\lambda} = \frac{\omega+\omega_0}{v_c} \Leftrightarrow \lambda = \frac{v_c}{f+F_0}, \\ v_c dp = u dp + p du = d\tilde{E} = h df \Rightarrow v_c dp = d(up) = d\tilde{E} = h df \Rightarrow \\ \Rightarrow v_c p = up + const. = \tilde{E} + E_0 = hf + hF_0 \Leftrightarrow (v_c - u)p = const. \Leftrightarrow \\ \Leftrightarrow p = \frac{const.}{v_c - u} = h\frac{f+F_0}{v_c} = \frac{\tilde{E}+E_0}{v_c} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{const.}(v_c - u) = \frac{v_c}{f+F_0} = \frac{h}{p} \end{array}\right\} \Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} \lambda = \frac{h}{p} = \frac{v_c}{f+F_0} = \frac{v_c / f}{1+\frac{F_0}{f}} = \lambda_0 \cdot \frac{c}{\gamma v} \leq \frac{v_c}{f},\ \lambda_0 = \frac{h}{mc} = const. \\ f = \frac{v_c}{\lambda} - F_0 = f_0 \cdot (\gamma - 1),\ f_0 = \frac{mc^2}{h} = Const., \\ u = \lambda f = \frac{\tilde{E}}{p} = v_c\frac{f}{f+F_0} = v_c\frac{1}{1+\frac{F_0}{f}} = \frac{\gamma}{1+\gamma}\cdot v = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} \leq v_c, \\ \tilde{E} = hf = \frac{v_c p}{1+\frac{F_0}{f}} = v_c p + E_0 = v_c p - hF_0,\ p = \frac{h}{v_c}(f+F_0) \geq \frac{hf}{v_c},\ hf + hF_0 = v_c p, \\ E_0 = v_c p(\frac{1}{1+\frac{F_0}{f}} - 1) = -v_c p(\frac{\frac{F_0}{f}}{1+\frac{F_0}{f}}) = -v_c p\frac{F_0}{f+F_0} = -hF_0\ . \end{array}\right.$$

The relations established until here should be in the background of Doppler Effect situations.

G) *The wave-mechanics picture of elementary particles (as described above and in the second chapter with equations from (2.5.1) to (2.11)), can additionally be upgraded and generalized. For instance, if we consider that some matter form (initially without a rest mass) aggregates as a spinning object, this should be certain kind of motional and wave energy form, mathematically presentable as a wave packet, serving as an mathematical equivalent to certain rotating particle, or as certain toroidal form of rotating spatially-distributed mass. Here, we are guided by the idea that rotation that creates closed toroidal form should be the mechanism of initial rest mass aggregation (since we already know that elementary particles always have their magnetic and mechanical, orbital and/or spinning moments). Here we are attempting to show that the very first, original elementary matter substance belongs to certain superposition of waveforms and fields (most probably composed of electromagnetic photons, fields, and waves). Such initial or primary "matter or mass substance", when being in a specific motion (with elements of rotation, and when creating standing waves on a closed path) produces stabilized particles or masses. Let us consider that rotating form in question does not make any (externally detectable) linear motion regarding the Laboratory System, or we can say that its center of mass, linear velocity equals zero,* $v_c = 0$ *, and that its rotational or spinning speed* ω *is constant. Tangential (group) velocity here is* $v = \omega R$ *. If we now express mentioned wave rotation as a linear motion of a certain equivalent particle along its circular path, we will have the following differential form of its motional energy:*

$$dE_{motional} = dE = d\tilde{E} = dE_{linear-motion} = dE_{rotational-motion} =$$
$$= \underline{vdp} = \omega Rdp = \underline{\omega dL} = c^2 dm = hdf\ ,\ dL = Rdp = n(\frac{h}{2\pi})\frac{dp}{p}, \quad (5.4.4)$$
$$\begin{Bmatrix} mv^2 = J\omega^2, J = mR^2, v = \omega R,\ 2\pi R = n\cdot\lambda,\ \lambda = h/p, u = \lambda f \\ L = J\omega,\ \theta = h/L, q\cdot\theta = 2\pi = qh/L\ , (n,q) = 1,2,3,... \end{Bmatrix}$$

Again, to underline, the condition which makes such conceptualization logical (like in (5.4.4)) is that when the average radius of rotation of specific particle R is much larger than any other dimension and radius of such equivalent rotating ring, or torus, moment of inertia of such thin walls object is in all cases equal to $J = mR^2$.

Now, we could imagine that initial rotating energy content ($\omega dL = vdp = c^2 dm$ *) starts making additional linear motion related to its Laboratory System (* $v_c dp$ *), having a specific non-zero <u>center of mass velocity</u>* $\underline{v_c \neq 0}$. *Here we are implicitly neglecting what is happening inside the rotating energy form, considering it (by observing externally) to be a particle that has certain rest mass content* $m = \tilde{m}$ *. The new motional energy picture, analog to (5.4.4), will be,*

$$dE = d\tilde{E} = v_c dp + \omega dL = c^2 d(\gamma m)$$
$$\begin{Bmatrix} p = \gamma m v_c\ , \gamma = (1 - v_c^2/c^2)^{-0.5}, \\ v_c = u - \lambda\frac{du}{d\lambda}, \lambda = \frac{h}{p}, u = \lambda f = \lambda\frac{\omega}{2\pi} \end{Bmatrix} \quad (5.4.5)$$
$$E_{motional} = \tilde{E} = E_{linear-motion} = E_{rotational-motion} = pu = \tilde{m}uv = \tilde{m}c^2 = hf$$

Also, assuming that every linear motion is only a particular case (or approximation) of specific curvilinear or rotational motion (with a sufficiently large radius of rotation), we could again make an equivalency between two aspects of such motion, as in (5.4.4),

$$dE = d\tilde{E} = v_c dp + \omega dL = \omega_c dL + mc^2 = c^2 d(\gamma m)$$
$$dE = d\tilde{E} = v_c dp + \omega_c dL^* = c^2 d(\gamma m)$$
$$v_c dp = \omega_c dL\ ,\ \omega dL = mc^2$$

(See also Chapter 4.1, Fig.4.1 and equations under (4.3))

H) *In cases of creation of real elementary particles, we should have some rotational, pure wave motion (with zero rest mass) that creates self-closed standing waves, and this way certain wave-packet becomes a stable elementary particle (like an electron). Such self-closed wave object observed externally looks much more as a particle that has its rest mass (being a carrier of its total energy). When the same particle is internally observed, there is no rest mass, and we will have the following total energy associated with such an object,*

$$E_{total} = E_{motional} = \tilde{E} = E_{linear-motion} = E_{rotational-motion} = pu = \tilde{m}uv = \tilde{m}c^2 = hf$$
$$E_{total}^2 = E_0^2 + p^2c^2 = p^2c^2 = (\tilde{m}c^2)^2 \Leftrightarrow pc = \tilde{m}c^2 \Leftrightarrow p = \tilde{m}c = \tilde{m}v \Leftrightarrow v = c = u \quad (5.4.6)$$

From (5.4.6), we see that the wave packet in question rotates with group (or tangential) velocity that is constant and equal to c (meaning that internal content of such object iare photons). Now, from the equivalency $pv = L\omega$ *, replacing group velocity with c, we get,*

$$\left(pv = L\omega,\ v = c = \omega_c R,\ L = n\frac{h}{2\pi} \right) \Leftrightarrow$$
$$pc = L\omega_c = n\frac{h}{2\pi}\omega_c = nhf_c = \tilde{m}c^2 = E_{total} = E_{motional} = hf, \qquad (5.4.7)$$
$$L = \frac{pc}{\omega_c} = n\frac{h}{2\pi}, f = nf_c, \omega_c = 2\pi f_c = const.\ c = Const.,$$
$$f = nf_c = \frac{\tilde{m}c^2}{h}, p = \tilde{m}c = \frac{hf}{c} = \frac{nhf_c}{c}, n = 1,2,3,...$$

We can find similar challenging concepts regarding the structure of elementary particles in a number of papers coming from Bergman, David L. and Lucas, Jr., Charles W. and their collaborators (see mentioned literature under [16 to 20]. [22], [25], [83], [88]).

What is explained here is that specific pure wave state, without any rest mass content (observed internally), under certain conditions transforms itself into a state that externally starts behaving as having rest mass. The fundamental laws governing behaviors and appearance of such matter state are as usual: Law of Total Energy Conservation, Law of Total Linear Momentum Conservation, and Law of Total Angular Momentum Conservation. Let us imagine that matter-wave state in question is a set of distributed matter-waves elements, which is passing certain transformation. The following conservation laws can describe such locally isolated system,

$$E_{total} = E = \sum_{(i)} E_i = \sum_{(j)} E_j = \gamma mc^2, \frac{\partial E}{\partial t} = 0,$$
$$\vec{P}_{total} = \vec{P} = \sum_{(i)} \vec{p}_i = \sum_{(j)} \vec{p}_j = \gamma m\vec{v}_c, \frac{\partial P}{\partial t} = 0, \qquad (5.4.8)$$
$$\vec{L}_{total} = \vec{L} = \sum_{(i)} \vec{L}_i = \sum_{(j)} \vec{L}_j = J\vec{\omega}_c = \gamma mR^2\vec{\omega}_c, \frac{\partial L}{\partial t} = 0,$$

Also, the total energy conservation should satisfy the following relations,

$$\begin{Bmatrix} E_{total}^2 = E_0^2 + P^2c^2 = E^2, \\ dE = \omega_c dL + v_c dP = c^2 d(\gamma m) \end{Bmatrix} \Rightarrow$$
$$E = \int dE = \int \omega_c dL + \int v_c dP = \gamma mc^2,$$
$$\int \omega_c dL = m_0 c^2 + L^2\omega_c^2 = mc^2, m_0 = const. \qquad (5.4.9)$$
$$\int v_c dP = (\gamma - 1)mc^2$$
$$\gamma = (1 - v_c^2/c^2)^{-0.5}$$

I) *Until here, we did not make any differentiation between the mechanical or angular rotation* $\omega_m = \omega_{gm}$ *and wave angular speed* $\omega = 2\pi f$ *, since we had only circular wave motion (without rest mass).*

Let us imagine that specific particle (which has non-zero rest mass) rotates, having tangential velocity $v = v_g = \omega_m R = \omega_{gm} R$ *, where the radius of rotation is R = const. and* $\omega_m = \omega_{gm}$ *is the mechanical, angular particle velocity (presenting number of full rotations per second), and let us find all particle and matter wave parameters associated to such movement. Practically, the same concept of a wave packet, which has its group and phase velocity, in cases or rotational motions should be analogically extended to the rotating wave packet that has a group and phase angular velocity, (using indexing: m (=) mechanical, f (=) phase, g (=) group), as for instance,*

$$\begin{gathered}
\mathbf{v} = \mathbf{v_g} = \mathbf{R}\omega_{\mathbf{gm}} = \frac{\mathbf{d}\omega}{\mathbf{dk}} (=) \text{ group wave velocity} (=) \text{particle, linear velocity}, \\
\omega_{\mathbf{gm}} = \frac{\mathbf{v}}{\mathbf{R}} = 2\pi \mathbf{f_{gm}} (=) \text{group angular velocity or frequency} (=) \text{particle angular velocity}, \\
\mathbf{u} = \mathbf{v_f} = \mathbf{R}\omega = \frac{\omega}{\mathbf{k}} = \lambda \mathbf{f} (=) \text{ phase, wave velocity}, \mathbf{Rk} = 1, \lambda = 2\pi\mathbf{R}, \\
\omega = \omega_{\mathbf{f}} = \frac{\mathbf{u}}{\mathbf{R}} = 2\pi\mathbf{f} (=) \text{angular wave frequency}, \quad \frac{\omega_{\mathbf{gm}}}{\omega} = \frac{\mathbf{f_{gm}}}{\mathbf{f}} = \frac{\mathbf{v}}{\mathbf{u}}, \\
\mathbf{k} = \frac{2\pi}{\lambda} = \frac{2\pi}{\mathbf{h}} \mathbf{p} (=) \text{ wave vector}, \tilde{\mathbf{E}} = \mathbf{hf}, \\
\left\{ \begin{array}{l} \left[\mathbf{v} = \mathbf{u} - \lambda \frac{\mathbf{du}}{\mathbf{d}\lambda} = \mathbf{u} + \mathbf{k}\frac{\mathbf{du}}{\mathbf{dk}} = \frac{\mathbf{d}\omega}{\mathbf{dk}} \right] \cdot \frac{1}{\mathbf{R}} \Leftrightarrow \\ \frac{\mathbf{v}}{\mathbf{R}} = \frac{\mathbf{u}}{\mathbf{R}} - \lambda \frac{\mathbf{d}(\frac{\mathbf{u}}{\mathbf{R}})}{\mathbf{d}\lambda} = \frac{\mathbf{u}}{\mathbf{R}} + \mathbf{k} \frac{\mathbf{d}(\frac{\mathbf{u}}{\mathbf{R}})}{\mathbf{dk}} = \frac{1}{\mathbf{R}} \frac{\mathbf{d}\omega}{\mathbf{dk}} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \omega_{\mathbf{gm}} = \omega - \lambda \frac{\mathbf{d}\omega}{\mathbf{d}\lambda} = \omega + \mathbf{k}\frac{\mathbf{d}\omega}{\mathbf{dk}} = \frac{1}{\mathbf{R}} \frac{\mathbf{d}\omega}{\mathbf{dk}} \\ \omega = \frac{\omega}{\mathbf{Rk}} = 2\pi\mathbf{f} \end{array} \right\} \\
(0 \le v \le c) \Rightarrow 1 \le \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = \frac{v}{u} \le 2 \\
(v << c) \Rightarrow v = 2u \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 2 \quad \textit{(See also (4.3), chapter 4.1, Matter Waves)} \\
(v \cong c) \Rightarrow (v = u) \cong c \Rightarrow \frac{\omega_{gm}}{\omega} = \frac{f_{gm}}{f} = 1
\end{gathered} \tag{5.4.10}$$

Matter waves associated with any particle in motion are practically defined by PWDC relations (see equations (4.2) - (4.3), Chapter 4.1, and Chapter 10., "10.00 DEEPER MEANING OF PWDC"). Particle mechanical, rotating and spinning properties are different compared to familiar frequency related, matter-wave properties. We should make a difference between mechanical rotation (number of mechanical revolutions about certain center), and orbital matter-waves frequency of associated matter waves. In cases of inter-atomic circular motions, where standing matter waves are an intrinsic structural property, conditions for creating standing waves are:

$$2\pi\mathbf{R} = \mathbf{n}\lambda \Rightarrow \mathbf{Rk} = \mathbf{n} = 1, 2, 3... \Rightarrow \omega_{\mathbf{gm}} = \frac{1}{\mathbf{R}}\frac{\mathbf{d}\omega}{\mathbf{dk}}, \frac{\omega_{\mathbf{gm}}}{\omega} = \frac{\mathbf{v}}{\mathbf{u}} = \frac{\mathbf{k}}{\omega}\frac{\mathbf{d}\omega}{\mathbf{dk}} \tag{5.4.11}$$

Such differentiation between mechanical and matter wave angular (frequency related) parameters has been often neglected. Instead of separating frequency-related matter-wave parameters from ordinary mechanical rotation parameters, some ad hoc and exotic postulates and theory-correcting and supporting statements (regarding elementary particles and atom structure) have been formulated. Anyway, additionally imposed, or postulated principles got legitimacy in Orthodox Quantum Mechanics (to explain challenging situations such as: gyromagnetic ratio, spin attributes, correspondence principle, orbital, and magnetic moments relations etc.). The example described with (5.4.11) should be appropriately generalized and harmonized to comply with Wilson-Sommerfeld rules; -see (5.4.1). In other words, quantization is not something universally applicable to all motions, fields, waves, and charges, without limits and specific structural conditions. We should have kind of resonant and standing-waves conditions that are favorable for creating quantization, usually appearing when stable, elementary matter domains are being assembled and mutually interacting (exchanging moments and energy) in many ways. The nature and shapes of matter-domains and natural forces, regarding their gearing, fitting, synchronization, and packing (both internally and externally) is closely related to standing waves and resonant structures. The attempts to apply Quantum Theory quantizing concepts, and to quantify (by default) all fields and waves known in physics in the same way and for all cases of different motions, are most probably confusing and incompletely defined objectives. In most of the cases regarding particle-wave duality phenomenology we only see emanations of rules of signal analysis and synthesis (known in spectrum analysis and digital signal processing under different signal-sampling and signal-recovery techniques such as we find in works from Shannon–Kotelnikov-Nyquist-Whittaker). An exact *and universal mathematical signals or wave-groups quantizing is often neglected or replaced by some universal quantization without structure and rules, as the contemporary Quantum Theory is promoting.*

Real quantization (where integers and discrete intervals show their importance) is only a modus of energy packing inside stable objects (when available packing space is limited). We will have manifestations of energy quantization also in situations when objects like atoms and elementary particles mutually exchange energy and different moments, but not all energy formats and all energy exchanges in our Universe have forms of, by integers quantifiable standing waves. For instance, in some of transient, progressive and space-time evolving wave-motions regarding particle interactions, we would not be able to implement clear and straightforward energy quantization concepts.

The author of this book considers that the big part of the success would be if certain old dogmas and ad hock postulates of Quantum Theory start evolving and changing towards some of here proposed directions. Eventually, researchers involved in the same field would start seriously considering many different and new ideas that in some ways diverge from the Orthodox Quantum Theory. Since Quantum Theory in many situations mathematically already works well, our task will be to transform inexplicable, unnatural, abstract, or postulated segments found there, into something that would be conceptually and mathematically much more tangible and better integrated into the rest of Physics. ♣]

5.1. Uncertainty in Physics and Mathematics

In a contemporary physics, the Uncertainty Principle is usually presented mostly in connection with Heisenberg's Uncertainty Relations. It is also, almost exclusively linked to (micro world) Planck's constant **h**. For real, correct, and full understanding of many forms of Uncertainty Relations, it is, for the time being, better to forget that Heisenberg made any invention regarding Uncertainty, since what we know and have from mathematics is already enough, and much more generally applicable.

In other words, we should know (or learn) that Uncertainty is not "married" almost exclusively with Statistical interpretation of Quantum Theory, and that it primarily belongs to mathematics (meaning it is universally applicable, presenting mutually conjugate absolute domain-lengths relations, valid both in micro-world of atoms and elementary particles, and in a macro-world of cosmic formations). Even Planck's constant **h**, which has its significant place in all micro-world events (regarding atoms, photons, and elementary particles), has its (analogical) macro-world equivalent in a new, much bigger constant **H** >> **h** (see chapter 2; -equations (2.11.12) - (2.11.21)). It is also the current case that Uncertainty, as presented in contemporary Physics (mostly in Quantum Theory), is often used as a supporting background for many oversimplifications, mystifications, and justifications of the number of logical, conceptual, and methodological uncertainties in Physics. *The here-upgraded concept of Uncertainties will show that this is a much more productive and more diversified concept compared to one presented and applied in the contemporary physics.*

A much more general approach to universally valid Uncertainty relations (in connection to quantum manifestations of all energy formats) should start from a band-limited, energy-finite wave function $\Psi(t)$. Here we shall treat the square of the wave function as a power: $\Psi^2(\mathbf{t}) = \mathbf{Power} = \mathbf{dE} / \mathbf{dt}$, but later, we could also transform, normalize and treat $\mathbf{\Psi(t)}$ as a dimensionless function without influencing the results of the analysis that follows.

Let us designate with $\mathbf{T}$ and $F = \Omega / 2\pi$ the absolute and finite time and frequency durations of a wave function $\Psi(t)$ and its spectral function $\mathbf{A(f)}$, as we find in (4.9), where ($t \in [\mathbf{T}]$, $0 < \mathbf{T} < \infty$, $-\infty < \mathbf{t} < \infty$), and ($\mathbf{f} \in [\mathbf{F}]$, $0 < \mathbf{F} < \infty$, $0 < \mathbf{f} < \infty$, $2\pi F = \Omega, 2\pi f = \omega$). We can also say that $\mathbf{T}$ and $\mathbf{F}$ are absolute (or total) time and frequency lengths of $\Psi(t)$ and its spectrum $\mathbf{A}(f)$).

Here we are considering the wave function, $\Psi(t)$ as an Analytical Signal form (first time introduced by Dennis Gabor, in connection with Hilbert transform, see **[7]** and **[8]**), as well as a finite, energy-limited function, both in its time or frequency domain. It is an advantage of Analytical Signals that they cover only natural domains of real-time and frequency: $-\infty < \mathbf{t} < \infty, \mathbf{0} \leq \mathbf{f} < \infty$. This is opposite to the traditional Signal Analysis (Fourier analysis), where frequency can take negative values, but in all other aspects Analytic Signals produces the same or equivalent results, as in the case of Fourier Signal Analysis, including producing additional dynamic, spectral signal properties, time-frequency dependent, which the Fourier analysis is not able to generate. Before we start developing concepts of Uncertainties, it would be very much recommendable to go to the chapter 4.0 of this book (Wave functions wave velocities and uncertainty relations) in order not to repeat already presented Uncertainties background.

Spectrum Analysis shows (without any doubts, and any applicability limits to the micro or macro world) general validity of the following Uncertainty relation (which is equivalent to (5.1)):

$$TF > \frac{1}{2},\ T\Omega > \pi\ . \tag{5.5}$$

This is the most general, "*master uncertainty framework*" that can later evolve towards Heisenberg micro-world Uncertainty where Planck constant **h** is the most relevant. Analogically, in cases of Uncertainty relations for periodical, stable planetary-systems motions, another Planck-like constant **H**>>**h** is relevant (see more in Chapter 2.). There is nothing here exclusively related to standard deviations and statistical meaning of signal durations. T and F are absolute and total signal durations. Of course, there are logical and clear situations (when we deal with a significant number of identical or similar members, events, or participants) where absolute signal durations could be, very practically, conveniently, and approximately, or statistically replaced with relevant standard deviations (but not with so much exclusivity and ontological weight as ambitiously postulated in the contemporary Quantum Theory).

If specific transformation (5.6) happens to a wave function $\Psi(t)$, changing its time and frequency lengths, **T** and **F**, for the amounts **Δt** and **Δf**, the same signal transformation will automatically influence all other space and energy-related parameters of $\mathbf{\Psi(x, t)}$ to change, producing similar Uncertainty relations, as given in (5.1). Then, an effective physical signal length **L** and signal wave energy $\tilde{\mathbf{E}}$ will also change, as for instance (the following results are taken from Chapter 4.0):

$$\left\{\begin{array}{l} T \to T \pm \Delta t > 0, F \to F \pm \Delta f > 0, L \to L \pm \Delta x > 0, K \to K \pm \Delta k > 0 \\ \Delta\tilde{E} = h\ \Delta f = \overline{v}\Delta p = \tilde{F}\Delta x, \tilde{F} = \dfrac{\Delta p}{\Delta t} = \text{force}, \\ 0 < \delta\, t \cdot \delta f = \delta\, x \cdot \dfrac{\delta k}{2\pi} = \delta\, x \cdot \delta f_x < \dfrac{1}{2} < F \cdot T = \dfrac{1}{2\pi} \cdot K \cdot L = F_x \cdot L \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \tilde{E} \to \tilde{E} \pm \Delta\tilde{E}, \\ \overline{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\delta\, x}{\delta\, t} = \dfrac{\delta\omega}{\delta k} = \dfrac{\delta\tilde{E}}{\delta p} \end{array}\right\} . \tag{5.6}$$

In cases when time and frequency changes are either positive or negative, we have:

$$T \cdot F > \frac{1}{2} \Leftrightarrow \left\{\begin{array}{l} (T + \Delta t) \cdot (F + \Delta f) > \dfrac{1}{2} \\ (T - \Delta t) \cdot (F - \Delta f) > \dfrac{1}{2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \left[T^2 - (\Delta t)^2\right] \cdot \left[F^2 - (\Delta f)^2\right] > \dfrac{1}{4} \\ T \cdot F + \Delta t \cdot \Delta f > \dfrac{1}{2} \\ T \cdot \Delta\, f + \Delta t \cdot F > 0 \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \begin{Bmatrix} 1-(\frac{\Delta t}{T})^2-(\frac{\Delta f}{F})^2+(\frac{\Delta t}{T})^2\cdot(\frac{\Delta f}{F})^2>\frac{1}{4} \\ 1+(\frac{\Delta t}{T})\cdot(\frac{\Delta f}{F})>\frac{1}{2T\cdot F}, T\cdot F>\frac{1}{2} \\ (\frac{\Delta t}{T})+(\frac{\Delta f}{F})>0 \end{Bmatrix} . \tag{5.7}$$

Let us additionally conceptualize the same idea (about sudden signal duration changes) using the moving particle Energy-Momentum 4-vector from the Minkowski-space of Relativity Theory, by (mathematically) introducing mutually coupled changes of total system energy and belonging total momentum, applying discrete, central differentiations method,

$$\begin{aligned} &\overline{P}_4=\overline{P}\left[\vec{p}=\gamma m\vec{v}, \frac{E}{c}=\gamma mc\right], \overline{P}^2=\vec{p}^2-\frac{E^2}{c^2}=-\frac{E_0^2}{c^2}, E_0=mc^2, E=\gamma E_0 \Rightarrow \\ &\vec{p}^2c^2+E_0^2=E^2, (p\to p\pm\Delta p)\Leftrightarrow(E\to E\pm\Delta E)\Rightarrow \end{aligned} \tag{5.7.1-1}$$

$$\begin{aligned} &\begin{Bmatrix} (p+\Delta p)^2c^2+E_0^2=(E+\Delta E)^2 \\ (p-\Delta p)^2c^2+E_0^2=(E-\Delta E)^2 \\ \overline{v}=\frac{\Delta x}{\Delta t}=\frac{\delta x}{\delta t}=\frac{\delta \omega}{\delta k}=\frac{\Delta \omega}{\Delta k} \end{Bmatrix} \Leftrightarrow \left\{ c^2\cdot p\ \Delta p=E\ \Delta E \Leftrightarrow \frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=\overline{v} \right\} \Rightarrow \\ &\Rightarrow \frac{\Delta x}{\Delta t}=\frac{\delta x}{\delta t}=\frac{\Delta E}{\Delta p}=c^2\frac{p}{E}=\overline{v}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}=\frac{\delta \omega}{\delta k}\leq c\ . \end{aligned} \tag{5.7.1-2}$$

What we can see from (5.7.1-2) is that sudden energy-momentum changes in a specific system (a moving particle, here) are causally related to its average center-of-mass velocity, which is at the same time equal to the system average group velocity. This now gives a new and more tangible meaning of Uncertainty Relations that should be well analyzed before we make other conclusions and associated statistical makeup. Before, we found that signal domain proportionality only dimensionally and quantitatively indicated that this could be the signal group velocity (or particle velocity), and now we can safely confirm that this is the real case.

The idea here is to show that so-called Uncertainty Relations are causally related to a velocity of matter waves propagation. The meaning of that is that at the same time when specific motional object or signal experiences a sudden change of its energy-related parameters, matter-waves are automatically created, and this produces results captured by Uncertainty Relations (indirectly saying that there is no real Uncertainty in its old and traditional meaning).

For instance, we can express the average group velocity $\overline{v}$ associated with the transformations (5.7.1-2) as:

$$\left\{u=\frac{\omega}{k}, \omega=ku\right\} \Rightarrow \Delta\omega=(k+\frac{1}{2}\Delta k)(u+\frac{1}{2}\Delta u)-(k-\frac{1}{2}\Delta k)(u-\frac{1}{2}\Delta u)=$$
$$=k\Delta u+u\Delta k \Leftrightarrow \bar{v}=\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k} \Leftrightarrow \quad (5.7.1\text{-}3)$$
$$\Leftrightarrow \left\{v=u+k\frac{du}{dk}=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt}=\text{immediate group velocity}\right\}.$$

Such group velocity (found using finite differences) is fully analog to its differential form where infinitesimal signal changes are involved. By merging average group velocity with Uncertainty Relations, we will again see that they are mutually comparable,

$$\left\{\begin{array}{l}\bar{v}=u+k\frac{\Delta u}{\Delta k}=\frac{\Delta\omega}{\Delta k}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta x}{\Delta t}=\text{average group velocity}\\ \text{and}\\ \left|\Delta x\ \Delta p\right|=\left|\Delta t\ \Delta\tilde{E}\right|=h\left|\Delta t\ \Delta f\right|>h/2, \Delta\tilde{E}=h\Delta f,\\ 0<\delta t\cdot\delta f=\delta x\cdot\delta f_x<\frac{1}{2}\leq F\cdot T=F_x\cdot L\leq\frac{1}{4\cdot\delta t\cdot\delta f}=\frac{1}{4\cdot\delta x\cdot\delta f_x}\end{array}\right\}\Rightarrow$$
$$\Rightarrow \bar{v}=\frac{\Delta x}{\Delta t}=\frac{\Delta E}{\Delta p}=h\frac{\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}=u+k\frac{\Delta u}{\Delta k}=\frac{\delta x}{\delta t}=\frac{\delta\omega}{\delta k}=\frac{dx}{dt}=\frac{d\omega}{dk}. \quad (5.7.1\text{-}4)$$

A very interesting fact regarding the average group velocity "$\bar{v}$" in (5.7.1-3) and (5.7.1-4), which could pass unnoticed, is that the full analogy between the expression for the average group velocity "$\bar{v}$" and the expression for the immediate group velocity "**v**" is not made as an approximation, automatically by formal and simple replacement of infinitesimal difference "d" with discrete, delta difference "Δ". The real development of the average group velocity (as given here) is based on applying symmetrical central differences to basic definitions of group and phase velocity, and it is fully correct. This shows that there is a deterministic connection between the physics of continuum and physics of discrete or finite steps (to support better this statement it would be necessary to devote certain time to learn about properties of central, symmetrical differences).

[♣ COMMENTS & FREE-THINKING CORNER:

Since we know relations between group and phase velocity (of certain specific wave packet) in connection with signal wavelength and frequency, as given in (4.2), we can find the absolute motional parameter frames where all the signal parameters, caused by transformation (5.6), should be expected, for instance:

$$\left\{ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = \frac{dE}{dp} = \frac{dx}{dt}, \lambda = \frac{h}{p}, dE = hdf = c^2 d(\gamma m), p = \gamma m v \right\} \Rightarrow$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = \frac{1}{\lambda_{max.} - \lambda_{min.}}\int_{\lambda_{min.}}^{\lambda_{max.}}(-\lambda^2\frac{df}{d\lambda})d\lambda = \frac{-1}{\Delta\lambda}\int_{f_{max.}}^{f_{min.}}\lambda^2 df = \frac{-h}{\Delta\lambda}\int_{f_{max.}}^{f_{min.}}\frac{1}{p^2}d(hf) =$$

$$= \frac{-h}{\Delta\lambda}\int_{[\Delta E]}\frac{1}{p^2}dE = \frac{-h}{\Delta\lambda}\int_{[\Delta v]}\frac{c^2}{(\gamma m v)^2}d(\gamma m) = \frac{-hc^2}{m\Delta\lambda}\int_{[\Delta v]}\frac{d\gamma}{\gamma^2 v^2} = \frac{-h}{m\Delta\lambda}\int_{[\Delta v]}\frac{dv}{v\sqrt{1-v^2/c^2}} =$$

$$= \frac{h}{2m\Delta\lambda}\left[\frac{1-\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right]_{V_{MIN.}}^{V_{MAX.}} =$$

$$= \frac{v_{min.}v_{max.}}{2\left[v_{max.}\sqrt{1-v_{min.}{}^2/c^2} - v_{min.}\sqrt{1-v_{max.}{}^2/c^2}\right]}\left[\frac{1-\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}}\right]_{V_{MIN.}}^{V_{MAX.}} \leq$$

$$\leq \frac{v_{min.}v_{max.}}{2\left[v_{max.}\sqrt{1-v_{min.}{}^2/c^2} - v_{min.}\sqrt{1-v_{max.}{}^2/c^2}\right]} = \frac{h}{2m\Delta\lambda},$$

$$\Delta\lambda = \lambda_{max.} - \lambda_{min.} = \frac{h}{\gamma(v_{min.})mv_{min.}} - \frac{h}{\gamma(v_{max.})mv_{max.}} =$$

$$= \frac{h}{m}\left[\frac{v_{max.}\sqrt{1 - v_{min.}{}^2/c^2} - v_{min.}\sqrt{1 - v_{max.}{}^2/c^2}}{v_{min.}v_{max.}}\right] \leq \frac{h}{2m\bar{v}},$$

$$\lambda_{max.} = \frac{h}{m}\left[\frac{v_{max.}\sqrt{1-v_{min.}{}^2/c^2}}{v_{min.}v_{max.}}\right], \lambda_{min.} = \frac{h}{m}\left[\frac{v_{min.}\sqrt{1 - v_{max.}{}^2/c^2}}{v_{min.}v_{max.}}\right],$$

$$\bar{\lambda} = \frac{h}{\bar{p}} = \frac{h}{m\bar{v}}\sqrt{1-\bar{v}^2/c^2} \leq \frac{h}{m\bar{v}}, \frac{\Delta\lambda}{\bar{\lambda}} \leq \frac{1}{2},$$

$$\lambda = \frac{h}{p} \Rightarrow p\Delta\lambda + \lambda\Delta p + \Delta p\Delta\lambda = 0,\ (1+\frac{\Delta p}{p})(1+\frac{\Delta\lambda}{\lambda}) = 1,\ \frac{\Delta\lambda}{\bar{\lambda}} = -\frac{\frac{\Delta p}{\bar{p}}}{1+\frac{\Delta p}{\bar{p}}}.$$

$$u_{min} = \lambda_{min.}f_{min.} = \frac{v_{min.}}{1+\sqrt{1-v_{min.}{}^2/c^2}},\ u_{max.} = \lambda_{max.}f_{max.} = \frac{v_{max.}}{1+\sqrt{1-v_{max.}{}^2/c^2}},$$

$$\bar{u} = \bar{\lambda}\bar{f} = \frac{h\bar{f}}{m\bar{v}}\sqrt{1-\bar{v}^2/c^2} = \frac{\bar{v}}{1+\sqrt{1-\bar{v}^2/c^2}},$$

$$f_{max.} = \frac{1}{\lambda_{max.}}\left[\frac{v_{max.}}{1+\sqrt{1-v_{max}^2/c^2}}\right] = \frac{m}{h}\left[\frac{v_{min.}v_{max.}}{(1+\sqrt{1-v_{max.}{}^2/c^2})\sqrt{1-v_{min}^2/c^2}}\right],$$

$$f_{min.} = \frac{1}{\lambda_{min.}}\left[\frac{v_{min.}}{1+\sqrt{1-v_{min}^2/c^2}}\right] = \frac{m}{h}\left[\frac{v_{min.}v_{max.}}{(1+\sqrt{1-v_{min}^2/c^2})\sqrt{1-v_{max}^2/c^2}}\right],$$

$$\bar{f} = \frac{m\bar{v}^2}{h(1+\sqrt{1-\bar{v}^2/c^2})\sqrt{1-\bar{v}^2/c^2}},$$

$$\Delta f = f_{max.} - f_{min.} \leq \frac{v_{max.}}{\lambda_{max.}} - \frac{v_{min.}}{2\lambda_{min.}} = \frac{m(v_{min.}v_{max.})}{h}(1 - \frac{1}{2\sqrt{1-v_{max}^2/c^2}}),$$

$$\overline{E}_k = \tilde{E} = \frac{\overline{p}\overline{v}}{1+\sqrt{1-\overline{v}^2/c^2}} = \frac{m\overline{v}^2}{(1+\sqrt{1-\overline{v}^2/c^2})\sqrt{1-\overline{v}^2/c^2}} = h\overline{f}, \quad m = \frac{E_{total}}{c^2},$$

$$v <<< c \Rightarrow$$

$$\overline{v} \cong \frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}, \frac{\Delta v}{\overline{v}} \cong 2\frac{(v_{max.}-v_{min.})^2}{v_{min.}v_{max.}},$$

$$\overline{u} = \overline{\lambda}\overline{f} \cong \frac{1}{2}\overline{v} \cong \frac{v_{min.}v_{max.}}{4(v_{max.}-v_{min.})},$$

$$\Delta\lambda \cong \frac{h}{m}\left[\frac{v_{max.}-v_{min.}}{v_{min.}v_{max.}}\right], \overline{\lambda} \cong \frac{h}{m\overline{v}} \cong \frac{2h}{m}\left[\frac{v_{max.}-v_{min.}}{v_{min.}v_{max.}}\right] \cong 2\Delta\lambda, \frac{\Delta\lambda}{\overline{\lambda}} \cong \frac{1}{2}, \tag{5.7.2}$$

$$\Delta f \cong \frac{m(v_{min.}v_{max.})}{2h}, \overline{f} \cong \frac{m}{2h}\overline{v}^2 \cong \frac{m}{2h}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2, \frac{\Delta f}{\overline{f}} \cong 4\frac{(v_{max.}-v_{min.})^2}{v_{min.}v_{max.}},$$

$$\overline{E}_k = \tilde{E} = \frac{m}{2}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2 = \frac{E_{total}}{2c^2}\left[\frac{v_{min.}v_{max.}}{2(v_{max.}-v_{min.})}\right]^2, \ m = \frac{E_{total}}{c^2}.$$

Particle-wave duality and Uncertainty relations will become even more interesting research subjects if we express the signal length variation from (5.6), in connection to signal wavelength variation as $\Delta x \cong const. \times \Delta\lambda$, or if the total signal length presents the integer multiple of the average signal half-wavelength, $L = n\frac{\overline{\lambda}}{2}$, $n = 1,2,3...$

Results and relations from (5.7.2) should also be applicable on explaining the spectral nature of the Black Body Radiation (and Planck's law). Whatever the meaning of results found in (5.7.2) is, in relation with the signal transformation (5.6) and (5.7), the important message regarding understanding Uncertainty relations is that we should always make clarification of what we are talking about. For instance, looking only from a mathematical point of view, everything regarding Uncertainty relations is clear, since we know that we are talking about absolute signal interval-lenghts relations between their time and frequency domains. When we say that our signal or wave function $\Psi(x, t)$ should represent a real object in motion, we should mathematically correctly apply Uncertainty relations considering relevant velocity, momentum, energy, etc. A part of confusion and misunderstandings regarding Uncertainty relations in Physics comes from a very much arbitrary methodology in connecting mathematical aspects of Uncertainty (between wave function domains durations), with equivalent real (geometry related) particles dimensions. *Uncertainty relations known from mathematics, when correctly applied, are easily understandable, but Uncertainties in microphysics (as presented in the contemporary Quantum theory) are still challenging and should be better elaborated.* If we now go backwards from (5.6) - (5.7.2) to (5.1) - (5.5), we will understand that Uncertainty relations related to the world of Physics should be reformulated and generalized in the frames of meanings of (5.6) - (5.7.2). This way, we will get more complete picture regarding real and generally valid Uncertainty relations and quantum nature in physics.

For instance, all relations and mathematical expressions, starting from (5.6) until (5.7.2) should also apply to any macro-object from our universe. Let us hypothetically imagine that with (5.6) and (5.7) we describe the total size and all other frames (or frontiers) of our universe, approximating it geometrically to be like an "expanding balloon". Since it looks that we know how the external diameter of that balloon should grow, based on Hubble's law (and astronomic observations), we could draw many challenging predictions and conclusions, just by correctly applying here relevant Uncertainty relations. We could also deductively conclude that certain step-stones and pillars of modern physics, presently accepted as unquestionable, should again become questionable. ♣]

Understanding Uncertainty relations in physics (presently still on a mathematical level) is also related to our choice of signal duration intervals. Until here, we have been using (or talking about) real, absolute, or total signal interval lengths. Now we will once more extend already established Uncertainty Relations of absolute signal duration intervals, taking into consideration corresponding *signal standard deviation intervals*.

Since the Orthodox Quantum Mechanics mostly deals with statistical distributions and probabilities, relevant interval lengths are represented by signal variance intervals, which are statistic or standard deviations of certain variables around their mean values. Consequently, mathematical expressions of basic Uncertainty Relations, when using variance intervals or statistical deviations, present another aspect of Uncertainty Relations (not mentioned before, but exclusively used in today's Orthodox Quantum Mechanics theory), and here, such approach will be integrated into a chain of all other, already known Uncertainty Relations. The statistics' concept of variance is used to measure the signal's energy spreading in time and frequency domains. For instance, for an energy-finite wave function, we can define the following variances (see **[7]**, pages: 29-37, **[8]**, pages: 273-277 and **[79],** pages: 57-60):

$$
\begin{aligned}
&(\sigma_t)^2 = \Delta^2 t = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty}(t-\langle t\rangle)^2\left|\overline{\Psi}(t)\right|^2 dt = \int_{-\infty}^{+\infty} t^2 \frac{\left|\overline{\Psi}(t)\right|^2}{\tilde{E}} dt - \langle t\rangle^2 < T^2,\\
&(\sigma_\omega)^2 = \Delta^2\omega = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}(\omega-\langle \omega\rangle)^2\left|A(\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{+\infty} \omega^2 \frac{\left|A(\omega)\right|^2}{\tilde{E}} d\omega - \langle \omega\rangle^2 < (2\pi F)^2,\\
&\omega = 2\pi f,\ \sigma_\omega = 2\pi\sigma_f,\ \tilde{E} = \left\|\overline{\Psi}(t)\right\|^2 = \int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\int_{0}^{+\infty}\left|A(\omega)\right|^2 d\omega,
\end{aligned}
\tag{5.8}
$$

where mean time and mean frequency should be found as:

$$
\langle t\rangle = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} t\left|\overline{\Psi}(t)\right|^2 dt,\ \langle \omega\rangle = \frac{1}{\pi\tilde{E}}\int_{0}^{+\infty}\omega\left|A(\omega)\right|^2 d\omega = 2\pi\langle f\rangle = 2\pi\overline{f}\,. \tag{5.9}
$$

If two functions, $\mathbf{\Psi(t)}$ and $A(\omega)$, form a Fourier-integral pair, then they cannot both be of short duration. This is supported by the scaling theorem,

$$
\Psi(at) \leftrightarrow \frac{1}{|a|}A(\frac{\omega}{a}), \tag{5.10}
$$

*where **"a"** is a real constant. The above claim, (5.10), also known as the **Uncertainty Principle**, can be given various interpretations, depending on the meaning of the term "duration".*

Using the time and frequency variances, (5.8), as the significant signal duration intervals, found for an energy-finite wave function $\Psi(t)$, it is possible to prove the validity of the following Uncertainty Principle (see **[7]** and **[8]**):

If $\sqrt{t}\Psi(t) \to 0$ for $|t| \to \infty$,
then:

$$2\pi\mathbf{TF} = \mathbf{T\Omega} > \mathbf{TF} > \sigma_t\sigma_\omega = 2\pi\sigma_t\sigma_f = \sqrt{(\Delta^2 t)(\Delta^2\omega)} = 2\pi\sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{2}, \quad (5.11)$$

$$\mathbf{TF} = \frac{\mathbf{T\Omega}}{2\pi} > \frac{\mathbf{TF}}{2\pi} > \frac{\sigma_t\sigma_\omega}{2\pi} = \sigma_t\sigma_f = \frac{\sqrt{(\Delta^2 t)(\Delta^2\omega)}}{2\pi} = \sqrt{(\Delta^2 t)(\Delta^2 f)} \geq \frac{1}{4\pi}$$

In the variance relations (5.11) we consider as apparent that absolute (or total) time and frequency durations, $\mathbf{T}$ and $\mathbf{F}$, can never be shorter than time and frequency variances, $\boldsymbol{\sigma_t}$ and $\boldsymbol{\sigma_f}$ (and usually, they should be much larger than $\boldsymbol{\sigma_t}$ and $\boldsymbol{\sigma_f}$). It is also clear that statistical, (5.11), and Quantum Mechanic's aspect of Uncertainty should be fully integrated (meaning correctly estimated) within absolute durations or interval values Uncertainty Relations, (5.7) - (5.7.2), as for instance,

$$\begin{aligned} &0 < \delta t \cdot \delta f = \delta x \cdot \delta f_x < \frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L \leq \frac{1}{4 \cdot \delta t \cdot \delta f} = \frac{1}{4 \cdot \delta x \cdot \delta f_x}, \\ &0 < \delta t \cdot \delta \tilde{E} = \delta x \cdot \delta p < \frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L \leq \frac{h}{4 \cdot \delta t \cdot \delta f} = \frac{h}{4 \cdot \delta x \cdot \delta f_x}. \end{aligned} \quad (5.12)$$

In cases when normal, Gauss amplitudes (or envelopes) distributions are applicable for certain wave packet (what means that 99% signal energy is captured by six times of standard deviation length, $6\sigma_{t,f,x,p,E}$) it should also be valid,

$$\begin{aligned} &\frac{1}{2} \leq \sigma_t \cdot \sigma_f = \sigma_x \cdot \sigma_{f\text{-}x} < 36 \cdot \sigma_t \cdot \sigma_f = 36 \cdot \sigma_x \cdot \sigma_{f\text{-}x} < F \cdot T = F_x \cdot L, \\ &\frac{h}{2} \leq 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = \sigma_x \cdot \sigma_p < 36 \cdot 2\pi\sigma_t \cdot \sigma_{\tilde{E}} = 36 \cdot \sigma_x \cdot \sigma_p < \tilde{E} \cdot T = P \cdot L. \end{aligned} \quad (5.12\text{-}1)$$

In other words, (5.12) extends (and explains) the meaning of Uncertainty relations (in connection with (5.7) and (5.11)) and presents a kind of universal discretization of a wave function. To be sure that here elaborated Uncertainty Relations are correctly applied, when *related to certain wave-packet integrity and stability*, group velocity of such wave packet should satisfy the extended relation, $v = \frac{\delta x}{\delta t} = \frac{\sigma_x}{\sigma_t} = \frac{\sigma_f}{\sigma_{f\text{-}x}} = \frac{L}{T} = \frac{F}{F_x} \leq c$ (see also equations (4.0.72) and (4.0.76) from Chapter 4.0, supporting the same form of group velocity).

In Mathematics, modern Telecommunications Theory and Digital Signal Processing practices we can find many methods and formulas for discretized signal representations, meaning that time-continuous signals, or wave functions, could be adequately represented (errorless, without residuals) if we just implement sufficiently short *time increments sampling* (of a particular continuous signal), and create discrete series of such signal samples. For instance, if a continuous wave function, $\boldsymbol{\Psi(t)}$ is frequency band-limited (at the same time it should also be an energy-finite function), by applying Kotelnikov-Shannon-Nyquist-Whittaker Sampling theorem, we can present a function $\boldsymbol{\Psi(t)}$, concerning its sample values $\boldsymbol{\Psi(n \cdot \delta t)}$, for instance as (see **[8]**),

$$\Psi(t)=a(t)\cos\varphi(t)=\sum_{n=-\infty}^{+\infty}\Psi(n\cdot\delta t)\frac{\sin\ \Omega(t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}=$$
$$=\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega\ (t-n\cdot\delta t)}{\Omega(t-n\cdot\delta t)}\cos\varphi(n\cdot\delta t),\Psi(n\cdot\delta t)=a(n\cdot\delta t)\cos\varphi(n\cdot\delta t), \qquad (5.12\text{-}2)$$
$$\delta t\le\frac{\pi}{\Omega}=\frac{1}{2F},$$

where Ω is the highest frequency in the spectrum of $\Psi(t)$, and we could consider that Ω is the total frequency duration of the signal $\Psi(t)$.

If in the relation (5.7) we take $\Delta t=\delta t$, where δt is the sampling time interval taken from (5.12), then it will be $\Delta f=\delta f$, where δf should be the frequency-sampling interval when applying the Kotelnikov-Shannon-Whittaker-Nyquist sampling theorem on $\mathbf{A(f)=A(\omega/2\pi)}$. It is also important to notice that δt and δf in (5.6) and (5.7), can take positive and negative values, but in the sampling process, they take only positive values. This way we can express the function $\mathbf{A(f)}$ concerning its sample values $\mathbf{A(n\cdot\delta f)}$, as for instance,

$$\mathbf{A(f)=\sum_{n=-\infty}^{+\infty}A(n\cdot\delta f)\frac{\sin 2\pi T(f-n\cdot\delta f)}{2\pi T(f-n\cdot\delta f)},\ \delta f\le\frac{1}{2T}\Leftrightarrow T\cdot\delta f\le\frac{1}{2},}$$
$$\mathbf{(T+\delta t)(F+\delta f)>\frac{1}{2},\ \frac{1}{2}<TF\le\frac{1}{4\delta t\cdot\delta f},\ \delta t\cdot\delta f\le\frac{1}{2}.}$$

$$2\delta t\cdot\delta f\le 4\delta t\cdot\delta f\cdot TF\le 1. \qquad (5.13)$$

Now, from (5.12) and (5.13) we can get even more general Uncertainty and energy expressions than before, such as,

$$|\delta t|\cdot|\delta f|\le\left(T\cdot|\delta f|\approx|\delta t|\cdot F\right)\le\frac{1}{2}\le\sigma_t\sigma_\omega=2\pi\sigma_t\sigma_f<TF\le\frac{1}{4\delta t\cdot\delta f}<2\pi TF,$$
$$\frac{|\delta t|\cdot|\delta f|}{TF}\le\left(\frac{|\delta f|}{F}\approx\frac{|\delta t|}{T}\right)\le\frac{1}{2TF}\le\frac{\sigma_t\sigma_\omega}{TF}=\frac{2\pi\sigma_t\sigma_f}{TF}\le\frac{1}{4\delta t\cdot\delta f\cdot TF}<2\pi,$$
$$|\delta t|<\sigma_t<T,\ |\delta f|<\sigma_f<F,\ \frac{|\delta t|}{T}\approx\frac{|\delta f|}{F},$$
$$\tilde{E}=\int_{-\infty}^{+\infty}\Psi(t)^2dt=\frac{1}{\pi}\int_0^{+\infty}|A(\omega)|^2\,d\omega=\delta t\cdot\sum_{n=-\infty}^{+\infty}|\Psi(n\cdot\delta t)|^2=\delta f\cdot\sum_{n=-\infty}^{+\infty}|A(n\cdot\delta f)|^2=$$

$$=\int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt=\frac{1}{2}\int_{-\infty}^{+\infty}\left|\overline{\Psi}(t)\right|^2dt=\frac{1}{2}\int_{-\infty}^{+\infty}\Psi(t)\Psi^*(t)dt=\frac{1}{2}\int_{-\infty}^{+\infty}a^2(t)dt=\tilde{p}u=h\overline{f}, \qquad (5.14)$$

$$\begin{aligned}\overline{f} &= \frac{\overline{\omega}}{2\pi} = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} f(t)\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\tilde{E}}\int_{-\infty}^{+\infty} \frac{\partial\varphi}{\partial t}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{2\pi^2\tilde{E}}\int_{0}^{+\infty} \omega\left|A(\omega)\right|^2 d\omega = \\ &= \sqrt{\frac{1}{2\pi^2 h}\int_{0}^{+\infty} \omega\left|A(\omega)\right|^2 d\omega} = \sqrt{\frac{1}{h}\int_{-\infty}^{+\infty} f(t)\left|\overline{\Psi}(t)\right|^2 dt} = \sqrt{\frac{1}{h}\int_{-\infty}^{+\infty} \frac{\partial\varphi}{\partial t}\left|\overline{\Psi}(t)\right|^2 dt}\ .\end{aligned} \tag{5.14}$$

The next possibility for analogical extending of uncertainty and energy relations (5.14), could be to introduce similar relations between linear and orbital mechanical moments and electromagnetic charges, in a similar way as (5.3) was created.

As we know from Signal Analysis, it should be clear that if a specific signal (or wave) has a limited or very short duration in its time domain, then its duration in its frequency domain is very large, and vice versa (see (5.10)). Contrary to such general (mathematical) knowledge, here (in (5.14)), we are talking about energy finite and/or limited duration signals in both (time and frequency) domains. This is mathematically not entirely correct, but under reasonable approximations (considering, for instance, 99% of the signal energy in both domains, and especially in cases of Gaussian-envelope signals) could be practically satisfied. Nature anyway (intrinsically) implements or produces an effective "signal filtering and time-frequency shrinking", during signal creation and propagation (also performing some signal modulations, shaping and attenuation) what eventually makes signal duration and its energy content limited and well localized, both in its time and frequency domain.

To illustrate what a finite and limited duration (of an elementary) wave function means in time and frequency domains, let us imagine that we can find (or calculate) an equivalent, averaged wave function, which will replace real (arbitrary shaped) wave-packet function, $\Psi(\mathbf{x},\mathbf{t})$ *. We will place this new (mathematically useful) wave function into a rectangular-shape-amplitude borders (in both, time, and frequency, this way creating some equivalent "rectangular frames", or molds). We shall also request that this elementary, finite-energy wavefunction (or wave-packet* $\underline{\Psi(\mathbf{x},\mathbf{t})}$ *) has energy equal to one energy quant* $\underline{\tilde{\mathbf{E}} = \mathbf{hf}}$*. For instance, the original signal amplitude in a time domain,* $\mathbf{a(t)}/\sqrt{\mathbf{2}}$*, will be replaced by its effective and constant amplitude,* $\overline{\mathbf{a}}/\sqrt{\mathbf{2}}$*. The real signal duration,* $\mathbf{T}$*, in a time domain, will be replaced by an effective signal duration,* $\overline{\mathbf{T}}$*. The real signal amplitude in a frequency domain,* $A(\omega)/\sqrt{\pi}$*, will be replaced by its effective and constant amplitude,* $\overline{\mathbf{A}}/\sqrt{\pi}$*. The real signal duration,* $\mathbf{F}$*, in a frequency domain will be replaced by effective signal duration,* $\overline{\mathbf{F}}$*. Also, the signal mean (central, or carrier) frequency,* f*, will be replaced by its effective central frequency* $\overline{\mathrm{f}}$*, (4.15), placed in the middle (like in a center of gravity) point of the interval* $\overline{\mathbf{F}}$*. Instead of rectangular signal frames or molds (as a method for signal averaging), we could also say that both time and frequency domain wave-forms of the same wave function would be like Gaussian pulses (or Gaussian window functions) because the Gaussian function is optimally concentrated, finite and limited in its joint time-frequency domain.*

Now, based on general wave-energy expressions found in (5.14), all relevant parameters of the one-quantum wave packet, of a valid (or average), rectangular shape elementary wave-packet, or signal, can be presented as:

$$\begin{aligned}
&\tilde{E} = \int_{[t]} [\Psi(t)]^2 \, dt = \frac{\bar{a}^2}{2}\bar{T} = \frac{\bar{A}^2}{\pi} 2\pi\bar{F} = 2\bar{A}^2\bar{F} = h\bar{f} = \overline{pu}, \ \bar{T}\bar{F} = \frac{1}{2}, \\
&\bar{T} = \frac{\sqrt{2}\bar{A}}{\bar{a}} = \left(\frac{2\bar{A}}{\bar{a}}\right)^2 \bar{F} = \frac{1}{2\bar{F}}, \ \bar{F} = (\frac{\bar{a}}{2\bar{A}})^2 \bar{T} = \frac{\bar{a}}{2\sqrt{2}\bar{A}} = \frac{1}{2\bar{T}}, \\
&\bar{f} = \frac{\bar{\omega}}{2\pi} = \frac{1}{2\pi^2 \tilde{E}} \int_0^{+\infty} \omega |A(\omega)|^2 \, d\omega = \sqrt{\frac{1}{2\pi^2 h} \int_0^{+\infty} \omega |A(\omega)|^2 \, d\omega}, \\
&\bar{\lambda} = \frac{h}{\bar{p}}, \ \bar{u} = \bar{\lambda}\bar{f} = \left\langle \frac{\omega}{k} \right\rangle = \left\langle \frac{\tilde{E}}{p} \right\rangle, \ \bar{v} = \left\langle \frac{\Delta\omega}{\Delta k} \right\rangle = \left\langle \frac{d\tilde{E}}{dp} \right\rangle, \\
&h = \frac{2\bar{A}^2}{h} \cdot \frac{\bar{F}}{\bar{f}} = \frac{\bar{a}^2}{2\bar{f}}\bar{T} = (\frac{\bar{a}}{2})^2 \frac{1}{\bar{f} \cdot \bar{F}} = 6.62606876 \text{ x } 10^{-34} \text{ Js}, \\
&\frac{\bar{F}}{\bar{f}} = \frac{1}{2}\left(\frac{h}{\bar{A}}\right)^2 = \frac{1}{2\bar{T}\bar{f}}, \\
&\Psi(x,t) = a(x,t) \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} \cos(\omega t - kx) \ (\Leftrightarrow) \text{wave packet}.
\end{aligned} \tag{5.14-1}$$

As we can see from (5.14-1), if the matter-wave packet has a higher mean frequency $\bar{f}$, its frequency width or duration $\bar{F}$ is shorter. Consequently, for <u>low and exceptionally low mean frequency matter-wave packets</u> (like in phenomenology related to gravitation, planetary and galactic systems), frequency duration $\bar{F}$ of corresponding wave-packets is exceptionally long.

With (5.14-1) we are again formulating conditions for signal discretization or defining meaning of elementary matter-waves domains (as being energy finite, optimally concentrated, Gaussian signals in all mutually conjugate domains), as addressed all over this chapter, since conditions and relations in (5.14-1), (5.2.1), (5.3) and (5.4.1) are mutually comparable and equivalent.

Another far-reaching aspect of uncertainty-relations (and signals quantifying) could be developed if instead of relatively stable, mean-frequency found in (5.14) and (5.9), we use immediate, time variable frequency **f(t)**, from the Analytical Signal model,

$$\begin{aligned}
&\omega(t) = \partial\varphi / \partial t = 2\pi f(t), \\
&(\overline{\Psi}(x,t) = \Psi(x,t) + j\hat{\Psi}(x,t) = a(x,t)e^{j\varphi(x,t)}, \hat{\Psi} = H[\Psi]),
\end{aligned} \tag{5.14-2}$$

what is left to be analyzed another time.

Regarding here-presented aspects of Uncertainty relations we should make a difference between Macro-Uncertainty Relations valid between total interval lengths of mutually coupled or conjugate spectral domains (expressed in absolute or statistical conjugate-variables intervals), and Micro-Uncertainty Relations between minimal signal sampling segments, when signal is "atomized" (or sampled) by means of Digital Signal Processing. Consequently, we should be able to formulate the complete set of United Micro and Macro Uncertainty

Relations, as well as merge them with the extended meaning of "de Broglie periodicity intervals" presented in T.5.4). The contemporary concept of Uncertainty Relations in Physics is still not in a full agreement with the here-described objectives and results (and it still presents more a confusing and uncertain, than clear view regarding that problematic).

5.2. Uncertainty Relations, Fields, and Transformation Domains

In modern physics (starting from Heisenberg) we often find oversimplified (and sometimes mystified) comments about Uncertainty relations without proper explanation between what kind of entities, values, phenomena, categories, signals, dimensions, domains, and tangible-physics items, such Uncertainty exist (see for instance T.5.1 and T.5.2). Most of microphysics literature is presenting Uncertainty relations almost exclusively as something typically valid for micro-world of elementary particles and other quantum entities, what is not correct, because general time-frequency domain, signal analysis does not assume any (a priory) size limitations on the “time-space-frequency” and “object-shape-size” represented by certain wave function.

Several crucial situations are explaining the roots and a background of Uncertainty relations. From mathematics, there is not 1:1 or point-to-point imaging, or correspondence between specific function in its Original and its Spectral domain (valid in both directions, whatever we take as Original or Spectral domain). This property first found and described in mathematics dealing with Signal Analysis, Spectrum Analysis, Fourier transformations, etc., is generally valid and applicable without limitations to all wavefunctions from micro and macro physics. From Physics point of view, the most significant achievement was the formulation of generally valid conservation laws (and associated universal principles) as for instance: Energy conservation, Momentum conservation, etc., found as consequences of space-time uniformity and isotropy (in isolated inertial systems). Euler-Lagrange-Hamilton Theory, as well as Quantum Mechanics differently and correctly contributed to the conclusion that time and energy domains are Original and Spectral domain to each other. Max Planck and Mileva and Albert Einstein also formulated the significant relation for photon as a wave-packet energy, $\tilde{\mathbf{E}} = \mathbf{hf}$, this way connecting energy and frequency (because this way doing, fully correct explanations of Photoelectric and Compton effects confirmed such concept). Einstein showed that there is a direct equivalence or proportionality between mass end energy as $\mathbf{E} = \mathbf{mc^2}$, and that the same relation extends to any form of energy, to particles, quasiparticles, fields, and waves. Luis de Broglie (indirectly and probably unintentionally) discovered another *<u>(Original Domain)-to-(Spectral Domain) couple</u>*, by correctly formulating his matter-wavelength $\lambda = h/p$, this way connecting position (distance, length, or spatial dimension) with linear momentum, leading to $p = hf_s = \hbar\mathbf{k}$ (where $\mathbf{f_s}$ is the spatial frequency).

Physics found that “Nature obeys Fourier Spectrum Analysis”, or that predictions of Spectrum Analysis are confirmable by experimental evidence (this way precisely connecting time and frequency, and position-momentum domains). It is obvious that Physics deals with two (mutually coupled and conjugate) worlds: the world of (our perceptible) Original domains, and the world of corresponding Spectral domains.

Using such concept as a guiding idea (combined with already developed analogies from earlier chapters of this book, as a predictive platform for generating new physics related concepts), we can again formulate several essential Original-to-Spectral, mutually conjugate domain couples, which should be the building blocks of our Universe, as for instance (see T.1.6, T.3.1, T.3.2, T.3.3 and T.5.1):

T. 5.5. Mutually conjugate variables

Original Domains $\leftrightarrow$	$\leftrightarrow$ **Spectral Domains**
Time = $\mathbf{t}$	**Energy** = $\tilde{\mathbf{E}}$ **, (and/or frequency** = $\mathbf{f} = \tilde{\mathbf{E}} / \mathbf{h}$**)**
Displacement = $\mathbf{x} = \mathbf{S}\dot{\tilde{\mathbf{p}}} = \mathbf{S}\tilde{\mathbf{F}}$, $(\tilde{\mathbf{F}} = \mathbf{force})$	**Momentum** = $\tilde{\mathbf{p}} = \tilde{\mathbf{m}}\dot{\mathbf{x}} = \tilde{\mathbf{m}}\mathbf{v}$
Angle = $\alpha = \mathbf{S}_{\mathbf{R}}\dot{\mathbb{L}} = \mathbf{S}_{\mathbf{R}}\tau$	**Angular momentum** = $\mathbb{L} = \mathbf{J}\dot{\alpha} = \mathbf{J}\omega$
Electric charge = $\mathbf{q}_{\mathbf{el.}} = \Phi_{\mathbf{el.}} = \mathbf{C}\dot{\mathbf{q}}_{\mathbf{mag.}} = \mathbf{C}\mathbf{i}_{\mathbf{mag.}}$	**Magn. charge** = $\mathbf{q}_{\mathbf{mag.}} = \Phi_{\mathbf{mag.}} = \mathbf{L}\dot{\mathbf{q}}_{\mathbf{el.}} = \mathbf{L}\mathbf{i}_{\mathbf{el.}}$

Since there is no chance to make 1:1, or point-to-point mapping, imaging, or correspondence between Original and Spectral domain points, there should exist certain intervals-relation between coupled domains, describing the amount of mutual interval matching or mismatching, named in physics the Uncertainty relations, (see (5.2), (5.3), (5.7) and (5.14)). Also, in physics we do not have any strong platform to say which domain is an Original, and which one is its Spectral domain (since both could coincidently exist, be equally important, mathematically, and experimentally verifiable). For instance, Quantum Mechanics (in connection with traditional Schrödinger's equation) formulates and exploits (at least) two of (bi-directional) Original to Spectral domain transformations (or associations), also found in T.1.1 and T.5.5, such as: (time)-(energy), given by $\mathbf{t} \leftrightarrow \mathbf{f}$, and (position)-(momentum), $\tilde{\mathbf{p}} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{x}}$. By analogy (see **T.5.5**), we could also imagine (or propose) to introduce two more associations (concerning rotation and electromagnetic field), for instance: (angle)-(angular momentum), $\mathbb{L} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \alpha}$, and (electric charge)-(magnetic charge), $\mathbf{q}_{\mathbf{mag.}} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{q}_{\mathbf{el.}}}$. Here is convenient to mention that independent, separate, and self-standing unipolar magnetic charges naturally do not exist (except as mutually coupled parts of magnetic dipoles).

Here we are in a strong position to explain the most interesting conceptual platform of this book predicting that the field of Gravitation should have its *complementing field* (at present still hypothetical), caused by mass rotation, in the same way as Electric and Magnetic fields are mutually dependent and complementary fields (see also (4.30) and (4.31)). As we can see from the analogies given in T.5.5, as well as from $\mathbf{q}_{\mathbf{mag.}} \leftrightarrow -\mathbf{j}\hbar\frac{\partial}{\partial \mathbf{q}_{\mathbf{el.}}}$, we should be able to find (analogically) the couple of *Gravitation-Rotational*, mutually conjugate charges, $\mathbf{q}_{\mathbf{gravitat.}}, \mathbf{q}_{\mathbf{rotat.}}$ (analog to electric and magnetic charges, $\mathbf{q}_{\mathbf{el.}}, \mathbf{q}_{\mathbf{mag.}}$), that will mutually respect similar relation as one valid between electric and magnetic charges. For instance, such charges will be an Original and Spectral domain to each other or satisfy the mapping and operator relation: $\mathbf{q}_{\mathbf{rotat.}} \leftrightarrow -\mathbf{j}\hbar \frac{\partial}{\partial \mathbf{q}_{\mathbf{gravitat.}}}$).

Direct and inverse Fourier transforms (of certain wave function), as relations between different charges (or between Original and Spectral domain-couples) should also satisfy Uncertainty Relations (5.2) and (5.5), and can be symbolically and analogically (on a simplified and intuitive way), generally presentable as:

$$F\left\{\Psi\left(\|S\|\right)\right\} = U\left(\|Q\|\right) = \int_{-\infty}^{+\infty} \Psi\left(\|S\|\right) \cdot e^{-j\|Q\|\cdot\|S\|} d\|S\| ,$$

$$F^{-1}\left\{U\left(\|Q\|\right)\right\} = \Psi\left(\|S\|\right) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} U\left(\|Q\|\right) \cdot e^{-j\|Q\|\cdot\|S\|} d\|Q\| ,$$

$$\left\{\begin{array}{l} \|S\| = \begin{pmatrix} t \\ x \\ \alpha \\ q_{el.} \\ q_{gravitat.} \end{pmatrix} = \begin{pmatrix} \text{time} \\ \text{position} \\ \text{angle} \\ \text{el.chrg.} \\ \text{grv.chrg.} \end{pmatrix}, \|Q\| = \begin{pmatrix} f ; (\tilde{E} = hf) \\ \tilde{p} ; (\lambda = h/\tilde{p}) \\ L \\ q_{mag.} \\ q_{rotat.} \end{pmatrix} = \begin{pmatrix} \text{frequency} \\ \text{momentum} \\ \text{ang.momnt.} \\ \text{mag.chrg.} \\ \text{rotat.charg.} \end{pmatrix}, \\ \|\Delta S\| \cdot \|\Delta Q\|^T = \|\Delta Q\| \cdot \|\Delta S\|^T \geq \frac{h}{2} \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \begin{pmatrix} h \cdot (\Delta t \cdot \Delta f) \\ \Delta x \cdot \Delta p \\ \Delta \alpha \cdot \Delta L \\ \Delta q_{el.} \cdot \Delta q_{mag.} \\ \Delta q_{gravitat.} \cdot \Delta q_{rotat.} \end{pmatrix} \geq \frac{h}{2}. \tag{5.15}$$

Delta intervals in relations (5.15) should be considered as total, absolute signal lengths or durations in both, mutually conjugate domains (not as standard deviations and statistical items).

It is interesting to notice that in the case of electric and magnetic charges (and fields) it is possible to have both, Original and Spectral domain equally and coincidentally present in the same, real-time (and in the same space), and that both create (electric and magnetic) fields around them. Is something like that possible for any other conjugate couple of Original-Spectral domains (5.15), it is the question to answer? What should be the Original-Spectral domain couple that is most relevant for gravitation ($\Delta q_{gravitat.}, \Delta q_{rotat.}$), is also the question to answer. Anyway, it would be difficult and illogical to show that **(t, x, α) (=) (time, length, angle),** from (5.15), are sources of some presently known or unknown fields. Also, we must find the answer about what should be the essential, minimal, and complete sets of elements of Original and Spectral, mutually conjugate domains ($\|\mathbf{S}\|, \|\mathbf{Q}\|$) relevant for the description of our universe.

It is almost needless to say that Uncertainty relations like (5.2), (5.3) and (5.14) apply to all the values found in (5.15), T.5.1, T5.2, and T.5.5, belonging to vectors of Original and Spectral domains ($\|\mathbf{S}\|,\|\mathbf{Q}\|$). This time we can create another generalization of uncertainty-relations (5.2) and (5.15), based on (4.32), as for instance:

$$\left\{\begin{array}{l} dP_{\Sigma} = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} dE(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t) \Rightarrow \Delta P_{\Sigma} = \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \Delta E_i \\ E_i = E(q_i, \dot{q}_i, \ddot{q}_i, \dddot{q}_i, ..., q_i^{(n)}, t), \\ dX_{\Sigma} = V_{\Sigma} dt = \frac{\sum_{(i)} dE_i}{dP_{\Sigma}} dt = \frac{\sum_{(i)} \dot{E}_i}{F_{\Sigma}} dt = \frac{\sum_{(i)} dE_i}{F_{\Sigma}} \Rightarrow \Delta X_{\Sigma} = \frac{\sum_{(i)} \Delta E_i}{F_{\Sigma}} \\ \mathcal{F}\left\{\Psi\left(\|X_{\Sigma}\|\right)\right\} = U\left(\|P_{\Sigma}\|\right) = \int_{-\infty}^{+\infty} \Psi\left(\|X_{\Sigma}\|\right) \cdot e^{-j\|P_{\Sigma}\| \cdot \|X_{\Sigma}\|} d\|X_{\Sigma}\|, \\ \mathcal{F}^{-1}\left\{U\left(\|P_{\Sigma}\|\right)\right\} = \Psi\left(\|X_{\Sigma}\|\right) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} U\left(\|P_{\Sigma}\|\right) \cdot e^{-j\|P_{\Sigma}\| \cdot \|X_{\Sigma}\|} d\|P_{\Sigma}\| \end{array}\right\} \Rightarrow$$

$$\|\Delta X_{\Sigma}\| \cdot \|\Delta P_{\Sigma}\| = \left| \frac{\sum_{(i)} \Delta E_i}{F_{\Sigma}} \cdot \sum_{(i)} \frac{\alpha_i}{\dot{q}_i} \Delta E_i \right| \geq \frac{h}{2}. \qquad (5.16)$$

We could also adjust all components of vectors $\|\mathbf{S}\|,\|\mathbf{Q}\|$ and $\|\mathbf{X}_{\Sigma}\|,\|\mathbf{P}_{\Sigma}\|$ from (5.15) and (5.16) to be Lorentz-covariant in Minkowski space (for instance to create 4-space vectors $\|\overline{\mathbf{S}}_4\|,\|\overline{\mathbf{Q}}_4\|$ and $\|\overline{\mathbf{X}}_{\Sigma-4}\|,\|\overline{\mathbf{P}}_{\Sigma-4}\|$, similar to (4.33) - (4.37)). Since the product between any couple of Lorentz-covariant vectors (in the Minkowski space) always presents an (inertial coordinate systems) invariant, it is possible to show that uncertainty relations, or products made of such 4-vectors (in fact made between mutually related, conjugate, absolute, total duration intervals of such 4-vectors),

$$\|\Delta\overline{S}_4\| \cdot \|\Delta\overline{Q}_4\| = \|\Delta\overline{X}_{\Sigma 4}\| \cdot \|\Delta\overline{P}_{\Sigma 4}\| = \underline{\text{Constant } (= \text{invariant})} \geq \frac{h}{2}, \qquad (5.17)$$

are no more uncertain relations (being equally applicable to the world of microparticles, or planetary systems and galaxies, considering that Planck **h**-constant will be replaced by another **H**-constant valid for planetary systems). Also, when creating a New Topology and New Metrics of our universe, we should find a way to incorporate and merge results and predictions from T.5.2, (4.32), (4.37), (5.15), (5.16) and (5.17) in it.

As an example, let us explore (5.17) as the case of uncertainty relations between absolute amounts or durations of energy $\tilde{E}$, time t, momentum p and signal length or position x. Until present, we know such uncertainty relations (without considering 4-vectors), as $\Delta t \cdot \Delta\tilde{E} = h \cdot (\Delta t \cdot \Delta f) = \Delta x \cdot \Delta p \geq \frac{h}{2}$. Let us now consider that momentum $\mathbf{p}$ is replaced by the corresponding 4-vector $\overline{P}_4 = (p, \frac{E}{c})$, and that the same Uncertainty Relation will be applicable between corresponding 4-vectors,

$$\left[p \to \overline{P}_4 = (p, \frac{E}{c})\right] \Rightarrow \left[\Delta p \to \Delta\overline{P}_4 = (\Delta p, \frac{\Delta E}{c})\right],$$
$$\left[\Delta p \cdot \Delta x \geq \frac{h}{2}\right] \Rightarrow \left[\left|\Delta\overline{P}_4 \cdot \Delta\overline{X}_4\right| \geq \frac{h}{2}\right] \Leftrightarrow \left|(\Delta\overline{P}_4)^2 \cdot (\Delta\overline{X}_4)^2\right| \geq \left(\frac{h}{2}\right)^2, \tag{5.18}$$
$$(\Delta\overline{P}_4)^2 = (\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2 = -\left(\frac{E_0}{c}\right)^2 = \text{invariant}.$$

There are at least two different options how to continue from (5.18). We can first consider that Δx is an ordinary, total signal length (in three-dimensional space), equal to $|\Delta x| = |\Delta\overline{X}_4|$, and get,

$$\left|(\Delta\overline{P}_4)^2 \cdot (\Delta\overline{X}_4)^2\right| = \left|(\Delta\overline{P}_4)^2 \cdot (\Delta x)^2\right| = \left\|\left[(\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2\right] \cdot (\Delta x)^2\right\| = \left|-\left(\frac{E_0}{c}\right)^2 \cdot (\Delta x)^2\right| \geq \left(\frac{h}{2}\right)^2 \Rightarrow$$
$$\Rightarrow \left[(\Delta p)^2 \cdot (\Delta x)^2 - \left(\frac{\Delta E}{c}\right)^2 \cdot (\Delta x)^2 = \left(\frac{E_0}{c}\right)^2 \cdot (\Delta x)^2 \geq \left(\frac{h}{2}\right)^2\right] \Rightarrow \tag{5.19}$$
$$\Rightarrow \Delta x = \frac{hc}{2\sqrt{c^2(\Delta p)^2 - (\Delta E)^2}} \geq \frac{hc}{2E_0} = \frac{hc}{2mc^2} = \frac{1}{2}\frac{h}{mc} = \frac{\lambda_c}{2}.$$

Another option is to consider that Δx can analogically be replaced by its Relativistic theory 4-vector, $\Delta\overline{X}_4 = (\Delta x, c\Delta t)$ and this will produce,

$$\left[x \to \overline{X}_4 = (x, ct)\right] \Rightarrow \left[\Delta x \to \Delta\overline{X}_4 = (\Delta x, c\Delta t)\right], (\Delta\overline{X}_4)^2 = (\Delta x)^2 - c^2(\Delta t)^2$$
$$\left|(\Delta\overline{P}_4)^2 \cdot (\Delta\overline{X}_4)^2\right| = \left\|\left[(\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2\right] \cdot \left[(\Delta x)^2 - c^2(\Delta t)^2\right]^2\right\| = \left|-\left(\frac{E_0}{c}\right)^2 \cdot \left[(\Delta x)^2 - c^2(\Delta t)^2\right]\right| =$$
$$= E_0^2 \cdot \left|(\Delta t)^2 - \frac{(\Delta x)^2}{c^2}\right| \geq \left(\frac{h}{2}\right)^2 \Leftrightarrow \left|c^2(\Delta t)^2 - (\Delta x)^2\right| \geq \left(\frac{hc}{2E_0}\right)^2 \Leftrightarrow \tag{5.20}$$
$$\Leftrightarrow \Delta x \geq \sqrt{\left|(\Delta x)^2 - c^2(\Delta t)^2\right|} \geq \frac{hc}{2E_0} = \frac{hc}{2mc^2} = \frac{1}{2}\frac{h}{mc} = \frac{\lambda_c}{2}.$$

Anyway, in both situations regarding relativistic interval length, we can see that minimal signal (or wave packet) length is equal to the half of Compton wavelength, $(\Delta x)_{minimal} = \frac{hc}{2E_0} = \frac{1}{2}\frac{h}{mc} = \frac{\lambda_c}{2}$. This is the same indicative conclusion as found earlier in (5.4) and (5.2.1), meaning that minimal length of certain signal (that represents moving particle) cannot be shorter than belonging Compton, resonant half-wavelength, concerning standing waves, atomized or discretized signal packing. In other words, Uncertainty Relations are defining the elementary building blocks of matter, or frontiers between a micro and macro world of Physics.

From (5.19) and (5.20) we could have some doubts regarding 4-vector associated to the total signal duration, but principal conclusions regarding the significance of

Compton wavelength and standing waves formatting of elementary matter domains will still stay valid. The definition of 4-dimensional relativistic space-time interval (as we find it in Relativity theory, and in this book) is maybe still speculative and controversial concept (not completely proven), and most probably that it can (and should) be modified and upgraded starting from 4-vector of relativistic velocity (which can be considered as correctly defined), as for instance,

$$\begin{aligned}
&\bar{V}_4 = (\gamma v, \gamma c) = \frac{d}{dt}\bar{X}_4 \Rightarrow d\bar{X}_4 = \bar{V}_4 dt = (\gamma v \cdot dt, \gamma c \cdot dt) \Rightarrow \\
&\Rightarrow (\Delta S)^2 = (\Delta \bar{X}_4)^2 = \left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2 - \left(c \int_{[\Delta T]} \gamma \cdot dt\right)^2, \ v = v(x,y,z,t), \ \gamma = 1/\sqrt{1 - v^2/c^2}, \\
&\left|(\Delta \bar{P}_4)^2 \cdot (\Delta \bar{X}_4)^2\right| = \left|\left[(\Delta p)^2 - \left(\frac{\Delta E}{c}\right)^2\right] \cdot \left[\left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2 - \left(c \int_{[\Delta T]} \gamma \cdot dt\right)^2\right]\right| = \\
&= \left|-\left(\frac{E_0}{c}\right)^2 \cdot \left[\left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2 - \left(c \int_{[\Delta T]} \gamma \cdot dt\right)^2\right]\right| = \left|\left(\frac{E_0}{c}\right)^2 \cdot \left[\left(c \int_{[\Delta T]} \gamma \cdot dt\right)^2 - \left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2\right]\right| = \\
&= E_0^2 \cdot \left|\left(\int_{[\Delta T]} \gamma \cdot dt\right)^2 - \frac{1}{c^2}\left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2\right| = \left(E_0 \int_{[\Delta T]} \gamma \cdot dt\right)^2 \geq \left(\frac{h}{2}\right)^2 \Leftrightarrow \\
&\Leftrightarrow \Delta S = \sqrt{\left|\left(\int_{[\Delta T]} \gamma c \cdot dt\right)^2 - \left(\int_{[\Delta T]} \gamma v \cdot dt\right)^2\right|} = c \int_{[\Delta T]} \gamma \cdot dt \geq \frac{hc}{2E_0} = \frac{h}{2mc} = \frac{\lambda_c}{2}.
\end{aligned} \tag{5.21}$$

ΔT in (5.21) is the total signal (or wave packet) duration in its time domain.

In Relativity Theory the same situation regarding challenging relations between 4-vectors of space-time and velocity are addressed by considering the proper time $\mathbf{t_0}$, which is replacing observer time, $t = \gamma t_0$, but (based on (5.19) – (5.21)) there is still a space to ask if the picture about correct foundations of space-time 4-vector is completed.

Similarly, we can address Uncertainty Relations from (5.15), (5.16) and (5.17) replacing normal vectors (from the three-dimensional space) with similar 4-vectors in the Minkowski space. ***Next decisive and essential step will be to unite Minkowski 4-vectors and Analytic Signal concepts (see more in Chapter 10).***

*[♣ **COMMENTS & FREE-THINKING CORNER:** If we continue to exploit the same idea regarding couples of Original and Spectral domains, we could reorganize and generalize all **analogy and symmetry** tables previously established in the first chapter of this book, as well as to generalize Schrödinger-like equations, (4.25), to be equally applicable in all Physics domains (Quantum Mechanics, Gravitation, Maxwell Electromagnetic Theory etc.). For instance, since square of the wave function (in this book) presents the power function, we know that power can be expressed (like in T.5.2) as the product of relevant velocity and force, or voltage and current, or angular velocity and torque, etc. Alternatively, in some situations we will have the sum of such power members (see also (4.30) and (4.31)). By applying generalized Schrödinger equation (4.25) on such wave functions we will be able to develop new wave equations (see also "**phase-group"** concept (4.19)), where currents, voltages, velocities, forces, torque, etc., are explicitly used (what effectively exists in Maxwell electromagnetic theory, and it was developed much before Schrödinger assembled his equation). If the wavefunction is considered only as a (normalized) probability distribution (defined in the 4-dimensional space-time domain), there is no easy way to diversify Schrödinger-like equations, since, for every new attribute, such as current, voltage, force, momentum, torque, etc., we should formulate a proper operator. Unfortunately, the founders of Orthodox Quantum Mechanics devoted most of their professional careers modeling and fixing the meaning of the wave function only in relation to a probability framework. Later, most of their well-obeying followers have just been repeating the same "verses from the bibles written by theory founders. a long time ago" (and surprisingly and fortunately, this concept still works well, but it should not be considered as the best, last, unique, and only possible option, as often being presented). **The modern Statistical Electrodynamics has already introduced a lot of similar, more implicitly than explicitly formulated doubts (like elaborated here) into a unique, final, and irreplaceable position of the Orthodox Quantum Mechanics.***

*Masses with different moments (m, p, and L) are important, but not exactly the types of Gravity-Rotational charges we are here talking about, and this is the area where current Gravitation Theory could be modified and upgraded (while respecting predictions based on electromechanical analogies). Consequently, we should pose the essential question about what the real sources of Gravity and Inertia are (if a static mass is not a sufficient attribute to describe all of such phenomena, according to (4.30) - (4.32), (5.15) and (5.16)). Most probably, the answer to such a question is in understanding interactions, like force expressions (2.1), (2.2) and (2.4), and maybe the same problem will be additionally and better explained within certain multidimensional space environment or coupling (see **[10]** and **[11]**). It is also clear that we cannot neglect the fact that mass always creates a gravitation field or space deformation around itself (as stated in Newton force law and Einstein General Relativity), but here we only propose (based on analogies and indicative insights) how to search for more general sources of gravitation that would be analog or symmetrical to electromagnetic field sources. ♣]*

5.3. Central Differences, Uncertainty and Continuum

There is another platform closely related to signals quantizing, Uncertainty relations, errors estimation, and to the possibility of creating different analogies, which is a direct consequence of mathematics of finite differences, and whose significance has not been exposed and exploited enough in physics. In table T.5.2, the replacement of infinitesimal differences with similar finite differences ($dt = \Delta t$, $dE = \Delta E$…) was obvious and directly made, just following the idea to create dimensional analogies. It will be good to pay attention to the background platform that makes such replacements possible and to know when this is entirely correct (see [82]).

If finite differences (of Δ - types) <u>belong to the class of central or symmetrical differences</u>, then, applying them to a large group of continual functions (often used in mathematical physics), we will obtain the same (or in some cases almost the same) results as in differential analysis with infinitesimal differences. This situation gives us a chance, whenever something like that is applicable, to transform many differential equations of mathematical physics, almost directly (by replacing $d (=) \Delta$ **,** $dy \to \Delta y$, $dx \to \Delta x$...), into simpler algebraic, "<u>quantified, discrete and finite" equations</u>, respecting the rules of central and finite differences. In many cases the same method also replaces higher levels of infinitesimal derivatives $\mathbf{d^n}$ with their simple analog and finite, differential $\mathbf{\Delta^n}$ **-** operators.

The central difference of the function $F(x)$ can be defined as $\Delta F(x) = F(x + \alpha \Delta x) - F(x - (1- \alpha)\Delta x)$, where $0 \leq \alpha \leq 1$. If $\alpha = 1/2$, then $\Delta F(x)$ presents central and symmetrical difference. There are cases of functions and differential equations where the first derivation $dF(x)/dx$, is identical (or almost identical) to $\Delta F(x)/\Delta x$, where $\Delta F(x)$ is the central and symmetrical difference, ***without applying $\Delta x \to 0$ (meaning that we can consider Δx = Const.).*** Based on the previously mentioned specifics of central differences, we can try to explain the relations between the Physics of Continuum and Quantum Physics, and provide the part of the answer why, where, when and how nature made quantization of its elementary entities.

The mathematical models of reversible, continual, smooth, and deterministic processes in physics (related to their differential equations) seem to belong to the family of functions where differential, central-symmetrical Δ^n-operators can replace infinitesimal derivatives d^n. This conditional (or samewhat hypothetical) statement should be taken as a starting platform for a new research task.

For instance, as can be found in chapter 4.0 (equations (4.0.73) to (4.0.76)), we can apply the symmetrical central differences method to determine mutual relations between relevant energy-momentum domains' variations. This is, in a few elementary steps, creating an almost completed picture of Uncertainty Relations, and at the same time reinforcing foundations of Particle-Wave Duality as follows,

$$\left.\begin{cases}\overline{P}_4 = \overline{P}\left[\vec{p} = \gamma m\vec{v}, \dfrac{E}{c} = \gamma mc\right], \\ \overline{P}^2 = \vec{p}^2 - \dfrac{E^2}{c^2} = -\dfrac{E_0^2}{c^2}, \\ E_0 = mc^2, \ E = \gamma E_0, \\ \vec{p}^2 c^2 + E_0^2 = E^2, \\ p \to p \pm \Delta p, \\ E \to E \pm \Delta E\end{cases}\right\} \Rightarrow \begin{cases}(p+\Delta p)^2 c^2 + E_0^2 = (E+\Delta E)^2 \\ (p-\Delta p)^2 c^2 + E_0^2 = (E-\Delta E)^2 \\ \overline{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{\delta x}{\delta t} = \dfrac{\delta\omega}{\delta k} = \dfrac{\Delta\omega}{\Delta k}\end{cases} \Leftrightarrow \left\{c^2 \cdot p\,\Delta p = E\,\Delta E \Leftrightarrow \frac{\Delta E}{\Delta p} = c^2 \frac{p}{E} = \overline{v}\right\} \Rightarrow$$

$$\Rightarrow \frac{\Delta x}{\Delta t} = \frac{\delta x}{\delta t} = \frac{\Delta E}{\Delta p} = c^2 \frac{p}{E} = \overline{v} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = \frac{\delta\omega}{\delta k} \leq c.$$

(4.0.73), (4.0.74)

$$\left.\begin{cases} u = \dfrac{\omega}{k}, \ \omega = ku, \\ k \to k \pm \Delta k, \\ u \to u \pm \Delta u \end{cases}\right\} \Rightarrow \Delta\omega = (k + \frac{1}{2}\Delta k)(u + \frac{1}{2}\Delta u) - (k - \frac{1}{2}\Delta k)(u - \frac{1}{2}\Delta u) = \quad (4.0.75)$$

$$= k\Delta u + u\Delta k \Leftrightarrow \begin{Bmatrix} \overline{v} = \dfrac{\Delta\omega}{\Delta k} = u + k\dfrac{\Delta u}{\Delta k} = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta x}{\Delta t} \\ = \text{average group velocity} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} v = u + k\dfrac{du}{dk} = \dfrac{d\omega}{dk} = \dfrac{d\tilde{E}}{dp} = \dfrac{dx}{dt} \\ = \text{immediate group velocity} \end{Bmatrix}.$$

$$\left.\begin{cases} \overline{v} = u + k\dfrac{\Delta u}{\Delta k} = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\Delta\tilde{E}}{\Delta p} = \dfrac{\Delta x}{\Delta t}, \\ |\Delta x\ \Delta p| = |\Delta t\ \Delta\tilde{E}| = h|\Delta t\ \Delta f| > h/2, \ \Delta\tilde{E} = h\Delta f, \\ 0 < \delta t \cdot \delta f = \delta\, x \cdot \delta f_x < \dfrac{1}{2} \leq F \cdot T = F_x \cdot L \leq \dfrac{1}{4 \cdot \delta t \cdot \delta f} = \dfrac{1}{4 \cdot \delta x \cdot \delta f_x} \end{cases}\right\} \Rightarrow$$

$$\Rightarrow \overline{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = h\frac{\Delta f}{\Delta p} = \frac{\Delta\tilde{E}}{\Delta p} = \frac{\Delta\omega}{\Delta k} = u + k\frac{\Delta u}{\Delta k} = \frac{\delta x}{\delta t} = \frac{\delta\omega}{\delta k} = \frac{dx}{dt} = \frac{d\omega}{dk}. \quad (4.0.76)$$

The important message in this short Uncertainty relations review is also to show that there is only one fundamental Uncertainty Principle equally valid in Mathematics, as well as in Quantum Mechanics and whole Physics.

We can also conclude that Uncertainty relations, as presently known in Physics, are still not sufficiently and generally treated, and that there is still a lot to be done in this field. The particle-wave duality theory, as well as Schrödinger's wave equation, should also be upgraded (to cover all matter-waves aspects regarding thermodynamics, gravity, particles and fields rotation, and electromagnetic theory, as initiated in (5.15) until (5.21)). What we are going to create eventually, are universally valid uncertainty relations between signal (or wave-packets) lengths, or durations (in relevant space, time, and frequency domains). This way, we will have, A) relations between absolute signal-durations values in different domains, B) between statistically found

mean-duration values, and relevant standard deviations in all domains, and C) relations considering optimal signal sampling intervals (in relation to Nyquist-Kotelnikov-Shannon-Whittaker signal sampling rules). Of course, all of that should properly comply and be integrated, arranged and compared with modern "Error Analysis" and "Central-Differences" methods, theory and practices (see more in [96]).

Here we could stop, concluding that now we have new elements for the profound conceptual understanding of the meaning and origins of Uncertainty Relations, and later we can try to formulate a more general concept regarding Uncertainty than presently practiced in Physics.

[♣ COMMENTS & FREE-THINKING CORNER: Examples, … in process… only a reminder

Analogies, if well and correctly established and applied, are the best possible platform for understanding and connecting different fields of physics. Knowing relevant analogies, we can (almost) start from every distinct physics theory and (try to) extend its most significant forms/laws to other fields.

*For instance, let us apply **central differences** on the simplest form of D. Bernoulli's fluid flow equation (valid for tube segment with negligible viscosity, laminar fluid flow):*

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{const.} = p_0 \qquad (5.22)$$

*where $\rho = \frac{\mathbf{m}}{\mathbf{V}}$ -is fluid density, **v** -is fluid speed, **g** -is gravitational constant, **p** -is pressure in the fluid segment, and **z** -is a vertical coordinate/position of the analyzed fluid segment. Apparently, (5.22) can be transformed as follows (**applying central, Δ differences, introduced above**):*

$$\begin{aligned}
&\frac{\frac{1}{2}mv^2}{V} + \frac{mgz}{V} + p = \frac{E_k}{V} + \frac{E_p}{V} + p = \frac{\sum_{(i)}(E_{ki} + E_{pi})}{V} + p = \text{const.} = p_0 \Rightarrow \\
&\Rightarrow \sum_{(i)}(E_{ki} + E_{pi}) + pV = p_0 V \Leftrightarrow \sum_{(i)} E_i + pV = p_0 V \Leftrightarrow \sum_{(i)} E_i + (p - p_0)V = 0 \\
&\Rightarrow \sum_{(i)} \Delta E_i + (p - p_0)(\Delta V) + (\Delta p)V + \Delta p \Delta V = \sum_{(i)} \Delta E_i + \Delta(pV) = \\
&= \sum_{(i)} \Delta E_i + \Delta(\frac{F}{S} S \cdot \delta x) = \sum_{(i)} \Delta E_i + \Delta(F \cdot \delta x) = 0 \text{ , } E_i = E_{ki} + E_{pi} \text{ .}
\end{aligned} \qquad (5.23)$$

*where **V** is the volume of the analyzed fluid/tube segment and, $\mathbf{E_k}$ and $\mathbf{E_p}$ are corresponding kinetic and potential energy of the fluid segment. The last part of (5.23) can be taken as the starting point for creating different analogous forms, where the meaning of the fluid and energy could be extended to something else, for instance to a fluid of electrons, a fluid of photons, phonons… Indeed, we cannot say that (5.23) is fully and immediately applicable to other fields phenomena in physics (before implementing necessary adjustments to it), but we can be sure that the creative process (based on analogies) has already started (even if (5.23), in some later step will be significantly upgraded). ♣]*

6. DIFFERENT POSSIBILITIES FOR MATHEMATICAL FOUNDATIONS OF MULTIDIMENSIONAL UNIVERSE

During long time Mathematical Physics has been creating different theoretical and conceptual platforms, to explain how our universe could be multidimensional, or populated with many mutually separated, not interacting, independent worlds, each of them having its own combination of dimensions. We still do not have practical and directly testable applications or confirmations in relation to an extended multidimensionality, but we have particularly good mathematical tools, indications and concepts suggesting that certain kind of multidimensionality could exist. Our natural, existential, testable, and perceptual reality shows that we live in a four-dimensional world (three of them spatial, which are all lengths, or all of them have the same nature, and one temporal dimension, all being mutually related and mutually orthogonal), and we are presently able to visualize and detect only such a world. We do not know precisely what higher or (for us) new dimensions of our universe could be (for instance, dimensions numbered as fifth, sixth, etc.), or what could be other independent worlds with other combinations of different dimensions.

Estimating logically and analogically, if higher and/or different dimensions really exist, we could say with certainty that such, still hidden reality, could produce some detectable effects on certain matter-states in our perceptual and experimentally accessible 4-dimensional world. For instance, in cases of some extreme conditions and perturbations coming from other multidimensional worlds, this way externally produced signals could penetrate our four-dimensional world influencing some of measurable parameters within *our four-dimensional world, affecting its motional, electromagnetic, and other matter states*. For instance, frequency, phase and amplitude of specific matter-waves could be on some way slightly excited or modulated by something what is happening in higher dimensions worlds, producing some new frequency harmonics, amplitude ripples, phase shifts, and different signal modulations of natural matter-waves known within our Universe, or we could detect some strange and inexplicable (but measurable) energy deficits, surpluses, spikes and/or other vacuum states fluctuations such as variations in dielectric and magnetic permeability, what will affect frequency and wavelengths of photons propagation... We could also estimate that everything what exist in our (3+1)-dimensional world is only a kind of projection or cross-section of certain multidimensional world (which has more than 3+1 dimensions, or different combinations of other, still unfamiliar dimensions).

Let us propose or imagine different hypothetical options where we could expect to find effects or imprints of multidimensional and multiverse worlds:

A) Possible candidates for finding detectable manifestations of worlds within higher dimensions could be so-called "Black Holes". We presently conceptualize black holes, as energy and masses, dominantly produced as gravitational-force sinks, being kind of dynamic vortex formations, analog to fluids sinks. Since we know that energy and momentum conservation laws should always be satisfied, analogically meaning that every vortex-sink (or black hole) should have its energy-inlet (or mass-input) and somewhere, on some way, it should also have an material output, or an accountable accumulation (for the same amount) of the energy-input (and the same being valid for input and output content of linear, angular, and electromagnetic moments as vectors). We also know that all "energy-flow loops", like electric currents and voltages circuits, or electromagnetic waves and photons emissions and circulations, including

All over this paper are scattered small comments placed inside the squared brackets, such as: [♣ **COMMENTS & FREE-THINKING CORNER…** ♣]. The idea here has been to establish intuitive and brainstorming, not-confirmed and freethinking corners for making fast comments, and presenting challenging ideas, that could be some other time developed towards something much more meaningful and more properly integrated with Physics.

mechanically operating circuits should be completely spatially closed, meaning that every of such "energy-moments flow circuits" should have its front and last ends, or sources and loads (see more about mentioned closed circuits concept in the first chapter of this book). If we apply the same closed-circuits concept on a "black-hole", we will realize that we still address or understand much more of its front-end "energy-moments and mass-flow inputs", and not very much of its "last-end outputs, and possible black-holes loads", (meaning that our present conceptualization about black holes is at least incomplete, or oversimplified, and maybe very much naïve and wrong, or not very much related to what we consider as Gravitation). Another important theoretical background involved here is related to Parseval's identity or theorem, and to Uncertainty Relations between corresponding original and spectral domains' durations (of a "black-hole" overall spatial-temporal and spectral situation), that should be clearly understood, mathematically well formulated, and always satisfied (see more about similar items in Chapters 5. and 10.). Of course, we could explore energy-moments inputs and outputs within different theoretical, mathematical, and observations-based assumptions or imaginative concepts. ***One of such speculative idea is that black holes could have connections, outputs (or loads) with some of higher (or different) dimensions worlds.*** Another option is that what we presently conceptualize as black holes is still an incomplete picture. Black holes could also be very much different, dominantly electromagnetic *matter-states formations* (compared to our present conceptualization), since static and rest masses are not at all the primary and dominant sources of Gravitation (as shown based on analogies, in the first and second chapter of this book). Only dynamic (including oscillatory and resonant) states of atoms and masses with associated forces, fields, mechanical, linear, angular, and electromagnetic moments, with involved or induced charges, currents, and voltages, are the real sources of Gravitation, but this is not fully equivalent to what Newton and A. Einstein theories about Gravitation are promoting (see more in Chapter 8.).

B) Casimir effect or force could also be one of manifestations of cross-frontiers communications between different multidimensional worlds. Casimir effect could be a simple manifestation of natural, wideband electromagnetic noise reception, and case of mutual coupling and resonant synchronization between, by frequency-range, mutually overlapping resonant states of parallel metal plates (see more in [103] and [104]), and maybe certain part of such electromagnetic background noise is coming from other multidimensional worlds.

C) Let us assume that atoms are kind of matter structures where electron clouds are "spherically or spatially" and bidirectionally exchanging photons (inwards and outwards, or internally and externally) with atoms from an external world of other matter states (outside placed atoms), and, "spherically-mirror-symmetrically" exchanging photons and/or other matter waves (internally) with relevant atom-nucleus states, as conceptualized in Chapter 8. of this book, creating ***closed and self-balanced standing waves, flow circuits*** *of involved power, energy, forces, charges, moments, and currents. This way, we effectively explain that atoms are naturally stable, internally, and externally balanced resonant structures, being well (and omni-directionally) connected and synchronized with the Universe, including being (hypothetically) connected with other multidimensional worlds. If we, on some way, create atomic explosions, implosions, extreme shocking and atoms disintegration conditions, or atoms fission and/or fusion effects, this could (at least partially) be like affecting, modulating, or braking communication channels between atoms and surrounding multidimensional worlds. This way we could directly affect or excite some of mentioned multiverse worlds. A sudden atom perturbation (or destruction) could send signals towards other multiverse worlds. By convenient monitoring and detecting of created transient and reflected "echo matter states", existing natural* *radiation, and measuring relevant spectral characteristics, we maybe find a way or indications to prove that other multidimensional worlds exist. Atoms' fission could also be an act of creating perturbative effects against other multiverse worlds if we give us a freedom to make such science-fiction-alike mental excursions. In addition, we could consider atoms as kind of (still somewhat hypothetical and hidden) resonating nodes serving as connections with other worlds. We could also hypothesize that active stars or suns are kind of unusual (or still not well understood) resonant, and bidirectional radiators and sinks (like antennas), or passthrough channels for cosmic (or electromagnetic) energy circulation between our Universe and other worlds. Here described (and still hypothetical) concept about atoms and stars nature and their mutual connectivity is associating on Rudjer Boskovic universal and natural force law, as well as on Nikola Tesla radiant (and resonating) energy bidirectional flow, responsible to keep atoms and other masses stable, to produce external effects of an attractive force of gravitation, and to create other internal effects of balancing repulsive/attractive, natural forces (as Tesla conceptualized his Dynamic Gravity theory).*

D) *Another manifestation of Multidimensional Universe could be related to a hypothetical existence of hidden and/or unknown variables of a phenomenology that naturally and dominantly still belongs to the world of only statistically or probabilistically manageable modeling and conceptualizations, where we still do not have any better way or access to analyze such situations. We could simply assume that stochastically behaving fluctuations of different material or vacuum properties are creating different matter states as products of something what is (at least partially) happening in a surrounding multidimensional environment.* Here we could consider as a relevant and an indicative example, *zero-point, vacuum energy fluctuations (ZPE) or effects*. Maybe we could find certain relevant and significant *signals detecting and processing method* that would selectively extract some signals arriving from other multidimensional worlds, meaning based on ZPE measurements.

E) New dimensions (or new worlds) initially created only mathematically, without any measurable and experimentally verifiable backing, could not be sufficiently realistic, or not sufficiently and practically manageable assumptions. Anyway, mathematics is very exactly explaining that multidimensional aspects of the Universe around us could exist and be hidden within temporal-spatial, phase-shifted, and mutually orthogonal energy-moments and/or matter-waves formations. Since Mathematics that comes from mastering tangible Physics (preferably without a Game theory concepts and non-provable postulations and assumptions) always presents the best language, operating framework, and logic of our Universe (and the same should be valid in relation to other, still hypothetical multidimensional worlds), whatever we could create/define/construct on the best, correct and natural mathematical way, regarding multidimensional worlds (or spaces), has also chances to exist, or to indicatively present an insight in certain reality that would be one day discovered. We should take care not to create totally artificial and only mathematical (games theory) concepts of multidimensionality without multilevel, analogical, and natural connections with the solid and universally applicable knowledge from Mathematical-Physics.

F) What already exist as a consistent and natural mathematical concept regarding establishing the foundations of extended multidimensionality in the world of Physics are Complex, Hyper-complex and Imaginary numbers, functions, signals, and relevant Analytic Signal Phasors. Such mathematical modeling is especially convenient to represent multidimensional spaces in Physics, as already and very successfully realized within Minkowski space of Special Relativity theory. On a similar way to Minkowski spatial-temporal 4-vectors modelling, we could shape the Analytic Signal matter-waves, creating particularly good and structured ways and models of spatial or dimensional extensions, especially when from basic and ordinary Analytic Signals, with one imaginary unit, we evolve towards Hypercomplex Analytic Signals and Phasors with many imaginary units. See more in Chapter 10.

G) The easiest way of simple analogical thinking related to understanding multidimensional spaces is to create cross-sections or projections of multidimensional structures towards lower levels of dimensionality. For instance, let us project a 3-dimensional object to a 2-dimensional object. Any cross section or projection of a 3-dimensional sphere $z = a = \text{const.} \Rightarrow x^2 + y^2 + z^2 = r^2$, will be a circle, or ellipse such as $x^2 + y^2 = r^2 - a^2 = r_1^2$ in a 2-dimensional plane. Analogically extending the same example, any cross section or projection of a 4-dimensional sphere $(x_1^2 + x_2^2 + x_3^2 + x_4^2 = r_2^2)$ will become a 3-dimensional sphere, ($x_4 = a = \text{const.}$, $x_1^2 + x_2^2 + x_3^2 = x^2 + y^2 + z^2 = r_2^2 - a^2 = r^2$) etc. Anyway, in our explorations of Nature (and in any other aspect of life), we are often applying analogical, inductive, and deductive conclusions, combined with interpolations, extrapolations and “cross-sections or projections”, inwards and outwards.

The simplest, analogically created basis of an extended, multidimensional universe (**while staying in the theoretical frames of Riemannian metrics and 4-vector or n-vector rules of Minkowski-Space of the Theory of Relativity**) is:

{(SPACE) & TIME} $\Leftrightarrow$

$$\left\{(x_1,x_2,...,x_n),t\right\} \Leftrightarrow \left\{(r,t),\ r = r(x_1,x_2,...,x_n)\right\} \Leftrightarrow (x_1,x_2,...,x_n,t) \Leftrightarrow (r_n,t). \tag{6.1}$$

There are many possibilities how to conceptualize and shape multidimensional or multi-coordinate basis (6.1), and here we will touch only a couple of them, that are sufficiently imaginative and challenging (mostly serving like brainstorming and indicative options). We can assume, as the first step towards multidimensionality, that every spatial dimension or coordinate from the multidimensional basis (6.1) could have its real and imaginary part, whatever that should mean,

$$\left\{\begin{matrix}\left[r(t) = (x_1,x_2,...,x_n,t) \Leftrightarrow (r,t)\right] \to \overline{r}(t) = (\overline{x}_1,\overline{x}_2,...,\overline{x}_n,t) \\ \overline{x}_i = x_{i(real)} + j\,x_{i(imaginary)},\ i = 1,2,3,...,n;\ j^2 = -1\end{matrix}\right\} \Rightarrow$$
$$\Rightarrow \begin{bmatrix}(\overline{x}_1,\overline{x}_2,...,\overline{x}_n,t) \Leftrightarrow (\overline{r},t) \Leftrightarrow \overline{r}(t), \\ \overline{r} = \overline{r}(\overline{x}_1,\overline{x}_2,...,\overline{x}_n),\ \overline{x}_i = \overline{x}_i(t)\end{bmatrix}. \tag{6.2}$$

Very similar idea already got its legitimacy in the Minkowski space of Relativity Theory, where the time scale is placed on the imaginary axis, as a way towards space and time unification, or the way to extend 3-dimensional spatial formations to 4-dimensional spatial-temporal formations. Practically, one extra dimension (here being a time) was associated mathematically to the very tangible and obvious 3-dimensional spatial coordinates, by means of added imaginary axis with a single imaginary unit (here j, j^2= -1). Using Minkowski-Einstein Relativity Theory formulation of complex energy-momentum 4-vectors, we are presently able to analyze, solve, or predict all impact interactions between particles, wave-groups (as photons), and other energy-momentum states in Physics. Such framework (of Minkowski space) is giving

us an example of time dimension understanding and its mathematical conceptualization (which is mathematically manageable by a "time dimension placed on an imaginary-units' axis"). This effectively means that temporal and spatial dimensions are mutually orthogonal and phase-shifted for $\pi/2$. Since spatial and temporal domains are mutually proportional or coupled, we should conclude that any spatial-temporal world should also have equal number of temporal and spatial dimensions or coordinates. Analogically following, we could place any new (still not discovered) dimension on an imaginary axis, or we could extend such concept to deal with many imaginary units, as in cases of Quaternions and Hypercomplex functions, and this way create new multidimensional structures. See more in Chapter 10. under "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors, in relation to matter-waves and particle-wave duality. "

1. Let us now introduce pragmatic convention (compatible with (6.1) and (6.2)) by saying that all measurable or detectable spatial dimensions (x, y, z), of our world ($(\overline{x}_1 = x_{1(real)} = x_1 = x), (\overline{x}_2 = x_{2(real)} = x_2 = y), (\overline{x}_3 = x_{3(real)} = x_3 = z)$) have only real parts from (6.2), and all other, like time, and still unknown and hidden spatial dimensions would have only imaginary parts from (6.2). **One of the brainstorming options here is to treat other dimensions on the same way as a time is treated in the Minkowski space of the Theory of Relativity.** Thus, multidimensional basis (6.2) can be modified as,

$$\begin{aligned}&(\overline{x}_1,\overline{x}_2,\overline{x}_3,\overline{x}_4,...,\overline{x}_n,t) \Leftrightarrow (\underleftrightarrow{x_{1(real)},x_{2(real)},x_{3(real)}},\underleftrightarrow{x_{4(imag.)},x_{5(imag.)},...,x_{n(imag.)}},t) \Leftrightarrow\\ &\Leftrightarrow (\underleftrightarrow{x_{1(real)},x_{2(real)},x_{3(real)}},\underleftrightarrow{jx_{4(imag.)},jx_{5(imag.)},...,jx_{n(imag.)}},jct) \Leftrightarrow\\ &\Leftrightarrow \left[(x_{1(real)},x_{2(real)},x_{3(real)}),\ j(x_{4(imag.)},x_{5(imag.)},...,x_{n(imag.)},ct)\right] \Leftrightarrow\\ &\Leftrightarrow \left[(x_{1(real)},x_{2(real)},x_{3(real)}),\ jct^*\right] \Leftrightarrow \left[(x_1,x_2,x_3),\ jct^*\right] \Leftrightarrow \left[(x,y,z),\ jct^*\right],\\ &ct^* = (x_{4(imag.)},x_{5(imag.)},...,x_{n(imag.)},ct).\end{aligned} \tag{6.3}$$

In fact, in (6.3) all higher-level dimensions are associated to an extended meaning of an effective time dimension $t^* = (\frac{x_{4(imag.)}}{c}, \frac{x_{5(imag.)}}{c},..., \frac{x_{n(imag.)}}{c}, t)$. Let us now (based on (6.3)), try to express the multidimensional space-time interval, as hypothetically being "*coordinates-invariant*", using the analogy with space-time interval from the Relativity theory, as,

$$\begin{aligned}&(\Delta S)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t^*)^2 =\\ &= (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t)^2 - (\Delta x_4)^2 - (\Delta x_5)^2 - ... - (\Delta x_n)^2.\end{aligned} \tag{6.4}$$

Alternatively, we can express the same interval in its infinitesimal form as,

$$\begin{aligned}&(dS)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2(dt^*)^2 =\\ &= (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - c^2(dt)^2 - (dx_4)^2 - (dx_5)^2 - ... - (dx_n)^2.\end{aligned} \tag{6.4-1}$$

Obviously, since we know that our four-dimensional universe is still the dominant and only measurable reality we master, there are at least two possibilities (1. a) and 1. b)) how to understand the space-time interval (6.4), to stay in agreement with our experience and contemporary Relativity theory. *We still do not have undoubtedly, completely verifiable empirical arguments confirming that relativistic space-time interval is always coordinates-invariant, and something like that is still presenting only very probable, but partially speculative and still challenging concept (see* **[49]; -** Universal Invariance: A Novel View of Relativistic Physics, T. E. Phipps, Jr.). *Another proposal* (6.17-1), *for the corrected definition of the space-time interval will be introduced later.*

Let us briefly explore the following brainstorming options in establishing mathematical foundations of multidimensional universes, based on space-time intervals.

1. a) One of options related to (6.3) is that higher dimensions of our or other universes (if any) always present extremely short or negligible time-space intervals,

$$(\Delta \mathbf{x}_4)^2 + (\Delta \mathbf{x}_5)^2 + \ldots + (\Delta \mathbf{x}_n)^2 \cong \mathbf{0} . \tag{6.5}$$

Or,

1. b) That higher dimensions somehow contribute to our real time-scale modification, or to creation of certain temporal or phase shift, $\mathbf{t}^* \cong \mathbf{t} + \tau$,

$$\begin{aligned} &\mathbf{c}^2(\Delta \mathbf{t}^*)^2 = \mathbf{c}^2(\Delta \mathbf{t})^2 + (\Delta \mathbf{x}_4)^2 + (\Delta \mathbf{x}_5)^2 + \ldots + (\Delta \mathbf{x}_n)^2, \\ &(\Delta \mathbf{t}^*)^2 = (\Delta \mathbf{t})^2 + \frac{(\Delta \mathbf{x}_4)^2}{\mathbf{c}^2} + \frac{(\Delta \mathbf{x}_5)^2}{\mathbf{c}^2} + \ldots + \frac{(\Delta \mathbf{x}_n)^2}{\mathbf{c}^2} = \left[\Delta(\mathbf{t} + \tau)\right]^2 . \end{aligned} \tag{6.6}$$

Anyway, both options, (6.5) and (6.6) effectively produce the well-known space-time interval (from the Relativity Theory),

$$(\Delta \mathbf{S})^2 = (\Delta \mathbf{x}_1)^2 + (\Delta \mathbf{x}_2)^2 + (\Delta \mathbf{x}_3)^2 - \mathbf{c}^2(\Delta \mathbf{t}^*)^2 . \tag{6.7}$$

In (6.6), all spatial dimensions are presented by their real parts, and only the effective time dimension (related to other higher, for us still hidden dimensions) is associated to an imaginary coordinate axis. Here, (6.6), presents the effective time-shift where invisible (hidden or higher) dimensions create mentioned time shifts,

$$(\Delta \mathbf{t}^*)^2 = (\Delta \mathbf{t})^2 + \frac{(\Delta \mathbf{x}_4)^2}{\mathbf{c}^2} + \frac{(\Delta \mathbf{x}_5)^2}{\mathbf{c}^2} + \ldots + \frac{(\Delta \mathbf{x}_n)^2}{\mathbf{c}^2} = \left[\Delta(\mathbf{t} + \tau)\right]^2 .$$

This could be an interesting intellectual-excursion option (at least for science fiction speculations about time traveling), because it gives the chances for parallel existence of many different worlds having a part of common, and a part of mutually different combinations of dimensions (and being mutually phase-shifted on the time scale, or within spatial-temporal coordinates, for different original and spectral intervals). Such phase-shifted worlds are mutually invisible and untouchable, almost non-interacting (like being mutually orthogonal functions), but if we are living in one of such worlds, the "*effective residual amount*" of all other phase-shifted worlds could (creatively and imaginatively) present for us kind of "***ether***", spatial matrix, or carrier fluid for propagation of *our-world matter waves* (like electromagnetic waves, gravitation etc.). Presently, we know that such "**spatial matrix or carrier fluid**" has measurable and very important electromagnetic material properties, such as dielectric permittivity and magnetic permeability of vacuum, ε_0 , μ_0 , which are mutually linked and producing the universal speed constant $c = 1/\sqrt{\varepsilon_0\mu_0}$, which is equal to the speed of photons in a vacuum. If constants ε_0 and μ_0 are for some reason fluctuating, speed of photons in such environment (at least measured as an averaged value) should still keep its constant value, meaning $\varepsilon_0\mu_0 = \varepsilon_1\mu_1 = ... = \varepsilon_x\mu_x = 1/c^2$, or frequency and wavelength of involved photons will also fluctuate as, $\varepsilon_x\mu_x = 1/c^2 = 1/(\lambda_x f_x)^2$. Generally applicable relation between group and phase velocity (u and v) is also applicable to photons, since we have,

$$\begin{bmatrix} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} \\ u = \lambda f = \lambda_x f_x = c = 1/\sqrt{\varepsilon_x\mu_x} = \text{constant} \end{bmatrix} \Rightarrow v = c = u = \text{constant}.$$

Practically, we assume or have indicative arguments that photons have the same group and phase velocity (v and u), which is equal to universal constant c = u = v = constant (equal to the speed of light). What can change or fluctuate here (in cases of perturbations) are relevant wavelength λ_x and frequency f_x, or dielectric and magnetic constants ε_x and μ_x. Here we can directly establish proportionality between the temporal Δt and spatial Δs durations or intervals, such as,

$$v = c = u = ds/dt \Rightarrow ds = c \cdot dt \Rightarrow \Delta s = c \cdot \Delta t,$$

without any need to refer to big inventions and postulates from Relativity theory.

Constant c is directly involved in everything else we consider as important in Physics, such as energy-momentum - mass - time - space relations. It is also very indicative and challenging fact that photons have only transversal oscillations of mutually perpendicular electric and magnetic field vectors. Considering analogies and comparisons between wave motions in gaseous, liquid, and solid states, we can conclude that in a solid, extremely hard, rigid, and strong (almost non-elastic) materials, transversal waves are dominant, and longitudinal waves almost do not exist there. Consequently, analogically concluding, mentioned ideal vacuum-state or photons carrier, alternatively being an ether or some spatial matrix, should be on some equivalent or analogical way infinitely strong and rigid medium (with extremely high Young modulus) without compressibility, for photons propagation, because photons manifest only as transversally oscillating waves. Maybe higher spatial dimensions (not directly and easily detectable by our sensors and other measurement instruments) are on some way relevant for such strange and exotic, ideally rigid spatial matrix. Of course, this is still a preliminary, hypothetical, and conditional, oversimplified brainstorming, because maybe we still do not know everything relevant about the nature of electromagnetic waves propagation.

Surprisingly and analogically, it is also possible to create acoustic or mechanical, fully transversal vibrations (orthogonal to a direction of propagation), oscillating and propagating in solid metals (wires and tubes) like photons in fiberoptic waveguides, which are reaching exceedingly long distances and almost without attenuation.

2. The options mentioned above could be made compatible or very similar to formalism of using Minkowski space in Relativity theory. In such situations we can use ordinary complex numbers or Analytic Signal functions (which have only one imaginary unit, $\mathbf{j}^2 = -1$) as a convenient time-axis "**marker**" to unite spatial and temporal domains. This way we realize rich mathematical processing regarding conservation laws in Physics (for instance, in conceptualizing de Broglie matter waves, or in facilitating mathematical processing of impact and scattering interactions). As the next advanced step, we can introduce new, Minkowski-equivalent multidimensional space, using **quaternions** or **hypercomplex functions**, starting again from (6.2), with (at least) three imaginary units. (See more regarding Hypercomplex Analytic functions in Chapter 4.0 under "4.0.2.6. Hyper-complex Analytic Signal", and in Chapter 10. under "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality").

For instance, (just to give certain intuitive background about possible applications of such "hypercomplex world"), contemporary physics is mathematically promoting that elementary subatomic particles are presentable as being composed of quarks. Assumed quarks are not detectable being on some way masked, hidden, and grouped as sets of three elements (including additional sets of three antiparticles (of their kind), and additional sets of three of them in different "colors" ..., and each of them with two of other attributes... This makes 36 quarks in total, until present, and probably much more in the future). This could be an oversimplified and imaginative brainstorming backing, with an intention to apply the Analytic, Hypercomplex wave functions (with never-ending number of imaginary-triplet-units), for realizing an innovative multidimensional modeling, with well-organized classification of wavefunctions pretending to represent structures of elementary particles.

In case of hyper-complex functions, the basic (ordinary) imaginary unit "$\mathbf{I}$" is composed of three, more elementary imaginary units, $\mathbf{i}, \mathbf{j}, \mathbf{k}$ (see below). Now, we could preliminarily (still like brainstorming) exercise how to mathematically formulate such multidimensional, extended Minkowski space, as follows,

$$\left\{(\overline{x}_1,\overline{x}_2,...,\overline{x}_n,t) \Leftrightarrow (\overline{r},t) \Leftrightarrow \overline{r}(t),\ \overline{x}_i = \overline{x}_i(t)\right\} \Rightarrow \left\{\overline{z},\ t\right\} \Leftrightarrow \overline{z}(t) \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} (\overline{x}_1,\overline{x}_2,\overline{x}_3,\ jct) \to (\overline{x}_1,\overline{x}_2,\overline{x}_3,\ Ict) \Rightarrow \left\{\overline{x}_1,\ \overline{x}_2,\ \overline{x}_3,\ c(it_i + jt_j + kt_k)\right\} \\ \left[\begin{array}{l} Ict\ =\ c(it_i +\ jt_j +\ kt_k), \\ (t^2 = t_i^{\ 2} +\ t_j^{\ 2} +\ t_k^{\ 2},\ \ t_i t_j k - t_i t_k j + t_j t_k i = 0,\ ??!!) \\ or,\quad Ict\ = ic_i t_i +\ jc_j t_j +\ kc_k t_k), \\ (c^2 t^2 = c_i^2 t_i^{\ 2} +\ c_j^2 t_j^{\ 2} +\ c_k^2 t_k^{\ 2},\ \ c_i c_j t_i t_j k - c_i c_k t_i t_k j + c_j c_k t_j t_k i = 0,\ ??!!) \end{array}\right], \\ I^2 = i^2 = j^2 = k^2 = -1, \\ i\cdot j = k,\ \ j\cdot k = i,\ \ k\cdot i = j,\ \ j\cdot i = -k,\ \ k\cdot j = -i,\ \ i\cdot k = -j, \\ e^{I\varphi} = \cos\varphi + I\cdot\sin\varphi \\ e^{i\cdot\varphi_i} = \cos\varphi_i + i\cdot\sin\varphi_i\ ,e^{j\cdot\varphi_j} = \cos\varphi_j + j\cdot\sin\varphi_j\ ,\ e^{k\cdot\varphi_k} = \cos\varphi_k + k\cdot\sin\varphi_k \end{array}\right\} \Rightarrow$$

$$\overline{z} = \left|\overline{z}\right| e^{I\varphi} = \left|\overline{z}\right| e^{i\varphi_i + j\varphi_j + k\varphi_k} = \left|\overline{z}\right| e^{i\varphi_i} e^{j\varphi_j} e^{k\varphi_k} = \left|\overline{z}\right| (\cos\varphi_i + i\cdot\sin\varphi_i)(\cos\varphi_j + j\cdot\sin\varphi_j)(\cos\varphi_k + k\cdot\sin\varphi_k) =$$
$$= r + a_i\cdot i + a_j\cdot j + a_k\cdot k = r + I\cdot A = \overline{z}_i + \overline{z}_j + \overline{z}_k = \left|\overline{z}_i\right| e^{i\varphi_i} + \left|\overline{z}_j\right| e^{j\varphi_j} + \left|\overline{z}_k\right| e^{k\varphi_k}, \left|\overline{z}\right|^2 = r^2 + A^2,$$
$$r = \left|\overline{z}\right|\cos\varphi,\ A = \left|\overline{z}\right|\sin\varphi,\ I\varphi = i\varphi_i + j\varphi_j + k\varphi_k\ ,$$

$$\Rightarrow \left\{\begin{array}{l} A\cdot I = a_i\cdot i + a_j\cdot j + a_k\cdot k = Ae^{I(\frac{\pi}{2}+2m\pi)} = I\cdot\left|\overline{z}\right|\sin\varphi = \\ = a_i e^{i(\frac{\pi}{2}+2n\pi)} + a_j e^{j(\frac{\pi}{2}+2p\pi)} + a_k e^{k(\frac{\pi}{2}+2q\pi)},\ m,\ n,\ p,\ q = 1,2,3,... \\ I = \frac{a_i}{A}\cdot i + \frac{a_j}{A}\cdot j + \frac{a_k}{A}\cdot k = e^{I(\frac{\pi}{2}+2m\pi)} = I\cdot\frac{\left|\overline{z}\right|}{A}\sin\varphi, \\ \varphi\cdot I = \varphi_i\cdot i + \varphi_j\cdot j + \varphi_k\cdot k, \\ I = \frac{\varphi_i}{\varphi}\cdot i + \frac{\varphi_j}{\varphi}\cdot j + \frac{\varphi_k}{\varphi}\cdot k = \frac{a_1}{A}\cdot i + \frac{a_2}{A}\cdot j + \frac{a_3}{A}\cdot k = e^{I(\frac{\pi}{2}+2m\pi)}, \\ \frac{\varphi_i}{\varphi} = \frac{a_i}{A}, \frac{\varphi_j}{\varphi} = \frac{a_j}{A}, \frac{\varphi_k}{\varphi} = \frac{a_k}{A},\ (A^2 = a_i^2 + a_j^2 + a_{3k}^2,\ \varphi^2 = \varphi_i^2 + \varphi_j^2 + \varphi_k^2\ ??!!) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{ll} \cos\varphi = & \cos\varphi_i\cos\varphi_j\cos\varphi_k - \sin\varphi_i\sin\varphi_j\sin\varphi_k, \\ \sin\varphi = & \left[\begin{array}{l} \frac{\varphi}{\varphi_i}(\cos\varphi_i\sin\varphi_j\sin\varphi_k + \sin\varphi_i\cos\varphi_j\cos\varphi_k), \\ \frac{\varphi}{\varphi_j}(\cos\varphi_i\sin\varphi_j\cos\varphi_k - \sin\varphi_i\cos\varphi_j\sin\varphi_k), \\ \frac{\varphi}{\varphi_k}(\sin s\varphi_i\sin\varphi_j\cos\varphi_k + \cos\varphi_i\cos\varphi_j\sin\varphi_k) \end{array}\right] \end{array}\right\}. \qquad (6.8)$$

What could be particularly interesting in the hypercomplex coordinate basis (6.8), is the possibility to unite linear and rotational (or torsional) aspects of certain motion within the same mathematical concept. Anyway, majority of elementary particles, quasiparticles, and matter waves have (apart from their linear or translational motion parameters) also rotational or angular attributes, like spin and orbital moments (see chapter 4.1; "4.1.2. De Broglie Matter Waves and Hidden Rotation"). For instance, mentioned torsional and spinning matter-wave parameters could be introduced in the following way (just to start a creative thinking about such modeling):

$$\begin{aligned} &\overline{z} = |\overline{z}| e^{I\varphi} = |\overline{z}| e^{I(\omega t \mp kr)},\ \omega = \frac{\partial \varphi}{\partial t},\ k = \frac{\partial \varphi}{\partial x}, \\ &\overline{x}_n = |\overline{x}_n| e^{i\varphi_n} = |\overline{x}_n| e^{i(\omega_n t \mp k_n x_n)}, \omega_n = \frac{\partial \varphi_n}{\partial t}, k_n = \frac{\partial \varphi_n}{\partial x}, n = 1,2,3. \end{aligned} \tag{6.8-1}$$

When relevant wave functions are conveniently presented in a hypercomplex space, described by (6.8), we would have a lot of additional freedom regarding mathematically rich modeling of motions, interactions, and analyzing energy-momentum states. For instance, using couples of mutually complex-conjugate functions, and algebraic sign variation in front of different imaginary units (+ or **–** sign), we will be able to present, or only mark and separate different energy-momentum states of particles, waves, antiparticles, etc., as for example,

$$\begin{aligned} &\overline{z}^* = r - a_1 \cdot i - a_2 \cdot j - a_3 \cdot k = r - I \cdot A = |\overline{z}| e^{-I\varphi} = \overline{z}_i^* + \overline{z}_j^* + \overline{z}_k^* = \\ &= |\overline{z}_i| e^{-i\varphi_i} + |\overline{z}_j| e^{-j\varphi_j} + |\overline{z}_k| e^{-k\varphi_k}, |\overline{z}^*|^2 = r^2 + A^2 = |\overline{z}|^2 = \overline{z} \cdot \overline{z}^*, \end{aligned} \tag{6.9}$$

whatever shows appropriate in a certain case of interest.

3. In order to reach higher level of unity and applicability of above-mentioned options **1.** and **2.** (for instance, in relation to the quantum mechanical wave function), already introduced complex functions (figuring in (6.2) - (6.9)), can also be treated as multidimensional **Hypercomplex Analytic Signal wave functions or Phasors** (see: eq. (4.9), **[7]** and **[8]**), and be presented in some of the following ways:

3. a) Using hypercomplex representation (6.8) and (6.9), we can transform the multidimensional basis (6.2) or (6.3), and express the ***Hypercomplex, Analytic Signal wave function,*** as follows,

$$\begin{aligned} &(\overline{r},t) \Leftrightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}},\ \underleftrightarrow{x_{4(imag)}, x_{5(imag)}, \ldots, x_{n(imag)}},\ t) \Rightarrow \\ &\Rightarrow (\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}},\ \underleftrightarrow{Ix_{4(imag)}, Ix_{5(imag)}, \ldots, Ix_{n(imag)},\ Ict}) \Leftrightarrow \\ &\Leftrightarrow \left\{ r(\underleftrightarrow{x_{1(real)}, x_{2(real)}, x_{3(real)}}), Ict^* \right\} \Leftrightarrow (r, Ict^*),\ I^2 = -1 \\ &r = r(x_1, x_2, x_3),\ t^* = t^*(t, \frac{x_4}{c}, \frac{x_5}{c}, \ldots, \frac{x_n}{c}). \end{aligned}$$

$$\overline{\Psi}(\mathbf{r},\mathbf{t}) = \Psi(\mathbf{r},\mathbf{t}) + \mathbf{I}\cdot\mathbf{H}\left[\Psi(\mathbf{r},\mathbf{t})\right] = \Psi(\mathbf{r},\mathbf{t}) + \mathbf{I}\cdot\hat{\Psi}(\mathbf{r},\mathbf{t}) =$$

$$= \overline{\Psi}_{\mathbf{i}} + \overline{\Psi}_{\mathbf{j}} + \overline{\Psi}_{\mathbf{k}} = \left|\overline{\Psi}(\mathbf{r},\mathbf{t})\right|\cdot\mathbf{e}^{\mathbf{I}\cdot\varphi(\mathbf{r},\mathbf{t})},\ \overline{\Psi}_{\mathbf{i,j,k}} = \Psi_{\mathbf{i,j,k}} + \begin{bmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{bmatrix}\cdot\hat{\Psi}_{\mathbf{i,j,k}} = \left|\overline{\Psi}_{\mathbf{i,j,k}}\right|\cdot\mathbf{e}^{\begin{bmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{bmatrix}\cdot\varphi_{\mathbf{i,j,k}}}, \quad (6.10)$$

$$\left|\overline{\Psi}(r,t)\right|^2 = \left[\Psi(r,t)\right]^2 + \left[\hat{\Psi}(r,t)\right]^2 = \Psi^2 + \hat{\Psi}^2 = \left|\overline{\Psi}\right|^2,\ \varphi(r,t) = \operatorname{arctg}\frac{\hat{\Psi}(r,t)}{\Psi(r,t)} = \varphi,$$

$$\left|\overline{\Psi}_{i,j,k}\right|^2 = \left[\Psi_{i,j,k}\right]^2 + \left[\hat{\Psi}_{i,j,k}\right]^2,\ \varphi_{i,j,k} = \operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi_{i,j,k}},\ \omega_{i,j,k} = \frac{\partial\varphi_{i,j,k}}{\partial t}$$

$$\left|\overline{\Psi}\right|^2 = \left|\overline{\Psi}_i\right|^2 + \left|\overline{\Psi}_j\right|^2 + \left|\overline{\Psi}_k\right|^2 = \Psi^2 + \hat{\Psi}^2 = \hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2 + \Psi_i^2 + \Psi_j^2 + \Psi_k^2,$$
$$\Psi^2 = \hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2,\quad \hat{\Psi}^2 = \Psi_i^2 + \Psi_j^2 + \Psi_k^2,$$
$$\Psi = \left|\overline{\Psi}\right|\cdot\cos\varphi = \Psi_i + \Psi_j + \Psi_k,\ \hat{\Psi} = \left|\overline{\Psi}\right|\cdot\sin\varphi = H\left[\Psi\right] = \hat{\Psi}_i + \hat{\Psi}_j + \hat{\Psi}_k,$$
$$H\left[\hat{\Psi}\right] = -\Psi = \hat{\Psi} + i\cdot\hat{\Psi}_i + j\cdot\hat{\Psi}_j + k\cdot\hat{\Psi}_\kappa,$$
$$\Psi_{i,j,k} = \left|\overline{\Psi}_{i,j,k}\right|\cdot\cos\varphi_{i,j,k},\ \hat{\Psi}_{i,j,k} = H\left[\Psi_{i,j,k}\right] = \left|\overline{\Psi}_{i,j,k}\right|\cdot\sin\varphi_{i,j,k},$$
$$H\left[\Psi_1\Psi_2\right] = (\Psi_1\Psi_2)\frac{\Psi_1\hat{\Psi}_2 + \hat{\Psi}_1\Psi_2}{\Psi_1\Psi_2 - \hat{\Psi}_1\hat{\Psi}_2},\ H\left[\frac{\Psi_2}{\Psi_1}\right] = (\frac{\Psi_2}{\Psi_1})\frac{\Psi_1\hat{\Psi}_2 - \hat{\Psi}_1\Psi_2}{\Psi_1\Psi_2 + \hat{\Psi}_1\hat{\Psi}_2},$$

$$I\cdot\hat{\Psi}(r,t) = i\cdot\hat{\Psi}_i + j\cdot\hat{\Psi}_j + k\cdot\hat{\Psi}_\kappa = e^{I(\frac{\pi}{2}+2m\pi)}\cdot\hat{\Psi}(r,t),$$
$$I\cdot\varphi(r,t) = i\cdot\varphi_i + j\cdot\varphi_j + k\cdot\varphi_k = e^{I(\frac{\pi}{2}+2m\pi)}\cdot\varphi(r,t),$$
$$I = i\cdot\frac{\varphi_i}{\varphi} + j\cdot\frac{\varphi_j}{\varphi} + k\cdot\frac{\varphi_\kappa}{\varphi} = i\cdot\frac{\hat{\Psi}_i}{\hat{\Psi}} + j\cdot\frac{\hat{\Psi}_j}{\hat{\Psi}} + k\cdot\frac{\hat{\Psi}_\kappa}{\hat{\Psi}} = e^{I(\frac{\pi}{2}+2m\pi)}, \quad (6.10)$$
$$\frac{\varphi_i}{\varphi} = \frac{\hat{\Psi}_i}{\hat{\Psi}},\ \frac{\varphi_j}{\varphi} = \frac{\hat{\Psi}_j}{\hat{\Psi}},\ \frac{\varphi_k}{\varphi} = \frac{\hat{\Psi}_k}{\hat{\Psi}},$$
$$\frac{\varphi_{i,j,k}}{\varphi} = \frac{\hat{\Psi}_{i,j,k}}{\hat{\Psi}} = \frac{H\left[\Psi_{i,j,k}\right]}{H\left[\Psi\right]} = \frac{\operatorname{arctg}\frac{\hat{\Psi}_{i,j,k}}{\Psi}}{\operatorname{arctg}\frac{\hat{\Psi}}{\Psi}},$$

$$\begin{bmatrix}\text{?!... to be verified}\\ \begin{pmatrix}\Psi = \Psi_i + \Psi_j + \Psi_k,\ \Psi^2 = \hat{\Psi}_i^2 + \hat{\Psi}_j^2 + \hat{\Psi}_k^2,\ \hat{\Psi}_i\hat{\Psi}_j + \hat{\Psi}_j\hat{\Psi}_k + \hat{\Psi}_i\hat{\Psi}_k = 0,\\ \hat{\Psi} = \Psi_i + \Psi_j + \Psi_k,\ \hat{\Psi}^2 = \Psi_i^2 + \Psi_j^2 + \Psi_k^2,\ \Psi_i\Psi_j + \Psi_j\Psi_k + \Psi_i\Psi_k = 0,\\ (\varphi = \varphi_i + \varphi_j + \varphi_k,\ \varphi^2 = \varphi_i^2 + \varphi_j^2 + \varphi_k^2,\ \varphi_i\varphi_j + \varphi_j\varphi_k + \varphi_i\varphi_k = 0)\end{pmatrix}\end{bmatrix}.$$

What we see from (6.10) is that important wave function elements (such as amplitude, phase, frequency, etc.) cannot be found if we do not take into account both, an original wave function Ψ, and its Hilbert couple $\hat{\Psi}$, meaning that in reality, both of such wave functions should coincidently exist (see Chapter 4.0 for more information about Analytic Signals).

The *hypercomplex analytic signal wave function*, which has an arbitrary number of imaginary units (higher than 3), could be expressed in a similar way, as the summation:

$$\overline{\Psi}(t) = \Psi(t) + I\,\hat{\Psi}(t) = a_0(t)e^{I\,\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\ \sin\varphi_0(t)\right] = \\ = a_0(t)e^{\sum_{(k)} i_k\varphi_k(t)} = \sum_{(k)} a_k(t)e^{i_k\varphi_k(t)} = \sum_{(k)} \overline{\Psi}_k(t)\ ,$$

or, as the multiplication,

$$\Psi(t) = \frac{a_n(t)}{2^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}),\ \hat{\Psi}(t) = \frac{a_n(t)}{(2i)^{n+1}}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\ , \\ \overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}, \tag{6.10-1}$$

$$\left[\begin{array}{l} \overline{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t)\ , \\ \cos\varphi_k = \frac{1}{2}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}),\ \sin\varphi_k = \frac{1}{2i}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)}), \\ \varphi_k(t) = \mathrm{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)},\ \varphi_0{}^2(t) = \sum_{(k)}\varphi_k{}^2(t),\ \ \omega_k(t) = \frac{\partial\varphi_k(t)}{\partial t} = 2\pi f_k(t), \\ a_k{}^2(t) = a_{k-1}{}^2(t) + \hat{a}_{k-1}{}^2(t) = \Psi_{k+1}{}^2(t) = \Psi_k{}^2(t) + \hat{\Psi}_k{}^2(t)\ , \\ a_0{}^2(t) = \left|\overline{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)} a_k{}^2(t) + 2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t),\ \ \forall i,j,k \in [1,n] \end{array}\right].$$

To present structure of different elementary particles, quasiparticles, wave-packets etc., it will be conceivable to establish certain well-operating practice how to use analytic, hypercomplex wave functions and phasors for an appropriate mathematical modeling with a deep meaning in Physics. Later, for instance, we may conclude that what we have been trying to isolate or define as a single elementary particle, or detectable entity, presents ***a structured world*** that has its unbounded, internally, and externally expanding structures and dimensions. Such structured body of an analytic hypercomplex wave function, having multiplicative energy levels (characterized by integers as, n = 1, 2, 3…) is recognizable from the expression:

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\},$$

where wave components going inwards and outwards (in mutually opposite directions) are marked by different signs in the exponent (such as $\mathbf{e}^{I\varphi_k(t)}$, $\mathbf{e}^{-I\varphi_k(t)}$). This is probably the main reason why our attempts to find and classify all elementary particles within dimensions of our universe will never end. By going into higher or deeper energy levels (regarding particle impacts and relevant products in collider accelerators), we are just opening a countless number of new matter-waves combinations of **mutually coupled**, amplitudes modulating, inwards and outwards expanding matter-waves of dynamically dependent and dualistic wave-particle structures. A much better strategy would be to find the right logical and conceptual mathematical structure of wave functions regarding how to understand and present them within naturally developing and evolving hypercomplex analytic wave functions and Phasors.

Here, we could also intuitively address elementary particles creation (when a particle with a stable rest mass is created). Certain multidimensional wave function (matter-wave) can be presented as an amplitude-modulated product of many cosine functions, each of them having different temporal and spatial frequencies $\omega_n(t) = \frac{\partial \varphi_n(t)}{\partial t} = 2\pi f_n(t)$ and $k_n(x) = \frac{\partial \varphi_n(t)}{\partial x} = 2\pi f_n(x)$. Such frequency harmonics in a proper environment could create self-closed, stable structures with standing waves, which would externally manifest as different stable particles.

It is interesting to notice that certain Hypercomplex function, for instance with three imaginary units, could easily be separated, or treated as the sum of three single, ordinary complex functions (each of them with only one imaginary unit). In addition, certain Hypercomplex function can also be presented as a product between three ordinary, simpler complex functions (where, again, every simple complex function has only one imaginary unit). This is giving a chance to address separately triadic or trinary structures (for instance structures like quarks). Mentioned simple complex functions (each of them with one imaginary axis) will have mutually orthogonal and different imaginary units. Here we try to open a window to mathematical understanding and analyzing of multidimensional and more-complex spatial structures and symmetries, when compared to much simpler dyadic or binary cases (which are like being positive or negative in certain sense, or having matter and anti-matter states, or being a simple mirror-symmetry states ...).

Remarks: *Hypercomplex-Quaternions and Analytic signal wave function modeling can assume that signal phase function is also presentable as a Hypercomplex function, having its real and imaginary parts, such as,*

$$\overline{\Psi}(r,t) = \overline{\Psi}(x,y,z,t) = \Psi(r,t) + I \cdot H\left[\Psi(r,t)\right] = \Psi(r,t) + I \cdot \hat{\Psi}(r,t) =$$
$$= \Psi(r,t) + i \cdot \hat{\Psi}_x(r,t) + j \cdot \hat{\Psi}_y(r,t) + k \cdot \hat{\Psi}_z(r,t) =$$
$$= \overline{\Psi}_i + \overline{\Psi}_j + \overline{\Psi}_k = \left|\overline{\Psi}(r,t)\right| \cdot e^{\overline{\varphi}(r,t)},$$
$$\overline{\varphi}(r,t) = \varphi_R(r,t) + I\,\varphi_I(r,t) = \varphi_R(r,t) + i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k = \left|\overline{\varphi}(r,t)\right| e^{\overline{\varphi}(r,t)},$$
$$I \cdot \varphi_I(r,t) = i \cdot \varphi_i + j \cdot \varphi_j + k \cdot \varphi_k,\ \ I^2 = i^2 = j^2 = k^2 = -1.$$

This will give us even larger mathematical modeling freedom.

3. b) Following a quite simple analogy, based on the knowledge that our universe has three spatial and one temporal dimension $(\mathbf{x_1, x_2, x_3, t})$, this situation has been easily generalized (in modern Physics) creating multidimensional spaces analogically extending the basis of the three spatial dimensions into n spatial dimensions

$(\mathbf{x_1, x_2, x_3, \ldots, x_n, t})$. This is the simplest choice, and only one of numerous possibilities to imagine what could be a good starting platform to analyze hypothetical and multidimensional universes. Most probably, we still do not know what the actual choice of the Universe is, (we could only think, or believe that a basis $(\mathbf{x_1, x_2, x_3, \ldots, x_n, t})$ is the most realistic multidimensional platform for a future exploration regarding multidimensionality). We have seen that every spatial coordinate of our universe $(\mathbf{x_1, x_2, x_3, t})$ is also presentable as having its own real and imaginary component $\left[(x_{1r},x_{1i}), (x_{2r},x_{2i}), (x_{3r},x_{3i}), t\right]$. It is also imaginable that a time component, $\mathbf{t}$, has its real and imaginary components, $\left[(x_{1r},x_{1i}), (x_{2r},x_{2i}), (x_{3r},x_{3i}), (t_r,t_i)\right]$ **thus creating modified Minkowski space using hypercomplex 4-vectors,** or we can introduce analogical n-vectors (as already mentioned), etc. From Hypercomplex Minkowski 4-vectors, or analogical n-vectors, we can develop corresponding Complex and Hypercomplex Analytic Signal Phasors, and profit from number of advantages of such Phasors in relation to Wave-Particle Duality (see more in Chapter 10.).

The most interesting situation (coming from Hypercomplex Analytic Signal functions) is that every single imaginary unit $\mathbf{i, j, k}$ from (6.8) - (6.10) can additionally split into three new imaginary units, creating new levels of endlessly growing (higher dimensional, or sub-dimensional) triplets, or something like fractals, such as,

$$\begin{aligned}
&i \rightarrow (i_{11}, i_{12}, i_{13})\ldots,\ j \rightarrow (j_{11}, j_{12}, j_{13})\ldots,\ k \rightarrow (k_{11}, k_{12}, k_{13})\ldots \\
&i_{mn} \rightarrow (i_{mn1}, i_{mn2}, i_{mn3})\ldots,\ j_{mn} \rightarrow (j_{mn1}, j_{mn2}, j_{mn3})\ldots,\ k_{mn} \rightarrow (k_{mn1}, k_{mn2}, k_{mn3})\ldots \\
&\overline{\Psi}(r, t) = \overline{\Psi}(x, y, z, t) = \Psi(r, t) + I \cdot H\left[\Psi(r, t)\right] = \Psi(r, t) + I \cdot \hat{\Psi}(r, t), \\
&\Psi(r, t) = \Psi_x(r, t) + \Psi_y(r, t) + \Psi_z(r, t),\ \hat{\Psi}_{x,y,z}(r, t) = H\left[\Psi_{x,y,z}(r, t)\right], \\
&I \cdot \hat{\Psi}(r, t) = i \cdot \hat{\Psi}_x(r, t) + j \cdot \hat{\Psi}_y(r, t) + k \cdot \hat{\Psi}_z(r, t) =
\end{aligned}$$

$$= \begin{bmatrix} i_{11} \cdot \hat{\Psi}_{x1}(r, t) + i_{12} \cdot \hat{\Psi}_{x2}(r, t) + i_{13} \cdot \hat{\Psi}_{x3}(r, t) \\ + \\ j_{11} \cdot \hat{\Psi}_{y1}(r, t) + j_{12} \cdot \hat{\Psi}_{y2}(r, t) + j_{13} \cdot \hat{\Psi}_{y3}(r, t) \\ + \\ k_{11} \cdot \hat{\Psi}_{z1}(r, t) + k_{12} \cdot \hat{\Psi}_{z2}(r, t) + k_{13} \cdot \hat{\Psi}_{z3}(r, t) \end{bmatrix}. \qquad (6.11)$$

Following the same patterns of brainstorming, we could imagine the basis of multidimensional universe extending each of the spatial coordinates into a new set of three elements (associating different, hypercomplex imaginary units to every triplet of spatial coordinates), as for instance:

$$(\mathbf{x_1, x_2, x_3, t}) \Rightarrow \left[(\mathbf{x_{11}, x_{21}, x_{23}}), (\mathbf{x_{12}, x_{22}, x_{32}}), (\mathbf{x_{13}, x_{23}, x_{33}}), \ldots (\mathbf{x_{1n}, x_{2n}, x_{3n}}), \mathbf{t})\right]. \quad (6.12)$$

Hypercomplex numbers and functions can be formulated in a more general way than in (6.8) and (6.9), using $\mathbf{n}$ elementary complex units (instead of three), and this way we can exercise new and more complex multi-dimensional coordinate systems...

*"**Experimenting**" with algebraic sign (**+** and/or **–**), with different combinations of hypercomplex imaginary units (**i, j, k**), and with couples of complex and complex-conjugate wave functions and their phase-shifted Hilbert couples mentioned in 3.a) and 3.b), we can try to address various **symmetry related structures** known in Physics.*

3. c) Until present, the time dimension **t** or **Ict** was implicitly considered as a homogenous and isotropic, single and an independent variable. Mathematically, we can show that the time dimension and absolute speed **c** could be treated as anisotropic, coordinates-dependent, physical (and vector) values, **at least in one of three different ways**, as follows,

$$\mathrm{I}\cdot ct=\begin{Bmatrix}(i_x ct+i_y ct+i_z ct),\ \text{or}\\ (i_x c_x t+i_y c_y t+i_z c_z t),\ \text{or}\\ (i_x ct_x+i_y ct_y+i_z ct_z),\ \text{or}\\ (i_x c_x t_x+i_y c_y t_y+i_z c_z t_z)\end{Bmatrix}\begin{pmatrix}\Rightarrow\\ (=)\end{pmatrix}\mathrm{I}\cdot\begin{Bmatrix}c,t, & \text{or}\\ (c_x+c_y+c_z),\ t & \text{or}\\ c,(t_x+t_y+t_z), & \text{or}\\ (c_x,c_y,c_z),(t_x,t_y,t_z) & \end{Bmatrix}$$

$$\mathrm{I}^2=i_x^2=i_y^2=i_z^2=-1,\ (i_x=i, i_y=j, i_z=k, x_1=x, x_2=y, x_3=z, \qquad (6.13)$$

$$c^2=c_x^2+c_y^2+c_z^2,\ t^2=t_x^2+t_y^2+t_z^2,$$

while keeping the relativistic space-time interval $\Delta\mathbf{S}$, and absolute speed and time amplitudes ($\mathbf{c}$, $\mathbf{t}$), unchanged,

$$(\Delta\mathbf{S})^2=(\Delta\mathbf{x}_1)^2+(\Delta\mathbf{x}_2)^2+(\Delta\mathbf{x}_3)^2-\mathbf{c}^2(\Delta\mathbf{t})^2=\mathbf{const.}.$$

In other words, there is a possibility that the constant and maximal, absolute speed $\mathbf{c}$ has variable and unlimited, mutually coordinate-dependent, velocity vector components $(\mathbf{c}_x,\mathbf{c}_y,\mathbf{c}_z)$, and that something similar is valid for the relevant and associated time dimension components $(\mathbf{t}_x,\mathbf{t}_y,\mathbf{t}_z)$. Maybe, some of speed components, hypothetically higher than **c,** could be mathematically (still hypothetically) related to entanglement effects in relation to "Bell's interconnections theorem", and to David Bohm's concept of particle-wave phenomenology of pilot waves. Another option radiating from similar ideas is that any Multidimensional Universe should have an equal number of mutually coupled, spatial and temporal components or dimensions, since in Relativity theory time and space are mutually linked and interdependent (and this should be a generally extendable situation).

In Chapter 4.1 of this book, we can find all relations between the group and the phase velocity applicable to any wave motion that has sinusoidal wave components and harmonic nature of its spectrum (see eq. (4.2)).

$$\left\{\begin{array}{l} v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp} = \frac{df}{df_s} = \frac{2u}{1+\frac{uv}{c^2}}\,, \\ u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} = \frac{f}{f_s} = \frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}} = \frac{E_k}{p} \Rightarrow \\ \Rightarrow \quad 0 \le 2u \le \sqrt{uv} \le v \le c\,, \\ d\tilde{E} = hdf = mc^2 d\gamma, \quad \frac{df}{f} = (\frac{dv}{v})\cdot\frac{1+\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v^2}{c^2}} \Rightarrow \frac{\Delta f}{\overline{f}} = (\frac{\Delta v}{\overline{v}})\cdot\frac{1+\sqrt{1-\frac{\overline{v}^2}{c^2}}}{1-\frac{\overline{v}^2}{c^2}} \end{array}\right\} \quad (4.2)$$

Relations (4.2) are also applicable to photons, electromagnetic waves, and speed of light, where the speed of light should correspond to the group velocity in (4.2). When an electromagnetic wave propagates in a vacuum and free space, all three velocities will become mutually equal, **v** = **u** = **c ≅ 300000 km/s**, and here should lie a part of the explanation what the meaning of the universal constant **c** is. Another challenging question is if we would be able to send part of the energy or meaningful information at speeds higher than **c**.

3. d) Whatever we establish as the mathematically most convenient basis of the multidimensional universe, it should be compatible with the energy aspects of dimensionality given earlier by (4.32), (5.15), (5.16), and (5.17). Complex or hypercomplex numbers and functions (in connection with a chosen coordinate basis) will usually serve as a good mathematical model, enabling elegant and concise formulations of multidimensional physics reality.

The principal question here is, on what grounds or arguments do we almost uniquely take (or impose) the option $(\mathbf{x_1, x_2, x_3, t}) \Rightarrow (\mathbf{x_1, x_2, x_3, \ldots, x_n, t})$ or $(x_1,x_2,x_3...,x_n\,,\ t_1,t_2,t_3...,t_n)$, as the basis of an n-dimensional universe? Do we really know the choice of Nature (regarding n-dimensionality)? The leading idea here is not to criticize the existing mathematical concepts regarding n-dimensionality (which could be quite correct), but also to explore other imaginable options, and to find out which one of them better fits into the actual structure of our n-dimensional Universe (all of that being still highly hypothetical and speculative). One of the options, still not mentioned here, is the Superstrings theory with curled spatial dimensions (but the author of this paper has the opinion that Hypercomplex Analytic Signals and n-dimensional Phasors are much more fruitful and promising mathematical framework for multidimensional reality explorations).

All the above-introduced options regarding the mathematical basic frames of a multidimensional universe can be generalized by modifying the multi-dimensional basis, given by (6.1), introducing hypercomplex spatial coordinates $(\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_n)$, where the time dimension is not explicitly present (but couples of spatial and temporal components always exist inside), such as:

$$\{\textbf{(SPACE) \& TIME}\} \Leftrightarrow \{(x_1,x_2,...,x_n)+t\} \Leftrightarrow (r,t) \Rightarrow (\overline{x}_1,\overline{x}_2,...,\overline{x}_n)\,. \quad (6.14)$$

In (6.14) the time dimension is created only from imaginary parts of generalized hypercomplex space coordinates $(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)$, or n-vectors in the following way:

$$\begin{aligned}
&\overline{x}_i = x_{i(real)} + I\, x_{i(imaginary)} = x_i e^{I\theta_{xi}},\ x_i = \sqrt{x^2_{i(real)} + x^2_{i(imaginary)}}\,, \\
&\theta_{xi} = \arctan\frac{x_{i(imaginary)}}{x_{i(real)}},\ i = 1,2,3,...,n;\ I^2 = -1, \\
&(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n) \Leftrightarrow \left\{ (x_{1(real)}, x_{2(real)}, ...x_{n(real)}), (x_{1(imaginary)}, x_{2(imaginary)}, ...x_{n(imaginary)}) \right\} \\
&\Leftrightarrow (r,t),\ r = r(x_{1(real)}, x_{2(real)}, ...x_{n(real)}),\ t = t(x_{1(imaginary)}, x_{2(imaginary)}, ...x_{n(imaginary)})
\end{aligned} \tag{6.15}$$

There are many of mathematical possibilities how to treat and analyze (6.15), operating with only one, three or more hypercomplex imaginary units. Practically, a kind of generalized multidimensional (hypercomplex) Minkowski n-space of the Relativity Theory is immediately recognizable in (6.15) and (6.16), when we introduce the following n-vector notation:

$$\begin{aligned}
&(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n) \Leftrightarrow \left\{ (x_{1(real)}, x_{2(real)}, ...x_{n(real)}), (x_{1(imaginary)}, x_{2(imaginary)}, ...x_{n(imaginary)}) \right\} \\
&\Leftrightarrow (r,ct),\ r = r(x_{1(real)}, x_{2(real)}, ...x_{n(real)}),\ t = \frac{1}{c}(x_{1(imaginary)}, x_{2(imaginary)}, ...x_{n(imaginary)})
\end{aligned} \tag{6.16}$$

The 4-dimensional relativistic space-time interval based on (6.16), unified with Uncertainty relations from Chapter 5, could be expressed as:

$$\begin{aligned}
&(\Delta S)^2 = (\Delta x_{1(real)})^2 + (\Delta x_{2(real)})^2 + (\Delta x_{3(real)})^2 + (\delta s)^2 - c^2(\Delta t)^2, \\
&(\delta s)^2 = (\Delta x_{4(real)})^2 + (\Delta x_{5(real)})^2 + ... + (\Delta x_{n(real)})^2 \geq \left[\frac{h}{2\Delta p}\right]^2 \approx 0, \\
&c^2(\Delta t)^2 = (\Delta x_{1(imaginary)})^2 + (\Delta x_{2(imaginary)})^2 + ... + (\Delta x_{n(imaginary)})^2, \\
&\delta s \cdot \Delta p = \Delta t \cdot \Delta\tilde{E} \geq \frac{h}{2}.
\end{aligned} \tag{6.17}$$

The definition of 4-dimensional relativistic space-time interval (as we find it presently in Relativity theory and in this paper; -see Chapter 5.; -equations (5.18) – (5.21) and Chapter 10.) is still somewhat speculative and challenging concept. It is not experimentally or undoubtedly proven, and most probably that it would be modified and upgraded using 4-vector of relativistic velocity (which can be considered as correctly defined), as for instance,

$$\begin{aligned}
&\overline{V}_4 = V(\gamma v, \gamma c) = \frac{d}{dt}\overline{X}_4 \Rightarrow d\overline{X}_4 = \overline{V}_4 dt = X(\gamma v \cdot dt, \gamma c \cdot dt) \Rightarrow \\
&\Rightarrow (\Delta S)^2 = \left(\int_{[\Delta T]} \gamma v \cdot dt \right)^2 - \left(c \int_{[\Delta T]} \gamma \cdot dt \right)^2,\ v = v(x,y,z,t) = v(x_1, x_2, x_3, t), \\
&t = t(x_4, x_5, x_6, ...x_n),\ \gamma = 1/\sqrt{1 - v^2/c^2}.
\end{aligned} \tag{6.17-1}$$

Certainly, in some future analyses regarding coordinate systems of multidimensional space, it must be shown which spatial coordinate frame ((6.1) - (6.17-1) ... or something like them, like in **[49]**) would be the most realistic for fruitful applications in physics.

Here we have been almost arbitrarily speculating with many of multidimensional "*space-time-frame*" options, similar or equivalent to Riemannian and Minkowski space and metrics of Relativity Theory, by exploring new possibilities in the case when ordinary complex functions are replaced by analytic hypercomplex functions. Present Minkowski 4-vectors concept implemented in Relativity theory is intellectually exciting and challenging, producing particularly good results regarding particles interactions, but most probably that its interpretation and practical implementations will evolve to something much more general (see relevant proposals in Chapter 10., under "10.1 **Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"**). In a similar fashion, the Quantum Mechanical wave function (of a multidimensional space) could be formulated as a convenient Analytic Signal, a hypercomplex Phasor function, crating options for integration between Relativistic and Quantum theory. The intellectually extrapolated and estimated superiority of properly developed, multidimensional, hypercomplex analytic signal functions should be obvious (to open-minded and visionary structured individuals) if we consider the overwhelming mathematical and waves-modeling superiority of an "ordinary Analytic Signal", based on Hilbert Transform, when compared to any other signal analysis established until the introduction of the Analytic Signal (see Chapters 4.0, 4.3 and 10.). Pity that Quantum Theory in its early steps did not establish the Wave Function as an Analytic Signal or Phasor. This is, probably, because the Analytic Signal concept was introduced just after Quantum Theory canonized and "over-glorified" its own, artificial wave functions mathematical processing. In addition, later, all of that was looking (to creators and supporters of such Quantum Theory) as something not really necessary, or too much, and too late to be taken into account to restart everything from zero, especially because Quantum Theory with all associated mathematics has been functioning sufficiently or very well (and too many of published references, recognitions, and prizes already materialized, that any restart from zero, with any new or better concept would not be easily defendable). The opinion of the author of this book about the same situation is that Quantum Theory will function equally well and much better in case if a Wave Function framework is established based on Hypercomplex Analytic Signals and Phasors (just to start with).

[♣ COMMENTS & FREE-THINKING CORNER: **STILL IN PROCESS ⇒**

For our Universe, we usually say that it has three spatial dimensions and conveniently, we add a time dimension. Three spatial dimensions are just spatial coordinates, having the same physical nature (being some distances measured in the same way using the same units). How such, ***identical nature coordinates***, could be separate or independent dimensions? Here is the Quote from the books ***Lone Pine Writings by Eric Dollard***:

"It is customary to consider space boundaries as a CUBIC, or third degree set of coordinates. The three coordinates are length, width, and height, taken from a corner of the cube. Think of a sugar cube, the sugar is the space and the corners define the boundaries. These three coordinates, length, width, and height are WRONGLY known as the three dimensions of our space. This is a major mind virus, and it is hard to erase it. There is only one dimension of space, SPACE, a metrical dimension. Any number of coordinates in any number of geometries can serve to define the boundaries of said space. The use of the cubic three is habitual. The

dimension of space is considered to exist in degrees or powers of a unit space dimension, here centimeters, l (lowercase L). So, we can say cubic centimeters, or square centimeters, etc."

6.1. Hypercomplex, In-depth Analysis of the Wave Function

Starting from the signal representation in the form of an analytical signal (**1.1**) and (**1.2**) from the chapter 4.0, (which reminds to a simple harmonic form of a signal), we can continue the same procedure of factorization unlimitedly, representing every new amplitude function a(t), by a new form of analytical signal. Thus, it will be:

$$\Psi(t) = \frac{1}{\pi}\int_0^\infty [U_c(\omega)\cos\omega t + U_s(\omega)\sin\omega t]d\omega = \frac{1}{\pi}\int_0^\infty [A(\omega)\cos(\omega t + \Phi(\omega))]d\omega =$$

$$= a(t)\cos\varphi(t) = \Psi_0(t) = a_0(t)\cos\varphi_0(t) =$$

$$= a_1(t)\cos\varphi_1(t)\cos\varphi_0(t) = \Psi_1(t)\cos\varphi_0(t) =$$

$$= a_2(t)\cos\varphi_2(t)\cos\varphi_1(t)\cos\varphi_0(t) = \Psi_2(t)\cos\varphi_1(t)\cos\varphi_0(t) =$$

....

$$= a_n(t)\prod_{i=0}^{n}\cos\varphi_i(t) = \Psi_n(t)\prod_{i=0}^{n-1}\cos\varphi_i(t) = a_n(t)\cdot\cos\varphi_0(t)\prod_{i=1}^{n}\cos\varphi_i(t),$$

$$\left[a(t) = a_0(t) = a_n(t)\prod_{i=1}^{n}\cos\varphi_i(t),\ \cos\varphi(t) = \cos\varphi_0(t)\right]. \qquad (6.18)$$

Previously obtained, factorized signal form reminds us of a multiple amplitude modulation of the signal, where every following level represents an amplitude function of the previous level, so in that sense, we can talk about the in-depth structure of the signal, whose structural levels may be presented by following functions:

$$\Psi_1(t) = a_0(t) = a_1(t)\cos\varphi_1(t) = |\overline{\Psi}(t)| = |\overline{\Psi}_0(t)| = \frac{\Psi_0(t)}{\cos\varphi_0(t)},$$

$$\Psi_2(t) = a_1(t) = a_2(t)\cos\varphi_2(t) = |\overline{a}_0(t)| = |\overline{\Psi}_1(t)| = \frac{\Psi_0(t)}{\cos\varphi_1(t)\cos\varphi_0(t)},$$

.... (6.19)

$$\Psi_n(t) = a_{n-1}(t) = a_n(t)\cos\varphi_n(t) = |\overline{a}_{n-2}(t)| = |\overline{\Psi}_{n-1}(t)| = \frac{\Psi_0(t)}{\prod_{i=0}^{n-1}\cos\varphi_i(t)}.$$

From (3.15) and (3.16) one may notice that following relations remain:

$$\Psi_{k+1}(t) = a_k(t) = a_{k+1}(t)\cos\varphi_{k+1}(t) = a_n(t)\prod_{i=k+1}^{n}\cos\varphi_i(t),\ k < n,$$

$$\prod_{i=1}^{n}\Psi_i(t) = \prod_{i=1}^{n}a_n(t)\cdot\prod_{i=1}^{n}\cos\varphi_i(t)\ ,\ \frac{\Psi_k(t)}{\Psi_{k+1}(t)} = \cos\varphi_k\ ,\prod_{i=0}^{n}\cos\varphi_i(t) = \frac{\prod_{i=0}^{n}\Psi_i(t)}{\prod_{i=1}^{n+1}\Psi_i(t)}\ , \qquad (6.20)$$

$$\frac{a_1}{a_0}\cos\varphi_1 = \frac{a_2}{a_1}\cos\varphi_2 = ... = \frac{a_n}{a_{n-1}}\cos\varphi_n = 1.$$

If we briefly address signal velocities, we can conclude that the phase velocity of the basic signal $\Psi(t)$ (as well as the common phase velocity of its components) is determined only by the $\cos\phi_0$. In addition, the group velocity of the signal $\Psi(t) = \Psi_0(t) = a_0(t)\cos\phi_0(t)$ propagation is determined only as the velocity of its amplitude function $a_0(t)$. By considering the factorization of the signal, one may conclude that there are series of partial group velocities, which by appropriate addition (like the relativistic determination of the velocity of the inertia center) must form the resulting (central, group) velocity of the amplitude member $a_0(t)$.

A problem that is interesting to be solved mathematically is the case of factorized signal presentation (6.18), where we could fix several $\cos\varphi_i(t)$ members in advance, giving them specific (or arbitrary) values for phase functions, regardless if such values could be found in the initially factorized signal form. Let us take the example where only three of such cosine-phase members (indexed with ***A***, ***B***, *and* ***C***) *are fixed by our intervention,*

$\Psi(t) = a_n(t)\cdot\prod_{i=0}^{n}\cos\varphi_i(t) = a_n(t)\cdot\cos\varphi_A(t)\cdot\cos\varphi_B(t)\cdot\cos\varphi_C(t)\cdot\prod_{i=3}^{n}\cos\varphi_i^*(t)$ *The question here is if we could still find the right product of remaining cosine members* $\prod_{i=3}^{n}\cos\varphi_i^*(t)$ *that will fit exactly to represent the same signal (without any error). Why and where could be such factorization interesting? Let us imagine that we intend to present the process of disintegration of certain microparticle, initially presentable by* $\Psi(t)$. *Based on experimental data, we may know (in advance) the resulting amplitude function* $a(t) = a_0(t) = a_n(t)\prod_{i=1}^{n}\cos\varphi_i(t)$ *of the same particle before its disintegration. In addition, knowing the experimental results of particle disintegration, we could find that three new particles (for the needs of this example) are clearly detectable (particles indexed with* ***A***, ***B***, *and* ***C***), *and that possibly we have certain signal residuals, or energy dissipation, producing some other elements, not easy to detect. Hypothetically, we will attempt to present all such residual elements using the factorization* $\prod_{i=3}^{n}\cos\varphi_i^*(t)$. *If here described scenario can be mathematically modeled to describe real situations from microphysics, this will pave a way to start organizing elementary particles in a much better way than presently known.*

Every structural level of the signal from (6.19) can be associated with correspondent complex, analytical signal, by the same procedure as given in (4.02),

$$\{\Psi_1(t)\} \rightarrow \overline{\Psi}_1(t) = a_0(t) + i_1\hat{a}_0(t) = a_1(t)e^{i_1\varphi_1(t)},$$

$$\{\Psi_2(t)\} \rightarrow \overline{\Psi}_2(t) = a_1(t) + i_2\hat{a}_1(t) = a_2(t)e^{i_2\varphi_2(t)}, \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\{\Psi_n(t)\} \rightarrow \overline{\Psi}_n(t) = a_n(t) + i_n\hat{a}_n(t) = a_n(t)e^{i_n\varphi_n(t)},$$

$$\{\Psi_n(t)\} \rightarrow \overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)}, \tag{6.21}$$

where $\hat{\Psi}_{k+1}(t) = \hat{a}_k(t) = H[a_k(t)]$.

One may notice that in (6.19), an indexed imaginary unit is introduced, which for now, should be understood, and interpreted, as an already known (common) imaginary unit:

$$I^2 = i^2 = j^2 = i_1^{\,2} = i_2^{\,2} = i_3^{\,2} = \ldots = i_n^{\,2} = -1, \tag{6.22}$$

however, later such indexed imaginary units will serve for evolving from a simple complex space to a hyper-complex space with many imaginary units.

The specificity of penetrating into the structural depth of the signal (or into the wave function) becomes obvious if we determine energies of the structural levels of the signal (in the same way as in (4.04)):

$$\begin{aligned}
&\tilde{E}_0 = \int_{-\infty}^{+\infty} \Psi^2(t)dt = \int_{-\infty}^{\infty} a_0^2(t)\cos^2\varphi_0(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_0^2(t)\,dt\ ,\\
&\tilde{E}_1 = \int_{-\infty}^{+\infty} \Psi_1{}^2(t)dt = \int_{-\infty}^{\infty} a_0^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_1^2(t)\,dt = 2\tilde{E}_0\ ,\\
&\tilde{E}_2 = \int_{-\infty}^{+\infty} \Psi_2{}^2(t)dt = \int_{-\infty}^{\infty} a_1^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_2^2(t)\,dt = 4\tilde{E}_0\ , \qquad (6.23)\\
&\text{............}\\
&\tilde{E}_n = \int_{-\infty}^{+\infty} \Psi_n{}^2(t)dt = \int_{-\infty}^{\infty} a_{n-1}^2(t)\,dt = \frac{1}{2}\int_{-\infty}^{+\infty} a_n^2(t)\,dt = 2^n\tilde{E}_0 = 2\tilde{E}_{n-1}, n = 0,1,2...\\
&(\tilde{E}_0 = (\gamma-1)m_0c^2 = \frac{\tilde{E}_n}{2^n} \Rightarrow m_0 = \frac{\tilde{E}_0}{(\gamma-1)c^2} = \frac{\tilde{E}_n}{2^n(\gamma-1)c^2}, \gamma = (1-v^2/c^2)^{-0.5}).
\end{aligned}$$

It is obvious that penetrating into deeper structural levels of the signal "**n**" demands consumption of more and more energy for factor 2^n, to isolate and analyze those levels (in a way like the analysis of nuclear and elementary particles).

Let us go back now to the structural levels of the signal in the form of (6.19). We will call the basic level of the signal in its complex form a hypercomplex analytical function of the signal, and we will represent it as in (4.02), apart from that, the complex unit "j" will be replaced by the hypercomplex, imaginary unit "I":

$$\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\sin\varphi_0(t)\right], \qquad (6.24)$$

where the hypercomplex, imaginary unit "I" is defined as a linear combination over the base of elementary hypercomplex units "$i_1, i_2, i_3, ..., i_n$", as:

$$\begin{aligned}
&I\varphi_0(t) = i_1\varphi_1(t) + i_2\varphi_2(t) + ... + i_n\varphi_n(t) = \sum_{k=1}^{n} i_k\varphi_k(t),\\
&I^2 = i_1{}^2 = i_2{}^2 = i_3{}^2 = ... = i_n{}^2 = -1,\\
&i_j i_k = 0, \forall j \neq k \Rightarrow \varphi_0{}^2(t) = \sum_{k=1}^{n} \varphi_k{}^2(t), \qquad (6.25)\\
&e^{i_k\varphi_k(t)} = \cos\varphi_k(t) + i_k\sin\varphi_k(t), \forall k \in [1, n].
\end{aligned}$$

According to the introduction of the hypercomplex wave function $\overline{\Psi}(t)$ in a previous way, it is possible to show the validity of the following relations:

$$\begin{aligned}
&\overline{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = a_0(t)e^{I\varphi_0(t)} = a_0(t)\left[\cos\varphi_0(t) + I\sin\varphi_0(t)\right] =\\
&= a_0(t)e^{\sum_{(k)} i_k\varphi_k(t)} = \sum_{(k)} a_k(t)e^{i_k\varphi_k(t)} = \sum_{(k)} \overline{\Psi}_k(t), \qquad (6.26)
\end{aligned}$$

where:

$$\begin{aligned}
&\overline{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} = \Psi_k(t) + i_k\hat{\Psi}_k(t)\,,\\
&\varphi_k(t) = \mathrm{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)},\ \varphi_0{}^2(t) = \sum_{(k)}\varphi_k{}^2(t),\\
&a_k{}^2(t) = a_{k-1}{}^2(t) + \hat{a}_{k-1}{}^2(t) = \Psi_{k+1}{}^2(t) = \Psi_k{}^2(t) + \hat{\Psi}_k{}^2(t),\\
&a_0{}^2(t) = \left|\overline{\Psi}(t)\right|^2 = \Psi^2(t) + \hat{\Psi}^2(t) = \sum_{(k)} a_k{}^2(t) + 2\sum_{(i\neq j)}\Psi_i(t)\Psi_j(t),\ \forall i,j,k \in [1,n]\,.
\end{aligned} \tag{6.27}$$

Based on the structural analysis of the signal, introducing its structural levels through (6.19), it is obvious that we are in a position that, if we know the function of the basic signal level $\Psi(t)$, we can determine its remaining (depth) levels of a higher order:

$$\Psi(t) = \Psi_0(t) \to \Psi_1(t) \to \Psi_2(t)\ ...\ \to \Psi_n(t), \tag{6.28}$$

and also go back from any level $\Psi_k(t)$, $0 < k \leq n$ (using inverse transformations) to the basic level of the signal $\Psi_0(t)$,

$$\Psi_k(t) \to \Psi_{k-1}(t) \to \Psi_{k-2}(t) \to ... \to \Psi_0(t)\,. \tag{6.29}$$

If we continue the procedure of inverse transformations from (6.29) in the same direction (backward), we will be able to determine sub-structural signal levels (or its history),

$\Psi_k(t) \to \Psi_{k-1}(t) \to \Psi_{k-2}(t) \to ...$	$\to \Psi_0(t) \to$	$\to \Psi_{-1}(t) \to \Psi_{-2}(t) \to ... \to \Psi_{-n}(t)$
depth levels of the signal	basic level	Sub-structural levels
(future of the signal, $\tilde{E}_k$)	(present state $\tilde{E}_0$)	(past of the signal, $\tilde{E}_{-k}$)

One can show that eigenenergies of the substructural signal levels (or its prehistory, i.e., energy spent to form that signal) will be, similarly to the formula for the energies of the structural (depth) levels (6.23), equal to

$$\tilde{E}_{-n} = 2^{-n}\tilde{E}_0\ \ , n = 0, 1, 2, ... \tag{6.30}$$

One may immediately notice that sum of all energies of the substructural or past signal levels is equal to the sum of a convergent series (converging towards $\tilde{E}_0$),

$$\sum_{k=1}^{n}\tilde{E}_{-k} = \sum_{k=1}^{n} 2^{-k}\tilde{E}_0 \leq \tilde{E}_0,\ \ (n \to \infty)\,, \tag{6.31}$$

which obviously represents the form of the Law of energy conservation, while the series of the energies of the higher or deeper structural signal levels $(n \to +\infty)$ is divergent, i.e.,

$$\sum_{k=1}^{n}\tilde{E}_k = \sum_{k=1}^{n} 2^k\tilde{E}_0 \to \infty,\ \ (n \to \infty)\,. \tag{6.32}$$

As we can see, from (6.32), in order to penetrate deeper and deeper into the structure of certain signal (or matter), we must spend more and more energy, and this is a never-ending process (similar to what we experience with modern particle accelerators).

Now we can give a parallel between the previously exposed structural analysis of the signal, which reduces to the factorization of the signal, and Fourier's analysis of a signal (which corresponds to the searching of appropriate sums of orthogonal functions). In a context of such classified signal analysis, we can picturesquely say that Fourier's analysis, or summation decomposition of a signal, is the planar or **"surface"** analysis of a signal in a framework of one structural signal level. Continuing the same line of associative classifications, signal factorization can be considered as structural, "**spatial or volumetric**" signal decomposition, or signal mining. From factorized signal dissolution, we could explore a possibility that similar method or model (conveniently mathematically modified) can be used for an analysis of the matter structure (of atoms and subatomic particles), as well as for processing and transferring data in telecommunications and for analyses of different signals that carry information about certain process.

ANALYTIC SIGNAL FORMS

$$\bar{H}[\Psi(t)] = \bar{\Psi}(t) = \Psi(t) + I\cdot\hat{\Psi}(t) = a(t)\cdot e^{I\cdot\varphi(t)} \;,\; \bar{H} = 1 + I\cdot H,\; I^2 = -1$$

AS SUPERPOSITION	AS MULTIPLICATION
$\bar{\Psi}(t) = \sum_{(k)} \bar{\Psi}_k(t) = \sum_{(k)} \Psi_k(t) + I\sum_{(k)} \hat{\Psi}_k(t)$	$\bar{\Psi}(t) = \Psi_n(t)\prod_{(i=0)}^{n-1} \cos\varphi_i(t) + I\Psi_n(t)\prod_{(i=0)}^{n-1} \sin\varphi_i(t)$

RELATIONS BETWEEN ADDITIVE AND MULTIPLICATIVE ELEMENTS

$$\Psi(t) = \sum_{(k)} \Psi_k(t) = \Psi_n(t)\prod_{(i=0)}^{n-1} \cos\varphi_i(t) = a_0(t)\cos\varphi_0(t) = a(t)\cos\varphi(t) = -H\left[\hat{\Psi}(t)\right]$$

$$\hat{\Psi}(t) = \sum_{(k)} \hat{\Psi}_k(t) = \Psi_n(t)\prod_{(i=0)}^{n-1} \sin\varphi_i(t) = a_0(t)\sin\varphi_0(t) = a(t)\sin\varphi(t) = H\left[\Psi(t)\right]$$

$$\bar{\Psi}_k(t) = \Psi_k(t) + i_k\hat{\Psi}_k(t) = a_k(t)e^{i_k\varphi_k(t)} \;, \Psi_k(t) = -H\left[\hat{\Psi}_k(t)\right],\; \hat{\Psi}_k(t) = H\left[\Psi_k(t)\right]$$

$$\bar{\Psi}(t) = \Psi(t) + I\hat{\Psi}(t) = \frac{a_n(t)}{2^{n+1}}\left\{\prod_{k=0}^{n}(e^{I\varphi_k(t)} + e^{-I\varphi_k(t)}) + \frac{1}{(i)^n}\prod_{k=0}^{n}(e^{I\varphi_k(t)} - e^{-I\varphi_k(t)})\right\}$$

Signal Amplitude

$$a(t) = a_0(t) = \left|\bar{\Psi}(t)\right| = \sqrt{\Psi^2(t) + \hat{\Psi}^2(t)} = a_n(t)\prod_{(i=1)}^{n} \cos\varphi_i(t) = \Psi_{n+1}(t)\prod_{(i=1)}^{n} \cos\varphi_i(t)$$

$$a_k(t) = \Psi_{k+1}(t) = \left|\bar{\Psi}_k\right| = \sqrt{\Psi_k^2(t) + \hat{\Psi}_k^2(t)} = a_{k+1}(t)\cos\varphi_{k+1}(t) = a_n(t)\prod_{(i=k+1)}^{n} \cos\varphi_i(t),\; k < n$$

$$a_{n-1}(t) = \Psi_n(t) = \left|\bar{\Psi}_{n-1}(t)\right| = \sqrt{\Psi_{n-1}^2(t) + \hat{\Psi}_{n-1}^2(t)} = a_n(t)\cos\varphi_n(t) = \frac{\Psi_0(t)}{\prod_{(i=0)}^{n-1} \cos\varphi_i(t)}$$

Signal Phase

$$\varphi_0(t) = \varphi(t) = \mathrm{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)} = \sqrt{\sum_{(k)} \varphi_k^2(t)},\; \varphi_k(t) = \mathrm{arctg}\frac{\hat{\Psi}_k(t)}{\Psi_k(t)}$$

$$I^2 = i_1^2 = i_2^2 = ... = i_n^2 = -1 \;,\; i_j i_k = 0, \forall\, j \neq k \;\; \text{(hypercomplex imaginary units)}$$

$$I\varphi_0(t) = I\varphi(t) = i_1\varphi_1(t) + i_2\varphi_2(t) + ... + i_n\varphi_n(t) = \sum_{k=1}^{n} i_k\varphi_k(t)$$

$$e^{i_k\varphi_k(t)} = \cos\varphi_k(t) + i_k\sin\varphi_k(t),\;\; \varphi_0^2(t) = \sum_{k=1}^{n}\varphi_k^2(t)$$

Signal instant frequency

$$\omega_i(t) = 2\pi f_i(t) = \frac{\partial\varphi_i(t)}{\partial t},\;\; i = 0,1,2,...k,...n$$

The parallelism between Time and Frequency Domains	Analytic Signal	
	Time Domain	**Frequency Domain**
Complex Signal	$\overline{\Psi}(t) = a(t)e^{j\varphi(t)}$ $= \Psi(t) + j\hat{\Psi}(t)$ $= \int_{(0,+\infty)} \overline{U}(\omega)e^{-j\omega t}d\omega$ $= \int_{(0,+\infty)} A(\omega)e^{-j(\omega t+\Phi(\omega))}d\omega$	$\overline{U}(\omega) = A(\omega)e^{-j\Phi(\omega)}$ $= U_c(\omega) - j\,U_s(\omega)$ $= \int_{[t]} \overline{\Psi}(t)\, e^{j\omega t}dt$ $= \int_{[t]} a(t)e^{j(\omega t+\varphi(t))}dt$
Real and imaginary signal components	$\psi(t) = a(t)\cos\varphi(t)$ $= -H[\hat{\psi}(t)],$ $\hat{\psi}(t) = a(t)\sin\varphi(t)$ $= H[\psi(t)]$	$U_c(\omega) = A(\omega)\cos\Phi(\omega)$ $U_s(\omega) = A(\omega)\sin\Phi(\omega)$
Signal Amplitude	$a(t) = \sqrt{\psi^2(t) + \hat{\psi}^2(t)}$	$A(\omega) = \sqrt{U_c^{\,2}(\omega) + U_s^{\,2}(\omega)}$
Instant Phase	$\varphi(t) = \mathrm{arctg}\frac{\hat{\psi}(t)}{\psi(t)}$	$\Phi(\omega) = \arctan\frac{U_s(\omega)}{U_c(\omega)}$
Instant Frequency	$\omega(t) = \frac{\partial\varphi(t)}{\partial t}$	$\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\omega}$
Signal Energy	$E = \int_{[t]} \lvert\overline{\psi}(t)\rvert^2 dt$ $= \int_{[t]} [a(t)]^2 dt$	$E = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert\overline{U}(\omega)\rvert^2 d\omega$ $= \frac{1}{\pi}\int_0^{\infty} [A(\omega)]^2 d\omega$
Central Frequency	$\Omega = \frac{\int_{[t]} \omega(t)\cdot a^2(t)dt}{\int_{[t]} a^2(t)dt}$	$\Omega = \frac{\int_0^{\infty} \omega\cdot[A(\omega)]^2 d\omega}{\int_0^{\infty} [A(\omega)]^2 d\omega}$
"Central Time"	$T = \frac{\int_{[t]} t\cdot[a(t)]^2 dt}{\int_{[t]} [a(t)]^2 dt}$	$T = \frac{\int_{[\omega]} \tau(\omega)\cdot[A(\omega)]^2 d\omega}{\int_{[\omega]} [A(\omega)]^2 d\omega}$
Standard Deviation	$\sigma_\Omega^2 = \frac{1}{\Delta t}\int_{[t]} \lvert\omega(t) - \Omega\rvert^2 dt$	$\sigma_T^2 = \frac{1}{\Delta\omega}\int_{[\omega]} \lvert\tau(\omega) - T\rvert^2 d\omega$
Uncertainty Relations	$\omega(t)\cdot\tau(\omega) \cong \Delta\omega\cdot\Delta\tau(\omega) \cong \sigma_\Omega\cdot\sigma_T \cong \Omega\cdot T \cong \Delta\omega\cdot\Delta t \cong \pi(!?)$	

Energy-momentum vectors and multidimensional Hypercomplex universe

The hypercomplex wave function is a much richer concept regarding the usual complex functions, and on the following example, we can show some more advantages of such representations of the wave function. Let us start from the previously defined hypercomplex wave function (6.24) and let us represent it in one of the possible (modified) forms:

$$\overline{\Psi}(t)=\Psi(t)+I\,\hat{\Psi}(t)=a_0(t)e^{I\,\varphi_0(t)}=a_0(t)[\cos\varphi_0(t)+I\ \sin\varphi_0(t)]=$$
$$=a_{01}(t)e^{I_1\varphi_{01}(t)}+a_{02}(t)e^{I_2\varphi_{02}(t)}+a_{03}(t)e^{I_3\varphi_{03}(t)}=$$

$$=\Psi_1(t)+\Psi_2(t)+\Psi_3(t)+i_1\,\hat{\Psi}_1(t)+i_2\,\hat{\Psi}_2(t)+i_3\,\hat{\Psi}_3(t)=$$
$$=\left[\Psi_{11}(t)+\Psi_{12}(t)+\Psi_{13}(t)\right]+\left[\Psi_{21}(t)+\Psi_{22}(t)+\Psi_{23}(t)\right]+\left[\Psi_{31}(t)+\Psi_{32}(t)+\Psi_{33}(t)\right]+$$
$$+\left[i_{11}\,\hat{\Psi}_{11}(t)+i_{12}\,\hat{\Psi}_{12}(t)+i_{13}\,\hat{\Psi}_{13}(t)\right]+\left[i_{21}\,\hat{\Psi}_{21}(t)+i_{22}\,\hat{\Psi}_{22}(t)+i_{23}\,\hat{\Psi}_{23}(t)\right]+$$
$$+\left[i_{31}\,\hat{\Psi}_{31}(t)+i_{32}\,\hat{\Psi}_{32}(t)+i_{33}\,\hat{\Psi}_{33}(t)\right]=$$
$$=\left[\Psi_{11}(t)+i_{11}\,\hat{\Psi}_{11}(t)\right]+\left[\Psi_{12}(t)+i_{12}\,\hat{\Psi}_{12}(t)\right]+\left[\Psi_{13}(t)+i_{13}\,\hat{\Psi}_{13}(t)\right]+$$
$$+\left[\Psi_{21}(t)+i_{21}\,\hat{\Psi}_{21}(t)\right]+\left[\Psi_{22}(t)+i_{22}\,\hat{\Psi}_{22}(t)\right]+\left[\Psi_{23}(t)+i_{23}\,\hat{\Psi}_{23}(t)\right]+$$
$$+\left[\Psi_{31}(t)+i_{31}\,\hat{\Psi}_{31}(t)\right]+\left[\Psi_{32}(t)+i_{32}\,\hat{\Psi}_{32}(t)\right]+\left[\Psi_{33}(t)+i_{33}\,\hat{\Psi}_{33}(t)\right]=$$
$$=a_{01}(t)e^{I_1\varphi_{01}(t)}+a_{02}(t)e^{I_2\varphi_{02}(t)}+a_{03}(t)e^{I_3\varphi_{03}(t)}=\ \dots\ ,$$

so that following relations stand:

$$\Psi(t)=\Psi_1(t)\ +\ \Psi_2(t)\ +\ \Psi_3(t),$$
$$I\,\hat{\Psi}(t)\ =\ i_1\,\hat{\Psi}_1(t)+\ i_2\,\hat{\Psi}_2(t)+\ i_3\,\hat{\Psi}_3(t)\ ,$$
$$\Psi_1(t)=\left[\Psi_{11}(t)\ +\ \Psi_{12}(t)\ +\ \Psi_{13}(t)\right],$$
$$\Psi_2(t)\ =\ \left[\Psi_{21}(t)\ +\ \Psi_{22}(t)\ +\ \Psi_{23}(t)\right],$$
$$\Psi_3(t)\ =\left[\Psi_{31}(t)\ +\ \Psi_{32}(t)\ +\ \Psi_{33}(t)\right],$$

$$i_1\,\hat{\Psi}_1(t)\ =\ \left[i_{11}\,\hat{\Psi}_{11}(t)+\ i_{12}\,\hat{\Psi}_{12}(t)+\ i_{13}\,\hat{\Psi}_{13}(t)\right],$$
$$i_2\,\hat{\Psi}_1(t)\ =\ \left[i_{21}\,\hat{\Psi}_{21}(t)+\ i_{22}\,\hat{\Psi}_{22}(t)+\ i_{23}\,\hat{\Psi}_{23}(t)\right],$$
$$i_3\,\hat{\Psi}_3(t)\ =\ \left[i_{31}\,\hat{\Psi}_{31}(t)+\ i_{32}\,\hat{\Psi}_{32}(t)+\ i_{33}\,\hat{\Psi}_{33}(t)\right]\ .$$

The previous model of the wave function becomes a more interesting one, with the following introduction of the rule of three-dimensional (as vectors) orthogonality of the imaginary axes:

$$i_1\times i_2\ =\ i_3\ ,\quad i_2\times\ i_3\ =\ i_1\ ,\quad i_3\times\ i_1\ =\ i_2\ ,$$
$$i_2\times i_1\ =-i_3\ ,\quad i_3\times\ i_2\ =-i_1\ ,\quad i_1\times\ i_3\ =-i_2\ ,$$
$$i_{1n}\times\ i_{2n}\ =\ i_{3n}\ ,\ i_{2n}\times\ i_{3n}\ =\ i_{1n}\ ,\ i_{3n}\times\ i_{1n}\ =\ i_{2n}\ ,$$
$$i_{2n}\times\ i_{1n}\ =-i_{3n}\ ,\ i_{3n}\times\ i_{2n}\ =-i_{1n}\ ,\ i_{1n}\times\ i_{3n}\ =\ -i_{2n}\ ,\quad n\ \in[1,2,3]\ .$$

....

It is obvious that (if we continue with the previous procedure) we can introduce different forms of analysis, synthesis, and divide the hypercomplex wave function into levels, which could probably be used in purpose of modeling of specific structures that are characteristic for the world of elementary, subatomic particles, or for establishing the universal field theory, etc.

Obviously, based on multilevel Hypercomplex structuring (using many of "imaginary triplets of units"), we should be able to address kind of multidimensional space that is different from most of the multidimensional concepts presently known. To make such concept of Hypercomplex Universe more practical and applicable to Physics, we could (analogically) follow the example of 4-vectors (known as the Minkowski space of Relativity Theory). We can simply formulate equivalent Hypercomplex functions, or Hypercomplex 4-vectors, by replacing the ordinary imaginary unit with triplets of Hypercomplex imaginary units. On the example of linear momentum vector, it is possible to demonstrate how we can create the Hypercomplex momentum, as follows (see (4.3-0)-p,q,r,s…, from the Chapter 4.1; paragraph 4.1.2.1. "Hypercomplex functions interpretation" and in chapter 10, "10.1

Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"),

$$\left\{\begin{array}{l} \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right) = \left(\vec{p}_i, i\frac{E_i}{c_i}\right) + \left(\vec{p}_j, j\frac{E_j}{c_j}\right) + \left(\vec{p}_k, k\frac{E_k}{c_k}\right) \\ \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) = \text{invariant} \Rightarrow \vec{p}^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c^2})^2 \Leftrightarrow E_0^2 + p^2c^2 = E^2 = (E_0 + E_k)^2 \\ \vec{p} = \vec{p}_i + \vec{p}_j + \vec{p}_k, E = E_0 + E_k, E_i = E_{0i} + E_{ki}, E_j = E_{0j} + E_{kj}, E_k = E_{0k} + E_{kk} \\ I^2 = i^2 = j^2 = k^2 = -1, ij = k,\ jk = i, ki = j, ji = -k, kj = -i, ik = -j \end{array}\right\} \Rightarrow \qquad (4.3\text{-}0)\text{-s}$$

$$\left\{\begin{array}{l} I\frac{E}{c} = i\frac{E_i}{c_i} + j\frac{E_j}{c_j} + k\frac{E_k}{c_k}, \\ \left(\frac{E}{c}\right)^2 = \left(\frac{E_i}{c_i}\right)^2 + \left(\frac{E_j}{c_j}\right)^2 + \left(\frac{E_k}{c_k}\right)^2 \\ \left(\frac{E_0}{c}\right)^2 = \left(\frac{E_{0i}}{c_i}\right)^2 + \left(\frac{E_{0j}}{c_j}\right)^2 + \left(\frac{E_{0k}}{c_k}\right)^2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} p^2 = \frac{E^2 - E_0^2}{c^2} = \frac{E_i^2 - E_{0i}^2}{c_i^2} + \frac{E_j^2 - E_{0j}^2}{c_j^2} + \frac{E_k^2 - E_{0k}^2}{c_k^2} = \\ = \vec{p}_i^2 + \vec{p}_j^2 + \vec{p}_k^2 \end{array}\right\}$$

In (4.3-0)-s, constants c_i, c_j, c_k *that have dimensions of speed (like the universal speed constant* $c \cong 3 \cdot 10^8 m/s$*), could also be equal to* $c = c_i = c_j = c_k$ *(but involved mathematics is giving chances for other options). Energies indexed with 0 and k in (4.3-0)-s are effectively paving the ways to explain creations of big numbers of products in impact reactions (zeros are indicating particles with rest masses, and k-indexing stands for kinetic energy states). Such extended energy-momentum framework can be later merged with universal Complex and Hypercomplex Analytic Signal representation of wave functions (leading to all famous wave equations of Quantum Theory), and with novel foundations of multidimensional Universe (see Chapter 4.3 and chapter 6, equations (6.10) - (6.13)). Presence of three imaginary units in (4.3-0)-s is intuitively igniting ideas about mutually coupled energy triplets such as three quarks, three anti-quarks etc., what could create another, more general and more precise concept of Super-symmetry in the world of microphysics (and significantly or essentially modify, enrich, and simplify the Standard Model).* ♣]

7. CONCLUSION

This book is dealing with innovative aspects of particle-wave duality, gravitation, and electromagnetic theory, addressed from the point of view of electric and mechanical analogies united with Continuous Symmetries. This is also a discussion document with objectives to initiate the revision of particle-wave dualism and gravitation theory, by exploring one specific system of overall electromechanical analogies (which is an extended ***mobility type of analogies;*** *-see more in Cha*pter 1.). The concept of universally applicable analogies as found in the first chapter of this book (see T.1.3 – T.1.8), is formulating the message that ***everything being on mentioned way analogical and symmetrical also presents the first indicative sign and step towards creation of the Unified Physics or Unified Field theory***. In this book, the first internally compatible, analogical, intuitive, and indicative steps are elaborated in the direction of creating Universal and United Field theory, which can be established based on:

1. *Analogies summarized from the similarity of relevant mathematical formula and relevant differential equations, addressing seemingly different (but mutually analogical) phenomenology in Physics,*
2. *Analogies coming from the same geometry, or the same topology, of mutually equivalent mechanical and electromagnetic oscillatory or resonant circuits and models,*
3. *Analogies created after comparing different energy-moments, and natural forces formulas, as well as comparing different Conservation Laws, known in Physics,*
4. *Knowing that* even Maxwell Electromagnetic theory is heavily, ontologically, and analogically, initially established based on Fluids Dynamics (see more in Chapter 3.).
5. *Considering that all waves, oscillating and resonance related phenomena, known in different domains of Physics, belong to the same family of matter waves, being naturally mathematically modeled and explicable using the same Complex Analytic Signal model (independently and regardless of any statistics and probability-based concepts; -see more in Chapter 4.0).*
6. *Considering that most of contemporary Physics theories describing events and constellations in our cosmic universe, regarding Gravitation, Planetary systems, Galaxies, Black holes, Neutron stars, and other Natural Forces ... are dominantly addressing kind of mechanical, electromagnetically neutral, A. Einstein and Newtonian-world side, almost neglecting electromagnetic side of the same Universe, but* ***in this book is shown that Mechanical, Electromagnetic, and Wave-Particle-Duality nature of our Universe cannot be separated (or mutually exclusive).***

Based on the established system of analogies, several innovative ideas are formulated, regarding possible generalization and unification of Gravitation, Maxwell Electromagnetic Theory, and Quantum Theory in the frames of new understanding of **P**article **W**ave **D**uality (see more in Chapters 4.0, 4.1, 4.2, 4.3, 8, 9, and 10.). Of course, we must be conscious that any conclusion based only on analogies is at the same time powerful, if correct, and it could be very risky and weak if it is not correctly developed and applied, and this is the reason why analogies in this book are taken only as an initial, indicative new ideas generator. A wider and more general platform for extending and exploiting the analogies in physics is to be found in exploiting Symmetries. The following, supporting statement is taken from the book [10] written by Michio Kaku (see also [11], from the same author): "… ***we are beginning to realize that nature, at the fundamental level, does not just prefer symmetry in a physics theory, nature demands it. Physicists now realize that symmetry is the key to constructing physics laws without disastrous anomalies and divergences***."

We need to be careful regarding correct foundations of relevant Symmetries and Analogies, and author of this book is proposing to apply the self-correcting and self-supporting (gradually advancing) strategy of exploiting united multilevel Analogies and Continuous Symmetries, using them as starting, indicative foundations for future Physics managements, predictions, advances, and generalizations.

The artificially assembled concept of **P**article **W**ave **D**uality, as it is known from our contemporary physics (or from the Orthodox Quantum theory), in this book is specifically updated, more generally reformulated and additionally clarified, becoming the concept of "Particle Wave Unity" (here formulated as **PWDC** (=) **P**article **W**ave **D**uality **C**ode). It is also shown that all important wave-equations and

Uncertainty relations of Orthodox Quantum theory can be logically, naturally, and step by step developed mathematically, without even knowing that present Quantum Theory exist, and without any of "*probability and statistics philosophy*" backing. We only need to start from slightly different mathematical and conceptual grounds, and to use better, already existing, more convenient mathematical theories and models (such as Signal Analysis based on Analytic Signal concept; -see more in Chapters 4.1 and 10.).

Wave-Particle Duality is equally applicable to a micro and macro world of Physics concerning situations with intrinsic periodicities, potentially hosting stable, self-closed, standing waves orbits and structures. Both, energy, or matter states in a temporal and spatial domain (including matter in joint temporal-spatial domains) can be sources of matter waves in different cases of motions and temporal-spatial perturbations. *Wave-Particle Matter Duality phenomenology also presents a "bridge" between the **linear motions with spatial translational symmetry**, and always and naturally associated **rotational motions with spatial rotational symmetry**. In addition, under linear motions of masses, we can analogically and phenomenologically associate linear motions of electrically charged particles being in certain electric field. The same way, under rotational motions of masses, we can analogically and phenomenologically associate rotational (spinning and helix) motions of electrically charged particles being in certain magnetic field. Moreover, spatial, and temporal dimensions or duration intervals of involved masses, electric charges and associated matter-waves are always mutually or proportionally dependent and connected (on a way as addressed in Relativity theory). Most of contemporary theories describing events and constellations in our cosmic universe, regarding Gravitation, Planetary systems, Galaxies, Black holes, Neutron stars and other Natural Forces ... are dominantly addressing kind of mechanical, geometry-related and electrically neutral and passive, Newtonian world side, almost neglecting electromagnetic side of the same Universe. Here (in this book) we see that Mechanical, Electromagnetic, and Wave-Particle-Duality nature of our Universe cannot be separated (or mutually exclusive). **We also have indications that the Electromagnetic Universe is dominant, and that Mechanics and Geometry related Universe is only the product of mentioned Electromagnetic reality.***

The specific innovative achievements, or just creatively formulated hypothetical ideas and indicative proposals radiating from this book can be briefly summarized as contributions related to:

1. New ways of seeing Gravitation as being related to stationary and resonant, standing matter-wave states of masses (or energy of natural fields agglomerations), within involved, mutually coupled, and mutually synchronized, linear, orbital, and spinning motions, also with associated electromagnetic properties. Static masses are not the principal and only source of gravitation, but electromagnetically charged or polarized masses, with linear and angular moments, are the real sources of Gravitation, supported by Translational and Rotational Symmetry, or with linear and angular moments and energy conservation. ***Atoms and masses in motion (including masses oscillations and other angular or torsional motions) with internally coupled linear and angular motion elements and associated electromagnetic complexity (meaning with coupled linear, angular, and electromagnetic moments and dipoles) are the real source of gravitation. Consequently, atoms, (<u>atomic field and universal natural force structure, emanating from atoms</u>, with dynamic properties) are real sources of gravitation, since any internal atoms' field structure is also extending and balancing externally (synchronizing with other atoms and masses of structurally vibrating Universe) and creating $1/r^2$ -dependent central forces*** (see more in Chapters 2., 8. and 10.).

   Schrödinger types of wave equations are also applicable to Gravitation (in relation to orbital planetary motions) since Schrödinger equation is the product of Classical Wave equation. Gravitation should be a specific manifestation of standing, electromagnetic and electromechanical waves' formations of an electromagnetic force acting between motional and mutually coupled electric and magnetic dipoles or moments, within global spatial-temporal proportionality (see more in Chapter 2.). Consequently, **Relativity theories should be redesigned and updated, becoming exclusive products of conveniently updated Electromagnetic theory** (see more in Chapter 3.). Gravitation is also the force between agglomerated macroscopic masses that are anyway composed of atoms and molecules, this way internally, naturally, and structurally having an intrinsic electromagnetic complexity, being composed of electric charges, with magnetic moments, and polarized electromagnetic dipoles or multipoles (each of them internally structured as standing waves resonators). Because of such internal electromagnetic complexity of masses, and permanent, global, linear, angular, and oscillatory motions within our Universe, structurally induced effects of electromagnetic forces are being created inside and around atoms and masses, producing effects of Gravitation. Gravitation as the recognizable and measurable effect disappears if "*supposed to be attracting masses*" and distances between them are becoming exceedingly small (for instance smaller than 100 micrometers, because after certain, minimal spatial *size-threshold,* there is not enough of electrically charged and polarized electromagnetic entities to create effects of currents, voltages, electric and magnetic induction, fluxes, and forces, we externally see, in average, as being effects of gravitation).

Shielding and manipulating Gravitation.

a) As presently known and tested, when applying certain closed cage, envelope, metallic, or solid shield, around any mass, we do not find its weight reduction. Gravitation is on some way penetrating applied shielding. This is supporting the concept that sources of Gravitation are extended atomic fields, producing electromagnetic dipoles polarization that is naturally becoming part of applied shielding, being conveniently aligned on the way that applied shield is effectively transparent for such electromagnetic polarization formations. Another speculative or hypothetical option is that electromagnetic field has additional field and force components (not covered by the contemporary Maxwell theory), penetrating metal electric and magnetic shields (see [152] William J. Hooper). This is implicating that around involved masses we should have specific background matter state, with omnidirectional vortex and spinning formations of an invisible and very fine fluid (or ether). This resembles the concepts of Nikola Tesla and Rudjer Boskovic regarding Gravitation and Radiant energy, which is also creating standing-waves formations between mutually attracting and conveniently polarized masses (respecting $1/r^2$, central Coulomb force, which effectively, or electromagnetically penetrates applied electromagnetic shielding).

b) There are also (speculative and hypothetical) indications that gravitation shielding, weight reduction, and some gravitation field-reflecting effects could be produced using shielding objects that have specific, spatial resonant structures. Russian professor Viktor Stepanovich Grebennikov, [149], described number of effects familiar to gravitation, levitation, and temporal-spatial matter-waves phenomenology of structurally periodic geometric structures created by insects, or as natural beehive cells, dry honeycomb structures, or objects artificially assembled on a similar way, creating weight reduction.

Why we know only about attractive forces between masses.

This is the result of our conceptualization of Gravitation. Presently we find only attractive forces of Gravitation because our concept and modelling of gravitation is still restrictive (meaning Newton and Einstein concepts are incomplete and restrictive). Stable macro masses are some agglomerations of smaller masses or involved atoms, imaginable as being created (or placed) in nodal zones of certain structurally resonating system with standing matter waves between involved masses. ***In such nodal zones we can naturally detect only attractive, agglomerating forces (or forces of Gravitation), but standing waves structures also have anti-nodal zones, where we can detect only repulsive (or antigravitational and negative energies forces;*** *-see more about analogical effects of ultrasonic levitation in [150 – 151]****).*** This again means that Nikola Tesla and Rudjer Boskovic formulated qualitatively indicative concepts about Gravitation and means that our Universe is structurally resonating.

Why Newton and Coulomb laws have the same mathematical form.

Because all such forces initially or essentially originate from electric and magnetic charges and polarizations, where we naturally apply Coulomb type of interactions, and Newton force between masses is mathematically identical or analogue to Coulomb law. We can also conceptualize and present masses as being directly proportional to involved (internal) electric and magnetic dipoles and charges, and this way, when we apply Newton law, we effectively apply Coulomb law (see more in the second chapter of this book).

Black or Dark matter, masses, and energies in relation to gravitation.

Mass and energy are not what we find only within solid boundaries (or rigid shape) of certain body, or "mass-energy-moments" formation. Spatially extended mass-energy formations should be better conceptualized and characterized. Different mass-energy aspects are part of dynamic, structurally resonating Universe.

Fundamental Electromagnetic, Nuclear and Gravitation forces, acting in our Universe, are (most probably) wrongly formulated in Physics, some of them still being as labels or indicative names on "black boxes" without an essential meaning and explanation. Much better explanation of nuclear forces would be discovered within electromagnetic, and vibrations-related forces, existing in nodal and antinodal interatomic zones of standing matter-waves structures (meaning in atom nucleus zone). Since an electromagnetic-periodicity and standing waves related structure describes well electron clouds around an atom nucleus, and Coulomb electromagnetic force is acting between positive charged nucleus and its negative-charged electrons cloud, consequently, atom nucleus should also have similar mirror-image, periodical structure of electromagnetic standing waves (see more in Chapter 8.).

The real nature of all forces in our Universe is universally related to spatial energy gradients (regardless of origins and "shapes" of the energy in question), and to effects of attraction and repulsion present around nodal and anti-nodal zones of standing matter waves that are shaping and resonating our Universe (here including waves in mechanics, fluid states, and electromagnetic waves). See more in Chapters 5. and 10. Consequently, we can summarize, as follows.

1. Overall matter in our universe can be seen in many of "packing or formatting shapes", of energy-momentum states, in forms of auto-synchronizing and self-organizing, self-closed and self-stabilized matter waves. Elementary matter entities like electrons and protons (or particles with rest masses) are specifically packed, as spatially self-closed electromagnetic, standing waves formations (like being composed of photons), structured in forms of resonators and electromagnetic dipoles (or multi-poles). The new, upgraded formulation of generalized (structural, spatial, and temporal) matter waves' periodicity in this book is partially complementing and replacing the old de Broglie concept of matter waves. The meaning here is that waving and oscillating properties are not only related to linear (spatial) elongations, but also to radial, orbital, angular, and torsional motions (both in temporal and spatial domains); -all of them essentially and ontologically belonging to mutually coupled electromagnetic and mechanical resonant systems, manifesting as waves and oscillations. All of that is also familiar to String theory concepts. Matter waves, as energy-momentum carrying states, which are not structured as self-closed and stabilized standing-waves resonators, are progressive waves and wave groups without initial rest mass attributes. What we presently see and understand as a mass, should be a much more subtle entity, which also has its (dynamic, motion-dependent, and wave-like) real and imaginary parts, essentially and intrinsically having wave-particle duality properties with associated electromagnetic complexity (see more in Chapter 10.). Foundations of wave-particle duality are originally established and causally related to electromagnetic, electromechanical, mechanical, and analogical parallelism between photons and moving particles (especially particles electromagnetically charged). Mater waves are all kind of signals, waves and oscillations existing in our Universe (dominantly present and known as mechanical and electromagnetic waves and signals), being all described and explicable using the same mathematical modeling (originally and ontologically based on Fourier signal analysis and synthesis, and later based on Complex and Hypercomplex Analytic Signal modelling). Even macrocosmic configurations like solar or planetary systems are creating macrocosmic matter-waves structures that are very much mathematically and analogically presentable like matter waves and electromagnetic structures in a microworld of atoms (see more in Chapter 2.). Everything mentioned until here are the reasons why we can successfully create and utilize number of electromechanical and other analogies in Physics (see more in Chapter 1.).

   Schrödinger second order differential wave equation is the simple case of Classical Wave Equation, when treating the square of Wavefunction as a Power-function (based on Parseval identity relations), where components of such Power-function are defined as Complex Analytic

Signals. Such modelling concept can be until certain level transformable and isomorphic with a probabilistic wavefunction (as practiced in Quantum theory), but it is mathematically richer, much clearer, and more deterministic. Consequently, ***one of important insights here are updated understanding of matter waves and generalized Schrödinger wave equation as a simple wave-function related product of Classical wave equation, when treating a wave function as a Complex Analytic Signal function (based on Hilbert Transform and D. Gabor model of the Analytic Signal; -****see more in Chapter 4.3****).*** De Broglie matter waves are shown to be causally and structurally linked to Analytic Signal wave function modeling, having the same properties (see more in Chapter 4.1 and 10.). Hypercomplex Analytic Signal wavefunctions could also be convenient for modelling elementary particles and multidimensional structure of matter. In addition, in this book, we can find more clarifications about matter waves velocities and wave equations (than in the contemporary Physics literature). Mathematically and analogically compared, the same Classical Wave equations are equally known in Mechanics and Electromagnetic theory, as well as in Quantum theory.

Atoms, molecules, and other masses are only different manifestations and formatting of electromagnetic states and field structures. Particles and atoms present self-closed and self-stabilized, standing matter-waves structures with intrinsic periodicities (where Mendeleev Periodic System of elements is an exceptionally good and striking, supporting evidence). Temporal periodicities are intuitively best explicable with sinusoidal and simple harmonic waves in a time domain. Spatial periodicities are structural periodicities like in cases of crystalline matter and fractals-like formations. Since spatial and time domains are mutually dependent and coupled, we can also find unified effects of temporal and spatial periodicities, manifesting as stationary, resonant, and standing-waves formations (reminding on String theory concepts). All of that is being valid, because of the complementary nature of united linear and rotational (or spinning) motions that behaves as complementarity and mutual coupling nature between electric and magnetic fields, and like mathematical connections between real and imaginary parts of an Analytic signal function (see more in Chapter 10.). Other aspects of temporal-spatial proportionality and Analytic Signal modeling of matter-waves are effects of electromagnetic, electromechanical and quantum synchronizations between mutually similar or identical resonators, including associated entanglement effects (with mutually overlapping spectral characteristics). Based on such mathematical and empirical foundations, we can create generalized platform for an advanced understanding of a time and multidimensionality in Physics. Number of a) "*Action-equal-to-Reaction*", b) "*Forces related to Induction and Mirror-Imaging Laws*", as well as c) "*in pairs coupled wave-equations solutions of mutually opposed wave groups, propagating inwards and outwards*", are giving us more complete understanding of spatial-temporal reality (compared to what we presently conceptualize about time and space in Physics). Consequently, our contemporary concepts about Black Holes, Dark Mass, and Dark Energy, Cosmic, Relict or Background radiation, Neutron stars, and matter structure, are still incomplete, somewhat arbitrary, and wrong, and should be fundamentally reestablished (see more in Chapter 10.).

Quantizing, or quantum nature of our Universe are naturally deterministic manifestations of a multi-resonant matter assembly of different resonant states and standing waves formations. Probabilistic and by Statistic realized modelling of such structures is completely opposite to mentioned deterministic and tangible background (of resonant and standing waves matter formations), but it also works well, when mathematical conditions for such modelling are involved. All stable matter states in our Universe are structural (spatial-temporal) combinations of more elementary, vibrating, and resonant states with dualistic wave-particle properties. This way we are coming closer to "***String theory"*** *concepts*. Since the **String theory is the most promising, universal, natural fields and forces unification platform, or concept**, it would be necessary to merge and unify here favoured mater structure concepts based on resonant and standing waves with contemporary String theory achievements (where the most important or background mathematical support should comply with Kotelnikov-Shannon-Nyquist-Whittaker signal analysis theory).

2. Uncertainty relations are equally applicable to a micro and macro world of physics. Only the way of understanding dimensions or domains durations should be appropriately reconsidered. Certainty relations (drawn from Uncertainty relations) are defining elementary building blocks of

matter when both of wavefunctions in mutually conjugate domains have Gaussian-Gabor (or similar, bell-shaped) envelopes. Uncertainty Relations initially known in Signal and spectrum analysis, and later implemented in micro-world Physics can be significantly extended to all matter states, valid for micro and macro-Universe (and should be entirely the same in Mathematics, Physics, Relativity and Quantum Theory). The elementary matter constituents, like elementary particles and atoms, are described in this book by an innovative understanding of extended ***Uncertainty Relations*** (which are in specific cases of stabilized matter formations becoming ***Certainty Relations***), clarifying a boundary spatial zone between mutually conjugate and optimally concentrated signals' energy, in relevant Original and Spectral domains (see more in Chapters 5. and 10.). Spatial and temporal domains of different material objects, structures and energy-moments states are mutually dependent, coupled and synchronously communicating states based on their resonant and spectral complexity and similarity.

In addition, it is demonstrated that Uncertainty Relations and quantizing are just a part or consequences of Shannon-Nyquist-Kotelnikov-Whittaker signals sampling, and signals reconstructing rules, where we can correctly address "elementary particles associated quantizing situations", and creation of standing matter-waves, when we rich minimal (atomized) matter domains. This way (based on standing waves attractive and repulsive force effects around nodal and anti-nodal zones) we can conceptualize and explain all natural forces (including Nuclear forces and Gravitation), what is implicating that contemporary explanation of natural, especially nuclear forces is incorrect, arbitrarily formulated, and being only like a declarative labeling without real explanations (just as placing some names on black boxes). Another emerging idea, or still hypothetical assumption, is that all ***natural forces are on some way originating from electromagnetic and structural complexity of matter in our Universe united with global or holistic, resonant vibrations and standing matter-waves.*** *Forces, wave-energy oscillations, and vibrations are not appearing only from electromagnetic and electromechanical sources and devices characterized with temporal periodicities, that are active only in a temporal domain. Different geometric shapes and spatial structures being vibratory-excited (like crystals, fractals, stones, and big masses, as planetary and galactic systems are) also characterized by spatial periodicities. Within such spatial structures it is also possible to create different natural radiation effects, as moments, forces, wave-energy resonant oscillations and vibrations (all of that being related to Uncertainty and Elementary Certainty relations; -see more in Chapters 5., 6., and 10.).*

Probability and Statistics (including Parseval's theorems, or identities between energies of signals in different spectral domains) are exceptionally good and universal mathematical tools for correct (and in-average) "mathematical accounting" regarding satisfaction of all conservation laws, presenting "in-average described trends and distributions" within statistical sets of identical (or remarkably similar) elements, when properly applied (being well applicable in all Natural and other sciences).

3. The bottom-line conclusion and significantly simplified statement about the nature of matter and masses in our universe is that in a space around us, we dominantly find electromagnetic fields, waves, photons and electrically charged states. Everything else what belongs to other particles, gaseous, liquid, solid and plasma states (including all masses, electrons, protons, neutrons, atoms, and molecules) are kind of condensed, specifically formatted, structurally packed and other short-living, electromagnetic energy states (essentially being conveniently packed photons). There are number of indications and facts saying that electrically charged elementary particles or all electric charges are also specific structural formations of photons, anyway, being analog to dynamic entities like mechanical linear and angular moments (see more in Chapters 1., and 3.). Between here mentioned active and dynamic electric charges, we should expect to have certain continuous flow of Radiant electromagnetic energy (as Nikola Tesla speculated; -see more in Chapter 8.). In addition, there are strong indications suggesting that neutron is specific structural and electromagnetic unity of a proton and an electron. Variety of all other subatomic, elementary, and cosmic particles, waves, signals, and energy-moments carrying states, should also (and most probably) be elements or products of electromagnetic states of matter, and products of different electromagnetic and mechanical interactions (see more in Chapters 3., 8., 9., and 10.).

4. *To get out of the box (which is our Universe), we could hypothetically imagine that our Universe is a big system, automate or machine, automatically controlled by certain powerful computer and by certain powerful software. Such software has its complex algorithm, which we could recognize thanks to what we learned about natural Conservation laws, Analogies (as described in the first Chapter of this book), and thanks to methods of sophisticated Mathematics we master. Our Universe, as a computer, should have its clock, or carrier frequency, synchronized with all other processes and operations within our Universe. Its algorithm should be something fixed or finite (already assembled and stable), meaning that with convenient mathematical operations (after investing sufficient efforts) we could develop, extract, axiomatically postulate, or calculate all kind of universal Constants known in Physics, define masses of planets, galaxies, atoms, and elementary particles… Of course, this is still kind of Metaphysics, or a Science-fiction option, but it is worth our attention and further investigations (see more in Chapter 10.).*

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Eric Dollard's older work:

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My Resources:

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- Book Recommendations & References

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- Wireless Giant of the Pacific - 381 pages

  This is a monumental book authored and compiled by Eric Dollard. It discusses his understanding of the Alexanderson antenna system, which operated with longitudinal electrostatic waves instead of transverse electromagnetic waves. That means it is a communication system that has instantaneous propagation with no time delay, thereby defeating Einsteinian physics.

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APPENDIX:

8. BOHR's ATOM MODEL, GRAVITATION AND PARTICLE-WAVE DUALISM
(Draft. Still in preparation)

Citation from: https://en.wikipedia.org/wiki/Atom

"In 1913 the physicist Niels Bohr proposed a model in which the electrons of an atom were assumed to orbit the nucleus but could only do so in a finite set of orbits, and could jump between these orbits only in discrete changes of energy corresponding to absorption or radiation of a photon.[15] This quantization was used to explain why the electrons' orbits are stable (given that normally, charges in acceleration, including circular motion, lose kinetic energy which is emitted as electromagnetic radiation, see *synchrotron radiation*) and why elements absorb and emit electromagnetic radiation in discrete spectra.[16]

Later in the same year, Henry Moseley provided additional experimental evidence in favor of Niels Bohr's theory. These results refined Ernest Rutherford's and Antonius Van den Broek's model, which proposed that the atom contains in its nucleus a number of positive nuclear charges that is equal to its (atomic) number in the periodic table. Until these experiments, the atomic number was not known to be a physical and experimental quantity. That it is equal to the atomic nuclear charge remains the accepted atomic model today.[17]

Chemical bonds between atoms were now explained, by Gilbert Newton Lewis in 1916, as the interactions between their constituent electrons.[18] As the chemical properties of the elements were known to largely repeat themselves according to the periodic law,[19] in 1919 the American chemist Irving Langmuir suggested that this could be explained if the electrons in an atom were connected or clustered in some manner. Groups of electrons were thought to occupy a set of electron shells about the nucleus.[20]..."

The basic structure of this chapter was formulated by the author of this book back in 1975, and many times after that supplemented with comments and new addenda. General synthesis and complete harmonization of this chapter have not been performed yet, but the principal message is still up-to-date and original enough... In fact, this Chapter should be considered as an extension or addition to Chapter 2. mostly elaborating about Gravitation.

Bohr's atom model also has deep roots in Rudjer Boskovic's works about universal natural force, [6]. Bohr's atom model could also be very instructive to understand Gravitation on a way that is intuitive and complementary, compared to what we have from I. Newton and A. Einstein theories. ***In addition, Dynamic Gravity Theory from Nikola Tesla is also familiar to Rudjer Boskovic natural force concepts, and in a close relation to the extended hydrogen atom model that will be presented here (see [97] and [117]).*** *Briefly summarizing, all atoms, masses, and other relatively stable mass-energy states in our Universe are mutually (bi-directionally, omni-directionally, and synchronously) communicating by creating and/or exchanging standing and stationary matter-waves. Such energy-exchange communications are, dominantly of an electromagnetic nature, and this is producing effects of gravitation, as sporadically, hypothetically and imaginatively announced and described by Nikola Tesla in his theory (unfortunately never integrally published, but easily and incompletely available on Internet, consisting on what Tesla said many times about it): http://teslaresearch.jimdo.com/dynamic-theory-of-gravity/). N. Tesla's and R. Boskovic's ideas about gravitation, universal natural force, and radiant energy are implicating that all atoms in our Universe should mutually communicate externally and internally (outwards and inwards), on a way to have closed-circuits flow of involved energy, mass, moments, currents, and voltage participants. Laws related to Newton action-reaction forces, and familiar electromagnetic induction laws, including quantum entanglement effects, and "in-pairs" solutions of classical wave equations, where we see that mutually opposed or in opposite directions propagating waves are always being created (at the same time, and in the same space, synchronously), are also supporting closed-circuits concepts. See more about such concepts and relevant analogies in the first chapter of this book, as well as in chapter 4.1 (example: Fig. 4.1.6, An Illustration of the Closed-Circuit Energy Flow).*

Original Bohr's atom model is modeling energy exchanges between stationary states of electrons by means of emission and absorption of photons. ***Here extended atom model will speculate with an existence of a similar energy exchanges between electrons' states and atom nucleus, as well as within atom nucleus states, and between atoms and external space (towards other atoms and masses), since electrons and protons are mutually attracting electromagnetically, and what happens in one group of them is immediately affecting the other.*** *This way, we will understand atoms as bidirectional or omnidirectional communication centers, being connected and active (internally and externally, or inwards and outwards), and this way presenting dynamically stabilized sets of resonators with standing-waves structured force-field formations between them (existing both on a micro and macrophysics scales). Such complex electromagnetic force-field is also creating stationary, spatially circular resonant states with self-closed standing-matter-waves structures (such as electron stationary states or orbits inside atoms), and kind of (radial or longitudinal) standing matter-waves structures between and among electron states and atom nucleus, extending the same atom-field structure towards infinity (in both directions, or inwards and outwards). On the same way as electron states or clouds have many resonant and quantized orbital and standing-waves states and stable structure, something similar should exist as an electromagnetic mirror-image within every atom nucleus, and naturally, such electrons and nucleus states are mutually communicating, continuously and bi-directionally (or even omni-directionally). Involved spatially structured force (inside and outside atoms) will be on some way like R. Boskovic universal natural force, where the long-distance force-part is acting within an atom's-outer-space (externally towards other atoms and masses) presenting Newton gravitational force. See more about such extended atom model concept later in this chapter under "8.3. Structure of the Field of Subatomic Forces".*

Mentioned bidirectional standing matter-waves structuring and communications among atoms corresponds to ideas of N. Tesla about radiant energy and associated action and reaction forces that are creating effects of gravitation. The same phenomenology, about N. Tesla radiative energy and sources of gravitation, should also be related to the M. Planck and A. Einstein Black Body Radiation (see more here: ***[100]****, M. Sonnleitner 1, 2, M. Ritsch-Marte 2, and H. Ritsch 1, Attractive Optical Forces from Blackbody Radiation, and in [114], [115] and [116]).* ***In addition, here is speculated that background or relict blackbody radiation (initially assumed or hypothesized to be an echo from a cosmic Big Bang), could simply be an ordinary, always, and everywhere present, radiation coming from energy exchanges within and between any atom nucleus (inside the same mass and between different, mutually separated masses). Such radiation is like being a spectral image, or mirror copy of a similar, M. Planck Black Body radiation coming from electrons' states; -see more in Chapter 9. about Blackbody Radiation).***

The hydrogen atom (as the simplest one among other atoms) presents almost perfect mini-laboratory for verifying basic elements and principles of the particle-wave dualism and even gravitation. The importance of Bohr's hydrogen atom model (although it is obsolete, very much artificial, incomplete, and surpassed by other, better, and competitive modelling) is in the fact that it gives results and predictions (related to the spectrum of light sources and light absorbers), which are perfectly accurate (see much more in [92]). From analyzing such results, one may make many conclusions about the particle-wave duality of matter (in a very elementary and analogically comprehensible way, which is not too far from a mechanistic and deterministic cognition of the world, with the possibility of "an intuitive visualization" of the model configuration). The purpose of the following analysis is to reinvestigate or summarize validity and logic of the particle-wave dualism concepts on the simplest level, applying them onto Bohr's hydrogen atom model (serving to identify and prove the **P**article **W**ave **D**uality **C**ode (or **C**oncept) = **PWDC**, as introduced in other chapters of this book; -see Chapters 4.1 and 10.). The following elaboration reveals the roots of the modern particle-wave duality concepts that are found in Bohr's atom model, and still enough care was not dedicated to that fact, neither in the time when Bohr's atom model was created, nor today. By discovering the true sources of particle-wave duality, we support the later development of those ideas. In that sense, the following elaboration sets and explains some dilemmas of the essential understanding of the particle-wave dualism (extracted from N, Bohr's model) that are still sufficiently up to date, although they are bypassed by formalism and concepts of modern Quantum Physics practices. **In the context with the previous chapters of this book, the following analysis of N. Bohr's hydrogen atom model will be limited only to the aspects of the model that check the foundations of the particle-wave dualism, i.e., to check and prove PWDC (as presented in Chapters 4.1 and 10. of this book).** Consequently, here, the original Bohr's model will be partly supplemented to give more of indicative results and conclusions regarding understanding a wave-particle duality.

It is the fact that Physics many times explained the limits of Bohr's atom model in comparison with Quantum Mechanics modeling, where the same problem is treated using probabilistic wave functions and Schrödinger's equation. Here, we will not attempt to challenge this well-known situation (regarding disadvantages and weak sides of Bohr's model comparing it to the contemporary quantum wave mechanics elaborations). The ultimate objective here will be to prove (by analyzing Bohr's hydrogen atom model) that essentials and step-stones of **P**article **W**ave **D**uality are in the following facts or statements (in this book classified as **PWDC**):

1 The matter waves or de Broglie waves are manifestations of all motional mass-energy states. The rest energy (and rest particle mass) does not belong to de Broglie matter-wave, or to a corresponding wave group (or to a wave packet that is associated to a moving particle). Stationary, stable, and periodical, non-forced and natural, orbital, and rotating (continuous and smooth) motions are inertial motions, such as orbital motions within atoms and planetary systems. Here we need to include oscillatory motions, since rotating and orbiting motions are often sources of oscillatory motions or can be mutually transformed. Such orbital motions are creating stationary, standing, stable, self-closed matter waves. Only non-inertial (sudden, accelerated) changes of motional energy (of a certain "mass-energy-moments" entity) directly create unbounded progressive de Broglie, or matter waves. All waves and oscillations known in Physics naturally belong to the same family of matter-waves.

2 De Broglie wavelength $\lambda = \frac{h}{p}$ and Planck-Einstein energy of the wave quantum $\tilde{E} = hf\ (= \tilde{m}c^2)$ are intrinsic and mutually compatible mathematical elements of relations between the group "$\mathbf{v}$" and phase velocity "$\mathbf{u}$" of de Broglie matter waves, $v = u - \lambda \frac{du}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{\tilde{E}}{p} - \frac{h}{p} d(\frac{\tilde{E}}{p})/d(\frac{h}{p})$ (also being mathematically presentable as wave groups, or wave packets). The group velocity is at the same time equal to the velocity of the particle that is represented by the same wave group. The phase velocity (of the wave group) is never higher than its group velocity, and never higher than the speed of light. See (4.0.73) - (4.0.76) from the chapter 4.0.

3 The **P**article **W**ave **D**uality of matter (**PWD**) is the consequence of dynamic and intrinsic coupling between linear and angular motions and their surrounding fields'

manifestations in any of two or multi-body systems. **PWD** is also in a close relation to all mutually coupled "action-reaction" and inertial forces between interacting and moving objects (regardless of the involved forces and fields nature present in two-body or multiple-body systems), while respecting all conservation laws of Physics (especially energy and momentum conservation laws). Here, we can again summarize the most relevant relations as already classified under **PWDC** (see more about PWDC in Chapters 4.1 and 10.):

$$\lambda = \frac{h}{\tilde{p}} = \frac{h}{p},\ \tilde{E} = E_k = hf = \hbar\omega = \tilde{m}vu = pu = h\frac{\omega}{2\pi} = \frac{h}{\tau} = (\gamma - 1)mc^2,\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p,$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E},\quad u = \lambda f = \frac{\tilde{E}}{p} = \frac{\omega}{k},\ v = v_g = u - \lambda\frac{du}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk},\ \tilde{m} = \gamma m,\ p = \gamma mv = \tilde{m}v = \tilde{p}\ .$$

Now, we will try to extract, identify, and prove the above-formulated elements of the **PWDC**, in relation to N. Bohr atom model.

A short summary of Bohr's atom model (for assumed non-relativistic velocities of electrons, $v << c$) is included in two of N. Bohr's postulates and in one hypothesis about the electrons motion in an atom-field.

Let it be (for the planetary atom model):
m – mass of an electron, **e** – charge of an electron, **n** – main quantum number, **h** – Planck's constant, **v** - orbital velocity of an electron, $\omega_m = 2\pi f_m$ - angular revolving frequency related to an electron rotation around its nucleus, **r** – radius of the electron orbit, $p = mv$ – linear, orbital momentum, **Z** – number of protons in an atom nucleus, ε – energy level (or wave energy) of the stationary electron orbit, $\mathbf{f_{12}}$ – frequency of the emitted or absorbed photon, and $\mathbf{f_s}$ – eigenvalue frequency of the stationary electron wave (in the orbit denoted with principal quantum number **n = 1, 2, 3,…**).

Bohr initially conceptualized or assumed that (in a hydrogen atom) an electron (as a particle) is quite mechanically rotating around its nucleus (a proton), meaning being in a natural, not-forced inertial and orbital motion. To impose certain stability to such planetary atom model, Bohr introduced the postulate of allowed, stationary and stable electron atom orbits (by direct analogy with gravitation-related planetary or solar systems). To be stable, stationary electron orbits must satisfy the condition of standing waves, or quantization of the involved orbital momentum (as planets rotating around a sun do), meaning that such mechanical rotation of an electron is respecting:

$$pr = mvr = n\frac{h}{2\pi},\quad n = 1, 2, 3, ... \qquad (8.1)$$

as well as respecting all other conservation laws of physics.

The second Bohr postulate is related to the conditions when an electron may emit (or absorb) the quantum of electromagnetic emission (a photon), during the change of the energy level of the electron between its two stationary orbits:

$$hf_{12} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = h\Delta f, \quad \varepsilon_2 > \varepsilon_1 \ . \tag{8.2}$$

Of course, something similar (or equivalent) is valid when a photon is either leaving or entering the atom space captured by electron orbits (meaning that certain electron will reduce or increase energy level of its stationary orbit). Such energy transitions and exchanges can be considered (or better to say approximated) as discrete, sudden, and fast changes.

Finally, Bohr assumes that electron orbits around the nucleus of the (hydrogen) atom are circular, and he sets the condition of the dynamic balance of the attractive and repulsive electrostatic forces between the electron (that revolves) and the static atom nucleus (very much analog to gravitational planetary systems situation):

$$\frac{1}{4\pi\varepsilon_0}\frac{e(eZ)}{r^2} = \frac{mv^2}{r} \, . \tag{8.3}$$

From the previous postulates and conditions (8.1), (8.2), and (8.3), one may get the elements that characterize the circular motion of the electrons around the atom nucleus, such as:

-a radius of an electron orbit,

$$r = r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2 Z} = \frac{v_n}{\omega_m}, \ \left(\frac{1}{n^2} = \frac{h^2 \varepsilon_0}{\pi m e^2 Z} \cdot \frac{1}{r_n} \right), n = 1,2,3,... \tag{8.4}$$

-orbital velocity of the electron,

$$v = v_n = \omega_m r_n = 2\pi f_m r_n = \frac{nh}{2\pi m r} = \frac{Ze^2}{2nh\varepsilon_0}, n = 1,2,3,..., \tag{8.5}$$

-the kinetic energy of the electron,

$$E_k = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r_n}, n = 1,2,3,..., \tag{8.6}$$

-and the potential (electrostatic) energy of the electron,

$$U = \int_{\infty}^{r} F dr = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = -2E_k = -mv^2 \ \ ,(F_\infty = 0) \, . \tag{8.7}$$

The total orbital energy of the electron (apart from its rest energy) in the interatomic field of a hydrogen atom is equal to the sum of its kinetic and potential energy:

$$\varepsilon_B = E_k + U = -E_k = -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{mZ^2e^4}{8n^2h^2\varepsilon_0{}^2} = -\frac{1}{2}mv^2 \text{ , } n = 1,2,3,... \text{ .} \quad (8.8)$$

Applying the second Bohr's postulate (8.2) onto the (8.8), one may find the energy of the electromagnetic emission of the atom, at the transition of its electron between two (internal) stationary energy levels $\varepsilon_2 > \varepsilon_1 , n_2 > n_1$,

$$hf_{12} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon_B = \frac{mZ^2e^4}{8h^2\varepsilon_0{}^2}(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}) = \frac{Ze^2}{8\pi\varepsilon_0}(\frac{1}{r_{n1}} - \frac{1}{r_{n2}}) = \Delta E_k \text{ .} \quad (8.9)$$

The indexing with "B" in the previous (and following) expressions is introduced to emphatically indicate that certain values directly and exclusively emanate from the originally established Bohr's atom model (because later, there will be other indexed symbols), due to the later comparison of Bohr's results with other available results.

An electron and an atom nucleus also revolve (as a two-body system) about their common center of mass. Therefore, the mass of the nucleus **M** should enter the equations. Sommerfeld has shown that equation (8.9) still stands if **m** is replaced by the so-called reduced mass μ (contributing with fine correction of (8.9); -see [9] and [92]),

$$\left\{\begin{array}{l} m \to m_r = \mu = \frac{mM}{m+M} \cong m \text{ , } \frac{M}{m} = 1836.13 \text{ , } \\ (m = m_e = \text{electron mass, } M = m_p = \text{nucleus mass}) \end{array}\right\} \Rightarrow \quad (8.9\text{-}1)$$

$$\Rightarrow hf_{12} = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon_B = \frac{\mu Z^2e^4}{8h^2\varepsilon_0{}^2}(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}) \text{ .}$$

Since the nucleus mass (or proton) is much greater than the electron mass, if we analyze the same system (nucleus-electron) in the Laboratory and in the center-of-mass system, **we can easily realize that the electron's stationary (or orbital) wave carries practically only the motional energy of the revolving electron**. Consequently, we can easily and with high certainty find that this stationary wave energy (in the case of non-relativistic electron velocities: see (8.1) - (8.9-1)) will be expressed as,

$$\begin{array}{l} \varepsilon = \varepsilon_s = hf_s = \frac{1}{2}\mu v^2 = \frac{1}{2}\mathbf{J}_e\omega_m^2 = \frac{1}{2}\mathbf{L}_e\omega_m = \frac{\mu Z^2e^4}{8h^2\varepsilon_0{}^2}\cdot\frac{1}{n^2} \cong \frac{1}{2}mv^2 \text{ , } \mathbf{J}_e = \mu r^2 = \mathbf{J}_m \text{ ,} \\ \Leftrightarrow v = \frac{Ze^2}{2h\varepsilon_0}\cdot\frac{1}{n} \text{ , } \mathbf{L}_e = \mathbf{J}_e\omega_m = \mathbf{L}_m = \mu r^2\omega_m = \frac{1}{\omega_m}\cdot\frac{\mu Z^2e^4}{4h^2\varepsilon_0{}^2}\cdot\frac{1}{n^2} = \frac{r}{v}\cdot\frac{\mu Z^2e^4}{4h^2\varepsilon_0{}^2}\cdot\frac{1}{n^2} = n\frac{h}{2\pi}. \end{array} \quad (8.9\text{-}2)$$

In (8.9-2) we are applying indexing "$_m$" to underline that here we are talking about almost an ordinary ***mechanical*** rotation (in the frames of the initial Bohr's atom model). Since an electron is assumed rotating, there is another index "$_e$" addressing electron (because later we will upgrade such atom model and introduce different indexing for another level of spinning, or rotation which will be differently associated to the same electron). Then, it will be shown and proven that (8.9-1) and (8.9-2) are fully correct (presenting a small and fine, but conceptually important correction of the original Bohr spectral formula (8.9)), and that it contains very essential information regarding the **PWDC** (see also (8.25) - (8.30)).

Originally, Bohr introduced the idea about particle-like inertial revolving of an electron mass and electron electric charge around the atom center (or nucleus), and until a certain level, such concept started producing verifiable and especially useful results in light spectrum measurements (usually measured in gaseous states). The experimentally found spectral lines of the emitting and absorption hydrogen atom spectrum (being also well-applicable to many other, heavier, and more complex atoms) are in the perfect agreement with theoretical predictions described by (8.9) and (8.9-1), confirming the validity of Bohr's atom model.

It is the fact that Bohr's model reveals little about the structure and nature of the atom and about the essence of the accompanying PWD phenomenology because it is based on two (not self-standing) postulates and one hypothesis. It is also the fact that the result (8.9) enabled excellent theoretical predictions of the experimentally obtained (light radiation or absorption) spectral distributions. Consequently, one may conclude that by some more comprehensive analysis of N. Bohr's model, we may probably recognize there some more fundamental physics-related essence still not exposed until present. This would additionally explain Bohr's postulates and shed some new light on the nature of the matter and the essence of the particle-wave dualism (in this paper, we call that still unexposed physical essence PWDC (=) Particle Wave Duality Code or Concept). Here mentioned, more fundamental physics-related picture, could be that all natural, not forced, orbital, periodic and inertial motions are hosting standing matter-waves formations.

Primarily, retrospectively looking, N. Bohr's atom model does not take explicitly into account possibilities of the wave interpretation of the nature of an electron. Nevertheless, the postulate (8.2) implies emission or absorption of the electromagnetic wave quantum, exploiting transitions of involved electrons between stationary orbits, which implicitly and indicatively qualifies that some accompanying, standing-wave properties, also relate to the orbital motion of electrons, even before the energy change occurs (8.9).

The moment of inertia of a rotating electron (initially considered a discrete particle) in the framework of Bohr's atom model is $\mathrm{J} = \mathrm{mr}^2$***. It will stay the same (again as*** $\mathrm{J} = \mathrm{mr}^2$***) if we consider that the same electron-mass rotates while being distributed around a thin-walled equivalent, hollowed torus (which covers the same space of electron orbit or path and has the same total mass m). Considering this, we have a certain freedom to make the next step in conceptualizing an electron as a helical matter-wave on a certain toroidal coil-form (see the second chapter of this book: "2.3.2. Rotation and stable rest-mass creation", T.2.4, T.2.5, T.2.6 and equations (2.11.2) and (2.11.3). See also chapter 4.1, Fig.4.1.2 – Fig.4.1.4 and T.4.3). Here we approach somewhat similar, significant, and original ideas introduced by M. Kanarev in [44], and C. Lucas and David L. Bergman in [16] - [22]. In fact, C. Lucas, Bergman and Kanarev's electron (and an atom) modeling looks more***

realistic and advanced, compared to the original Bohr's atom model, and to atom models and concepts of contemporary Quantum Theory. Anyway, all of them are producing big part of identical or similar (indicative and challenging), very correct results regarding what can be verified experimentally with spectroscopic measurements, electrons energy levels, relevant frequencies, orbital, and magnetic moments etc. (See on Internet somewhat familiar electron magnetic field concept about Henry Augustus Rowland effect around rotating conductor, as presented by Jean de Climont. An excellent modeling of the helical electron (very much compatible with in this book conceptualized helical matter waves) can be found in [108]; -Oliver Consa. "g-factor and Helical Solenoidal Electron Model". Since historically N. Bohr's model was the first one of sufficiently successful atom models, we will try to draw maximum of PWDC-relevant conclusions from it. Later, an attempt will be made to find other of unifying, updating, and migrating areas to merge the most successful and best aspects of Bohr, Kanarev, Bergman and Lucas's atoms and electron structure concepts (see [120]). In fact, if we make an intellectual digression towards the Ptolemaic geocentric planetary system, we will find that such erroneous planetary model or concept was producing significantly good mathematical results (also verifiable by astronomic observations) regarding describing mutually synchronized and periodic planetary motions. This was possible thanks to many of introduced assumptions about an additional gearbox-like associated rotating, and natural periodical motions of planets and moons from our planetary system. This also made possible to compensate original conceptual errors with mathematical association of artificially invented and added, non-existent, periodic, circular, and (mutually synchronized) rotating motions of every planet and Sun (around planet Earth), eventually predicting sufficiently correct planetary and satellite motions (but, of course, on a complicated and artificial way, by compensating initial model mistakes with new, results self-correcting mistakes). Something like that could also be applicable or comparable until certain level to a situation when we analyze differences between the original Bohr's model and Bergman-Lucas-Kanarev-Climont-Consa... models. Thanks to mutually coupled, involved intrinsic and structural periodicities, and mutually synchronized motions, circular and standing matter-waves structured formations of atom constituents, we can formally explain an atom structure on conceptually different ways, one of them with the additional assumption that the electron and other atom constituents are composed of electromagnetic photons trapped in self-closed, synchronized, and stabilized vortex formations.

Nevertheless, most of mentioned models are mathematically producing the same or very similar, and experimentally verifiable spectral results, meaning that differences between directly mutually related and linearly dependent energy levels of electrons and photons transitions are always the same; -see more in Chapter 10.). Something similar should also be valid for Quantum theory hydrogen atom model based on solving Schrödinger equation, thanks to intrinsic, mutually related, temporal, and spatial periodicities, and existence of structural standing electron waves within real atoms (meaning that Quantum theory atom model could also be well operating, but still being somewhat kind of Ptolemaic construction). Ironically, the original N. Bohr's atom model conceptually has just something or little to do with a real atom

modeling, but analogically it has significantly more applicability to innovative modeling of solar or planetary systems, as presented in the second chapter of this book (see: "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"). As a generally valid conclusion, we could say that certain inertial state of matter (or system of atoms, particles etc.), that has number of (mutually well-coupled) intrinsic and structural periodicities, with circular, spinning and repetitive, mutually synchronized periodical motions (and resonant or standing waves formations), could be relatively well described, or approximated with number of mathematical and maybe artificial, Ptolemaic models and concepts (that are mutually transformable by respecting certain deterministic mapping or mutual functional transformations and imaging). Anyway, only one of them will be conceptually and topologically or dynamically, naturally correct, and realistic, physics-relevant model (but mathematically, all such models could operate sufficiently well). Between mentioned "Ptolemaic models" and relevant energy-momentum states (of the same matter-wave structure, like atoms are), thanks to stable and mutually coupled, synchronized periodicity relations between them, we will be able to make mutual (and deterministic) functional mapping, imaging, and mathematical, spatial transformations (being similar as it was mathematically constructed and proven by optimized Ptolemy-models conceptualization, mathematically falsely showing that the planet Earth could be the center of our Universe on certain modeling way). This way thinking, we could speculate that planetary and orbital N. Bohr atom model is not the best presentation of the atom's real spatial structure, but it is still useful. Anyway, it works sufficiently well, since it is well and causally (with strong periodicity relations and mapping, imaging, and functional transformations) connected to other, much better atom models, which are naturally belonging to, or respecting the same periodicity situations. This way, we can still make number of relevant conclusions within the framework and mathematics of an erroneous N. Bohr's atom model and create new conclusions that would also be relevant within certain, much better, atom model (since mentioned, periodicity-related mapping, imaging, and functional transformations will correctly and causally transform one framework or set of results into another). This should be the case of other, competitive, and better atom models, introduced by M. Kanarev in [44], by Charles Lucas, Bergman, and coworkers in [16] - [22], and Oliver Consa in [120].

Quantizing in Physics is mathematically related to different numerical quantifications and counting (using integers), and to simple arithmetic relations between different (mutually coupled) energy-moments, stationary and resonant states of matter in mentioned systems with intrinsic periodicities and standing or resonating waves formations.

Anyway, we now know that humans can make big conceptual mistakes, which are sometimes, with implemented (or imposed) convenient mathematical modeling and assumptions, working sufficiently correctly (thanks to embedded and intrinsic, mutually linked, natural periodicities). We are keeping alive some of mentioned Ptolemy like theories or relics, as originally and officially recognized during a long time, as it was the case with the geocentric planetary system (and obviously, we are

still not ready to recognize that this could be the cases of certain contemporary theories within our modern Physics).

Let us start from an intuitive, simplified geometric concept of the interpretation of **stationary** or stable and inertial-motion electron orbits, (which we directly connect with the assumed existence of stable orbital electron waves, limiting ourselves, for now, only on circular orbits and stable, inertial motions). We can say that the stationary electron orbit is the orbit to which the stationary (standing) electron wave belongs, such that the mean perimeter of that orbit is equal to the integer number of the wavelengths of the stationary electron wave, i.e.,

$$2\pi r = n\lambda_s , \ \ n = 1,2,3,\dots . \qquad (8.10)$$

Indexing with "$_s$" should remind us that we are talking about stationary electron orbits and stationary energy states.

Comparing the condition of "*stationarity*" (8.10) and Bohr's postulate (8.1), one may notice a direct and indisputable analogy from which it takes only one step to the posing of the de Broglie's wave hypothesis (which, surprisingly, was formulated almost 10 years later),

$$2\pi r = n\frac{h}{mv} = n\frac{h}{p} = n\lambda_s , \qquad (8.11)$$

i.e., the wavelength of an orbital electron wave equals:

$$\lambda_s = \frac{h}{p} \ . \qquad (8.12)$$

Conditions of such particle-wave duality and "*stationarity*", (8.10) – (8.12), are later generalized by Sommerfeld (1915, about 10 years before Luis de Broglie eventually realized that here we should have electron or matter waves), extending such standing-waves concept to any orbit-shape and any degree of freedom of its internal (spatial, linear and orbital) moments "p_k" and coordinates "q_k" -related energy states, as for instance,

$$\oint p_k dq_k = n_k h, \ \ n_k \in [1, 2, 3, ...] . \qquad (8.11\text{-}1)$$

All here-mentioned conceptual upgrading of N. Bohr atom model contributed (later) to smaller results improvements related to spectral formula (8.9-1), showing new evolutional directions for advanced atoms modeling, and additionally supporting here-underlined statements about **PWDC**. Of course, ideas about standing electron (or electromagnetic) waves, (8.10) – (8.12) and (8.11-1) are also conceptually supporting and explaining Bohr's postulates, establishing spatially and temporally synchronized and stabilized atom structure.

Quantizing of electron momentum and quantum nature of atomic structure (in many other aspects) is kind of conceptual misinterpretation, because the essential for describing stability of atom structure are different spatial and structural formatting involving resonant states with standing, self-closed, inertial motions of matter waves, where integer numbers of half-wavelengths are something natural, and only possible structuring and counting situation. Bohr-Sommerfeld moments quantizing is another equivalent way to describe spatial standing-waves formations. Anyway, since the time of N. Bohr's atom model foundations, Quantum Theory is overwhelmingly elaborating different aspects of quantum nature of matter and atoms, what is a consequence of different standing waves packing, where integer numbers are what counts energy and moments formations (see more in Chapter 10.). Real quantizing in Physics is much more related to Kotelnikov-Shannon-Nyquist-Whitaker signals (or waveform) synthesis and decomposition.

It seems clear that electron, on its stationary orbit (whatever that means), is a kind of mixed or dual entity (having mutually equivalent and commuting properties of a particle and a wave). To literal identification of such dual object, in this book, we will use the following wave expressions and symbols for the wavelength and wave energy of (de Broglie) matter waves: $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$, $(E_k \to \tilde{E},\ p \to \tilde{p})$. In order to support the use of wave symbols, $\tilde{E}$, $\tilde{p}$, we can remember that energy $E_f = \tilde{E}_f$, mass $m_f = \tilde{m}_f$, and wavelength λ, of an electromagnetic energy quantum or photon, can be given by analogical relations (see T.4.0. Photon – Particle Analogies from Chapter 4.1),

$$\tilde{E}_f = hf = (m_f - m_{f0})c^2 = m_f c^2 = \tilde{m}_f c^2 = E_{kf},\ \tilde{p}_f = \frac{hf}{c} = p_f,\ \ \tilde{m}_f = m_f = \frac{hf}{c^2},\ \lambda_f = \frac{h}{\tilde{p}_f},$$

moreover, all of them are effectively (and particle-wave analogically) proved valid and applicable in analyses of Photoelectric, Compton and other familiar effects (without explicit usage of here proposed wave symbolic).

With previous results, (8.11) - (8.12), and (8.9-2), it is implicated that Bohr's atom model and de Broglie's matter-wave hypothesis have become mutually complementary with mathematical elements that simultaneously support and prove the correctness of both concepts.

For now, for the assumed electron wave we may say that we know only its wavelength. Of course, we know that the difference between the eigenenergies of two stationary electron waves, (8.9) very precisely predicts emissive or absorptive atom spectral lines, but we still do not know the absolute amounts of the energies of stationary electron waves ($\varepsilon = \varepsilon_s = hf_s$), which take a part in that difference. Yet, formally, we may write Bohr's postulate (8.2) in a form that takes into consideration (for now unknown) orbital stationary frequencies of that wave, so it will be,

$$hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B\ . \qquad (8.13)$$

In addition, we will try to determine (in absolute values) all elements of the stationary electron wave, such as its phase and group velocity, frequency, wavelength, and energy. Of course, the term stationary electron wave is quite justified with the fact that atom can absorb or emit an electromagnetic wave (as a light quantum, or photon), i.e., it is likely that appropriate interferences and superposition of the two familiar wave phenomena will happen. Thereupon a possibility intuitively obtruded is that an electron in orbital revolving around the atom nucleus possesses, besides particle attributes, and some wave attributes (like a photon). Naturally, an electron is no longer a particle localized in some point, but some (helically shaped) standing wave structure of distributed “electromagnetic mass” and distributed charge of an electron across the perimeter of the corresponding stationary orbit (which we will here call de Broglie's matter wave). *See on Internet somewhat similar electron magnetic field concept about Henry Augustus Rowland effect around rotating conductor, presented by Jean de Climont.*

Until here we did not care about relativistic mass-energy velocity dependencies since the first objective was to recognize and prove the most important relations belonging to the **P**article **W**ave **D**uality **C**ode (= **PWDC**), $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$. There are some other elements of **PWDC** that will be mentioned later (More about **PWDC** foundations can be found in Chapter 4.1 and Chapter 10.).

To identify the orbital eigenfrequency f_s of the electron wave (before any energy jumps, emissions, or photons absorptions), it is necessary to assume the nature of that wave, i.e., what energy content the electron stationary wave exists on.

For example, we may express the total energy of an electron in orbital motion as,

$$\varepsilon_t = mc^2 , \quad (m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 , \ m_0 = \text{const.}), \tag{8.14}$$

while its total motion energy is,

$$\varepsilon_k = (m - m_0)c^2 = (\Delta m)c^2 = \frac{mv^2}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{pv}{1 + \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\mathbf{L}_m \omega_m}{1 + \sqrt{1 - \frac{v^2}{c^2}}}, \tag{8.15}$$

$$p = mv = \gamma m_0 v, \mathbf{L}_m = \mathbf{J}_m \omega_m = mr^2 \omega_m = 2\pi m r^2 f_m = mvr, pv = \mathbf{L}_m \omega_m .$$

We may notice at once that energy ε_B from (8.8), which we consider as the motional energy of the electron in the electric field of the atom, must be included in some amount in (8.14) and/or (8.15). In addition, we will notice that we may now represent Bohr’s postulate (8.2) by the difference of two energy levels (which are in direct and mutually translated dependence, i.e., which mutually differ in some constant) in several ways, using energy expressions (8.8), (8.13), (8.14), or (8.15):

$$hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B = \Delta\varepsilon_t = \Delta\varepsilon_k\,. \qquad (8.16)$$

In expressions (8.14)-(8.16) and later on, electron's non-relativistic kinetic or total energy E_k, E, is simply transformed into its relativistic motional or total energy $\varepsilon_k, \varepsilon$, implicitly indicating that relativistic motional and/or total energy could (or should) have more of complex electromagnetic nature, containing particle and wave manifestations of electron's energy, for instance.

If we are not sure which one of the previous energies ($\varepsilon, \varepsilon_B, \varepsilon_t, \varepsilon_k$) from (8.16) is in an absolute amount equal to the energy of the stationary electron wave $\varepsilon = \varepsilon_s = hf_s$, then we could not determine what the frequency of the stationary electron wave is. The thing we know at the start (or may determine) is the set of all possible assumptions (where some of them figure only temporarily, fictively, and dimensionally as frequencies) among which at least one will represent the required, realistic stationary, eigenfrequency f_s. Later, by elimination, we will find the real eigenfrequency (in its absolute amount) that satisfies the selection criterion (particularly by satisfying the criterion of group and phase velocity connection, and other elements of **PWDC**).

Now (in process of searching for real, orbital eigenfrequency f_s) we may quote the following "candidate" frequencies (connected to the energy states of electrons on stationary orbits):

- the frequency of the mechanical rotation of the electron as a particle, around the atom nucleus, which is found in (8.4), (8.5) and (8.15),

$$f_m = \frac{\omega_m}{2\pi} = f_B^{'} = \frac{v}{2\pi r} = \frac{mZ^2e^4}{4n^3h^3\varepsilon_0^{\,2}} = \frac{2\varepsilon_B}{nh} = \frac{2}{n} f_B \;, \qquad (8.17)$$

- the frequency that is dimensionally found in (8.8),

$$f_B = f_B^{''} = \frac{\varepsilon_B}{h} = \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^{\,2}} = \frac{n}{2} f_B^{'} = \frac{n}{2} f_m \,, \qquad (8.18)$$

- the frequency that is obtained based on the total energy of the electron (8.14),

$$f_t = \frac{\varepsilon_t}{h} = \frac{mc^2}{h}\,, \qquad (8.19)$$

- and the frequency that is obtained from the motional energy (8.15),

$$f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h}\,. \qquad (8.20)$$

Considering all previous frequencies from (8.17) to (8.20), the second Bohr's postulate (8.16) formally becomes:

$$\begin{aligned} & hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_2 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_B = \Delta\varepsilon_t = \Delta\varepsilon_k = \\ & = \frac{1}{2} h(n_2 f_{B2}{}' - n_1 f_{B1}{}') = h(f_{B2} - f_{B1}) = h\Delta f_B = \\ & = h(f_{t2} - f_{t1}) = h\Delta f_t = \\ & = h(f_{k2} - f_{k1}) = h\Delta f_k \ . \end{aligned} \tag{8.21}$$

As one may see, in (8.21) several different frequencies (that are translated, mutually and linearly) figure here, which leads to the equality of their differences.

Now one may determine the phase velocity of a stationary electron wave, as a product of its corresponding wavelength, (8.12), and some of the frequencies from (8.17) to (8.20). Thus, it is possible to determine several phase velocities, from which only one (real and exact) will be required to show which electron wave (orbital) frequency (from (8.21)) is the most relevant:

$$\begin{aligned} & u_m = \lambda_s f_m = \lambda_s f_B{}' = u_B{}' = \frac{Ze^2}{2n^2 h\varepsilon_0} \ , \\ & u_B = \lambda_s f_B = \lambda_s f_B{}'' = u_B{}'' = \frac{n}{2} u_B{}' = \frac{n}{2} u_m = \frac{Ze^2}{4n^2 h\varepsilon_0} \ , \\ & u_t = \lambda_s f_t = \frac{c^2}{v} \\ & u_k = \lambda_s f_k = c\sqrt{\frac{m - m_0}{m + m_0}} . \end{aligned} \tag{8.22}$$

Phase velocities from (8.22) are mutually very much different, but the physical essence of the electron orbital wave should produce (or defend) only one of them. To establish the needed selection criterion, let us start from fact that the orbital velocity of an electron (8.5) is equal to the group velocity of the stationary electron wave associated with it. Then, we will search (among possibilities from (8.22)) for the phase velocity of a stationary electron wave as the phase velocity that satisfies the general equation of the group and phase velocity relation of (any) wave group,

$$v = u - \lambda \frac{du}{d\lambda} = -\lambda^2 \frac{df}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{\tilde{E}}{p} - \frac{h}{p} d(\frac{\tilde{E}}{p})/d(\frac{h}{p}) . \tag{8.23}$$

The equation (8.23) is obtained from definitional expressions for group and phase velocity of the wave group, which are generally valid for any form of harmonic wave motions (See more in (4.0.73) - (4.0.76) from the Chapter 4.0),

$$v = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}} \ , \ u = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}} = \lambda f \ , \ \omega = 2\pi f \ , \ k = \frac{2\pi}{\omega} . \tag{8.24}$$

Therefore, it is obvious that equation (8.23) may become the key arbiter in the explanation of the energy essence of the electron wave (i.e., in finding its real phase velocity and orbital frequency).

In fact, the relations (8.23) and (8.24) also belong to **PWDC** (**P**article **W**ave **D**uality **C**ode), including $\tilde{E} = hf$, $\lambda = \frac{h}{\tilde{p}}$, too.

One may check and prove that equations and relations (8.23) - (8.24) describe a real stationary electron wave that has:

-phase velocity (see (8.22)),

$$u_s = u_k = \lambda_s f_k = \lambda_s f_s = c\sqrt{\frac{m-m_0}{m+m_0}} = c\sqrt{\frac{\gamma-1}{\gamma+1}} = \frac{v}{1+\sqrt{1-(\frac{v}{c})^2}},\ m = \gamma m_0, v = v_s,$$

-orbital eigenfrequency,

$$f_s = f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} = n\frac{f_m}{2}(1+\frac{u^2}{c^2}) = n\frac{mZ^2e^4}{8n^3h^3\varepsilon_0{}^2}(1+\frac{u^2}{c^2}),$$

-energy,

$$\varepsilon_s = hf_s = \varepsilon_k = (\Delta m)c^2 = muv, \qquad (8.25)$$

-and wavelength,

$$\lambda_s = \frac{h}{p} = \frac{h}{c\sqrt{m^2 - m_0{}^2}} = \frac{h}{m_0 c\sqrt{\gamma^2 - 1}}.$$

Due to obtaining greater evidence in checking previous solutions, considering the cases when the orbital velocity of an electron is relatively small, $v << c$, previous expressions from (8.25), after adequate approximations, are reduced to the results known from the original Bohr's atom model. In addition, one also obtains some new values (that Bohr's model does not generate) and which are typical for the wave concept of electron motion. Therefore, it is:

- the phase velocity of the electron wave,

$$u_s = u_k = \lambda_s f_k = \lambda_s f_s = \frac{\omega}{k} = c\sqrt{\frac{m-m_0}{m+m_0}} = \left.\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}\right|_{v<<c} \approx \qquad (8.26)$$

$$\approx \frac{1}{2}v = \frac{1}{2}v_s = u_B = \frac{n}{2}u_m = \frac{Ze^2}{4nh\varepsilon_0},$$

-the group velocity of the electron wave (see (4.0.17), (4.0.26), and (4.0.28)),

$$v = \frac{d\omega}{dk} = \left.\frac{2u_s}{1+\frac{u_s^2}{c^2}} = v_s\right|_{v<<c} \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0} \qquad (8.27)$$

-the frequency of the orbital electron wave (see (4.0.31) and (4.0.39)),

$$\begin{aligned} f_s = f_k = \frac{\varepsilon_k}{h} = \frac{(\Delta m)c^2}{h} = \left. n\frac{f_m}{2}(1+\frac{u^2}{c^2})\right|_{(v,u)<<c} \approx \\ \approx \frac{mv^2}{2h} = f_B = f_B'' = \frac{\varepsilon_B}{h} = \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^2} = \frac{n}{2}f_m , \end{aligned} \qquad (8.28)$$

-the wavelength of the orbital electron wave (see (4.7), (4.8), and (8.12)),

$$\lambda_s = \frac{h}{p} = \left.\frac{h}{c\sqrt{m^2 - m_0^2}} = \frac{2\pi r}{n}\right|_{v<<c} \approx \frac{2nh^2\varepsilon_0}{mZe^2} , \qquad (8.29)$$

-and energy of the stationary electron wave (see T4.1 and (4.0.8)),

$$\begin{aligned} \varepsilon_s = hf_s = \varepsilon_k = (\Delta m)c^2 = muv = \left.\frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} = nh\frac{f_m}{2}(1+\frac{u^2}{c^2})\right|_{(u,v)<<c} \approx \\ \approx \frac{1}{2}mv^2 = \varepsilon_B = \frac{mZ^2e^4}{8n^2h^2\varepsilon_0^2} . \end{aligned} \qquad (8.30)$$

The conclusion that obtrudes from the previous analysis is that sources of the wave nature of matter (here of a uniformly orbiting electron in natural, inertial, and not-forced motion) are included only in a motional energy, and that a rest mass of an electron does not enter the content of the accompanying wave energy. This is against the official interpretation found in modern quantum theory, which considers a total energy and rests mass of a particle as elements of the wave-equivalent of the particle. Almost all possible misapprehensions and errors connected with questions about particle-wave dualism (in the atom world) originate from the differences of the corresponding relative energy levels (8.21) equal to the differences of their absolute energy levels. One could not claim in advance, which levels are absolute, and which relative (because all of them are “mutually linearly-translated or shifted” for some constant energy level). As we have already seen, in a close relationship with the mentioned situation there was also the relation between the group and phase velocity ((8.23) and (8.24)), because this was the decisive selection criteria to identify electron matter waves (among several options).

In fact, the results (8.25) - (8.30) are completely in agreement with (8.9-2), mutually confirming, complementing, and supporting each other, and fully describing the PWDC

(**P**article **W**ave **D**uality **C**ode). The same expressions also explain the essential relations and intrinsic connections between mechanical rotation and frequency of de Broglie matter waves (when electron mass is treated as the reduced mass, as in (8.9-1)).

***[♣ COMMENTS & FREE-THINKING CORNER**: “Gravitostatic versus electrostatic analogy”*

The same results and conclusions (as in (8.26) - (8.30) and earlier in (2.11.14)) can be formulated almost directly, analogically, and much faster, if we consider that in any solar system, the Sun analogically presents a nucleus or proton, and planets are like electrons orbiting around. This is analogically applicable, when the solar system can be approximately treated as a “2-body problem”, and if we exploit the mathematical identity between the electrostatic Coulomb force in the hydrogen atom, and Newton’s static gravitational force, and if we systematically apply the following substitutions:

$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M,\ e \Leftrightarrow m,\ \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \rightarrow GmM\right\}.$$

Doing this way, we can (analogically) reproduce all relevant results known from hydrogen atom analyzes, and link them to planetary motions in solar systems, where (analogically) M is the mass of the sun, m is mass of a certain planet, H is a macrocosmic planetary constant analog to Planck constant h, and G is gravitational constant (see also in Chapter 2., (2.4-4.1) - (2.4-4.3) as supporting arguments to mas-electric charge proportionality).

Much more of such background we can find in [63] Arbab I. Arbab, [67], and in other familiar publications from Johan Hansson. "Gravito-static versus electrostatic analogy" should not be only a mathematical curiosity, coincidence, and an “academic-discussion option”, after we consider as realistic the possibility that solar system elements (meaning planets and sun) are mutually electrically and magnetically polarized like mutually attracting electric and magnetic dipoles and multi-poles (since there are electromagnetic fields and forces around them). Such electromagnetic polarization assumption is already presented in the second chapter of this book (see 2.2. Generalized Coulomb-Newton Force Laws; -equations from (2.3) until (2.4-10)).

Of course, here analogy means more than mathematical similarity or equality, since in case of gravitation within planetary (or solar) systems relevant results are also verifiable by astronomic measurements and other theoretical and experimental analyzes. Consequently, in astronomy and gravitation-related phenomenology we are dealing with verifiable scientific facts, and the consequences are that gravitation and associated electromagnetic complexity are coincidently present and mutually coupled, at least in cases of solar or planetary systems. ***The intentions here are to find tangible and reasonable grounds showing that sources of gravitation are not masses, but atoms and associated forces, fields and different mechanical and electromagnetic moments emanating from an internal atom structure. This way conceptualizing, all atoms and masses in our Universe are mutually connected, synchronized, and communicating (by exchanging matter waves, electromagnetic waves, and photons). This is also the reason or explanation how macrocosmic systems like solar systems could analogically behave as atoms*** *(see much more about mentioned analogies in Chapter 2., under (2.11.14), T.2.3.3, T.2.3.3-a, T.2.3.3-1 and later). This cannot be only an arbitrary and random-event, unimportant and meaningless coincidence.*

As we know, N. Bohr’s Planetary Atom Model is upgraded, and additionally modeled by applying Schrödinger’s equation and knowledge about particle-wave duality theory (here elaborated as PWDC), meaning that we should be able, because of applicability of mentioned “Gravito-static versus electrostatic analogy” (based on $\boxed{\frac{Ze^2}{4\pi\varepsilon_0} \rightarrow GmM}$*), to analogically apply Schrödinger equation and relevant wave functions to similar planetary and other astronomic situations. Such analogy is not at all establishing unique, non-doubtful, definitive grounds to say that probabilistic methodology of quantum theory is relevant here (opposite to what certain authors are implicitly forging as the undisputable fact). Schrödinger equation is simply applicable here mostly because stable, closed orbits (of planets or electrons), hosting periodical, and* *standing waves, are creating necessary conditions for formulating such equation (nothing to connect with probabilities).*

From elaborations in Chapter 2. (see analyzes around equations from (2.4-4) until (2.4-4.3)) we already have strong indications about how electric charges and mass of the same object are directly proportional. Let us now apply the following, analogical substitutions to quantizing expressions relevant for non-relativistic orbiting electrons, $v << c$, as in (8.26) - (8.30), to create analogical quantizing expressions for orbiting planets (see (2.11.14) in the second Chapter), as follows,

$$\left\{\frac{Ze^2}{4\pi\varepsilon_0 r^2} \Leftrightarrow \frac{GmM}{r^2},\ Ze \Leftrightarrow M,\ e \Leftrightarrow m, \frac{1}{4\pi\varepsilon_0} \Leftrightarrow G, h \Leftrightarrow H\right\} \Rightarrow \left\{\frac{Ze^2}{4\pi\varepsilon_0} \to GmM\right\} \Rightarrow$$

phase velocity of an electron wave	***phase velocity of a planetary wave***
$u_s \approx \frac{1}{2}v = \frac{Ze^2}{4nh\varepsilon_0}$ ***(8.26)***	$u_n \approx \frac{1}{2}v = \frac{\pi GmM}{nH}$ ***(2.11.14)***

group velocity of the electron wave	***group velocity of the planetary wave***
$v_s \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0},$ ***(8.27)***	$v_n \approx 2u_n = \frac{2\pi GmM}{nH}$ ***(2.11.14)***

frequency of the orbital electron wave	***frequency of the orbital planetary wave***
$f_s \approx \frac{mZ^2e^4}{8\varepsilon_0{}^2 \cdot n^2h^3}$ ***(8.28)***	$f_n \approx \frac{2\pi^2G^2m^3M^2}{n^2H^3}$ ***(2.11.14)***

wavelength of the orbital electron wave	***wavelength of the orbital planetary wave***
$\lambda_s = \frac{h}{p} \approx \frac{2nh^2\varepsilon_0}{mZe^2}$ ***(8.29)***	$\lambda_n = \frac{H}{p} \approx \frac{nH^2}{GMm^2}$ ***(2.11.14)***

energy of the stationary electron wave	***energy of the stationary planetary wave***
$\varepsilon_s = hf_s \approx \frac{1}{2}mv^2 = \frac{mZ^2e^4}{8\varepsilon_0^2 \cdot n^2h^2}$ ***(8.30)***	$\varepsilon_n = \tilde{E}_n = Hf_n \approx \frac{1}{2}mv^2 = \frac{2\pi^2G^2m^3M^2}{n^2H^2}$ ***(2.11.14)***
radius of the electron orbit	***radius of the planetary orbit***
$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2Z}$ ***(8.4)***	$r_n = \frac{n^2H^2}{4\pi^2Gm^2M}$ ***(2.11.14)***

All left-side results, parameters, and values [(8.26)) - (8.30) and (8.4)] are related to an orbiting electron, and right side takes in account only certain analogical (orbital) planetary system parameters (See much more in the second chapter of this book under (2.11.14)). Since magneto-static forces between two magnets are also respecting Coulomb force law involving all spinning and rotating, electrically charged entities inside an atom, we should be able to create analogical N. Bohr type atom model counting only on magnetic moments of its electrically charged, motional elements. In fact, here presented analogies are underlining that we are dealing with electromagnetically and electromechanically coupled and synchronized resonators, and that such properties, or nature of matter, are spread from the micro world of atoms, until the macro world of galaxies. This is very much familiar with N. Tesla ideas about Dynamic Gravity Theory (see [97]). ♣]

Bohr's second postulate (8.2) now becomes entirely determined (as a difference between absolute stationary energy levels),

$$\tilde{E}_f = hf_{12} = h(f_{s2} - f_{s1}) = \varepsilon_{s2} - \varepsilon_{s1} = \Delta\varepsilon_s = \Delta\varepsilon_k \,, \qquad (8.31)$$

because it is obvious, that it relates only to the (relativistic) motional energy of an electron and to the orbital eigenfrequency of the electron (8.28). Of course, applying Bohr's postulate (8.2), i.e. (8.31) to the frequency (8.28), with an approximation $v << c$, one gets the known result for the emissive spectral distribution of the hydrogen atom (8.9).

Anyway, in a short time after N. Bohr, Sommerfeld, Luis de Broglie, and others upgraded and perfected, as much as they could, the initial planetary atom model, E. Schrödinger analogically and intuitively formulated his famous wave equation. This equation integrated, upgraded, and justified all the here-exercised particle-waves quantizing and standing waves foundations and opened amazing mathematical ways to analyze atom world differently. This was the turning point when any new improvements, updates, and developments of the old atom modeling have been prematurely and unnecessarily stopped, bypassed, and abandoned (missing to draw what has still been precious there).

Accuracy and applicability of the spectral formula (8.9) is far too great, yet one could consider Bohr's model not sufficiently current and completely surpassed (especially now, after we realize that new and significant upgrading and remodeling is still possible). On the contrary, analyzing old planetary atom model we return to the origins of the atom science and face with some essential truths regarding particle-wave duality that apparently remained hidden (until present). By this occasion, we also face with the possibility of the revision of some states valid hitherto, which have arisen during the construction of the concept of the particle-wave dualism (giving new chances to significantly revitalize and update N. Bohr's atom model).

We take into account that electron revolving around atom nucleus is also spinning around its circular stationary path, with a spinning frequency $f_s = \omega_s / 2\pi$, and having spinning moment $\mathbf{L}_s = \mathbf{J}_s\omega_s$ (performing helical or spiral motion, thus enveloping certain toroidal form, like we find in (4.3-0)-(4.3-1.2), Fig.4.1.1, Fig.4.1.4 and T.4.3.; -chapter 4.1). Now, we can express such orbital, motional electron energy as,

$$\varepsilon_s = hf_s = \varepsilon_k = \tilde{E} = (\Delta m)c^2 = (\gamma - 1)mc^2 = muv = \frac{mv^2}{1 + \sqrt{1 - \frac{v^2}{c^2}}} =$$

$$= \frac{pv}{1 + \sqrt{1 - v^2/c^2}} = \frac{\mathbf{L}_m\omega_m}{1 + \sqrt{1 - v^2/c^2}} = \frac{\mathbf{L}_s\omega_s}{1 + \sqrt{1 - v^2/c^2}} =$$

$$= \frac{2\pi\mathbf{L}_m f_m}{1+\sqrt{1-v^2/c^2}} = \left.\frac{2\pi\mathbf{L}_s f_s}{1+\sqrt{1-v^2/c^2}} = nh\frac{f_m}{2}(1+\frac{u^2}{c^2})\right|_{v<<c} \approx$$

$$\approx \pi\mathbf{L}_m f_m = \pi\mathbf{L}_s f_s = \frac{1}{2}mv^2 = \varepsilon_B = nh\frac{f_m}{2} = \frac{mZ^2e^4}{8n^2h^2\varepsilon_0{}^2}\,, \tag{8.30-1}$$

$$pv = \mathbf{L}_m\omega_m = \mathbf{L}_s\omega_s\,, \mathbf{L}_s = \frac{h}{\pi} = \frac{f_m}{f_s}\mathbf{L}_m \approx \frac{n}{2}\mathbf{L}_m\,.$$

With (8.30-1) we are already merging with concepts introduced by M. Kanarev found in [44], and Charles Lucas and his colleagues in [16] - [22]. *Here is also convenient to mention that Lucas-Bergman atom model is too seriously (and wrongly) criticized from some of non-creative mainstream authorities of present Quantum theory. It is considered as not experimentally verified (but anyway it is producing better results and predictions than contemporary N. Bohr atom model, and the official Quantum theory). Probably, the most correct statement here should be that all presently known atom models (starting from N. Bohr's model, and later ones, including elaborations from this book), are different options of "Ptolemaic conceptualizations" of systems with intrinsic, structural periodicities and internal self-closed, mutually synchronized standing matter-waves structures. All of them are producing sufficiently good (mutually similar) and experimentally verifiable results tanks to involved intrinsic periodicities, and mutual transformability or mutual and deterministic mapping.* It is difficult to accept that something as isolated, orbiting and spinning particles (with rest masses) exist inside atoms (as electrons, neutrons, and protons are). *It is much more probable that atoms (including their internal constituents) are specific, self-stabilized, closed resonant formations of standing, electromagnetic waves with certain internal dynamics. Electrons, protons, and neutrons are wave-particle, dualistic formations or states that can be detected as particles after being on some way excited, extracted or expulsed from atoms.*

---------------- (This part, from here, should be additionally modified) --------------

Let us try, to completely associate Bohr's atom model (in the context of the previous interpretation) with the general concept of the particle-wave dualism, which is presented in Fig.4.1, Chapter 4.1 and within relations from (4.9) to (4.22), Chapter 4.3. In the expression for the second Bohr's postulate, (8.31) figures a stationary orbital energy of the electron, which is, in fact, equal to the total (relativistic) motional energy of the electron (4.12). Of course, we may now consider that the electron has its orbital and stationary state (for which the main quantum number **n** is associated) with the total motional energy,

$$\varepsilon_{kn} = E_{kn} + \tilde{E}_n = \varepsilon_{sn}\,. \tag{8.32}$$

If we now say that the electron moved from its stationary state $n = n_2 = 2$, to its second stationary state $n = n_1 = 1$, and there emitted a photon $\tilde{E}_f = hf_{12}$, the second Bohr's postulate (8.31) may be developed as,

$$\tilde{E}_f = hf_{12} = h(f_{s2} - f_{s1}) = \Delta\varepsilon_k = (E_{k2} + \tilde{E}_2) - (E_{k1} + \tilde{E}_1) =$$
$$= (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1) = \Delta E_k + \Delta\tilde{E} \qquad (8.33)$$

As the emitted photon, $\tilde{E}_f = hf_{12}$, is a form of wave energy that leaves the atom overall (and not only one of its stationary orbits), but we also have a case of action and reaction, or better to say, a case of shooting a missile from a gun called an atom. It is obvious that the atom will in such case experience something like a reactive recoil, i.e., then we must apply the general balance of energy and impulse of the particle-wave dualism, which are given by expressions (4.14) and (4.20), Chapter 4.3, but it is also clear that this general balance of energy will differ from the balance represented by (8.33). Of course, the difference between the total balance of the overall atom energy, and the case described by (8.33) is most likely, quantitatively, negligibly small, considering the relations of total energies among the electron, atom nucleus, and the emitted photon. Of course, this should not be forgotten (or neglected) due to the qualitatively understanding of the situation in the full complexity of the particle-wave dualism.

Let us now determine the most general case of the balance of an excited atom energy that an outgoing photon will emit. Of course, an atom, i.e., some of its electrons, before emitting a photon was excited (and according to the natural assumption, it will emit a photon in the next moment, and then return to its basic state). Such atom may be characterized by its total motional energy that is equal to the sum of the motional energy of its excited electron, and the motional energy of the whole atom (let us say, almost the motional energy of its nucleus). So, state of an atom before de-excitation may be presented as:

$$\varepsilon_{ka2} = E_{k2} + \tilde{E}_2 + E_{ka2} + \tilde{E}_{a2} , \qquad (8.34)$$

where $E_{k2} + \tilde{E}_2$ is total stationary motional energy of the excited electron, as given in (8.33), and $E_{ka2} + \tilde{E}_{a2}$, is total motion energy of an atom as a whole (or let us say, significantly close to a motional energy of the atom nucleus), which was neglected until now (or we could start from an assumption that it was equal to zero). In a way analogous to the previous, the state of the whole situation after de-excitation (emitting a photon) may be expressed by the resulting motional energy:

$$\varepsilon_{ka1} = E_{k1} + \tilde{E}_1 + E_{ka1} + \tilde{E}_{a1} + hf_{12} , \qquad (8.35)$$

where $E_{k1} + \tilde{E}_1$ is a total stationary motional energy of an electron in a new stationary orbit, and $E_{ka1} + \tilde{E}_{a1}$ is a total motional energy of an atom (or let us say remarkably close to the motional energy of the atom nucleus) after de-excitation.

Let us now apply the law of conservation of total energy (4.14) to the cases given by (8.34) and (8.35), if the rest energy (or rest mass) of an atom (before and after de-excitation) did not change, wherefrom one will get:

$$\begin{aligned}\varepsilon_{ka2} - \varepsilon_{ka1} &= E_{k2} + \tilde{E}_2 + E_{ka2} + \tilde{E}_{a2} - (E_{k1} + \tilde{E}_1 + E_{ka1} + \tilde{E}_{a1} + hf_{12}) = \\ &= (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1) + (E_{ka2} - E_{ka1}) + (\tilde{E}_{a2} - \tilde{E}_{a1}) - hf_{12} = 0 \ .\end{aligned} \tag{8.36}$$

It is now possible from (8.36) to determine the energy of the emitted photon as,

$$\begin{aligned}\tilde{E}_f = hf_{12} &= (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1) + (E_{ka2} - E_{ka1}) + (\tilde{E}_{a2} - \tilde{E}_{a1}) \approx \\ &\approx (E_{k2} - E_{k1}) + (\tilde{E}_2 - \tilde{E}_1).\end{aligned} \tag{8.37}$$

One may notice that the second Bohr's postulate (8.33) is identical to the situation described by (8.37), under a condition that all forms of the motional energy of an atom as a whole, $(E_{ka2} - E_{ka1}) + (\tilde{E}_{a2} - \tilde{E}_{a1})$, may be neglected or mutually annulled. This is correct (because the mass of an electron is negligible considering the mass of the atom nucleus, and rest masses of the electron and the nucleus did not change). If we now apply the law of conservation of the impulse (4.20), the atom (under the same assumptions as hither) would receive approximately the impulse that corresponds to the impulse of the emitted photon with an opposite sign.

The nature of the wavelength of the total stationary electron wave (that is expressed through the wave impulse of an electron) remains to be explained. During the previous analysis, de Broglie's definition of the wavelength was used for the wavelength of the electron wave (see (8.12), (8.25), (8.29)), compared with the fact that in this book we also exercise with the new definition of that same wavelength, through the wave impulse. **It is obvious that in the case of Bohr's atom model, the equality of the absolute values of the particle and wave impulse of an electron that is in motion in its stationary orbit, i.e., $|p| = |\tilde{p}|$ is achieved. We can explain this situation in the following way. If the atom is in the state of rest, or, let the velocity of its center of mass equals zero, then orbital and stationary revolution of an electron around the atom nucleus is very much balanced in the sense that some uniform, center-symmetrical, spatial distribution of the electron mass (across the whole area of the stationary orbit) exists. The total impulse of the electron**, P_e, **which is equal to the vector sum of its impulse as a particle p_e , and its wave impulse $\tilde{p}_e$, must be equal to zero (in the opposite case, if it is not equal to zero, the center of the atom mass would not be in the state of rest),**

$$P_e = p_e + \tilde{p}_e = p + \tilde{p} = 0 \tag{8.38}$$

From (8.38) we have:

$$\Delta p + \Delta \tilde{p} = 0\,,\; p = -\tilde{p} \;\Rightarrow\; |p| = |\tilde{p}| \;\Rightarrow\; \lambda_s = \frac{h}{|\tilde{p}|} = \frac{h}{|p|} \tag{8.39}$$

In reality, when we search for the de Broglie's wavelength (of some energy state) we should know that every stable, uniform and stationary state (or state of relative rest) regarding certain «Laboratory system» may be characterized either by its particle p*, or by its wave momentum,* $\tilde{p}$ *(* $\lambda = \frac{h}{\tilde{p}}$ and/or $\lambda = \frac{h}{p}$*. Substantial differences (as "vectors or scalar-like") between the wave and particle momentum originate in transient regimes when the certain state experiences a modulation of its motion*. According to (8.39), quantizing the orbital momentum of an electron, or applying the first Bohr's postulate, (8.1), in context with the wave attributes of an electron is more logical to present by quantizing its wave orbital momentum,

$$\tilde{m}vr = \tilde{p}r = n\frac{h}{2\pi}\,. \tag{8.40}$$

Instead of (8.38), for the stable atom in the state of rest, we should consider the total momentum of the atom P_a (including the nucleus momentum $P_p = p_p + \tilde{p}_p$). The result or the conclusion above will approximately stay the same, since the nucleus of the stable atom has a much bigger mass than the electron, and it will be almost in the state of rest (comparing to a revolving electron: see also (8.9-1) and (8.9-2)),

$$\begin{aligned} &P_a = P_e + P_p = (p_e + \tilde{p}_e) + (p_p + \tilde{p}_p) = 0 \Rightarrow \\ &\Rightarrow P_e = p_e + \tilde{p}_e \cong 0 \text{ and } P_p = p_p + \tilde{p}_p \cong 0 \Rightarrow \\ &\Rightarrow \Delta p_e + \Delta\tilde{p}_e \cong 0 \text{ and } \Delta p_p + \Delta\tilde{p}_p \cong 0\,. \end{aligned} \tag{8.38-1}$$

As we know, the electron and the nucleus both revolve around their common center of mass. Consequently, the total orbital momentum of a stable and neutral atom $\mathbf{L_a} = L_p + L_e$ (in the state of rest, analogously to (8.38-1)) should also be equal to zero,

$$\begin{aligned} &L_a = L_e + L_p = (\ell_{\mathbf{e}} + \tilde{\ell}_{\mathbf{e}}) + (\ell_{\mathbf{p}} + \tilde{\ell}_{\mathbf{p}}) = 0 \Rightarrow \\ &\Rightarrow (\Delta\ell_{\mathbf{e}} + \Delta\tilde{\ell}_{\mathbf{e}}) + (\Delta\ell_{\mathbf{p}} + \Delta\tilde{\ell}_{\mathbf{p}}) = 0, \end{aligned} \tag{8.38-2}$$

implicitly accepting that between an atom nucleus and the revolving electron there is always certain energy-momentum coupling (with electromagnetic wave energy exchanges). Of course, here we should also consider angular spinning moments of involved participants (presently neglected).

With previous results and conclusions, N. Bohr's atom model fits into the general concept of the particle-wave dualism of this book, which was the purpose of former analysis. ***In addition, here we are creating grounds showing that certain matter-waves, energy-momentum exchanges, and communications should naturally exist between electron and nucleus stationary states.***

8.1. New Aspects of Atom Configuration from the view of the Bohr's Model

By further exploration of the same problematic, one may draw other interesting consequences that result from the richness of the content of N. Bohr's atom model. For example, frequency of the quantum of inter-orbital, electron exchange for stationary orbits with big quantum numbers is approximately equal to:

$$f_{12} = \frac{mZ^2e^4}{8h^3\varepsilon_0^{\,2}}\left(\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right) \approx \frac{mZ^2e^4}{4n^3h^3\varepsilon_0^{\,2}} = f_m \approx \frac{2f_s}{n} \quad , \tag{8.41}$$

implying that it is possible to use the following approximations:

$$n_1 = n_2 + 1 \;>>\; 1\,, \quad n_1 \approx n_2 \approx n \approx \sqrt{n_1 n_2} \approx (n_1 + n_2)/2\,, \quad v << c\,.$$

Looking from the other side, frequency of the stationary electron wave (in its orbit) is:

$$\begin{gathered} f_s = \underline{nf_m \frac{1}{2}(1 + \frac{u^2}{c^2})}\Big|_{v<<c} \approx \frac{mZ^2e^4}{8n^2h^3\varepsilon_0^{\,2}} = \frac{n}{2}f_m \\ \frac{n}{2} \le \frac{f_s}{f_m} \le n \;,\; \frac{1}{2} \le \frac{f_s}{nf_m} \le 1 \;,\; 0 \le (u, v) \le c. \end{gathered} \tag{8.42}$$

One may notice that for <u>*border cases*</u> (8.41), when **n** is a great number, the frequency of the quantum of the electron emission is approximately equal to the corresponding orbital, mechanical (or rotational) frequency of an electron treated like a particle, f_m , (see (8.17)). Here, quantum mechanical principle of correspondence is evoked, by which N. Bohr intended to show where the borders between a macroscopic and quantum treatment of the matter are, in the world of physics. However, one should pay attention that wave frequency or electromagnetic quantum frequency does not have very much in common with the mechanical-revolution number of the particle around some center, (8.42), and this again represents something else for a stationary electron wave (the only common term here is the term frequency, and both frequencies may be dimensioned using the same units). **In fact, Bohr's principle of correspondence is now corrected, and it differs from what Bohr claimed.**

Based on the expressions found for the group and phase velocity of an electron, (8.25), (8.26), and (8.27), as well as based on (4.0.27), (4.0.28), and (4.0.29), one can check that the following relations between group and phase velocity of an electron wave remain valid:

$$0 \le u_s \le v \le c, \;\; 0 \le u_s^{\,2} \le u_s v \le v^2 \le c^2. \tag{8.43}$$

The most interesting is that by solving differential equations that connect group and phase velocity of an electron wave (which may be found in the same form as in expressions (4.0.27), (4.0.28), and (4.0.29)) we may come to adequate spectral distributions of radiating energies that remind us on the Planck's law of heated blackbody emission.

Let us start from the expression for the group velocity of an electron wave (8.27) and let us connect it with (8.43), where we get:

$$v = \left.\frac{2u_s}{1+\frac{u_s^2}{c^2}}\right|_{(u,v)<<c} \approx 2u_s = \frac{Ze^2}{2nh\varepsilon_0} < \frac{Z_{max}e^2}{2n_{min}h\varepsilon_0} < c \ , \ n_{min} = 1,$$

$$\Rightarrow Z_{max} < \frac{2c\varepsilon_0 h}{e^2} = \frac{1}{\alpha} = \frac{2h}{\mu_0 ce^2} = 137.03604 \ , (\varepsilon_0\mu_0 = \frac{1}{c^2}) \qquad (8.44)$$

$$\Rightarrow \alpha Z_{max} < 1.$$

In the previous expression, α is "thin (or fine) structure", the universal physical constant. Also, from (8.44) one may conclude that the maximal possible protons' number of some element is $Z_{max} \leq 137$. Since 137 is not a much bigger number than hitherto known maximal protons number of the last known element from the Periodic Table, one may expect to discover around twenty new elements of the Periodic Table. Of course, concluding based on the previous (simplified) analysis is approximative by its character to be on the proof-level, but it is logical, non-contradictory and interesting, in the same time putting more light on the nature of the thin structure constant.

Since in the Bohr's planetary atom model some circular motion of electrons (or their stationary waves) is immanently present, it is important to determine mechanical and magnetic orbital moment of the electron (in the stationary orbit).

Mechanical orbital moment or torque of the electron is,

$$L = L_e = J_e\omega_m = \gamma mvr = \gamma m\omega_m r^2 = 2\gamma m\pi r^2 f_m \, , \, J_e = \gamma mr^2 \, , \qquad (8.45)$$

and the magnetic moment of the electron is,

$$M = I_e S = I_e \pi r^2 \, , \qquad (8.46)$$

where I_e is an elementary current of a rotating electron across its orbit, and S is the surface encompassed by that orbit.

If we now determine the elementary current of a rotating electric charge as the product of the electron charge **e** and corresponding frequency by which that charge really orbits f_e, there will be,

$$I_e = ef_e \Rightarrow M = \pi r^2 ef_e \, . \qquad (4.47)$$

The relation between the magnetic and mechanical moments of an electron is called the gyromagnetic ratio and it has great significance in describing the magnetic properties of materials:

$$\frac{M}{L} = \frac{\pi r^2 e}{2\pi r^2 \gamma m}\frac{f_e}{f_m} = (\frac{e}{2\gamma m})\frac{f_e}{f_m} \, . \qquad (8.48)$$

There remains an open question, how to treat the frequency $\mathbf{f_e}$ by which the charge orbits around the atom nucleus. In general case, we have very few possibilities, such as:

a) That the electron charge and its mass are always concentrated in the same spatial spot, i.e., that there is some compact material object or wave-group that orbits around the atom nucleus by its mechanical frequency, so there will be $f_e = f_m$, or

b) That electron in the stationary orbit is a somewhat different form of spatially distributed mass and electric charge (across the whole orbit area) when the notion of the mechanical rotational frequency of the electron mass differs from the frequency by which the distributed electric charge of the electron orbits $f_e \neq f_m$. Consequently, we may consider that the orbiting frequency of the electron is equal to the frequency of its stationary electron wave $f_e = f_s$, (see (8.42)), and

c) That all previously mentioned frequencies are mutually dependent and different $f_e \neq f_m \neq f_s$.

In the first case, if the electron charge would orbit by its mechanical rotation frequency around the atom nucleus, $f_e = f_m$, the gyromagnetic ratio would be:

$$\frac{M}{L} = (\frac{e}{2\gamma m})\frac{f_e}{f_m} = \frac{e}{2\gamma m} = \frac{e}{2m}\sqrt{1-\frac{v^2}{c^2}}\,, \quad 0 < \frac{M}{L} \leq \frac{e}{2m}\,,\ 0 \leq v \leq c\,, \tag{8.49}$$

which is the known (old Quantum theory) case found in the existing textbook literature.

In the second case, when the electron charge forms a stationary orbital electron wave and orbits by eigenfrequency of that wave around the atom nucleus, $f_e \approx f_s$, the gyromagnetic ratio will be:

$$\left\{\left(\frac{M}{L} = (\frac{e}{2\gamma m})\frac{f_e}{f_m} = \frac{n}{2}(\frac{e}{2\gamma m})(1+\frac{u_s^2}{c^2})\frac{f_e}{f_s} = \frac{n}{2}(\frac{e}{2m})(1-\frac{u_s^2}{c^2})\frac{f_e}{f_s}\right), (0 \leq u_s \leq c)\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{c} \boxed{0 < \frac{M}{L} \leq \frac{n}{2}(\frac{e}{2m})\frac{f_e}{f_s} \cong \frac{n}{2}(\frac{e}{2m})}, \\ \left(\begin{array}{l} f_s = nf_m\frac{1}{2}(1+\frac{u^2}{c^2}), f_m = 2f_s / n(1+\frac{u^2}{c^2}), \frac{1}{2}nf_m \leq f_s \leq nf_m\,,\ 2\frac{f_s}{n} \leq f_m \leq \frac{f_s}{n}, \\ L = n\hbar,\ \ \hbar = \frac{h}{2\pi}, \mu_B = \frac{eh}{4\pi m} = \frac{e}{2m}\hbar \end{array}\right) \end{array}\right\}. \tag{8.50}$$

There is an obvious difference between possible treatments of the gyromagnetic ratio, depending on how we treat the frequency of an electron charge rotation around the atom nucleus (8.49) and (8.50), and in what relation the orbital eigenfrequency of the electron (de Broglie's) wave is with the (mechanical) rotation frequency of its mass and charge. It is probable that in this area we can search for the weak elements of Bohr's atom model and early quantum theory because the frequency of the mechanical orbiting of an electron was wrongly related with the frequency of its orbital (de Broglie's) wave. The point here is that the so-called mechanical rotational frequency represents the revolutions number of the material point around some center of rotation in the unit of time, and that the frequency of the wave motion is characterized through relations (8.23) and (8.24). It is obvious that here we have two different notions and phenomena for which in certain situations an identical term has been used, such as ***frequency***, which later leads to all other correcting and complementing consequences and to the ***correspondence principle*** found in the quantum physics literature. Sometimes is hard to make nuance between the senses of certain close notions (that are dimensionally equal).

Of course, after the previous plot, we can make the situation with the gyromagnetic ratio more complex, introducing the spin or rotation of the electron (around itself), producing a magnetic moment that will be superimposed to the total magnetic momentum of the electron (that orbits around the nucleus), etc. Analyzing the problem of the gyromagnetic ratio in a previous way, easily leads to the hypothesis that an electron addressed integrally as part of the atom shell is not a particle that orbits around the nucleus anymore. More probably is that an electron is some diffused and distributed particle-wave and mass-energy-momentum state, which binds to itself characteristics of rotation, but not anymore in a sense of motion of strictly spatially localized particles, which is a way quantum mechanics took. We may only add that electron-mass distribution and electron-charge distribution involved in such orbital motion, most likely comply with mutually dependent but different distribution laws. The confirmation of the previous opinion is also in the fact that submitting masses or atoms to high pressures (or low temperatures) leads to their phase transformations of the material properties and to body and energy restructuring of their electron shells. In addition, we may find expressions different from those done hitherto for the electron radius, or for the area an electron occupies. In (8.50) and before we have velocity dependent expressions capturing the interval $0 \leq (u, u_s, v) \leq c$, but we are still addressing only electrons. Briefly, we can give an intuitive and oversimplified understanding, or statement of electron states in relation to characteristic boundary velocities, such as: If or when $0 \leq (u, u_s, v) << c$, electron states are closer to particle states, and when $0 <<< (u \cong u_s \cong v) \cong c$, such electron states should be kind of distributed electromagnetic field and matter-wave energy states. Of course, Bohr's atom model is not sufficiently rich to draw better and more precise conclusions. Anyway, electrons in different situations and different energy-momentum states are different formatting or packing states of a certain specifically structured electromagnetic field. The way in which mentioned electromagnetic field is structured will create positive and negative electric charges, as well as familiar matter and antimatter entities.

The most important implication of the situation mentioned above is related to PWDC, and to the situation, that linear motion and rotation (or spinning) are intrinsically coupled (like couplings of the electric and magnetic field).

[♣ The aim of such seemingly exceeded analyses (like in this book) is to point out that in essence there is not only a problem of different conceptualization of the structure of matter from the point of view of classical mechanics or quantum wave mechanics, but that we also know insufficiently relevant facts, laws, concepts and involved mathematical models, i.e., their logical links and mutual couplings. It could also be that some theoretical concepts (we are still considering as correct) are partially set incorrectly from the very beginning, to construct those, in a certain historical moment, unknown links. **Of course, erroneous theoretical postulates must be added onto by convenient, new erroneous concepts to become self-corrected complying with experimental results (like in updated Ptolemaic geocentric theory), unless we are ready to reject primary and basic theoretical hypothesis.** The most ungrateful moments during writing such books (like this one) is that they may be dogmatically criticized from the point of view of the existing, mainstream theoretical concepts and models. Such models have already entered the history of certain scientific opinion, encyclopedias, and learning textbooks, and seemingly fit into the experimental database. Namely, it is known that for certain relevant set of facts, a good enough model may be formed, which locally connects all known facts, but that in essence, the same model is bad if it is later considered from some substantially broader set of relevant facts, which will be discovered after new experimental practices. Intellectual rigidity, the inertia of thinking and dogmatic compliance with the adopted standpoints that have not changed for years, represents a great obstacle in introducing new ideas that undermine the structure of the formed models. Because of that, especially in physics, it happens that some models or theories are simply exhausted at the certain moment, abandoned, and replaced by new ones. Simply, some of old models serve no more any new objective of scientific predictions, and they cannot explain or predict all empirically known facts in a certain domain (as it was the case with the Ptolemy geocentric system). Of course, this has good and bad sides, because all abrupt jumps are risky and problematic since scientifically formulated knowledge demands continuity of the cognitive process from simpler models towards more complex, which approximately imply (or capture) their predecessors.

Citation concerning Historical Atom Models Evolution taken from Lucas-Bergman website, http://www.commonsensescience.org/atom_models.html:

Bohr's Atom
In 1913, Niels Bohr proposed a theory of the hydrogen atom (the simplest of all atoms) consisting of one heavy proton in the center with one lighter electron in orbit around the proton. Bohr supposed

- *That electrons move in circular orbits around the atomic nucleus.*
- *Only certain orbits are permitted.*
- *That in these permitted orbits, the electrons would not radiate (would not create radio waves).*
- *That light of certain colors (and wavelengths) would be created when the electron (of its own power) changed orbits.*

These postulates were entirely arbitrary and even violated the established laws of electricity and magnetism. In spite of this, physicists still use the Bohr model (when it is convenient).

Parson's Magneton Theory of the Atom
"By 1915, A. L. Parson knew that the Bohr model of the atom could not be real, so he developed and even experimented on a model of the atom where the electrons were not point-sized particles that orbit around the atomic nucleus. In Parson's atom, the electrons in the shells surrounding the nucleus were rings of charge (with the shape of a toroid or donut). Since the electrostatic charge at the surface of these rings is rotating, each electron is a tiny magnet. In 1918, Dr. H. Stanley Allen of the University of Edinburgh discussed the arguments in favor of the ring electron, showing how it removed many outstanding difficulties of other theories of the atom. In spite of its superiority, Parson's magneton model of the atom did not become popular (but a modern version, the Lucas model of the atom, has now been introduced).

De Broglie's Model of Matter: Particle-Wave Duality
"About 1924, Louis de Broglie proposed that all particles of matter (from single atoms to large objects) moving at some velocity would have the properties of a wave. Today, most physicists take this farther and say that all material objects are waves until they are measured or observed in some way. When this takes place, the wave is said to collapse and turn into an object. An example of this notion of reality is given by the famous Cornell physicist N. David Mermin who says, "We now know that the moon is demonstrably not there when nobody looks."

Schrodinger's Wave Model of the Atom
"In 1925, soon after de Broglie had put forward his ideas, Schrodinger used them [to write] a wave equation to describe this new mechanics of particles." Schrodinger's model of the atom is not a physical model (where an object has size, shape, and boundaries)

but is a mathematical model (an equation where objects are point particles.) The equation is useful to predict some properties of objects (or atoms) but is not able to describe the object (or atom) itself.

Dirac's Model of the Atom

"The Dirac model is an equation that includes imaginary numbers. It is not an attempt to describe the objective reality of the physical electron but to predict the various levels of energy that the electron may have at various times. There appears to be a serious problem with the Dirac model, which has the electrons orbiting the nucleus. Another major assumption of the Dirac theory is that the statistical version of quantum wave theory or quantum mechanics is valid. Unlike previous quantum theories of the atom that used real numbers (such as the Bohr model and Schrodinger model), the terms and imaginary numbers in Dirac's equation do not correspond to measurable quantities.

Standard Model of Elementary Particles

"The Standard Model of Elementary Particles is not a description of the atom. However, we must mention it now because, in modern theory, atoms are not only waves but also when measured, the atoms change into objects composed of elementary particles. In modern physics, the important components of the atoms are electrons, protons, and neutrons. The Standard Model considers electrons to be true elementary particles, either waves or point particles with inherent properties of mass, magnetism, spin, and stability. However, in modern Quantum Theory, protons are supposed to be composed of quarks; and neutrons are thought to be composed of a different combination of quarks. Another important part of the Standard Model is that forces between these elementary particles are supposed to be carried by other particles that move back and force randomly between the material particles:

- *photons are supposed to be particles that carry forces between electrons,*
- *mesons are supposed to be particles that carry forces between protons and neutrons,*
- *gluons are supposed to be particles that carry forces between the quarks (which are supposed to be inside protons and neutrons).*

Lucas Model of the Atom

"In 1996, while still a high school student, Joseph Lucas introduced his model of the atom. In this model, electrons, protons, and neutrons are all based on Bergman's Spinning Charged Ring Model of Elementary Particles (a refinement of Parson's Magneton). The Lucas Model of the Atom is by far the most successful of all models of the atom ever proposed. It is a physical model that shows where electrons are located throughout the volume of the atoms. This model predicts the "magic numbers" 2, 8, 18, and 32 of electrons in the filled shells and can predict why the Periodic Table of the Elements has exactly seven rows. The Lucas model also predicts the structure of the nucleus and correctly predicts hundreds of nuclide spins.

R.J. Haüy – G.P. Shpenkov – V. Christianto atom model (see [85] and [86])

As we know, all atoms are composed of electrons, protons, and neutrons. Proton and neutron have similar masses, almost equal to the mass of hydrogen atom, and to the sum of masses of an electron and a proton.

Mass of an electron	$9.109389700 \times 10^{-31}$ kg
Mass of proton	$1.672623100 \times 10^{-27}$ kg
Mass of an electron + proton	$1.673534039 \times 10^{-27}$ kg
Mass of hydrogen atom	$1.673534000 \times 10^{-27}$ kg
Mass of neutron	$1.674928666 \times 10^{-27}$ kg

It is (still hypothetically) imaginable that we could present a neutron like certain specific combination of an electron and proton. A hydrogen atom is already composed of an electron and proton. Within such an imaginative framework, we could (hypothetically) say that neutron and hydrogen atoms are only mutually different (exotic and specific) combinations, formatting or packing of an electron and a proton. Consequently (still hypothetically), we could say that, since hydrogen atom is on some way uniting an electron and a proton, and this could also be (imaginatively) presented as certain packing state of a neutron, all other atoms are simply assembled of different packing and formatting combinations of hydrogen (or neutron like) atoms. The same hypothetical thinking we could support if we analyze what can happen when a photon collides with an atom: We can get only photoelectrons (explained by M. Maric-Einstein Photoelectric Effect), we can get reduced-energy photon, and in some cases an expulsed, moving electron (known as Compton Effect), or we can produce only a couple of electron-positron in case when incident photon has sufficiently high energy. Such situations we could explain (or analyze) as we find in almost all Quantum Theory and Physics books, but we could also imagine that inside atoms electron states are presenting different energy-momentum and spatial, standing matter and resonant waves formations, compared to free, spatially concentrated, and much more corpuscular electron-body formations outside atoms. Practically, we could say that electrons inside atoms are standing waves of specific electromagnetic energy (like photons formations), distributed around the atom nucleus. When we succeed to increase and expulse enough of such electromagnetic energy outside an atom (after an impact between an external photon and atom), we are effectively forcing such surplus of ejected energy to create new electrons (Photoelectric effect), or new electrons and new photons (Compton Effect), or electrons and positrons. In other words, here we are explaining that electrons and positrons are different structural formations (packing and formatting) of photons, or specific formations of electromagnetic energy. Empirically verifiable situations could be topologically different, depending on where and how electrons are placed or created; -if found inside atoms, electrons are self-closed, standing electromagnetic waves, and if forced to go outside atoms, electrons are getting much more of corpuscular, spatially localized nature. On a familiar imaginative (and hypothetical) way we could say that certain specific wave packing (or coupling) of an electron and a proton will either create hydrogen atom or a neutron. Consequently, we could again say that effectively, all atoms are specific packing and formatting states of hydrogen atoms (or being also packing states of neutrons, or essentially and ontologically we could say that certain electromagnetic energy packing is behind everything). The familiar concept about such composite atoms' structure (***but not explained as here***) can be found in [85] and [86]. See also citation below:

Citation: A THEORETICAL PREDICTION OF MOLECULAR AND CRYSTAL STRUCTURES, ***Leonid G. Kreidik[1] and George P. Shpenkov[2]***

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Keywords: *Haüy's molecules, wave equation solutions, intra-atomic space, crystal structure*

Analyzing the structure of crystals at the end of 18th century, R.J. Haüy [1] has concluded that it is necessary to consider atoms as elementary molecules, an internal structure of which is closely related to the crystal shape of solids. Particles, constituent these elementary molecules, must be coupled with strong bonds, which we call the multiplicative bonds. Then it is reasonable the ordinary molecules with relatively weak bonds to call the composite or additive molecules; e.g. if deuterium D is the multiplicative molecule then the hydrogen molecule H_2 *is the additive one.*

As masses of atoms are multiple to the neutron (hydrogen atom) mass then, following Haüy's ideas, it was reasonable to suppose that the atom, as the elementary Haüy's molecule, is the neutron multiplicative molecule. According to this model, using the wave equation, the problem on the distribution of matter (neutrons) in the elementary Haüy's molecules has been solved.

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Based on elementary multiplicative Haüy's molecules, it is easy to predict the possible structures of both the crystals and additive molecules that is presented in the extended report.

1. R.G. Haüy, Essai d'une theorie sur la structure des crystaux, Paris, (1784).
2. L.G. Kreidik, G.P. Shpenkov, Alternative Picture of the World, V.1-3, Bydgoszcz (1996).
3. S. Flugge, ed., Handbuch der Physik, Encyklopedia of Physics, V.30, Springer-Verlag (1957).

Modern atom physics is correctly explaining that inside an atom, between positive and negative electric charges (like between protons and electrons) we have in action only electromagnetic or electrodynamic forces, meaning that between positively charged atom nucleus and surrounding electrons dominant force is Coulomb's electrostatic force. Of course, forces between magnetic elements or moments of an atom constituents are also belonging to the same family of electromagnetic or electrodynamic forces. Since atom nucleus (as conceptualized in contemporary physics) has protons and neutrons, and we know that such a nucleus is very stable in all cases of stable atoms, we could search for a new force that is much stronger than repulsive Coulomb force between protons, to have something what keeps all protons and neutrons together in a small space. Instead of solving such conceptual problem correctly, technically, and scientifically, modern physics simply postulated that it should exist a strong ***nuclear force***, (still without certain reliable and verifiable mathematical modeling), which is so strong and dominant that repulsive Coulomb force (between protons) can be neglected. This has been easy to say, and presents an oversimplified intuitive statement without any explanation, or this is a simple real problem replacement with something else, still unknown, but good sounding and hypothetically promising, like sticking a label with some name on certain black box. For unstable (radioactive) atoms, which are slowly disintegrating, transmuting, and decaying, it was, also on a similar over-simplistic way easy to introduce existence of a weak nuclear force between nucleus constituents (again without a good mathematical modeling). Again, one problem was simply replaced, and wrongly or superficially solved, by a simple and hypothetical statement, or with an arbitrary labeling. Later, an army of well-obeying and not-critical disciples and followers of such teachings continued to use mentioned hypothetical and incomplete concepts and terminology, and developed present-days, sufficiently well-operating atom physics, combining and exploiting new experimental results, number of added and healthy mathematical practices, conservation laws, event probabilities, uncertainty relations, experimentally known results, etc., practically correcting and masking initial, oversimplified and arbitrary assumptions about nuclear forces. We still repeat in all physics books that our universe has four fundamental forces (Electromagnetic, Gravitational, Strong and Weak Nuclear ones), but in practical calculations where such forces should be in action, we are well mastering only electromagnetic forces, since gravitational forces are not relevant on the level of atoms, and Strong and Weak Nuclear forces do not have any, or at least modestly developed and elaborated field/forces structure and valid mathematical forms, like in case of Maxwell's electromagnetic theory where such items are known and mathematically manageable. In addition, for the force of gravitation, we started to see (at least who is interested to see, and who would not damage his existential position with following some new and original insights) number of publications indicating that this could be some unusual manifestation of electromagnetic forces.

The opinion of the author of this book is that all atom-constituents, like electrons, protons, neutrons etc. are presenting certain well packed and mutually stabilized, essentially electromagnetic, spatial standing matter-waves structures (at least when mutually coupled inside atoms). This way, it is becoming clear that atom nucleus is not structured in a form of distinct or mutually glued particles, but rather being as some complex electromagnetic field structure (like well combined matter-wave packet or group) that still manifests its positive electric charge, equal to the total charge of involved protons. Another vision of the author of this book is that electrons' envelope (or cloud) and nucleus are on some way mutually and spatially mirror-symmetrical, or structurally similar and, of course, being electromagnetically coupled. Starting from here, we should conceptualize new theory about nuclear fields and forces (in relation to atoms).

A much more radical and explicit statements, with a lot of supporting and convincing arguments against contemporary concepts in relation to Strong and Weak Nuclear forces, and against the existence of corpuscular and distinct neutrons inside an atom nucleus can be found in [91], Charles W. Lucas, Jr. "Is There Any Truth in Modern Physics"? February 2015. Volume 18, number 1; -The Journal of Common-Sense Science. The author simply presented the evidence that neutron is certain specific formation or combination of an electron and a proton, based only on electrodynamic interactions (without any evidence of involvement of Strong and Weak Nuclear forces). Some of the ideologically colored statements (not strictly related to atoms) found in the same paper, from the same author, could be simply omitted and neglected, without diminishing other only physics-related statements and conclusions.

Citations from [91]: "A proper examination of atomic and nuclear data shows that there is no strong force holding the positively charged protons together in the nucleus and no weak force governing nuclear decay. There is only one force in the nucleus, which is the electrodynamic force.

... A second analysis of the NIST isotopic masses of various nuclei has discovered that there is no data in the entire table of isotopic masses to justify (1) the existence of neutrons inside nuclei, (2) the existence of the Strong Force inside nuclei to keep the protons bound together in the nucleus, and (3) the existence of the Weak Force controlling the decay of various nuclei. The only evidence for the electrodynamic force is found".

Citation from [93], F.A. Gareev, I.E. Zhidkova. “We come to the conclusion that the all atomic models based either on the Newton equation and the Kepler laws or on the Maxwell equations or on the Schrödinger and Dirac equations achieved reasonable agreement with experimental data. We can only suspect that these equations are grounded on the same fundamental principle(s) which is (are) not known or these equations can be transformed into each other. … Bohr and Schrodinger assume that the laws of physics that are valid in the microsystem do not hold in the micro world of the atom. We think that the laws in macro and the micro world are the same”. ♣]

8.2. Bohr Atom Model and the Stationary States of Nucleus

Until now, a somewhat static hydrogen atom model has been discussed, if electron orbits around the stable and immobile nucleus (what is almost correct, considering the ratio of the mass of the nucleus and the mass of the electron), but it is more precise to say that an electron and a proton orbit around the common center of mass.

As an illustration of the platform for merging de Broglie matter waves and N. Bohr’s atom structure, let us analyze a hydrogen atom in the center-of-mass coordinate system, like the one shown in Fig.4.1 (from Chapter 4.1). Let us apply (4.1), (4.2), and (4.3) (from Chapter 4.1) to describe the movement of the electron and the proton around their common center of gravity, and to describe their associated de Broglie waves, assuming that the electron and atom nuclei are treated as rotating particles (creating slightly modified Bohr’s model), having the following characteristics: m_e-electron mass, m_p-nucleus or proton mass, $v_e = \omega_{me} r_e$ -electron velocity around the common center of mass, $v_p = \omega_{mp} r_p$ -proton velocity around the common center of mass, r_e-radius of revolving electron, r_p -radius of revolving proton, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$ -mechanical, revolving frequency of an electron and proton, $\lambda_e = h/\gamma_e m_e v_e = h/p_e$ -de Broglie wavelength of electron wave, $\lambda_p = h / \gamma_p m_p v_p = h / p_p$ -de Broglie wavelength of proton wave, f_e -de Broglie frequency of an electron wave, f_p -de Broglie frequency of a proton wave, $u_e = \lambda_e f_e$ -phase velocity of the de Broglie electron wave, and $u_p = \lambda_p f_p$ -phase velocity of a de Broglie proton wave. In addition, the following basic relations also belong to the above-described Bohr’s hydrogen atom model: $\gamma_e m_e r_e^2 = \gamma_p m_p r_p^2$, $\gamma_e m_e r_e^2 \omega_m = \gamma_p m_p r_p^2 \omega_m$, $\gamma_e m_e v_e r_e = \gamma_p m_p v_p r_p$, $n\lambda_e = 2\pi r_e$, $n\lambda_p = 2\pi r_p$ (see **[4]** regarding the same situation).

In a few steps, the following correct relations (between all the parameters described above, see (4.2) and (4.3) from Chapter 4.1), valid for Bohr’s atom model, can easily be found (see also **[4]**):

$$\frac{m_p}{m_e} = \frac{\gamma_e}{\gamma_p}(\frac{r_e}{r_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{v_e}{v_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{\lambda_e}{\lambda_p})^2 = \frac{\gamma_e}{\gamma_p}(\frac{u_e}{u_p})^2 \cdot (\frac{f_p}{f_e})^2 = \frac{\gamma_e}{\gamma_p}(\frac{u_e}{u_p})^2 \cdot (\frac{\tilde{E}_p}{\tilde{E}_e})^2 =$$

$$= \frac{\gamma_e v_e \lambda_e}{\gamma_p v_p \lambda_p} = \frac{\gamma_e v_e r_e}{\gamma_p v_p r_p} = \frac{m_p c^2}{m_e c^2} = 1836.13, \; f_m = f_{me} = f_{mp} = f_{m(e,p)} = \omega_m / 2\pi,$$

or for $(v_e, v_p, u_e, u_p << c) \Rightarrow$

$$\frac{m_p}{m_e} = 1836.13 \cong \left[(\frac{r_e}{r_p})^2 = (\frac{v_e}{v_p})^2 = (\frac{\lambda_e}{\lambda_p})^2 = (\frac{u_e}{u_p})^2 \cdot (\frac{f_p}{f_e})^2 = \frac{v_e \lambda_e}{v_p \lambda_p} = \frac{v_e r_e}{v_p r_p} \right],$$

$$f_e = n\frac{f_m}{2}(1 + \frac{u_e^{\,2}}{c^2}) = n\frac{m_e Z^2 e^4}{8n^3 h^3 \varepsilon_0^{\,2}}(1 + \frac{u_e^{\,2}}{c^2}) \cong (f_p \text{, for } u_e << c, \text{or } u_e = c),$$

$$f_p = n\frac{f_m}{2}(1 + \frac{u_p^{\,2}}{c^2}) = n\frac{m_e Z^2 e^4}{8n^3 h^3 \varepsilon_0^{\,2}}(1 + \frac{u_p^{\,2}}{c^2}) \cong (f_e \text{, for } u_p << c, \text{or } u_p = c),$$

$$1 \le \frac{v_e}{u_e} = n \cdot \frac{f_m}{f_e} = \frac{2}{(1 + \frac{u^2_e}{c^2})} = 1 + \sqrt{1 - \frac{v^2_e}{c^2}} \le 2,$$

$$1 \le \frac{v_p}{u_p} = n \cdot \frac{f_m}{f_p} = \frac{2}{(1 + \frac{u^2_p}{c^2})} = 1 + \sqrt{1 - \frac{v^2_p}{c^2}} \le 2, \; n = 1, 2, 3, ...$$

$$\frac{f_p}{f_e} = \frac{1 + \frac{u^2_p}{c^2}}{1 + \frac{u^2_e}{c^2}} = \frac{1 + \sqrt{1 - \frac{v^2_e}{c^2}}}{1 + \sqrt{1 - \frac{v^2_p}{c^2}}} = \frac{1 + \frac{1}{\gamma_e}}{1 + \frac{1}{\gamma_p}} = \frac{v_e}{v_p}\frac{u_p}{u_e}. \tag{4.4}$$

It is important to underline that the revolving mechanical frequency of an electron and proton around their common center of mass, $\omega_{me} = \omega_{mp} = \omega_m = 2\pi f_m$, is something that should not be mixed or directly (quantitatively) associated to de Broglie matter-wave frequency of the (stationary or orbital) electron and/or proton wave/s, i.e., $\omega_m = 2\pi f_m \neq (\omega_e = 2\pi f_e \neq \omega_p = 2\pi f_p)$, $f_{e,p} \le n \cdot f_m = f_{e,p} \cdot (1 + \sqrt{1 - v^2_{e,p} / c^2}) \le 2f_{e,p}$. *From (4.4) we can also conclude that a wave energy of the stationary electron wave (* $hf_e = (\gamma_e\text{-}1)m_e c^2 = \gamma_e m_e v_e u_e = p_e u_e = E_k$*) is fully equal to the electron's motional energy (meaning that the rest electron mass or its rest energy have no participation in this energy).* Obviously, a relation like (4.4) should be valid for planets rotating around their suns, except that a mass ratio is a different number (and planets in their solar systems should have their associated macrocosmic de Broglie waves). For instance, our planet Earth rotates around its sun and at the same time is spinning around its own planetary axis, performing similar motion as presented in Fig.4.1 (but numerical values of the de Broglie wavelength and frequency are meaningless for such big objects, if microworld Planck constant is relevant in such cases). For macrosystems like solar systems, there is another Planck-analog and much bigger constant H >>> h, with similar meaning as microworld Planck constant h (see more in Chapter 2.).

In any stable atom, there is relatively strong electrostatic (or better to say electromagnetic) coupling and spatial mirror-imaging symmetry between electrons (or electron clouds) and protons in an atom nucleus. This is related to electric and magnetic Coulomb-type forces. Consequently, it should be obvious that the atom nucleus also has some orbital form (or structure) of complex standing waves with motional energy (as its inherent, internal motional energy, including the adequate rest energy, too). In addition, we already know that all atomic and subatomic entities (like electrons, protons, and neutrons) have orbital and spin moments and associated magnetic moments, which are manifesting like permanent magnets. Mentioned magnetic moments are also mutually communicating (or coupling) between the nucleus and its electrons envelope, because magnets could mutually attract or repeal, depending on their positions and magnetic fields polarity (again, mathematically respecting Coulomb type forces). It is now logical to conclude that atom nucleus and belonging (or surrounding) electrons should, on some deterministic ways, electromagnetically communicate and manifest synchronized relations of mutual (like a spherical mirror) symmetry, imaging, and mutual action-reaction or electromagnetic induction effects, between relevant matter-wave groups or standing waves formations. Even if we say that Lucas-Kanarev atom model could be better than N. Bohr's atom model, mentioned electromagnetic communication and structural symmetry (or mutual structural analogy) should still exist in both models. Whatever we consider that is happening with electron clouds (regarding absorptions and emissions of photons and electrons), should have an analog picture, balance, echo, or influence on a similar and synchronous nucleus activity and on its energy structure (because of their electromagnetic fields strong couplings). If we, on some mechanical, electrical, or electromagnetic way, excite an electrons-cloud of a certain atom, its atom nucleus should feel or receive the echo or opposite momentum of the same action (because all known conservation laws should be properly and entirely satisfied, and action and reaction forces and mutual inductions are always synchronously working). It is another question if we would be able to sufficiently excite and destabilize nucleus states only by exciting electrons' clouds (what is maybe possible by creating and stimulating certain resonant state of electrons). On a familiar electromagnetic way, all atoms and molecules inside certain macro particle are mutually coupled (as described by van der Waals forces), meaning that macro particle can be externally agitated or excited, and all belonging nuclear particles should electromechanically and electromagnetically "feel" such excitation. The differences between vibratory electrons' states and states of macromolecular structures are related to their characteristic spectral, resonant, or modal frequencies. For instance, macromolecular vibrational states are covering lower frequency band than electron states, and nucleus states cover higher frequency band than electron states, and all of them are mutually, continuously, and electromagnetically coupled and communicating in both directions in cases of excitations. Something analogical should be valid for planets within any solar system. In fact, described bi-directional electromagnetic communications within and between stationary electron and nucleus states should be extendable (inwards and outwards) between all atoms, other masses and cosmic formations, this way creating *closed circuits flow situations of electromagnetic and mechanical currents and streaming. This is the concept we will follow here in remodeling and extending N. Bohr's atom model.*

Let us now observe possible communication nucleus-electrons in the laboratory system, and in the center-of-mass system. Since we are interested only in aspects of photon emissions and absorptions, we know that in such cases there is not an involvement of the rest masses (similarly as in (8.36) and (8.37)). In this case, the law of conservation of the total energy is identical to the law of conservation of the motional energy, and we can apply it in the following way (as in (4.0.44) and (4.0.47)):

$$\varepsilon_{kj} + \varepsilon_{ke} = \varepsilon_{kc} + \varepsilon_{je} \Leftrightarrow E_{kj} + \tilde{E}_j + E_{ke} + \tilde{E}_e = E_{kc} + \tilde{E}_c + E_{je} + \tilde{E}_{je}, \tag{8.51}$$

This part, starting here, should be reformulated, and generalized.

In the following expressions, we will use the following symbols and values:

m_e, M_p -masses of an electron and a proton,

v_e, v_j, v_c -velocities of the electron, the nucleus, and the mass-center of the electrons-nucleus system,

$v_c = \frac{m_e v_e + M_p v_j}{m_e + M_p} \approx (\frac{m_e}{M_p})v_e + v_j$ -the center of mass velocity.

Now, the corresponding energies (given by their non-relativistic representatives),

$\varepsilon_{kj} = E_{kj} + \tilde{E}_j$ - the total energy of the motion of the nucleus in the laboratory system,

$\varepsilon_{ke} = E_{ke} + \tilde{E}_e \approx \frac{1}{2} m_e v_e^2$ - the energy of the motion of the electron in the laboratory system,

$$\varepsilon_{kc} = E_{kc} + \tilde{E}_c \approx \frac{1}{2}(M_p + m_e)v_c^2 \approx \frac{1}{2} M_p v_c^2 \approx$$
$$\approx (\frac{m_e}{M_p})\frac{1}{2} m_e v_e^2 + \frac{1}{2} M_p v_j^2 + m_e \mathbf{v_e v_j} \approx (\frac{m_e}{M_p})\varepsilon_{ke} + \frac{1}{2} M_p v_j^2 + m_e \mathbf{v_e v_j}$$
- the energy of motion of the mass-center of the electron-nucleus system,

$\varepsilon_{je} = E_{je} + \tilde{E}_{je} = \varepsilon_r \approx \frac{m_e M_p}{2(m_e + M_p)} \left| v_e - v_j \right|^2 \approx \frac{1}{2} m_e v_e^2 \approx \varepsilon_{ke}$ -relative energy of the motion of the electron towards the nucleus, or electron-nucleus reaction energy (see (4.0.62)).

In non-relativistic case (when all figuring velocities of the nucleus and the electron are negligible regarding the light speed), the balance of energy (8.51) becomes,

$$\varepsilon_{kj} + \frac{1}{2}m_e v_e^{\ 2} \approx \frac{1}{2}(M_p + m_e)v_c^{\ 2} + \frac{m_e M_p}{2(m_e + M_p)}\left|\mathbf{v_e} - \mathbf{v_j}\right|^2 . \tag{8.52}$$

In the case of the hydrogen atom, it is logical to assume that $v_j << v_e$, $m_e << M_p$, so (8.51) or (8.52) becomes:

$$\begin{aligned} \varepsilon_{kj} &= E_{kj} + \tilde{E}_j \approx \frac{1}{2}M_p v_j^{\ 2} + (\frac{m_e}{M_p})\frac{1}{2}m_e v_e^{\ 2} + m_e \mathbf{v_e}\mathbf{v_j} \approx \\ &\approx \frac{1}{2}M_p v_j^{\ 2} + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e}\mathbf{v_j}. \end{aligned} \tag{8.53}$$

In previous expressions, one should distinguish the total motional energy ε_k from the energy of particle motion E, in the same way, as differentiated in (4.12). If we start from the fact that photoexcitation of the atom has no interference with the rest mass of the nucleus, i.e., that it is relevant only for the electron stationary states in the shell, which is correct in essence, then one may transform (8.53) in the following way:

$$\begin{aligned} \varepsilon_{kj} &= E_{kj} + \tilde{E}_j \approx \frac{1}{2}M_p v_j^{\ 2} + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e}\mathbf{v_j} \approx \\ &\approx E_{kj} + (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e}\mathbf{v_j}, \\ &\Rightarrow \tilde{E}_j = (\frac{m_e}{M_p})\varepsilon_{ke} + m_e \mathbf{v_e}\mathbf{v_j} \approx (\frac{m_e}{M_p})\varepsilon_{ke} . \end{aligned} \tag{8.54}$$

From (8.54) we can determine the total motional energy of an electron,

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})(\varepsilon_{kj} - E_{kj} - m_e \mathbf{v_e}\mathbf{v_j}) . \tag{8.55}$$

In the first approximation, starting from the fact that the atom nucleus observed as a particle, is effectively at rest. An imaginable velocity of the nucleus motion is almost equal to zero, and undoubtedly far smaller than the velocity of the electron (thereby, the particle energy of the motion of the nucleus, E_{kj}, will be equal to zero) the previous relation, (8.55), will become,

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})\varepsilon_{kj} \ \ , \ (E_{kj} + m_e \mathbf{v_e}\mathbf{v_j} \approx 0)$$

$$\varepsilon_{ke} \approx (\frac{M_p}{m_e})(\varepsilon_{kj} - E_{kj} - m_e \mathbf{v_e v_j}). \tag{8.56}$$

We may now ask ourselves what happens if an atom is hit (excited) by a photon or emits a photon, i.e., we can apply the second Bohr 's postulate (8.31) on (8.56), so one gets,

$$\begin{aligned} &\tilde{E}_f = hf_{e12} = h(f_{s1} - f_{s2}) = \varepsilon_{s2} - \varepsilon_{s1} = \Delta\varepsilon_s = \Delta\varepsilon_k = \\ &= \Delta\varepsilon_{ke} \approx (\frac{M_p}{m_e})\Delta\varepsilon_{kj} = 1836.13\Delta\varepsilon_{kj} \approx 1836.13 \cdot \frac{\gamma_p}{\gamma_e} \cdot \frac{(u_p\tilde{E}_e)^2}{(u_e\tilde{E}_p)^2}\Delta\varepsilon_{ke} \approx 1836.13 \cdot \frac{(u_p\tilde{E}_e)^2}{(u_e\tilde{E}_p)^2} \cdot \Delta\varepsilon_{ke}, \\ &(\frac{M_p}{m_e} \approx \frac{\Delta\varepsilon_{ke}}{\Delta\varepsilon_{kj}}) = 1836.13 = \frac{\gamma_e \cdot (u_e\tilde{E}_p)^2}{\gamma_p \cdot (u_p\tilde{E}_e)^2} \approx \frac{(u_e\tilde{E}_p)^2}{(u_p\tilde{E}_e)^2}, \\ &\Rightarrow m_e\Delta\varepsilon_{ke} \approx M_p\Delta\varepsilon_{kj}, \ \frac{\tilde{E}_e{}^2\Delta\varepsilon_{ke}}{\gamma_e u_e{}^2} = \frac{\tilde{E}_p{}^2\Delta\varepsilon_{kj}}{\gamma_p u_p{}^2}, 1836.13 \cdot \gamma_p \cdot (u_p\tilde{E}_e)^2 = \gamma_e(u_e\tilde{E}_p)^2 . \end{aligned} \tag{8.57}$$

Starting from the fact that every change of motional energy is accompanied by the creation of the corresponding part of wave energy (neglecting the change of internal energy or rest mass of particles, which is at this time quite justified, we may formally transform (8.57) into

$$\begin{aligned} &m_e\Delta\varepsilon_{ke} \approx M_p\Delta\varepsilon_{kj} \Leftrightarrow m_e hf_{e12} \approx M_p hf_{j12} \\ &\Leftrightarrow m_e f_{e12} \approx M_p f_{j12}. \end{aligned} \tag{8.58}$$

Based on (8.57) and (8.58) a conclusion obtruded is that every inter-orbital change of stationary energy of an electron is accompanied by some similar response in a form of a coincident change of (some symmetrical) stationary energy levels of the nucleus (regardless of what it meant), especially because of strong Coulomb and magnetic moments forces acting between them. All those changes of the stationary levels are accompanied by emission or absorption of photons (in the electron shell, as well as around the atom nucleus). In fact, (8.58) represents the reaction or echo of the atom nucleus to the change that takes place in the atom electrons shell. This is logical and quite reasonable as well as from an angle of the law of conservation of impulses.

Of course, if there is some wave reaction of the atom nucleus to the change that hits its electron shell, this will be vectors (impulses) with the same direction, and with opposite senses.

Since the masses of the electron and the proton are known, it is possible to express (8.58) in a form of numerical relations,

$$f_{j12} \approx 5.446 \cdot 10^{-4} f_{e12}, \ f_{e12} \approx 1836.1 \cdot f_{j12} . \tag{8.59}$$

From (8.58) and (8.59) we see that it is likely that there are radiating and absorptive spectra of the atom nucleus that are synchronous and coincident with absorptive and radiating spectra of the electron shell with known relations of proportionality between corresponding frequencies. Imaginatively, analogically, and creatively thinking, we could (one day) create LASERs based on stimulation and excitation of nuclear atom states, and since all atoms and masses should mutually communicate like coupled resonators, we could analogically extend LASER applications to gravitation.

There is another question; -how to detect previously predicted (still hypothetical) nucleus spectrum, and whether some interferences between the nucleus-spectrum and electron spectrum occur or not. **It is immediately noticeable that hypothetical nucleus spectrum (or spectral ECHO of the nucleus (8.59)) will be in the infrared and microwave part of the spectrum (partly overtaking the area of millimeter wavelengths *<u>and cosmic, background or relict radiation</u>*). Another possible consequence is that Planck's law of blackbody radiation would be supplemented by a similar law of radiation, which is translated into a domain of lower frequencies for a factor** $5.446 \cdot 10^{-4}$ **(**or $f_{j12} \approx \boxed{5.446 \cdot 10^{-4}} \cdot f_{e12}$**), with a much smaller intensity and greater frequency density of corresponding wave components.** Perhaps, on this occasion, we could also pose a question about the existence of communication between stationary energy levels of an atom nucleus and its electrons' shell, manifesting as some quantized spectra of photons, which covers frequency interval of the background cosmic radiation.

We can now present (8.59) in a complete form that defines the combinational spectrum of the electromagnetic emission or absorption of the atom, using (8.9) or (8.41),

$$f_{e12} = \frac{m_e Z^2 e^4}{8h^3 \varepsilon_0^{\ 2}} \left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right),$$

$$f_{j12} \approx \left(\frac{m_e}{M_p}\right) \frac{m_e Z^2 e^4}{8h^3 \varepsilon_0^{\ 2}} \left(\frac{1}{n_1^{\ 2}} - \frac{1}{n_2^{\ 2}}\right). \tag{8.60}$$

Knowing the relations between masses of an electron and a proton, as well as between frequency and the wavelength of the electromagnetic quantum (based on calculations from (8.60)), it is possible to obtain the following estimates for frequency and wavelengths of hydrogen spectrum,

$$\frac{f_{e12}}{f_{j12}} = \frac{\lambda_{j12}}{\lambda_{e12}} \approx \left(\frac{M_p}{m_e}\right) \approx 1836.1,$$

$$f_{e12} \leq 3.2898417 \cdot 10^{15}\,\mathrm{Hz},\ \lambda_{e12} \geq 0.091126714\mu\mathrm{m}, \tag{8.61}$$

$$f_{j12} \leq 1.7917551 \cdot 10^{12}\,\mathrm{Hz},\ \lambda_{j12} \geq 167.31776\mu\mathrm{m}.$$

Since the hydrogen atom is one of the most prevalent elements in the cosmic universe, the previously given quantitative relations (8.61) should be almost universally valid. If we like to include all possible atoms heavier than the hydrogen atom, we could broaden the meaning of the relation (8.58), considering that the nucleus mass of any atom is equal to the sum of masses of the protons and the neutrons that constitute the nucleus, so we shall have:

$$Z \cdot m_e f_{e12} \approx \left[ZM_p + (A-Z)M_n \right] f_{j12} \approx AM_p f_{j12} . \tag{8.62}$$

If we assume that all possible atoms and their isotopes (known and still unknown) will be incorporated with atom number $A \leq 6Z$, $Z_{max.} \leq 137$, and (8.58), (8.62) and (8.44) will generate the following relations,

$$\frac{f_{e12}}{f_{j12}} = \frac{\lambda_{j12}}{\lambda_{e12}} \leq 6(\frac{M_p}{m_e}) \approx 6 \cdot 1836.1 = 11016.6 ,$$
$$f_{e12} \leq Z \cdot 3.2898417 \cdot 10^{15} \text{Hz} \cong 4.507083129 \cdot 10^{17} \text{Hz} , \lambda_{e12} \geq \frac{1}{Z} \cdot 0.091126714 \mu\text{m} \cong 0.665158 \text{nm} , \tag{8.63}$$
$$f_{j12} \leq \frac{Z}{6} \cdot 1.7917551 \cdot 10^{12} \text{Hz} \cong 4.091174145 \cdot 10^{13} \text{Hz} , \lambda_{j12} \geq \frac{6}{Z} \cdot 167.31776 \mu\text{m} \cong 7.327785109 \mu\text{m} .$$

At the end, one could pose a question: Is it the relict, background radiation of the universe an exclusive reflection of the primary cosmic explosion (**BIG-BANG**), or it is maybe connected (or superimposed) with continuously present, intrinsic "*blackbody familiar «**ECHO**» and noise radiation from atoms nuclei*" (as previously described within (8.60) – (8.63))? If we try to apply general relations of "***elementary certainty***" (4.0.64) from chapter 4.0, and (5.1), from chapter 5., i.e., Relations of Uncertainty, to intervals of the relevant spectral and time duration of the background radiation of the universe, as known from the measurements of such background radiation, and from other theoretical estimates, it will become evident that those relations cannot be convincingly or mathematically satisfied. In other words, the current pattern of the primary cosmic explosion has significant, at best for that concept, quantitative faults (even though it is generally considered that the Big-Bang hypothesis is conceptually and by measurements well justified).

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*Familiar analyzes, concepts, and conclusions (at least qualitatively) can be found in the works of Dr. George Shpenkov, addressing cosmic background radiation, and electromagnetic exchanges between atom nucleus and electrons' shell (see literature references under **[85],** organized and summarized by Victor Christianto. Similar and much wider problematic (also related to matter-wave equations and atom models) is discussed and elaborated by Victor Christianto in his article **[86]:** "Review of Schrödinger Equation & Classical Wave Equation". V. Christianto is an Independent Researcher; -URL: http://www.sciprint.org, Email: victorchristianto@gmail.com or admin@sciprint.org . Phone: (62) 341-403205 or (62) 878-59937095.).*

Citation from [86]: *"Kreidik & Shpenkov derive microwave background radiation of hydrogen atom based on Shpenkov's interpretation of classical wave equation. They conclude that the Microwave Background Radiation, observed in Cosmos, apparently is the zero-level (background) radiation of all atoms in the Universe. Following their dynamic model, the H-atom is a paired dynamic system with the central spherical micro-object of a complicated structure (proton) and the orbiting electron. The electron in H-atom under the wave motion exchanges the energy with the proton constantly at the fundamental frequency* ω_e*. This exchange process between the electron and proton has the dynamic equilibrium character. It is represented by a system of radial standing waves, which define "zero level exchange" in a dynamically stable state of the atom. At p = 0, they obtain* $\lambda = 0.106267\,\text{cm}$*, and then they can find an estimate of the absolute temperature of zero level of radiation:*

$$T = \frac{0.290\,\text{cm} \cdot \text{K}}{\lambda} = 2.7289\,\text{K} \cong \Delta\text{K} \tag{22}$$

Where $\Delta = 2\pi \lg e = 2.7288$ *is the measure of the fundamental period (fundamental quantum measures). The temperature obtained coincides with the temperature of "relict" background measured by NASA's Cosmic Microwave Background Explorer (COBE) satellite to four significant digits (2.725 ±0.002K). The concept of zero level radiation of H-atoms* *question quantum mechanical probabilistic model, which excludes an electron's orbital motion along a trajectory as a matter of principle. "*

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Bohr's hydrogen atom-model is a quite simple one, often experimentally tested and proven applicable in spectral characterizations. By combining N. Bohr's planetary model with the concept of de Broglie matter waves (see Fig. 4.1 and equations (4.1), (4.2) and (4.3), from Chapter 4.1), we are indirectly testing and proving the hypothesis (of this book) claiming that every linear motion should be accompanied with certain mater-field spinning, and such spinning naturally creates de Broglie matter waves, producing correct result as in (4.4).

We know that electrons and positrons should also be specific, self-standing, stabilized formations like packets of photons (or specifically structured, standing electromagnetic waves), and that electrons and positrons can be fully transformed into photons (as largely elaborated in Chapter 4.1 of this book). In addition, based on Compton and Photoelectric effects, we know that electrons, positrons, and photons also behave as particles or waves, respecting **PWDC** rules (as summarized in Chapters 4.1 and 10.). Here we are anyway intuitively gravitating around different motional and formatting aspects of an electromagnetic field phenomenology. Consequently, we could consider an atom as a structure of a certain set of mutually coupled, electromagnetic resonant states or resonators. One part of such resonant states belongs to the atom nucleus, and the other, mirror-set of coupled energy states, belongs to its electron-cloud states. ***There is a third part of an <u>external atom space</u> where gravitation acts as the consequence of radiant electromagnetic energy exchanges between resonant states of atoms and other masses***. All such resonant states can absorb or emit photons with very precise, mutually matching frequencies, what results in discrete (quantized) internal and external energy exchanges, where, again, mutually transformable photons, electrons, nucleus states and their matter-waves, are involved. The real meaning of quantizing in the microworld of atoms and subatomic structures is that every microworld entity presents certain set of specific, well-defined, and stable resonant states (equivalent to standing waves generating oscillatory circuits). Mentioned oscillatory configurations are mutually (electromechanically and electromagnetically) coupled and communicating internally and externally (like coupled piezoelectric and magnetostrictive elements), by emitting, absorbing, and exchanging specific electromechanical and electromagnetic matter-waves and photons (including electrically charged entities). In our measurements and analyzes, we observe such matter waves as spinning particles, electrons, photons etc. Of course, stable resonant states are also presentable as self-closed standing waves formations. The quantum nature behind this reality is just the consequence of coupling and interactions between excited resonant circuits and created matter waves, operating on series of specific resonant frequencies, and this is slightly different, compared to what Quantum Theory is promoting as quantizing. Of course, that between self-standing resonators all energy exchanges are fixed, or being quantized amounts.

8.3. Structure of the Field of Subatomic and Gravitation related Forces

The idea here is to upgrade (mathematically, intuitively, and hypothetically) the concept of N. Bohr's, or any other similar atom model, from an angle of a field theory. Atoms are elementary building blocks or content of macro-world matter that has mass properties with non-zero rest masses. Atoms are also sets of resonators or oscillating formations with self-stabilized and periodicities-related structures of standing matter waves. Atoms are continuously mutually communicating (internally and externally) by exchanging (at least) photons. Between atoms and other masses agglomerations exist the force of gravitation among electromagnetic forces, and most probably that gravitation also has its deep ontological roots in electromagnetic fields and forces that are an extension of atomic field-forces. All kind of mutually similar, or *by spectral characteristics overlapping resonant and oscillating structures*, with intrinsic, internal, and structural periodicity properties are mutually and continuously communicating (being acoustically, electromechanically, and electromagnetically coupled), this way creating specific forces and fields, around and internally. This coupling-activity also presents *atoms structure, atomic fields, and forces mutual synchronizations, and extension* towards bigger, agglomerated masses, effectively making those bigger masses as kind of augmented (united, synchronized and superimposed) "macro atoms structures", and this is the essential origin or link between an internal *atomic field-force* and Gravitation.

Starting from results as found in this chapter under (8.1) – (8.50), and in second Chapter, under (2.11.14), we can imaginatively and mathematically assemble what could be the structure of underlaying and universal, atomic-field force (acting internally inside atoms, and externally outside atoms). Let us assume that all occurrences connected with stationary energy states of electrons and an atom nucleus take place in the space where a specific and complex structure of mentioned material force (for instance, described qualitatively as Rudjer Boskovic Universal Natural Force) exists. We will denote, in spherical coordinates, the function of such complex (atomic) force as $\mathbf{F(r,\theta,\phi,t)}$, where: $\mathbf{r}$ is the radius, $\boldsymbol{\theta,\phi}$ are angular coordinates, and $\mathbf{t}$ is a time coordinate.

We assume that the function of the force of an atomic field is continuous and limited (or finite):

$$\mathrm{Lim}\ \mathrm{F(X)}_{x\to A} = \mathrm{F(A)},\quad \mathrm{Lim}\ \mathrm{F(r)}_{r\to R\leq\infty} = 0 \qquad (8.64)$$

Let $\mathrm{R}(\mathrm{r},\theta,\phi,\mathrm{t})$ be the radius vector of electrons' orbits of inter-atomic states in an atomic field. The element of an electron matter-wave orbit described by the radius vector $0 \leq \mathbf{r} \leq \mathbf{R(r,\theta,\varphi,t)} < \infty$ may be presented as,

$$\mathbf{dR^2 = dr^2 + r^2\cdot d\theta + r^2\cdot \sin^2\theta\cdot d\phi,\quad 0\leq\theta\leq 2\pi,\quad -\frac{\pi}{2}\leq\phi\leq\frac{\pi}{2}.} \qquad (8.65)$$

We should always have in mind that any of modern and still applicable atom models (starting from Bohr's model), as created, known, or initiated from Rutherford

scattering experiments, has some central zone where positively charged atom nucleus is (with protons and neutrons structured as mater waves), and an external nucleus-envelope, or "electrons cloud zone", or specifically structured, negatively charged "electrons field states". Both (nucleus states and electron matter-wave states) have electric, magnetic, orbital and spin moments, and electromagnetic dipole attributes (meaning that atoms are dynamic, internally coupled resonant and standing-waves structures in certain close to perpetual motion). Mentioned <u>electromagnetic-nature electron states</u> are mutually coupled and interacting, meaning that everything what happens within electron states or clouds should (analogically and synchronously) have a certain mirror-image response or mapping, being a similar structure with periodicity properties within atom-nucleus states. That means we should be able to describe atom nucleus resonant states and structure on a similar way as describing electron cloud structure.

Let us start with describing internal atom-field structure and properties. Now we can give a simplified definition of a stationary electron orbit $\mathbf{C_n}$. This is an orbit whose perimeter $\mathbf{a_n}$ is equal to the mean value of the wavelength $\overline{\lambda}_n$, of a stationary, or standing electron wave that covers orbit $\mathbf{C_n}$, multiplied by the main quantum number $\mathbf{n}$. This results from the first Bohr's postulate, as well as from the meaning of standing waves, and relates to de Broglie's wavelength; -see (8.10) - (8.11), and familiar conceptualization in Chapter 10.),

$$
\begin{aligned}
&a_n = \oint_{C_n} dR = \oint_{C_n} dr = n\overline{\lambda}_n = n\frac{h}{\tilde{p}_n},\\
&\overline{\lambda}_n = \frac{1}{n}\oint_{C_n} dR = \frac{1}{n}\oint_{C_n} dr = \frac{a_n}{n} = \frac{1}{n}\sum_{i=1}^{n}\lambda_{ni} = \frac{h}{\tilde{p}_n}.
\end{aligned}
\tag{8.66}
$$

If we imagine that states of an atom-nucleus should be a kind of spherical or spatial mirror image of electron states, because of Coulomb forces acting between them, certain similar standing-waves situation within a nucleus (as described with (8.66)), should also exist there.

A much better replacement for (8.66), for any stable periodical motion on a closed and stationary orbit, was introduced by Wilson-Sommerfeld action integral (as the more general quantifying rule, see [9]), applied over one period of the motion. Here, the same Wilson-Sommerfeld action integrals will be formulated in a bit modified form (to be consistent with the concept of Particle-Wave Duality presented in this paper) as,

$$
\begin{aligned}
&\oint_{C_n} p_q dq = \left[\oint_{C_n} d(p_q q) - \oint_{C_n} q dp_q\right] = n_q h = \frac{\tilde{E}'_{nq}}{f'_q} = \frac{hf'_{nq}}{f'_q}, \tilde{E}'_{nq} = hf'_{nq} = hn_q f'_q = \left[\oint_{C_n} p_q dq\right]\cdot f'_q,\\
&\left[\oint_{C_n} d(p_q q) = 0, \oint_{C_n} q dp_q = -n_q h,\ f'_{nq} = n_q f'_q\right],
\end{aligned}
$$

$$\oint_{C_n} L_q dq = \left[\oint_{C_n} d(L_q q) - \oint_{C_n} q\, dL_q\right] = n_q h = \frac{\tilde{E}"_{nq}}{f"_q} = \frac{hf"_{nq}}{f"_q}, \tilde{E}"_{nq} = hf"_{nq} = hn_q f"_q = \left[\oint_{C_n} L_q dq\right] \cdot f"_q$$

$$\left[\oint_{C_n} d(L_q q) = 0, \oint_{C_n} q\, dL_q = -n_q h, f_{nq} = n_q f"_q\right],$$

$$q = (r, \theta, \varphi, ...), r = r(x, y, z...), n_q = \text{integer} (= 1, 2, 3, ...).$$

In the Wilson-Sommerfeld action integral we generalized different momentum states (concerning linear motion and rotation), by using electromechanical analogies (as established in the first chapter of this book), we can (at this time still analogically and hypothetically) extend the Wilson-Sommerfeld' action integral to electric and magnetic charges, as follows,

$$\left.\begin{array}{l}\oint_{C_n} \Phi_{electr.} d\Phi_{magn.} = \left[\oint_{C_n} d(\Phi_{electr.}\Phi_{magn.}) - \oint_{C_n} \Phi_{magn.} d\Phi_{electr.}\right] = n_{electr.} h = \frac{\tilde{E}_{n-electr.}}{f_{electr.}} = \frac{hf_{n-electr.}}{f_{electr.}}, \\ \oint_{C_n} \Phi_{magn.} d\Phi_{electr.} = \left[\oint_{C_n} d(\Phi_{electr.}\Phi_{magn.}) - \oint_{C_n} \Phi_{electr.} d\Phi_{magn.}\right] = n_{magn.} h = \frac{\tilde{E}_{n-magn.}}{f_{magn.}} = \frac{hf_{n-magn.}}{f_{magn.}},\end{array}\right\rangle \Rightarrow$$

$$\oint_{C_n} d(\Phi_{electr.}\Phi_{magn.}) = 0, \oint_{C_n} \Phi_{magn.} d\Phi_{electr.} = -n_{electr.} h, \oint_{C_n} \Phi_{electr.} d\Phi_{magn.} = -n_{magn.} h$$

$$n_{electr.} = -n_{magn.}, f_{n-electr.} = n_{electr.} f_{electr.}, f_{n-magn.} = n_{magn.} f_{magn.},$$

$$\tilde{E}_{n-electr.} = hf_{n-electr.}, \tilde{E}_{n-magn.} = hf_{n-magn.}, (n_{electr.}, n_{magn.}) = \text{integer} (= 1, 2, 3, ...) .$$

By mathematical and analogical manipulation of Wilson-Sommerfeld action integrals, here, an attempt is made to introduce the idea that Planck's quantization law $\tilde{E} = nhf$ *is incorrectly interpreted, even by Planck, and that better interpretation should be based on* $\tilde{E} = hf(n)$*). More precisely, in this paper is already underlined that a mechanical revolving frequency of a particle should not be mixed with the associated matter-wave frequency (or with the frequency of an associated de Broglie matter wave; -see equations (8.28) and (8.42)), as the consequence of the correct treatment of the phase and group velocity). The same comment should be automatically addressed to Planck's postulate regarding the energy of simple harmonic oscillators (in relation to blackbody radiation). Of course, the intention of the author of this book is to say that modern Quantum Mechanics concept of phase velocity (of de Broglie matter waves) is not fully correct. Luckily, Planck and Einstein produced a smart, tricky and, only as the result, correct mathematical function for the energy density of a blackbody radiator, and this result has been so good that quantization law* $\mathbf{\tilde{E} = nhf}$ *was easily and wrongly interpreted as correct. Here, we also see a quite simple and basically correct picture of the quantum nature of matter, as the consequence of a proper gearing or resonant fitting between involved internal field properties, by creating self-closed standing matter waves, which can be characterized by integers. Natural laws of quantizing in Physics should be based on Kotelnikov-Shannon-Nyquist-Whitaker signal sampling and reconstructing methods (see more in literature under [57], [109], [110], and [111]).*

We may now define the energy of the stationary electron wave (i.e., reformulating the second Bohr's postulate) through the work of the atomic field force (4.1.64), as well as using the wave function of the stationary electron wave (see (4.0.1), to (4.0.5) from Chapter 4.0, and equations from (4.9) to (4.10-5) from Chapter 4.3), on the following way,

$$\tilde{\varepsilon}_{sn} = \oint_{C_n} \mathbf{F} d\mathbf{R} = \int_{C_n:[\Delta t]} \Psi_n^{\,2}(t)dt = \int_{C_n:[\Delta p]} v_n dp = \iint_{\sigma_n} (\nabla \times \mathbf{F}) d\sigma = hf_{sn} \neq 0 \,, \quad (8.67)$$
$$\Rightarrow \nabla \times \mathbf{F} \neq 0 \,.$$

We shall determine the energy of an electromagnetic quantum of the inter-orbital exchange (by absorption or emission of photons), as (see (4.0.1), to (4.0.5) from Chapter 4.0, and equations from (4.9) to (4.10-5) from Chapter 4.3),

$$\tilde{\varepsilon}_{m,n} = \oint_{[C_m - C_n]} \mathbf{F} d\mathbf{R} = \int_{[\Delta t]} \Psi_{m\text{-}n}^{\;2}(t)dt = \int_{[\Delta p]} v_{m\text{-}n} dp = h(f_n - f_m) = hf_{mn} \neq 0 \,, \quad (8.68)$$
$$\Rightarrow \nabla \mathbf{F} \neq 0 \,.$$

Again, it is good to say that stationary energy levels C_m and C_n could be inside of an atom among its electron and/or nucleus states.

Now, let us go back to the atomic force field $\mathbf{F(r,\theta,\phi,t)}$. From (8.67) and (8.68) we see that $\nabla \times \mathbf{F} \neq 0$ and $\nabla \mathbf{F} \neq 0$, so according to the general classification of fields in differential geometry, we must consider force field $\mathbf{F(r,\theta,\phi,t)}$ as a complex vector field, which may be always decomposed in two more elementary vector fields that are,

$$\mathbf{F}(r, \theta, \varphi, t) = \mathbf{F_1}(r, \theta, \varphi, t) + \mathbf{F_2}(r, \theta, \varphi, t) \,, \quad (8.69)$$

from which the first is **a potential field**:

$$\nabla \times F_1 = 0 \,, \; \nabla F_1 \neq 0 \,, \quad (8.70)$$

and the second is a **solenoidal field**:

$$\nabla \times F_2 \neq 0 \,, \; \nabla F_2 = 0 \,, \quad (8.71)$$

therefore, instead of (8.67) and (8.68), we may introduce new definition that replaces them,

$$\tilde{\varepsilon}_{sn} = \oint_{C_n} \mathbf{F}_2 d\mathbf{R} = \int_{C_n:[\Delta t]} \Psi_{2n}^{\;2}(t)dt = \int_{C_n:[\Delta p]} v_{2n} dp =$$
$$= \iint_{\sigma_n} (\nabla \times \mathbf{F}_2) d\sigma = hf_{sn} \neq 0 \,, \quad (8.72)$$
$$\Rightarrow \nabla \times \mathbf{F}_2 \neq 0 \,,$$

and

$$\tilde{\varepsilon}_{m,n} = \oint_{[C_m - C_n]} \mathbf{F}_1 d\mathbf{R} = \int_{[\Delta t]} S_{m\text{-}n}(t)dt = \int_{[\Delta t]} \Psi_{m\text{-}n}^{\;2}(t)dt = \int_{[\Delta p]} v_{m\text{-}n} dp = \quad (8.73)$$
$$= h(f_n - f_m) = hf_{mn} \neq 0 \,, \;\Rightarrow\; \nabla \mathbf{F}_1 \neq 0 \,.$$

Here, we should explore the possibility that mentioned potential and solenoidal vector fields (F_1 and F_2) are, when presented using Hilbert transform and an Analytic Signal model, mutually coupled and phase-shifted electric and magnetic fields. After applying proper mathematical processing and dimensional arrangements we could try to prove that wave functions of certain mutually-coupled electric and magnetic force or field function behave as $\Psi(t) \simeq F_1(t)$ and $\hat{\Psi}(t) \simeq F_2(t)$, meaning creating certain electromagnetic-field Analytic signal (similar to Pointing vector) that is representing the total force function $\mathbf{F}(r, \theta, \varphi, t) = \mathbf{F_1}(r, \theta, \varphi, t) + \mathbf{F_2}(r, \theta, \varphi, t)$. *See more about Analytic Signals in Chapter 4.0.*

To treat an atom as a stable structure (both internally and externally), one should add the condition of balance of the potential energy of all attractive and all repulsive forces what secures atom stability, inwards and outwards, or internally and externally, such as,

$$\oiiint_{\left\{ \begin{array}{l} R\in[0,\infty], \\ \theta\in[0,2\pi], \\ \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right] \end{array} \right\}} \mathbf{FdR} = \mathbf{0} . \qquad (8.74)$$

The condition of a balance of potential energy of all attractive and all repulsive forces (8.74) within and around an atom may be supplemented by a hypothesis about the existence of permanent communication (by an interchange of electromagnetic quanta), between stationary states of a nucleus and electrons shell of an atom including similar external exchanges between atoms and other masses. Involved energy-moments states should communicate synchronously and coincidently, in both directions, within the internal space of an atom (i.e. could not be noticed in the external atom space, if an atom is really and totally neutral, self-standing, stable, not-connected to other atoms, and non-excited), but since atoms are also mutually attracting and agglomerating thanks to $1/r^2$ "external" and central forces, we know that standing matter-waves field structures should also exist in atoms' external space between mutually separated masses. Here we could profit from a similar conceptualization, hypothetically saying that mentioned (bidirectional) electromagnetic, quantized exchanges between atom nucleus and its electron shell are also extendable almost endlessly towards infinity of an outer atom space (meaning inside and deeply outside of atoms, towards other atoms, masses, and cosmic constellations). In other words, all atoms (and oscillating masses) are on such way mutually synchronized and connected with our Universe, continuously radiating, and receiving electromagnetic waves, externally and internally (while creating standing matter-waves structures between them). That also means that all other compact mases, pieces of metals, stones, monuments, pyramids, and mountains, including alive species of plants and animals are on some way internally, externally, and mutually synchronized, active, dynamic, and on different levels alive and connected. This is like having atoms (and other masses) as continuous sources and sinks of Tesla's radiative energy and qualitatively structured as Rudjer Boskovic Universal Natural Force. Since we know that between mases is acting Newton $1/r^2$ gravitation force, and this is at the same time analogue to Coulomb force between electric charges, we could significantly simplify the same situation, visualizing gravitationally attracting masses as attraction between two electric charges, or between two permanent magnets (where each mass has certain, directly proportional amount of available electric-dipoles charges, or elementary permanent magnets, conveniently aligned to experience $1/r^2$ attractive force. See more in Chapter 2. under: "2.1.1 More about Newtonian Gravitation based on analogies., 2.2. Generalized

Coulomb-Newton Force Laws., and 2.2.1. WHAT THE GRAVITATION REALLY IS”). The Universe reaction to such radiative energy, omnidirectional communications, are action and reaction forces, always acting synchronously. Mentioned external reaction-forces are like cohesion forces and *belong to attractive effects of Gravitation (and in some cases to other electromagnetic interactions). Practically, all atoms, different particles, bigger objects, and our Universe are, permanently and bi-directionally, communicating by radiating, and exchanging electromagnetic energy, while receiving, an echo of attractive Gravitational forces, as conceptualized in Nikola Tesla’s Dynamic Gravity, and in Rudjer Boskovic’s universal natural force law theories, [97] and [6].*

Most probably, Casimir effect is also a familiar and measurable manifestation of mentioned "radiant energy flow” or a physical-vacuum fluctuations (see [103] and [104]). The next chapter of this book (9. BLACK BODY RADIATION & PHOTONS) is additionally addressing similar ideas.

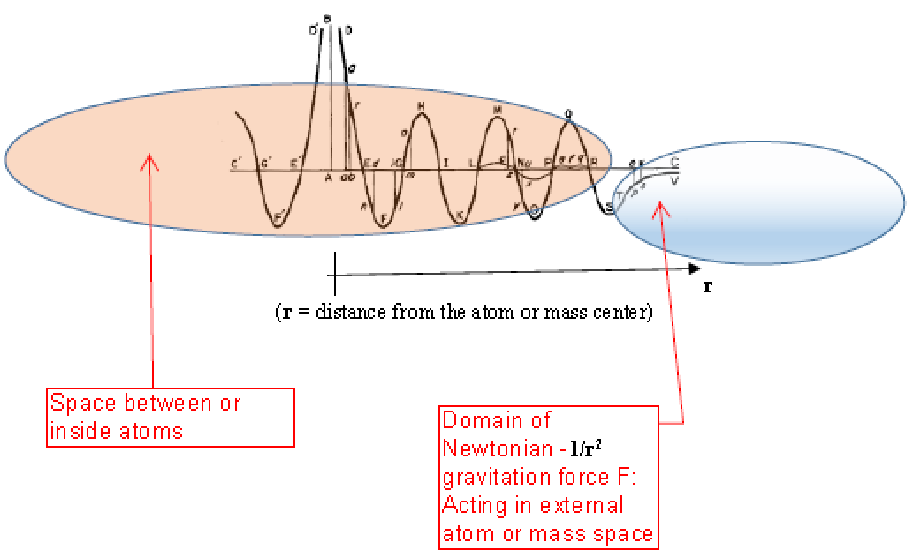
Embryonic roots of such interpretation of an atomic force field are found in the publications of Rudjer Boskovic about universal natural force ([6], Principles of the Natural Philosophy), as well as in certain papers published in «Herald of Serbian Royal Academy of Science» between 1924 and 1940 (J. Goldberg 1924; V. Žardecki 1940). Nikola Tesla’s, [97], dynamic theory of Gravitation is also close to Rudjer Boskovic’s unified natural force (see the picture below), and to the here-elaborated concept, summarized with (8.74). Another source of familiar ideas we can find in [73], Reginald T. Cahill, Dynamical 3-Space; -Emergent Gravity and in [117], from Jean de Climont.

Briefly summarizing we could say that subatomic and interatomic field of forces $\mathbf{F}(r, \theta, \varphi, t)$ is on a similar structural and dynamic way extending outside atoms (and between separated masses) and such externally manifesting fields are mutually synchronizing, coupling, communicating, and exchanging matter waves and photons (between any of two or more atoms and/or bigger masses). This coupling (being essentially of an electromagnetic nature; -see somewhat familiar ideas in [33] and [71]), is creating Newton force of gravitation. Of course, here introduced force field $\mathbf{F}(r, \theta, \varphi, t)$ described by (8.64) – (8.47) could also be different and more complex, being guided by some much better atom modeling compared to quite simple N. Bohr’s atom model, but the idea and conclusions about natural forces and our world structure would still stay the same as already elaborated. Atoms are anyway, presenting resonant and standing-waves structures, and effects of mutual synchronizations and couplings (including entanglement effects) are known and typical within such structures. Quantizing in Physics is causally related to atomic and other standing matter-waves structures. To support mentioned coupling and synchronizing between atoms and masses, we also need to have certain intermediary fluidic medium, or material carrier of involved matter waves, which could be named as an ether. In addition, we should upgrade our conceptualization and understanding of electromagnetic fields on a fully symmetrical and mutually integrated and mutually dependent electric and magnetic fields, as suggested in the third Chapter of this book, since different induction laws and Lorentz forces are responsible for electro-mechanical couplings between atoms, masses, and mechanical and electromagnetic moments, what we can consider as effects of Gravitation. Here we extend or extrapolate fields inside atoms towards macro masses. In the second Chapter (about Gravitation), we can find that macrocosmic systems (such as planetary systems), are analogically behaving or structuring as atoms (see “2.3.3. Macro-Cosmological Matter-Waves and Gravitation”), this way closing the conceptualization loop about the same idea.

One may just imagine how more picturesque, interesting, and richer would be the physics if the previously introduced ideas (or hypotheses) have not been completely missed, bypassed, or ignored in modern physics.

Apart from stable and neutral atoms, we also know that non-stable, naturally radioactive atoms exist, meaning that standing-waves structured, electromagnetic atomic force or field $\mathbf{F}(\mathbf{r}, \theta, \phi, \mathbf{t})$ *as defined by (8.67) - (8.74), can in number of cases be, on some way, unstable. This also means that we should be able to externally apply certain (artificially created) electromagnetic field, acting around naturally radioactive matter, and manipulate its natural instability and radioactivity, by channelling, or stabilizing, neutralizing, transmitting, transmuting, or additionally destabilizing treated atoms, or stimulating radiations from such atoms.*

Citation from the internet; http://www.irb.hr/50g/eng/3/index.htm: "Rudjer Boskovic was the first to realize that the interatomic forces were something else than a sheer gravitation. In his main work, Theory of Natural Philosophy (Venice, 1763) he developed the concept of the atom as a point-like center of force. To explain cohesion on a microscopic level, he postulated the existence of forces between molecules, whose direction and intensity are distance dependent. According to Boskovic's ideas, forces (between solid particles and atoms) change the sign infinitely many times with decreasing distance (between solid particles and atoms); this requires oscillating potentials, or infinitely many relative minima, which is the essence of the nuclear picture and of quantum stationary states. His ideas exerted a strong influence on the leading physicists and chemists of the 19th century. Indeed, more than a century later such forces were introduced into modern physical chemistry and are known today as London-van der Waals forces. Rutherford took Boskovic's atom as a cornerstone of his, today canonic, picture of the atom".



Rudjer Boskovic's universal force function between solid particles and atoms

Traditional N. Bohr atom model is describing only electromagnetic, or photons and electrons related exchanges between electrons energy states and external atom space, and inside or within different energy levels of electrons. Here belongs Compton and Photoelectric effect, secondary emissions, lasers, diffraction effects, atoms ionizing etc.

The extended atom model (as conceptualized here) is underlining (still hypothetically) bidirectional electromagnetic or photonic energy exchanges between electrons' energy states and on some way mirror-symmetrical nucleus energy states, as well as similar energy exchanges inside nucleus energy states. Possible candidates involved in such matter waves energy-exchange events are cosmic and high-energy radiation, X and Gamma rays, electron-positron creation and/or annihilation, spontaneous radioactive emissions, and similar phenomenology.

In addition, here extended atom model is also (hypothetically) addressing bidirectional electromagnetic matter-waves and other matter states as energy-exchanges (or communications) between all internal atom energy levels (including nucleus states), and between atoms and external or surrounding masses, what supports or creates effects of Gravitational force. Closely related to this part of electromagnetic exchanges are infrared and microwave radiation (as well as lower frequency electromagnetic phenomena). See Chapter 9. (about Black Body Radiation) where familiar additional and complementary elaborations are presented.

We could summarize the following analogical, indicative, and intuitive elements supporting the concept that Gravitation is being the manifestation of all masses and atoms coupling, such as:

1. *Atoms are electromagnetically polarizable. Electric and magnetic dipoles and moments already exist or can be created within atoms. Atoms, electrons, protons, and neutrons (including other subatomic and elementary particles) naturally have magnetic and spin moments, including orbital moments, and Coulomb type of forces also exist between such entities. Coulomb force law is mathematically the same for electric charges and dipoles, as well as for magnets. All of such or analogical statements can also be attributed to planets within solar systems.*
2. *Atoms can be externally excited or ionized by receiving and emitting electromagnetic radiation or photons, meaning that atoms and other masses are mutually communicating.*
3. *Newton law of Gravitation (between distant masses) is mathematically analogical or the same as Coulomb law between electromagnetic charges.*
4. *Compact masses are agglomerations of mutually (and electromagnetically) coupled and synchronized atoms, where involved "atomic fields and forces" are mutually superimposing and integrating (on certain way reacting as "united macro atoms" with averaged and analogical properties of single atom constituents).*
5. *Elementary particles like electron, proton, and neutron, inside atoms, are only a kind of an easy and oversimplified presentation based on analogies and convenient mapping between moving-particles and matter-wave packets. Mentioned elementary particles, when being inside atoms, are specifically structured matter-waves or standing waves fields formations. Very convenient mapping, functional transformation, and imaging between mentioned elementary particles and corresponding matter waves is realized based on **PWDC** or based on unity and complementarity between linear and angular motions (as already described in Chapters 4.1, 10., and in this chapter). There is a striking and predictive analogy between Bohr's atom model and planetary systems modelling (see much more in the second Chapter of this book under (2.11.14)). In fact, mentioned analogies are underlining that here we are dealing with electromagnetically and electromechanically coupled and synchronized resonators, and that such properties, or nature of matter, are spread from the micro world of atoms, until the macro world of galaxies. This is very much familiar with N. Tesla ideas about Dynamic Gravity Theory and Rudjer Boskovic concept of Universal Natural Force (see [97]). All that presents an innovative concept about Gravitation, where mases of planets are in states of permanent mutual electromagnetic energy exchanges and coupling, being at the same time transmitters and receivers of electromagnetic energy, and where relevant solar system formation is creating standing waves fields between the sun and its planets, including many of additional imaginative and challenging excursions towards Tesla's Dynamic Gravity and other domains of modern Physics. Similar ideas can be found in [144], Poole, G. (2018) Cosmic Wireless Power Transfer System and the Equation for Everything (see more in Chapter 2). The most important references regarding Tesla Dynamic Gravity and R. Boskovic Universal Natural Force can be found on Internet under the following links:* https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/ and

6. *The fact (based on linear and angular moments conservation) that uniformly spinning disks or gyroscopes are maintaining and stabilizing any spatial position and orientation where being initially placed (of course being submersed in a gravitational field), is telling us a lot about the nature of the "extended atomic field" that should be more completely described and decoded in relation to Gravitation and electromagnetic and electromechanical couplings between "mass-moments-energy" entities (see more in Chapter 2. Under "2.2.1. WHAT THE GRAVITATION REALLY IS"). ♣]*

Citation from [144] "**Abstract**: By representing the Earth as a rotating spherical antenna several historic and scientific breakthroughs are achieved. Visualizing the Sun as a transmitter and the planets as receivers the solar system can be represented as a long wave radio system operating at Tremendously Low Frequency (TLF). Results again confirm that the "near-field" is Tesla's "dynamic gravity", better known to engineers as dynamic braking or to physicists as centripetal acceleration, or simply (g). …

A new law of cosmic efficiency is also proposed that equates vibratory force and pressure with volume acceleration of the solar system. Lorentz force is broken down into centripetal and gravitational waves. …

Spherical antenna patterns for planets are presented and flux transfer frequency is calculated using distance to planets as wavelengths. The galactic grid operates at a Schumann Resonance of 7.83 Hz, …

The Sun and the planets are tuned to transmit and receive electrical power like resonating Tesla coils". …

……………………..

Citation (from Common Sense Science, Charles W. Lucas, Jr. Statements related to irrelevant ideological items are simply omitted):

"**Experimental Evidence to Support Boscovich's Atomic Model**. ... It started with the discovery of solitons ... Solitons are long lasting semi-permanent standing wave structures with a stable algebraic topology. [9] The soliton can exist in air or water as a toroidal ring. Solitons in water are usually formed in pairs known as a soliton and anti-soliton. Their structure is weak, and they decay away after 10 or 20 minutes.

Bostick's Plasmons. Winston Bostick (1916-1991), the last graduate student of Nobel Prize winner Arthur Compton (1892-1962), experimentally discovered how to make "plasmons" or solitons from the electromagnetic field within an electromagnetic plasma. [10] These structures were strong compared to solitons in air and water. Solitons have very long lifetimes and cannot be destroyed by normal processes in nature. Bostick proposed that electrons were just simple solitons and positrons were contrary or anti-solitons. All other elementary particles were built of more complex geometrical structures, such as dyads, triads, quatrads, etc. All plasmons or solitons in the electromagnetic field are of the same shape, i.e. a toroidal ring. The plasmon was of very great strength. Bostick tried to create a bottle from plasmons to hold controlled thermo-nuclear fusion. All materials known to man up to that time slowly disintegrated when exposed to controlled thermonuclear fusion. Only the plasmon was strong enough, but Bostick failed to succeed in building a bottle from plasmons...

Hooper's Electromagnetic Field Experiments. The nature of the plasmon, electromagnetic soliton was more completely revealed by another modern-day scientist, William J. Hooper [11]. He discovered that charged elementary particles, such as the electron, were not only made of the electromagnetic field, but variations in the field around them due to their structure extends to great distances. This same feature is also observed about solitons in water.

Hooper [11] also discovered that there are three types of electric and magnetic fields. One of these types is due to velocity effects from Lenz's Law causing it to have the property that it cannot be shielded. Thus, portions of the electromagnetic field exist everywhere in the universe.

References:

9. May, J. P., **Stable Algebraic Topology**, p. 1 (1966). http://www.math.uchicago.edu/~may/PAPERS/history.pdf

10. Bostick, Winston H., "Mass, Charge, and Current: The Essence and Morphology," **Physics Essays, Vol. 4, No. 1**, pp. 45-59 (1991).

11. Hooper, W. J., **New Horizons in Electric, Magnetic, and Gravitational Field Theory** (Electrodynamic Gravity, Inc., 543 Broad Blvd., Cuyahoga Falls, OH 44221, 1974), preface. http://www.rexresearch.com/hooper/horizon.htm"

Of course, to complete, make more precise and revive the previously described atom model, it would be more proper to analyze the structure of an atomic field using the modified, generalized Schrödinger's equation as elaborated from (4.22) to (4.28), which is formulated in chapter 4.3 of this paper:

$$\left\{\begin{array}{l} \frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta\overline{\Psi} + L\overline{\Psi} = 0 \ ; \ \Delta\overline{\Psi} = (\frac{\tilde{E}}{L})\frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2} = jk(\frac{\tilde{E}}{L})\nabla\overline{\Psi} \ ; \ (\frac{L}{\hbar})^2\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0, \ \tilde{E} \Leftrightarrow E_k \\ \frac{L}{\hbar}\overline{\Psi} = j\frac{\partial\overline{\Psi}}{\partial t} = -\frac{\hbar}{L}\frac{\partial^2\overline{\Psi}}{\partial t^2}; \ \frac{\partial\overline{\Psi}}{\partial t} + u\nabla\overline{\Psi} = 0; \overline{\Psi} = \overline{\Psi}(t,r), \ j^2 = -1 \\ (\frac{u}{v} = 1 + \frac{\lambda}{v}\frac{du}{d\lambda} = 1 + \frac{h}{\tilde{p}v}\frac{du}{d\lambda} = \frac{hf}{\tilde{p}v} = \frac{\hbar\omega}{\tilde{p}v} = \frac{\tilde{E}}{\tilde{p}v}, \ \tilde{E} = \tilde{m}uv = \tilde{p}u = hf = \hbar\omega = E_k, \ \lambda = \frac{h}{\tilde{p}}, u = \lambda f = \frac{\omega}{k}) \end{array}\right\} \Rightarrow$$

$$\frac{\hbar^2}{\tilde{m}}(\frac{u}{v})\Delta = \frac{(\hbar u)^2}{\tilde{E}}\Delta = \frac{\tilde{E}}{k^2}\Delta = j(\frac{\tilde{E}^2}{kL})\nabla = \frac{\hbar^2}{L}\frac{\partial^2}{\partial t^2}, \Delta = jk(\frac{\tilde{E}}{L})\nabla, \ \tilde{E} - U_p \leq L < \infty, \quad (4.25)$$

$$S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i ..., t)dt = \text{extremum} \ ,$$

When using the wave equation (4.25) we should not forget that its wave function has a physical dimension related to power (and it does not represent only the probability of finding certain energy state in some part of the atomic field space). Modified Schrödinger equation (4.25) is kind of Classical, second order, differential wave equation, where wave function $\overline{\Psi}$ is formulated as a Complex, Analytic Signal function, based on Hilbert transform, as elaborated in the chapters 4.0 and 4.3.

[♣ Within solar systems, we really have orbiting and spinning planets, but inside stable atoms we only have spatial formations of electromagnetic fields and waves that are respecting number of periodicities (meaning periodical motions characterized with regular and repeatable, temporal, and spatial intervals), what is typical for standing waves. Anyway, we can find number of useful, indicative and well-applicable mathematical analogies when comparing solar systems and N, Bohr atom model (see more in Chapter 2, about gravitation), which is basically wrong (for describing atoms), but it still works sufficiently well until certain level, thanks to involved intrinsic periodicities and analogies that can be presented on number of ways (like in the Ptolemy geocentric system; -meaning such model was very much wrong, but it also works on some useful way). There are other atom models seemingly being much better when compared to N. Bohr atom model, but all of them are producing numbers of identical results regarding (emissions and absorptions) spectral situations, because of involved, mutually coupled and synchronized periodicities.

Certain driving source that creates vibrations and waves inside atoms should also exist (to support and maintain internal standing-waves electromagnetic field structure), but we still do not know how to specify it, or where to place it. One of logical assumptions could be that all atoms and other macro masses in our Universe are mutually coupled and synchronized, and that the whole Universe is like a very big atom, on some way rotating and oscillating, most probably in an extremely low frequency zone. There are different points of view regarding atom and planetary systems modeling and applying analogies between them, where standing-waves formations are always something common (see much more in Chapter 2). Of course, there are still many unanswered questions here.

If one looks from a different angle how the structural modeling of an atom-field is done (starting from (8.64) to (8.75)), he may conclude that something similar may be a general approach to modeling applicable to any elementary particle (with proper respect to relevant physical, mathematical and quantum particularities). For example, one may always pose questions if the field of some elementary particle is "solenoidal", "potential" or "complex", if there are a particle nucleus and particle shell, what kind of communication between them exist, and/or what kind of communication of that elementary particle with outer space exist, etc. Then, we *can address relevant group and phase velocity of involved matter-wave elements, find involved amplitude and phase functions (in time and frequency domain), and apply relevant relations of uncertainty (see chapter 5.). To connect all previous steps and this way define the model of certain elementary particle successfully and coherently, this procedure should properly fit the frames of particle-wave dualism (4.9) - (4.22).*

Although modern quantum physics presents atoms and elementary particles in a complex and quantitatively predictable way, one may yet conclude that this is not nearly the analysis that is described or proposed in this book. In addition to previous statements, to give some arguments about how it is possible that seemingly quite different and mutually opposing modeling and conceptualization of atomic matter structures could produce useful and verifiable results we may ask ourselves what happens when we axiomatically assume/name/make/forge the wave function to behave as a probability function. We will mathematically (by normalization, etc.) completely conform it to functions of probability and statistical distributions (that stands for sets with many identical participants). This way, we create (in average) localization of domains where certain energy-momentum, force or field entity exists (with respect to all spatial, time, and other relevant parameters and conservation laws). So, it is no wonder that such probabilistic treatment of wave functions (as known in the contemporary quantum theory), enriched with associated mathematics, which is securing compliance with known conservation laws, is also producing useful and particularly good results (only in average, and without having immediate, temporal-spatial phase information).

Apart from theoretical elaborations and modeling of atoms, what we know empirically is that atoms can absorb and emit electromagnetic energy in a form of single-frequency (or narrow-frequency band) photons. We also know that sufficiently high-energy photon could produce an electron-positron couple, and that electron-positron collision can disintegrate (or annihilate) both producing photons (and something analogical and even more complex is also applicable to any couple of other particles and antiparticles). From the Compton and Photoelectric effects, we know that photons and electrons are mutually interacting like particles, being also dualistically presentable as mutually interacting matter-waves. All phases in mechanical motions with energy-momentum interactions are accompanied with heat energy dissipation (again related to infrared radiation of electromagnetic energy). The most logical and very probable conclusion is that atoms (electrons, protons, neutrons…) are certainly complex and self-stabilized forms of electromagnetic energy, composed of self-closed standing-waves structures that are (internally and externally), mutually communicating by photons and electrons exchanges. We usually associate such situations to energy quantizing, but this is more about an optimal energy packing and energy exchanges within multi-resonant and standing-waves structures.

Also, for gravitation related fields and forces the most important should be linear moment p and angular moment L of involved mass-energy states (including associated electromagnetic moments and dipoles). Surprisingly, we do not find an analogical indication that a static mass m should be the primary and unique source of gravitation, but on some way vibrating mass is for sure the important part or source of Gravitation. We can find a striking and verifiable analogy between N. Bohr atom model and solar systems. Thanks to wave particle duality of matter, we can also address atom (and solar systems) models using wavefunctions and Schrödinger equation, or by defining proper atomic field or force, as we find in (8.64) - (8.73). Practical predictive meaning of such analogical and intuitive revelations is that I. Newton and A. Einstein theories about Gravitation, involving electromagnetic phenomenology, should be one day significantly updated. The other brainstorming insight here is that electric charges (or electrons and protons) should be kind of dynamic, motional energy states (like mechanical and electromagnetic moments), since all other mutually analogical mechanical and electromagnetic moments and energy-flux entities are also kind of moving or dynamic states (see T.1.3 in the first chapter of this book). In Physics, presently we still wrongly consider electric charges as stable, or fixed and having only static parameters (as number of Coulombs). Consequently, we could expect certain continuous electromagnetic energy exchange (or N. Tesla's Radiant energy flow, within a surrounding ether fluid) between any of positive and negative charges, or between conveniently aligned electromagnetic dipoles of micro and macro masses, this way creating necessary background for explanation of Gravitation. ♣]

9. BLACKBODY RADIATION, GRAVITATION AND PHOTONS

(JUST BRAINSTORMING DRAFT… IN PROCESS)

In this book, we address (M. Planck and A. Einstein) Blackbody Radiation in the context of Nikola Tesla Radiant energy, in relation to real sources of Gravitation, **[97]**, and related to Universal Natural Force-field elaborated by Rudjer Boskovic in **[6]**. The most important references regarding Tesla Dynamic Gravity, and R. Boskovic Universal Natural Force, can be found on Internet under the following links:
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ru%C4%91er-josip-bo%C5%A1kovi%C4%87-1711-1787/
https://teslaresearch.jimdofree.com/dynamic-theory-of-gravity/ .
Familiar phenomenology about radiative-energy related sources of gravitation is recently described in **[100]**. Most probably that Casimir effect is also certain manifestation of the "Radiant energy flow" (see [103] and [104]). The most significant phenomenology regarding Radiant energy and Gravitation (including thermal radiation) is related to the concept (or hypothesis) of structurally resonating Universe and standing-waves formations between all cosmic masses and energy concentrations (see more in Second chapter of this book under, "*Natural Forces based on Standing Waves, Resonating Universe*").

Everything in our Universe is receiving and radiating certain content of "*mass-energy-moments*" entities such as electromagnetic and cosmic rays, including charged and electromagnetically neutral particles, mechanical or acoustical and seismic vibrations, thermal motions, electromagnetic atmospheric discharges, and/or other planetary, geomagnetic oscillations and perturbations, what is producing number of secondary matter-waves and/or harmonics with measurable mechanical and electromagnetic manifestations (including heat radiation). Practically any atom, molecule, mass, …, produces and absorbs large spectrum of different mater waves, vibrations, and oscillations, being in causal relation with mentioned natural, radiating and receiving, or Radiant energy phenomenology (all that dominantly or ontologically having an electromagnetic origin).

All mentioned natural radiation, oscillations, or signals are anyway mutually coupled and superimposed (or appearing at the same time within and around the same radiating objects or masses). What makes a practical difference for us is how we detect certain signal in certain frequency range, in relation to used sensors, or measurement instruments (since we cannot easily measure in all natural frequency ranges). When analyzing some *matter-waves radiating object*, we simply neglect that when we externally detect mechanical or electromagnetic waves, somewhere internally (inside the same object), in its atomic or molecular structure, there is an activity of synchronous electromagnetic and/or thermal oscillations (and vice versa), mostly because atoms and molecules intrinsically have number of electromagnetically charged, rotating, and spinning, mutually connected states (but in different frequency ranges). For certain frequency intervals, it is easier and more convenient to detect mechanical (or acoustical) signals, and for other frequency intervals it is more natural and easier to detect electromagnetic signals (just because we do not have good sensors for all frequency ranges). All mentioned natural vibrations, emissions, and absorptions of certain object (being characterized mechanically, acoustically, electromechanically, electromagnetically, thermally…) are mutually coupled, and mutually spectrally overlapping (internally and externally) in some of involved temporal and spatial frequency ranges, and this is anyway happening within any of mutually interacting objects or masses. If we monitor only one part of some large-band spectrum, such as its thermal, blackbody (also electromagnetic) radiation, we should understand that this spectrum also (and naturally) has superimposed spectral contributions from other, phenomenologically related events (naturally being mechanical, electromechanical, or electromagnetic).

In other words, natural radiative, electromagnetic, and acoustic field emissions, and absorptions of atoms (see more in Chapters 8., and 10.), including excitation of other forms of involved resonators, produces matter-waves radiation (or radiant N. Tesla's energy), followed with a creation of active mechanical and electromagnetic mass-energy entities such as ionized atoms, photons, and other field-charges (and vice-versa). On the same way, certain excitation, or vibration (of solid matter structures) in a temporal domain will produce corresponding "*energy-mass-frequency effects*", or emissions in a spatial domain (and vice-versa), what contemporary Blackbody theory is not addressing. Masses and all atoms in our oscillating and resonating Universe are anyway mutually connected, at least thanks to a surrounding electromagnetic field, and tend to become by "matter-waves synchronized", while agglomerating, resonating and "energy-*moments communicating"*. If we consider M. Planck, Black Body electromagnetic radiation as an "***external radiation***" (radiating from a blackbody cavity towards surrounding, external, and almost an empty space), then we could analogically understand and conceptualize effects within heated masses, as being influenced by an "***internal blackbody radiation***". In fact, here we generalize (external and internal) heat radiation as originating from exited and oscillating atoms (being in different states of motions).

Quantum entanglement effects between involved matter entities are also contributing the universal tendency of resonant and standing matter waves to synchronize. *Here, under an extended meaning of resonant effects, we should consider both temporal and spatial resonances (or spectral distributions).*

All mentioned motions and radiations that are acting like matter-waves, conceptually present N. Tesla's ***Radiative, or Radiant Energy flow***. Atoms, as dominantly electromagnetic multi-resonant structures, being internally *self-stabilized standing waves formations*, *are anyway extending such micro-world field structures externally*, towards other atoms and masses (see more in Chapter 8. under "8.3. Structure of the Field of Subatomic and Gravitation related Forces"). Always when photons' emissions and absorptions are manifesting, unavoidably we also have some activity and alignment of involved electrically charged particles or dipoles with magnetic and spinning properties. Mentioned situations (where electromagnetic and/or thermal ***Blackbody Radiation*** is always present), should be on some causal way related to the force of Gravitation, as N. Tesla speculated (since in both cases the common situation are the involved masses, resonating-Universe, and the same *radiative energy flow, including the same mathematical form of Coulomb-Newton forces*). On the example of our Sun, we know that such blackbody object has only about 6000 °C on its surface, and between one and two million of °C in its corona zone, meaning that such natural blackbody radiation is also being specifically temperature-energy increased and modulated by the rising velocity of structural standing matter-waves between the Sun and its planets, until the high vacuum-state zone around the Sun (see more about the same concept in the second chapter of this book under, "*Natural Forces based on Standing Waves, Resonating Universe*").

We cannot simply neglect imaginative visions of Nikola Tesla about something what we still do not understand or master well, because without creativity, imagination, innovations, and specific engineering contributions of N. Tesla, the big part of our contemporary (and planetary) technology and industry will not function. Existing and technically proven heritage, creations, and results of N. Tesla are simply giving certain indicative, (very positive and very high) legitimacy, confidence, and credibility to his still not well-understood and maybe not adequately and completely formulated visions, and still not-materialized imaginative concepts, one of them being connection between radiative energy and gravitation. If Tesla said something like that (about his Dynamic Gravitation) on several occasions, during a long period of time, there is a very big probability and expectation that this is essentially or conceptually correct vision, because Tesla was the visionary whose visions have been in too many cases materialized as initially formulated). He was (on certain strange way) well connected and synchronized with "*knowledge data banks*" of our Universe, at least much better and stronger connected than others are.

♣ *If we carefully read many of imaginative and hypothetical statements N. Tesla gave regarding* ***Radiant energy*** *(see under [97]) this should be some fine-particles or fluidic-mass flow of extremely small and electromagnetically polarizable matter formations with electric and magnetic properties, behaving like some ideal gas (or ether). Such etheric energy in a form of fluid flow is circulating between or through all masses in our universe, practically manifesting a gravitational force (like experiencing an equivalent-to-gravitation, attractive force effect in A. Einstein, upwards accelerated elevator). This way we described conceptual and imaginative grounds of N. Tesla's Dynamic Gravity theory, regardless we still do not have the complete picture about the Radiant energy flow and ether (but we do know that even an absolute vacuum still has measurable dielectric and magnetic permeability (of involved and still unknown content), and that electromagnetic waves or photons are traveling through such vacuum without problems, meaning that the vacuum state is a tangible fluidic matter). It is obvious that, at least, part of mentioned Tesla's radiant energy is an electromagnetic radiation from heated masses, including all kind of associated, already known, and detectable radiation in forms of photons, electrons, positrons, cosmic rays, neutrinos, different matter waves, plasma states and streaming of other non-specified participants. We can say that blackbody and any thermal radiation also belong to Tesla's or cosmic radiant energy (see more in [128]). Consequently, heat-related radiation should make local perturbations on surrounding "gravitational" force. Gravitation anyway should have electromagnetic roots (as hypothesized in this book, and in many publications from others), because Newton force law is mathematically analog to an electromagnetic Coulomb-type force between conveniently polarized, electromagnetic charges, moments, and dipoles, also being influenced by global or holistic cosmic rotation and spinning (see more in Chapters 1., 2., and in [114], [115], [116], [117] and [128]). Consequently, strongly heated masses will not locally respect Newton law of gravitation. In cases of heating, local thermal and electromagnetic field perturbations and streaming of waves and particles will be created. This will locally influence "radiant-energy flow and associated field of gravitation". Consequently, since heat is causally related to kinetic and/or wave energy of involved moving participants, and creates blackbody electromagnetic radiation, gravitation is strongly related to associated thermodynamic and electromagnetic field perturbations, which are also influenced and shaped by standing matter-waves between gravitational masses and* *concentrated energy formations (see more about Natural Forces and Resonating Universe in Chapter 2. of this book).*

If we like to address an analogical and simplified situation, to visualize mentioned radiative energy flow, let us observe certain vessel with water, placed on a heating plate. ***Water will anyway "thermodynamically" evaporate on any temperature (also below freezing temperature) since the environmental or background radiant energy flow and thermal motions are always present around. Natural liquid evaporation should be, at least partially, influenced by some omnipresent, surrounding radiant energy flow. Of course, temperature of evaporated-particles, in front of a free heated-water surface, will gradually and smoothly increase with the water temperature increase until reaching the boiling temperature of water.*** *By hot-plate heating, molecules, and matter waves of thermal motion in water are getting enough energy to jump or stream out of water (and this way we are also influencing a radiant energy flow). Once when water boiling starts, we will observe certain uniform and stable vapor-flow situation since, still not-evaporated, liquid water mass will tend to keep the same temperature during boiling, until the whole quantity of water would evaporate. Here, we already have sufficiently intuitive and indicative facts to search for an analogy between liquids evaporation, radiant energy flow, and black body radiation. Even solid-state mases, like metals, are on some way changing their mass content during sufficiently long-time intervals. For instance, the International Prototype of the Kilogram (made from platinum-Iridium alloy) also experiences its extremely small mass variations during relatively long time periods. Some of reasons for measurable mass variations are predictable and known, but there are still unknown or inexplicable reasons, probably related to an environmental, radiant energy flow (and to Resonating Universe).*

In other words, heat-related phenomenology of matter states starts with random mechanical motions and oscillations of atoms, molecules, and with creation of matter-waves of involved participants (including particles and quasiparticles). With the increase of kinetic energy or temperature of matter constituents of certain solid-body cavity, produced electromagnetic or thermal radiation will start manifesting as Blackbody radiation. Produced thermal and radiative energy flow effects, combined with standing-waves effects of resonating Universe, should locally affect the force of gravitation (by affecting temperature, kinetic energy, motional states and spatial distribution of involved atoms, molecules, and formations of surrounding electromagnetic radiation). We can also say (based on analogical comparisons with liquids evaporation) that strong heating of certain mass should locally create an additional radiant and matter-waves energy flow, since involved molecules, atoms and other participants are also kind or oscillating objects with number of resonant states, and such vibrating entities, when excited, are naturally creating and emitting matter waves. Practically, already existing, surrounding electromagnetic field structure that is (initially and still hypothetically) responsible for manifesting effects of gravitation (as Tesla conceptualized) should be on some way locally modified by strong heating and additional radiative energy streaming, and it should be possible to measure deviations or perturbations of the local, Newtonian force of gravitation versus temperature (as published in [101]). ***Forces of any kind are either related to spatial gradients of relevant mass-energy-density, or to temporal derivatives of involved mechanical and electromagnetic moments*** *(see more about natural forces in Chapter 10.). Of course, N. Tesla primarily described purely electromagnetic situations, methods and* *experiments that are producing and affecting radiant energy flow, and since we know that blackbody radiation is an electromagnetic-waves radiation, the consequence is that gravitation, heat, and electromagnetism should be mutually related. At the same time, we could hypothesize that stars or suns (including "black-holes") are some locally dominant (spatial-temporal) resonant nodes of radiant energy flow that is the part of a larger-scale of cosmic, electromagnetic (standing waves) field structure (existing among and between involved masses, stars, surrounding planets, galaxies, and the Universe as a whole). Another source of Radiant energy is related to an energy flow between electrically charged particles and other electromagnetic dipoles. In the first chapter of this book (addressing analogies) we can find that electric charge by its nature should also have some dynamic or motional properties, like mechanical moments have. Consequently, between two electric charges (or dipoles) should always and continuously exist certain bidirectional electromagnetic energy flow, this way also influencing the force of Gravitation. There is a chance that Tesla intuitively had such electromagnetic energy flow in mind, when describing Radiant energy and Dynamic Gravity.* ♣

In the first chapter of this book (about multilevel Analogies), we also and analogically established that real sources of gravitation (and other natural forces) should be mutually coupled natural field-charges, such as linear and angular moments, electric charges and magnetic flux, or magnetic moments (in states of different and relative, linear, and angular motions and oscillations). Consequently, only static (or standstill), electromagnetically neutral masses cannot be real and unique gravitational charges or sources (as assumed in Newton and A. Einstein theories of Gravitation), but dynamically (internally and externally) active atoms, mutually coupled with other atoms and masses, are (see more in Chapters 1., and 8.). Blackbody radiation is addressing an external electromagnetic radiation of any heated, solid mass cavity. Heated cavity internally has many mutually coupled (and relatively fast synchronized) electromagnetic charges, plasma states and heat-energy participating, moving, and oscillating particles, including matter wave formations with linear and angular mechanical moments. Since blackbody cavity (externally) is a kind of solid-mass cavity, we could also say that internally, number of simple harmonic (equivalent to mass-spring and capacitance-inductance) oscillators are getting excited (by heating), and continuously emitting and absorbing photons. From such point of view, we could say that Blackbody radiation has its specific spectral signature, as a spatially and temporally dependent field structure, and consequently carries an information that is related to a local field of gravitation. Consequently, it is a time to update the wider understanding of heat energy, temperature and associated (blackbody) electromagnetic radiation, since present (mostly mechanistic) thermodynamic picture about heat energy and temperature could be incomplete (see much more in exciting and challenging revelations under [126], from Dr. Sorin Cezar Cosofret). *It is also empirically known that for distances between masses being in a micrometers range (or shorter) there is no more meaning to speak about any gravitational force. Gravitational force is not detectable below certain small, threshold-distance between involved masses and atoms (since for formation of gravitation we need to have interactions and couplings between many atoms, involved field charges, angular and linear moments, and electromagnetic field entities of agglomerated atoms, acting internally and externally). Consequently, gravitation should be an "external, non-contact, macro-masses attraction effect", which manifests as a coupling effect only between relatively large masses. Subatomic and interatomic, electromagnetic (and other) internal* *communications and energy-momentum exchanges (between involved atoms and "moments-energy-mass" entities) does not belong to typically gravitational effects. From a global (and spectral) point of view, gravitation should be causally linked to external, natural radiative communications and couplings between atoms, masses, and surrounding, oscillating universe (for instance, being linked to phenomenology that is creating red, infrared, microwaves radiation, and lower frequency electromagnetic radiation), like what N. Tesla speculated (see more in [128]).* *Mutual influence and dependence between any heated mass and surrounding gravitational force is already experimentally verified, meaning that by heating, we can influence locally the force, and spatial shape of a surrounding gravitation field (see below).*

Quote *(The Negative Temperature Dependence of a Gravity is a Reality; Professor Alexander L. Dmitriev and Sophia A. Bulgakova, [101]): "Temperature dependence of force of gravitation - one of the fundamental problems of physics. The negative temperature dependence of weight of bodies is confirmed by laboratory experiments and like Faraday phenomenon in electrodynamics is a consequence of natural "conservatism" of a physical system, its tendency to preserve a stable condition. Realization of experimental research of an influence of the temperature of bodies on their gravitational interaction is timely and, undoubtedly, will promote the progress of development of physics of gravitation and its applications".*

Here, we will try to draw important conclusions from Blackbody and cosmic, radiative energy exchanges to address Gravitation as a force dominantly related to electromagnetic phenomenology and to standing waves of resonating Universe (see more in the second Chapter of this book). Conceptualization of electromagnetic energy quants or photons is already elaborated in Chapter 4.1, under "4.1.1.1. Photons and Particle-Wave Dualism" and summarized in "T.4.0. Photon – Particle Analogies". ***Gravitational attraction should also be an effect of standing and stationary matter-waves between mutually attracting masses, since in nodal zones of mentioned standing waves, only attractive force effects exist (and in anti-nodal zones of the same field structure, only repulsive forces exist), as already known in relation to acoustic resonators and/or ultrasonic levitation (see more in [150-151]).***

Since we know the final, resulting, and on some way mathematically fitted and averaged M. Planck function of spectral content of the Blackbody or electromagnetic radiation in its frequency (and wavelengths) domain, we could also try to find its time-domain related wave-functions (or spatial shapes of involved matter waves).

The integral forms of Stefan-Boltzmann and Wien's radiation laws of a Blackbody heated to the temperature **T** presents a surface power density **R(T)** of emitted electromagnetic waves (or photons) from that Blackbody,

$$R(T) = R_T = \int_0^\infty dR(f,T) = d\tilde{P}/dS = \sigma \cdot T^4 (=) \left[\frac{W}{m^2}\right],$$

$$\tilde{P} = \sigma \cdot T^4 \cdot S = R(T) \cdot S = \text{Power radiated in } [W] = \left[\frac{J}{s}\right],$$

$$\lambda_{peak} = \frac{2.898 \cdot 10^{-3}}{T} \ (=) \ [m] \ (=) \ \text{Peak Wavelength at maximum intensity},$$

$$\sigma = \text{Stefan's Constant} = 2\pi^5 k^4 / 15 c^2 h^3 = 5.67 \cdot 10^{-8} \left[\frac{W}{m^2 K^4}\right], \qquad (9.1)$$

$$S = \text{Relevant (radiating) surface } [m^2],$$

$$T = \text{Surface Temperature of the body } [K].$$

Planck successfully created (or better to say mathematically fitted, based on experimental data and some intuitive assumptions) the electromagnetic, spectral radiance formula **R(f, T)** of a body at absolute temperature T as,

$$dR(f,T) = \frac{2\pi f^2}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} df \ (=) \left[\frac{W}{m^2 \cdot sr}\right] \Rightarrow$$

$$\frac{dR(f,T)}{df} = R_f(f,T) = \frac{2\pi f^3}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} (=) \left[\frac{Ws}{m^2 \cdot sr}\right], \qquad (9.2)$$

$$\frac{dR}{d\lambda} = R_\lambda(\lambda,T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} (=) \left[\frac{W}{m^3 \cdot sr}\right]$$

where **k** is the Boltzmann constant, **h** the Planck constant, and **c** the speed of light in the medium, whether certain material or vacuum.

Citation (see http://en.wikipedia.org/wiki/Planck%27s_law): The spectral radiance can also be measured per unit wavelength instead of per unit frequency. These distributions represent the spectral radiance of blackbodies—the power emitted from the emitting surface, per unit projected area of an emitting surface, per unit solid angle, per spectral unit (frequency, wavelength, wavenumber, or their angular equivalents). Since the radiance is isotropic (i.e. independent of direction), the power emitted at an angle to the normal is proportional to the projected area, and therefore to the cosine of that angle as per Lambert's cosine law, and is non-polarized.

------------------------------------Citation took from the internet (http://physics.info/planck/):

Let us try to derive the blackbody spectrum. Planck's law is the formula for the spectral radiance of an object at a given temperature as a function of frequency or wavelength. It has dimensions of power per solid angle per area per frequency or power per solid angle per area per wavelength.

$$\frac{dR(f,T)}{df} = R_f(f,T) = \frac{2\pi f^3}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} (=) \left[\frac{Ws}{m^2 \cdot sr}\right],$$

$$\frac{dR(\lambda,T)}{d\lambda} = R_\lambda(\lambda,T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} (=) \left[\frac{W}{m^3 \cdot sr}\right]$$

When these functions are multiplied by the total solid angle of a sphere (4π steradian) we get the spectral irradiance. This function describes the power per area per frequency or the power per area per wavelength.

$$\frac{dR(f,T)'}{df} = 4\pi R_f(f,T) = \frac{8\pi^2 f^3}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} (=) \left[\frac{W}{m^2 \cdot Hz}\right],$$

$$\frac{dR(\lambda,T)'}{d\lambda} = 4\pi R_{\lambda}(\lambda,T) = \frac{8\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} (=)\left[\frac{W}{m^3}\right]$$

When these functions are integrated over wavelength, the result is either the irradiance or the power per area.

$$R(T) = \frac{2\pi^5 k^4}{15h^3c^2}T^4$$

The pile of constants in front of the temperature is known as Stefan's constant.

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2} = 5.67040\times 10^{-8}\frac{W}{m^2K^4}$$

Multiplying the irradiance by the area gives us the essence of the Stefan-Boltzmann law.

$$\tilde{P} = \frac{2\pi^5 k^4}{15h^3c^2}\cdot S = 5.67040\times 10^{-8}\cdot S = \sigma\cdot T^4\cdot S = R(T)\cdot S = \text{Power radiated in } [W] = \left[\frac{J}{s}\right]$$

Then we apply the first derivative test to the wavelength form of Planck's law to determine the peak wavelength as a function of temperature.

$\lambda_{max} = \frac{hc}{k}\frac{1}{xT}$, where x is the solution of the transcendental equation $\frac{xe^x}{e^x-1}-5=0$, x = 4.96511...

Then we combine all the constants together and we get the Wien displacement law, $\lambda_{max} = \frac{b}{T}$, b = 2.89777... mmK.

The total power and *peak radiating force* of a heat radiation from the Blackbody Surface **S** can be found as,

$$\tilde{P}(t) = \Psi_{bb}^2(t) = d\tilde{E}/dt = \int_{[S]} R_T dS = R_T S = S\int_0^{\infty} dR(f,T) = S\sigma T^4 (=)[W]$$

$$\Rightarrow \tilde{P}(t) = d\tilde{E}/dt = \tilde{F}\cdot dr/dt = \tilde{F}\cdot v = \tilde{F}\cdot c = S\sigma T^4 \Rightarrow \tag{9.3}$$

$$\Rightarrow \text{Radiative Force } (=)\ \tilde{F} = S\sigma T^4/c (=)[N],$$

and where $\Psi_{bb}(t) = a_{bb}(t)\cos\varphi_{bb}(t)$ is still unknown, time domain wave function of the Blackbody radiator (here and later recognizable by indexing “bb”). Radiated heat or electromagnetic energy from a Blackbody (based on Parseval’s theorem), after a time-period Δt, would be,

$$\tilde{E} = \int_{-\infty}^{+\infty}\tilde{P}(t)dt = \int_{-\infty}^{+\infty}\Psi_{bb}^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|U_{bb}(\omega)|^2 d\omega = \frac{1}{\pi}\int_0^{+\infty}|A_{bb}(\omega)|^2 d\omega = S\Delta t\int_0^{+\infty} dR(f,T) =$$

$$= S\Delta t\int_0^{+\infty}\frac{dR(f,T)}{df}df = S\Delta t\int_0^{+\infty}\frac{2\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1}df = S\cdot\Delta t\cdot\sigma T^4 (=)[Ws = J], \tag{9.4.1}$$

$$d\tilde{E} = \left[S\int_0^{+\infty} dR(f,T)\right]dt = \left[S\int_0^{+\infty}\frac{2\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1}df\right]dt = \Psi_{bb}^2(t)dt = S\cdot\sigma T^4 dt = \tilde{F}dr.$$

If the wave energy and wave function in question is related only to one (single) photon, or single wave-packet, we could simplify mathematics with $\tilde{E} = hf \Rightarrow d\tilde{E} = h\cdot df$. At least, we could also say (based on experimental experiences) that a time-domain wave function of a Blackbody electromagnetic radiation is presenting certain stable, stationary, continuous signal with relatively wide time duration and limited frequency duration (since Uncertainty Relations are universally valid). Here we will try to determine wave functions as realistic signals, with dimensions of power (analog to *electromagnetic energy, Poynting vector concept*), as introduced in the chapters 4.0 and 4.3 (not at all as being only stochastic and non-dimensional functions).

In (9.3) and (9.4.1) we also introduced the matter-wave radiative force $\tilde{F}$, which is acting aligned with Blackbody Radiation. ***Since action and reaction forces are always simultaneously and coincidently present, we should expect that reaction and attractive force (towards radiating body) is the Newton, Coulomb, or N. Tesla and Rudjer Boskovic force of Gravitation. All kind of thermal radiation, or evaporation, combined with other electromagnetic radiation are presenting certain energy flow, or an equivalent mass flow. Other part of internal, balancing, repulsive (and like standing waves oscillating) forces, as described by R. Boskovic Universal Natural Force Law, are involved in creating atoms and masses agglomerations*** *(see more in Chapter 8. under "8.3. Structure of the Field of Subatomic and Gravitation related Forces").*

Existence of Radiant energy flow is on some way like A. Einstein accelerated elevator, when a person inside such elevator will feel the force equivalent to Gravitation (see **[6], [97]**, and **[100]**). It is also known that Rudjer Boskovic supported Isaak Newton in formulating the Law of Gravitation, by promoting his concepts of Universal Natural Force Function. Later, Nikola Tesla was a devoted and enthusiastic sympathizer or Rudjer Boskovic's ideas, and he announced on several occasions his (never finalized and not published) theory of Dynamic Gravity (probably motivated by R. Boskovic ideas).

From (9.4) it is possible to determine or quantify the Blackbody spectral amplitude function, such as (see chapter 4.0 for relevant mathematics),

$$A_{bb}^2(\omega) = \pi\ S\ \Delta t\left[\frac{dR(f,T)}{d\omega}\right] = \frac{1}{2} S\ \Delta t\left[\frac{dR(f,T)}{df}\right] = S\ \Delta t \frac{\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1} =$$

$$= S\cdot\Delta t\cdot\frac{\pi}{\lambda^2}\cdot\frac{\tilde{E}}{e^{\tilde{E}/kT}-1} = S\cdot\Delta t\cdot\frac{h}{8\pi^2c^2}\cdot\frac{\omega^3}{e^{h\omega/2\pi kT}-1} = \frac{1}{2}\cdot\frac{d\tilde{E}}{df} = \pi\frac{d\tilde{E}}{d\omega} =$$

$$= U_c^2(\omega) + U_s^2(\omega)\ (=)\ \left[Ws^2 = Js\right],$$

$$A_{bb}(\omega)e^{-j\Phi_{bb}(\omega)} = \iint_{[-\infty,+\infty]} \overline{\Psi}_{bb}(t)e^{j\omega t}dt = U(\omega) = U_c(\omega) - jU_s(\omega),$$

$$\overline{\Psi}_{bb}(t) = \Psi_{bb}(t) + j\hat{\Psi}_{bb}(t) = a_{bb}(t)e^{j\varphi_{bb}(t)} = \int_{(0,+\infty)} A_{bb}(\omega)e^{-j(\omega t+\Phi_{bb}(\omega))}d\omega,$$

$$\hat{\Psi}_{bb}(t) = H\left[\Psi_{bb}(t)\right] = a_{bb}(t)\sin\varphi_{bb}(t),\ \Psi_{bb}(t) = a_{bb}(t)\cos\varphi_{bb}(t), a_{bb}(t) = \sqrt{\Psi_{bb}^2 + \hat{\Psi}_{bb}^2},$$

– – – – – – – – – – – – – –

$$d\tilde{E} = \Psi^2(t)dt = \hat{\Psi}^2(t)dt = \left|\overline{\Psi}(t)\right|^2 dt = \left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 d\omega = [\frac{A(\omega)}{\sqrt{\pi}}]^2 d\omega =$$

$$= P(t)dt = \frac{h}{2\pi}d\left[\frac{\Psi(t)\dot{\hat{\Psi}}(t) - \dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)}\right],\ h = const.,$$

$$\tilde{E} = \int_{[T]}\Psi^2(t)dt = \int_{[T]}\left[a(t)\cos\varphi(t)\right]^2 dt = \int_{[T]} a^2(t)dt,$$

$$P(t)=\frac{d\tilde{E}}{dt}=\Psi^2(t)\ (\Leftrightarrow)\left[\frac{a(t)}{\sqrt{2}}\right]^2\ (=)\ [W]\ ,t\in(-\infty,+\infty)\ ,$$
$$P(\omega)=\frac{d\tilde{E}}{d\omega}=\frac{\Psi^2(t)}{d\omega/dt}=\left[\frac{A(\omega)}{\sqrt{\pi}}\right]^2\ (=)\left[Js=Ws^2\right],\ \omega\in(0,+\infty),$$
$$\Psi(t)=a(t)\cos\varphi(t)=-H\left[\hat{\Psi}(t)\right]\ ,\ \hat{\Psi}(t)=a(t)\sin\varphi(t)=H\left[\Psi(t)\right],$$
$$\overline{\Psi}(t)=a(t)e^{j\varphi(t)}=\Psi(t)+j\hat{\Psi}(t),$$
$$a(t)=\sqrt{\Psi^2(t)+\hat{\Psi}^2(t)}\ \ ,\ a^2(t)=\frac{\Psi(t)\dot{\hat{\Psi}}(t)-\dot{\Psi}(t)\hat{\Psi}(t)}{\overline{\omega}},\ \varphi(t)=\arctan\frac{\hat{\Psi}(t)}{\Psi(t)},$$
$$\dot{a}(t)=\frac{\partial a(t)}{\partial t}=\frac{\psi(t)\cdot\dot{\psi}(t)+\hat{\psi}(t)\dot{\hat{\psi}}(t)}{a^2(t)}=a(t)\cdot\mathrm{Re}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right],\tag{9.4.2}$$
$$f(t)=\frac{\omega(t)}{2\pi}=\frac{1}{2\pi}\frac{\partial\varphi(t)}{\partial t}=\frac{1}{2\pi}\dot{\varphi}(t)=\frac{1}{2\pi}\frac{\Psi(t)\dot{\hat{\Psi}}(t)-\dot{\Psi}(t)\hat{\Psi}(t)}{a^2(t)}=\frac{1}{2\pi}\mathrm{Im}\left[\frac{\dot{\overline{\Psi}}(t)}{\overline{\Psi}(t)}\right].$$

In cases of single photons, could be,

$$\left[\omega=\overline{\omega}=2\pi\overline{f},\ c=\overline{\lambda}\overline{f}\ ,\ \tilde{E}=h\overline{f},\ d\tilde{E}=hd\overline{f}\ \right]\Rightarrow$$
$$\Rightarrow A_{bb}^2(\overline{\omega})=U_c^2(\overline{\omega})+U_s^2(\overline{\omega})=\frac{1}{2}\cdot\frac{d\tilde{E}}{df}=\pi\frac{d\tilde{E}}{d\omega}=\frac{h}{2}(=)[Js]\ .\tag{9.5.1}$$

Based on the amplitude spectral function $A_{bb}(\omega)$ we should be able to explore a family of possible time-domain wave functions $\Psi_{bb}(t)=a_{bb}(t)\cos\varphi_{bb}(t)$ that comply with Planck's radiation law (as given in its frequency domain). In fact, this way we are searching for most general analytic expressions or wave functions, or wave packets and photons (formulated in a time and frequency domain). In addition, from theoretical and experimental achievements of M. Planck, A. Einstein, and Compton, we know that an elementary wave quant or wave packet in a form of photon has the energy $\tilde{E}=hf$. We also know that a photon has certain mean carrier frequency and a narrow frequency bandwidth (or frequency duration). Based on experimental experiences, we know that photons really have relatively short time and frequency durations with Gaussian amplitude envelopes (being band-limited, temporally, spatially, and spectrally). We assume that (an external) blackbody radiation, or its wave function $\Psi_{bb}(t)$, is composed of many single photons, $\psi_i(t)=a_i(t)\cos\varphi_i(t)$. Anyway, the energy conservation here is defined by Parseval's theorem, and considering a quantum nature of photons, and matter-waves superposition, we can formulate the following simplified relations,

$$\Psi_{bb}^2(t)=\left[a_{bb}(t)\cos\varphi_{bb}(t)\right]^2=\sum_{(i)}n_i\psi_i^2(t)\Rightarrow\tilde{E}_i=\int_{-\infty}^{+\infty}\psi_i^2(t)dt=hf_i=p_iu_i\ ,$$
$$\tilde{E}=\int_{-\infty}^{+\infty}\Psi_{bb}^2(t)dt=\frac{1}{\pi}\int_0^{+\infty}\left|A_{bb}(\omega)\right|^2d\omega=\int_{-\infty}^{+\infty}\left[\sum_{(i)}n_i\psi_i^2(t)\right]dt=\sum_{(i)}n_i\tilde{E}_i=h\sum_{(i)}n_if_i\ ,\tag{9.5.2}$$
$$n_i\in[1,2,3,...]\ .$$

It is much more realistic that any matter-waves emission, superposition, and interference effects would comply with Kotelnikov-Shannon-Nyquist-Whittaker and Analytic signal concepts about signals analysis, synthesis, sampling, and signals reconstruction.

Here (in (9.3) – (9.5.2) and later), we are using not-normalized wave functions (having dimensions in SI units), without introducing statistical and probabilistic assumptions, to stay connected with realistic, measurable, experimentally verifiable, and conceptually clear, tangible physics.

If blackbody radiation wave functions are sufficiently close to Gaussian-envelope waves, we could say that in certain frequency interval $\Delta f = F$ equal to six times of involved standard frequency deviation ($F = \Delta f = 6\overline{\sigma}_f$), we will find 99% of mentioned radiative energy, as follows,

$$0.99 \cdot \tilde{E} = S\Delta t \int_0^{+\Delta f} dR(f,T) = S\Delta t \int_0^{6\overline{\sigma}_f} \frac{2\pi f^2}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} df = 0.99 \cdot S \cdot \Delta t \cdot \sigma T^4 (=) [Ws = J],$$

and 100% of the same energy is equal to,

$$\tilde{E} = S\Delta t \int_0^{+\infty} dR(f,T) = S\Delta t \int_0^{+\infty} \frac{2\pi f^2}{c^2} \cdot \frac{hf}{e^{hf/kT} - 1} df = S \cdot \Delta t \cdot \sigma T^4 (=) [Ws = J],$$

We should be able (with proper mathematical processing) to find (for 99%-energy content) the corresponding frequency interval $F = \Delta f = 6\overline{\sigma}_f$, and this will be almost the total frequency length (frequency duration or interval) of the radiative energy function. We could also select that total time duration T^* of the same radiative function (for 99% of its energy) will be $\Delta t = T^* = 6\overline{\sigma}_t$. Now we can apply Uncertainty relations, and get the temporal duration of the radiative signal as,

$$\tilde{E} \cdot T^* = h \cdot F \cdot T^* \geq \frac{h}{2} \Leftrightarrow F \cdot T^* \geq \frac{1}{2} \Leftrightarrow T^* = 6\overline{\sigma}_t \geq \frac{1}{2F} = \frac{1}{12\overline{\sigma}_f},$$

and if we really measure the total radiated energy during the calculated time interval ($T^* \geq \frac{1}{2F} = \frac{1}{12\overline{\sigma}_f}$), we will get,

$$0.99 \cdot \tilde{E} = 0.99 \cdot S \cdot T^* \cdot \sigma T^4 \geq 0.99 \cdot S \cdot \frac{1}{2F} \cdot \sigma T^4 \cong 0.5 \frac{S}{F} \cdot \sigma T^4 (=) [Ws = J], \ (F = 6\overline{\sigma}_f \geq \frac{1}{2T^*}).$$

Here we use T^* as the time interval, and T as the temperature.

Let us now imagine that a spherical “black body”, with the external radius R, has certain (total) content of heat (and radiative) energy $\tilde{E}$, which can be considered as an internal matter-waves energy.

We could also approximate the total spatial length of the same body as being $L = 2R$. Resulting (total) linear momentum of the same "blackbody" radiator will be $\tilde{P}$. Proper time duration of the same "blackbody" object will be T^*. Now we can exploit relations between absolute domains durations or lengths, $T^* \cdot \tilde{E} = L \cdot \tilde{P}$ (see (10.2-2.2) from Chapter 10).

$$\left[\begin{array}{l} T^* \cdot \tilde{E} = L \cdot \tilde{P} \\ \tilde{E} \cong 0.5 \dfrac{S}{F} \cdot \sigma T^4 = 0.5 \cdot S \cdot T^* \cdot \sigma T^4 = 2\pi R^2 \cdot T^* \cdot \sigma T^4 \\ L = 2R = C \cdot T^* [m],\ S = 4\pi R^2 [m^2],\ V = \dfrac{4\pi}{3} R^3 [m^3] \end{array}\right] \Rightarrow$$

$$\Rightarrow T^* \cdot \tilde{E} = 2R \cdot \tilde{P} \Leftrightarrow T^* \cdot 2\pi R^2 \cdot T^* \cdot \sigma T^4 = 2R \cdot \tilde{P}$$

$$\Rightarrow \pi R \cdot (T^*)^2 \cdot \sigma T^4 = \tilde{P} \Leftrightarrow \frac{4\pi R^3}{C^2} \cdot \sigma T^4 = \tilde{P} \Rightarrow$$

$$\Rightarrow \tilde{P} = \frac{4\pi R^3}{3} \cdot \frac{3\sigma T^4}{C^2} = \frac{3\sigma}{C^2} \cdot V \cdot T^4 [kg \cdot m]$$

We know that all "blackbody" objects are radiating matter-waves (or electromagnetic) energy, which has its own (resulting) linear momentum $\tilde{P}$. This should be the most important part of "radiant energy", as speculated by N. Tesla? On a similar way, instead of considering only a heat energy as a wave energy, we could conceptualize natural radioactivity, considering radioactive emission as an internal wave energy of unstable atoms.

....

Since we know universally valid relations between group and phase velocity, and we know de Broglie wavelength expression for wave packets and matter waves, including photons (see about PWDC in the chapter 4.1), we should be able to explore velocity-dependent <u>spectral distribution of quantized matter waves</u> (that are inside a Blackbody cavity), as for instance,

$$\left\{\begin{array}{l} \lambda = h/p,\ \tilde{E} = hf = pu = E_k,\ u = \lambda f = \dfrac{v}{1 + \sqrt{1 - (\frac{v}{c})^2}} = \dfrac{\omega}{k} = \dfrac{\tilde{E}}{p}, \\ v = \dfrac{2u}{1 - \frac{uv}{c^2}} = u - \lambda \dfrac{du}{d\lambda} = -\lambda^2 \dfrac{df}{d\lambda} = \dfrac{d\omega}{dk} = \dfrac{d\tilde{E}}{dp},\ \omega = 2\pi f, \\ 0 \leq 2u \leq \sqrt{uv} \leq v \leq c \end{array}\right\} \Rightarrow \qquad (9.5.3)$$

$$\Rightarrow d\tilde{E}_i = hdf_i = v_i dp_i = d(p_i u_i) = c^2 d\tilde{m}_i = dE_{ki} = \psi_i^2(t)dt = \tilde{F}dr.$$

The familiar problematic and thinking are already initialized in Chapter 4.1 of this book (see T.4.1). In other words, Planck thermal radiation law should also be a consequence of relations between group and phase velocities, interactions, superposition, diffractions, and interferences of involved wave packets.

4.1. Interacting and coupled wave groups inside the Blackbody cavity

<table>
<tr><td colspan="2">

$$\left\{\begin{array}{l} v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda}, \\ u = \frac{v}{1+\sqrt{1-v^2/c^2}} = \frac{u-\lambda\frac{du}{d\lambda}}{1+\sqrt{1-v^2/c^2}} = \frac{-\lambda^2\frac{df}{d\lambda}}{1+\sqrt{1-v^2/c^2}} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \frac{du}{u} = -\left(\frac{d\lambda}{\lambda}\right)\sqrt{1-v^2/c^2} = \frac{df}{f}\cdot\frac{\sqrt{1-v^2/c^2}}{1+\sqrt{1-v^2/c^2}} \\ \frac{df}{f} = -\left(\frac{d\lambda}{\lambda}\right)(1+\sqrt{1-v^2/c^2}) \end{array}\right\}$$

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Non-relativistic group velocities:
(Lower and moderate temperatures)

$$v << c \Rightarrow v \cong 2u,\ \sqrt{1-v^2/c^2} \cong 1,$$

$$\left\{\frac{du}{u} \cong -\frac{d\lambda}{\lambda} \cong \frac{df}{2f}\right\} \Rightarrow$$

$$\left\{\ln\left|\frac{u}{u_0}\right| \cong -\ln\left|\frac{\lambda}{\lambda_0}\right| \cong \frac{1}{2}\ln\left|\frac{f}{f_0}\right|\right\} \Rightarrow$$

$$v \cong 2u = 2\lambda f$$

</td><td>

Relativistic group velocities:
(Very high temperatures)

$$v \approx c \Rightarrow v \approx u \approx c,\ \sqrt{1-v^2/c^2} \cong 0$$

$$\frac{du}{u} \cong 0\ ,\ \frac{df}{f} \cong -\frac{d\lambda}{\lambda}$$

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</table>

The matter-wave states inside a Blackbody cavity are not only photons and electromagnetic standing waves. There are electrons, ions, and other charged and neutral (micro and macro) particles, matter waves, secondary emissions, scattering products, and plasma states inside a Blackbody cavity. As we know, all structured and sufficiently stabilized matter like atoms, electrons, protons, neutrons etc. are presenting internally (or structurally) quantized states, meaning being well packed, self-closed standing matter waves formations (what is also applicable on planetary or solar systems, as we can see in the second chapter of this book). We also know that by heating produced electrons and photons are on some direct way mutually synchronized and coupled, mutually convertible and communicating, since electrons are emitting and absorbing photons, thanks to quantized, resonant and standing-waves electrons' structure. Such internal and motional matter wave states (produced by thermal motions and associated electromagnetic activity) should naturally cover the large interval of velocities, temperatures, and frequencies. Because of mentioned blackbody reality (having many different participants and their interactions, interferences, and superposition), we should not blindly consider that original assumptions and development of Planck radiation law (9.2) is something that is fully conceptually clear, "Physics and applied mathematics realistic", and exceptionally relevant. *Since we know that the final mathematical formula (for Blackbody radiation) established by Planck is very much correct, we should consider applied logic, assumptions, and mathematics only as a particularly practical and effective mathematical curve-fitting. This is just a kind of successful, well-improvised mathematical fitting or interpolation based on* ***in advance known*** *results.* Consequently, we are still far from the final answer about time, frequency and spatial shapes of quantized wave functions generated by a Blackbody radiation.

Only externally radiated photons, outside Blackbody cavity have constant group and phase velocity $v \cong u \cong \lambda f \cong c$, but this is probably not the case inside the blackbody cavity. Consequently, wave functions of involved quantized wave packets (including photons) should be mutually different (by shapes in a time and frequency domains, and by their physical nature) inside and outside the blackbody cavity.

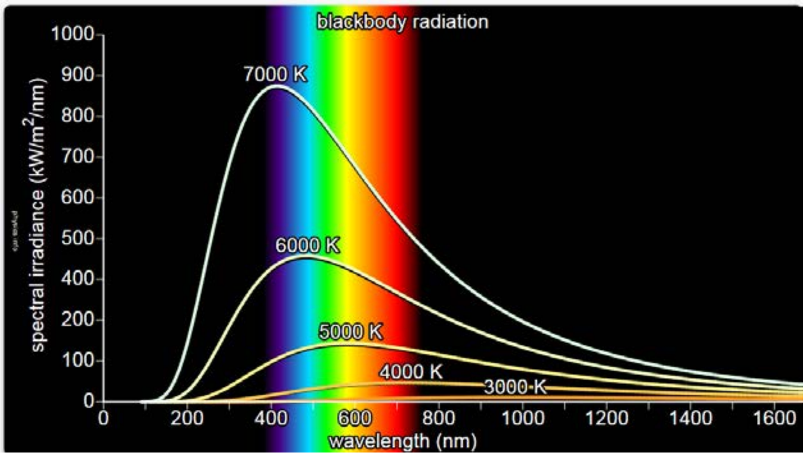
The facts about blackbody radiation and possible relations to N. Tesla Dynamic Theory of Gravity

(taken from http://physics.info/planck/): The Physics Hypertextbook Opus in profectus

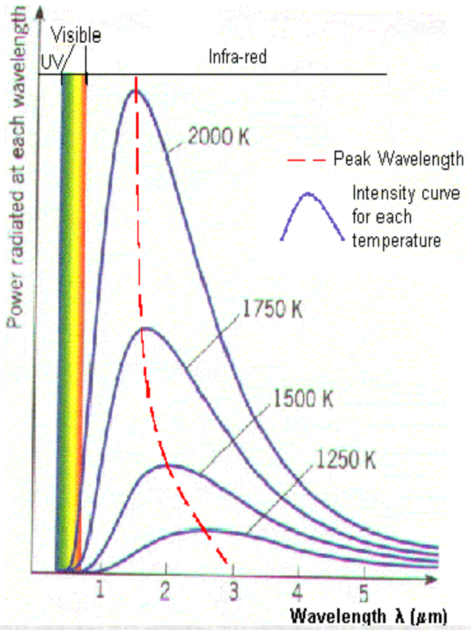
- We know the shape of the distribution,
- The peak shifts according to Wien's law,
- The Stefan-Boltzmann law describes the total power output.

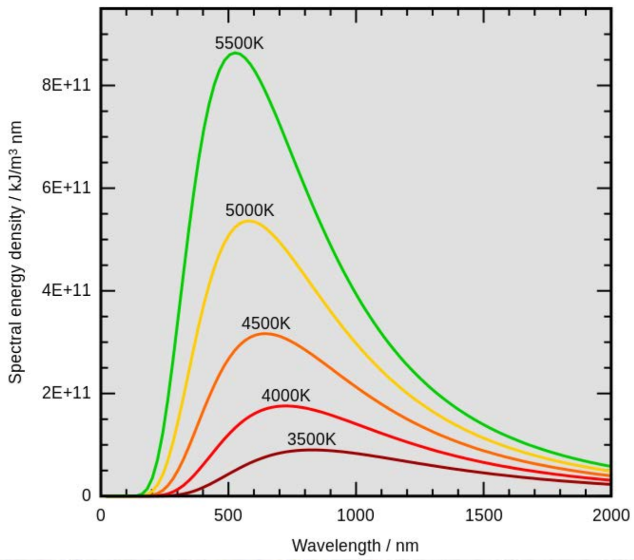
For instance, Wien's Displacement Law is in a full agreement with Planck's radiation law (9.2), and presents the maximal wavelength $\lambda_{max.}$ of external Blackbody (electromagnetic or photons) radiation at which the monochromatic emissive power has a maximum (see the pictures below),

$$\begin{aligned}
&\lambda_{max.} = \frac{2897.8 \cdot 10^{-6}}{T} = \frac{h}{\tilde{p}_{min}} = \lambda_{peak} (=) [m], \quad T = T_{min.} \ (=)[K] \\
&u = \lambda f \leq \lambda_{max.} \cdot f \leq c, \Rightarrow u = \frac{E_k}{p} = \frac{\tilde{E}}{\tilde{p}} \leq \frac{f}{T} \cdot 2897.8 \cdot 10^{-6} \leq c \ (=) \left[\frac{m}{s}\right], \ p = \tilde{p} = p_{min.}, \\
&\Rightarrow f \leq \frac{cT}{2897.8 \cdot 10^{-6}} = f_{max}, \ E_k = \tilde{E} = pu = hf \leq \frac{hcT}{2897.8 \cdot 10^{-6}} \leq c\tilde{p}_{max} = \tilde{E}_{max} (=) [J], \\
&\Rightarrow \tilde{p}_{max} \geq \frac{hT}{2897.8 \cdot 10^{-6}} = \frac{h}{\lambda_{max.}} = \tilde{p}_{min} \left(= \frac{hT}{2897.8 \cdot 10^{-6}} \right) \Rightarrow \\
&T = \frac{2897.8 \cdot 10^{-6}}{h} \tilde{p}_{min} = T_{min} \leq \frac{2897.8 \cdot 10^{-6}}{h} \tilde{p}_{max} = \frac{2897.8 \cdot 10^{-6}}{ch} \tilde{E}_{max} = \frac{2897.8 \cdot 10^{-6} \cdot f_{max}}{c} (=) [K]
\end{aligned} \tag{9.6}$$

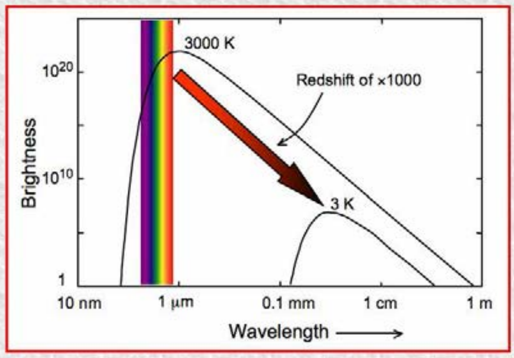


Taken from the Internet; -Blackbody Radiation
(http://physics.info/planck/): The Physics Hypertextbook Opus in profectus





Redshift Preserves a Thermal Spectrum



Redshift preserves a black body spectrum, simply lowering the temperature in proportion to the scale factor: $T_{now} = a_{emit} T_{emit}$. For example, the CMB was created at 3000K, at a scale factor of a = 10^{-3}, so today's CMB has spectrum T = 10^{-3} %#215; 3000 K = 3 K.

The graphs show:

- As the temperature increases, the peak wavelength emitted by the Blackbody decreases.
- As temperature increases, the total energy emitted increases, because of the total area under the curve increases.
- Blackbody radiation (https://en.wikipedia.org/wiki/Wien's_displacement_law) as a function of wavelength for various absolute temperatures. Each curve is seen to peak at a somewhat different wavelength; Wien's law describes the shift of that peak in terms of temperature.
- **Wien's displacement law** states that the Blackbody radiation curve for different temperatures peaks at a wavelength inversely proportional to the temperature. The shift of that peak is a direct consequence of the Planck radiation law, which describes the spectral brightness of Blackbody radiation as a function of wavelength at any given temperature. However, it had been discovered by Wilhelm Wien several years before Max Planck developed that more general equation, and describes the entire shift of the spectrum of Blackbody radiation toward shorter wavelengths as temperature increases.
- Formally, Wien's displacement law states that the spectral radiance of Blackbody radiation per unit wavelength peaks at the wavelength λ_{max} given by: $\lambda_{max.} = \frac{b}{T}$,

  where *T* is the absolute temperature in kelvin. *b* is a constant of proportionality called *Wien's displacement constant*, equal to 2.8977729(17)×10^{-3} m K[1].

Frequency-dependent formulation: For spectral flux considered per unit frequency df (in hertz), Wien's displacement law describes a peak emission at the optical frequency f_{max} given by:

$$f_{max} = \frac{\alpha}{h} kT \approx (5.879 \times 10^{10}\ \text{Hz/K}) \cdot T$$

where $\alpha \approx 2.821439...$ is a constant resulting from the numerical solution of the maximization equation, k is the Boltzmann constant, h is the Planck constant, and T is the temperature (in kelvin). This frequency does not correspond to the wavelength from the earlier formula, which considered the peak emission per unit wavelength.

Blackbody radiation does not depend on the type of object emitting it. An entire spectrum of blackbody radiation depends on only one parameter, the temperature, T.

A Blackbody is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy).

Wien's displacement law

- Shows how the spectrum of Blackbody radiation at any temperature is related to the spectrum at any other temperature. If we know the shape of the spectrum at one temperature, we can calculate the shape at any other temperature.
- A consequence of Wien's displacement law is that the wavelength at which the intensity of the radiation produced by a Blackbody is at a maximum, λ_{max}, it is a function only of the temperature.

Taken from the Internet (http://de.slideshare.net/FanyDiamanti/blackbody-radiation-11669156): BLACKBODY RADIATION. Authors: DISUSUN OLEH, Rahmawati Th. Diamanti, Ivone Pudihang, Recky Lasut, Aulinda Tambuwun, Deyvita Montolalu

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Conclusions and hypothetical proposals regarding N. Tesla Dynamic Force of Gravity:

1. *The speculative opinion of the author of this book is that N. Tesla radiative energy, [97], is closely related to the Blackbody Radiation (and to other aspects of electromagnetic, cosmic, and electromechanical radiation). Our Universe or Cosmos dominantly presents an electromagnetic and electromechanical oscillating Universe (manifesting as different cosmic matter formations, particles, masses, atoms, solid, liquid and plasma states, with dualistic wave-particle and different matter waves states).*

2. *Let us consider that all masses, atoms, and objects in our Universe are mutually and structurally resonating and communicating (inwards and outwards), both like well-coupled mechanical and electromagnetic resonators by exchanging electromagnetic and mechanical radiation and vibrations. For simplified conceptual understanding of Gravitation, we can, (analogically and naturally) extend N. Bohr's atom model to have quantized electromagnetic energy exchanges and communications, not only within electron orbits and electron-clouds and externally, but also internally, between electrons and atom nucleus, as well as within atom nucleus, between any segment of an atom and its external environment (towards other masses and atoms). This way we are approaching R. Boskovic concept of Universal Natural Force, [6] (see more in Chapters 8, and 2). Consequently, all atoms and masses in our Universe are*

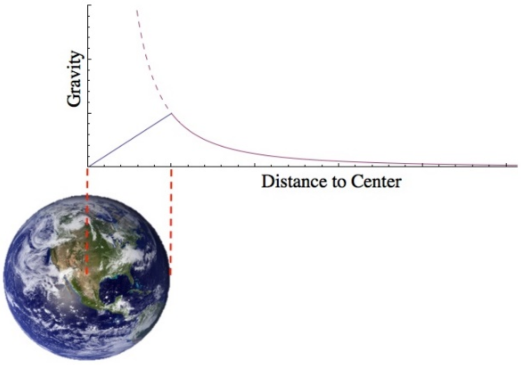
mutually synchronized, and connected, synchronously radiating, and receiving electromagnetic and other matter waves energy (and involved mechanical and electromagnetic moments), and here we simply speculate that such radiative energy also has close relations to Blackbody Radiation. At least, thermal electromagnetic radiation in its infrared domain can be created by stationary resonant states of mentioned, internal mechanical and electromagnetic resonant states of heated masses and its atoms. We know that spectral content and overall spectral complexity of any object has at least two superficially different manifestations. One is its electromagnetic spectrum, dominantly covering what Planck's Blackbody radiation law is describing, but it also extends to (often not easy detectable) exceptionally low and extremely high frequencies of acoustic and electromagnetic oscillations (what we often neglect in our elaborations, because all electromagnetic vibrations are at the same time producing mechanical or acoustic vibrations). Of course, some of mechanical, acoustic and phonons related vibrations also exist in a domain covered by Blackbody electromagnetic radiation but being not easily detectable (or still not specifically measured). The main message here is to say that Blackbody and Electromagnetic radiation (of certain body) are covering the same frequency intervals as associated acoustic, mechanical and phonons vibrations, only having different spectral distributions (with different amplitudes in different frequency intervals). As we know (see more in the Chapters 6. and 10.), spatial and temporal dimensions and spectral characteristics, regarding durations and sizes of mutually conjugate spectral distributions of any energy-momentum state are mutually coupled, corelated and proportional. That means all of them should cover the same frequency intervals, both temporally and spatially defined (of course, having different amplitude-frequency distributions and resonant modes), somewhat like what we can get using Finite Elements Analysis to analyze resonant states of certain object. Our electromagnetic and mechanical Universe is anyway well united, regardless we still do not have proper fields unification theory.

3. *The domain where gravitational force (produced by radiative energy within stationary and standing matter waves), should be in certain relation to Blackbody Radiation, being valid for all matter waves with wavelengths higher than* $\lambda_{max.} = \frac{b}{T} = \frac{2897.8 \cdot 10^{-6}}{T}$. *Consequently, where matter waves (meaning electromagnetic waves, or Blackbody Radiation), participating in communications between atoms and other masses, have lower frequencies (compared to frequencies of photons and electromagnetic exchanges inside an atom), this should be related to gravitation. See the pictures below (next pages), where estimated domains of gravity-related electromagnetic, bidirectional, radiative energy exchanges are encircled. See also the extended Bohr's atom model in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces", where similar concept about inwards and outwards electromagnetic energy exchanges (inside and outside all atoms) is presented, associating on ideas of N. Tesla and R. Boskovic. In the second Chapter, "*2.2.1. WHAT THE GRAVITATION REALLY IS ", we can find additional arguments about radiative energy and gravitation.

Black body radiation and Gravitation force comparisons

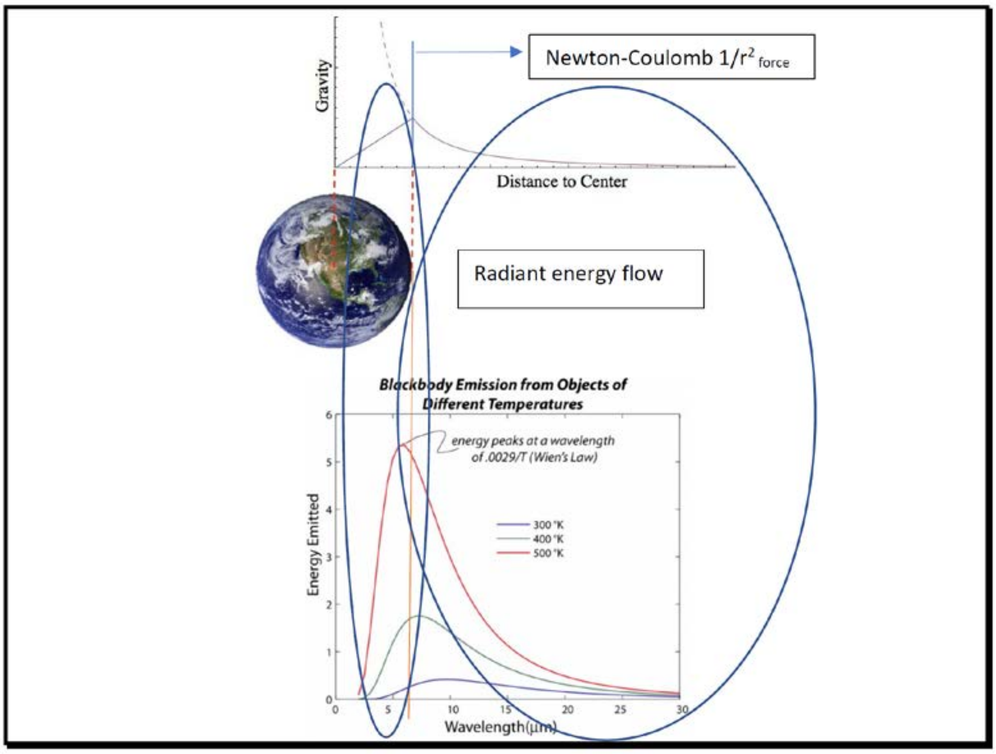
Let us start from the following Citation from Internet:
https://www.askamathematician.com/2018/06/q-why-does-gravity-pull-things-toward-the-center-of-mass-whats-so-special-about-the-center-of-mass/
Q: Why does gravity pull things toward the center of mass? What's so special about the center of mass? | Ask a Mathematician / Ask a Physicist

.... "if you're inside of a sphere of mass, only the layers below you count toward the gravity you feel. So, the closer you are to the center, the less mass is below you, and the less gravity there is. If you were in an elevator that passed through the Earth, you'd know you were near the center because there'd be no gravity. With the same amount of mass in every direction, every atom in your body would be pulled in every direction equally, and so not at all. A sort of gravitational tug of war stalemate.
Newton's Universal Law of Gravitation applies to everything (hence the "universal"). But instead of applying to the centers of mass of pairs of big objects, it applies to every possible pair of pieces of matter. In the all-to-common-in-space case of spheres, we can pretend that entire planets and stars are points of mass and apply Newton's laws to those points, but only as long as we don't need perfection (which is usually). As soon as things start knocking into each other, or the subtle effects of their not-sphere-ness become important, you're back to carefully tallying up the contribution from every chunk of mass".



"As you approach a sphere of mass gravity increases by the inverse square law, but drops linearly inside the sphere. If the density varies (which it usually does), that straight line will bow up a bit."

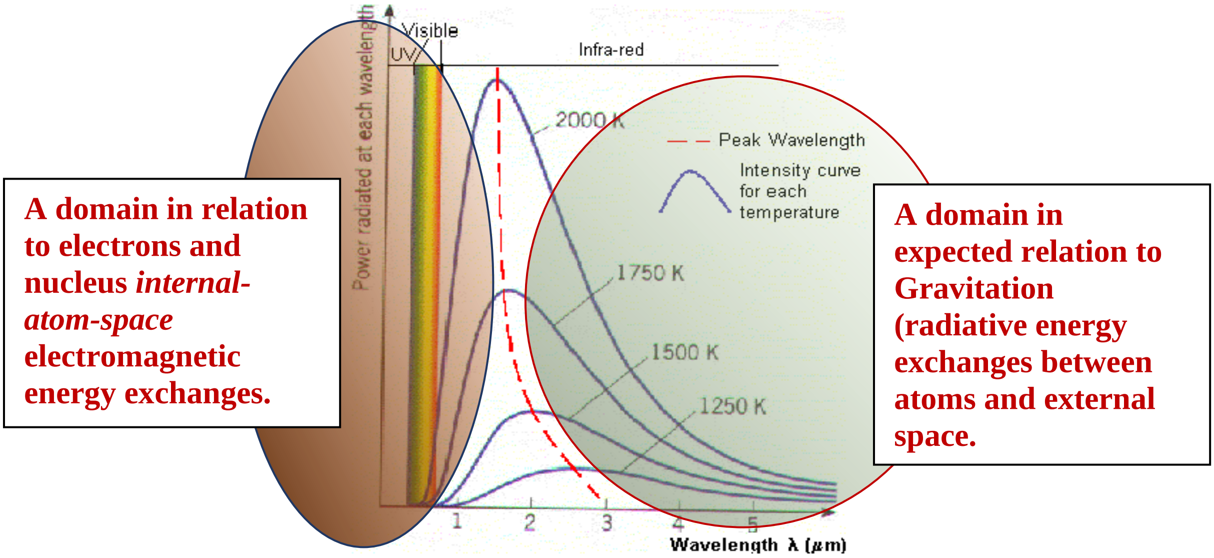
Let us now, creatively, imaginatively, and with a big doze of intellectual flexibility, make an indicative and analogical comparison between *Gravity force curve* and curve of Blackbody electromagnetic emission, as illustratively presented on the picture below,

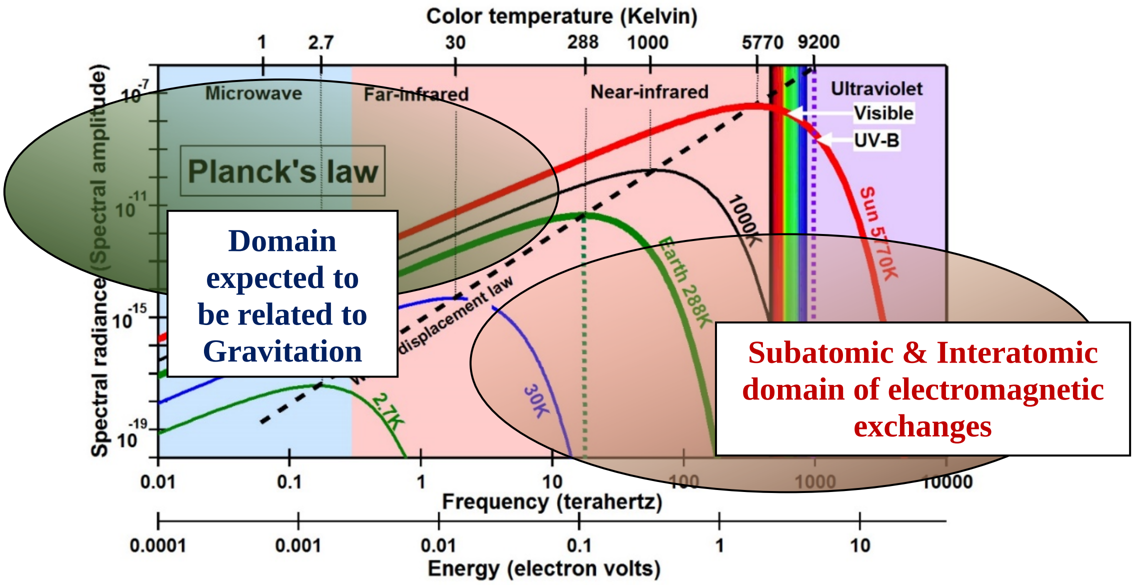


Of course, in case of gravitation, horizontal axis is the distance between ***attracting masses centers***, and Blackbody emission curve is related only to wavelengths of radiated emission, and comparison and correlation between relevant curves presents a challenging task, but intuitively we can feel some encouragement for establishing certain conceptual comparison. ***Basically, here we are hypothetically suggesting that Blackbody radiation is involved in Tesla's Radiant energy flow***.

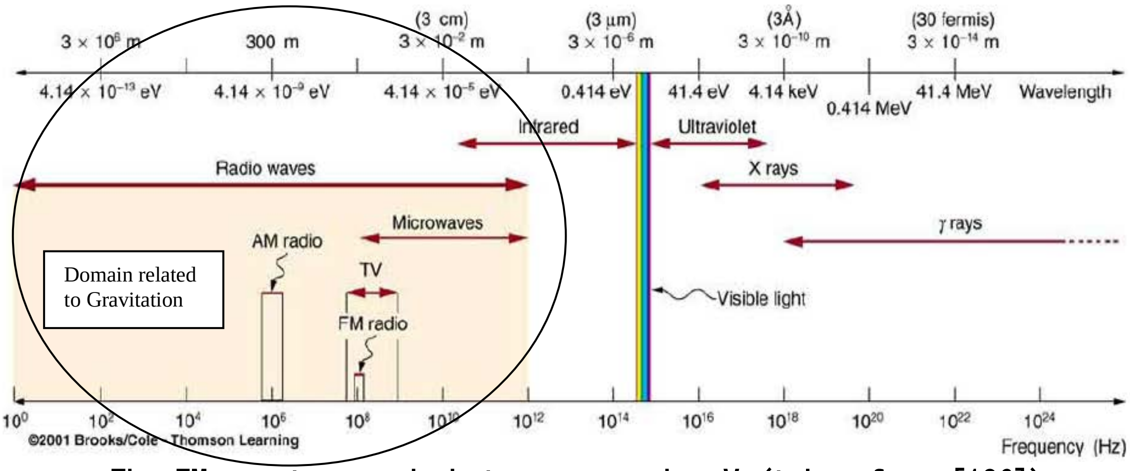
By continuing with similar brainstorming, we can intuitively (and hypothetically) create some conceptual and analogical assignments, or assumptions, indicating where we could expect detection of gravitation-related electromagnetic emissions (see the following illustrations with relevant and indicative comments).

Read more at http://phys.org/news/2013-07-blackbody-stronger-gravity.html#jCp
http://phys.org/news/2013-07-blackbody-stronger-gravity.html, **[100]** M. Sonnleitner[1,2], M. Ritsch-Marte[2], and H. Ritsch[1]:
"Perfectly non-reflective objects, called blackbodies, produce blackbody radiation when at a uniform temperature. Although the properties of blackbody radiation depend on the blackbody's temperature, this radiation has always been thought to have a net repulsive effect. Now in a new study, scientists have theoretically shown that blackbody radiation induces a second force on nearby atoms and molecules that are usually attractive and, quite surprisingly, even stronger than the repulsive radiation pressure. Consequently, the atoms and molecules are pulled toward the blackbody surface by a net attractive force that can be even stronger than gravity. The new attractive force—which the scientists call the "blackbody force"—suggests that a variety of astrophysical scenarios should be revisited".

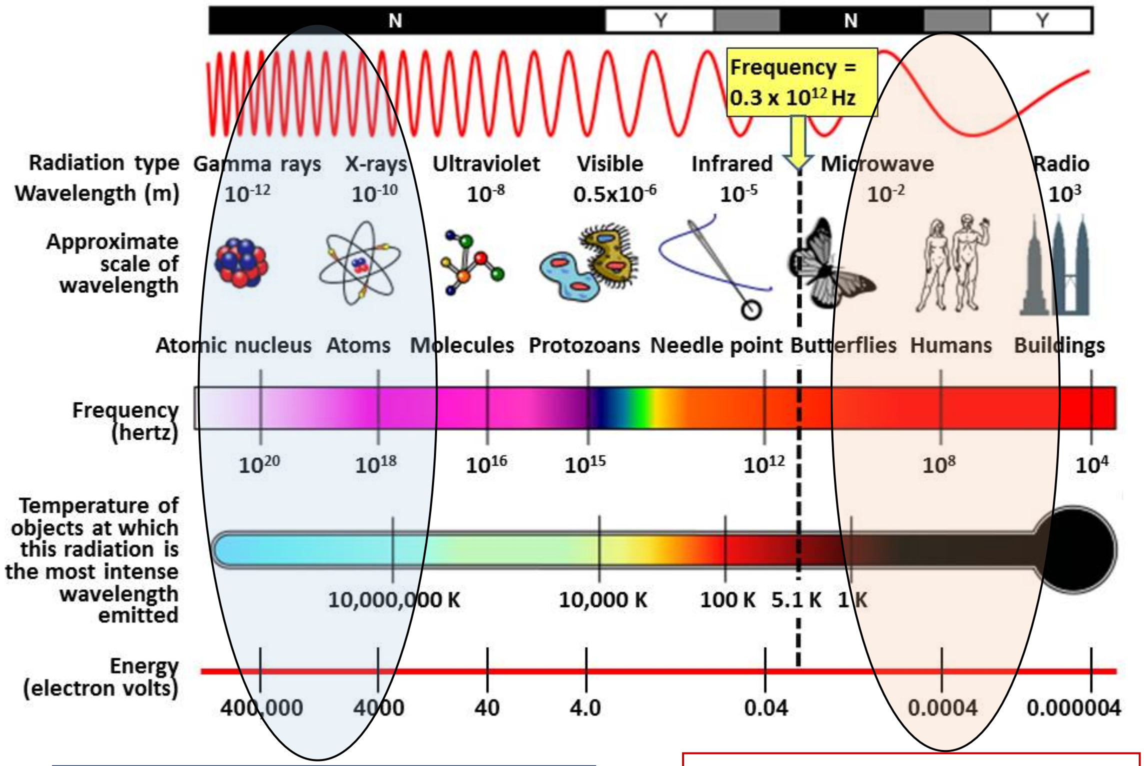




Blackbody Radiation and Gravitation
(taken from https://ozonedepletiontheory.info/energy-not-additive.html)



The EM spectrum and photon energy in eV (taken from [126])



Here is high energy, internal atomic and nuclear, electromagnetic and cosmic radiation

Here we expect to find effects of gravitational force and by electromagnetic energy produced mechanical effects

Properties of the Electromagnetic Spectrum in relation to Gravitation
(from **http://ozonedepletiontheory.info/ImagePages/EM-spectrum-properties.html**
https://ozonedepletiontheory.info/energy-not-additive.html)

Citation from https://ozonedepletiontheory.info/energy-not-additive.html: Planck's law describes the spectral radiance (the energy contained in electromagnetic radiation at each frequency) radiated by a Blackbody of matter as a function of the temperature of the surface of the body (in degrees Kelvin). Note that when a temperature is increased, the spectral radiance is increased for each frequency and the peak spectral radiance moves to a higher frequency.

Thermal energy

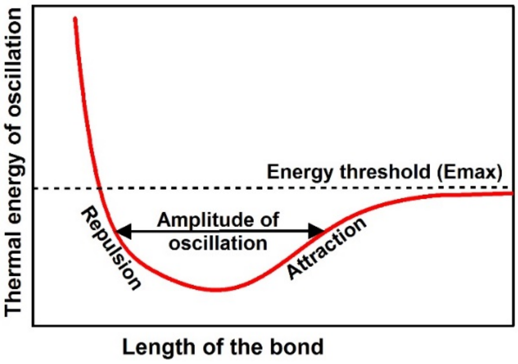
The chemical bonds that hold atoms together to form matter are not rigid. They are observed to oscillate about an energy minimum between electrodynamic repulsive forces pushing the atoms apart and electrodynamic attractive forces pulling the atoms together. As the thermal energy of oscillation increases, the amplitude of oscillation increases until at some energy threshold (E_{max}) the bond comes apart. Energy (frequency) may increase in discrete steps as higher and higher normal modes (frequencies) of oscillation are activated. One such anharmonic atomic oscillator exists for every normal mode of oscillation of every degree of freedom of every bond in the matter. Each oscillator has a characteristic resonant frequency and an amplitude of oscillation, the latter of which increases with increasing temperature (thermal energy).

Because the oscillator is asymmetric, the average length of the bond increases with increasing thermal energy so that the volume of the matter expands with increasing temperature, something that is well observed.

An atomic oscillator of this type is asymmetric (anharmonic) because the force of repulsion increases very rapidly as like charges are pressed together whereas the force of attraction decreases much more slowly with distance as opposite charges separate.

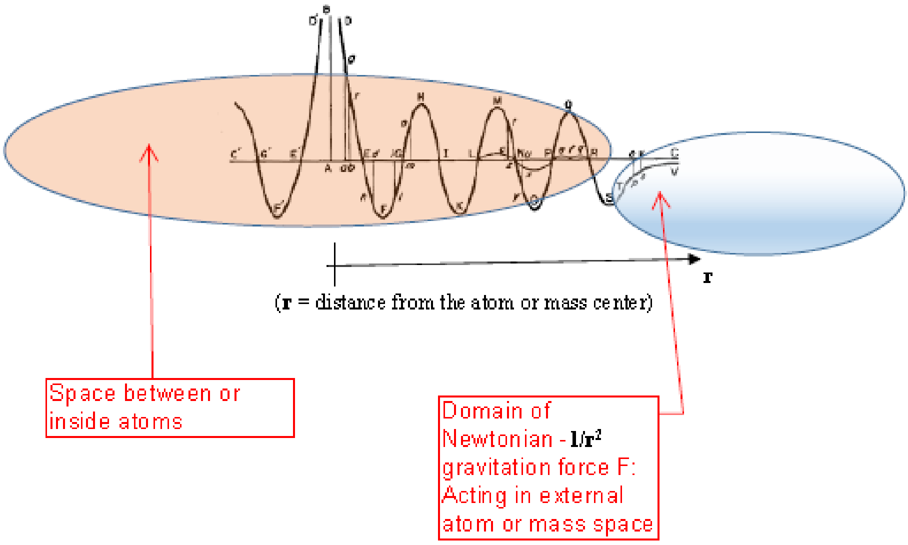
An atomic oscillator of this type is frictionless and thus can oscillate for an exceptionally long time. The only way to add energy to such an oscillator, or to subtract energy from it, is through resonance. When the amplitude of oscillation at a specific frequency of one bond is larger than the amplitude of oscillation at the same frequency of an adjacent bond, the bond with the larger amplitude will "give up" energy to the bond with the lower amplitude until the amplitudes for both bonds are equal. The rate of energy transfer increases with increasing difference in amplitudes.

We can find that atomic and molecular chemical bonds between matter particles are also surprisingly familiar and like R. Boskovic Natural Force (see the pictures below).



The chemical bonds that hold atoms together to form matter are observed to oscillate about an energy minimum between electrodynamic repulsive forces pushing the atoms apart and electrodynamic attractive forces pulling the atoms together. As the thermal energy of oscillation increases, the amplitude of oscillation increases until at some energy threshold (E_{max}) the bond comes apart (taken from the Internet: https://ozonedepletiontheory.info/energy-not-additive.html).

R. Boskovic Universal Natural Force Law (see below) also describes familiar kind of standing-waves structured forces.



Rudjer Boskovic's Universal Natural Force Function

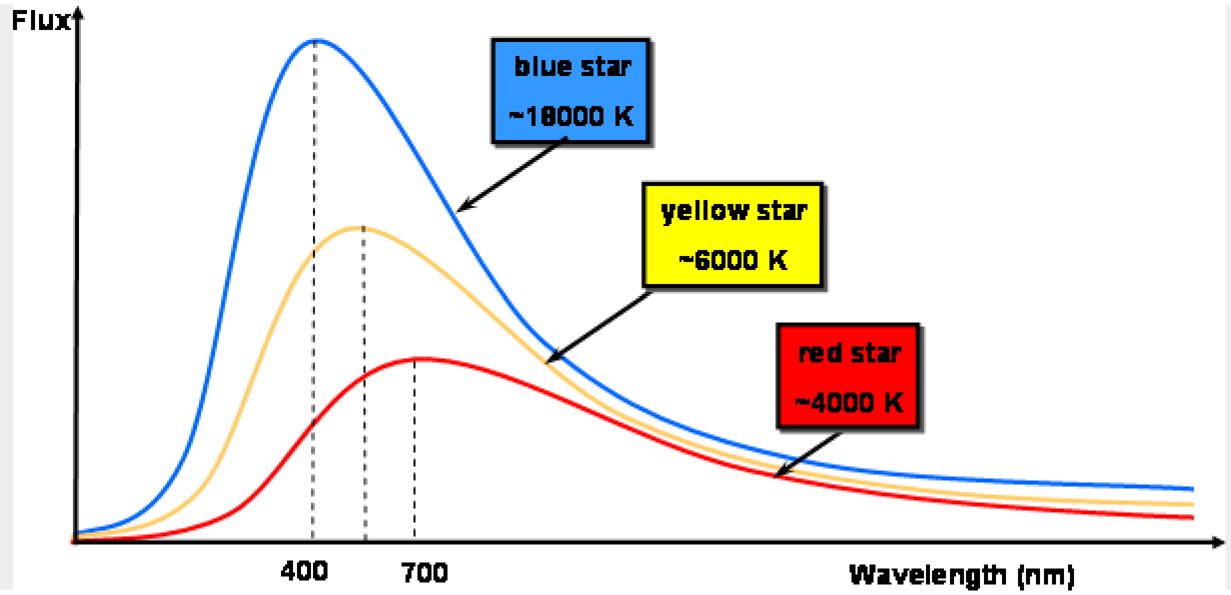
If we consider that force of gravitation is kind of electromagnetic force, naturally such forces are both respecting the same Coulomb-Newton $1/r^2$ force law (since this is the same force). In cases we focus our attention only on what we consider as gravitation, we can see that this is only the attractive-force part of the R. Boskovic universal natural force curve (on the right picture side). *Usually, we expect that natural forces have mutually balanced positive and negative, or attractive and repulsive values or sides, and for Newtonian gravitation we still did not find where its repulsive force part is. In case of Rudjer Boskovic universal natural force it is obvious that repulsive, balancing, alternating forces are in the oscillatory part of masses and atoms' structure (on the left picture side) being structured as periodical, resonant, standing matter-waves or "standing masses and energy agglomerations and distributions".* We can also experimentally demonstrate existence of such attractive and repulsive forces with ultrasonic resonators (where nodal zones are manifesting only attractive forces, and anti-nodal zones are manifesting only repulsive forces). See more about gravitation and R. Boskovic in the second chapter of this book.

....

Uncertainty Relations (or inequalities, between mutually original and spectral domains) from mathematics are universally valid and applicable to micro and macro world of Physics (and not obligatory or necessarily linked only to probabilities and statistical distributions of microphysics events, as the contemporary Quantum Theory is implicating). Consequently, we can make an additional estimation of the durations or domains lengths of Blackbody radiation (regarding its energy-content, time, frequency, and spatial durations). Consequences of properly applied Uncertainty Relations would influence different and innovative understanding of Cosmology, Background or Relict Radiation, Big-Bang assumptions, natural forces, matter structure, Gravitation etc. (… will be continued...). See more in Chapters 6. And 10.

Citation from, [100]:
http://phys.org/news/2013-07-blackbody-stronger-gravity.html#jCp
Scientific Article: M. Sonnleitner, et al. "Attractive Optical Forces from Blackbody Radiation." *PRL* 111, 023601 (2013). DOI: 10.1103/PhysRevLett.111.023601

The blackbody objects are perfect not-reflectors that produce constant radiation when they are at a uniform temperature. Thus, the properties of a blackbody depend on its temperature, thinking that this radiation would have a repulsive effect. Now a new, scientists have demonstrated theoretically that blackbody radiation induces a second force in atoms and molecules that are near its surface, which is attractive and stronger than the repulsive radiation pressure. Consequently, the atoms and molecules are pulled to the surface of the blackbody by a force, which may be greater than gravity. The new attractive force - what scientists describe as "*blackbody force*" - suggests that a variety of astrophysical scenarios need to be revisited.



Scientists identify a few interesting results in their formulation. First, this force decays with the third power of the distance to the blackbody ($F \propto 1/r^3$). Second, it is stronger for small objects. Third, the force is stronger for warmer objects. Above a few thousand degrees Kelvin, the force of attraction changes to repulsion.
In their study, scientists demonstrated that the strength of the blackbody in a grain of dust, at a temperature of 100 K, is much stronger than the gravitational force in this grain. However, for a massive star at a temperature of 6000 K blackbody force is much weaker than the gravitational force.

9.1. Wave Function of a Single Photon

Since Planck's radiation law says that the heat radiation is composed of elementary wave packets or energy quanta, called photons, we could try to determine the most probable waveforms describing such photons (in a temporal, spatial, and frequency domains) and see up to which level such objective is mathematically manageable.

In Quantum theory, it is generally accepted that a single photon has wave energy equal to the product of Planck's constant **h** and photon's frequency **f**. Let us analyze the additional mathematical background of Planck's radiation law, regarding superposition of photons with energies as, $\tilde{\mathbf{E}} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega$. We know that initially such radiation law was established almost empirically as the best curve fitting or estimation (without more general theoretical grounds), ***guided by experimentally already known results***. Photon should be an electromagnetic wave state, or wave packet, known as the carrier of one quantum of energy ($\tilde{\mathbf{E}} = \mathbf{hf}$), and it should be presentable using certain time-domain wave function $\psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$ that can be expressed in the form of an Analytic signal, being frequency narrow-band, and simple-harmonic, Gaussian amplitude-envelope wave function $\psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\omega\mathbf{t}$. Here, we try to create the framework, which will define the family of (elementary or basic), *photons wave functions*, or wave-packets that intrinsically or naturally carry one quantum of energy (to support Planck's radiation law, and to verify if the concept of quantized photons is always applicable). See much more about photons conceptualization in Chapter 4.1 under "**4.1.1.1. Photons and Particle-Wave Dualism** ", and as summarized in "T.4.0. Photon – Particle Analogies".

The Analytic Signal presentation gives the chance to extract immediate (or instant) signal amplitude $\mathbf{a}(\mathbf{t})$, phase $\varphi(\mathbf{t})$, and frequency $\omega(\mathbf{t}) = \frac{\partial\varphi(\mathbf{t})}{\partial\mathbf{t}} = 2\pi\mathbf{f}(\mathbf{t})$. Let us also extend and test the meaning of Planck's energy quantum ($\tilde{\mathbf{E}} = \mathbf{hf}$), where instead of constant photon frequency $\mathbf{f} = \omega/2\pi$ (valid for a single photon), we will consider the mean wave frequency, $\overline{\omega} = \langle\omega(\mathbf{t})\rangle = 2\pi\langle\mathbf{f}(\mathbf{t})\rangle = 2\pi\overline{\mathbf{f}}$, of the time-domain, arbitrary photon wave function $\Psi(\mathbf{t})$.

In other words, we assume that photons or elementary energy quanta, as wave packets, have limited, short time and frequency durations. Practically, this way we are formulating the concept that a wave energy quantum or package is represented by a ***frequency band limited*** wave function. It is also obvious that such time-domain elementary wave functions (carrying one quantum of energy, $\tilde{\mathbf{E}} = \mathbf{hf}$) could be used as "basis functions" for relevant Spectral Signal Analysis and Synthesis. The platform for analytic expressions of such elementary wave functions will be certain Analytic Signal wave function (see (4.0.47) - (4.0.54) from the Chapter 4.0), with the Planck energy formula, as follows.

$$\tilde{E} = \int_{[T]} \Psi^2(t)dt = \int_{[T]} [a(t)\cos\varphi(t)]^2\, dt = \int_{[T]} a^2(t)dt = \frac{h}{2\pi}\overline{\omega} = h\overline{f},\ \frac{\tilde{E}}{\overline{\omega}} = \frac{d\tilde{E}}{d\omega} = \frac{h}{2\pi},$$

$$\omega(t)=\frac{\partial\varphi(t)}{\partial t}=2\pi f(t)\quad(\Leftrightarrow)\quad \bar{\omega}(t)=\left\langle\frac{\partial\varphi(t)}{\partial t}\right\rangle=2\pi\left\langle f(t)\right\rangle=2\pi\bar{f}=\bar{\omega},$$

$$\bar{\omega}=\frac{1}{T}\int_{[T]}\omega(t)dt\quad(\Leftrightarrow)\quad\frac{\frac{1}{T}\int_{[T]}\omega(t)\cdot a^2(t)\cdot dt}{\int_{[T]}a^2(t)\cdot dt}\quad(=)\quad\tilde{E}\frac{2\pi}{h}\Rightarrow$$

$$\Rightarrow\frac{\tilde{E}}{\bar{\omega}}=\frac{\left[\int_{[T]}a^2(t)\cdot dt\right]^2}{\frac{1}{T}\int_{[T]}\omega(t)\cdot a^2(t)\cdot dt}=\frac{\int_{[T]}a^2(t)\cdot dt}{\frac{1}{T}\int_{[T]}\omega(t)dt}=\frac{a^2(t)\int_{-\infty}^{+\infty}a^2(t)dt}{2\left[\Psi(t)\dot{\hat{\Psi}}(t)-\dot{\Psi}(t)\hat{\Psi}(t)\right]}=\frac{h}{2\pi}=\text{Const.}\qquad(9.8)$$

We still do not know analytic forms for a photon wave functions, neither in its time or frequency domain, and the only thing we know is that each of such wave functions should have the energy content of one energy quant (meaning hf). *Obviously, such elementary energy-quant or matter-wave has the same finite energy in its time and frequency domains. It should be on some way, well defined, and optimally concentrated both in its time and frequency domains, meaning that it should be from the family of Gaussian pulses, Gaussian window functions, including familiar Gaussian and analytic signal wavelets, because the Gaussian envelope or amplitude wave functions are optimally concentrated and finite in their joint time and frequency domains.*

Planck's photon energy we can determine as an energy of a photon analytic signal wave-function in a time and frequency domains. For instance, we can first calculate mean photon frequency, and later Planck photon energy, based on mentioned mean frequency.

$$\bar{\omega}=\left\langle\omega(\mathbf{t})\right\rangle=\left[\begin{matrix}\dfrac{\int_{[t]}\mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}{\int_{[t]}\mathbf{a}^2(\mathbf{t})\mathbf{dt}}\\ \textbf{and / or}\\ \dfrac{\mathbf{1}}{\Delta\mathbf{t}}\int_{[t]}\omega(\mathbf{t})\mathbf{dt}\end{matrix}\right]=2\pi\left\langle\mathbf{f}(\mathbf{t})\right\rangle=2\pi\bar{\mathbf{f}}\,,\qquad(9.9)$$

$$\tilde{\mathbf{E}} = \mathbf{hf} = \frac{\mathbf{h}}{2\pi}\omega \Rightarrow \tilde{\mathbf{E}} = \frac{\mathbf{h}}{2\pi}\overline{\omega} = \mathbf{h}\overline{\mathbf{f}} \Rightarrow$$

$$\Rightarrow \left\{ \tilde{\mathbf{E}} = \frac{\mathbf{h}}{2\pi} \frac{\int_{[t]} \mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}}{\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}} = \int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}, \text{ or } \tilde{\mathbf{E}} = \frac{\mathbf{h}}{2\pi}\frac{\mathbf{1}}{\Delta\mathbf{t}}\int_{[t]} \omega(\mathbf{t})\mathbf{dt} = \int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt} \right\} \Rightarrow \quad (9.10)$$

$$\Rightarrow \left\{ \frac{\left[\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}\right]^2}{\int_{[t]} \mathbf{a}^2(\mathbf{t})\omega(\mathbf{t})\mathbf{dt}} = \frac{\mathbf{h}}{2\pi} = \mathbf{Const.}, \quad \text{or} \quad \frac{\int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}}{\frac{\mathbf{1}}{\Delta\mathbf{t}}\int_{[t]} \omega(\mathbf{t})\mathbf{dt}} = \frac{\mathbf{h}}{2\pi} = \mathbf{Const.} \right\}.$$

Depending on how we define the mean frequency, we should be able to find the family of specific wave functions, $\Psi(\mathbf{t}) = \mathbf{a}(\mathbf{t})\cos\varphi(\mathbf{t})$, that can describe any photon or some other elementary quantum of wave energy in a time domain. All such elementary wave functions (at least in a micro-world of Physics) should have at least one common characteristic, which is that each of them has the energy content equal to a Planck's photon energy quantum, $\tilde{\mathbf{E}} = \mathbf{h}\overline{\mathbf{f}} = \int_{[t]} \Psi^2(\mathbf{t})\mathbf{dt} = \int_{[t]} \mathbf{a}^2(\mathbf{t})\mathbf{dt}$. Continuing this way, we should be able to see how much universally applicable Planck's photon energy law ($\tilde{\mathbf{E}} = \mathbf{hf}$) could be regarding quantifying energy of arbitrary-shaped wave functions. It is also clear that by creating a set of specific additional conditions and objectives, we can search and eventually find certain mathematical family of wave functions that satisfy mentioned quantizing conditions. Energy relation $\tilde{E} = Hf$ is even applicable to planetary systems, but the constant "H" is not equal to Planck constant "h" (see the second chapter: 2.3.3. Macro-Cosmological Matter-Waves and Gravitation). Moreover, similar relations (with certain H constant) are also applicable in vortex flow meters, related to fluid flow vortices, as speculated in Chapter 4.1; -see equations (4.3-0), (4.3-0)-a, (4.3-0)-b.

The principal objective here is to formulate (or find) the family of elementary wave functions that could present all quantized matter waves in our universe, or to conclude that something like that is generally impossible. In fact, Quantum Theory already uses the concept of quantized energy wave packets. This mathematically works well in explaining number of experimental facts in Physics (some of them, well known, are Compton and Photoelectric effect and Blackbody Radiation law), although we still do not know their analytic signal forms in a temporal or spatial domain (and the fact is that in many cases we do not need to know them). The important question here is, do we really have sufficiently good ideas, mathematical models, and concepts, about what photons really are.

The other message radiating from such elaborations (as presented in this book) is to challenge the assumption that famous background radiation (explained as a product of Big Bang cosmic explosion) is kind of natural, continuous thermal or electromagnetic radiation of all matter in our universe (see more in Chapter 8.).

To illustrate what a finite and limited duration (of an elementary and narrow-band) wave function means in time and frequency domains, let us imagine that we can find (calculate) an equivalent, averaged wave function, which will replace real (arbitrary shaped) wave-packet function, $\Psi(\mathbf{x},\mathbf{t})$ *. Let us realize this by placing new wave function into a rectangular-shape amplitude borders (in both time and frequency "rectangular frames" and molds). We shall also request that this elementary, finite wave function (or wave packet* $\Psi(\mathbf{x},\mathbf{t})$ *) has the energy equal to one energy quantum* $\tilde{\mathbf{E}} = \mathbf{hf}$*. For instance, the real signal amplitude in a time domain,* $\mathbf{a(t)}/\sqrt{2}$*, will be replaced by its effective and constant amplitude,* $\bar{\mathbf{a}}/\sqrt{2}$*. The real signal duration,* $\mathbf{T}$*, in a time domain will be replaced by effective signal duration,* $\bar{\mathbf{T}}$*. The real signal amplitude in a frequency domain,* $A(\omega)/\sqrt{\pi}$*, will be replaced by its effective and constant amplitude,* $\bar{\mathbf{A}}/\sqrt{\pi}$*. The real signal duration,* $\mathbf{F}$*, in a frequency domain will be replaced by effective signal duration,* $\bar{\mathbf{F}}$*. Also, the signal mean (central, or carrier) frequency,* $\mathbf{f}$*, will be replaced by its effective central frequency* $\bar{\mathbf{f}}$*, (4.15), placed in the middle (center of gravity) point of the interval* $\bar{\mathbf{F}}$*. Instead of rectangular signal frames or molds (as a method of an effective signal averaging), we could also say that both time and frequency domain expressions of the same wave function would be certain equivalent Gaussian pulses (Gaussian window functions) because the Gaussian function is optimally concentrated in the joint time-frequency domain.*

Now, based on wave energy expressions found in (5.14), and on "Kotelnikov-Shannon-Whittaker-Nyquist Sampling and Signals Recreating Concepts" all relevant parameters of an effective (and averaged), rectangular elementary band-limited wave-packet signal (being one-quantum of wave energy), can be presented as:

$$\Psi(x,t) = a(x,t)\frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}\cos(\omega t - kx)\ (\Leftrightarrow)\ \text{wave packet},$$
$$\omega = 2\pi f\ (=)\ \text{carrier frequency}\ (>>> \underline{\Delta\omega})$$
$$\underline{\Delta\omega} = 2\pi\Delta f\ (=)\ \text{wave-packet frequency bandwidth}\ (<<< f)\ .$$

If this is an elementary wave packet and presents an energy quant, like photons are, we should have,

$$\tilde{E} = \int_{[\Delta t]} \Psi^2(t)dt = \frac{1}{\pi}\int_0^{+\infty} |A(\omega)|^2\, d\omega = \int_{[\Delta\omega]} |A(\omega)|^2\, d\omega = h\bar{f}\ .$$

Conceptualization of photons is already elaborated in Chapter 4.1, under "4.1.1.1. Photons and Particle-Wave Dualism" and summarized in "T.4.0. Photon – Particle Analogies".

We also know (see Chapter 5. "Quantizing and Kotelnikov-Shannon, Whittaker-Nyquist Sampling Theorem") that time and frequency duration of such elementary and optimal sampling quant should be,

$$\Delta t \cdot \Delta f = T \cdot F = \frac{1}{2} \Rightarrow$$

$$\tilde{E} = \int_{[\Delta t]} \Psi^2(t)dt = \int_{[\Delta\omega]} |A(\omega)|^2 d\omega = = \frac{\bar{a}^2}{2}\bar{T} = \frac{\bar{A}^2}{\pi}\bar{F} = \frac{\bar{A}\bar{a}}{2\sqrt{\pi}} = h\bar{f} = \overline{pu},$$

$$\bar{T} = \frac{\bar{A}}{\bar{a}\sqrt{\pi}} = \frac{1}{2\bar{F}} =, \ \bar{F} = \frac{\bar{a}\sqrt{\pi}}{2\bar{A}} = \frac{1}{2\bar{T}}, \ \overline{TF} = \frac{1}{2},$$

$$\bar{f} = \frac{\bar{\omega}}{2\pi} = \frac{1}{2\pi^2\tilde{E}}\int_0^{+\infty} \omega|A(\omega)|^2 d\omega = \sqrt{\frac{1}{2\pi^2 h}\int_0^{+\infty} \omega|A(\omega)|^2 d\omega},$$

$$\bar{\lambda} = \frac{h}{\bar{p}}, \ \bar{u} = \bar{\lambda}\bar{f} = \left\langle \frac{\omega}{k} \right\rangle = \left\langle \frac{\tilde{E}}{p} \right\rangle, \ \bar{v} = \left\langle \frac{\Delta\omega}{\Delta k} \right\rangle = \left\langle \frac{d\tilde{E}}{dp} \right\rangle, \tag{5.14-1}$$

$$h = \frac{\bar{A}\bar{a}}{2\bar{f}\sqrt{\pi}} = \frac{\bar{A}^2}{\pi}\cdot\frac{\bar{F}}{\bar{f}} = (\frac{\bar{a}}{2})^2\frac{1}{\bar{f}\cdot\bar{F}} = 6.62606876 \text{ x } 10^{-34} \text{Js},$$

$$\bar{F} = (\frac{\bar{a}}{2})^2\frac{1}{h\bar{f}} = \frac{\bar{a}\sqrt{\pi}}{2\bar{A}} = \frac{1}{2\bar{T}}, \ \frac{\bar{F}}{\bar{f}} = (\frac{\bar{a}}{2})^2\frac{1}{h\bar{f}^2} = \frac{\bar{a}\sqrt{\pi}}{2\bar{A}\bar{f}} = \frac{1}{2\bar{T}\bar{f}}.$$

As we can see from (5.14-1), in chapter 5, if a quantizer (narrow-band) matter-wave packet has a higher mean frequency $\bar{f}$*, its frequency width or total duration* $\bar{F}$ *is shorter. Consequently, for low and extremely low mean frequency matter-wave packets (like in phenomenology related to gravitation, planetary and galactic systems), frequency duration* $\bar{F}$ *of corresponding wave-packets is exceptionally long.*

With (5.14-1) we are again formulating conditions for signal atomization or defining meaning of elementary matter-waves domains (as being finite, optimally concentrated, Gaussian, and analytic signals in all mutually conjugate domains), as discussed all over this chapter, since conditions and expressions in (5.14-1), (5.2.1), (5.3) and (5.4.1) are mutually comparable and equivalent.

Time and frequency domains wave functions of elementary quanta of energy (like photons) should also belong to a family of finite-energy Gaussian pulses and Gabor wavelets since such waveforms guarantee the best possible time-frequency resolution (and such wave-packet or signal can be well defined and localized both in its time and frequency domain, what should correspond to a photon case).

Also, since content of a blackbody radiation or its spectrum, $A^2(\omega) = S\,\Delta t\frac{\pi f^2}{c^2}\cdot\frac{hf}{e^{hf/kT}-1}$*, should be certain superposition of elementary narrow-band-limited quanta, or photons, or simply specific superposition of more elementary wave functions, we should be able to apply Kotelnikov-Nyquist-Shannon-Whitaker signals sampling and reconstruction concepts and this way find credible analytic forms of space-time dependent wave functions of elementary quanta (like photons). In addition, every single photon or corresponding wave-packet is known as a narrow frequency-band-limited signal.*

It is also logical to imagine that mentioned elementary wave-packet functions should comply with Classical and Schrödinger wave equation, and energy content of such wave-packet functions should be $\tilde{E}_i = hf_i$ *because we are already and successfully using such conceptualization (for instance in explanations of Compton and*

Photoelectric effects). Consequently, the wave function of such wave packet should be presentable as $\psi(t,x)=\psi(\omega t-kx)$*, because such analytic-signal wave functions are satisfying Classical and Schrödinger wave equations, and from such wave equations, we can get group and phase velocity as:*

$$u=\lambda f=\frac{\omega}{k}=\frac{\tilde{E}}{p}=\frac{v}{1+\sqrt{1-(\frac{v}{c})^2}}\ ,\ v=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=\frac{2u}{1-\frac{uv}{c^2}},$$

$$\lambda=h/p,\ \tilde{E}=hf=pu\,,\omega=2\pi f$$

If we additionally explore relations (5.14-1), we can get a group velocity of an elementary matter wave packet or photon as,

$$h\cdot\Delta t\cdot\Delta f=h\cdot T\cdot F=\Delta x\cdot\Delta p=L\cdot P=\frac{h}{2},$$
$$\Rightarrow\frac{\Delta x}{\Delta t}=\frac{h\Delta f}{\Delta p}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{h}{2\Delta p\cdot\Delta t}=v=c,\Rightarrow\frac{h}{2c}=\Delta p\cdot\Delta t. \quad (5.14\text{-}2)$$

In (5.14-2), Δx is an optimal, elementary, spatial sampling length, Δp is an elementary sampling momentum, and L and P are the total spatial signal length and total signal momentum. What we could see from (5.14-1) and (5.14-2) is that an elementary, single energy quantum (as a matter wave packet or photon) should have predictable and calculable characteristics coming directly from unavoidable mathematical relations from Signal and Spectral Analysis, Parseval's theorem and Uncertainty or Certainty relations.

The question here is, do we really deal (in Physics) with the universal law of naturally quantized wave packets, or with discrete energy exchanges between discretized resonant and standing waves building blocks that are presenting matter constituents. Such standing waves entities in different situations absorb or emit wave packets as acceptable energy surplus or deficit amounts. This produces a false impression that mentioned wave items are always intrinsically and universally quantized. In other words, the stable matter presents only certain spatial and complex, standing waves formations inside resonant-like energy packed structures (naturally countable as integer numbers of wavelengths, time periods etc., what is kind of quantizing). It would also be logical that a communication between such structures looks to an external observer like an exchange of quantized energy amounts (what should not be generalized to all kinds of energy exchanges, impacts, scattering, radiations, and waving). The position of the author of this book is that present Quantum Theory concepts regarding quantizing are too much and arbitrarily simplified, often overstated, and sometimes too widely generalized, appearing also in places where something like that is not universally applicable (like in a relatively open space propagations). Much better theoretical grounds for explaining universal quantizing are Kotelnikov-Nyquist-Shannon-Whittaker sampling and signal reconstruction concepts.

The far-reaching consequences of above-presented ideas are to prove that with controllable matter-wave energy flow, it would be possible to influence structural integrity and stability of matter states. For instance, this could be realized by stimulating or modulating certain resonant frequency or modulating a set of characteristic natural frequencies, this way agitating, amplifying, and modifying resonant, mechanical, and structural properties of certain object.

It is also known that by applying Parseval's identity we can connect temporal, spatial and relevant frequency domains of the signal energy, this way formulating another form of Energy Conservation Law. The most challenging aspect of Parseval's identity is related to our knowledge (from Physics or Quantum Theory) that photons or wave packets (that are without rest mass) under certain conditions could be transformed into particles having non-zero rest masses (like electrons and positrons; -see more in Chapter 4.1). Effectively, by searching for a wave function and wave-energy spectral characteristics, we touch the ontological grounds of matter structure. Another universal law of Nature is set of "***mathematical Uncertainty Relations***" (between total or absolute durations of mutually conjugate, original, and spectral domains), and associated "***CERTAINTY" relations*** showing when standing matter waves start packing and creating particles, as already mentioned in this and other chapters, summarized by $\Delta t \cdot \Delta f = T \cdot F = \frac{1}{2}$ (see more in Chapters 5. and 10.). Contemporary Uncertainty principles and relations in Physics, essentially based almost exclusively on Probability and Statistics, is something that should be appropriately modified and updated. Well known, and universally valid, mathematical Uncertainty Relations (apart from Statistics) should be the real and best origin or source for establishing Heisenberg's Uncertainty concepts (and not at all vice-versa).

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To close the loop regarding testing the applicability of Planck's photon energy formula ($\tilde{\mathbf{E}} = \mathbf{h}\overline{\mathbf{f}}$), the mean frequencies previously found using a time signal domain could be also linked to the signal frequency domain. This is the result of applying Parseval's identity, and it will give us the chance to reinforce the most appropriate definition of the mean frequency, by proving the validity of the following expressions:

$$\begin{aligned}
&\bar{\omega} = \langle \omega(t) \rangle = 2\pi \langle f(t) \rangle = \frac{\int_0^\infty \omega \cdot [A(\omega)]^2 d\omega}{\int_0^\infty [A(\omega)]^2 d\omega} = \frac{\int_{[t]} \omega(t) \cdot a^2(t) dt}{\int_{[t]} a^2(t) dt} = \\
&= \frac{2}{h} \int_0^\infty [A(\omega)]^2 d\omega = \frac{2\pi}{h} \int_{[t]} a^2(t) dt, \quad \sigma_{\bar{\omega}}^2 = \frac{1}{\Delta t} \int_{[t]} |\omega(t) - \bar{\omega}|^2 dt \Rightarrow \\
&\Rightarrow \frac{\left[\int_0^\infty [A(\omega)]^2 d\omega \right]^2}{\pi \int_0^\infty \omega \cdot [A(\omega)]^2 d\omega} = \frac{\left[\int_{[t]} a^2(t) dt \right]^2}{\int_{[t]} \omega(t) \cdot a^2(t) dt} = \frac{h}{2} \frac{\int_{[t]} a^2(t) dt}{\int_0^\infty [A(\omega)]^2 d\omega} = \frac{h}{2\pi} = \text{Const.}
\end{aligned} \tag{9.11}$$

Now, we can exercise the applicability of different (energy quantum) elementary functions. Any of such elementary-quantum functions should also be presentable as an Analytic Signal function, as for instance:

$$\left\{ \Psi(x,t) = a(x,t)\frac{\sin(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}\cos(\omega t - kx)\ (\Leftrightarrow)\ \text{wave packet} \right\} \rightarrow$$

$$\Psi(t) = a(t)\cos\varphi(t) = \omega^{0.5}A(t)\cos\varphi(t),\ [a(t)]^2 = \omega[A(t)]^2,$$

$$\int_{[t]}[\Psi(t)]^2 dt = \int_{[t]}[a(t)]^2 dt = \int_{[t]}\omega[A(t)]^2 dt = \omega\int_{[t]}[A(t)]^2 dt =$$

$$= \frac{h\omega}{2\pi} = hf,\ \ f = \frac{\omega}{2\pi},\ \omega = \langle\omega(t)\rangle = \overline{\omega},\ h = \text{const.},\ \int_{[t]}[A(t)]^2 dt = \frac{h}{2\pi} \Rightarrow \tag{9.12}$$

$$\frac{\left[\int_{[t]} a^2(t)dt\right]^2}{\int_{[t]}\omega(t)\cdot a^2(t)dt} = \frac{\left[\int_{[t]} a^2(t)dt\right]^2}{\omega\int_{[t]} a^2(t)dt} = \frac{1}{\omega}\int_{[t]} a^2(t)dt = \int_{[t]}[A(t)]^2 dt = \frac{hf}{2\pi f} = \frac{h}{2\pi} = \text{Const.}$$

The condition $[a(t)]^2 = \omega[A(t)]^2$ is chosen simply because this looks (for the time being) like the only available mathematical option to get logical results regarding single-quantum energy, $\int_{[t]}[\Psi(t)]^2 dt = hf$, and presently we do not have any better explanation.

By creating an analogy with immediate (or instant) frequency definition $\omega(t) = \frac{\partial\varphi(t)}{\partial t},\ \varphi(t) = \text{arctg}\frac{\hat{\Psi}(t)}{\Psi(t)}$, we could also analyze the meaning of the analogous time expression in a frequency domain, $\tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\omega},\ \Phi(\omega) = \arctan\frac{U_s(\omega)}{U_c(\omega)}$, and try to find how they are mutually related. For instance, $\omega(t) = \frac{\partial\varphi(t)}{\partial t}\ (\leq ? \geq)\ \tau(\omega) = \frac{\partial\Phi(\omega)}{\partial\omega}$ (will most probably be as a kind of Uncertainty relations, well known in Quantum Theory and Spectrum Analysis, for instance, $\omega(t)\cdot\tau(\omega) \geq \pi$, or $\Delta\omega(t)\cdot\Delta\tau(\omega) \geq \pi$, or $\sigma_{\overline{\omega}}\cdot\sigma_T \geq \pi$ …).

10. PARTICLES AND SELF-CLOSED STANDING MATTER WAVES

The universally valid assumptions, principles, and concepts regarding Matter Waves and Wave-Particle Duality understanding, as introduced in Chapters 2., 4.0., 4.1, and 4.3. (*besides always applicable and valid Conservation laws, Continuous Symmetries, and Variational Principles of Least or Stationary Action, including Hamiltonian and Lagrangian mechanics*), spread all over this book, are:

1. Our Universe is already united and synchronized (among numbers of mutually compatible, complementary, and often analogical entities, structures, and matter state options), regardless we still do not have certain sufficiently good, General Unification Fields Theory in Physics. Mathematics is the universal language, framework, and logic of our Universe when representing tangible (experimentally manageable) items, without real need for using postulated or artificially created mathematical environment. Artificially assembled mathematical theories (operating mostly with virtual, imaginative, abstract, symbolic, and stochastic entities) could be very much limited, misleading, and confusing, when applied in Physics (and unfortunately, something like that we frequently find in our contemporary Physics). Such (abstract, game-theory, statistics, and probability theory related strategy) is like making functional mapping, imaging, and transformations from an intuitive world of realistic, tangible objects and measurable items or signals (or matter waves and matter states), transforming them into another domain populated with abstract and virtual entities. Then, we need to operate with newly created, virtual and abstract items, using an artificially (and by some consensus) assembled and postulated mathematical theory, or game tools. Eventually, after producing some of intermediary results, we need to revert backwards to a real world of tangible items of Physics (using relevant inverse mathematical transformations, mapping, and imaging), and, after all of that, we expect to produce realistic, precise, good, and unique results. This is what modern Orthodox Quantum theory is performing and promoting, but since mentioned mapping, imaging and functional transformations are not deterministic, and not of 1:1 type, lot of arbitrary, fuzzy, exotic, and unclear descriptions, structures, dilemmas, questions and conclusions would surface, and we are not able to revert, fully, naturally, and correctly to the world of realistic and tangible physics. In such cases, we are also not able to use our natural and universally valid (conceptually tangible) mathematical logic and healthy intuition, and our creativity and conclusions based on such practices are significantly limited, unnatural and rich with absurd questions, dilemmas, and unrealistic statements (as it is the case within the present mathematical framework of Quantum theory). Even such unnatural Quantum theory still works sufficiently well (in its self-defined frames), judging by glorifications from the theory founders, followers, and most of contemporary mainstream physicists. Anyway, it looks only superficially that modern Quantum theory is very sophisticated, mathematically rich, and efficient, well-operating theory. It is using very generally valid mathematical tools, descriptions, postulations, and abstract definitions, which are obvious, anyway always valid, and (statistically, in average) applicable almost on everything around us, or in Physics (and almost nothing of that is uniquely specific or new). Quantum theory (**QT**) is also kind

of framing and useful ideas channeling (mostly already known and taken from well working existing mathematics and physics), but specifically, empirically, and conceptually, QT is almost trivial, since to produce useful results based on its own foundations, it heavily relays on independently and already well-developed, self-standing and powerful, always well working theories, like Signal and Spectral Analysis, Probability, Statistics, and Mechanics. Orthodox QT is almost like a hidden tautology, since to work properly and to produce meaningful results it should always and strongly refer to mentioned deterministic, grandiose theories, laws, and foundations of Mathematics and Classical Physics, and to always valid, deterministic conservation laws of Nature, being conveniently modified on certain mathematically manageable way, where Signal Analysis, Statistics and Probability theories are a dominant framework. Contemporary QT is (unnecessarily and overwhelmingly) complicating our relation to Physics, or to Universe around us, and it gives kind of false impression that it presents self-standing, very original, and stable theory with strong and unique, generally valid ontological foundations. QT is addressing results of events with big number of participants, valid only “statistically and in average terms”, without significant predictive power. Most of achievements of Quantum theory are first discovered experimentally, intuitively and using other deterministic and tangible methods based on Classical Physics, and then, afterwards, retroactively assigned, or on some way presented as being predicted and explained using only the Orthodox Quantum theory.

This situation looks like well-known tale about the emperor who walked naked, and everybody was devotionally saying that the emperor is well dressed (since tailors instructed the audience that only bottom-line, stupid, and ignorant people will not see how the emperor is well dressed). Anyway, an innocent, ignorant and naive child said that the emperor is naked, but everybody else seriously ignored and neglected such childish statement. Of course, the natural beauty is in elegancy and simplicity, but here tailors had something else in their minds.

*Citation from https://en.wikipedia.org/wiki/The_Emperor%27s_New_Clothes: "**The Emperor's New Clothes**" (Danish: Kejserens nye klæder) is a short tale written by Danish author Hans Christian Andersen, about two weavers who promise an emperor a new suit of clothes that they say is invisible to those who are unfit for their positions, stupid, or incompetent – while in reality, they make no clothes at all, making everyone believe the clothes are invisible to them. When the emperor parades before his subjects in his new "clothes", no one dares to say that they do not see any suit of clothes on him for fear that they will be seen as stupid. Finally, a child cries out, "But he isn't wearing anything at all!" The tale has been translated into over 100 languages.[1]*

Similar ambiguous, compromising, and apologetic approach to “*ignorant and childish situations and challenging and too much progressive statements*” (in our society and contemporary mainstream Physics related environment) has been often much safer than being literally dismissed, on some way punished, sacrificed, or burned. For instance, in our relatively recent past, when somebody *sincerely* and based on arguments, said that the Sun is in the center of our planetary system, he risked being severely punished (since Earth was ideologically predefined to be in the centre of our Universe). This frequently happened in a period of the catholic church inquisition activities, and during dominancy of Ptolemy geocentric, planetary concept (and such Ptolemy geocentric-concept teaching resisted during almost 1000 years, until eventually our modern science corrected it). We should be careful when associating to certain domain of Physics an abstract, internally rich, and consistent, superficially grandiose, and somewhat complicated, but trivial mathematical processing (where triviality here means, being always valid, correct, and applicable almost to everything around us, in its frames of definition, since such approach is claiming almost nothing new and extremely specific). This is, for instance, the case of Mechanics, Statistics, Probability and Signal Analysis theories in relation to Quantum theory. Quantum theory, conveniently, creatively, philosophically, mathematically, and imaginatively merged Statistics and Probability with generally valid (and already existing) Signal and Spectral analysis, with known Conservation laws, and generally valid Variational principles (including the framework of Hamilton-Lagrange mechanics). This way (by consensus created, and artificially assembled) Orthodox Quantum theory is profiting on everything what is anyway already known, correct and stable property of natural sciences and mathematics, and bringing almost nothing essentially new to Physics. Only an abstract, unnatural, and complicated, but mathematically consistent and operational framework (of non-deterministic mapping between real Nature and Quantum theory virtual creations) has been constructed, and later glorified (by theory founders and their followers) as something with eternal, unique, essential, and ontological values, as being produced only by the Quantum theory. Contemporary, orthodox quantum theory is almost explicitly claiming that everything, even absurdities and miracles, are statistically possible events within certain probability (which in some cases could be infinitely small), and this is a trivial and always valid statement. Of course, the level of mathematical complexity and sophistication of modern Quantum theory is remarkably high (this way masking implemented trivialities). However, its real contribution in mastering the micro world of Physics is, in most of cases, related to something what is first time discovered empirically, using a healthy and deterministic (or tangible) intuition, logic and analogical thinking (based on Classical Physics), or in some cases as results of arbitrary experimenting. Then, such known, obvious, and already obtained results are in many cases intentionally and retrospectively assigned to (inexistent in the first steps) Quantum theory predictions, being backwards explained or described by Quantum theory concepts. Many of achievements of the contemporary Orthodox Quantum theory are segmented and hybridized with big number of particularly developed mathematical processing practices, combined with Classical Mechanics, Signal Analysis, Curve Fitting methods, and other well-known mathematical modeling methods. Quantum theory also includes "*by consensus accepted instructions, patchwork convictions*", and convenient statements formulated to answer critical questions, without its own, real, new, self-standing and independent, well, and deeply founded, investigative, innovative, logical, and creative natural science foundations.

2. Big and extraordinarily successful theories about Nature should have grandiose elegancy and simplicity, and smooth unity with a remaining (tangible and deterministic) body of science. Thermodynamics (for instance) also and heavily relays on Statistics and Probability theory, but it is still very much deterministic theory. The founders of Quantum theory (and most of their modern-days successors) prudently, honestly, and sincerely elaborated the tricky answers and comments addressing possible critical questions (coming from non-

believers) by proudly saying that "*Nobody really understands why Quantum theory works well*", but anyway it works very well, and based on number of obtained and correct results we see that its predictions should also and always be totally correct, and in agreements with all measurements and experiments. Consequently, we need to accept the Orthodox Quantum theory as it is, since it works exceptionally well. *What works exceptionally well there, are already applied Classical and modern Mechanics, Mathematics, Statistics, Probability theory, Signal and Spectral analysis, Spatial, Temporal and Spectral Proportionality between different domains, and everything related to Conservation laws (of course when being appropriately and well-packed within certain theory, even with one artificially assembled).* Contemporary Orthodox Quantum theory (historically, during its development) domesticated several more of exotic stepstones and peculiarities such as implicitly implementing an influence or importance of "*divine inspiration*", when some of essential discoveries are being postulated (like in the case of Schrödinger's equation), or like describing some very good mathematical tools, creations, concepts, models and functions (which are supporting Quantum theory assumptions) as unique and "*brilliant minds*" revelations (like in the case of P. Dirac). The leaders, guardians, believers, and editors of such mainstream Quantum theory teachings are systematically prescribing and implementing rules and suggestions such as, that "*it is useless to critically reinvestigate foundations of present Quantum theory, since most of such items are already published, officially recognized as good references, and considered as solved and closed chapters, and there is nothing essentially new to be added or discovered here*". Every attempt to rearrange or change foundations of Orthodox Quantum theory is simply dismissed, forbidden, and rejected. Of course, presently we have so many publications (in Physics) where so many items, terms and concepts are on different ways mentioned, superficially explained and mixed (especially in micro physics and associated mathematics), that on a first look, everything of that matter (being old or new) looks so mutually similar, known, or *déjà vu*. Everybody who attempts to bring there something essentially challenging and innovative is not welcome, being rejected or aggressively misrepresented as an ignorant, free-lance promoter of already published, but unclear and not sufficiently distinctive options. Only well-obeying participants, who are completely and non-critically accepting foundations of Orthodox Quantum theory and contributing only with some minor extensions of existing mainstream teaching, are welcome.

For instance, there are challenging, inspiring, provocative, indicative, and sufficiently well-supported comments and fundamental critics regarding weak sides of contemporary (orthodox) Quantum and Nuclear physics often publicized by Dr. Sorin Cezar Cosofret (see more under [126]), but such, little bit aggressive, non-diplomatic, oversimplified and essentially correct statements and insinuations, are being a priory and immediately dismissed, without being analyzed. Here, (as an illustration) are listed some shocking or surprising (but essentially correct) statements and comments regarding our modern Physics, formulated by Dr. Sorin Cosofret (citation from the author's email):

"-Any form of energy must have a carrier.
-Vacuum cannot be a carrier nor a reservoir for energy.
-Dark matter and dark energy are either incomplete or nonsensical concepts.
-There is no similitude between the comportment of a solid body and the comportment of a gas.
-Only a „supplementary" central force can explain the discrepancies between the observed and calculated motion of a particle around a center of force.
-Any material body which performs a periodic motion around a center of force must be acted compulsory by the force of a vortex or something similar.
-Gravitational force cannot generate or maintain alone a rotational motion.
-The so-called electric force cannot generate or maintain alone a rotational motion.
-No signal before dark age epoch can be observed by a later observer (assuming that a Big Bang existed, of course)
-It is demonstrated that no credible dark matter distribution can explain the rotational motion in the outskirts of galaxies.
-Dark matter is found to be in complete contradiction with experimental reality.
-The recombination process of a mixture of elements cannot be a single step and single stage temperature process.
-The variation of physical parameters for a gas mixture generates only a mass transfer if the specific conditions for deposition or condensation are not reached".
-A gas can only perform a forced rotational motion.

3. All phenomena we know as waves, vibrations and oscillations should belong to the same physics framework and respect the same mathematical modeling and wave equations (such as Analytic Signal model, and Classical Wave equation; -see more in the chapters 4.0 and 4.3). Solid matter states, particles, atoms, molecules, and other stable masses (with non-zero rest masses) are specifically assembled and stabilized matter waves structures, being effectively composed of self-closed, spatial standing matter-waves' formations, different oscillators, and resonators. On a microworld level, mentioned standing-waves formations are presenting superposition and interferences of more elementary matter-waves, or wave functions (most probably, originally, and essentially being electromagnetic waves, which are at the same time coupled with, or creating mechanical and acoustical waves). Electromechanical and electromagnetic couplings and mutual synchronization of mentioned standing matter-waves resonant states is penetrating from a micro world of atoms, towards the macro-Universe. Stochastic, Probability and Possibility-based, non-dimensional waves (as proudly and vocally advertised and practiced in modern Quantum theory), do not belong here (except as useful and challenging mathematical abstractions and tools, when statistically defendable). We are too often and incorrectly, or sometimes arbitrarily, using the names like quantum theory, quantum physics, quantum phenomenology, quantum nature, and quantum effects..., implicating that everything in a microphysics-world should intrinsically and structurally (or ontologically) have mentioned discrete, quantum, or quantizing attributes. In this book under quantizing, we primarily address finite and resonating matter structures, where self-closed and stable standing matter-waves are on some way involved, very much analogical to analyzing spectrum of natural, or modal resonant frequencies (and associated harmonics) of certain solid body, which can be visualized after applying the Finite Elements Analysis methods. All matter structures, like atoms, molecules, and other matter states, are conceptually, mathematically, imaginatively, and effectively presentable as sets, or ensembles of some real, physical, oscillatory, and resonant, electromagnetic, electromechanical, and mechanical, elementary mass-spring, or inductance-capacitance oscillators (of course with

damping elements). This is very much analogical, symmetrical, or comparable situation to Fourier spectral analysis, where we can find that any kind of spatial-temporal function (or an arbitrary-shaped matter state in motion) can be conveniently decomposed on simple-harmonic, sinusoidal elementary components (or waves). ***Mechanical, or acoustical oscillations and vibrations, or audio signals and music, can also be created by applying different signal-modulating techniques on laser beams and plasma states (or signals); -See relations under (10.2-2.4) and literature references from [133] until [139].*** Mentioned elementary physical oscillators or resonators, are mutually and naturally communicating, or interacting (by exchanging discretized amounts of energy, moments, and different fields-charges, in different forms of matter-waves), what is mathematically presentable or accountable with involved series of (by integers-indexed) elementary harmonic waves (since standing-waves naturally have integer numbers of wavelengths or half-wavelengths). This is explicable because mutually communicating and specific-shape resonating structures are internally composed of limited number of standing and self-closed matter-waves formations, and optimal or most effective communications between such mutually coupled resonant structures, or sets of resonators, are maximized in spectral areas where natural resonant frequencies of involved participants are mutually overlapping (regardless of distances between them). This looks as dealing with discretized or quantized states, or discrete amounts of energy, charges, and mechanical and electromagnetic moments, since every finite, well-defined geometry, or specific resonator shape, has certain limited (or by integers countable) number of well-defined resonant frequencies and wavelengths. *Here, we should not forget that in Mathematics, we already have very exact and precise universal theory, explaining quantizing, as signals sampling, discretization, and signals reconstruction, and this should be the real and best foundation of everything what we address as the quantum phenomenology and Uncertainty Relations in Physics (related to Kotelnikov-Shannon-Nyquist-Whittaker signal analysis; -see more in [57, 58, and 59]).* We will see that familiar standing wave quantizing is also applicable to astronomic structures, like solar or planetary systems, and galaxies (see more in Chapter 2.). From such point of view, the whole Physics or Nature around us is quantized (but there are too many of other, not quantized, open-space situations, where standing waves are not involved). Very exact, precise, and repeatable mathematical quantizing in the whole world of microphysics (is related to Kotelnikov-Shannon-Nyquist-Whittaker signal analysis theory), and the other, not completely mathematically supported, essentially and ontologically, by consensus established, probabilistic nature of the same micro-world (like in Orthodox Quantum theory), are still not enough in a mutual harmony. *Micro and Macro World, quantizing, or quantum nature of our Universe are naturally deterministic manifestations of a multi-resonant assembly of different resonant states and standing waves formations. Probabilistic and by Statistic realized modelling of such structures is completely opposite to mentioned deterministic and tangible background (of resonant and standing waves matter formations), but it also works well, when mathematical* *conditions for such modelling are involved.* Here, we could say that all *stable matter states* in our Universe are structural (spatial-temporal) combinations of more elementary, vibrating, and resonant states with dualistic wave-particle properties, some of them being considered in Physics as elementary particles. This way we are coming closer to "***String theory" concepts***, except those strings in "String theory" are the simplest, elementary vibrating entities (or energy agglomerations), when compared to most of elementary particles, atoms, and masses configurations. Since the ***String theory is the most promising, universal, natural fields and forces unification platform, or concept***, it would be very much beneficial to merge and unify here favoured mater structure concepts based on resonant and standing waves with contemporary String theory achievements (where the most important or background mathematical support should comply with Kotelnikov-Shannon-Nyquist-Whittaker signal analysis theory). Dualistic Wave-Particle concepts associated to whole matter in our Universe already started to surface (unintentionally and unknowingly) when Jean-Baptiste Joseph Fourier created grounds for Fourier analysis and harmonic analysis showing that:

A) Time-domain arbitrary-shaped functions can be decomposed on elementary (time-dependent) sinusoidal-waves' components, this way creating relevant temporal-frequency spectral functions. Later, Shannon-Kotelnikov-Nyquist-Whittaker (with signal sampling and reconstruction theory; - see more in [57, 58, and 59]), and Dennis Gabor (with his Analytic Signal model; -see literature under [7], [57]) generalized and perfected the same joint time-frequency domain signal analysis and synthesis creating powerful mathematical grounds for wave-particle duality and matter waves understanding and processing. In parallel, Luis de Broglie, M. Einstein, M. Planck, E. Schrödinger, W. Heisenberg, and many others contributed to our conceptual understanding that matter is waving, oscillating, creating specifically stabilized structures of matter waves in forms of elementary particles, interacting between particles and waves in any combination (like in mechanics), radiating and absorbing matter waves …

B) Spatial-domain arbitrary shaped functions can also be (on a similar way like in a Time-Domain analysis, for instance, by replacing time with spatial length) decomposed on elementary sinusoidal functions, again creating relevant spatial-frequency spectral distributions. Here, we can again, almost on the same way (as in a time-domain), apply similar Shannon-Kotelnikov-Nyquist-Whittaker signal sampling and reconstruction theory, and Dennis Gabor Analytic Signal model (by replacing a time by a relevant spatial dimension). Again, thanks to Luis de Broglie, M. Einstein, M. Planck, E. Schrödinger, W. Heisenberg, and many others, we started to understand that space and time are mutually related, on certain direct way being connected and mutually dependent, and consequently, mater waves concept and wave-particle duality got its final and generalized theoretical shape. ***This implicates that energy-moments characterized matter-***

waves and motional states can be equally well created from perturbations, excitations, geometric-shape and size complexity and periodicity, and oscillations produced or existing either in a temporal or spatial domain (or in both of them). See relations under (10.2-2.4) and literature references from [133] until [139].

4. Statistics and Probability theory and familiar concepts are universally and well applicable to any set with a sufficiently big number of mutually identical or very similar elements, presenting the mathematical toolbox for analyzing numerical and functional structures, participants distributions and trends, within a certain system of events (being equally useful and applicable in all natural and other sciences). We should not associate some exclusively significant, ontological, conceptual, unique, and qualitative meaning and modeling power of statistics and probability of *mass data processing* to microphysics, regardless of the universal predictive, and numerical fitting or modeling power such mathematical tools have. Modern Quantum theory or Microworld-physics should not have as its "*conceptual front-end*", by Probability theory and Statistics defined foundations. It is preferable, logical, and normal to apply Probability and Statistics as its final or "*last-end*", to present relevant and expected results and trends on the best possible way. Using certain creative imagination and convenient mathematical skills, we can mimic and replace conservation laws known in Physics with Statistics and Probability laws and concepts, what effectively happened within present Quantum theory. Conceptual front-end and core elements of the future and renovated Quantum theory and Microworld physics should manifest: In natural and proper application of *Conservation laws and Parseval Identity, in respecting Variational Principles of Least or Stationary Action, in using Complex Analytic Signals or Phasors for all matter-waves modeling, including Complex Classical and Schrödinger Wave Equations based on Analytic Signal wave function, and in applying other* ***PWDC*** *facts (see more about PWDC little bit later, under "*10.00 DEEPER MEANING OF PWDC"). Such approach is still not practiced within the contemporary Quantum theory.

   If we neglect all temporal-spatial phase information of motional states, in our very first steps (when foundations of the Orthodox Quantum theory are paved), and we prematurely give exclusively a probabilistic and statistical meaning to wave functions (for presenting matter waves), instead of using full mathematical and modeling power of Complex Analytic signals and phasors, we will limit our modeling, conceptualization and imagination regarding applications in micro physics. Of course, later, to make such theory functional, we need to effectively replace conservation laws with conservation of total probability, and to introduce number of new, "convenient *makeup with corrective mathematical steps*" to get mathematically well operating Quantum theory within a probability and statistics framework. Probability theory and statistics are universal mathematical processing tools for describing situations with big numbers of identical or similar participants, and this is always valid, or always applicable mathematical toolbox, on almost everything in our Universe (when conditions for such approach exist). But we should not forget that such universally applicable tools cannot be exclusive front-end and first-steps, ontological and fundamental grounds of certain natural-science theory, where we deal with motions and waves.

If we do not neglect temporal-spatial phase information of motional states, wave functions and matter waves, and we apply very natural modeling of involved wavefunctions with relevant Analytic Signal Phasors, we will still have chances later to use probability and statistics toolbox whenever mathematically defendable and useful. However, when we are using temporal and spatial phase information, we can easily understand and describe universal matter tendency towards resonant or vibrational synchronization and ***"extended-meaning-of-entanglement relations"*** between different objects, matter states and systems having similar, identical, or overlapping spectral signatures, what is perfectly manageable when relevant wavefunctions are presented as Complex Analytic Signals or Phasors…

Citation from https://en.wikipedia.org/wiki/Bose%E2%80%93Einstein_condensate: A ***Bose–Einstein condensate*** *(**BEC**) is a state of matter (sometimes called the fifth state of matter) which is typically formed when a gas of bosons at low densities is cooled to temperatures very close to absolute zero (-273.15 °C). Under such conditions, a large fraction of bosons occupy the lowest quantum state, at which point microscopic quantum phenomena, particularly wavefunction interference, become apparent macroscopically. A BEC is formed by cooling a gas of extremely low density, about one-hundred-thousandth (1/100,000) the density of normal air, to ultra-low temperatures. This state was first predicted, generally, in 1924–1925 by Albert Einstein[1] following a paper written by Satyendra Nath Bose, although Bose came up with the pioneering paper on the new statistics.[2]*

5. The resulting, holistic, universal, effective, and total quantity of linear, rotational, and other motions of different forms of matter and waves in our Universe are always on some way mutually neutralized, balanced and compensated on a macro scale (or being equal to zero, when presentable as different moments, vectors, or mutually opposed charges). We also know that ***every Action is always equal to the relevant Reaction***, in relation to different induction laws (as known in mechanics, electromagnetism and everywhere else). *Solutions of all (well-known, second order, differential) wave equations are always appearing in pairs of signals or wave groups (propagating inwards and outwards, or in mutually opposed spatial and temporal directions, since time and space domains are mutually coupled and transformable). Natural wave phenomena, vibrations, and oscillations (in different media) are always presentable with mutually similar wave equations, including Schrödinger equation, all originating from Classical Wave equation (see more in Chapter 4.3). Classical wave equation based on using complex Analytic signals, in relation to real matter-waves, also and always, has as solutions (at least) two, or "many-in-pairs", mutually coupled and phase-shifted wave groups, always propagating in mutually opposite, or inwards and outwards directions.* ***Quantum entanglement phenomenology should be closely related to here-mentioned Action-Reaction and Spatially-Temporally symmetric and mutually coupled states, thanks to overlapping of relevant spectral characteristics.*** *For instance, the difference between matter and anti-matter*

states or particles is such, that both states are almost identical matter-waves packets or wave groups but being mutually phase shifted for π (or 180°), this way mutually cancelling or annihilating both when being in contact.

6. Our Universe could be a part of a Multiverse, composed of many of mutually phase-shifted worlds, where phase shifting is related to spatial and temporal dimensions (behaving like mutually orthogonal wave functions that could be matter-waves and energy-momentum caring entities). What presents our original, tangible and experimentally verifiable world of Physics is the principal and only reality we presently know, and everything else, belonging to a domain of (probabilistic, non-detectable) phase-shifted, orthogonal, or higher dimensions of matter formations, could be imaginatively considered as an ether, or just as a remaining, spatial, and temporal background structure of our Universe. For instance, an absolute vacuum state, or very empty space (in our 3+1-dimensional world), should be anyway filled with mentioned ether (with still unknown content). The reasonable assumption is that this ether should have properties like an ideal gaseous fluid. In addition, such ether has its own, measurable, and verifiable electric and magnetic permeability. This is confirming the speed of electromagnetic waves in a vacuum $C = 1/\sqrt{\varepsilon_0 \mu_0}$, and implicates that our Universe has essentially and ontologically an electromagnetic nature. Casimir effect belongs to the same phenomenology related to vacuum states, and matter states couplings and communications. Obviously, ether should be composed of dielectric and magnetic entities, as Nikola Tesla creatively speculated, [97]. Familiar ideas and concepts are increasingly surfacing on the Internet; -see, for instance, “Jean de Climont, Editions d'Assailly”; - ref. [117]. How to understand and exploit much better such hidden reality, fluidic and etheric matter, or still little bit strange problematic, will stay an ongoing project during certain time. What other (detectable or measurable) parameters (if any) of an ether are, is also a challenging question to answer.

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The following citations are presenting kind of universally applicable, advanced, state of the art position of the contemporary Physics (taken apart from not so much conceptually clear and artificially assembled statistical, Orthodox Quantum theory):

*“**Principles of Physics,** From Quantum Field Theory to Classical Mechanics*
https://doi.org/10.1142/9056 | February 2014. Pages: 444
Author: Jun Ni (Tsinghua University, China). ISBN: 978-981-4579-39-1

We start from the following five basic principles to construct all other physical laws and equations. These five basic principles are: (1) Constituent principle: the basic constituents of matter are various kinds of identical particles. This can also be called locality principle; (2) Causality principle: the future state depends only on the present state; (3) Covariance principle: the physics should be invariant under an arbitrary coordinate transformation; (4) Invariance or Symmetry principle: the spacetime is homogeneous; (5) Equi-probability principle: all the states in an isolated system are expected to be occupied with equal *probability. These five basic principles can be considered as physical common senses. It is very natural to have these basic principles. More important is that these five basic principles are consistent with one another. From these five principles, we derive a vast set of equations which explains or promise to explain all the phenomena of the physical world.*

Karen Markov, Ph.D. Physics, Yale University (1991)
Answered Oct 29 2019 ·

What are some fundamental principles of physics every person should learn?

This is a remarkably difficult and profound question. It asks for the 10 most important pillars of physics. My personal view on it would be

1. ***Principle of least action:*** *this principle in variation theory defines how equation of motion look like and how they are related to symmetries of a so-called action function(al). Principle of least action - Wikipedia Newton's equation F=ma is a direct consequence of it.*
2. ***Conservation of energy and momentum:*** *a consequence of (1) if one assumes time and spacial homogeneity. Laws of physics and outcome of experiments don't depend when and where the experiment happened. This relationship of symmetries and conservation law is the essence of Noether's theorem. I would call this the second especially important principle. Noether's theorem - Wikipedia*
3. ***Constant speed of light****: defines how 4 dimensional space-time works and is the basis of special relativity. Speed of light - Wikipedia*
4. ***Strong equivalence of gravitational and acceleration (mass):*** *one of the foundations of general relativity. Stating that gravitation and acceleration is the same (on small scales). Equivalence principle - Wikipedia*
5. *Growth of entropy with time or* ***Second Law of Thermodynamics****: the foundation of thermodynamics and statistical physics. It links probability, reversibility and macroscopic variables. It also states that time flows in one direction(!). Equivalence principle - Wikipedia*
6. ***Heisenberg's uncertainty principle****: stating that you cannot measure certain quantities accurately at the same time. It describes the quantum nature of the universe on small scales. Uncertainty principle - Wikipedia*
7. ***CPT symmetry:*** *all physical laws and systems are invariant under reversal of time, parity and charge. This symmetry is well observed and limits the possible theories of the universe. It states that antimatter and matter behave in the same way. CPT symmetry - Wikipedia*

These 7 principles are the pillars of modern physics. They shape how the world works on a very basic level. If the world were a computer, this would be the lowest level of kernel code".

Too often we can find number of glorifying wording and descriptive statements about Quantum Physics or Orthodox Quantum theory (QT) that are on a very similar way surfacing when somebody of convinced QT believers is talking about QT. The stronger arguments of such people are: It works well, mathematically produces good results and verifiable predictions, and nobody cares what kind of reality and conceptuality is behind. Author of this book introduced his short comments, addressing such modern-days narrative and idealized statements about QT, as follows.

For Quantum Theory is often said that explains how everything in the world of Physics works, and that this is also presently the best description we have about matter structure and involved natural forces. This is only conditionally correct, and it is an oversimplified, too wide, and too ambitious statement. Quantum theory is much more about specifically customized Mathematics than Physics. If Signal Analysis, Statistics and Probability theory, including the natural body of existing mathematics works well, Quantum Theory also works well. This theory (QT) is the well-made and very much artificial (meaning not natural) hybrid, or an assembly of mentioned independently and already well-developed mathematical theories, including elements of a Game theory with implemented ad hoc assumptions and postulations, "accepted as correct by consensus" among certain group of physicists (or QT founders). In the background of Quantum Physics are also all conservation Laws of Physics that are anyway valid in all Natural Sciences. There are alternative and better mathematical modelling and concepts that can replace contemporary QT, but official and mainstream authorities in QT are suppressing or ignoring such attempts ...

Lot of arbitrary statements are being simply formulated when somebody of QT mainstream followers is talking about contemporary QT. Such positive and optimistic statements are often without very specific and unique meaning and applicability, intended to be very much valid for everything around us, and offering mostly verbal and apologetic glorifications and descriptions. There are many independent, alternative, and complementing explanations, theories, and mathematical modelling, already developed, or could easily be further developed, for replacing what is mentioned here about QT. Present QT is developed, postulated, and by "consensus among friends" accepted and formalized. Number of other QT concepts and theories with similar modelling power and significance exist, saying that not too much is unique, united, fixed, and stable here. One common part of all quantum mechanics theories was first developed and postulated in the 1920s by Niels Bohr, Werner Heisenberg, Erwin Schrödinger, and others. There is a common opinion and tendency to make QT much more comprehensible and universal for Physics, on a way to be combined with other elements of physics, mainly with A. Einstein's special theory of relativity, which explains what happens when objects move very fast – what creates number of quantum field theories.

To realize such unification, first, QT should be appropriately united with Electromagnetic theory (EM), Mechanics, thermodynamics, and fluids dynamics. In parallel, EM should be upgraded to the level that will generate new, updated Relativity theory (RT). Then old and new RT can be re-established, and eventually, the new QT could be assembled. Everything else are very much naive and arbitrary or incomplete attempts and statements.

Principal contemporary quantum field theories (three of them) deal with three of the four fundamental forces responsible for all matter interactions, such as: electromagnetism, which explains how atoms hold together, the strong nuclear force, which only descriptively explains the stability of the nucleus at the heart of the atom, and the weak nuclear force, which also descriptively explains why some atoms experience radioactive decay.

Most probably that almost everything about strong and weak nuclear forces is essentially wrong. This is like placing labels or names on black boxes, without offering real explanations, and how something works. Electromagnetism (EM) is the real and tangible force with strong and good mathematical modelling and verifiable experimental background. All other nuclear forces including Gravitation will most probably be some derivatives or consequences of an innovated and updated EM. Another complementing approach to forces is to understand force effects around nodal and anti-nodal zones of structured standing waves of matter.

Modern QT claims that Standard model (created based on mentioned quantum field theories) is the most accurately tested picture of matter's basic working that's ever been invented. Standard model also has lot of arbitrary constants and fuzzy talking, presenting a big mess of everything what is about matter content. It is simply and artificially assembled based on assumed nuclear and electromagnetic forces with lots of unanswered residuals and exceptions. If one day we find that certain of four of present days Natural forces are fundamentally wrong, contemporary Standard model will fail. Good mathematical modelling and much better conceptual picture are still missing here. Standard model cannot be considered as a very serious and stable (long lasting) theory, but on some intellectually hard to accept and descriptive way it is working in its own frames (full of holes and logically unclear elements). All of that regarding QT fields is complex, confusing, messy matter, with very low probability of generated results, with missing natural conceptualization, and with inconvenient mathematical modelling. Anyway, without good conceptual and generalizing picture, and proper mathematical modelling, nothing stable and significant can be created in relation to Standard model. As QT is formulated and practiced now, it is like walking in a new unexplored space, being blind and deaf. Lot of fuzzy, paradoxical, illogical, ambiguous, and confusing statements can be formulated based on present days QT, such as: Schrödinger's cat, and Entanglement effects of quantum particles being able to affect each other instantaneously even when they are far away from each other (as A. Einstein said, "spooky action at a distance"). The emerging and challenging technologies such as ultra-secure quantum cryptography and ultra-powerful quantum computing belong also here. From unnatural and improper theory foundations we can expect only confusing characterizations and predictions. Of course, something revolutionary innovative and new is missing here, what will replace contemporary QT.

Leading gravity theory is still the A. Einstein general theory of relativity, essentially non-quantum theory, not involving particles (being not much better compared to I. Newton Gravitation). Number of unsuccessful efforts are invested to bring gravity under the QT, and this way to explain all fundamental physics within the one "theory of everything". This is still not producing results, since first, EM should be updated until the level when new (updated) RT can be generated. Then, very much new QT should be established, and eventually we will be able to analyse what General Unified theory could be. In parallel, cosmological measurements indicate (to naive participants of such ideas) that over 95 per cent of the universe consists of dark matter and dark energy, presenting *a hypothetical assumption for which we have no explanation within the Standard model. Understanding of cosmological measurements guided with incomplete concepts and improper mathematical modelling will produce almost arbitrary and contradictory statements and strange insights. Here we have much more of "dark and oversimplified thinking" than about dark matter and real and bright concepts about Gravitation. Something new is also missing here (regarding central forces and Gravitation).*

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This Chapter 10. connects all chapters of this book on a convenient mathematical and logical way by addressing the most important, common concepts, foundations and relations of wave-particle duality, and associated energy-moments aspects of different motions and matter states. In this book we assume (or conceptualize) that matter, or mass is composed of self-stabilized elementary matter-waves in number of standing-waves formations, creating elementary particles, atoms and other stable structures or bigger masses... In other words, we will explore what could be the real meaning of standing matter waves packing, or formatting in relation to matter structure, matter-waves, and wave-particles duality.

WAVE EQUATIONS

Here, we will take as a sufficiently generally valid and easily presentable, to analyze progressive and standing, plane waves, as being (in most of the cases) elements of all matter-waves, since with plane waves we can clearly, easily, and on a simple way, describe and define all-important matter wave properties. Self-closed Standing matter waves will be (mathematically) linked to relatively stable matter structures such as atoms, different particles, and planetary systems. Progressive waves will present wavelike energy-momentum communication means between mentioned stable matter structures. Later, analogically we can address other types of waves, following the structure and methodology established based on plane waves. Plane waves are waves where analyzed field or wave front (or displacement) function only depends on one spatial coordinate (here, x) in the direction of the wave propagation, and on time t, such as: $\Psi(x,t)$, $p(x,t), v(x,t)$,... For instance, we can present a plane wave, which is an oscillatory displacement wave function as,

$$\Psi(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - u\cdot t)\right] = A\sin\left[2\pi(\frac{x}{\lambda} - \frac{u}{\lambda}\cdot t)\right] =$$
$$= A\sin\left[2\pi(\frac{x}{\lambda} - f\cdot t)\right] = A\sin(k\cdot x - \omega\cdot t).$$

A more general form of a wave function $\Psi(x,t)$ with an initial phase constant Φ that shifts the same wave is, $\Psi(x,t) = A\sin(k\cdot x - \omega\cdot t + \Phi)$.

The wave number k and the angular frequency $\omega = 2\pi f = 2\pi\frac{u}{\lambda}$ are analogically defined as being directly dependent on belonging spatial and temporal periodicities (or periods) such as wavelength λ and time period $T = \frac{1}{f}$, meaning

$$k = 2\pi f_x = \boxed{2\pi\cdot\frac{1}{\lambda}},\ \omega = 2\pi f_t = 2\pi f = \boxed{2\pi\cdot\frac{1}{T}}.$$

Plane waves could also be considered as being good approximations of spherical waves, at large distances from a point wave source. Plane waves $\Psi(x,t)$ are known to be solutions of a second order, homogenous, Classical, partial differential wave equation (equally known in acoustics, electromagnetism, mechanical oscillatory motions like a string and membranes oscillations, in fluids and plasma electromagnetic waving and oscillations, etc.). The generally known form of Classical wave equation (valid and applicable to any kind of wave and oscillatory motions in Physics) is,

$$\Delta\Psi(x,t) - \frac{1}{u^2}\frac{\partial^2\Psi(x,t)}{\partial t^2} = 0.$$

Schrödinger equation

Essentially, Schrödinger wave equation could be simply developed from Classical, second order differential wave equation (see how in Chapter 4.3). Anyway, Schrödinger exercised a more complicated development process and successfully **postulated** his equation, which is still one of the most important matter waves equations in Physics. We can find (relatively recent) publications where it is shown mutual transformability or equivalency between Classical wave equation and Schrödinger equation (including electromagnetic waves equation). *This indicates that fundamental and generally applicable wave equation (in a micro and macro world of physics) is the Classical wave equation, being especially successful if wave function is modelled as an (non-probabilistic) Analytic Signal* (see also the following supporting publications: [105] Himanshu Chauhan, Swati Rawal and R K Sinha. WAVE-PARTICLE DUALITY REVITALIZED: CONSEQUENCES, APPLICATIONS AND RELATIVISTIC QUANTUM MECHANICS, [85] George Shpenkov, and in [86] Victor Christianto, "Review of Schrödinger Equation & Classical Wave Equation"). Schrödinger equation is producing correct results in many cases, but, for instance, there are spectral lines of hydrogen atom that cannot be predicted using Schrödinger equation. Read citation below (from **[145],** Joseph Lucas and Charles W. Lucas, Jr. A Physical Model for Atoms and Nuclei - Part 3):

Citation: The Toroidal (atom) model is extended in this article to describe the emission spectra of hydrogen and other atoms. … The resulting model accurately predicts the same emission spectral lines as the Quantum Model. including the fine structure and hyperfine structure. Moreover it goes beyond the Dirac Quantum Model of the atom to predict 64 new lines or transitions in the extreme ultraviolet emission spectra of hydrogen that have been confirmed by the extreme Ultraviolet Physics Laboratory at Berkeley from its NASA rocket experiment data [5]…

Schrödinger wave equation (with probability wave function interpretation) works very well mostly for relatively small and sufficiently isolated or single micro-world items. In cases of big compounds of atoms and other forms of matter it works in principle, but with increased complexity and difficulties regarding mathematical processing, and in some cases, it would not produce useful results (read the following citation from **[146]**).

*Citation: **Quantum theory cannot consistently describe the use of itself. Daniela Frauchiger1 & Renato Renner1***
Quantum theory provides an extremely accurate description of fundamental processes in physics. It thus seems likely that the theory is applicable beyond the mostly microscopic domain in which it has been tested experimentally. Here, we propose a Gedankenexperiment to investigate the question whether quantum theory can, in principle, have universal validity.
The idea is that, if the answer was yes, it must be possible to employ quantum theory to model complex systems that include agents who are themselves using quantum theory.

Analysing the experiment under this presumption, we find that one agent, upon observing a particular measurement outcome, must conclude that another agent has predicted the opposite outcome with certainty. The agents' conclusions, although all derived within quantum theory, are thus inconsistent. This indicates that quantum theory cannot be extrapolated to complex systems, at least not in a straightforward manner.

The probabilistic (non-dimensional) interpretation of a wave function, and associated mathematical processing is a kind of results smoothing and averaging, (meaning being not a real-time evidence), and could create other limitations and challenging, or paradoxical situations regarding understanding Reality. Reality cannot be correctly, fully, and naturally addressed with an artificial mathematical modelling (when using probability wave function). In this book the wave function, modelled as an Analytic Signal, is related to real-time signal (or matter waves) power, what presents much more natural, richer, and very productive matter waves modelling compared to probabilistic modelling (see more about such non-probabilistic matter-wave function in Chapter 4.0 of this book).

Wave function properties

The **wave function phase velocity u** is the velocity of a point on the wave $\Psi(x,t)$ that has certain constant phase (for example, its crest), and is given by relations,

$$u = \lambda \cdot f = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{\tilde{E}}{p},$$

and on a similar way we define and develop **group wave-function velocity v** as the velocity of the envelope of a wave $\Psi(x,t)$ (that has certain constant amplitude), and it is given by relations (see more in Chapters 4.0 and 4.1),

$$v = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d\tilde{E}}{dp} = \frac{dE_k}{dp} = u - \lambda\frac{du}{d\lambda} = u + k\frac{du}{dk} = -\lambda^2\frac{df}{d\lambda}.$$

What is interesting and important to underline here, is that solutions of Classical wave equations are always composed of (at least) two wave components or wave groups, (***being in an intrinsic and entanglement coupling***). Such wave packets are propagating in a mutually opposite directions, or inwards and outwards from certain vibrations source since ***action and reaction effects and forces, and different induction laws are always and coincidently manifesting*** (see more in Chapter 4.3). This is also the manifestation of always valid Conservation laws and Symmetries in Physics. Later, we will present such wave groups or functions, propagating in mutually opposed directions, as $\psi^+ = \sum_{[n]} \psi_i^+$ and $\psi^- = \sum_{[n]} \psi_i^-$.

From a point of view of Spectral, Fourier and Analytic Signals analysis, and wave functions modeling, plane waves (when being solutions of Classical wave equation) are (for instance) presentable as,

$$\begin{aligned}
&\Psi(x,t) = \Psi(kx - \omega t) + \Psi(kx + \omega t) = \\
&= \sum_{[n]} \psi_i(kx - \omega t) + \sum_{[n]} \psi_i(kx + \omega t) = \sum_{[n]} \psi_i^+ + \sum_{[n]} \psi_i^-, \\
&\text{or, } p(x,t) = p(kx - \omega t) + p(kx + \omega t), \\
&\text{or, } v(x,t) = v(kx - \omega t) + v(kx + \omega t), \ \ldots
\end{aligned}$$

General solutions of Classical wave equations (when $\Psi(x,t)$ is formulated as a Complex and Analytic signal wave function $\overline{\Psi}(x,t)$) are also presentable as a linear summation or integral superposition of elementary plane-wave elements in a complex form, such as,

$$\overline{\Psi}(x,t) = a(x,t)\cdot e^{I(kx-\omega t)} + b(x,t)\cdot e^{I(kx+\omega t)} = \overline{\psi}^{(+)}(x,t) + \overline{\psi}^{(-)}(x,t),\ \overline{\psi}^{(+,-)}(x,t) = \left|\overline{\psi}^{(+,-)}\right|\cdot e^{I\left[\varphi_{(+,-)}\right]}$$

$$\Rightarrow \overline{\Psi}(x,t) = \left|\overline{\psi}^{(+)}\right|\cdot e^{I\left[\varphi_{(+)}\right]} + \left|\overline{\psi}^{(+)}\right|\cdot e^{I\left[\varphi_{(-)}\right]} + \left|\overline{\psi}^{(-)}\right|\cdot e^{I\left[\varphi_{(+)}\right]} + \left|\overline{\psi}^{(-)}\right|\cdot e^{I\left[\varphi_{(-)}\right]},\ I^2 = -1.$$

In this book, we will always consider that a complex wave function $\overline{\Psi}(x,t)$ will be modeled as an Analytic Signal function (see much more in the chapter 4.0, under "4.0.11. Generalized Wave Functions and Unified Field Theory" and "4.0.12. Evolution of the RMS concept"). This will be very much useful, natural, and most general way to describe and mathematically define all wave parameters and properties in a joint space-time and corresponding frequency domains (being applicable to any kind of waves, including de Broglie matter waves; -See more about such wave function options under (4.0.82) in Chapter 4.0)). The square of such (dimensional and tangible) wave function presents a signal (or matter-wave) power (or temporal energy flow), and it has rich mathematical structure and processing. It can be creatively and mathematically transformed (or normalized) to be similar (or isomorphic) to Orthodox Quantum theory probabilistic wave function properties and associated mathematical processing, but it has number of advantages and new insights (see more about proposed power-related wave functions in Chapter 4.3. under "4.3.3. Probability and Conservation Laws"). Arbitrary shaped, and energy-finite wave functions are always presentable (based on Fourier analysis) as ordinary summations or integral superposition of mentioned elementary and plane, simple-harmonic and sinusoidal waves. In physics, *we should pay particular attention to narrow frequency-band signals, wave packets, or wave groups, which are Gaussian-Gabor amplitude-shaped signals, like photons and other familiar matter waves (among elementary particles) are, since Nature dominantly tends to make signals' synthesis more energy-rationally and faster, using (readily available) narrow-band wave groups (like known in Kotelnikov-Nyquist-Shannon-Whittaker signal analysis). Such Gaussian-shaped wave-packets are well localized, energy-finite and band-limited in both of temporal and spatial and belonging spectral domains*, meaning that relevant Uncertainty relations for such signals would be like "Standing waves atomized" relations between corresponding domains (with an equality sign between domain durations).

Gaussian-Gabor (time and frequency limited) wave packet or pulse, which has a finite and band-limited spectral content (or frequency carrier, for instance being as a chirp or burst like signals), has its total (or absolute, non-statistical) time and frequency limited widths, or durations, here presented as T and F. *Since we are dealing with a Gaussian-Gabor pulse, in a time domain, its frequency-domain amplitude will also have Gaussian-signal, bell-curve envelope, be energy and domain-duration limited,* and based on this, we can (in all related situations), determine or measure relevant (total and absolute) temporal and frequency signal durations, T and F. Generally valid relation between the total (or absolute) temporal and frequency durations of such (in both domains energy-finite, time, and frequency band-limited) Gaussian-Gabor pulses or signals is, $T \cdot F \approx 1$. In other situations, when analyzed signal is energy-finite, but not time and frequency band-limited signal, we will have, $T \cdot F > 1$.

If Gaussian signal is propagating in certain material media and being detected with convenient sensors in different locations, every of receiving sensors will detect signals having similar Gaussian envelopes (as the incident, source signal). In case of every specific sensor, we will be able to measure certain temporal and frequency width or duration (of received signals), which will again satisfy basic uncertainty domains relation such as, $T_i \cdot F_i \approx 1 \, , \, i = 1, 2, 3, ..., n \;\Rightarrow\; F_i \approx \frac{1}{T_i}$.

Consequently, measured temporal durations T_i could be different, but in every new situation we will always have, $T_i \cdot F_i \approx 1 \, , \, i = 1, 2, 3, ..., n$. If we now summarize the same situation for one specific sensor, we can define the ***propagating-media structure quality factor*** as,

$$Q_s = \frac{F_{initial}}{F_{measured}} , \begin{pmatrix} T_{initial} \cdot F_{initial} = T_{measured} \cdot F_{measured} \approx 1, \\ \frac{T_{initial}}{T_{measured}} \cdot \frac{F_{initial}}{F_{measured}} = 1 \end{pmatrix} \Rightarrow 0 < \left(Q_s = \frac{F_{initial}}{F_{measured}} = \frac{T_{measured}}{T_{initial}} \right) \leq 1 \, .$$

If Q_s is close to one (1), monitored structure is homogenous, isotropic, and stable, and if Q_s would rich certain small-value-threshold, this could be considered as having dissipative and/or anisotropic media. For instance, we could conveniently present mentioned narrow-band wave-groups or wave-packets also as Analytic Signals, being the superposition of elementary (simple-harmonic and sinusoidal) plane waves, such as,

$$\overline{\Psi}(x,t) = \begin{Bmatrix} \sum_{(n)} \left[a(k_n) \cdot e^{I(k_n x - \omega_n t)} + b(k_n) \cdot e^{I(k_n x + \omega_n t)} \right] = \sum_{(n)} \overline{\psi}_n \\ \text{or} \\ \int_{[\Delta k]} \left[a(k) \cdot e^{I(kx - \omega t)} + b(k) \cdot e^{I(kx + \omega t)} \right] dk = \int_{[\Delta k]} \overline{\psi} dk \end{Bmatrix} = \overline{\Psi}^{+}(kx - \omega t) + \overline{\Psi}^{-}(kx + \omega t) = \left| \overline{\Psi} \right| \cdot e^{I\Phi}$$

$$R_e \left[\overline{\Psi}(x,t) \right] = \Psi(x,t) = A(x) \frac{\sin(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t - \underline{\Delta k}\, x)} \cos(\omega t - kx) + B(x) \frac{\sin(\underline{\Delta\omega}\, t + \underline{\Delta k}\, x)}{(\underline{\Delta\omega}\, t + \underline{\Delta k}\, x)} \cos(\omega t + kx)$$

$$\omega = 2\pi f \; (=) \text{carrier frequency } (>>> \underline{\Delta\omega})$$

$$\underline{\Delta\omega} = 2\pi\Delta f \; (=) \text{wave-packet frequency bandwidth } (<<< f) \, .$$

Wave-particle duality can be conceptualized on a similar way, if we imagine that any stable particle with certain non-zero rest mass is physical or tangible, spatial assembly of many elementary oscillators, or resonant, mechanical, acoustic, electromagnetic, and electromechanical circuits, such as presented on the Fig.1.1. from the first chapter.

An Analytic Signal Wave Function $\overline{\Psi}(x,t)$ (created based on D. Gabor model using Hilbert transform; -see literature under [7], [57]), and corresponding Complex, Classical Wave equation, are very much and naturally convenient to represent de

Broglie matter waves, when combined with **PWDC** facts (chapter 4.1 and 4.3), as follows,

$$\Delta\overline{\Psi}(x,t)-\frac{1}{u^2}\frac{\partial\overline{\Psi}^2(x,t)}{\partial t^2}=0 \Leftrightarrow \left[\Delta\Psi-\frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2}=0\,,\,\Delta\hat{\Psi}-\frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2}=0\right]$$

$$\left[\begin{array}{l}\overline{\Psi}(x,t)=\left|\overline{\Psi}\right|\cdot e^{I\Phi}=\overline{\Psi}^{+}(x,t)+\overline{\Psi}^{-}(x,t)=\Psi(x,t)+I\cdot\hat{\Psi}(x,t)=\\ =\left[\Psi^{+}(x,t)+I\cdot\hat{\Psi}^{+}(x,t)\right]+\left[\Psi^{-}(x,t)+I\cdot\hat{\Psi}^{-}(x,t)\right]=\\ =\left[\Psi^{+}(x,t)+\Psi^{-}(x,t)\right]+I\cdot\left[\hat{\Psi}^{+}(x,t)+\hat{\Psi}^{-}(x,t)\right],\ \hat{\Psi}(x,t)=H\left[\Psi(x,t)\right],\ I^2=-1\,.\end{array}\right]$$

Such Complex, Classical (and based on Analytic Signal) Wave function is naturally and easily (without any assumption and artificial mental and mathematical hybridization, where missing equation elements should be by some divine inspiration attached), producing Schrödinger and other, familiar wave equations known in Quantum theory when merged with **PWDC** (see more about PWDC in the chapters 4.1, 4.3, 8., and later in this chapter). The meaning and relevant mathematics of wave functions in this book is initially established (or just creatively introduced) in Chapter 4.0, under "4.0.11. Generalized Wave Functions and Unified Field Theory", being different from probabilistic quantum theory wave function, but still producing equivalent or better results than quantum mechanical wave function.

Again, we should underline, solutions of Classical and Complex Wave Equations (including any form of Schrödinger equation) are, at least, two Complex Analytic-Signal wave-functions, or two wave-packets, $\overline{\Psi}^{+}(x,t)$, $\overline{\Psi}^{-}(x,t)$ ***, propagating in mutually opposite directions (or inwards and outwards), being also in a mutual resonant and/or entanglement coupling.*** Unfortunately, or thanks to specific wave emitting situations, we often see or take into consideration only one of the mentioned wave components (for instance, propagating outwards, or in a positive direction).

We should not forget that spatially (and omni-directionally), somewhere around and inside a wave source, and on some specific way, many (in pairs coupled) wave-groups should also exist propagating in mutually opposite, negative, positive, or mixed (inwards and outwards'), temporal and spatial directions. Solutions of Ordinary and Complex, Classical Wave and Schrödinger equations (including Dirac's equation) always have forms as oppositely propagating waves combinations (part of them, mathematically propagating along negative time, or in what we could consider as a past time, being conceptually challenging options to understand). *Here is the natural place to imaginatively think about conceptualizing antimatter states as wave groups traveling in a negative, past-time direction (being phase shifted), as Feynman did within his famous interactions' diagrams. We could also speculate with hypothetical assumption that matter and antimatter states (or relevant matter-wave groups) are mutually orthogonal functions, like real and imaginary parts of a Complex Analytic Signal function are, what could conceptually explain why we are living in an ordinary mater Universe, and we do not see, interfere, or overlap with a partner antimatter Universe.*

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Citation taken from: https://simple.wikipedia.org/wiki/Feynman_diagram.
A **Feynman diagram** is a diagram that shows what happens when elementary particles collide.[1] Feynman diagrams are used in quantum mechanics. A Feynman diagram has *lines* in different shapes—straight, dotted, and squiggly—which meet up at points called *vertices*. The vertices are where the lines begin and end. The points in Feynman diagrams where the lines meet represent two or more particles that happen to be at the same point in space at the same time. The *lines* in a Feynman diagram represent the probability amplitude for a particle to go from one place to another.

In Feynman diagrams, the particles are allowed to go both forward and backward in time. When a particle is going backward in time, it is called an antiparticle. The meeting points for the lines can also be interpreted forward or backwards in time, so that if a particle disappears into a meeting point, that means that the particle was either created or destroyed, depending on the direction in time that the particle came in from.
……………………….
Feynman diagrams are named after Richard Feynman, who won the Nobel Prize in Physics. His diagrams are very simple in the case of quantum electrodynamics (QED), where there are only two kinds of particles: electrons (little particles inside atoms) and photons (particles of light). In QED, the only thing that can happen is that an electron (or its antiparticle) can emit (or absorb) a photon, so there is only one building block for any collision. The probability amplitude for the emission is very simple—it has no real part, and the imaginary part is the *charge* of the electron.

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Matter waves and matter states synchronization

There are many ways how synchronization between two or many motional objects, masses or atoms is naturally happening. Here, we are talking about phase-frequency synchronization between motional states, with mutual fitting, and coupling, by means of relevant matter-waves communications (since all motional, energy-momentum states are creating matter waves). Even biological species or alive matter are on different ways mutually synchronizing. Mentioned matter waves can have an electromagnetic or mechanical nature. Other forms (still not mentioned, unknown or hypothetical) of matter waves involved in synchronization are not excluded.

For instance (let us take a quite simple example), a small piece of metal has certain electrical, mechanical, and other atomic or chemical content. If we add to this piece of metal another piece of the same metal, by fusion or melting, new bigger piece (after solidification) will again have the same electric and mechanical properties as the first small piece, and all atoms of new mass will be mutually well fitted, packed and electromagnetically coupled. Internal or interatomic coupling and unification here we consider also as the act of electromagnetic synchronization between involved atoms. Such natural synchronization creates all kind of macro masses, planetary systems, and galaxies. *Since gravitation always works between masses, that means that we can also expect certain level of synchronization between any of mutually separated masses (and we know that all solar or planetary and galactic systems are in certain mutual and periodical synchronization).* If “internal” or interatomic synchronization is essentially electromagnetic one, this indicates that any “external”, non-mechanical or non-contact synchronization between separated masses should also involves some form of electromagnetic coupling. Intensity of coupling between two different and separated masses (beside Newtonian attraction that depends on distances between involved masses) is also related to spectral or resonant content of involved masses, since *atoms are ensembles of resonators or resonant states, meaning that coupling or synchronization level is the strongest where spectral characteristics of both masses are overlapping*. In conclusion, we can say that different aspects of synchronization between atoms and masses are always active in our Universe and presenting natural fact or law. *Wave-Particle Duality regarding relevant matter waves spreads from atoms and subatomic entities towards macro-systems created when involved micro-participants enter in certain mutually synchronized state.*

Here, in addition, we could draw the concept that *boundary zone between micro and macro cosmos manifestations and interpretations are atoms.* Everything smaller than atoms and inside atoms largely belongs to the micro word of physics, and bigger agglomerations and compounds of atoms and mases are presenting the world of macro physics (including planetary systems and galaxies). Of course, we can additionally elaborate, and supplement just described concept, but for a simplified, intuitive, and indicative understanding here we have sufficiently good starting platform. It also looks reasonable to estimate that all atoms and masses in our Universe should on some way and level be mutually connected, coupled, and tending to be synchronized. Entanglement effects could also be understood as a kind of immediate communication between coupled energy-momentum states, where entanglement (in its wider and still hypothetical meaning) is also an active, real-time information carrying channel. Gravitation and electromagnetic fields are presently the best candidates to explain non mechanical couplings between masses (and most probably that gravitation is a specific manifestation of certain electromagnetic field formation). All of that, just elaborated, also associates on Nikola Tesla's Radiant energy flow, and on his "Dynamic gravity theory", and on Rudjer Boskovic "Universal natural force".

Here associated effects of ***synchronization, resonant coupling and entanglements*** *(between different matter-wave entities with mutually overlapping spectral characteristics) are explicable based on Analytic Signal modeling (see more in Chapter 4.0). For instance, real part of the wave function* $\overline{\Psi}(x,t)$ *is composed of two wave components propagating in mutually opposed directions* $\Psi^{+}(x,t)+\Psi^{-}(x,t)$ *and its imaginary part also has two of such wave components* $\hat{\Psi}^{+}(x,t)+\hat{\Psi}^{-}(x,t)$*, meaning that what we detect, analyze, measure on one side of waves propagation, it will be directly and immediately transferred or present on the opposite side of propagation.* ***If we apply Parseval's theorem on the same situation (to find total wave-groups energy), we will find that all mentioned wave groups with positive and negative indexing of mutually opposed waves propagation will synchronously (in real, present time) contribute to such total energy, effectively making superposition of all past and future matter-wave states of certain real or present-time event. Or saying the same differently, any present-time state or event is created from its past and future states where spatial and temporal domains are infinite in both positive and negative or inwards and outwards directions,*** $x \in (-\infty,+\infty)$, $t \in (-\infty,+\infty)$. *In other words, this is an explanation or strong background of synchronization, resonant coupling, and "extended-meaning-of-entanglement" effects, both valid and applicable in a micro and macro world of physics (including biological, living species), based on real-time, or immediate satisfaction of conservation laws, including satisfaction of different mechanical and electromagnetic, action-reaction, and induction laws. Coherence between two wave functions or signals is the measure of resonance and mutual synchronization (see more about coherence factors in Chapter 4.0. around definitions (4.0.83), (4.0.87) and (4.0.109)).*

Another support to explanation of entanglement and synchronization effects is the fact that by creating an Analytic Signal function, by applying Hilbert transformation, we are in fact creating two mutually orthogonal functions (phase shifted for $\pi/2$*). If we creatively combine Minkowski space-time 4-vectors with Complex Analytic Signal modeling (see more, later, at the end of this chapter under "****Minkowski space foundations, 4-vectors and Hypercomplex Analytic Signal****"), we will find that Hilbert transform is effectively (or mathematically) penetrating in higher dimensions of certain multidimensional space, practically addressing one or many additional dimensions belonging to analyzed matter-wave situation. Since Minkowski energy-momentum 4-vectors are incredibly successful in describing and mastering all kind of impact and scattering interactions, we need to admit that consequences about mentioned spatial-temporal dimensional relations, synchronizations and extensions are essentially correct.*

We can start conceptualization of (de Broglie) matter waves from **PWDC** premises established based on analyzes of Compton, Photoelectric, Bragg diffraction, Continuous spectrum of X-rays, and number of familiar effects and situations, such as Secondary Emissions, Blackbody radiation, diffraction phenomena with atomic and molecular rays, atom modeling etc. All mentioned effects and facts are belonging to

PWDC = **P**article-**W**ave **D**uality **C**ode, or **C**oncept; where **PWDC** is the abbreviation created in this book for the matter-waves and wave-particle duality concept, or facts, as established and elaborated in the chapters 4.1, 4.3, 2.3.3, 8, and elsewhere in this book. See also [92] and [105] as the particularly good historical and logical literature resume and background about foundations, deviations, and necessary rectifications of wave-particle duality concepts.

Consequently, we can present every moving particle (cinematically, only regarding its motional energy attributes, but not taking into account its rest mass content, if exist; -see the picture below), as the superposition of elementary wave functions $\psi(x,t)$ or $\overline{\psi}(x,t)$ ***(described by its energy-momentum properties, combined with mentioned PWDC facts; -see more in the chapters 4.1 and 4.3, and later in this chapter, under (10.1)). Simple-harmonic and plane-wave elements superposition will create a dualistic, wave-group particle-equivalent, or kind of Gaussian wave packet that represents a moving particle as,***

$$\left\{\begin{array}{l} \psi(x,t) = a(x,t)\cdot\cos 2\pi(\frac{x}{\lambda}-\frac{t}{T}) + b(x,t)\cdot\cos 2\pi(\frac{x}{\lambda}+\frac{t}{T}) = a(x,t)\cdot\cos(kx-\omega t) + b(x,t)\cdot\cos(kx+\omega t), \\ \text{or} \\ \overline{\psi}(x,t) = a(x,t)\cdot e^{I(kx-\omega t)} + b(x,t)\cdot e^{I(kx+\omega t)} = \overline{\psi}^{+}(x,t) + \overline{\psi}^{-}(x,t),\ I^2=-1,\ \psi(x,t) = R_e\left[\overline{\psi}(x,t)\right] \end{array}\right\} \Rightarrow$$

$$\Rightarrow \overline{\Psi}(x,t) = \left\{\begin{array}{l} \sum_{(n)}\overline{\psi}_n,\ \text{or} \int_{[\Delta k]}\overline{\psi}dk, \\ \text{or summation of wave-groups} \\ \left|\overline{\psi}(x,t)\right|\frac{\sin(\underline{\Delta\omega}\,t-\underline{\Delta k}\,x)}{(\underline{\Delta\omega}\,t-\underline{\Delta k}\,x)}e^{I(\omega t\pm kx)} \end{array}\right\} = \overline{\Psi}^{+}(kx-\omega t) + \overline{\Psi}^{-}(kx+\omega t) = \left|\overline{\Psi}\right|\cdot e^{I\Phi}\ .$$

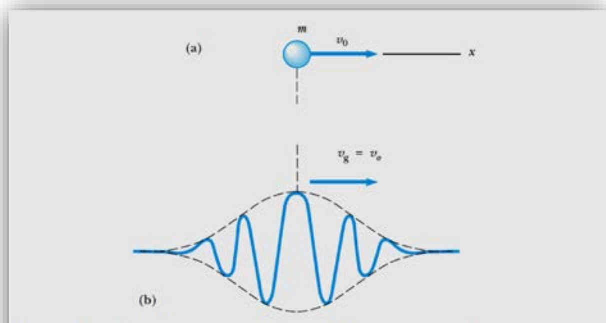


Figure 5.17 Representing a particle with matter waves: (a) particle of mass m and speed v_0; (b) superposition of many matter waves with a spread of wavelengths centered on $\lambda_0 = h/mv_0$ correctly represents a particle.

When we conceptualize de Broglie, matter waves, or wave packets that are associated to moving particles, we should always have in mind to follow analogical mathematical conceptualization of photons compared to particles, almost on the same way as already elaborated in Chapter 4.1. under "4.1.1.1. Photons and Particle-Wave Dualism ".

We could also present the same, simple Analytic Signal, or Hypercomplex Analytic Signal, Complex Wave Functions, or wave packets, as finite or infinite multiplications of mutually coupled and mutually modulating, harmonic wave functions (see more in Chapter 4.0), such as,

$$\overline{\Psi}(\mathbf{t}) = \Psi_n(\mathbf{t})\prod_{(i=0)}^{n-1}\cos\varphi_i(\mathbf{t}) + I\Psi_n(\mathbf{t})\prod_{(i=0)}^{n-1}\sin\varphi_i(\mathbf{t}) = \Psi_n(\mathbf{t})\prod_{(i=0)}^{n-1} e^{I\cdot\varphi_i(\mathbf{t})},\ I^2 = -1 .$$

*This kind of mathematical presentation and processing of wave functions could be in some cases better than a superposition of waves based on Fourier-Shannon-Nyquist-Kotelnikov harmonic elements summation, but it is still not sufficiently (mathematically) developed and associated with practical signals processing techniques. Since Hypercomplex (or quaternions modeled) Analytic Signal Waveform $\overline{\Psi}(\mathbf{t})$ has, at least three mutually orthogonal imaginary units (or axes), this could be exploited as a mathematical modeling platform for presenting elementary particles and states (such as photons, electrons, neutrinos and quarks, including their antimatter mirror pairs), as well as to be expanded towards higher spatial-temporal dimensions, and Supersymmetry concepts with natural modeling of new elementary particles-partners consisting of similar higher levels, or more energetic and short living particles or states (like muons, leptons etc.; -See more of relevant mathematical backing, and energy related structural hierarchy of matter waves in Chapter 6., under **"6.1. Hypercomplex, In-depth Analysis of the Wave Function"**). Contemporary Supersymmetry theory is intuitively and mathematically based on binary, bipolar or dualistic concepts about symmetry, for instance characterized by using + (plus) and – (minus) signs for fields charges and for different moments and spin numbers, moving in a mutually opposite directions (of a space and time), having a mirror symmetry for an object and its mirror image, conceptualizing matter and antimatter particles and states … Supersymmetry based on quaternions or hypercomplex wave functions has three imaginary units or three mutually orthogonal spatial-temporal domains. This is immediately generating imaginative ideas about trinary, tripolar, or "three ways structured mater-wave" concepts about symmetry. But far-reaching consequences here are that all our present understanding of matter structure, natural forces, elementary particles, and multidimensionality of our Universe would significantly change and evolve.*

10.00 DEEPER MEANING OF PWDC

The real founder of Wave-Particle Duality (still not recognized under such title) is Jean-Baptiste Joseph Fourier, who created Fourier Analysis, showing that all temporal and spatial, periodic, and aperiodic functions (presenting motional states and geometric formations) can be decomposed on sums of simple harmonic, sinusoidal functions, or recomposed from such components (of course, by respecting relevant mathematical conditions and procedures). Later, Hungarian, and American scientist Denis Gabor significantly modified and optimized Fourier Analysis introducing his Analytic Signal concept, becoming as generally applicable mathematical model (or wave function) for all kind of matter-waves, oscillatory and other wave motions. In addition, Luis de Broglie discovered the relations between properties of motional particles and waves. Effectively, functional mapping and equivalence or transformations between moving particles and matter-wave groups are defined by **PWDC.** In the same time, **PWDC** is essentially describing relations and unity between linear and angular (or rotational) motions. Of course, historical, or temporal succession and order of mentioned theoretical contributions, foundations, and inventions (from Fourier to Luis de Broglie) was messier with number of other participants and contributors (such as: Schrödinger, Heisenberg, N. Bohr, Sommerfeld, Max Planck, A. Einstein, and Mileva Maric, Rudjer Boskovic, and others). Now we know that specific **PWDC** properties (as introduced in Chapter 4.1 and summarized in "T.4.0. Photon – Particle Analogies") are connecting a moving particle and its (limited frequency-band, and energy-finite) matter-wave equivalent (or wave-packet), with de Broglie wavelength λ, matter wave or particle kinetic energy $E_k = \tilde{E}$, with particle or an equivalent wave-packet group velocity v, its phase velocity $u = \lambda f$, and mean carrier frequency $f = \frac{\omega}{2\pi} = \frac{1}{T}$, as follows,

$$\begin{Bmatrix} \text{PWDC} \\ \text{particle} \\ \text{wave} \\ \text{duality} \\ \text{code} \end{Bmatrix} (\Leftrightarrow) \left\{ \begin{array}{l} \bar{\Psi}(x,t) = \left|\bar{\Psi}\right| \cdot e^{I\varphi}, \lambda = \frac{h}{p},\ \tilde{E} = E_k = hf = \hbar\omega = \tilde{m}c^2 = m^* vu = pu = h\frac{\omega}{2\pi} = \frac{h}{T} = (\gamma - 1)mc^2,\ k = \frac{2\pi}{\lambda} = \frac{2\pi}{h}p, \\ d\tilde{E}_i = c^2 d(\gamma\tilde{m}_i) = hdf_i = v_i d\tilde{p}_i = d(\tilde{p}_i u_i) = -dE_{ki} = -c^2 d(\gamma m_i) = -v_i dp_i = -d(p_i u_i), \\ \omega = \frac{d\varphi}{dt} = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E} = 2\pi f, u = \lambda f = \frac{\tilde{E}}{p} = \frac{\omega}{k},\ \mathbf{L}\omega = pv, v = v_g = u - \lambda\frac{du}{d\lambda} = \frac{d\tilde{E}}{dp} = \frac{d\omega}{dk},\ m^* = \gamma m, p = \gamma mv = m^* v, \\ m^* = m + \tilde{m} = \gamma m = m / \sqrt{1 - v^2/c^2}, \tilde{m} = m^* - m = \gamma m\left(1 - \sqrt{1 - v^2/c^2}\right) = \tilde{E}/c^2, p = m^* v = (m + \tilde{m})v = mv + \tilde{m}v, \\ \\ \qquad \Delta\bar{\Psi}(x,t) - \frac{1}{u^2}\frac{\partial\bar{\Psi}^2(x,t)}{\partial t^2} = 0 \Leftrightarrow \left[\Delta\Psi - \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = 0,\ \Delta\hat{\Psi} - \frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0\right] \Rightarrow \\ \left\{ \begin{array}{l} R_e\left[\bar{\Psi}(x,t)\right] = \Psi(x,t) = A(x)\frac{\sin(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t \pm \underline{\Delta k}\ x)}\cos(\omega t - kx) + B(x)\frac{\sin(\underline{\Delta\omega}\ t \mp \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t \mp \underline{\Delta k}\ x)}\cos(\omega t + kx) = \\ = A(x)\frac{\sin(x \pm v \cdot t)}{(x \pm v \cdot t)} \cdot \cos\left[\frac{2\pi}{\lambda}(x - u \cdot t)\right] + B(x)\frac{\sin(x \mp v \cdot t)}{(x \mp v \cdot t)} \cdot \cos\left[\frac{2\pi}{\lambda}(x + u \cdot t)\right] = \\ = a(x,t) \cdot \cos\left[\frac{2\pi}{h}(p \cdot x - \tilde{E} \cdot t)\right] + b(x,t) \cdot \cos\left[\frac{2\pi}{h}(p \cdot x + \tilde{E} \cdot t)\right] = a(x,t) \cdot \cos\left[\frac{2\pi}{\lambda}(x - u \cdot t)\right] + b(x,t) \cdot \cos\left[\frac{2\pi}{\lambda}(x + u \cdot t)\right] \end{array} \right. \end{array} \right\}, \tag{10.1}$$

See also relevant background, later in the same chapter under (10.2-3), and in Chapter 4.0, under (4.0.34) and (4.0.35), including the situation regarding an equivalence and coupling between a spinning object and its "*linear-moment thrust-force,* $\mathbf{L}\omega = pv$", around equations (2.11.3-1) and (2.11-4), in Chapter 2):

Probabilistic Wave function, $\Psi(x,t)$*, as presently used and interpreted in Orthodox Quantum theory, has no immediate amplitude and phase information that is dependent on spatial and temporal variables. To get certain relevant (but still not immediate) spatial or combined spatial-temporal distribution, we need to rely on probabilistic and statistical expectations, based on a countless number of repetitive events, until we start recognizing some averaged wave-like shapes, or wave properties, produced as superposition, diffraction, and interference effects between such matter wave* $\Psi(x,t)$ *and its surroundings.* $\Psi(x,t)$ *is a wave packet that is replacing moving particle, where we apply PWDC properties described by (10.1).*

Contrary, if we treat a wave function $\bar{\Psi}(x,t)$ *as an Analytic Signal, which is a complex function or* ***Phasor*** *(as established in this book, meaning being non probabilistic,* $\Psi^2(x,t) (=) \text{Power}$*), we know from the very beginning that such wave function explicitly has very rich and immediate, spatial and temporal amplitude and phase information in all of its domains, as described by* ***PWDC*** *properties. Mentioned* ***Phasor*** *will be causally related to or connected with* ***PWDC*** *facts as summarized under (10.1). In addition, mentioned phase information (of an Analytic signal wavefunction) is essentially and completely supporting formulation of de Broglie hypothesis, and gives a platform for an easy development of Schrödinger equation without any exotic and artificial postulation (see more in Chapter 4.3). From such instant phase function, we can extract de Broglie wavelength and other matter-wave parameters (see more, later in the same chapter under (10.1.1), and in "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"). Sooner or later, we also need to accept that an ideally empty space or total vacuum still has certain fluidic matter content, or an ether, which is the carrier of electromagnetic and*

other matter waves, facilitating matter waves propagation and interactions with other matter states, and contributing to dualistic matter-wave properties.
PWDC, as elaborated in this book, has a lot of common-sense grounds (and it could be creatively extended) with "Many worlds Interpretation" of Quantum theory, and with de Broglie-Bohm and Both-particle-and-wave united view (which is already quoted in Chapter 4.1, as taken from https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality#cite_note-6).

Here (within **PWDC** framework, as summarized with (10.1)), it is important to underline that all relevant parameters and attributes of wave packets, wave groups, matter-waves, moving particles and corresponding wave functions, both in micro and macro world of Physics, will be essentially presented or modeled as:

1° Complex Analytic Signal functions or Phasors (being in a direct relation to relevant signal-power functions). See good examples in Chapter 4.0, around (4.0.82) and later. Such (Analytic Signal) modeling, when merged with **PWDC** from (10.1), is almost directly producing Schrödinger's and all other familiar wave equations of micro world physics, all of them originating from Classical, second order, differential wave equation (without need for any divine inspiration, guesswork, patchwork postulates, statistics and probability backing). Analytic Signal modeling presents the best native mathematical environment for conceptualizing de Broglie matter-waves and wave-particle duality. See more in the chapters 4.0, 4.1 and 4.3.

2° In addition, relevant Einstein-Minkowski 4-vectors, or complex energy-momentum relations can also be understood and conveniently presented as Complex Analytic Signals or Phasors, analog to Phasors in electrical-circuits theory. See how to establish such Complex Phasors modeling in Chapters 4.0, 6, and later in this chapter, under 10.1 "Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality". From such Complex and Analytic-Signal Phasors and Wave Functions, we can directly support and extract de Broglie Mater-Waves properties, as specified in (10.1). In fact, Einstein-Minkowski 4-vectors and associated Phasors can serve as the particularly good complement or replacement for de Broglie matter-waves hypothesis (see equations from (10.1.1) to (10.1.5), later in this chapter). From different Phasor functions, (resulting from Minkowski-Einstein 4-vectors), we can create relevant matter-waves Complex and Analytic Signal wavefunction and develop family of second order differential equations equivalent to Schrödinger equation (see more in Chapter 4.3). ***In Chapter 2., addressing gravitation and macro world of Physics, we can see that de Broglie matter waves concept and here elaborated PWDC are equally well applicable on solar systems (see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation"***

From a phase function of matter-wave Phasors, we can also find temporal and spatial frequencies and signal-durations relations between corresponding spatial and temporal domains (or we can find relevant spatial-temporal periodicity parameters). In addition, it will become clear that spatial-temporal integrity and mutual, direct, and stable domains proportionality $\boxed{\Delta x = C \cdot \Delta t}$ should naturally exist concerning stable and mutually synchronized energy-momentum entities (as invented or postulated in A. Einstein Relativity theory), as follows,

$$\begin{bmatrix}\Psi(t)\to A(\omega)\\ \Psi(x)\to A(k)\end{bmatrix}\Rightarrow\begin{bmatrix}\Psi(x,t)\to A(k,\omega),\\ \varphi(=)\text{phase}(=)\omega t\pm kx\end{bmatrix}\Rightarrow\overline{\Psi}(x,t)=\left|\overline{\Psi}\right|\cdot e^{I\varphi}\Rightarrow$$

$$\left.\begin{matrix}\omega=\omega_t=\left|\frac{\partial\varphi}{\partial t}\right|=\left|\frac{\partial\varphi_1}{\partial t}\right|=\left|\frac{\partial\varphi_2}{\partial t}\right|=2\pi f_t=\frac{2\pi}{T}=2\pi f,\\ k=\omega_x=\left|\frac{\partial\varphi}{\partial x}\right|=\left|\frac{\partial\varphi_1}{\partial x}\right|=\left|\frac{\partial\varphi_2}{\partial x}\right|=2\pi f_x=\frac{2\pi}{\lambda}=2\pi\frac{p}{h}\end{matrix}\right\}\Rightarrow\left\{\begin{matrix}u=\frac{\omega}{k}=\frac{f_t}{f_x}=\frac{\lambda}{T}=\lambda f_t=\frac{\tilde{E}}{p}=\frac{\partial x}{\partial t}\\ v=v_g=\frac{d\omega}{dk}=\frac{d\tilde{E}}{dp}=\frac{dx}{dt},\ \lambda=\frac{h}{p}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{matrix}dk\cdot dx=d\omega\cdot dt\\ \frac{2\pi}{h}dp\cdot dx=2\pi df\cdot dt\\ \frac{\omega}{k}=\frac{\partial x}{\partial t},\frac{d\omega}{dk}=\frac{dx}{dt}\end{matrix}\right\}\Rightarrow\left\{\begin{matrix}dp\cdot dx=hdf\cdot dt=dE\cdot dt\Leftrightarrow\Delta p\cdot\Delta x=\Delta E\cdot\Delta t,\\ dE=hdf=d\tilde{E}=dE_k,\\ \omega\cdot dt=k\cdot dx=2\pi f\cdot dt=\frac{2\pi}{\lambda}\cdot dx\Rightarrow\lambda f\cdot dt=dx,\\ \text{if }\lambda f=u=C=\text{Const.}\Leftrightarrow dx=C\cdot dt\Rightarrow\boxed{\Delta x=C\cdot\Delta t}\end{matrix}\right\}. \quad (10.1.1)$$

Or, if we have some additional angular motion added to the same moving object (see (4.8-4) from Chapter 4.2), we could analogically find group and phase angular velocity of such motion as,

$$\omega_{ph.}=\frac{\tilde{E}}{\tilde{L}}=2\pi f_{ph}(\equiv)\text{ angular phase velocity},\ \ \omega_g=\frac{dE}{dL}=\frac{d\tilde{E}}{d\tilde{L}}=2\pi f_g(\equiv)\text{ angular group velocity}.$$

Since spatial and temporal dimensions or durations of any energy-moments state are always mutually linked and related, it is logically and analogically natural that “Planck-Einstein-Maric” energy-quant $\tilde{E}=hf,\ dE=hdf=d\tilde{E}=dE_k$, initially defined for a time domain, has its spatial domain quantized energy defined on a similar way, meaning that another Planck-like constant h_x is also involved here, as follows,

$$\tilde{E}=\left\{\begin{matrix}hf=h_tf_t,\ dE=hdf=h_tdf_t=d\tilde{E}=dE_k\\ hf=h_xf_x,\ dE=hdf=h_xdf_x=d\tilde{E}=dE_k\end{matrix}\right\},\left\{\begin{matrix}(h_t,h_x)=\text{constants}\\ h=h_t=\mathbf{6.62607004}\times\mathbf{10}^{-34}\ m^2kg/s\\ c=\mathbf{299\,792\,458}\ m/s\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow d\tilde{E}=h_tdf_t=h_xdf_x=h_xd\left(\frac{1}{\lambda}\right)=-h_x\frac{d\lambda}{\lambda^2}\Leftrightarrow\frac{h_t}{h_x}df_x=-\frac{d\lambda}{\lambda^2},\ \alpha=\frac{h_t}{h_x}=\frac{h}{h_x}=\frac{1}{c}=\text{Const.}\Rightarrow$$

$$\Rightarrow\alpha(f_t-f_{t0})=\alpha(f-f_0)=\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)=(f_x-f_{x0})\Rightarrow$$

$$\Rightarrow h_x=h_t\frac{df_t}{df_x}=h\frac{df}{df_x}=-h\frac{df}{d\lambda}\lambda^2=hv\le h\cdot c=\mathbf{1.985445824}\times\mathbf{10}^{-25}\left[m^3kg/s^2\right]\Rightarrow$$

$$\Rightarrow\frac{df_t}{df_x}=\frac{h_x}{h_t}=\frac{1}{\alpha}=\text{Const}\Rightarrow\alpha\cdot df_t=df_x\Rightarrow\alpha(f_t-f_{t0})=(f_x-f_{x0}),\ (f_{t0},f_{x0})=\text{constants}.$$

In conclusion, spatial and temporal elementary wave-packet, or energy quant of the same “energy-moments event”, are mutually equal and analogically respecting the same “Planck-Einstein-M.Maric” law $hf=h_xf_x,\ dE=hdf=h_xdf_x=d\tilde{E}=dE_k$. In addition, modelling and explanation of Photoelectric and Compton effects will also benefit ...

3° We also need to consider that "**PWDC-relevant**" wavefunctions always have, at least, two (mutually coupled) analytic signal wave components, (such as $\overline{\Psi}^{+}(x,t)+\overline{\Psi}^{-}(x,t)$), propagating in mutually opposite, or inwards and outwards directions, such as,

$$\overline{\Psi}(x,t) = \Psi + I\cdot\hat{\Psi} = \left|\overline{\Psi}\right|\cdot e^{I\varphi} = \left|\overline{\Psi}_{1}\right|\cdot e^{I\varphi_{1}} + \left|\overline{\Psi}_{2}\right|\cdot e^{I\varphi_{2}} = \overline{\Psi}^{+}(x,t)+\overline{\Psi}^{-}(x,t),$$
$$\hat{\Psi} = H\left[\Psi\right], \varphi_{1,2} = kx \mp \omega t, I^{2} = -1 .$$

These are general solutions of Classical and Schrödinger, second order, differential wave equations, and at the same time ***this (about mutually opposed waves) is kind of manifestation of all action-reaction and induction laws known in physics.***

Here is the place to notice similarity of Analytic Phasors modeling with de Broglie and D. Bohm pilot wave model, since any Phasor or Complex Analytic Signal wave function can be created when we take an original signal function $\Psi(x,t)$ ***and add its imaginary part,*** $\hat{\Psi}(x,t) = H\left[\Psi(x,t)\right]$***. Complex Phasors can also be created starting from Minkowski 4-vectors from relativity theory. This way we are also generating the matter-wave phase function*** $\varphi(x,t) = \mathrm{arctg}(\hat{\Psi}/\Psi) = \omega t \mp kx + \varphi_{0}$***, which generates all PWDC properties (as summarized in (10.1)), and serves in developing non-probabilistic, power-related wave function and Schrödinger equation, which is mathematically the same as the original Schrödinger equation (see more in Chapter 4.3).***

Citation from: https://en.wikipedia.org/wiki/Wave%E2%80%93particle_duality#cite_note-6. "The pilot wave model, originally developed by Louis de Broglie and further developed by David Bohm into the hidden variable theory proposes that there is no duality, but rather a system exhibits both particle properties and wave properties simultaneously, and particles are guided, in a deterministic fashion, by the pilot wave (or its "quantum potential") which will direct them to areas of constructive interference in preference to areas of destructive interference. This idea is held by a significant minority within the physics community.[39]

De Broglie himself had proposed a pilot wave construct to explain the observed wave-particle duality. In this view, each particle has a well-defined position and momentum, but is guided by a wave function derived from Schrödinger's equation. The pilot wave theory was initially rejected because it generated non-local effects when applied to systems involving more than one particle. Non-locality, however, soon became established as an integral feature of quantum theory and David Bohm extended de Broglie's model to explicitly include it".

In the resulting representation, also called the de Broglie–Bohm theory or Bohmian mechanics,[18] the wave-particle duality vanishes, and explains the wave behavior as a scattering with wave appearance, because the particle's motion is subject to a guiding equation or quantum potential.

....

Very pragmatic and realistic consideration (based on respecting Conservation Laws) is that *Physics and **PWDC** relevant wave functions* should also be finite, energy-content limited, and well defined (or localized) in both of their temporal and spatial domains, like band-limited Gabor-Gaussian Wave Packets. Determination of elementary domain size, signals integrity, stability, and signal durations in all the mutually conjugate domains should be secured by respecting Parseval's Theorem and/or Identity, and relevant mathematical Uncertainty Relations, as presented in Chapter 5. of this book. ***If some solid particle (like a metal ball with well-defined shape), is in motion (meaning, it has linear and angular moments and kinetic energy), spatial and temporal durations, or dimensions of such particle,***

measured from the position of Classical Mechanics, will be different, compared to temporal and spatial size of the same particle when analyzed as an equivalent Matter-Wave-Packet. Real spatial-temporal size or dimensions, durations, and shape of a certain object (in different domains) are motion-dependent, as creatively elaborated in A. Einstein Relativity theory.

4° Every energy-moments described kinematic processes, including involved currents, voltages, forces, and velocities characterized systems, should be parts of certain closed electromagnetic and mechanical, or fluidic circuits (see much more in the first chapter of this book, elaborating Analogies). We always need to find and understand where input or source-energy elements (or front-end elements) are, supplying certain "black box system" with electro-mechanic excitation, and where its last-end or load elements are. Unfortunately, in our theoretic and practical analyzes in Physics, we often start from conceptualizing isolated, hanging, open-frame, and not at all closed (and completely fixed) circuits. Of course, conservation laws are forcing us to consider that everything what has certain input should have balanced outputs. We still consider (in Classical Mechanics and Physics) that angular and linear moments of the same motion are being mutually separated and independently conserved, what is not the case in relation to inertial and stationary motions (which are also particular or asymptotic cases of orbital and periodical matter-waves situations). All mechanical, electromechanical, electromagnetic, and fluidic systems in cases of temporal and spatial fluctuations or transformations are producing and receiving matter waves in different forms (as being dominantly acoustic waves, or mechanical vibrations, electromagnetic waves, voltages and currents, plasma-related waves etc.). All mentioned matter waves manifestations are on some level mutually coupled and balanced (or united), but in some cases we analyze only certain kind of dominant waving or oscillating manifestation, and neglect other, (associated) waves` manifestations. We should never forget the fact that solutions of Classical wave equations are always present as pairs of <u>mutually opposed</u>, propagating wave groups.

*5° Wave-Particle Matter Duality phenomenology presents a "bridge" between the **linear motions with spatial translational symmetry**, and always and naturally associated **rotational motions with spatial rotational symmetry**. In addition, under linear motions of masses, we can analogically and phenomenologically associate linear motions of electrically charged particles being in certain electric field. The same way, under rotational motions of masses, we can analogically and phenomenologically associate rotational (spinning and helix) motions of electrically charged particles being in certain magnetic field. Moreover, spatial, and temporal dimensions or duration intervals of involved masses, electric charges and associated matter-waves are always mutually or proportionally dependent and connected (on a way as addressed in Relativity theory). Most of contemporary theories describing events and constellations in our cosmic universe, regarding Gravitation, Planetary systems, Galaxies, Black holes, Neutron stars and other Natural Forces ... are dominantly addressing kind of mechanical, electrically neutral and passive, Newtonian world side, almost neglecting electromagnetic side of the same Universe. Here (in this book) we see that Mechanical, Electromagnetic, and Wave-Particle-Duality nature of our Universe cannot be separated (or mutually exclusive).*

6° Energy and wave-packets quantizing in Physics should also be part of universally valid mathematical signal sampling and reconstruction concepts, as established by

"Kotelnikov-Shannon-Whittaker-Nyquist". See more in [57, 58, and 59]. Later, if really needed, we will still have a lot of productive space and freedom to make non-dimensional mathematical normalizations, and to play with an additional, probabilistic, and statistical mathematical modeling. Anyway, **PWDC** grounds should be (initially and primarily), established like just underlined until here, and not at all axiomatically and by postulating whatever works locally and temporarily, and helps or serves a certain limited purpose.

Familiar to micro-world matter-waves concept is also applicable to macro matter waves of astronomic objects (as elaborated in the second chapter of this book; -see "2.3.3. Macro-Cosmological Matter-Waves and Gravitation").

Interpretations about what a matter-wave function conceptually, dimensionally, and qualitatively should be, are still creating waves, dilemmas, and uncertainties in modern physics (since the concept of non-dimensional probability and possibility waves, as practiced in Quantum theory, is very unusual and something artificial, or only being a mathematical, game-theory model). Analogical or mathematically identical appearance of the same wave function and Classical wave equations, regarding matter waves in many domains of Physics (or in many different experimental and observable situations), is impossible without the real existence of the carrier matter-medium, fluid, or spatial matrix, where matter waves are being created and propagating. Matter waves (as presented in this book), including quantum theory wave function, should be all waves and oscillations we know in Physics (since something being only mathematical, or like nothing tangible, cannot oscillate within nothing). Matter waves are always related to kinetic or motional energy (or power) of a certain dynamic undulatory process, or interaction, and to its motion participants. We cannot create matter waves without having certain (known or still unknown) carrier fluid somewhere in the background. Such carrier medium or ether could also be imaginatively and mathematically, conceptualized as the manifestation of some higher-dimensional reality. Anyway, mathematically well-operating and universally useful concepts are developed regarding wave functions, where still the best and most natural model, directly addressing all waves known in Physics (and exclusively practiced in this book) is the Complex Analytic Signal model.

In a contemporary-physics literature, complex wave functions are often presented only as being mathematically useful, "*__and calculations-beneficial, auxiliary mathematical method__*", tools or model, to solve different Classical and other wave equations easier and faster (and this is correct). If we will use only, by an Analytic signal modeled wave function, or wave groups, such as,

$$\begin{aligned}&\overline{\Psi}(x,t)=\left|\overline{\Psi}\right|\cdot e^{I\Phi}=\overline{\Psi}^{+}(x,t)+\overline{\Psi}^{-}(x,t)=\Psi(x,t)+I\cdot\hat{\Psi}(x,t)=\\&=\left[\Psi^{+}(x,t)+I\cdot\hat{\Psi}^{+}(x,t)\right]+\left[\Psi^{-}(x,t)+I\cdot\hat{\Psi}^{-}(x,t)\right]=\\&=\left[\Psi^{+}(x,t)+\Psi^{-}(x,t)\right]+I\cdot\left[\hat{\Psi}^{+}(x,t)+\hat{\Psi}^{-}(x,t)\right],\ \hat{\Psi}(x,t)=H\left[\Psi(x,t)\right],\ I^{2}=-1\ ,\end{aligned}$$

we will come to the conclusion that both, real part ($Re\left[\overline{\Psi}(x,t)\right]=\Psi^{+}(x,t)+\Psi^{-}(x,t)=\Psi(x,t)$), and corresponding imaginary part ($Im\left[\overline{\Psi}(x,t)\right]=\hat{\Psi}^{+}(x,t)+\hat{\Psi}^{-}(x,t)=\hat{\Psi}(x,t)$), of the same complex wave function, or Complex Analytic signal ($\overline{\Psi}(x,t)$), are realistic, natural matter-wave components, also existing coincidently and synchronously at the same time and space (see much more about Analytic signal properties in Chapter 4.0). The reasons for such statement are:

a) Total signal energy of $\Psi(x,t)$ is equal to the total signal energy of $\hat{\Psi}(x,t)$, or to the half of the total signal energy of $\overline{\Psi}(x,t)$ (see (4.0.4)). Here are the grounds for understanding self-organizing, auto-synchronization and entanglement effects in physics.

b) The common signal-phase, or wave function phase, and immediate frequency, group, and phase velocity functions can be found only by knowing (and using) both $\Psi(x,t)$ and $\hat{\Psi}(x,t) = H[\Psi(x,t)]$.

c) Energy and power of an Analytic signal wave function $\overline{\Psi}(x,t)$, is dynamically balanced, coupled and synchronously fluctuating between $\Psi(x,t)$ and $\hat{\Psi}(x,t)$. Or formulating analogically, this is as electromagnetic energy in resonating L-C circuits (meaning between electric inductance and capacitance), or in mechanically resonating, oscillatory, *potential, and kinetic* energy exchanges, as in mass-spring and pendula systems, or as coupling of electric and magnetic field components within an electromagnetic wave or photon, while a total involved energy is conserved (meaning constant).

d) Consequently, both wave packets $\Psi(x,t)$ and $\hat{\Psi}(x,t)$ should always and coincidently exist (on some self-organized, auto-synchronous way) in all oscillatory, resonant, and matter-wave phenomena, since Conservation laws, universal laws of inertia, and ***associated action and reaction laws or forces***, including induction related forces, currents, and voltages, are always working, both in mechanics and electrodynamics. If we see, measure, and consider as relevant only one of the mentioned wave components ($\Psi(x,t)$ or $\hat{\Psi}(x,t)$), this is only saying that our modeling and understanding of the same matter-wave and particle-wave duality situation is still incomplete, or only partially implemented. Here (within (10.1), (10.2), a), b), c) and d) are the most important grounds of Wave-Particle Duality (in this book summarized as **PWDC** facts).

e) N. Tesla and R. Boskovic's ideas about gravitation, universal natural force, radiant energy, and here-formulated PWDC are implicating that all atoms and masses in our Universe should mutually communicate externally and internally (outwards and inwards) to have closed circuits of involved energy, mass, moments, currents, and voltages (as explained in the first chapter about analogies). *Mechanical, or acoustical energy, moments, oscillations and vibrations, or audio signals and music, can be created and transferred by applying different signal-modulating techniques (in a temporal and spatial domain) on laser beams and dynamic plasma states (or signals); - See relations under (10.2-2.4) and literature references from [133] until [139].* Laws related to Newton action-reaction forces, and similar electromagnetic induction laws, including quantum entanglement effects, are in harmony with solutions of classical wave equations, where we see that mutually opposed, self-synchronized, and in different directions (in-pairs) propagating waves, are always being created. Effects of spontaneous and natural synchronization are known in mechanical, electromechanical, electromagnetic, and biological systems, what is also supporting the necessity of closed-circuits concepts. See more about such concepts and existing analogies in the first chapter of this book, as well as in the chapters 4.1, 4.3, 8 and 9 (example: Fig. 4.1.6, An Illustration of the Closed-Circuit Energy Flow). See also [117]. *Spontaneous, intrinsic effects of self-synchronizations are also characteristic for all* *biological and living entities (including humans), being initiated, maintained, and stimulated based on structural memory effects in liquids, on by light (or electromagnetically produced) excitation effects, including other mechanical vibrations, music … Coherence between two wave functions or signals is the measure of resonance and mutual synchronization (see more about coherence factors in Chapter 4.0. around definitions (4.0.83), (4.0.87) and (4.0.109)). Additionally, in cases of humans, synchronization and entanglement effects are manifesting as a complex, multiparameter chain of interactions and couplings related to ideological concepts, sport-related competitions, and different forms of art, creativity, and motivation. In present days, Internet is also becoming enormously powerful tool, information-source, and media for different connections and synchronizations among humans and surrounding equipment and technological processing.* In all cases of synchronizations and entanglements, we have significant part of mutually overlapping spectral characteristics of connected and communicating entities.

7° Finally, we can try to clarify what really means wave-particle duality. We are often using examples of photons behaving like particles (in analyses of Photoelectric, Compton, Bremsstrahlung, and familiar effects). To be sure that certain photon would behave as a particle, we need to compare wavelength of this photon (which has zero rest mass) with the wavelength, width, or physical size of the real interacting particle (with non-zero rest mass), when we analyze interactions between them. If photon has shorter mean wavelength and sufficiently short spatial duration (meaning it should have sufficiently high mean frequency), such photon will make impacts and scattering with a real particle, dominantly behaving as a particle. But, if the photon has much larger wavelength compared to the interacting particle physical size, or with its de Broglie wavelength, then such photon will behave as a wave (meaning no typical mechanical impacts and scatterings would happen). Photon is always a (Gaussian-Gabor-Kotelnikov-Shannon-Nyquist) wave packet, which in just described (size related) conditions react like a particle (or like a wave). Similar conceptualization is valid for all other kinds of matter-waves or wave groups. Wave-Particle dualistic properties are strongly related to the relevant wavelengths and physical sizes relations between mutually interacting participants (and relevant spatial, temporal, and spectral domains of such interaction participants are also mutually coupled, symmetrical and dependent). Wave groups will behave as particles only under here described conditions (related to wavelengths and physical size comparison). In different combination of spatial or sizes relations, the same photon or wave group would behave more as a particle, or more as a wave, but Planck's energy expression will still be applicable in all cases of familiar wave packets and photons ($\tilde{\mathbf{E}} = \mathbf{hf}$). See much more of familiar and supporting comments in Chapter 4.1 regarding "single, double and multiple slits diffractions of waves and particles".

Imaginary and virtual (not-measurable and not-dimensional) probability waves have no place here, since such conceptualization belongs to an abstract or game-theory mapping, resulting in creating almost metaphysical and unrealistic models, but it is still particularly (mathematically) useful within certain limited boundaries (when properly mathematically combined and supported with universal conservation laws and Euler-Hamilton-Lagrange concepts and principles).

*[♣ **Freethinking comments**: Here is a very convenient place to try to address or explain how light or photons' speed is always constant and equal to* $\mathrm{C} \cong \mathbf{300'000}\,\mathrm{km/s}$. *Let us imagine certain omnidirectional light source (or lamp) that is (mechanically) moving in one direction with certain*

constant, linear speed equal to V. Such light source is naturally radiating mutually entangled (or coupled) photons in a direction of its mechanical motion and in the opposite direction, $\bar{\Psi}^{+}(x,t)$, $\bar{\Psi}^{-}(x,t)$, like we already stated for the solutions of Classical wave equations. If we insist that only Galilean velocity transformations should be naturally applicable here (as everywhere else), we will (for instance) have that photonic wave group $\bar{\Psi}^{+}(x,t)$ is propagating with the speed $(C+V)\,[km/s]$, and its opposite, and coupled photonic wave group, $\bar{\Psi}^{-}(x,t)$, will propagate with $(C-V)\,[km/s]$. In fact, when using an Analytic signal model, we will have four wave-groups, such as, $\bar{\Psi}(x,t)=\left[\Psi^{+}(x,t)+\Psi^{-}(x,t)\right]+I\cdot\left[\hat{\Psi}^{+}(x,t)+\hat{\Psi}^{-}(x,t)\right]$. Let us be interested only for real wave elements $\left[\Psi^{+}(x,t)+\Psi^{-}(x,t)\right]=R_e\left[\bar{\Psi}(x,t)\right]$, and let us now calculate the average wave group's speed of $R_e\left[\bar{\Psi}(x,t)\right]$ as $\langle V\rangle=0.5\cdot\lfloor(C-V)+(C+V)\rfloor=C$. We see that regardless of (mechanical motion) speed V, the resulting wave-groups' speed (of electromagnetic waves) is always C. Let us now determine relevant center of mass velocity of $R_e\left[\bar{\Psi}(x,t)\right]$ (if we consider that light source or lamp will be in the center of mass position, and an independent observer (in the state of rest in a laboratory system) is monitoring and/or calculating mentioned center of mass velocity. Again, we will get that relevant center of mass velocity V_c will be equal to C, regardless of V, since,

$$V_c=\frac{(hf/c^2)\cdot(C-V)+(hf/c^2)\cdot(C+V)}{2\cdot(hf/c^2)}=C.$$

If we take an imaginary part of the same complex wave function, $\left[\hat{\Psi}^{+}(x,t)+\hat{\Psi}^{-}(x,t)\right]$ we will draw the same conclusions. As we know, Albert Einstein intuitively and logically, or by elimination (since nothing else worked well) concluded the same. Different consequences and dubious complexity of such axiomatic postulations and thinking are resulting in what we presently have as Relativity theory (meaning, mathematically, certain satisfactory and well-working solution is created, using Lorentz transformations, but this is still conceptually not completely explained theory). The biggest problem here (in a contemporary Physics literature) is that in most of the wave motions analyzes is systematically forgotten or omitted that Classical wave equation should always have (at least) two wave groups, mutually coupled, synchronized, and propagating in mutually opposite directions), such as $\bar{\Psi}(x,t)=\left|\bar{\Psi}\right|\cdot e^{I\Phi}=\bar{\Psi}^{+}(x,t)+\bar{\Psi}^{-}(x,t)$, what could count a lot for conceptual understanding of matter waves. Most probably, that Michelson-Morley experiment should also be analyzed using here elaborated ideas (and maybe the old ether concept could be revitalized). ♣]

The larger framework for addressing particles interactions, motional states, rest masses, and all involved energies and moments is supported with well-known relativistic theory 4-vector energy-momentum relations, and perfectly connected and compatible (or mutually provable as a cause and consequence) with all **PWDC** relations of Matter Waves (10.1), such as,

$$\left[\begin{array}{l}\left(\begin{array}{l}\left\{P_4^2=(p,\frac{E}{c})^2=\text{invariant}\right\}\Rightarrow p^2-\frac{E^2}{c^2}=-\frac{E_0^2}{c^2},\ p=\gamma mv,\ \frac{E}{c}=\gamma mc,\\ E=E_{tot.}=\gamma mc^2=E_0+E_k=E_0+\tilde{E},\ E_0=mc^2,\ E_k=E-E_0=(\gamma-1)mc^2,E=\gamma E_0,\\ v_g=v=\frac{dE}{dp}=\frac{dE_k}{dp}=\frac{d\tilde{E}}{dp}=\frac{d\omega}{dk}=u-\lambda\frac{du}{d\lambda}=-\lambda^2\frac{df}{d\lambda}=\frac{df}{d(\frac{1}{\lambda})}=\frac{2u}{1+\frac{uv}{c^2}},\frac{df}{v}=d(\frac{1}{\lambda}),\\ u=\lambda f=\frac{\omega}{k}=\frac{E_k}{p}=\frac{\tilde{E}}{p}=\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}=\frac{\tilde{E}}{\tilde{p}},\ \omega=2\pi f,\ \gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\end{array}\right)\Rightarrow\\ \quad\Rightarrow\left(\begin{array}{l}\lambda=\frac{h}{p}=\frac{h}{\gamma mv}=\frac{h}{\tilde{m}v}=\frac{h}{\tilde{p}},\ k=\frac{2\pi}{\lambda}=\frac{2\pi}{h}p=\frac{2\pi}{h}\tilde{p},\\ dE=dE_k=d\tilde{E}=vdp=hdf=c^2d(\gamma m)=c^2d\tilde{m}\end{array}\right).\end{array}\right]\Leftrightarrow$$

$$\Leftrightarrow\left[\begin{array}{l}\vec{p}^2c^2+E_0^2=E^2,\ (p\to p\pm\Delta p)\Leftrightarrow(E\to E\pm\Delta E)\Rightarrow\\ \left\{\begin{array}{l}(p+\Delta p)^2c^2+E_0^2=(E+\Delta E)^2\\ (p-\Delta p)^2c^2+E_0^2=(E-\Delta E)^2\\ \overline{v}=\dfrac{\Delta x}{\Delta t}=\dfrac{\Delta\omega}{\Delta k}\end{array}\right\}\Leftrightarrow\left\{c^2\cdot p\ \Delta p=E\ \Delta E\Leftrightarrow\dfrac{\Delta E}{\Delta p}=c^2\dfrac{p}{E}=\overline{v}\right\}\Rightarrow\\ \Rightarrow\overline{v}=\dfrac{\Delta x}{\Delta t}=\dfrac{\Delta E}{\Delta p}=c^2\dfrac{p}{E}=h\dfrac{\Delta f}{\Delta p}=\dfrac{\Delta\tilde{E}}{\Delta p}=\dfrac{\Delta\omega}{\Delta k}\leq c.\end{array}\right]\Rightarrow$$

$$\Rightarrow\left[\begin{array}{l}\left\{\begin{array}{c}u=\dfrac{\omega}{k},\ \omega=ku\\ (k\to k\pm\Delta k/2)\Leftrightarrow(u\to u\pm\Delta u/2)\end{array}\right\}\Rightarrow\Delta\omega=(k+\frac{1}{2}\Delta k)(u+\frac{1}{2}\Delta u)-(k-\frac{1}{2}\Delta k)(u-\frac{1}{2}\Delta u)=\\ =k\Delta u+u\Delta k\Leftrightarrow\overline{v}=\mathrm{Lim}\left\{\dfrac{\Delta\omega}{\Delta k}=u+k\dfrac{\Delta u}{\Delta k}=\left(\dfrac{\Delta x}{\Delta t}=\dfrac{\Delta E}{\Delta p}=c^2\dfrac{p}{E}=h\dfrac{\Delta f}{\Delta p}=\dfrac{\Delta\tilde{E}}{\Delta p}\right)\right\}_{\Delta\to 0}\Rightarrow\\ \Rightarrow\left\{v=u+k\dfrac{du}{dk}=\dfrac{d\omega}{dk}=\dfrac{d\tilde{E}}{dp}=\dfrac{dx}{dt}=\text{instant group velocity}\right\}.\end{array}\right]\quad(10.2)$$

Relations within (10.2) are demonstrating that matter waves and particle-waves duality concepts are mathematically mutually well integrated and fully compatible with relativistic particle energy-momentum picture. This is mutually reinforcing and confirming the conceptual and theoretical power of both entities, meaning being non-contradictorily applicable to matter waves (or wave groups) and to relativistic properties of moving particles (see much more about the same items in chapter 4.1). The significance of relations (10.2) is even bigger because wave-group velocity and matter waves formulas (10.1) are now independently developed, not considering any of relativistic theory concepts and mathematical practices (see about group and phase velocity in chapter 4.0). ***Very much familiar concepts, ideas, and conclusions (concerning PWDC) can be found in [105], Himanshu Chauhan, Swati Rawal, and R K Sinha. WAVE-PARTICLE DUALITY REVITALIZED CONSEQUENCES, APPLICATIONS, AND RELATIVISTIC QUANTUM MECHANICS***.

Let us (for instance) make a digression towards Signals and Spectrum analysis (based on Integral Fourier and Analytic Signal transformations) and consider that one of the relevant time-domain signals (or function) for addressing particle-wave duality is the linear moment, being a time-dependent function, $p=mv=p(t)$. Direct Fourier transform of such signal will be, $F[p(t)]=\overline{P}(f)$, where $P(f)$ presents "*temporal*" spectrum of $p(t)$. The inverse Fourier transform will again reproduce the original time-domain signal $F^{-1}[\overline{P}(f)]=p(t)$ (see basic integral signals transformations and associated definitions, starting from (4.0.1) to (4.0.5-1), T.4.0.1 and (4.0.13), as presented in chapter 4.0).

Before we make the next step, let us introduce definitions (or just symbols) of mutually analogical "*temporal and spatial*" frequencies ω_t, ω_x as, $\omega_t=\dfrac{2\pi}{T}=\dfrac{2\pi}{h}\tilde{E}=2\pi f_t=\omega,$

$\omega_x = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p = 2\pi f_x = k$, having the same meanings as in (10.1) and (10.2). Now we can define Fourier pair of time and frequency domain functions as,

$$F[p(t)] = \overline{P}(\frac{\omega_t}{2\pi}) = \overline{P}(f_t) = \int_{-\infty}^{\infty} p(t) e^{-j2\pi f_t t} dt, \ \omega = \frac{2\pi}{T} = \frac{2\pi}{h}\tilde{E} = 2\pi f_t = \omega_t,$$
$$F^{-1}\left[\overline{P}(f_t)\right] = p(t) = F^{-1}\left[\overline{P}(\frac{\omega_t}{2\pi})\right] = \int_{-\infty}^{\infty} \overline{P}(\frac{\omega_t}{2\pi}) e^{j2\pi f_t t} df_t = \int_{-\infty}^{\infty} \overline{P}(f_t) e^{j2\pi f_t t} df_t .$$

We can also claim that linear moment is dependent on its spatial coordinate x, being the path (or axis) in a direction of its propagation, $p = mv = p(x)$. Direct Fourier transform of such spatial-domain signal will be, $F[p(x)] = \overline{P}(f_x)$, where $\overline{P}(f_x)$ presents "*spatial spectrum*" of $p(x)$. The inverse Fourier transform will again reproduce the original spatial-domain signal $F^{-1}\left[\overline{P}(f_x)\right] = p(x)$.

$$F[p(x)] = \overline{P}(\frac{\omega_x}{2\pi}) = \overline{P}(f_x) = \int_{-\infty}^{\infty} p(x) e^{-j2\pi f_x x} dx, \ \omega_x = 2\pi f_x = k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} p,$$
$$F^{-1}\left[\overline{P}(f_x)\right] = p(x) = F^{-1}\left[\overline{P}(\frac{\omega_x}{2\pi})\right] = \int_{-\infty}^{\infty} \overline{P}(\frac{\omega_x}{2\pi}) e^{j2\pi f_x x} df_x = \int_{-\infty}^{\infty} \overline{P}(f_x) e^{j2\pi f_x x} df_x .$$

Linear moment should be dependent both on its temporal and spatial dimensions t and x, as $p = p(x,t)$. Consequently, a linear moment will also be more generally presentable as a two-dimensional Fourier integral transform pair,

$$\overline{P}(f_x, f_t) = F[p(x,t)] = A_p(f_x, f_t) e^{j\Phi(f_x, f_t)} = A_p(f_x, f_t) e^{j\frac{2\pi}{h}(p \cdot x - \tilde{E} \cdot t)} = A_p(f_x, f_t) e^{j(k \cdot x - \omega \cdot t)}$$
$$p(x,t) = F^{-1}\left[\overline{P}(f_x, f_t)\right] ,$$

and since temporal and spatial integrity, signal shape and its basic properties (at least for stable energy-momentum matter states) should be on certain way united, stable, and preserved (based on always valid Conservation Laws of Physics), it is expectable that temporal and spatial dimensions and periods of certain signal should also be mutually dependent, proportional, or causally related. This can be demonstrated or realized by using mathematical Uncertainty Relations and relevant group and phase velocity relations (of the particle or wave group in question), as we can see in (10.1) and (10.2), and in the chapter 4.0, starting from (4.0.60) until (4.0.72), and in the chapter 5.0, from (5.1) until (5.4.1). Of course, we can (on a similar way) expose the spatial, spectral and periodicity importance of angular motions and angular and spinning moments (as practiced in the Fourier and Statistical Optics, and in this book founded in the first chapter by "T.1.8 Generic Symmetries and Analogies of the Laws of Physics" and in the fifth chapter by "T.5.4. Wavelength analogies in different frameworks").

We can safely say that real inventors or founders of wave-particle duality (or founders of its essential mathematical grounds) are Jean Baptiste Joseph Fourier and Dennis Gabor (see more in Chapter 4.0). Fourier or more realistic (but sufficiently similar) Analytic Signal integral transformations are describing mutually conjugate original and spectral signal domains, which are not only abstract mathematical definitions, but much more something what our Nature, Universe or Physics are fully respecting and experimentally manifesting or following (without exceptions). Analytic signal spectral functions are more practical and realistic than similar Fourier spectral functions since only positive frequencies are considered. Mathematics (if not artificially defined as an abstract game of imposed and postulated rules and definitions) is generally considered as the best natural and exact language of Physics, and mathematical conclusions, implications, predictions, and extrapolations should be taken as something either known or waiting to be discovered and experimentally confirmed in Physics. This is the case with Fourier and Analytic signals spectral functions. For instance, if our relevant signal is linear momentum $p(x, t)$, being in a stable inertial motion (meaning being constant and preserved), there are its spectral functions showing that certain frequency or periodicity related nature is associated to such signal (both regarding its temporal and spatial domains). In other words, this is our particle-wave duality of matter states, being also independently and differently discovered and confirmed in Physics. In other words, we should not construct, postulate, or invent probability-based framework of something that is grandiosely, and on a rich and powerful (but simple) way explicable, and supported by real (not postulated or imaginatively and abstractly defined) Mathematics. Under spectral distributions, pictures, and domains of original (physics related) signals, we often (and almost exclusively) understand only time-to-frequency-periodicity or spectral domain relations and transformations. However, here we can see that similar spectral periodicity exist also in a typically spatial, configuration and geometry related coordinate terms. In addition, both, spatial and temporal dimensional properties, velocities, and periodicities are mutually related and deterministically transformable. This is securing the unity and stability of our Universe. For instance, “Double Slits Interference” explanation has been one of the important step stones of Particle-Wave Duality theory (in this book abbreviated as **PWDC**). Let us ask ourselves what double or multiple slits situations, from a spatial point of view presented in this book is? Of course, this should be a certain case of already mentioned spatial or topological periodicity like in crystalline, fractals and similarly structured matter states (that naturally and without particle-wave probabilistic assumptions) manifests with interference and superposition, wavelike signal patterns, or spatial spectral images.

Consequently, all other, artificial, strange, unusual, extraordinary, imaginative, and phantasy-based game-theory concepts and data sets mapping, like exclusively and ontologically stochastic and probability-based matter-waves, and some of Orthodox Quantum theory concepts, could only be conditionally valid and applicable in their own framework. This is correct description of the state of art of present Quantum theory, regardless how successful, well fitted, merged, and hybridized with number of grandiose self-standing, non-disputable mathematical and physics-related concepts and theories it is. Probability theory and Statistics will stay forever as powerful, universally applicable tools for analyzing and modeling of processes with a big number of almost identical participants (or matter states) within any science, and should not be considered as having an essential, exclusive, and unique ontological significance only for Quantum theory.

Several dualistic particle-wave manifestations, impacts, scatterings, diffractions, interferences, and similar effects (already mentioned in chapter 4.1) should be primarily considered, analyzed, and understood, within a spatial and temporal periodicity framework (based on relevant spectral distributions in both directions, and mutually coupled). Then, all motional energy-moments states interactions should be mathematically analyzed both in a relevant Laboratory and Center of a mass system since Laboratory system is describing what we (or our intelligence, sensors, and available observers) see, but Center of mass system is showing what Physics or Universe respects. Of course, Conservation laws of Physics should always be satisfied (in a real space-time environment). Here we should not forget that surrounding static frames, holders, diffraction plates, double or multiple slits plates, prisms, targets etc. are also interaction participants (in a Center of a mass framework, and from the point of view of mentioned spatial periodicity items).

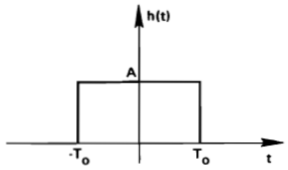
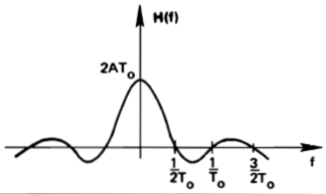
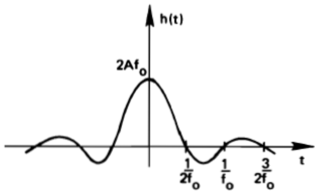
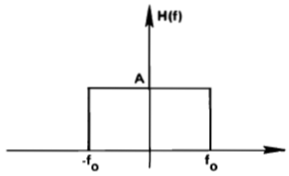
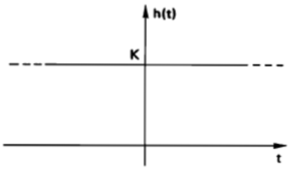
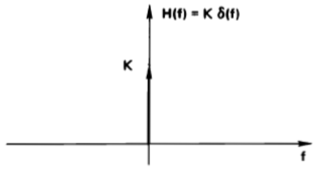
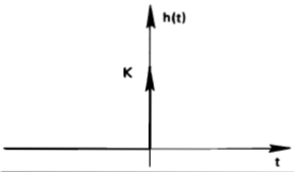
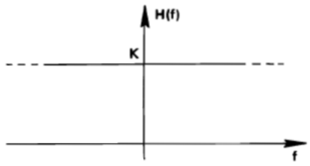
For instance, signals or wave functions with temporal periodicity will produce (or be synthetized from) many of spectral, frequency-dependent elementary simple-harmonic signals. Spatial 2D and 3D objects with geometric or spatial periodicity (like metals, crystals, different fractals, liquid memory states and related spatial structures, or double or multiple-slits diffraction plates), when properly agitated or excited, will also produce number of spatial and temporal matter-wave elements with linear or angular moment's properties (without the need for probability-based modeling). Naturally, temporal, and spatial periodicities are mutually connected and synchronized, where such couplings are explicable by Uncertainty Relations and group and phase velocity relations, or with PWDC relations, as elaborated under (10.1) and later.

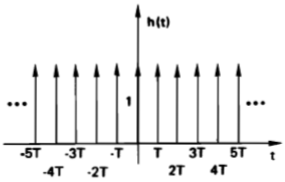
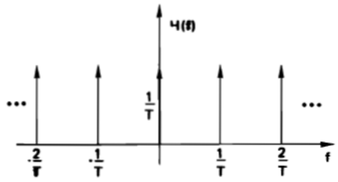
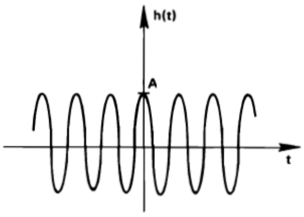
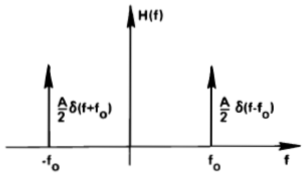
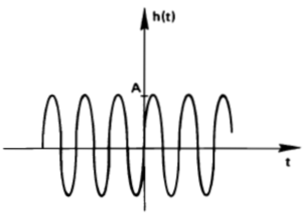
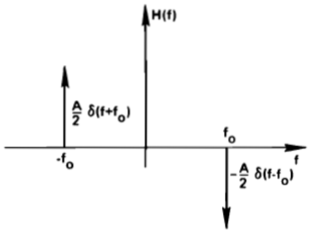
Then, we need to find out if analyzed particle-wave duality manifestations behave as electromagnetically neutral (or compensated) states, like neutral masses, or we may have associated effects of additional electric and magnetic charges and fields. Presence of electromagnetic charges, fields, and dipole or multi-pole polarizability (of interaction participants), when in certain motion, will change significantly and qualitatively the way that we should analyze such matter-waves related interactions (because of additional electromagnetic fields and waves interactions). In addition, there should be a difference if surrounding frames, holders, targets, diffraction plates, and screens are metallic (highly electrically conductive) or electric isolators, or have specific dielectric, magnetic, electrostrictive and magnetostrictive properties. We often consider experiments with electrons' diffractions, scattering, superposition, and interference effects, as confirmations that typical particles are behaving like waves, but we almost intentionally forget or neglect that most probably, electrons are specific electromagnetic waves energy (or photons) packed states, and there is no wonder that such photonic states in certain situations easily manifest their wave-like nature.

Present Orthodox Quantum theory foundations and analyzes of basic matter-waves and particle-waves duality situations is oversimplifying and omitting here mentioned (real and immediate joint time-frequency) interactions-associated situations and facts. This is creating unclear and incomplete models and concepts, with somewhat metaphysical grounds and logical uncertainties and dilemmas, where number of unnatural and strange questions, postulates and almost metaphysical explanations find their places, while intentionally forging probability and statistics-based modeling and concepts as the only and best applicable framework, which (by consensus decided) should have ontological grounds or meaning. In the contemporary Quantum theory, we consider $[\Psi(x,t)]^2$ as a probability that certain matter or energy state can be found in certain segment of space. Higher probability here means that chances to detect certain state in certain location are higher. Sum of all probabilities related to the same event are equal to one (1), and this is equivalent to satisfying the energy conservation law. We could also and much more naturally consider $[\Psi(x,t)]^2$ as a power (or spatial power density) distribution or wave function, and where such power density has its peaks, there we should expect to detect relevant matter states. Sum of all such time-integrated power functions (related to the same event) can only correspond to the 100% of a relevant total energy, and this way we could make analogy or parallelism between probabilistic and power related wave functions (of course, after applying convenient mathematical normalization and processing).

In the reference under [106], Brigham, E. Oran, we can find many of Fourier transform, time-frequency domain-pairs that are naturally associating on realistic situations regarding basic particle-wave duality understanding (see **Fig.10.1**). If we simply replace the left side's ***time-domain*** with similar space-domain, and right-side ***frequency-domain*** with its analogical spatial-frequency domain, we will cover all single, double, and multiple slits analogical (diffraction-related) situations. These are situations where certain variable or function (like linear moment) is constant, periodical, sinusoidal or impulse-like, and where its spectral domain has a wave-like shape or vice versa (see pictures under **Fig.10.1**). The same conclusions (and integral transformation shapes in both directions) are valid if we simply and mutually replace pictures of original and spectral domains (or relevant temporal and spatial variables) and designate them differently.

Examples of mutual, original, and spectral signal shapes in both directions (based on Fourier integral transformation), shown on **Fig.10.1**, encircled with red-color frames, are potentially presenting matter-wave or wave-group states (or packets) that are similar or equivalent to stable, particle-like states having finite energy and limited temporal, spatial and spectral durations in all their domains. In such cases **Uncertainty Relations** could be considered or formulated as **Certainty Relations** (using sign of equality instead of inequality; -see more in Chapter 5.). Mentioned energy states could be photons, elementary particles, atoms … If we create similar original-to-spectral presentations based only on Analytic Signal modeling, negative frequencies will be avoided, but we will again have the same conclusions and almost the same signal shapes, as shown on **Fig.10.1**.

Time Domain		Frequency Domain
	$h(t) = A \quad \lvert t\rvert < T_0$; $= \frac{A}{2} \quad \lvert t\rvert = T_0$; $= 0 \quad \lvert t\rvert > T_0$ ⬭ $H(f) = 2AT_0 \frac{\sin(2\pi T_0 f)}{2\pi T_0 f}$	
	$h(t) = 2Af_0 \frac{\sin(2\pi f_0 t)}{2\pi f_0 t}$ ⬭ $H(f) = A \quad \lvert f\rvert < f_0$; $= \frac{A}{2} \quad \lvert f\rvert = f_0$; $= 0 \quad \lvert f\rvert > f_0$	
	$h(t) = K$ ⬭ $H(f) = K\delta(f)$	
	$h(t) = K\delta(t)$ ⬭ $H(f) = K$	

Time Domain		Frequency Domain
	$h(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ ⬭ $H(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$	
	$h(t) = A\cos(2\pi f_0 t)$ ⬭ $H(f) = \frac{A}{2}\delta(f - f_0) + \frac{A}{2}\delta(f + f_0)$	
	$h(t) = A\sin(2\pi f_0 t)$ ⬭ $H(f) = -j\frac{A}{2}\delta(f - f_0) + j\frac{A}{2}\delta(f + f_0)$	

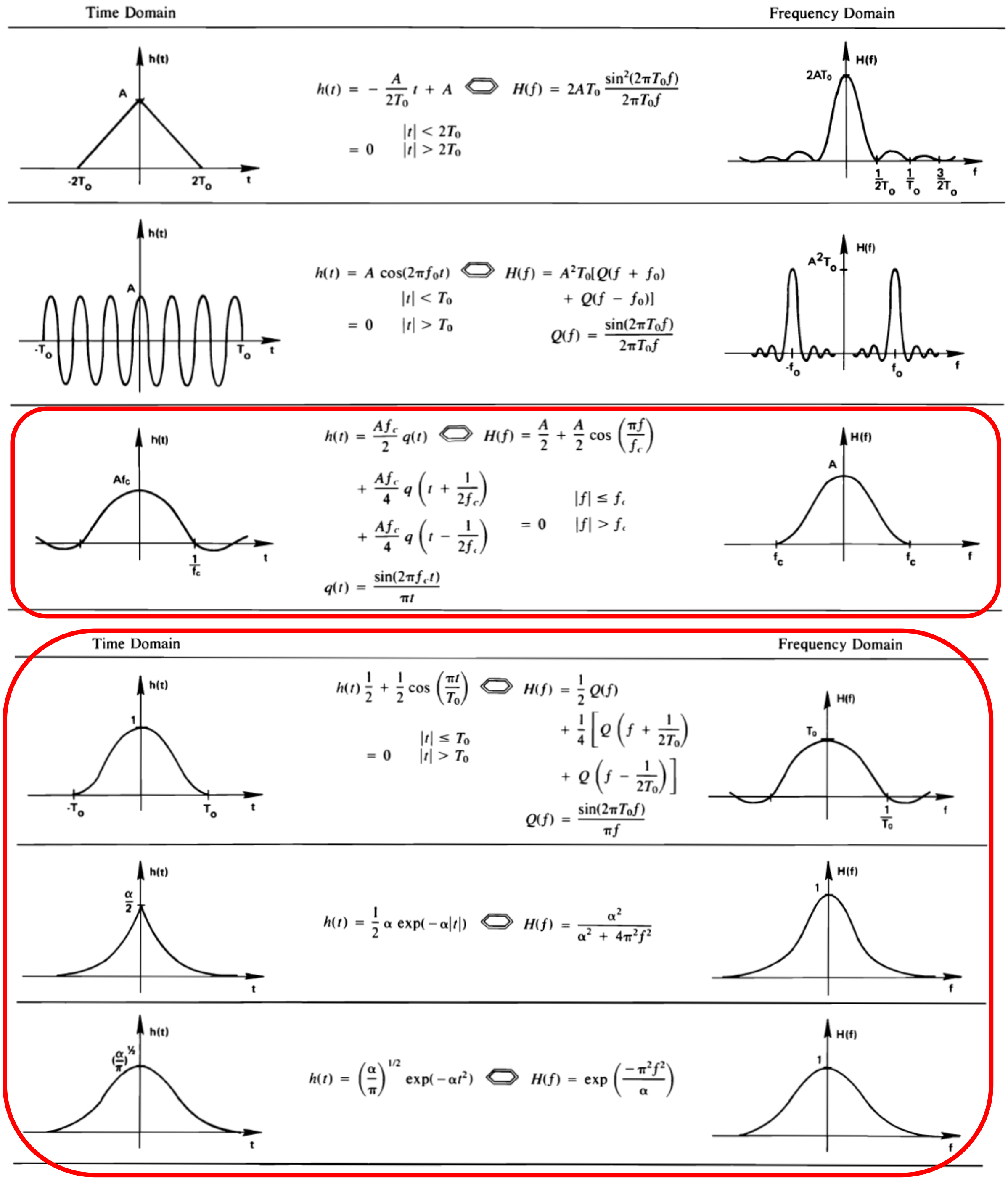


Fig.10.1, Fourier transform with time-frequency domain pairs
(Taken from [106], Brigham, E. Oran**)**

We also know that if a matter-wave or wave-packet is presented in the form of an Analytic Signal (introduced by D. Gabor, [57], see more in Chapter 4.0), we can determine a natural frequency and phase functions of such wave packet. From such phase function, we can extract relevant wavelength, and find out that all the mentioned waves-related parameters can be treated as de Broglie, matter waves properties, as shown in (10.1) and (10.2). Practically, the Analytic Signal and de Broglie matter waves belong to the same **PWDC** situation (meaning should be united in the frames of **PWDC**). The Analytic Signal wave function, here, is related to kinetic energy and power of the analyzed motional state. Such a wave function is always composed of two mutually phase-shifted wave functions that are equally realistic and coincidently present within the same wave motion. For instance, in an electromagnetic field or matter-wave, electric and magnetic field vectors are mutually coupled, complementing, and creating Pointing's vector; -See more of relevant examples of wave functions in a form of an Analytic Signals at the end of Chapter 4.0 under: "4.0.11. Generalized Wave Functions and Unified Field Theory").

We can also notice that Analytic Signal concept, de Broglie matter-waves, and **PWDC** situations universally belong (on the same mathematical way) both to a micro and macro world of physics, meaning there is not an essential difference in applied mathematical modeling. Only what we should differentiate (or treat and measure differently) is the actual localization, dimensions, shapes and sizes of matter waves and associated particles or bodies, to be able to apply the same Uncertainty Relations on a micro or macro world scale (meaning on small moving particles and/or big masses). In other words, mass is not defined or localized only as a static, stable, and by its solid-shape, and geometry-bounded object.

Whole family of Schrödinger and familiar wave equations of Quantum theory can be developed easily, smoothly and without artificially inventing and attaching missing parts of such equations, if we formulate relevant wave functions as complex analytic signal functions and phasors (see much more in Chapter 4.3).

There is another, much more elementary and independent (but still universally applicable), "non-mechanical", or better to say "wave-mechanical" method to develop the same expressions for group and phase velocity of certain wave packet, as elaborated in Chapter 4.0 of this book (see: "4.0.5. Wave Packets and mathematical strategies in formulating Wave Velocities", with equations from (4.0.6) until (4.0.33)). The existence of multiple, independent, and different methods leading to the same results (regarding waves velocities) is showing importance and universality of such results, as well as additionally reinforcing foundations of particle-wave duality and matter waves concepts. This way, we are also making theoretical cross-platforms connections and unifications in Physics. We should not forget that Physics and Nature are already, harmonically, and smoothly, united in many ways; -only our specific theories, tools and concepts about Physics and Nature could not be well united. Particle-Wave duality properties and joint spatial-temporal reality and transformability of our motional world are significantly contributing to mentioned unity.

Within the concept that wave, and kinetic energy are mutually equal ($\tilde{E} = E_k = hf$) we can find that matter-wave phase speed, u is always lower than relevant wave-group or particle speed v (see more in chapter 4.1), because,

$$\left\{\begin{array}{l}\left[\begin{array}{l}E_k=\tilde{E}=hf=\frac{pv}{1+1/\gamma},\gamma=1/\sqrt{1-\frac{v^2}{c^2}},m=\gamma m_0,\\ f=\left(\frac{E_k}{h}\right)_{v<<c}=\frac{\frac{mv^2}{2}}{h},\lambda=\frac{h}{mv}\Rightarrow u=\lambda f=\frac{v}{2},\text{ or}\\ f=\left(\frac{E_k}{h}\right)_{0<v<c}=\frac{(\gamma-1)m_oc^2}{h}=\frac{(m-m_0)c^2}{h},\lambda=\frac{h}{\gamma mv}\Rightarrow u=\lambda f=\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}\\ (0\le v\cong 2u<<c)\Rightarrow E_k=\tilde{E}=hf\cong\frac{1}{2}mv^2=\frac{1}{2}pv\end{array}\right]\\ \left(u=\frac{E_k}{p}=\frac{v}{1+\sqrt{1-\frac{v^2}{c^2}}}=\frac{\frac{dE_k}{dp}}{1+\sqrt{1-\frac{v^2}{c^2}}},\ v=\frac{dE_k}{dp}\right)\Rightarrow\\ \frac{dE_k}{E_k}=\frac{dp}{p}\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)=\left\{\begin{array}{l}\left(\frac{dE_k}{E_k}\cong 2\frac{dp}{p}\right)_{v<<c}\Rightarrow E_k\cong\frac{mv^2}{2}\\ \left(\frac{dE_k}{E_k}\cong\frac{dp}{p}\right)_{v\approx c}\Rightarrow E_k\approx cp\end{array}\right\},\\ \left[\begin{array}{l}\left(\begin{array}{c}v\cong u\cong c,\ \tilde{m}=\gamma m_0=m\\ E_0=m_0c^2,p_0=m_0c)=\text{constants}\end{array}\right)\Rightarrow\left(\frac{\tilde{E}}{p}\cong\frac{d\tilde{E}}{dp}\right)\Leftrightarrow\frac{dp}{p}\cong\frac{d\tilde{E}}{\tilde{E}}\Rightarrow\\ \tilde{E}=hf=E_k\cong\frac{E_0}{p_0}p\cong cp,\ v=\frac{d\tilde{E}}{dp}\cong\frac{E_0}{p_0}\cong\frac{E_a}{p_a}=\frac{E_b}{p_b}...\cong u=\lambda f=\frac{\tilde{E}}{p}\cong c\end{array}\right]\end{array}\right\}\Rightarrow \tag{10.2-1}$$

$$\Rightarrow 0\le u<2u\le\sqrt{uv}\le v\le c.$$

This looks like certain part of matter wave is retarded or traveling behind moving particle, what is somewhat strange because intuitively we are closer to the concept that moving particle and its matter wave should mutually overlap within the same space-time-frequency intervals.

*From the same exercise, we can also conclude that an optimal localization (and maximal concentration) of all relevant particle and wave attributes and characteristics is achievable only when group and phase velocity are mutually similar and close to the maximal speed **c**. This is valid for photons and fast moving microparticles), and this is implicitly indicating which self-stabilized and self-closed matter waves are creating stable particles.*

We could hypothetically try (just for comparison) what will happen if a particle total energy and its matter wave energy are mutually equal, $E_t=\tilde{E}=\gamma mc^2=\tilde{m}c^2$*, as for example,*

$$f=\frac{E_t}{h}=\frac{\gamma mc^2}{h},\lambda=\frac{h}{\gamma mv}\Rightarrow\left(u=\lambda f=\frac{c^2}{v},0\le v\le c\right)\Rightarrow u>c.$$

This time, the situation gets very strange because phase speed will be higher than particle speed, and even higher than light speed c, and this is one reason more to eliminate such option as unrealistic, meaning that total particle energy is not equal to the matter-wave-packet energy that is representing the same particle. The other basic and strong reasons to consider that only kinetic particle energy creates a relevant wave group are consequences of analyzes of Compton and familiar impact and scattering effects, and satisfaction of basic conservation laws (See chapter 4.2 of this book, "4.2.2. Example 3: Elastic collision photon-particle").

Obviously, we have much more arguments (as elaborated in the chapters 4.1 and 4.2) to consider that kinetic and matter-wave energy of moving particles are mutually equal, $(\tilde{E}=E_k=hf)\Rightarrow(0\le u<2u\le\sqrt{uv}\le v\le c)$. ***The way to understand what kind or part of matter-***

wave group is retarded (or delayed), and following the particle or wave group, is to take into consideration that all moving particles should be in certain (mutual) field-force and matter waves relations and couplings with their environment, creating kind of two or multi-body situations. Even if we do not see (or measure) any presence of a second body, certain well-hidden matter-wave and fields relation should exist between a single moving particle and its environment. This way, it is imaginable that some part of a matter wave in question is propagating behind moving particle, where phase speed u ***is lower than particle or group speed,*** $0 \leq u < 2u \leq \sqrt{uv} \leq v \leq c$.

Another supporting situation to explain the existence of a retarded part of a matter wave group is that linear and spinning or angular motions (of the same particle) are mutually complementary and united, having natural tendency to create closed-line, circular paths (like in cases of atoms), this way potentially hosting standing waves formations (like in cases of stable planetary systems; -see "2.3.3. Macro Cosmological Matter Waves and Gravitation" in chapter 2.). Within mentioned self-closed lines and standing waves formations, the meaning of retarded waves is becoming an important conceptual part of involved structural matter-wave groups' periodicity.

We also know that general solutions of differential, Classical Wave Equations (including similar complex, analytic signal, and generalized Schrödinger equations) are always presented with two wave functions propagating in mutually opposed directions, what we are, too often and simply neglecting as not enough operational and not well conceptually supported in Physics (see much more in chapter 4.3). Such two-component wave group can explain velocity and spatially retarded matter wave's nature. In addition, we could imagine (or exercise) that mentioned wave groups also have mutually opposed (or mutually canceling) angular and/or spin moments, this way coming closer to the understanding of the real nature of matter waves and wave-particle duality).

What should be much more general and realistic, united modeling of particle-wave duality is the framework of Analytic Signal functions (as established by D. Gabor; - see much more in the chapters 4.0 and 4.1). By using the Analytic Signal model creatively and with certain mathematical and dimensional arrangements, we should be able to present every matter-wave function of certain motion, its power, field, and force as a couple of mutually phase-shifted and Hilbert transform related wave functions $\Psi(t)$ ***and*** $\hat{\Psi}(t)$. ***Mentioned couple creates a complex analytic signal function*** $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t) = (1+jH)\cdot\Psi(t)$, ***where both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***present real, natural, and detectable items, like combination and coupling of electric and magnetic field vectors that is creating an electromagnetic field. On a similar way, every linear motion of particles and waves presented with a wave function*** $\Psi(t)$, ***should be automatically followed by synchronously created analytic signal couple*** $\hat{\Psi}(t)$, ***again creating a complex analytic signal*** $\overline{\Psi}(t) = \Psi(t) + j\hat{\Psi}(t)$, ***which has all matter-wave properties that are products of Analytic Signal modeling (such as de Broglie wavelength, frequency, phase etc.). Both*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***should present detectable physics items. For instance, a very good example of*** $\Psi(t)$ ***and*** $\hat{\Psi}(t)$ ***as coupled matter waves is where*** $\Psi(t)$ ***presents certain linear motion, and*** $\hat{\Psi}(t)$ ***presents a spiral, spinning or helix, de Broglie matter wave of a field existing in a space around the path of the original linear motion of*** $\Psi(t)$***. De Broglie matter waves have the frequency and phase described by the corresponding Analytic Signal model, as defined in chapter 4.0, by equations (4.0.2). From the Analytic Signal frequency and phase, we can determine de Broglie matter-wave wavelength*** $\lambda = h/p = u/f$ ***and belonging group and phase velocity*** $v = u - \lambda du/d\lambda = -\lambda^2 df/d\lambda$. ***Relevant Analytic Signal wave functions that are naturally describing de Broglie matter waves are power and motional energy-related functions, including corresponding field and force functions. By creating normalized, non-dimensional wave functions, we will be able to address Quantum Theory approach to***

the same problematic. This will be the background for explaining matter-waves and particle-wave duality in this book.

Now we can summarize matter waves and particle-wave duality concept as:

1. ***Propagating matter waves can create momentum-energy entities (such as atoms, nuclear and elementary particles and other energy states) with stable (non-zero) rest masses in cases if and when such matter waves will transform to self-closed and stabilized standing-waves structures*** (having an integral number of half-wavelengths, $n\frac{\lambda}{2}$, $n = 1, 2, 3...$, or $n\bar{\lambda} = 2\pi\bar{r}$, $n \cdot \theta = 2\pi...$, as summarized in T.4.2. "Wavelength analogies in different frameworks" in chapter 4.1). Freely propagating photons (as wave packets) could become electrons or other particles (with non-zero rest masses) if under certain conditions such wave formation is forced to become self-closed, stabilized, and standing waves structured formation. This way, motional wave energy (without rest mass attributes) could be transformed into a moving particle with a rest mass. This is usually related to elementary and subatomic particles formations).

   Certain relatively stable dynamic state of matter, or a system of particles, like an inertial motion that has number of intrinsic and structural periodicities, (including orbital, circular, spinning, and repetitive, periodical motions, and standing waves formations), can be relatively well described, or approximated, with number of mutually isomorphic, analogous, and equivalent mathematical models (being like Geocentric, Ptolemaic concepts). Anyway, only one of them will be the most correct and realistic model. Quantizing in Physics is mostly related to energy-finite, spatially-temporally limited entities, and to different counting and quantifications of atomized (or discretized) matter states, such as number of half-wavelengths, number of revolutions, number of periods, spinning frequency... (when we are using integers), and to simple arithmetic relations between integers-characterized states of matter in mentioned systems with intrinsic and standing-waves related periodicities. As examples for quantized structures, we could take Bohr's and Lucas-Bergman atom models (see chapter 8), and Planetary systems (see chapter 2; -2.3.3.).

*Here, we can propose an extended meaning of **inertial motions**, as motions characterized with mutually coupled and convertible, linear and angular moments, being natural, self-sustaining, self-closed, stable, smooth, and space-time periodical (see more in chapters 2 and 4.1). Such inertial (circular or orbital) motions are always creating stable, self-closed standing matter-waves formations along relevant orbits. Consequently, stable rest masses are also kind of "internally frozen or captured" inertial states of combined angular and linear moments. Since standing matter waves always have integer (countable) number of wavelengths and time-periods, this is the primary and most important background of quantizing in Physics. Why and how we can identify natural inertial motions to be parts of stable orbital motions we can also find in [36], and from number of elaborations presented in this book (and in other publications) we know that this is equally valid for the micro-world of atoms and macro-world of planetary systems (see in Chapter 2, "2.3.3. Macro-Cosmological Matter-*

Waves and Gravitation"). Effectively, here we connect or identify stable standing matter waves formations with natural inertial motions.

2. Any wave function can be presented and analyzed in its time and frequency domain, and its minimal space-time-frequency durations in mentioned domains are respecting "Uncertainty Relations" (see Chapter 5. UNCERTAINTY). Regarding the size of elementary particles and other (standing waves) stabilized particles, mathematical Uncertainty (between corresponding time, space, and frequency domain intervals) is effectively transformed into a Certainty, where an inequality sign, "$\geq$, or $\leq$" is transformed into an equality sign, "=". ***As elaborated in Chapter 5., we can conclude that metrics and energy formatting of Nature, regarding its elementary parts (such as atoms and elementary particles), has come to certain, conditionally non-divisible (and minimal) units of building blocks, this way realizing optimal matter waves packing (or formatting) with minimal and finite domain intervals.*** Such elementary matter building blocks (or narrow-banded matter-wave packets, effectively being like Gabor-Gaussian Wave Packets) should satisfy the following "resonant gearing and fitting, CERTAINTY relations, or optimal packing conditions" (for more explanations see (5.3) in the chapter 5.),

$$\begin{aligned}&\left(\Delta q_{mag.}\cdot\Delta q_{el.}\right)_{min.}=\left(\Delta\alpha\cdot\Delta L\right)_{min.}=h\cdot\left(\Delta t\cdot\Delta f\right)_{min.}=\left(\Delta x\cdot\Delta p\right)_{min.}=\\&=\left(\Delta t\cdot\Delta\tilde{E}\right)_{min.}=c^2\left(\Delta t\cdot\Delta m\right)_{min.}=\left(\Delta s_1\cdot\Delta s_2\right)_{min.}=h/2,\ (\Delta x)_{min.}=\frac{\lambda}{2}.\end{aligned}\tag{5.3}$$

<u>In (5.3) and later, we deal only with total and absolute durations of the same signal or wave function in all of its mutually original and spectral domains (not at all with relevant probabilities and statistical standard deviations).</u>

From Uncertainty Relations (as presented in chapter 5. and from (5.3)) we can also draw conclusions about space-time mutual and direct proportionality. For instance, relations relevant for the same energy-moments state, are effectively securing its structure, integrity, stability, and relatively constant shape, since original and spectral functions in time and frequency domains are mutually replaceable, on the same way as analogical, original, and spectral domains of spatial (or geometry related) signals are being mutually connected by relevant group and phase velocity relations. See more of supporting arguments in this chapter under (10.1), (10.2), and (4.2) from the chapter 4.1, as follows,

$$\begin{pmatrix}\Delta x\cdot\Delta p=\Delta t\cdot\Delta E=h\cdot\Delta t\cdot\Delta f\geq h/2\Leftrightarrow\Delta x\cdot\Delta\tilde{p}=\Delta t\cdot\Delta\tilde{E}=h\cdot\Delta t\cdot\Delta f\geq h/2,\\ \Delta q_{mag.}\cdot\Delta q_{el.}=\Delta\alpha\cdot\Delta L=h\cdot\Delta t\cdot\Delta f=\Delta x\cdot\Delta p=\Delta t\cdot\Delta\tilde{E}=c^2\Delta t\cdot\Delta m\end{pmatrix}\Rightarrow$$
$$v=\frac{\Delta x}{\Delta t}=\frac{\Delta\tilde{E}}{\Delta p}=\frac{\Delta\omega}{\Delta k}=h\frac{\Delta f}{\Delta p}\ (=\text{group velocity})\Leftrightarrow v\Delta t=\Delta x=\frac{\Delta\omega}{\Delta k}\Delta t=h\frac{\Delta f}{\Delta p}\Delta t.$$

Probability and Statistics, here, could have some secondary, peripheral, or accessorial meaning and applicability, mostly related to better final modeling and results presentation (when conditions for such modeling exist). In (5.3), (10.1) and (10.2), all relations between different functions, variables, and involved parameters are mutually deterministic, clear, analytic, explicit, and not at all probabilistic (but there is always a chance to interpret such situations, relations, and variables statistically and probabilistically, by applying certain convenient and necessary mathematical makeup, patchwork, and an artificial hybridization). Of course, not all matter states and matter-wave packets are stable and keeping mutually proportional spatial and temporal durations and shape (of relevant variables), and such states can be different short-living transients, and/or dispersive waves and energy-moments states. In addition, we will eventually conclude that every stochastic conceptualization and modeling of particle-wave-duality problematic should be much more statistically deterministic (like in Thermodynamics), than ontologically and Orthodox-Quantum-Theory probabilistic and non-deterministic.

The quite common mistake in such analyzes and modeling (about Uncertainty Relations) is that we often and wrongly consider the applicability of such relations mostly statistically and related only to a micro-world of Physics, what is mathematically not founded (since Mathematics is the language and logic of the whole Universe).

To additionally expose the space-time symmetry, related to mutual replacements of original and spectral domains, let us analyze analogically formulated consequences starting from Einstein-Minkowski 4-vectors. The most useful, and in a countless number of cases verified, is an energy-momentum 4-vector,

$$\overline{\mathbf{P}}_4 = (\mathbf{p}, \frac{\mathbf{E}}{\mathbf{c}}) \Rightarrow \mathbf{p}^2 - \frac{\mathbf{E}^2}{\mathbf{c}^2} = -\frac{\mathbf{E}_0^2}{\mathbf{c}^2} = \text{ invariant } (= \text{ constant}).$$

We will also exploit the same uncertainty relations (as formulated in Chapter 5) applicable on certain wave group, or wave packet, or simply on the same $\overline{\mathbf{P}}_4 = (\mathbf{p}, \frac{\mathbf{E}}{\mathbf{c}})$, narrow-banded energy-momentum matter-wave group,

$$\Delta x \cdot \Delta p = \Delta t \cdot \Delta E \left(\geq \frac{h}{2} \right).$$

To expose and underline analogies and symmetry (between original and spectral domains), we can formulate or expand the following relations,

$$\left\{ \begin{array}{l} \omega_x = k = 2\pi f_x = \boxed{2\pi \cdot \frac{1}{\lambda}} = 2\pi \cdot \frac{\tilde{p}}{h} = \frac{2\pi}{h} \cdot \tilde{p} = \frac{\tilde{p}}{\hbar} \\ \omega_t = \omega = 2\pi f_t = \boxed{2\pi \cdot \frac{1}{T}} = 2\pi \cdot \frac{\tilde{E}}{h} = \frac{2\pi}{h} \cdot \tilde{E} = \frac{\tilde{E}}{\hbar} \\ \Delta x \cdot \Delta \tilde{p} = \Delta t \cdot \Delta \tilde{E} \geq \frac{h}{2},\ v = \frac{\Delta x}{\Delta t} = \frac{\Delta \tilde{E}}{\Delta \tilde{p}} \\ \Delta E = \Delta \tilde{E},\ \Delta p = \Delta \tilde{p} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \Delta \tilde{p} = \hbar \cdot \Delta k, \Delta \tilde{E} = \hbar \cdot \Delta \omega \\ \Delta x \cdot \Delta k = \Delta t \cdot \Delta \omega \geq \pi = \frac{h}{2\hbar}, \\ \Delta k = 2\pi \Delta f_x,\ \Delta \omega = 2\pi \Delta f_t, \\ \Delta x \cdot \Delta f_x = \Delta t \cdot \Delta f_t \geq \frac{1}{2}, \\ v = \frac{\Delta x}{\Delta t} = \frac{\Delta f_t}{\Delta f_x} = \frac{\Delta \omega}{\Delta k} = \frac{\Delta \tilde{E}}{\Delta \tilde{p}} = \frac{d\omega}{dk} = \frac{dE}{dp} \end{array} \right\}.$$

Let us now (based on analogical assumptions) exercise space-time symmetry, connectivity, proportionality, and possibility of mutual original and spectral domain replacements (here underlined with $\boxed{\Delta x} \leftrightarrow \boxed{\Delta t}$ and $\boxed{\hbar \Delta k = \Delta p} \leftrightarrow \boxed{\Delta \tilde{E} = \hbar \Delta \omega}$). We already know that energy and momentum are mutually related ($\mathbf{E} \leftrightarrow \mathbf{p}$) or connected within invariance of $\overline{\mathbf{P}}_4 = (\mathbf{p}, \frac{\mathbf{E}}{\mathbf{c}})$. From A. Einstein-Minkowski Relativity mathematics we also know that space-time interval should be invariant regarding different reference or coordinates (and inertial) systems presentations, $(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = \text{const.}$. Now, we will create a couple of new, analogical (look-like, somewhat hypothetical) 4-vectors that are only formally like $\overline{\mathbf{P}}_4 = (\mathbf{p}, \frac{\mathbf{E}}{\mathbf{c}})$, and draw interesting consequences about space and time intervals' relations, as follows,

$$\left\{\begin{array}{l} \overline{P}_4 = (p, \frac{E}{c}) \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} = \text{invariant} \\ \boxed{\Delta x} \cdot \Delta p = \boxed{\Delta t} \cdot \Delta E,\ E_0 = mc^2 \\ (\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = \text{const.} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \overline{X}_4 = (x, ct) \Rightarrow (\Delta x)^2 - c^2(\Delta t)^2 = (\Delta x_0)^2 - c^2(\Delta t_0)^2 = -(\Delta s)^2 \\ \left[\begin{array}{l} \overline{T}_4 = (t, \frac{x}{c}) \Rightarrow (\Delta t)^2 - \frac{(\Delta x)^2}{c^2} = (\Delta t_0)^2 - \frac{(\Delta x_0)^2}{c^2} = \text{invariant} \\ \overline{E}_4 = (E, cp) \Rightarrow E^2 - c^2p^2 = E_0^2 = \text{invariant} \end{array}\right] \\ \frac{(\Delta s)^2}{(\Delta t)^2} = c^2 - \frac{(\Delta x)^2}{(\Delta t)^2} = c^2(1 - \frac{v^2}{c^2}), \Delta s = c\Delta t\sqrt{1 - \frac{v^2}{c^2}},\ v = \frac{\Delta x}{\Delta t} \end{array}\right\} \Rightarrow$$

$$c^2 = \frac{(\Delta x_0)^2}{(\Delta t_0)^2} + \frac{(\Delta s)^2}{(\Delta t_0)^2} = \frac{(\Delta x)^2}{(\Delta t)^2} + \frac{(\Delta s)^2}{(\Delta t)^2} = v^2 + \frac{(\Delta s)^2}{(\Delta t)^2} \Rightarrow (1 - \frac{v^2}{c^2}) = \frac{(\Delta t_0)^2 - \frac{(\Delta x_0)^2}{c^2}}{(\Delta t)^2} = \frac{1}{c^2}\frac{(\Delta s)^2}{(\Delta t)^2} \cong \frac{(\Delta t_0)^2}{(\Delta t)^2} \Rightarrow$$

$$\Delta t = \frac{\Delta x}{v} = \frac{\sqrt{(\Delta t_0)^2 - \frac{(\Delta x_0)^2}{c^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t_0\sqrt{1 - \frac{1}{c^2}\frac{(\Delta x_0)^2}{(\Delta t_0)^2}}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{\Delta s}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \cong \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}},\ \Delta s = c\Delta t_0\sqrt{1 - \frac{1}{c^2}\frac{(\Delta x_0)^2}{(\Delta t_0)^2}} \cong c\Delta t_0$$

Indexing with "0" is indicating that the corresponding variable belongs to the standstill rest state (or relevant center of mass, inertial system). On some way, here we implicitly consider that moving energy-momentum state in question is like a certain narrow-banded matter-wave packet with finite and short durations, both in its time and frequency domain. For instance, if such motional matter-state has its total spatial length (or size) in the direction of its propagation equal to Δx, and if we observe the same state between its temporal points T_1, T_2 and corresponding spatial points X_1, X_2, where $\Delta X = X_2 - X_1 \gg \Delta x,\ \Delta T = T_2 - T_1 \gg \Delta t$, we will, approximately (or in average) consider that,

$$\left(\frac{\Delta x}{\Delta t} \cong \frac{\Delta X}{\Delta T}\right) \cong \left(v = \frac{dx}{dt} = u - \lambda\frac{du}{d\lambda}\right), \text{ or } \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2 + \left(\frac{\Delta z}{\Delta t}\right)^2} \cong$$

$$\cong \sqrt{\left(\frac{\Delta X}{\Delta T}\right)^2 + \left(\frac{\Delta X}{\Delta T}\right)^2 + \left(\frac{\Delta X}{\Delta T}\right)^2} \cong \left(v = \frac{dr}{dt} = u(1 + \sqrt{1 - v^2/c^2})\right).$$

Practically, here we are on some way dealing mostly with linear and non-dispersive, *stable structure and shape* matter-states, where we can experience regular (or linear and simple) interference and superposition effects, and symmetry between original and spectral signal domains, like known in Analytic Signal and Fourier integral transformations.

In fact, here we still need to think more creatively and imaginatively, and develop or give proper formulations of involved spatial, temporal, and spectral items, and formulate an overall (or better) explanation, but we can already see that the idea about spatial and temporal proportionality and connectivity (or specific mutual symmetry) is very much incorporated in Relativity theory. Here we also present another (analogical and symmetry-related) insight that is testing the skeleton of Relativity theory.

For instance, we could pose a question if (supposed to be) an invariant space-time interval from Relativity theory is "*only approximately invariant*" or always constant, since $\Delta s \cong c\Delta t_0$. Larger and richer mathematical framework to address matter structure will be to create similar Minkowski 4-vectors or n-vectors and corresponding Phasor's mathematics with Hypercomplex Analytic Signals.

Spatial-temporal and joint time-space-frequency reality,

duality, proportionality, and mutual spatial-temporal transformability can also be imaginatively addressed if we consider that, at least local, infinitesimal, or differential space-time connections and relations are like relations between an Analytic signal, real and imaginary wavefunctions (which are by Hilbert transform mutually conjugate and phase shifted) couple, including that PWDC conditions are also satisfied (see (10.2) and later in this chapter).

We still do not know what the time essentially is, but we can easily operate with time intervals or time intervals durations in relation to corresponding spatial intervals. In the four-dimensional space-time world (x, y, z, t) we know that all spatial and temporal axes are mutually orthogonal. Orthogonality of time axis we realize mathematically based on significant success and applicability of Minkowski 4-vectors from Relativity theory, where time axis is an imaginary numbers axis. If instead of ordinary complex function with one real and one imaginary member, we apply hypercomplex dimensional framework (with three imaginary units or axes), we will create the platform that time could also have three temporal and mutually orthogonal components or axes, (x, y, z, t_x , t_y, t_z).

Here, still like an intuitive and over-simplified brainstorming (before establishing better mathematical foundations), time domain is considered as a kind of Hilbert transform of a space-domain (or vice versa), what means that temporal and spatial domains are (on certain way), mathematically (or functionally) mutually orthogonal and phase shifted for $\frac{\pi}{2}$ (similar as practiced in Minkowski space 4-vectors), as for instance, intuitively simplified and improvised on the following way,

$$\begin{bmatrix} x = ut,\ \Delta x = u\Delta t, dx = udt \\ u = \lambda f\ (=)\text{phase velocity} \\ \partial x^2 = \partial (ut)^2 = u^2 \partial t^2 \\ \partial t^2 = \partial (\frac{x}{u})^2 = \frac{1}{u^2}\partial x^2 \end{bmatrix}, \begin{bmatrix} \Delta\overline{x} = \Delta x + I\cdot\Delta\hat{x} = \Delta x + I\cdot H[\Delta x] = \Delta x + I\cdot u\cdot H[\Delta t] \\ \Delta\hat{x} = H[\Delta x] = H[\Delta (ut)] = u\cdot H[\Delta t],\ I^2 = -1 \\ \overline{x} = \|\overline{x}\|\cdot e^{I\varphi(t)} = \sqrt{(\Delta x)^2 + (\Delta\hat{x})^2}\cdot e^{I\varphi(t)} = I\cdot\|\overline{x}\| \\ \varphi(t) = \operatorname{arctg}\frac{u\cdot H[t]}{ut} = \operatorname{arctg}\frac{H[t]}{t} = \frac{\pi}{2} \end{bmatrix}.$$

If we apply proposed spatial-temporal relations, or $\mathbf{t}$ and $\mathbf{x}$ replacements, such as, $\Delta x = u\Delta t,\ dx = udt$, $\partial x^2 = \partial (ut)^2 = u^2\partial t^2$, $\partial t^2 = \partial (\frac{x}{u})^2 = \frac{1}{u^2}\partial x^2$, effectively meaning that spatial and temporal durations and/or dimensions are mutually replaceable, as $x \rightleftarrows t$, in the Complex Classical Wave equation (or in the Complex Schrödinger equation), we can prove that in a few simple mathematical steps we will again get the same Classical wave equation, as follows,

$$\left\{ \begin{matrix} \boxed{\Delta\overline{\Psi}(x,t) - \frac{1}{u^2}\frac{\partial\overline{\Psi}^2(x,t)}{\partial t^2} = 0} \Leftrightarrow \left[\Delta\Psi - \frac{1}{u^2}\frac{\partial^2\Psi}{\partial t^2} = 0,\ \Delta\hat{\Psi} - \frac{1}{u^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0\right] \\ \& \\ \boxed{x = ut,\ \partial x^2 = \partial (ut)^2 = u^2\partial t^2,\ \partial t^2 = \partial (\frac{x}{u})^2 = \frac{1}{u^2}\partial x^2, x \rightleftarrows t} \end{matrix} \right\} \Rightarrow \boxed{\Delta\overline{\Psi}(x,t) - \frac{1}{u^2}\frac{\partial\overline{\Psi}^2(x,t)}{\partial t^2} = 0}.$$

(10.2.1a)

The deeper meaning of presented brainstorming exercise is that the same matter waves could be created, synchronously and coincidently, both in a time and in a spatial domain (since

Classical Wave Equation did not change its form when we mutually replaced temporal and spatial variables t and x). Consequently, we could say that time and space are very much symmetrical, connected and mutually transformable, and we can also draw similar conclusions for relations between relevant temporal and spatial spectral domains. We know that the same Classical wave equation as known in mechanics is also describing electromagnetic waves. *Such kind of thinking could lead us to explain and understand better "double-slit interference effects" and the nature of wave-particle duality (see more in Chapter 4.1).*

Let us go back to basics of dimensionality where all relevant dimensions or axes should be mutually orthogonal. Analytic Signal model (or Hilbert integral transformation) is also and naturally producing mutually orthogonal functions. For instance, let us start from spatial-temporal wave function $\Psi(x,t)$. This function or signal, based on our detection or measurements methodology we can present as two wave functions (once only in a spatial, and second time only in a temporal domain), as follows,

$$\Psi(x,t) \rightarrow \begin{bmatrix} \Psi(x),\ t = const \\ \Psi(t),\ x = const \end{bmatrix}.$$

Here we assume (or accept) that spatial and temporal dimensions should be mutually orthogonal. Consequently, if we know certain time-domain, mathematically formulated function $\Psi(t)$, its corresponding or associated (phase shifted and orthogonal), spatial-domain function is Hilbert transform of the original time-domain function, producing $\Psi^*(x)$ and vice versa.

One of indicative ways how this could be mathematically formalized or still intuitively and hypothetically presented using Hilbert transform and the Analytic Signal model, is as follows,

$$\left\{ \begin{bmatrix} \Psi(t) \\ \text{temporal domain} \\ \cdots\cdots\cdots \\ \Psi(x) \\ \text{spatial domain} \end{bmatrix} \rightarrow H\begin{bmatrix} \Psi(t) \\ \Psi(x) \end{bmatrix} = \begin{bmatrix} \Psi^*(t) \\ \Psi^*(x) \end{bmatrix} (=) \begin{bmatrix} \Psi^*(x) \\ \bar{\Psi}(x) = \Psi(x) + I\cdot\Psi^*(x) \\ \text{spatial domain} \\ \cdots\cdots\cdots \\ \Psi^*(t) \\ \bar{\Psi}(t) = \Psi(t) + I\cdot\Psi^*(t) \\ \text{temporal domain} \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{\Psi}(x,t) = \Psi^*(t) + I\cdot\Psi^*(x) \\ \text{spatial} - \text{temporal} \\ \cdots\cdots\cdots \\ \bar{\Psi}(x,t) = \Psi^*(x) + I\cdot\Psi^*(t) \\ \text{spatial} - \text{temporal} \\ \cdots\cdots\cdots \\ \bar{\Psi}(x,t) = \Psi(x,t) + I\cdot\Psi^*(x,t) \end{bmatrix} \right\}$$

or

$$\left\{ \begin{matrix} \Psi(x,t) \rightarrow H[\Psi(x,t)] (=) \Psi(x) + \Psi(t) \rightarrow H[\Psi(x,t)] = \Psi(t,x) \Rightarrow \\ \Rightarrow \bar{\Psi}(x,t) = \Psi(t) + I\cdot\Psi(x) = \Psi_x(t) + \Psi_t(t) + I\cdot[\Psi_t(x) + \Psi_x(x)] \end{matrix} \right\}$$

or

$$\left\{ \begin{matrix} \begin{bmatrix} \Psi(x,y,z) \\ \text{spatial domain} \end{bmatrix} \rightarrow H[\Psi(x,y,z)] = \begin{bmatrix} \Psi(t_x,t_y,t_z) \\ \text{temporal domain} \end{bmatrix} \Rightarrow \\ \Rightarrow \begin{bmatrix} \bar{\Psi}(x,y,z,t_x,t_y,t_z) = \Psi(t_x,t_y,t_z) + I\cdot\Psi(x,y,z) \\ \text{spatial-temporal} \end{bmatrix} \end{matrix} \right\}, I^2 = -1 .$$

(10.2.1b)*

Temporal and spatial orthogonality between mutually related functions $\Psi(t)$ and $\Psi(x)$, and $\Psi^*(t)$ and $\Psi^*(x)$ is also associating on mutual orthogonality between an electric and magnetic field functions, as in cases of electromagnetic waves, what could be an additional (still hypothetical, indicative, simplified and improvised) insight into understanding what time and space are. In fact, everything in our temporal-spatial world is on some way well-packed, integrated and mutually dependent regarding temporal and spatial spectral pictures of the same event, and there is certain causal, functional, or by integral-transformations explicable mapping between them (here realized using Hilbert transform).

To be more explicit, clear, and indicative, let us analogically address the energy of the same spatial-temporal wave function from different points of view (of course using Parseval's identity, or wave energy definition; -see about Parseval identity in Chapter 4.0, equations (4.0.4)).

$$\Psi(x,t) \rightarrow \begin{bmatrix} \Psi(x),\ t = const \\ \Psi(t),\ x = const \end{bmatrix} \rightarrow \tilde{E} = E_k = \begin{bmatrix} \int_{[\Delta t]} \Psi^2(t)dt = \frac{1}{\pi} \int_{[\Delta\omega_t]} \left|A(\omega_t)\right|^2 d\omega_t \\ \int_{[\Delta x]} \Psi^2(x)dx = \frac{1}{\pi} \int_{[\Delta\omega_x]} \left|A(\omega_x)\right|^2 d\omega_x \\ \int_{[\Delta x, \Delta t]} \Psi^2(x,t)dx \cdot dt = \frac{1}{\pi} \int_{[\Delta\omega_x, \Delta\omega_t]} \left|A(\omega_x)\right|^2 d\omega_x \cdot d\omega_t \end{bmatrix}$$

$$d\tilde{E} = \begin{bmatrix} \Psi^2(t)dt = \frac{1}{2}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\left|A(\omega_t)\right|^2 d\omega_t = \frac{1}{2}a^2(t)dt \\ \Psi^2(x)dx = \frac{1}{2}\left|\overline{\Psi}(x)\right|^2 dx = \frac{1}{\pi}\left|A(\omega_x)\right|^2 d\omega_x = \frac{1}{2}a^2(x)dx \\ \Psi^2(x,t)\cdot dx \cdot dt = \frac{1}{2}\left|\overline{\Psi}(x,t)\right|^2 dx \cdot dt = \frac{1}{\pi}\left|A(\omega_x,\omega_t)\right|^2 d\omega_x \cdot d\omega_t = \frac{1}{2}a^2(x.t)\cdot dx \cdot dt \end{bmatrix} = \begin{bmatrix} h_t\, df_t \\ h_x df_x \end{bmatrix} = \begin{bmatrix} v_t dp_t \\ v_x dp_x \\ \omega_t dL_t \\ \omega_x dL_x \end{bmatrix} = c^2 d\tilde{m}$$

$$\begin{bmatrix} \frac{\partial \tilde{E}}{\partial t} = \Psi^2(t) \\ \frac{\partial \tilde{E}}{\partial x} = \Psi^2(x) \\ \frac{\partial^2 \tilde{E}}{\partial x \cdot \partial t} = \Psi^2(x,t) \end{bmatrix} = Power(=)[W], \quad \omega_t = \omega,\ \omega_x = k,$$

(10.2.1b)**

What is creatively, innovatively and analogically presented with (10.2.1b)* and (10.2.1b)** on a very much simplified way, will become the source or inspiration for number of challenging ideas and projects regarding wave-particle duality and spatial-temporal nature of our Universe understanding and relevant relations. Here we also find that energy can be stored in spatial configurations (what in the language of Mechanics presents potential energy), and motional, time dependent, dynamic energy is what we consider as kinetic energy of particles, and matter-waves energy of wave motions. Spatial (or static, spatial configuration dependent energy) is well connected and balanced with other (time-dependent) motional types of energy.

In addition, we can be very much certain that electromagnetic and associated mechanical or spatial vibrations and waves are also synchronously created, coupled and/or detectable (in parallel with mechanical motions or oscillations of masses), giving us a platform to develop concepts and technologies related to "extended-meaning-of-entanglement" effects, gravitational waves and maybe new methods of communications and masses transport. Mechanical, ultrasonic, or acoustical energy, moments, oscillations and vibrations, or audio signals and music, can also be created and transported by applying different signal-modulating techniques (both in spatial and temporal domain) on laser beams and dynamic plasma states (or signals); - See relations under (10.2-2.4), literature references from [133] until [139], and familiar relations (3.7-1) and (3.7-2) from the third chapter of this book.

We could also (and still hypothetically) imagine that by producing convenient conditions of specifically created electric and magnetic fields around certain heavy or macro-mass, combined with an external, *vibrational excitation, effects of* ***gravitational attraction, could be weakened or cancelled*** *(thanks to complexity of electromagnetic and mechanical interactions based on (3.1). (3.2), (3.3) and (3.4), as elaborated in Chapter 3.). The orientation of internal electric and magnetic dipoles and moments of threated mass should be changed on a way to become perpendicular to a direction of natural gravitational force from mentioned mass towards the biggest mass in its neighborhood (to weaken natural gravitation). Mentioned external vibrational mass excitation could be realized mechanically, acoustically, ultrasonically and using, by frequency-range well-selected infrared and/or microwaves radiators. Mechanical and/or acoustical excitation should cover lower frequency range of natural, mechanical resonant frequencies of the mass under vibrational treatment. External electromagnetic infrared excitation should have wavelengths that are of the same order as an average-size of internal mass constituents, which are atoms, molecules, and other agglomerated mass-grains. In addition, we should conveniently apply low frequency amplitude and frequency modulation on the carrier, infrared, or ultrasonic electromagnetic wave/s, to cover the range of dominant natural (or mechanical) frequencies of the macro-mass in question, this way mechanically resonating the same mass. Such effects of vibrational agitation belong to "MMM technology", - see [140], European Patent Application (related to MMM technology); -EP 1 238 715 A1. Multifrequency ultrasonic structural actuator).*

From generally valid Uncertainty relations between all important signal domain (or moving particle and its equivalent wave-group) total and absolute, non-statistical durations (see (5.2) in Chapter 5.), we can conclude ***what the nature of time-duration is***, as follows,

$$\left[\Delta q_{mag.}\cdot\Delta q_{el.}=\Delta\alpha\cdot\Delta L=h\cdot\Delta t\cdot\Delta f=\Delta x\cdot\Delta p=\Delta t\cdot\Delta\tilde{E}=c^2\Delta t\cdot\Delta m\geq h/2,\ \Delta E=c^2\Delta m,\ i_{el.}=\frac{\Delta q_{el.}}{\Delta t}\right]\Rightarrow$$
$$\Rightarrow\left[\Delta E=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta t}=\frac{\Delta\alpha\cdot\Delta L}{\Delta t}=\frac{\Delta x\cdot\Delta p}{\Delta t}=\overline{v}\cdot\Delta p=h\Delta f=c^2\Delta m=P\Delta t=\Delta\tilde{E}\geq\frac{h}{2\Delta t}\right]\cdot\frac{\Delta t}{\Delta E}\Rightarrow \qquad (5.2)$$
$$\Rightarrow\Delta t=\frac{\Delta q_{mag.}\cdot\Delta q_{el.}}{\Delta E}=\frac{\Delta\alpha\cdot\Delta L}{\Delta E}=\frac{\Delta x\cdot\Delta p}{\Delta E}\geq\frac{h}{2\Delta E}=\frac{h}{2c^2\Delta m}=(\Delta t)_{min.}.$$

The idea about spatial-temporal unity, transformability and coupling of mutually associated and phase shifted matter waves is grandiose, but ways and methods how to elaborate such concepts should be better addressed in our contemporary physics (and natural philosophy). *Generally valid is that if any of delta differences (Δ) or intervals in (5.2) is being agitated, oscillated or in resonance, this will create different, mutually synchronized matter waves.* Our present intuitive, perceptual, and philosophic concepts about time and space can be still considered as being over-simplified, bipolar, bidirectional, or binary, what means that for us time could be an evolving situation (or an impression) that has its past, present or future states (like positive and negative values on certain linear scale, continuous and infinite in both, mutually opposed directions. Zero time point is our present time, but since such timeline or axis is infinite in any direction, such understanding of present time means that zero pint could be everywhere …). Something similar could be concluded for one-dimensional spatial situations, meaning that we easily understand or conveniently arrange what positive and negative spatial directions, or mutually opposite directions are. At the same time, we know that our space is three-dimensional, created, or explicable geometrically on a way that all three spatial coordinates or length-dimensions are mutually, mathematically, and geometrically, perpendicular (or orthogonal). This means, we could have an infinite number of possible spatial directions (and this cannot be conceptualized only as one bipolar or bidirectional, rectilinear-axis case). Since time and space are mutually related, something similar (or multidimensional) should also be valid for a time directions or dimensions (as for instance, three spatial dimensions should be linked to three temporal dimensions). Our extraordinarily successful way for mathematical introduction, explanation, or temporal extension of our present three-dimensional spatial world towards four-dimensional spatial-temporal world has been to conceptualize new, time-related dimension or axis as being orthogonal to all other (already existing and known) spatial dimensions. In Relativity theory and Minkowski space-time mathematics with relativistic 4-vectors (which is the only what is essentially good and worth an attention in Relativity theory), new temporal dimension is introduced using an imaginary-units axis being orthogonal to other three of spatial dimensions. Now is no more defendable that we have only past, present, and future as bipolar temporal states of matter in motion, meaning that a time, at least mathematically, could also occupy number of spatial-temporal directions (thanks to introducing only one imaginary units' axis). If we now create similar unification and extension within certain multidimensional spatial and temporal situation, using hypercomplex numbers, where we have at least three imaginary units, our understanding of time complexity and multidimensionality is becoming much richer with omni-directional and infinite number of phase-related, spatial-temporal options and directions, like in a spatial domain (see more, later in this Chapter, under: "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality"). Of course, Classical wave equation always has number of possible (coupled and in-pairs-created) solutions (as matter waves) propagating in mutually opposite, or inwards and outwards, spatial-temporal directions, and this way, complexity and unity of spatial and temporal dimensions is becoming richer, both mathematically and conceptually. *We are always living in a present time (and space), but certain, here relevant spatial, electromagnetic, or matter waves components or signals are anyway travelling around (or on some way existing and being memorized) around a present-time-space reality (globally and macroscopically, mutually, and energetically canceling their relevant, past, and future related matter waves zones, leaving behind only an eternal and evolving present-time-space situation).* Past, present and future temporal states are anyway directly connected to related spatial dimensional situations, where we conveniently define (or agree) what negative, zero-spot, and positive directions are. Any spatial or temporal wave function, wave packet, or wave group, which has limited duration in all its domains (like a particle), and at the same time has like Gaussian bell-curve envelope, can be mathematically decomposed (using Fourier and/or Kotelnikov-Shannon-Nyquist-Whittaker signals' analysis and synthesis) on simple-harmonic, elementary (sinusoidal) waves that are on both sides endless and continuous, penetrating towards positive and negative infinities. *Fourier superposition of such elementary and (omni-directionally), endless, sinusoidal waves can create arbitrary-shaped, real, and energy-finite signals that are by durations limited in all their temporal, spatial and spectral domains. We can derive such understanding of time, as elaborated in Chapter 4.0, with equations (4.0.1) - (4.0.5), where we find that any real object, signal shape, or wave packet materializes as an entity that has certain (real space-time) finite energy only after superposition of all its simple-harmonic temporal and spatial components that are covering positive and negative, or spatial ant temporal, past and future infinity ($x \in (-\infty, +\infty),\ t \in (-\infty, +\infty)$). If we now start from a combined spatial-temporal (or multidimensional) wavefunction or Phasor, which again has Gaussian amplitude or envelope, the temporal-spatial meaning of positive, zero and negative, or only temporal, future, present and past situations, and events is becoming much more complex, like operating with spatial-temporal, four-dimensional (or multidimensional and hypercomplex) Phasors.*

Another option to speculate about time and space is related to how we create spatial and temporal wave functions, signals, and/or wave groups (for representing or replacing motional particles by such wave groups). In all mentioned cases, we can imagine that in the background is either Fourier or Analytic signal superposition of elementary simple harmonic or sinusoidal waves. Such simple harmonic, elementary waves are traveling with phase velocity $u = \frac{\partial x}{\partial t} = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}}$ in both, positive and negative, spatial, and temporal directions, spreading towards positive and negative infinity. However, after Fourier or Analytic Signal superposition, we will realize that this way created wave-group has its group velocity $v = \frac{dx}{dt} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}}$, and in this case only spatial dimension could be represented as having positive and negative direction, towards infinity, but temporal dimension, this time is defined only on the positive part of temporal axis (from certain zero-time spot, towards positive infinity). This can be mathematically presented (when observing a motional situation with constant phase and group velocity), as follows (see more of supporting background in Chapter 4.0),

$$u = \frac{\partial x}{\partial t} = \frac{\omega}{k} = \frac{\tilde{E}}{\tilde{p}} \Rightarrow \begin{pmatrix} \partial x = u\partial t \\ u \cong \text{const.} \end{pmatrix} \Rightarrow x = x_0 + ut,\ x \in (-\infty, +\infty), t \in (-\infty, +\infty),\ \omega \in \begin{bmatrix} (-\infty, +\infty) \\ \text{or} \\ [0, +\infty) \end{bmatrix}$$

$$v = \frac{dx}{dt} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{d\tilde{p}} \Rightarrow \begin{pmatrix} dx = vdt \\ v \cong \text{const.} \end{pmatrix} \Rightarrow x = x_0 + vt,\ x \in (-\infty, +\infty), t \in [0, +\infty),\ \omega \in [0, +\infty) \tag{10.2.1c}$$

$$0 \le u \le \frac{v}{2} \le \sqrt{uv} \le v \le c,\ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda},\ x_0 = \text{Const.},\ \tilde{E} = hf = E_k,\ v <<< c \Rightarrow v \cong 2u$$

$$\Rightarrow 0 \le dx \le udt \le \frac{v}{2}dt \le \sqrt{uv} \cdot dt \le vdt \le cdt.$$

Infinities in Physics

Meaning of infinities, or what is finite or infinite in Physics depends on our theoretical, mathematical, and observational point of view. This effectively depends in which system of reference we are analyzing certain situation. For instance, if we can describe certain process or state of matter with certain function f(x, n) = f[x(n)], n = 1, 2, 3, … we may be able to find $\lim_{n\to\infty} f(x(n)) = L$. This could also be related to multidimensional set of functions describing certain more complex situation as, $\lim_{n\to\infty} f_i(x, y, z, t, n) = L_i, i = 1, 2, 3, ...$ If we (as an observer) are "submerged inside such multidimensional set of variables", for us some of coordinates or dimensions (x, y, z, t, n) could appear as being unlimited, but in reality, outside of the same dimensional frame, and for another observer, mentioned infinities could be finite.

For instance, meaning of a time for us (within our spatial dimensional matter state f(x,y,z)) appears as something eternal, being unlimited, and/or endlessly spreading towards infinity, but if somebody from a higher dimensional world could observe us, our total time related interval or domain could be limited. We also know that temporal and spatial dimensions in our Universe are mutually related. Consequently, we still do not know if our time domain is unlimited.

Functional transformations and mapping could also describe such situations where certain coordinate is unlimited in one system of reference or coordinates, and limited in the other, like in Möbius transformations and Riemann Sphere concepts (described with linear coordinate transformations between z and z', where z' = (az + b) / (cz + d)). That means that our understanding of what is limited, or it spreads towards infinity, cannot be concluded, or analyzed based only on different subjective feelings and different philosophical elaborations. We need to use sophisticated mathematics for proper infinity understanding.

If we are not sure what the time is, we can still mathematically operate and make conclusions based on time intervals (or time differences) and on relevant velocities, as something what we can precisely measure.

If we are not sure how to understand a time as one additional and specific dimension of our 4-dimensional Universe (x, y, z, t), we could follow and extend the logic pattern knowing that (x, y, z) are mutually orthogonal, and consequently, the time dimension should also be orthogonal to mentioned spatial coordinates or dimensions, and we know that Hilbert transform is creating such mutually orthogonal functions.

For making valid analyzes, modeling, predictions, descriptions, and conclusions in Physics, we need to relay on Mathematical Logic, and on "Natural Mathematics". Natural Mathematics here means mathematics directly or causally developed or deduced based on measurements, experiments and concepts that are already confirmed during longer time, and from different conceptual platforms (meaning this is mathematics universally applicable in all domains of Physics). Orthodox Quantum theory mathematics is largely artificially and unnaturally constructed, and hybridized with grandiose, natural, and self-standing theories like Signal and Spectral Analysis, Statistics, Probability, and Mechanics, to serve certain purpose, mostly in its own, self-defined domain. This is still questionable, and challenging situation, or unique example in our modern Natural sciences.

Now we have more of real facts to think about how to understand the meaning of time, and this problematic is becoming much more tangible, interesting, and challenging. *At least, we can conclude that only philosophical understanding and explanations of the nature and meaning of time (based only on our intuition, perception, verbal, and intellectual constructions) is not enough and not scientifically acceptable, being inferior compared to relevant mathematical analyzes.*

For instance, we know that some of humans have sufficiently indicative and correct, predictive, and visionary insights, both related to past and future situations. Here, we could speculate that such "specifically assembled", on some way mentally exceptional members of humankind could behave as very unusual, selective, and sensitive receivers of matter (or electromagnetic) waves that belong to a past or future time-space events, this way manifesting extraordinary and exotic, *telepathic, and visionary insights and predictions which are in some cases shown to be sufficiently or surprisingly correct. As we know, Nikola Tesla was one of such people.*

Now, we could also mathematically and hypothetically speculate about time-traveling options. For instance, R. Feynman, with his interactions' diagrams successfully conceptualized antimatter states (on a practical, simple, and useful way) as wave groups traveling in a negative, past-time direction (see more here: https://www.britannica.com/science/Feynman-diagram).

Explaining the nature of time is the particularly challenging and ongoing project in modern Physics. Let us analyze circular, closed-line motion in a two-dimensional Descartes space (x, y). In such spatial domain (on a flat surface) we start a motion from certain point on a closed circular line, and following the same line, after certain time, we will again arrive to the same starting point. This is obvious and easy to understand or conceptualize for a self-closed spatial domain. Since we know that temporal and spatial domains are mutually connected and directly proportional, we can analogically create certain "time-traveling circular line", and start moving along this line in one direction, passing through *present, past and future time zones*, and eventually we will arrive to the same present-time-related starting point. Let us mathematically exercise with such imaginative and exciting options. The equation of a closed circular line in a (x, y) spatial plane is producing the following relations,

$$\left(x^2+y^2=R^2=const.\right)\Rightarrow\begin{pmatrix}xdx+ydy=0\\ x\Delta x+y\Delta y=0\end{pmatrix}\Rightarrow\begin{pmatrix}x\frac{dx}{dt}+y\frac{dy}{dt}=0\\ x\frac{\Delta x}{\Delta t}+y\frac{\Delta y}{\Delta t}=0\\ x\cdot v_x+y\cdot v_y=0\\ v^2=v_x^2+v_y^2\end{pmatrix}$$

If we now create a fully analogical, closed "*circular timeline*", since spatial and temporal domains are directly proportional, we will get,

$$\begin{pmatrix}t_x^2+t_y^2=T^2=const.\\ x=ct_x, y=ct_y, c=const\\ \Delta x=c\Delta t_x, \Delta y=c\Delta t_y\end{pmatrix}\Rightarrow\begin{pmatrix}t_xdt_x+t_ydt_y=0\\ t_x\Delta t_x+t_y\Delta t_y=0\end{pmatrix}\Rightarrow\begin{pmatrix}t_xdx+t_ydy=0\\ t_x\Delta x+t_y\Delta y=0\\ t_x\cdot v_x+t_y\cdot v_y=0\\ v^2=v_x^2+v_y^2\end{pmatrix}\Rightarrow\begin{bmatrix}t_x^2+t_y^2=\left(\frac{R}{c}\right)^2\\ R=cT\end{bmatrix}$$

We see here that a time, like a space (x, y), should have associated time components (t_x, t_y). Since we also know that our temporal and spatial domains are mutually orthogonal (existing synchronously in parallel), we could place a time domain on an imaginary axis of the united space-time domain, as very successfully practiced in the Minkowski-Einstein 4-vectors and Relativity theory mathematics.

$$x=Ict_x, y=Ict_y, R=IcT, \Delta x=Ic\Delta t_x, \Delta y=Ic\Delta t_y, I^2=-1.$$

Analogical time-space relations and conclusions can be now extended and summarized for the four-dimensional, united spatial and temporal domain, (x, y, z), (t_x, t_y, t_z), as follows,

$$\left(x^2+y^2+z^2=R^2=const.\right)\Rightarrow\begin{pmatrix}xdx+ydy+zdz=0\\ x\Delta x+y\Delta y+z\Delta z=0\end{pmatrix}\Rightarrow\begin{pmatrix}x\cdot v_x+y\cdot v_y+z\cdot v_z=0\\ v^2=v_x^2+v_y^2+v_z^2\end{pmatrix}$$

$$\begin{pmatrix}t_x^2+t_y^2+t_z^2=T^2=const.\\ x=ct_x, y=ct_y, z=ct_z, c=const\\ \Delta x=c\Delta t_x, \Delta y=c\Delta t_y, \Delta z=c\Delta t_z\end{pmatrix}\Rightarrow\begin{pmatrix}t_xdt_x+t_ydt_y+t_zdt_z=0\\ t_x\Delta t_x+t_y\Delta t_y+t_z\Delta t_z=0\end{pmatrix}\Rightarrow$$

$$\Rightarrow\begin{pmatrix}t_x\cdot v_x+t_y\cdot v_y+t_z\cdot v_z=0\\ v^2=v_x^2+v_y^2+v_z^2\end{pmatrix}\Rightarrow\begin{bmatrix}t_x^2+t_y^2+t_z^2=\left(\frac{R}{c}\right)^2\\ R=cT\end{bmatrix}$$

$$x=ict_x, y=jct_y, z=kct_z, R=IcT, \Delta x=ic\Delta t_x, \Delta y=jc\Delta t_y, \Delta z=kc\Delta t_z, I^2=i^2=j^2=k^2=ijk=-1, ij=k, jk=i, ki=j\ ...$$

Of course, here, instead of using an ordinary and simple complex analytic signal function with one imaginary unit, we will exploit a Hypercomplex analytic signal function (or quaternions) model with three mutually orthogonal complex units i, j and k, to represent a Hypercomplex Minkowski space with three temporal components. This way, we open a window towards new understanding of Symmetry and higher dimensional spaces in Physics. ***Here we arrived close to an exciting and challenging possibility to analyze what means space-time traveling in any of positive, negative, or differently specified direction*** *(what will stay as an open and ongoing project)*. See more of similar challenging and hypothetical speculations and analyzes at the end of this chapter under: "10.1 Hypercomplex Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality".

Another, almost like science fiction prediction, is that periodicity-characterized spatial structures like crystals and fractals, can analogically be found, or created in a time domain (since time and space domains, and their relevant spectral domains are mutually strongly related and causally linked; -see more in **[129]**). We could metaphorically say that music signals are oscillations in a time domain, and crystals are different form of music produced in a spatial domain. Consequently, certain crystals, most probably, can be used as detectors of coupled gravitational and/or electromagnetic waves (not only based on an old fashion amplitude demodulation method using a diode detector).

*Citation (from [129], **Frank Wilczek,** "Crystals in Time" in Scientific American 321, 5, 28-36 (November 2019), doi: 10.1038/scientificamerican1119-28, View This Issue*

- ***"Crystals are orderly states of matter in which*** *the arrangements of atoms take on repeating patterns. In the language of physics, they are said to have "spontaneously broken spatial symmetry."*
- ***Time crystals, a newer concept, are states of matter*** *whose patterns repeat at set intervals of time rather than space. They are systems in which time symmetry is spontaneously broken.*
- ***The notion of time crystals was first proposed in 2012****, and in 2017 scientists discovered the first new materials that fully fit this category. These and others that followed offer promise for the creation of clocks more accurate than ever before. …….*

The next step in our understanding of crystals is occurring now, thanks to a principle that arose from Albert Einstein's relativity theory: space and time are intimately connected and ultimately on the same footing. Thus, it is natural to wonder whether any objects display properties in time that are analogous to the properties of ordinary crystals in space. In exploring that question, we discovered "time crystals." This new concept, along with the growing class of novel materials that fit within it, has led to exciting insights about physics, as well as the potential for novel applications, including clocks more accurate than any that exist now". (End of citation)

Now we could better understand somewhat hidden and unusual theoretical grounds of Quantum Physics, Richard Feynman diagrams, anti-matter states and imaginative P. Dirac fluctuations of elementary and virtual particles in any vacuum state.

Citation from: https://en.wikipedia.org/wiki/Dirac_sea "The **Dirac sea** is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930[1] to explain the anomalous negative-energy quantum states predicted by the Dirac equation for relativistic electrons (electrons traveling near the speed of light).[2] The positron, the antimatter counterpart of the electron, was originally conceived of as a hole in the Dirac sea, before its experimental discovery in 1932.[nb 1]

In whole theory, the solutions with negative time evolution factors[*clarification needed*] are reinterpreted as representing the positron, discovered by Carl Anderson. The interpretation of this result requires a Dirac sea, showing that the Dirac equation is not merely a combination of special relativity and quantum mechanics, but it also implies that the number of particles cannot be conserved.[3] Dirac sea theory has been displaced by quantum field theory, though they are mathematically compatible".

P. Dirac creatively and imaginatively hypothesized that micro-world, elementary particles and different matter-wave entities originating from an infinite sea of particles could be found on many different places at the same time (of course, with certain probability), as spontaneously and randomly appearing and disappearing events, and R. Feynman, also imaginatively and very successfully, presented such situations including interactions among elementary particles and different matter-waves with his diagrams.

Citation from: https://en.wikipedia.org/wiki/Feynman_diagram "In theoretical physics, **Feynman diagrams** are pictorial representations of the mathematical expressions describing the behavior of subatomic particles. The scheme is named after its inventor, American physicist Richard Feynman, and was first introduced in 1948. The interaction of sub-atomic particles can be complex and difficult to understand intuitively. Feynman diagrams give a simple visualization of what would otherwise be an arcane and abstract formula. As David Kaiser writes, "since the middle of the 20th century, theoretical physicists have increasingly turned to this tool to help them undertake critical calculations", and so "Feynman diagrams have revolutionized nearly every aspect of theoretical physics".[1] While the diagrams are applied primarily to quantum field theory, they can also be used in other fields, such as solid-state theory”.

Something like that (very much virtual, arbitrary, exotic, and imaginative, but anyway productive), what Dirac and Feynman created, is much better explicable mostly thanks to fields-coupled, resonant and entanglement, self-synchronization effects between mutually opposed (inward and outward propagating) waves, when modeled using Complex Analytic Signal Phasors. Coupled waves or wave groups (created in pairs) are solutions of all relevant Complex and Ordinary, second order, Classical Wave equations (including Schrödinger equation). In addition, we know that even an empty space or an idealized total vacuum state are still presenting certain waves-carrier-medium with measurable electromagnetic properties (behaving like an ideal fluid, being familiar to old ideas about ether). If the “old ether” is confirmed being non-existent, as conceptualized in the original version of Relativity theory (what even A. Einstein, much later renounced), it is still necessary to have in mind that certain more imaginative and maybe still hidden fluid or material background state should exist (in a state of an ideal vacuum), being the carrier of electromagnetic and other matter-waves. It is very instructive and good to know that Maxwell-Faraday Electromagnetic theory is analogically, essentially, and intuitively developed by having in mind real-world fluid-dynamics (see more in Chapter 3. of this book and in publications from Tsutomu Kambe [123]). Waves without a carrier fluid or certain carrier matter state are simply impossible. If we now extend such binary-symmetry ideas to Quaternion or Hypercomplex based Analytic Signal Phasors, we will enter the much richer world of (at least) trinary-concept of Supersymmetry and new understanding of matter structure, natural forces, and multidimensionality.

10.01 CERTAINTY AND UNCERTANTY RELATIONS IN PHYSICS

Uncertainty relations, or quantum uncertainty principle are basic, elementary, and always valid products of standard Signals and Spectral Analysis, or generally valid Mathematics. Briefly saying, Uncertainty relations are mathematical relations between temporal, spatial and spectral durations of the same signal (or wave packet) in different, mutually conjugate (original and spectral) domains. Later created, or associations and adaptations of mentioned Uncertainty relations to Quantum theory are just subordinated and slightly or appropriately modified and well-known mathematical Uncertainty relations (see more in Chapter 5). There are many aspects of Uncertainty relations known in mathematics and physics, and all of them are mutually related (basically equivalent) and can relate to optimal signal sampling and reconstruction concepts (like in Shannon-Kotelnikov-Nyquist-Whittaker theory), or "*in-average*" connected with statistical definitions of domain durations (like in contemporary Quantum theory). Uncertainty relations could also be mathematically connected with finite differences and error-analysis. By analyzing ***Uncertainty relations***, we can define and understand minimal sizes, or atomized and discretized domains-durations existing in physics as stable objects (such as atoms, elementary particles, or solar systems and galaxies), what we here consider as ***Certainty relations*** (see much more in Chapter 5.).

Problems regarding understanding macro-world or macro-masses Uncertainty Relations (as well as all other Uncertainty Relations) are related to our understanding of the size, length, spatial boundaries, geometry and/or dimensional lengths and temporal durations of real masses and matter waves. Briefly to say, the spatial (and matter-wave or electromagnetic) size of a mass-energy-momentum matter-wave that represents the same stable or solid mass is different and bigger (or much larger) compared to perceptual and directly measurable spatial or geometric boundaries of the (solid and stable) rest mass in question. One of the reasons for such spatial spreading is the fact that group and phase velocity of any matter-wave are mutually different. We often forget that matter-wave equivalent (wave group) of a moving particle (what all particles and masses in our Universe are) is much fuzzier and wave-like spread, or spatially distributed (than its original and solid, finite-boundaries particle carrier), and it is naturally, "by fields and waves connected or synchronized" with all surrounding waves, particles, and masses. This is making possible that Uncertainty Relations are equally applicable both on micro and macro world of Physics since we apply such relations only on relevant matter-waves (not at all directly on solid particles where spatial or geometric, mechanical boundaries are much sharper, and precisely defined in a smaller spatial zone). For wave-like formations or wave packets, it is normal and natural to comply with universally valid mathematical uncertainty relations (between their original and conjugate spectral domain durations, regardless of involved sizes or dimensions).

We simply (and wrongly) apply mechanical, motional, and geometry-related items from our visually perceptible macro world of solid particles (with precisely defined spatial dimensions) to a world of matter-wave formations, regarding uncertainty relations.

Many naïve interpretations of Uncertainty behaviors (or properties) of our world are coming from such initial geometry-related misinterpretations (since solid body size or spatial boundaries are usually smaller than "matter-wave and electromagnetic size and boundaries of the same body"). In addition, what we should consider as an intrinsic property of our Universe (of stable, inertial, and self-closed or standing-waves formations and motions) is that for micro and subatomic world entities and photons we are always using Planck constant h. For analogical situations in the macro world of planetary and galactic systems (with some circular or elliptic paths periodicity), which are also self-closed, standing matter-waves formations (if we neglect global cosmic motions), we need to apply much bigger Planck-like constants H_i (see more about macro-cosmological Planck-like constants and planetary systems quantizing in the second chapter in "2.3.3. Macro-Cosmical Matter-Waves and Gravitation "). We should always have in mind that all continuous symmetries and conservation laws of Physics are valid and applicable both in a world of micro and macro particles or real masses and matter waves. Only, our way of conceptualizing and mathematical processing should be appropriately adjusted. In addition, we should not forget that all natural or electromechanical systems and motions belong to closed and multidimensional (or spatial) networks with front and last ends, meaning having sources and loads (or inputs and outputs), like in electric circuits' theory (see more in the first chapter). Probability framework of the contemporary Quantum Theory is only an acceptable and convenient method of "in-average" mathematical modeling that is considering conservation laws of our Universe and wave-particle-duality complexity on a way by calculating averaged values and statistical properties and motional (auto-organizing) trends of interaction participants. We should avoid ontological mystifications in relation to Probability and Statistics as presently practiced in Quantum Theory. Here presented understanding of Uncertainties is coming closer to Nikola Tesla and Rudjer Boskovic's theoretical concepts about structure and forces of our Universe (read about "Dynamic Gravity Theory" ideas from N. Tesla). See much more under [97] and [117].

The skeleton or framework of realistic quantized energy exchanges and particles packing (or formatting) in Physics should also be very closely related to signal analysis and synthesis described by Kotelnikov-Shannon-Nyquist-Whittaker concepts of signals discretization and/or information or signals recovery. In addition, Parseval's identity between a time and frequency domain of certain wave function or signal is equivalent to energy conservation in Physics and it is always valid. We also know that signals' spectral (or frequency) analysis is causally related to number of effects known in Physics and Mechanics, addressing on a different way what we know as motional energy, power, and momentum (experienced as superposition, interferences, refractions, reflections, scatterings... of wave functions and/or wave groups). We cannot avoid implementing mentioned grandiose and brilliant mathematical concepts that are universally valid, well proven, and currently used in different communications and information or energy transfer technologies. We only need to find the most natural and correct way to integrate Kotelnikov-Shannon-Nyquist-Whittaker and Analytic Signal concepts into particle-wave duality, or in matter waves and quantum theory. For instance, we could say for certain signal or wave packet that (with particularly good approximation) has a total time-domain duration **T**, and total frequency spectrum duration **F**. For such wave packet we could also say (for instance) that it has 99% or 99.99%... of signal energy inside intervals **T** and **F**, and we can make discretization or quantization of such signal, both in its time and its frequency domain, by applying Kotelnikov-Shannon-Nyquist-Whittaker theory under the following conditions,

$$\left\{TF \geq \frac{1}{2}, (\Delta t)_{max.} = \Delta t \leq \frac{1}{2F}, (\Delta f)_{max.} = \Delta f \leq \frac{1}{2T}, F = F_t\right\} \Rightarrow$$
$$\left\{0 < \Delta t \cdot \Delta f \leq \frac{1}{2} \leq TF \leq \frac{1}{4\Delta t \cdot \Delta f}\right\} \Rightarrow \left(\Delta t \cdot \Delta f\right)_{min.} = TF_t = \frac{1}{2}. \quad (10.2\text{-}2)$$

Here, Δt and Δf are time and frequency sampling intervals necessary to make total signal recovery after operating with Kotelnikov-Shannon-Nyquist-Whittaker discretization in both domains (see more in [57, 58, and 59]). Consequently, we could say that proper (elementary) quantum of certain arbitrary and energy finite signal (effectively being like Gabor-Gaussian Wave Packet) are signal elements or signal elementary wave functions with Δt and Δf, as time and frequency sampling durations. Based on Kotelnikov-Shannon-Nyquist-Whittaker theory we can also formulate the family of analytic elementary wave functions of such quantized signals, considering such signals as wave groups with group and phase velocity mutual relations as $v = u - \lambda\frac{du}{d\lambda} = \frac{d\tilde{E}}{dp}, u = \lambda f = \frac{\tilde{E}}{p}...$ (see the Chapter 5.). When we rich absolute limits such as, $\left(\Delta t \cdot \Delta f\right)_{min.} = TF_t = \frac{1}{2}$, we are addressing elementary particles, or relatively stable elementary matter domains, masses, and other elementary standing matter waves (or building blocks of our Universe); -this way practically explaining the real nature of Uncertainty Relations and Quantization in Physics.

Since spatial and temporal matter states and motions (and their relevant spectral domains) are always mutually coupled and dependent, we can analogically imagine that **L** and $\mathbf{F_x}$ are the total spatial duration and total spatial frequency interval of the same matter state, and Δx, Δt and Δp are its minimal sampling and signal reconstructing lengths and momentum, and we can extend (10.2-2) to,

$$\left(\Delta t \cdot \Delta f\right)_{min.} = TF_t = \frac{1}{2} \Rightarrow$$
$$\left[\begin{array}{l} h\left(\Delta t \cdot \Delta f\right)_{min.} = \frac{h}{2\pi}\left(\Delta t \cdot \Delta\omega\right)_{min.} = \left(\Delta t \cdot \Delta E\right)_{min.} = \boxed{\left(\Delta x \cdot \Delta p\right)_{min.}} = \\ = \frac{h}{2\pi}\left(\Delta x \cdot \Delta k\right)_{min.} = hTF_t = \boxed{hLF_x} = \frac{h}{2} \end{array}\right]_{\Delta \to d \to 0} \Rightarrow \quad (10.2\text{-}2.1)$$
$$\Rightarrow \left[h\left(dt \cdot df\right) = \frac{h}{2\pi}\left(dt \cdot d\omega\right) = \left(dt \cdot dE\right) = \boxed{\left(dx \cdot dp\right)} = \frac{h}{2\pi}\left(dx \cdot dk\right)\right] / dt =$$
$$= \left[h \cdot df = \frac{h}{2\pi} \cdot d\omega = dE = d\tilde{E} = v \cdot dp = dE_k = \frac{h}{2\pi} v \cdot dk = c^2 dm\right]$$

To be more general, we could safely consider that stabilized matter states (like stable particles and atoms) should qualitatively and conceptually present matter waveforms or wave-packets with certain (total) motional or wave energy content equal to $\tilde{E} = E_k$, with certain resulting momentum equal to $\tilde{P} = P >>> \Delta p$, with the total spatial length $L >>> \Delta x$, with the total temporal duration $T >>> \Delta t$, and be presentable with the following domains, and Nyquist-Shannon-Kotelnikov sampling Δ-intervals relations,

$$h(\Delta t \cdot \Delta f)_{min.} = (\Delta t \cdot \Delta E)_{min.} = \boxed{(\Delta x \cdot \Delta p)_{min.}} = hTF_t = \boxed{hLF_x} = \boxed{T \cdot \tilde{E} = L \cdot \tilde{P}} = \text{constant} \Rightarrow$$

$$\begin{cases} (\Delta t \cdot \Delta f)_{min.} = TF_t = LF_x = \overline{\text{constant}}, f = f_t = \omega / 2\pi, \rightarrow \boxed{L = \frac{F_t}{F_x} \cdot T \cong V^* \cdot T,\ V^* = \text{const.}} \\ (\Delta t \cdot \Delta E)_{min.} = (\Delta x \cdot \Delta p)_{min.} = \underline{\text{constant}}, \end{cases}$$

$$\begin{cases} (\Delta E)_{min.} = \left(\frac{\Delta x}{\Delta t} \cdot \Delta p\right)_{min.} = \bar{v} \cdot \Delta p \Leftrightarrow dE = v \cdot dp, \bar{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta E}{\Delta p} = \frac{\Delta \omega}{\Delta k} \Leftrightarrow v = \frac{dx}{dt} = \frac{dE}{dp} = \frac{d\omega}{dk} \\ (\Delta E)_{min.} = \left(\frac{\Delta x}{\Delta t} \cdot \Delta p\right)_{min.} = (\Delta x \cdot \Delta p)_{min.} \cdot \frac{1}{\Delta t} (=) h \cdot f \Rightarrow dE = hdf = vdp, \\ \left(\frac{\Delta E}{\Delta t}\right)_{min.} = \left(\frac{\Delta x}{\Delta t} \cdot \frac{\Delta p}{\Delta t}\right)_{min.} = \bar{v} \cdot \frac{\Delta p}{\Delta t} \Leftrightarrow \frac{dE}{dt} = v \cdot \frac{dp}{dt} = v \cdot F (=) \text{Power, where } F (=) \text{force}, \\ F = \frac{\Delta E}{\Delta x} = \frac{\Delta p}{\Delta t} = \nabla E = \dot{p} = m^* \dot{v} \ \begin{pmatrix} \text{analog to torque} \\ \Leftrightarrow \end{pmatrix} \tau = \frac{\Delta E}{\Delta \theta} = \frac{dL}{dt} = \dot{L} = J\dot{\omega} \end{cases} \quad (10.2\text{-}2.2)$$

Respecting mobility-type electromechanical analogies (see Chapter 1.), velocity v corresponds to certain voltage $u = \frac{\Delta \Phi}{\Delta t}$, and force $F = \frac{\Delta p}{\Delta t}$ (or torque $\tau = \frac{\Delta E}{\Delta \theta} = \frac{dL}{dt}$) corresponds to certain electric current $i = \frac{\Delta Q}{\Delta t}$, and we could qualitatively, dimensionally (and when reasonable, or as a brainstorming and indicative exercise), extend (10.2.2) to,

$$\text{Power } (=) \frac{\Delta E}{\Delta t} = \bar{v} \cdot \frac{\Delta p}{\Delta t} \Leftrightarrow \frac{dE}{dt} = v \cdot \frac{dp}{dt} = v \cdot F = \omega \cdot \tau (=) u \cdot i = \frac{\Delta \Phi}{\Delta t} \cdot \frac{\Delta Q}{\Delta t} \Rightarrow$$
$$\Rightarrow \Delta E = v \cdot \Delta p = \omega \cdot \Delta J = \frac{\Delta \Phi \cdot \Delta Q}{\Delta t}, \quad (10.2\text{-}2.3)$$

where Φ and Q are relevant magnetic flux and involved electric charge (when applicable). Here we should not forget that only Gaussian (or similar) amplitude shaped matter-wave packets (or energy-moments states) are well localized in all their original and spectral domains. *Since temporal, spatial and relevant spectral domains (of all motional states and wave functions) are mutually linked and proportional, it is reasonable to speak about temporal and spatial periodicity and temporal and spatial resonant states. One of special and important resonant states is* ***combined temporal-spatial, resonant, and standing waves situation, when, at the same time, mutually synchronous, temporal, and spatial resonance happens****. Such standing-waves' states can create stable particles with non-zero rest masses in cases when being self-closed. If mentioned standing matter-wave structure is in a form of open-ends waves (meaning not self-closed), such formations are becoming emitters of matter waves. Nodal and antinode zones of standing waves are locally manifesting effects of attractive and repulsive forces because this way gradients of vibrational energy and masses density are being created towards nodal and/or antinode zones.*

This could be an alternative explanation for natural forces we know in the world of Physics.

If any of delta-intervals, or total duration values from (5.2), (10.2-2) - (10.2-2.4), starts being on some way oscillatory-modulated, perturbated or vibrated, all other delta-intervals (or signal domain durations) will immediately and synchronously create matter waves, vibrate, resonate, and new vibrating spots can be created based on object's total spectral (both temporal and spatial) complexity. For instance,

$$\left\{\begin{array}{l}(\Delta x \text{ or } \Delta t) \Rightarrow [(\Delta x \pm \delta x) \text{ or } (\Delta t \pm \delta t)] \\ h(\Delta t \cdot \Delta f)_{min.} = (\Delta t \cdot \Delta E)_{min.} = \boxed{(\Delta x \cdot \Delta p)_{min.}} = hTF_t = \boxed{hLF_x} = \boxed{T \cdot \tilde{E} = L \cdot \tilde{P}} = const\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} h[(\Delta t \pm \delta t)\cdot(\Delta f \mp \delta f)]_{min.} = [(\Delta t \pm \delta t)\cdot(\Delta E \mp \delta E)]_{min.} = \\ = \boxed{[(\Delta x \pm \delta x)\cdot(\Delta p \mp \delta p)]_{min.}} = h(T \pm \delta T)\cdot(F_t \mp \delta F) = \boxed{h(L \pm \delta L)\cdot(F_x \mp \delta F_x)} = \\ = \boxed{(T \pm \delta T)\cdot(\tilde{E} \mp \delta E) = (L \pm \delta L)\cdot(\tilde{P} \mp \delta \tilde{P})} = const \end{array}\right\} \Rightarrow \quad (10.2\text{-}2.4)$$

$$\Rightarrow h(\delta t \cdot \delta f)_{min.} = (\delta t \cdot \delta E)_{min.} = \boxed{(\delta x \cdot \delta p)_{min.}} = hTF_t = \boxed{hLF_x} = \boxed{T \cdot \tilde{E} = L \cdot \tilde{P}} = const$$

We can also extend on a similar way the same Uncertainty or Certainty perturbative and oscillatory relations, considering total (non-statistical, or absolute) associated electromagnetic charges and angular moments and displacements, as already presented in Chapter 5., under (5.2) and later,

$$\Delta q_{mag.} \cdot \Delta q_{el.} = \Delta\alpha \cdot \Delta L = h \cdot \Delta t \cdot \Delta f = \Delta x \cdot \Delta p = \Delta t \cdot \Delta\tilde{E} = c^2 \Delta t \cdot \Delta m \geq h/2. \quad (5.2)$$

In engineering practice, we have such effects of significant mutual and bidirectional energy transformations in relation to piezoelectric and magnetostrictive effects. Oscillations, *perturbations,* or absolute durations in any of domains from (10.2-2.4) and (5.2) should also produce EMP (electromagnetic pulse) effects, like in cases of nuclear explosions.

Citation taken from: https://www.businessinsider.fr/us/nukes-electromagnetic-pulse-electronics-2017-5 "Nuclear blasts trigger an effect called electromagnetic pulse, or EMP. EMP can disrupt or even destroy electronics from miles away. Blasts miles above a country like the US might severely damage its electric and telecommunications infrastructure.
A nuclear detonation creates plenty of terrifying effects, including a blinding (and burning) flash of light, a building-toppling blast wave, an incendiary fireball, and radioactive fallout that can drift for hundreds of miles.
But there's a lesser-known consequence of a nuclear explosion that can drastically expand its damage zone: an electromagnetic pulse, or EMP. EMPs are rapid, invisible bursts of electromagnetic energy. They occur in nature, most frequently during lightning strikes, and can disrupt or destroy nearby electronics. However, nuclear EMPs — if a detonation is large enough and high enough — can cover an entire continent and cripple tiny circuits inside modern electronics on a massive scale, according to US government reports. The power grid, phone and internet lines, and other infrastructure that uses metal may also be prone to the effects, which resemble those of a devastating geomagnetic storm ".

This way analyzing, based on (10.2-2.4) and (5.2), with small spatial, mechanical, electromechanical and/or electromagnetic vibrations of certain signal-domain-duration, in some cases of matter states, we should be able to produce stronger oscillations of all other mutually related and conjugate domains' *durations, or we can produce significant displacements, forces and/or mechanical and electromagnetic moments, and create different matter waves (like in cases of electrostrictive and magnetostrictive phenomenology). This can be additionally and analogically compared, understood, or arranged like what we realize in mechanics with a lever or a hydraulic press (serving as mechanical force or amplitude amplifiers or transformers). For instance, specific spatial configurations of macro elements composed of big masses with a polycrystalline structure, being in mutual vicinity, when conveniently vibrated or excited (mechanically, electromechanically, or by cosmic rays, thermal and seismic vibrations, electromagnetic discharges, or other planetary oscillations and perturbations …) could produce secondary matter-waves with measurable mechanical and electromagnetic manifestations. Russian professor Viktor Stepanovich Grebennikov, [149], described number of effects familiar to gravitation, antigravitation, and temporal-spatial matter-waves phenomenology of geometric, periodicities-related structures created by insects, as natural beehive cells or dry honeycomb structures, or objects artificially assembled on a similar way. Other similar examples relate to coincidental, continu**ous mechanical, ultrasonic and electromagnetic radiation or waves (with specific, objects-geometry-related frequencies) measured around big pyramids (see [141], ELECTROMAGNETIC PROPERTIES OF PYRAMIDS FROM POSITIONS OF PHOTONICS, and [142], Hrvoje Zujić, Electromagnetic mechanism of the ultrasound on the Bosnian Pyramid of the Sun (Visoćica Hill). In other words, natural radiative electromagnetic, mechanical, and acoustic field emissions of atoms and other masses (see more in Chapter 8.), and an excitation of specific geometric, crystalline, or spatial matter forms with structural spatial periodicity (or resonators) could produce (or amplify) dynamic effects of matter waves radiation followed with creation of active mechanical and electromagnetic oscillations, moments, ionized atoms, and oscillating field charges (and vice-versa). On the same way, certain excitation, or vibration (of solid matter structures, including planets and galaxies) in a temporal domain, will produce corresponding "energy-moments-mass-frequency effects", or different matter waves emissions and vice-versa. For instance, Schumann resonances and related mechanical, acoustic, and electromagnetic oscillations are good examples for such phenomenology.***

*Citation from Wikipedia, https://en.wikipedia.org/wiki/Schumann_resonances. The **Schumann resonances** (**SR**) are a set of spectrum peaks in the extremely low frequency (ELF) portion of the Earth's electromagnetic field spectrum. Schumann resonances are global electromagnetic resonances, generated and excited by lightning discharges in the cavity formed by the Earth's surface and the ionosphere.[1]*

*Here are the reasons and explanations why we detect such effects around different spatial forms of micro and macro masses, and other resonators, such as spatial crystalline forms, pyramids, high mountains ... One of good examples of such mutually linked and coupled spatial-temporal, electromagnetic and acoustic domains-durations, perturbations, and matter waves emissions is also known as the old-fashion **Crystal Radio Receiver** or detector.*

See more about Crystal Radio Receivers here https://en.wikipedia.org/wiki/Crystal_radio.
Citation from Wikipedia.

………………….. Citation: "A **crystal radio receiver**, also called a **crystal set**, is a simple radio receiver, popular in the early days of radio. It uses only the power of the received radio signal to produce sound, needing no external power. It is named for its most important component, a crystal detector, originally made from a piece of crystalline mineral such as galena.[1] This component is now called a diode.

…………..

Crystal radios receive amplitude modulated (AM) signals, and can be designed to receive almost any radio frequency band, but most receive the AM broadcast band.[20] A few receive shortwave bands, but strong signals are required. The first crystal sets received wireless telegraphy signals broadcast by spark-gap transmitters at frequencies as low as 20 kHz.[21][22]

As a lesser known feat, a different design of crystal radios can also been constructed specifically for receiving frequency modulated (FM) VHF broadcast signals.[23]

……………………………

Crystodyne

In early 1920s Russia, Oleg Losev was experimenting with applying voltage biases to various kinds of crystals for manufacture of radio detectors. The result was astonishing: with a zincite (zinc oxide) crystal he gained amplification.[33][34][35] This was negative resistance phenomenon, decades before the development of the tunnel diode. After the first experiments, Losev built regenerative and superheterodyne receivers, and even transmitters.

A crystodyne could be produced in primitive conditions; it can be made in a rural forge, unlike vacuum tubes and modern semiconductor devices. However, this discovery was not supported by authorities and soon forgotten; no device was produced in mass quantity beyond a few examples for research.

………………….

The earliest crystal receivers did not have a tuned circuit at all, and just consisted of a crystal detector connected between the antenna and ground, with an earphone across it.[1][68] Since this circuit lacked any frequency-selective elements besides the broad resonance of the antenna, it had little ability to reject unwanted stations, so all stations within a wide band of frequencies were heard in the earphone[50] (in practice the most powerful usually drowns out the others). It was used in the earliest days of radio, when only one or two stations were within a crystal set's limited range". End of citation.

……………………..

[♣ Free thinking, brainstorming zone. ***To get out of the box (which is our Universe)****, we could hypothetically imagine that our Universe is like a big electromechanical system, or machine, automatically controlled by certain powerful computer and software. Such software has its complex algorithm, which we could recognize thanks to what we learned about natural Conservation laws, Analogies (as described in the first Chapter of this book), recognizable patterns we see around us, and thanks to methods of sophisticated Mathematics we already have. Our Universe as a computer (just described) should have its clock, or carrier frequency, synchronized with all other of its modules, functional blocs, processors, and operations existing within our Universe. Its algorithm should be something fixed or finite (already made and stable), like repeating typical patterns and predefined procedures, meaning that with convenient numerical operations (after investing sufficient intellectual and mathematical efforts) we could develop, extract, axiomatically postulate, or calculate all kind of universal Constants known in Physics, including masses of planets, galaxies, atoms, and elementary particles… Of course, this is still kind of Metaphysics, or a Science-fiction assumption, but it is worth our attention and further investigations.*

There are several empirically or otherwise known patterns describing ***how natural matter structures, or masses composed of atoms and molecules, are being created***, organized, growing, mutually communicating, and propagating, such as:

1. Growing as crystalline structures,
2. Expanding as fractals,
3. Following Fibonacci numbers and "Golden Ratio" proportions for structural expansion (see more here: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#section2),
4. Evolving towards respecting certain spatial symmetry and periodicity,
5. Being internally structured as self-closed, spatial, standing matter-waves, resonant formations, and mutually synchronizing externally,
6. Being combined or composite structures based on already mentioned patterns (from 1. to 5.),
7. Complying with number of Electromechanical Analogies and Symmetries.
8. Being mutually connected by matter-created fields, and by *matter-waves communicating*, while respecting Classical differential wave equation.

Of course, all mentioned structural packing and expanding macro matter manifestations (from 1. to 8.) originate from atoms, and as we know, all atoms (including subatomic entities) are internally presenting sets or ensembles of self-closed standing mater-waves resonators (being mutually synchronized). Atoms are already organized and systematized within the Periodic Table of Elements as structures and entities with number of intrinsic periodicities and analogies. This is advancing the conclusion that microstructures composed of atoms are on some way directly and synchronously, *real-time communicating* or being (bidirectionally and omni-directionally) connected with other macro structures also composed of atoms (what is important to exist in the background, to keep structural matter properties stable, as they naturally are; -see more of similar ideas in Chapter 8.). In simplified terms, mechanical objects are limiting, boundary and asymptotic cases of "*matter-waves energy packing*", or most probably of electromagnetic energy packing. Now we could search what such synchronization and communication-means between masses and atoms are, by extending our elaborations towards wave-particle duality, creation of matter waves, entanglement effects, and *resonant spectral domains coupling*. Known natural forces and fields are causally linked to the same problematic, and it is evident that everything what we address in Physics starts from atoms. Generally valid rules and mathematical models applicable here are all Conservation laws, and Spectral Analysis and Synthesis theory, based on J. Fourier and D. Gabor Analytic signal theory and concepts (see relevant achievements from J. Fourier, D. Gabor, Kotelnikov, Shannon, Nyquist, and Whittaker from the following references: [8], [57], [58], [59], [79], [109], [110], and [111]). *Quantizing in Physics is simply related to discretized and countable "matter-wave-packets" or "wave groups exchanges" between stabilized (limited size) matter domains and structures like atoms and other matter states with masses.* Stabilized matter domains (or masses) are always created from self-closed standing matter waves, being kind of spatial resonant structures. Such structures are mutually communicating by exchanging (emitting and/or receiving) different matter-wave packets with quantized (or discretized) energy amounts (like photons). Probability and Statistics here are simply good mathematical tools and well-organized presentation methods, when conditions for such mathematical processing exist.

Sufficiently familiar approach, or **hypothetical concept about our Universe being a kind of natural supercomputer** (or an automate with its axiomatic rules, or following an algorithm with fixed structure) is also addressed as Samkhya. *Samkhyayoga or Sankhya is an ancient axiomatic, natural, Vedic philosophy addressing AXIOMATIC PRINCIPLES IN RELATION TO PHYSICS: It unifies the two contentious concepts of materiality and spirituality by demonstrating through precise mathematics that all manifestation is a hologram or the very embodiment of spirituality. Samkhya has an extraordinary scientific content (most probably created about 30'000 to 40'000 years ago). In our cultural and historical heritage, it is described within about 70 verses, addressing axiomatically our Universe, Mathematics and Physics knowledge.* Read the following exiting citations from [148]:

"The confidence, that this theory is precise and correct, comes only from the fact that its numerical parameters match those of physics with a better-than-acceptable order of accuracy*. These comparative numbers are shown later in a tabulated form.*

....

The holographic mode of manifestation is proved mathematically by showing that all phenomenon is bound simultaneously by a spectrum of seven states and released sequentially by one mode. The enigma in science, why sound, light, particulate, molecular, atomic, nuclear, and sub-particle level have a periodicity spectrum, is resolved axiomatically. The above parameters are some of the 'easy to understand' aspects that differed from physics. ***The mathematical aspects of this theory are all encompassing, profound and complete in all respects. The unified solutions are derived internally and matched accurately to provide numerical answers to every known and unknown stable parameter in physics and cosmology. It has its own system of internal proof by matching six alternate derivations to 25 decimal places.***

Sankhya enables the tabulation of the entire cosmic manifestation parameters similar to any mathematical log table or almanac, with tlie certainty there will be no phenomena found falling outside it. This aspect is not possible in science today. How do we know Sankhya is right? Differentiating the Sankhya-derived-mass of the universe by its smallest displacement leaves a precise single unit-angular displacement value of the very first interaction. Such accuracy is possible only in the realm of the divine! Sankhya also gives equally accurate numerical solutions to both scientific and holistic problems in phenomenon. The latter process by itself is an extraordinary confirmation of Sankhyan supremacy, for science has deliberately closed its eyes to it and believes holistic perception does not exist!

An outstanding feature of Sankhya is that no measured or empirical inputs are required, and the axiomatic theory starts by manipulating the interactions between two objects in various ways. Explaining briefly, Sankhya is based on counting only oscillatory interactions as a ratio of a standard & axiomatic cycle of 10 counts which are dimensionless, scale-invariant, coherent, synchronous, reflection invariant and symmetric. Though space has substantial qualities identified as the Purusha State, it cancels out, as all measurements are relative comparisons through its smallest unit, the moolaprakriti. So the Purusha's basic qualities are not mathematically relevant in defining phenomenon. Any Sankhya equation is always the algebraic sum of three gunas as tama (strong force), raja (weak force and gravity) and satwa (electromagnetic force) or a ratio of tama/(raja into satwa.). Hence, all equations compare only three real dimensions. There are three cyclic states to define time and are governed by three principles, simultaneity, self- similarity and relativity and these have scalar (full force), tensor (stress dependent force) and vector (time dependent force) characteristics respectively. All of space is always in a dynamic oscillatory state, at an axiomatic rate of 296575967 oscillations per cycle of 10 oscillations or 299792458 oscillations at a meter wavelength/second, which equals the velocity of light in vacuum. The extraordinary fallout from deriving the holographic oscillatory state is that it corrects velocity of light in physics relativistically by the solar orbital v0elocity in the galaxy by the factor 1.010845. Michelson & Morley detected this corrected value, but no one realized that it was relevant and though the experiments failed, their results displayed a doppler blue shift in frequency. Hence, the frequency of light in the solar system cannot be constant. ....” ♣]

10.02 MEANING OF NATURAL FORCES

Now, (based on (10.2-2.2) and (10.2-2.3)), it is possible to elaborate the wider and mutually analogical meaning of linear force $F=\frac{\Delta E}{\Delta x}=\frac{\Delta p}{\Delta t}=\frac{dp}{dt}$, and angular force or torque $\tau=\frac{\Delta E}{\Delta\theta}=\frac{dL}{dt}$, and to create united concept of natural forces.

1. Any kind of force (or current of certain field charges, or moment entities) is analogically presentable as the first temporal derivation of involved linear $F=\frac{dp}{dt}$, angular $\tau=\frac{dL}{dt}$, or electromagnetic moment, and/or corresponding field charges (see more about ***force-current*** analogies in the first chapter).

2. Any kind of force and torque is also (analogically and dimensionally) presentable as the spatial gradient of relevant *motional energy distribution*, such as, $F=\frac{dE}{dx}(=)\nabla_{(x,y,z)}E$, $\tau=\frac{dE}{d\theta}(=)\nabla_{(\theta...)}E$. Energy here is effectively kinetic energy, since $dE_{tot.}=dE=dE_k$.

3. Of course, based on 1. and 2. we can easily develop less general, traditional, Newtonian force definition as a mass multiplied by corresponding acceleration, $F=m\cdot\dot{v}=m\cdot a=dp/dt$, $\tau=J\cdot\dot{\omega}=J\cdot\alpha=dL/dt$.

4. In cases of mechanically-spinning objects like gyroscopes (with zero initial linear moment), we need to consider (still hypothetically) that such spinning (if accelerated, having a torque $\vec{\tau}=\frac{d\vec{L}}{dt}$) could also generate certain linear (or axial) force-thrust $\vec{F}=\frac{d\vec{p}}{dt}$, and vice versa (what is also analog to an electromagnetic photon situation, as summarized in Chapter 4.1, and this also supports the conceptual meaning of wave-particle duality, and presents a part of explanation of Gravitation, as elaborated in Chapters 2. and 8.), as follows,

$$\boxed{\vec{L}\vec{\omega}=\vec{p}\vec{v}}\Rightarrow d\vec{L}\cdot\vec{\omega}+\vec{L}\cdot d\vec{\omega}=\vec{p}\cdot d\vec{v}+\vec{v}\cdot d\vec{p}\Leftrightarrow\frac{d\vec{L}}{dt}\cdot\vec{\omega}+\vec{L}\cdot\frac{d\vec{\omega}}{dt}=\vec{p}\cdot\frac{d\vec{v}}{dt}+\vec{v}\cdot\frac{d\vec{p}}{dt}\Leftrightarrow$$

$$\boxed{\begin{array}{l}\vec{\tau}\cdot\vec{\omega}+\vec{L}\cdot\vec{\alpha}=\vec{p}\cdot\vec{a}+\vec{v}\cdot\vec{F}\Rightarrow\vec{\omega}\left(\vec{\tau}+\frac{L}{\omega}\cdot\vec{\alpha}\right)=\vec{v}\left(\frac{p}{v}\cdot\vec{a}+\vec{F}\right)\Leftrightarrow\vec{\omega}\left(\vec{\tau}+J^{*}\cdot\vec{\alpha}\right)=\vec{v}\left(m^{*}\cdot\vec{a}+\vec{F}\right)\\ \Rightarrow\omega\tau=vF=v\frac{dp}{dt}=\omega\frac{dL}{dt},\ L\cdot\alpha=p\cdot a,\ \vec{\tau}=\frac{d\vec{L}}{dt},\ \vec{F}=\frac{d\vec{p}}{dt},\vec{\alpha}=\frac{d\vec{\omega}}{dt},\ \vec{a}=\frac{d\vec{v}}{dt},\end{array}}$$

where $\vec{\alpha}=\frac{d\vec{\omega}}{dt}$, $\vec{a}=\frac{d\vec{v}}{dt}$ are angular and linear accelerations of the same spinning body. Here we should not forget that *magnetic field effects* will follow spinning of masses, explaining some of unusual and extraordinary behaviors of gyroscopic motions of interatomic and planetary formations.

5. We also know that force manifestations are ***related to standing-waves field structures*** *(like in cases of ultrasonic or acoustic levitation)*. For instance, attractive forces are detectable in the vicinity of nodal zones (minimal or zero amplitude

oscillations) of standing waves. Repulsive forces are detectable in the vicinity of zones with maximal amplitudes. This is equally valid for mechanical and electromagnetic oscillations and waves. *High Power Mechanical, Ultrasonic or acoustic energy, moments, forces, oscillations and vibrations, or audio signals and music, can be created and transferred by applying different signal-modulating techniques on laser (or other) beams and dynamic plasma states, using laser and plasma states as carriers for lower frequency mechanical vibrations (or signals); - See relations under (10.2-2.4) and literature references from [133] until [139], and under [140] "European Patent Application (related to MMM technology), EP 1 238 715 A1". We can also create mechanical (or ultrasonic) vibrations in solid metal wires and beams, analogically to how electric and magnetic fields or photons are propagating, being transversal or orthogonal to a direction of propagation along metal wires (being flexural or bending oscillations). Such photons-propagation analog acoustic waves can propagate as electric or acoustic currents along any geometry and shape of perpendicularly oscillating metal wires (such as in coils, solenoidal forms, zig-zag shapes etc.) and transfer significant acoustic or mechanical vibrations energy on a long distance, regardless of wires length, and almost without any significant attenuation.*

In fact, standing waves are creating different accelerations around nodal and anti-nodal zones, and naturally creating energy and mass gradients, or "temporal *moments' derivations"* towards mentioned nodal zones, meaning that conditions 1. and 2. regarding basic force definition are, this way, created and satisfied (thanks to spatial-temporal coupling and symmetry). Effects of acoustic and ultrasonic levitation in fluids are demonstrating mentioned forces. Between atoms and masses in our macro-Universe, we also have effects of specific *standing-waves structured* natural force effects (which are an extension of internal atomic fields and forces towards external atoms and masses) that are creating effects of Gravitation (see more in Chapter 8., under "8.3. Structure of the Field of Subatomic and Gravitation related Forces".

6. Another meaning of natural forces is the attraction and repulsion between equally charged or opposite field charges and moments (valid both for mechanical and electromagnetic, charges and moments). Here we can place Newtonian and Coulomb $1/r^2$ forces. See much more about electromechanical analogies in the first chapter of this book. For instance, for gravitation $1/r^2$ related fields and forces the most important should be (on some way involved) linear moment p and angular moment $\mathbf{L}$. Surprisingly, we do not find an analogical indication that a static mass m should be the primary and unique source of gravitation, but we have indications that on some way intrinsically vibrating mass should be important for creating effects of Gravitation. Practical predictive or indicative meaning of such analogical revelations is that I. Newton and A. Einstein theories about Gravitation should be one day significantly updated. For instance, we know that cohesion, adhesion, and Van der Vaals forces are essentially based on electromagnetic field effects, but empirical manifestations of such forces for a "macro world observer" could look like typically mechanical attractive forces. The other brainstorming insight here is that electric charges (or electrons and protons) should be kind of dynamic, motional energy states (like mechanical and electromagnetic moments), since all other mutually analogical mechanical and electromagnetic moments and flux entities are also kind of moving or dynamic states (see T.1.3 in the first chapter of this book). As we know, in a contemporary Physics we still wrongly consider

electric charges as stable, or fixed and static parameters (meaning as number of Coulombs). Consequently, we could expect certain continuous electromagnetic energy exchange (like N. Tesla Radiant energy flow, within an ether fluid) between any of positive and negative electric charges, or between conveniently aligned electromagnetic dipoles, this way creating necessary background for explanation of Gravitation.

Natural forces may also be deeply and ontologically related to always valid Uncertainty Relations, both in micro and macro cosmos (see (10.2-2) - (10.2-2.2) and (10.2-2.3)). By an intellectual rigidity and inertia, we usually stick to the historical, Newtonian force definition (as the product between certain mass and its acceleration), but as we can see, much better and more general is to start from spatial and temporal energy and moments gradients or derivations. This (under 1., 2., 3., 4., 5., and 6.) will be much better starting platform or conceptual framework to explain Gravitation and other natural forces (including nuclear forces), instead of making oversimplified and unverifiable assumptions, axiomatic statements, or just giving some specific names to still unknown and "***expected-to-exist***" forces. Until here (from 1. to 6.) we addressed only the simplest forms of linear, angular, central, and oscillations-and standing-waves related forces, giving more attention to analogical, dimensional, and hypothetical aspects related to understanding natural forces, not considering that in all such *forces related situations* we often have potential fields and energies (what will additionally enrich and extend natural forces complexity). Such, much more general or universal forces definition and mathematical processing (considering potential energies) is based on Lagrangian and Hamiltonian mechanics.

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Citation from: ***https://www.sciencedaily.com/releases/2014/12/141219085153.htm, literature reference [107].***

"Patrick Coles, Jedrzej Kaniewski, and Stephanie Wehner made the breakthrough while at the Centre for Quantum Technologies at the National University of Singapore. They found that 'wave-particle duality' is simply the quantum 'uncertainty principle' in disguise, reducing two mysteries to one.

Wave-particle duality is the idea that a quantum object can behave like a wave, but that the wave behavior disappears if you try to locate the object. It is most simply seen in a double slit experiment, where single particles, electrons, say, are fired one by one at a screen containing two narrow slits. The particles pile up behind the slits not in two heaps as classical objects would, but in a stripy pattern like you'd expect for waves interfering. At least this is what happens until you sneak a look at which slit a particle goes through -- do that and the interference pattern vanishes.

The quantum uncertainty principle is the idea that it is impossible to know certain pairs of things about a quantum particle at once. For example, the more precisely you know the position of an atom, the less precisely you can know the speed with which it is moving. It is a limit on the fundamental knowability of nature, not a statement on measurement skill. The new work shows that how much you can learn about the wave versus the particle behavior of a system is constrained in exactly the same way.

Wave-particle duality and uncertainty have been fundamental concepts in quantum physics since the early 1900s.

In earlier papers, Wehner and collaborators found connections between the uncertainty principle and other physics, namely quantum 'non-locality' and the second law of thermodynamics. The tantalizing next goal for the researchers is to think about how these pieces fit together and what bigger picture that paints of how nature is constructed".

From (10.2-2.1), (10.2-2.2) and (10.2-2.3), we can, on some way philosophically and conceptually, conclude that stable, self-standing and sustainable or long-lasting recognizable (motional) matter states (or relevant wave packets) have certain predictable, micro and macro domains unity and flexibility, summarized as:

a) If the wave packet in question will experience certain temporal modulation or perturbation during the time-duration $(\Delta t, T)$, this will produce corresponding energy perturbation $(\Delta E, \tilde{E} = E_k)$ and vice-versa, keeping their product stable or constant, like $\Delta t \cdot \Delta E = T \cdot \tilde{E} = const.$. Similar is valid for the product of relevant total domains durations, such as $\left(T \cdot F_t,\ L \cdot F_x,\ L \cdot \tilde{P}\right) \cong const. \cdots$

b) If certain spatial modulation or perturbation Δx will happen, this will produce predictable momentum perturbation Δp and vice-versa, keeping their product stable or constant, $\Delta x \cdot \Delta p = L \cdot P = Const.$. Similar is valid for the product of relevant total intervals durations L and F_x. Temporal and spatial total lengths (or dimensions) of certain stable and motional matter state are anyway mutually dependent and strongly or causally linked (like $L \cong V^* \cdot T,\ V^* = const.$), what is also established on a different way in A. Einstein Relativity theory. Let us explore relation between total signal durations or lengths of mutually conjugate spatial and temporal domains of certain signal or wave packet, $T \cdot \tilde{E} = L \cdot \tilde{P}$. For having stable moving objects, signals, and wave packets, we know that condition of stability is to have direct proportionality between relevant temporal and spatial durations (such as $L = const \cdot T$). When we make signals' sampling and signals reconstruction, based on Kotelnikov-Nyquist-Shannon theory, we know that selected (and maximal) sampling intervals Δt and Δx should be,

$$\begin{pmatrix} \Delta t = \frac{1}{2f_{t-max}} = \frac{1}{2F_t},\ \Delta x = \frac{1}{2f_{x-max}} = \frac{1}{2F_x} \\ T = N \cdot \Delta t,\ L = N \cdot \Delta x,\ N\ (=)\ integer \end{pmatrix} \Rightarrow \begin{pmatrix} T \cdot \tilde{E} = L \cdot \tilde{P} \Leftrightarrow \Delta t \cdot \tilde{E} = \Delta x \cdot \tilde{P} \Rightarrow \\ V^* = \frac{L}{T} = \frac{\Delta x}{\Delta t} = \frac{\tilde{E}}{\tilde{P}} = \frac{F_t}{F_x} = u = const. \end{pmatrix}.$$

This is supporting constant phase speed $V^* = u = const.$ of all elements of the stable wave packet in question. If $u = const. = C\,(=)\ \text{speed of light}$, this is on some way implicating involvement of an electromagnetic nature in the background. In fact, here we could imaginatively establish that "***characteristic, proper-time duration***" T of a moving particle (that has its wave packet replacement with spatial length L, and the same temporal duration T), can be uniquely defined as, $T = L / C,\ L = C \cdot T,\ C = const.$. The same idea is later expanding towards 4-dimensional spatial-temporal basis, such as, (x, y, z, iCt), $i^2 = -1$.

c) In cases of combined and accelerated motions, if temporal $(\Delta t, T)$ and spatial durations $(\Delta x, L)$ would be affected synchronously, then all other ΔE, Δp, T, F_t, L, and F_x would synchronously experience directly proportional perturbations (and vice-versa).

d) We could extend similar thinking and conclusions in relation to involved masses, electric charges and magnetic fluxes, or electric voltages and currents (when applicable and reasonable). We can also extend similar elaborations to orbital, rotating and spinning motions and associated angular moments (see supporting literature sources under [36], Anthony D. Osborne, Department of Mathematics, Keele University, Keele, Staffordshire ST5 5BG, UNITED KINGDOM & N. Vivian Pope, "Llys Alaw", 10 West End, Penclawdd, Swansea, West Glamorgan SA4 3YX, UNITED KINGDOM). It looks that all, angular, orbital, spinning, circular, moments, and motions in our Universe have a big (still hypothetical) chance to be mutually connected by specific immediate, synchronous cosmic and holistic links (acting almost immediately, regardless of cosmic distances between such entities). That means (if correct and verifiable), that in specifically favorable conditions we could profit from such "*energy-angular-moments, holistic connections"*. This is like having convenient (direct and immediate) connection with countless number of spinning, rotating and orbiting elements, masses or spinning Flywheel energy storage (such as atoms, solar systems, galaxies...), and maybe benefit on some way from such options (in creating new devices for communications and energy draining or harvesting). In other words, all mechanical, electromagnetic, and other perturbations of any matter state are, synchronously and mutually connected and interrelated on a cosmic level. ***The Universe is ringing, oscillating, and responding on every single temporal, spatial, electromagnetic, mechanical, and other perturbation***. The matter around us is always united and presentable combining different aspects of electromagnetic, electromechanical, mechanical, spatial, and temporal manifestations. Because of such unity, we can create number of Analogies as elaborated in the first chapter of this book.

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*We know (from Fourier analysis) that we can decompose any wave group on number of elementary, simple-harmonic sinusoidal waves or functions, but signal analysis and synthesis "**applied by our Universe**" is using somewhat different and optimized elementary and harmonic signal-basis functions, effectively realizing signals' atomizing and quantizing, when different particles and wave-groups states are being created and when mutually communicating. Here, we intend to find or formulate the optimal and natural signal shape of an elementary wave packet, or wave energy quant, by respecting the Analytic Signal concept (to use it as a basis of elementary waves for signals analysis, quantizing, decomposition, reconstruction, qualification, quantification and different presentations in a time and frequency domain). If such elementary wave-packets are naturally and conveniently combined, self-stabilized and captured in the certain form of standing waves, as self-closed structures, we can get particles (or atoms) with stable rest masses. **If we would like to visualize such situations, this will look much more as finite elements analysis of resonant states of structures under stationary vibrations, than to oversimplified quantizing concepts as used in the contemporary Quantum theory.***

Based on wave energy expressions (5.14) and on "Kotelnikov-Shannon, Whittaker-Nyquist Sampling and Signals Recovering Concepts" (see references in [57, 58, and 59], and in Chapter 4.0, under equations (4.0.34, (4.0.5-2), (4.0.35)) all relevant parameters of an effective, elementary band-limited wave-packet or signal (or one-quantum of wave energy, such as photon), can be formulated as,

$$\Psi(x,t) = a\sum_{(i)}\cos(\omega_i t - k_i x) = a \cdot R_e\left[\int_{\left(k_0-\frac{\Delta k}{2}\right)}^{\left(k_0+\frac{\Delta k}{2}\right)} e^{j(\omega t - kx)}dk\right] =$$

$$= a(x)\frac{\sin(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}\cos(\omega t - kx)\ (\Leftrightarrow)\ \text{wave packet},$$
$$\omega = 2\pi f\ (=)\ \text{carrier frequency}\ (>>>\underline{\Delta\omega}), \qquad (10.2\text{-}3)$$
$$\underline{\Delta\omega} = 2\pi\underline{\Delta f}\ (=)\ \text{wave-packet frequency bandwidth}\ (<<< f),$$
$$v = d\omega/dk = \underline{\Delta\omega} / \underline{\Delta k},\ u = \omega / k.$$

If $\Psi(x,t)$ *is an elementary narrow-frequency-band, like Gaussian Wave Packet (which has a mean frequency* $\overline{f}$*), and presents an energy quant, we should also consider that its energy is,*

$$\boxed{\begin{matrix}\tilde{E} = E_k = h\overline{f} \\ (h = \text{const.})\end{matrix}} = \int_{[\Delta t]}\Psi^2(t)dt = \frac{1}{2}\int_{[\Delta t]}a^2(t)dt = \frac{1}{\pi}\int_0^{+\infty}|A(\omega)|^2 d\omega = \int_{[\Delta t,\Delta p]} vdp = \int_{[m]} c^2 d\tilde{m}\ ,\ \tilde{m} = \gamma m.$$

Kotelnikov-Shannon signal decomposition and/or synthesis also has similar Gaussian elementary wave components or basis functions. This looks like having a superposition of elementary matter-wave packets, like (10.2-3)), such as (see more in chapter 4.0):

$$\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty}\Psi(n\cdot\delta t)\frac{\sin\Omega(t - n\cdot\delta t)}{\Omega(t - n\cdot\delta t)} =$$
$$= \sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t - n\cdot\delta t)}{\Omega(t - n\cdot\delta t)}\cos\varphi(n\cdot\delta t),\ \Psi(n\cdot\delta t) = a(n\cdot\delta t)\cos\varphi(n\cdot\delta t), \qquad (4.0.34)$$

Where Ω *is the highest frequency in the spectrum of* $\Psi(t)$*, and we could consider that* Ω *is the total frequency duration of the signal* $\Psi(t)$*.*

Since sampling frequency-domain of signal-amplitude, Ω_L*, is always in a lower frequency range than the frequency range of its phase function* $\Omega = \Omega_H$*, and since the total signal energy is captured only by the signal amplitude-function, we should also be able to present the same signal as:*

$$\Psi(t) = a(t)\cos\varphi(t) = \sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t - n\cdot\delta t)}{\Omega(t - n\cdot\delta t)}\cos\varphi(n\cdot\delta t) =$$
$$= \left[\sum_{n=-\infty}^{+\infty}a(n\cdot\delta t_L)\frac{\sin\Omega_L(t - n\cdot\delta t_L)}{\Omega_L(t - n\cdot\delta t_L)}\right]\cdot\left[\sum_{n=-\infty}^{+\infty}\cos\varphi(n\cdot\delta t)\frac{\sin\Omega_H(t - n\cdot\delta t)}{\Omega_H(t - n\cdot\delta t)}\right], \qquad (4.0.35)$$
$$\overline{\Psi}(t) = \sum_{n=-\infty}^{+\infty}a(n\cdot\delta t)\frac{\sin\Omega(t - n\cdot\delta t)}{\Omega(t - n\cdot\delta t)}e^{j\varphi(n\cdot\delta t)} = a(t)e^{j\varphi(t)},$$

We also know (see (5.14-1), *Chapter 5. "Quantizing and Kotelnikov-Shannon, Whittaker-Nyquist Sampling Theorem") that effective time duration* $\overline{T} = \Delta t$*, frequency* *duration* $\overline{F} = \Delta f$ *, of an elementary and optimal sampling energy quant,* $\Delta\tilde{E} = \tilde{E} = hf$ *, should be,*

$$\left\{\begin{array}{l} \Delta t \cdot \Delta f = T \cdot F = \overline{T}\overline{F} = \frac{1}{2},\ \Delta t = \overline{T},\ \Delta f = \overline{F},\ \ \overline{\Psi}(t) = a(t) \cdot e^{j\varphi(t)}, \\ d\tilde{E} = \Psi^2(t)\,dt = \frac{1}{2}\left|\overline{\Psi}(t)\right|^2 dt = \frac{1}{\pi}\left|A(\omega)\right|^2 d\omega = \frac{1}{2}a^2(t)dt = h\,df = vdp = c^2 d\tilde{m} \\ \Delta\tilde{E} = \frac{1}{2}\overline{a}^2(t)\Delta t = \frac{1}{\pi}\left|\overline{A}(\omega)\right|^2 \Delta\omega = h\Delta f = v\Delta p = c^2\Delta m = \tilde{E} = \text{one quant} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \tilde{E} = E_k = \int_{[\Delta t]} \Psi^2(t)dt = \frac{1}{\pi}\int_{[\Delta\omega]} \left|A(\omega)\right|^2 d\omega = \frac{\overline{a}^2}{2}\overline{T} = 2\overline{A}^2\overline{F} = \frac{\overline{A}\overline{a}}{2\sqrt{\pi}} = h\overline{f} = \overline{pu},$$

$$\overline{T} = \frac{\sqrt{2}\overline{A}}{\overline{a}} = \left(\frac{2\overline{A}}{\overline{a}}\right)^2 \overline{F} = \frac{1}{2\overline{F}},\ \overline{F} = (\frac{\overline{a}}{2\overline{A}})^2\overline{T} = \frac{\overline{a}}{2\sqrt{2}\overline{A}} = \frac{1}{2\overline{T}},\ \frac{\overline{F}}{\overline{f}} = \frac{1}{2}\left(\frac{h}{\overline{A}}\right)^2 = \frac{1}{2\overline{T}\,\overline{f}},$$

$$h = \frac{2\overline{A}^2}{h}\cdot\frac{\overline{F}}{\overline{f}} = \frac{\overline{a}^2}{2\overline{f}}\overline{T} = (\frac{\overline{a}}{2})^2\frac{1}{\overline{f}\cdot\overline{F}} = 6.62606876 \text{ x } 10^{-34}\,Js,$$

$$\overline{f} = \frac{\overline{\omega}}{2\pi} = \frac{1}{2\pi^2\tilde{E}}\int_0^{+\infty}\omega\left|A(\omega)\right|^2 d\omega = \sqrt{\frac{1}{2\pi^2 h}\int_0^{+\infty}\omega\left|A(\omega)\right|^2 d\omega}\ , \tag{5.14-1}$$

$$\overline{\lambda} = \frac{h}{\overline{p}},\ \ \overline{u} = \overline{\lambda}\,\overline{f} = \left\langle\frac{\omega}{k}\right\rangle = \left\langle\frac{\tilde{E}}{p}\right\rangle,\ \ \overline{v} = \left\langle\frac{\Delta\omega}{\Delta k}\right\rangle = \left\langle\frac{d\tilde{E}}{dp}\right\rangle.$$

Bottom line explanation of the quantum nature of our Universe is that all stable energy-moments defined states and masses in motions are somewhat quasi-resonant, self-closed standing-matter wave's structures. Such states are mutually communicating and interacting by exchanging kind of "Gaussian-Kotelnikov-Shannon" wave packets or quants of energy-moments matter-wave states, such as $\overline{\psi}(x,t) = \left|\overline{\psi}(x,t)\right|\frac{\sin(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}e^{I(\omega t \pm kx)}$***. Quantizing of matter has two aspects, such as, Energy-communications between all matter states (mining micro and macrocosmic states) are not continuous but more like sending or exchanging discrete packets of energy (or in some cases masses can be exchanged), and if we associate some integers to such quants, this can be related to standing-waves wavelengths counting, or just counting of relevant wave packets. The real and mathematically valid theory about matter quantizing is much closer to "Kotelnikov-Nyquist-Shannon" sampling, analysis, and signals synthesis (or reconstruction) than to what present Quantum theory is promoting. Another familiar aspect of quantum nature within our Universe is that many stable structures, such as solar systems, atoms and elementary particles are self-closed and spatially self-stabilized, standing matter-wave formations, with number of involved structural, temporal, and spatial periodicities, which can be formulated involving integers. Such self-stabilized and self-closed structures are mutually communicating (by exchanging energy, mass and moments amounts, or quants) that have forms of "Gaussian-Kotelnikov-Shannon-Nyquist" wave packets, and energy*** $\tilde{E}_i = h\overline{f}_i$***. It is obvious that contemporary presentation of quantizing (as we find it in Quantum theory) is still very much oversimplified, imprecise, and unclear, being on the level to say that quantizing in Nature exist. See more***

in [57, 58, and 59]. Of course, by applying Euler-Lagrange-Hamilton equations and concepts, based on Variational Principle of Least or Stationary Action, are arranging realistic and optimal situations in the same field.

Consequences of particle-wave duality conceptualization (as summarized under 1., 2., and 3.) are that certain particle (such as an electron, positron, proton…) could be created from a specific matter-wave group (initially without having energy-momentum elements with rest masses), on a spatially finite, closed line or self-closed multi-dimensional domain, where matter-wave in question will create stable standing-waves field structure. The mean (or median) perimeter C_n of mentioned self-closed spatial domain (taken as an oversimplified case) will have an integral number of matter-waves wavelengths, $C_n = n\lambda$ and integral number of associated, elementary time periods $n\tau = \frac{n}{f}$, where $n = 1, 2, 3, ...$ In reality, such spatially closed, energy-finite and spatially limited standing-waves domains, will not be created only around ideal circles or perfect toroid. We should also consider that values of relevant matter wavelengths and belonging elementary time periods (as conceptualized here) should be conveniently treated only as averaged values,

$$\lambda = \frac{h}{p} \Rightarrow d\lambda = -h\frac{dp}{p^2} \Rightarrow \bar{\lambda} = \frac{C_n}{n} = \frac{1}{n}\oint_{C_n} d\lambda = -\frac{h}{n}\oint_{C_n}\frac{dp}{p^2} = \frac{h}{\bar{p}},$$
$$\tilde{E} = hf = \frac{h}{\tau}(=)\tilde{m}c^2 \Rightarrow \tau = \frac{h}{\tilde{E}} \Rightarrow \bar{\tau} = \frac{1}{\bar{f}} = \frac{1}{n}\oint_{C_n} d\tau = -\frac{h}{n}\oint_{C_n}\frac{d\tilde{E}}{\tilde{E}^2} = \frac{h}{\bar{\tilde{E}}}. \tag{10.3}$$

In very simple cases, when self-closed standing waves structure is on a circle with radius **r**, perimeter of such circle will be $C_n = n\bar{\lambda} = 2\pi\bar{r} = n\frac{h}{\bar{p}} \Rightarrow \bar{L} = \bar{p}\cdot\bar{r} = n\frac{h}{2\pi}$, meaning that quantizing and particles formation in the world of microphysics is effectively related to packing of *integer numbers of half (or integer) matter-wavelengths* $\frac{\lambda}{2}$, when stable, self-closed standing waves and resonant structures are being created. Atoms, elementary particles, photons, and other stabilized matter-waves formations are (mutually and naturally), communicating by exchanging similar quantized and minimal energy-momentum wave packets. The physical size of elementary particles is directly proportional to integer numbers of half wavelengths, $\frac{\lambda}{2}$, of relevant matter-wave or wave group. It is also indicative that whenever we can successfully apply Planck's constant h (or certain macrocosmic constant H), for process modeling (or interaction description), this is a direct sign that we are dealing with some spatially closed (circular, elliptic, toroidal, spherical ...) objects, particles, fields, waves formations, and structures where matter waves are creating standing waves. ***The same situation will be much clearer and naturally understandable if we analogically address Modal and Finite Elements Analysis when searching for natural resonant frequencies (or resonant states) of a certain body. What we get with such analyses are usually some structures of standing waves***

(corresponding to different resonant situations). In cases of atoms and minimal or elementary and atomized matter constituents, we should also come to such standing waves and self-stabilized resonant structures. This is a big (theory and practices related) part of the Quantum World of Physics. In addition, here we should include matter-waves communications between mentioned standing-waves and resonant structures that are respecting Kotelnikov-Shannon, Whittaker-Nyquist Sampling, and signal reconstruction rules.

One of the most probable or credible form of matter waves involved in creating stable elementary particles (like an electron, proton, positron…) should be certain form of electromagnetic wave (meaning specifically structured form of mutually coupled (and packed) electric and magnetic field components, because of known mutual transformability and very indicative interactions between them). In other words, elementary particles that have measurable rest masses (like an electron, positron, proton…) can be created from specifically packed, electromagnetic energy-momentum carrying matter-wave groups, which do not have initial rest masses. The supporting reasons for such statement are elaborated in chapter 4.1 of this book (as well as in [17] to [22], [44] and [84]). From many experiments, we know that particle-wave duality and matter waves have been on some way recognized, detected, and theoretically or mathematically confirmed. For instance, such experiments are Compton and Photoelectric effects, Secondary Emissions, Blackbody radiation, diffraction phenomena with atomic and molecular rays etc. We know that involved reaction participants (in many of mentioned cases) have electric charges, magnetic moments, and intrinsic spin characteristics with constant gyromagnetic ratios. The fact is that in most of the relevant mathematical analyses of mentioned dualistic matter manifestations, contemporary physics simply neglects electromagnetic charges, moments and spinning. We often take only kinematic, mechanical parameters of certain linear motion (like involved velocities, masses, and energies), but, spinning and magnetic moments are extremely significant for existence, explanation, and understanding of matter waves, and should not be neglected.

Practically, the creation of certain stabilized, self-standing n-dimensional body or particle is related to n-dimensional standing-waves packing including angular and electromagnetic standing-waves building blocks. Such multidimensional and discretized packing is the background of everything that we understand under matter quantization. What is significant here is that Planck constant **h** is essentially involved in all quantization cases related to micro-world physics of atoms, photons, and subatomic entities. We also know that similar quantization with different macrocosmic Planck-analog constant **H** also exist in macrophysics of stable and inertial astronomic, planetary systems, which are also self-closed standing matter-waves formations (see in the Chapter 2, “2.3.3. Macro-Cosmological Matter-Waves and Gravitation”. Quantizing in Physics is primarily, essentially, and ontologically incorporated in, or related to different inertial and stabilized matter structure systems and constituents, where standing waves granulation and packing is involved, and relevant mathematics dealing with such quantization belongs to “Kotelnikov-Shannon, Whittaker-Nyquist Sampling Theory”. See more in [57, 58, and 59].

In modern microphysics, we often use terminology like Quantum World and Quantum Physics for everything that belongs to the micro-world of atoms and

subatomic entities. The whole Micro and Macro Universe are the Quantum World of interconnected resonating structures of standing matter waves and masses agglomerations, where "Kotelnikov-Shannon, Whittaker-Nyquist Sampling, and Signals Recovery Theory" is creating rules and logic of quantization and "energy-moments" communications. At least two of four of fundamental forces of our Universe (as presently established) are most probably consequences of associated attracting and repulsing effects within different nodes of mentioned standing waves structuring, and consequently, contemporary conceptualization of four fundamental forces is expected to evolve significantly (or to be changed completely).

Effectively, after the stable particle is created, motional matter-wave energy of self-closed standing waves in question is being transformed into particle rest mass or captured by such particle mass (and any surplus of non-consumed energy-momentum will create macro motion of such particle, or it will be radiated in some form of matter waves). This is the meaning of energy formatting or packing (See also "**4.3.8. Mass, Particle-Wave Duality and Real Sources of Gravitation"** in chapter 4.3 of this book, as the additional conceptual support).

Now we can combine closed-domain standing waves concept (10.3) with other significant relations, and wave function expressions starting from (10.1), belonging to **PWDC**, to elaborate and underline advantages of operating with wave functions. This way we will extend an elementary and simplified understanding of standing waves (on a circularly closed line in one plane) towards generalized three-dimensional, or multi-dimensional spatial coordinates and time-dependent standing waves structures (see chapter 4.3, starting from equations under (4.9-0)), as follows,

$$
\begin{aligned}
&\overline{\lambda} = \frac{h}{\overline{p}} = \frac{C_n}{n} = \frac{1}{n}\oint_{C_n} d\lambda = -\frac{h}{n}\oint_{C_n}\frac{dp}{p^2} = -\frac{h}{n}\oint_{C_n}\frac{d\tilde{E}}{vp^2} = -\frac{h}{n}\oint_{C_n}\Psi^2\frac{dt}{vp^2} = -\frac{h}{n}\oint_{C_n}\Psi^2\frac{vdt}{(pv)^2},\\
&\overline{\tau} = \frac{1}{\overline{\overline{f}}} = \frac{h}{\overline{\overline{\tilde{E}}}} = \frac{1}{n}\oint_{C_n} d\tau = -\frac{h}{n}\oint_{C_n}\frac{d\tilde{E}}{\tilde{E}^2} = -\frac{h}{n}\oint_{C_n}\frac{vdp}{(pu)^2} = -\frac{h}{n}\oint_{C_n}\Psi^2\frac{dt}{(pu)^2},\ n = 1, 2, 3, \dots,\\
&(v <<< c, p = mv, m \cong const.) \Rightarrow \overline{\lambda} = \frac{h}{m\overline{v}}, \overline{\tau} = \frac{h}{(\frac{m\overline{v}^2}{2})} = \frac{1}{\overline{\overline{f}}},\ \overline{u} = \overline{\lambda}\overline{f} = \frac{\overline{v}}{2},\\
&(v \approx c, p \approx \frac{hf}{c}, m \approx \frac{hf}{c^2}) \Rightarrow \overline{\lambda} = \frac{c}{\overline{\overline{f}}}, \overline{\tau} = \frac{1}{\overline{\overline{f}}},\ \overline{u} = \overline{\lambda}\overline{f} \approx c,\\
&\overline{u} = \overline{\lambda}\overline{f} = \frac{\oint_{C_n}\Psi^2\frac{vdt}{(pv)^2}}{\oint_{C_n}\Psi^2\frac{dt}{(pu)^2}} = \frac{\overline{\overline{\tilde{E}}}}{\overline{p}},\ \Psi = \Psi(x, y, z, t) = \Psi(r, t).
\end{aligned}
\tag{10.4}
$$

The meaning of the wave function $\Psi = \Psi(x, y, z, t) = \Psi(r, t)$, $\Psi^2(t) = \frac{d\tilde{E}}{dt}$, from (10.1) – (10.4) and elsewhere in this book, is essentially related to relevant signal or matter-wave Power, and not at all significantly related to Quantum Theory, or statistics and probability theory practices (see much more about Power in the chapter 4.0, under equations (4.0.82) and later). Power and Analytic signal related wave function (see below) is much richer and more informative regarding understanding and describing particle-wave duality and matter wave states, than Quantum Theory probabilistic and statistics-based concepts.

$$\left\{\begin{array}{l} \{d\tilde{E} = hdf = dE_k = c^2 d(\gamma m) = vdp = d(pu)\} / dt \\ \Leftrightarrow \left\{\Psi^2(t) = \frac{d\tilde{E}}{dt} = h\frac{df}{dt} = c^2 \frac{d(\gamma m)}{dt} = \frac{d(pu)}{dt} = v\frac{dp}{dt} \, (=)[W]\right\}, \\ v = u - \lambda\frac{du}{d\lambda} = -\lambda^2\frac{df}{d\lambda} = u + p\frac{du}{dp} = \frac{d\omega}{dk} = \frac{d\tilde{E}}{dp} = h\frac{df}{dp}, \; u = \lambda f = \frac{\omega}{k} = \frac{\tilde{E}}{p} = \frac{hf}{p} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} POWER = \frac{d\tilde{E}}{dt} = \frac{1}{2}|\overline{\Psi}(t)|^2 = \frac{1}{\pi}|A(\omega)|^2\frac{d\omega}{dt} = \frac{1}{2}a^2(t) = \frac{1}{2}|\overline{s}_1 \cdot \overline{s}_2|^2 = \frac{1}{2}(a_1 \cdot a_2)^2 = \\ = h\frac{df}{dt} = v\frac{dp}{dt} = c^2\frac{d\tilde{m}}{dt} = s_1 \cdot s_2 \, (=)[W], \\ \overline{\Psi}(t) = |\overline{\Psi}(t)| \cdot e^{I\varphi} = a(t) \cdot e^{I\varphi} = \overline{s}_1 \cdot \overline{s}_2 = a_1 \cdot a_2 \cdot e^{I(\varphi_1 + \varphi_2)}, \, \overline{s}_{1,2} = a_{1,2} \cdot e^{\varphi_{1,2}}, \\ |\overline{\Psi}(t)| = a_1 \cdot a_2 = a(t), \; \varphi = \varphi_1 + \varphi_2, \\ \omega = 2\pi f = \frac{d\varphi}{dt} = \frac{d\varphi_1}{dt} + \frac{d\varphi_2}{dt} = \omega_1 + \omega_2 = 2\omega_c, \; \omega_1 = \omega_2 = \omega_c. \end{array}\right\} \Rightarrow$$

It is also the fact that certain (well selected) isomorphic and convenient mathematical transformations, connections, normalizations mapping, assumptions, and simplifications (applied on mentioned wave function) could be arranged (while respecting universally valid conservation laws), to materialize contemporary and probabilistic Quantum Theory concepts, which are also operational and predictive in analyzing different interactions, matter structure and particle-wave duality). To profit from very rich mathematical treasury regarding signal analysis, we will consider that matter-waves wave function $\Psi(r,t)$ should be (at least) an Analytic Signal function (in relation to Hilbert Transform), as introduced and elaborated in the chapters 4.0 and 4.3 of this book.

Now we can extend self-closed standing waves quantized packing concept (on same closed line C_n) to be applicable to arbitrary shaped closed domains, as follows,

$$\begin{array}{l} \overline{\lambda} = \frac{h}{\overline{p}} = \frac{C_n}{n} = \frac{1}{n}\oint_{C_n} d\lambda \Rightarrow \overline{p}C_n = \overline{p}\oint_{C_n} d\lambda = nh \Rightarrow \\ \Rightarrow \oint_{C_n} pdr = -h\overline{p}\oint_{C_n}\frac{dp}{p^2} = -h\overline{p}\oint_{C_n}\frac{d\tilde{E}}{vp^2} = -h\overline{p}\oint_{C_n}\Psi^2\frac{dt}{vp^2} = -h\overline{p}\oint_{C_n}\Psi^2\frac{vdt}{(pv)^2} = nh, \\ \overline{\tau} = \frac{1}{\overline{f}} = \frac{h}{\overline{\tilde{E}}} = \frac{1}{n}\oint_{C_n} d\tau \Rightarrow \overline{\tilde{E}}\oint_{C_n} d\tau = nh \Rightarrow \oint_{C_n}\tilde{E}dt = nh \Rightarrow \oint_{C_n} pdr = \oint_{C_n}\tilde{E}dt = nh, \\ \Psi^2(t) = \frac{d\tilde{E}}{dt} \Rightarrow \overline{\tilde{E}} = \oint_{C_n}\Psi^2(t)dt = h\overline{f} = -\frac{(\overline{\tilde{E}})^2}{n}\oint_{C_n}\frac{vdp}{(pu)^2} = -\frac{(\overline{\tilde{E}})^2}{n}\oint_{C_n}\Psi^2\frac{dt}{(pu)^2} = \\ = \frac{\overline{f}}{n}\oint_{C_n} pdr = \frac{\overline{f}}{n}\oint_{C_n}\tilde{E}dt = -\frac{(\overline{\tilde{E}})^2}{n}\oint_{C_n}\frac{d\tilde{E}}{\tilde{E}^2} = \tilde{m}c^2 \Rightarrow h\overline{f} = \tilde{m}c^2 \Leftrightarrow \frac{c}{\overline{f}} = \frac{h}{\tilde{m}c} = \overline{\lambda}_c. \end{array} \quad (10.5)$$

What we can find in (10.4) and (10.5), in addition to quantizing conditions, is that Compton wavelength $\overline{\lambda}_c$ of certain elementary particle corresponds to the averaged wavelength of its internally packed standing wave, under the condition that such internal standing wave should propagate (on its self-closed orbit) with the speed of photons,

$$\overline{\lambda}_c = \frac{c}{\overline{f}} = \frac{\overline{\lambda f}}{\overline{f}} = \frac{h}{\tilde{m}c} = \overline{\lambda} \,. \tag{10.6}$$

The privileged assumption practiced here is to treat mentioned self-closed domains of standing waves as being structured around thin and narrow, large-diameter toroid, because of helix-spinning nature of matter waves with number of advantages and valid approximations related to such toroidal and gyroscopic envelope forms (including conveniences of associated mathematical processing and simple links to basic, generally applicable physics). Similar assumptions and concepts can be found in publications of C. Lucas, David L. Bergman, Paul J. Wesley, Dennis P. Allen, Dr. Günther Landvogt and M. Kanarev (see chapter 4.1, and literature references from [17] to [22], [36], [44] and [84]).

Of course, since matter-wave function $\Psi = \Psi(x, y, z, t) = \Psi(r, t)$ from (10.5), which should present stabilized elementary particles, is (preferably and most probably) distributed around the certain thin toroidal envelope, where C_n presents the corresponding median or mean, central line, the more proper will be to use spatial (multidimensional) integration in (10.5). The consequences of geometry generalization are that self-closed standing waves field structure in question will be more complex, multidimensional, ***and produce other integers (or quantum numbers) supporting associated angular and spatial quantizing***, like in Wilson-Sommerfeld action integrals. Effectively, (10.5) has such integrals (see chapter 5. UNCERTAINTY, around equations (5.4.1) - (5.4.3)). Originally, Wilson-Sommerfeld action integrals are established and applied in the framework of N. Bohr atom model, but here we can analogically and more naturally apply such integral quantizing on matter-waves that are creating elementary particles (without consuming other particles with rest masses) like shown in (10.5). If such initially created elementary particles are electrons and protons, and if we consider neutron as the specifically coupled state between an electron and a proton, we already have building blocks of all atoms, molecules, and matter in our Universe. Varieties of all other particles and quasi-particles known in a micro-world of physics are mostly short-living and unstable products of collisions and surrounding field's interactions, interferences, and superposition of initial elementary particles and matter waves, presenting transient states and sub-structural formations of electrons, protons, neutrons, and their antiparticles. Some of them are only indirectly estimable and strongly conceptually linked to involved mathematical modeling (such as Quarks).

Wilson-Sommerfeld action-integrals concepts are also and analogically well applicable on stable solar or planetary systems (as elaborated in chapter 2. of this book).

Anyway, all such standing-waves entities are satisfying energy-momentum conservation and Continuous Symmetry laws (see chapter 1. About Continuous Symmetries), giving us chances to introduce, invent, structure, or see an unlimited number of real and virtual participants and products of different interactions.

We do not have problems regarding increasing and never-ending number of newly created or newly experimentally discovered or so-called elementary particles and quasiparticles within Quantum theory guided practices; -our real problems could be in practicing wrong or confusing concepts and artificially, weakly, or ambiguously structured framework and mathematical modeling about Particle-Wave Duality.

From the mathematical and modeling point of view we could safely say that all mentioned elementary particles, photons, physics relevant matter-waves and their wave functions $\Psi = \Psi(x,y,z,t) = \Psi(r,t)$ *should have finite spatial, temporal and frequency localizations or concentrations in a form of Gaussian and Analytic Signal pulses and should be mathematically processed by windowed Fourier transform or Gabor transform; see [79].*

Wilson-Sommerfeld action integrals (see [9], and chapter 5. UNCERTAINTY, around equations (5.4.1) - (5.4.3)), related to any periodical motion on a self-closed stationary orbit, applied over one period of a motion, present the kind of general quantifying rule (for all self-closed standing waves, which are energy carrying structures) that was also successfully used in supporting N. Bohr's Planetary Atom Model. By analogical extension of Wilson-Sommerfeld action integrals to all "CHARGE" elements found in T.5.3 and (5.2.1), we can formulate the following quantifying or quantizing expressions (again between mutually conjugate variables) that are also in agreement with "Periodicity of de Broglie Wave Intervals" from T.5.3, presenting important metrics of different elementary energy states:

Metrics of Elementary Particles:

$$\left\{\begin{array}{l} 2\left|\Delta q_{mag.}\cdot\Delta q_{el.}\right|_{min.} = 2\left|\Delta\alpha\cdot\Delta\mathbf{L}\right|_{min.} = 2h\cdot\left|\Delta t\cdot\Delta f\right|_{min} = 2\left|\Delta x\cdot\Delta p\right|_{min.} = 2\left|\Delta t\cdot\Delta\tilde{E}\right|_{min.} = \\ = 2c^2\left|\Delta t\cdot\Delta m\right|_{min.} = 2\left|\Delta s_1\cdot\Delta s_2\right|_{min.} = h, \\ \left\{\lambda = \frac{h}{p} = \vec{\lambda},\ \tilde{E} = hf = h\frac{1}{\vec{T}} \Leftrightarrow \vec{T} = \frac{h}{\tilde{E}}\right\} \Rightarrow \left\{\vec{\theta} = \frac{h}{\mathbf{L}},\ \vec{\Phi}_{el.} = \frac{h}{\Phi_{mag.}},\ \vec{\Phi}_{mag.} = \frac{h}{\Phi_{el.}}, \vec{s}_1 = \frac{h}{s_2}\right\}, \\ [X] = [q_{mag.}, q_{el.}, x, \alpha],\ [Q] = [q_{el.}, q_{mag.}, p, \mathbf{L}],\ \Phi_{el.} = q_{el.},\ \Phi_{mag.} = q_{mag.}. \end{array}\right\} \Leftrightarrow$$

$$\Leftrightarrow \left\{\begin{array}{l} \vec{\Phi}_{mag.}\cdot\Phi_{el.} = \vec{\Phi}_{el.}\cdot\Phi_{mag.} = \vec{\lambda}\cdot p = \vec{\theta}\cdot\mathbf{L} = \vec{T}\cdot\tilde{E} = ... = \vec{s}_1\cdot s_2 = h, \\ (\vec{\Phi}_{mag.}\cdot\Phi_{el.})_1 + (\vec{\lambda}\cdot p)_2 + (\vec{\theta}\cdot\mathbf{L})_3 + (\vec{T}\cdot\tilde{E})_4 + ... + (\vec{s}_1\cdot s_2)_m = mh,\ m = 1,2,3... \end{array}\right\} \Rightarrow$$

$$\Rightarrow [\,\vec{\mathbf{X}}\,][\,\mathbf{Q}\,] = [\,\mathbf{h}\,] \Rightarrow$$

Wilson-Sommerfeld action integrals

$$\left\{\begin{array}{l} \oint_{C_n} p_\lambda d\lambda = n_\lambda h,\ \oint_{C_n} \mathbf{L}_\theta d\theta = n_\theta h, \\ (n_\lambda, n_\theta) = \text{integers } (=1,2,3,...) \end{array}\right\} \wedge \left\{\begin{array}{l} \oint_{C_n} \Phi_{el.} d\Phi_{magn.} = n_{el.} h,\ \oint_{C_n} \Phi_{mag.} d\Phi_{el.} = n_{mag.} h, \\ \oint_{C_n} \tilde{E}_n dt = nh,\ (n_{el.}, n_{mag.}, n) = \text{integers} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{cases} \oint_{C_n} [X] d[Q]^T = \oint_{C_n} [Q] d[X]^T = \oint_{C_n} \tilde{E} dt = n \cdot [X] \cdot [Q]^T = [n] \cdot [Q] \cdot [X]^T = [n]h, \ n = 1,2,3,... \\ [X]^T = [X(t)]^T = \begin{bmatrix} q_{mag.} \\ q_{el.} \\ x \\ \alpha \end{bmatrix}, [Q]^T = [Q(t)]^T = \begin{bmatrix} q_{el.} \\ q_{mag.} \\ p \\ L \end{bmatrix}, [n] = \begin{bmatrix} n_{el.} \\ n_{mag.} \\ n_{\lambda} \\ n_{\theta} \end{bmatrix} \end{cases} \quad (5.4.1)$$

Until here, we are still not addressing complex particles, like atoms and their agglomerations. Initially involved linear moment and speed (of a matter-wave group forced to create standing waves) are something not externally visible, measurable, or recognizable since such parameters belong to internal, orbital, self-closed lines, or self-closed spatial domains where certain matter wave is creating standing waves. We are still addressing only (elementary) particles creation from pure standing matter-waves, without the involvement of other already existing and stable rest masses. Once, after certain elementary particle is created as a stable (standing waves structured) entity $m = \tilde{m}$, it will be externally measurable or detectable as a particle with a rest mass, and it can move as a compact object in a surrounding (external or laboratory) space.

The indicative implications of Wilson-Sommerfeld action integrals, and from the way how Schrödinger equation is developed in this book (see Chapter 4.3) is supporting the concept that stable, elementary particles and matter-wave groups (like electrons, protons, neutrons etc.) should have circular, self-closed, or toroidal spatial structure, all of that being familiar to Bergman, David L., and Lucas, Jr., Charles W., "Physical Models of Matter," and "Spinning Charged Ring Model of Elementary Particles" (see more in [16] to [22]).

Of course, particle created on such way again has (new) dualistic and "external" matter-wave properties (but no more as internally structured standing waves), and it will be simply characterized by its externally detectable parameters, such as mass, linear and angular moments, and kinetic energy (and in some cases with electric and magnetic charges, potentials, currents…). New, "external" particle speed, momentum, kinetic energy, and matter-waves wavelength values (see (10.7)) will be related to a motion of mass $m = \tilde{m}$ in the laboratory system (and should not be confused or mixed with some of similar, "internal", orbital, standing-wave parameters found in equations from (10.1) to (10.6)), as follows,

$$\begin{aligned} & m = \tilde{m}, p = \gamma m v, E_k = \tilde{E} = (\gamma - 1)mc^2 = E - E_0 = hf, \lambda = \frac{h}{p}, \gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} \\ & P_4(p, \frac{E}{c}) = \text{invariant to ref. system} \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}, E_0 = mc^2, E = \gamma mc^2. \end{aligned} \quad (10.7)$$

Until we have only certain matter wave (still not transformed or structured in a particle that has rest mass), we can apply the total energy equivalency between wave and effective mass-energy in some of the forms as,

$$\begin{aligned} & E_{total} = \tilde{E} = E_k = \tilde{m}c^2 = hf, \\ & dE_{total} = d\tilde{E} = dE_k = c^2 d\tilde{m} = hdf. \end{aligned} \quad (10.8)$$

Once when the same wave is structured as a stabilized, self-closed (circular), standing waves formation, we can consider that rest mass is created (meaning that former $\tilde{m}$

will start its new, kinematic existence as $m_0 = \tilde{m} = m$, having also certain (external) speed and kinetic energy, and if we monitor such particle as a whole, externally (in a Laboratory system), we will consider that $dm_0 = dm = 0$. The kinetic energy of such particle is now presenting associated, de Broglie, or matter wave energy,

$$\begin{aligned} &E_{total} = mc^2 + \tilde{E} = mc^2 + E_k, \ \tilde{E} = E_k \Rightarrow \\ &dE_k = d\tilde{E} = dE_{total} = vdp = hdf, \ p = \gamma mv. \end{aligned} \tag{10.9}$$

Matter wave frequency in (10.9) is no more comparable or equal to the frequency from (10.8), meaning that now (10.9) starts to be a relevant framework. In most of our analyses of particles motions and interactions with other particles (in Physics or Mechanics), we are starting from (10.7), meaning from already created particles (and not asking what kind of standing waves structures are packed inside).

For macro-mass formations like stabilized planetary and galactic systems with circular, elliptic or other self-closed orbits, and similar to toroidal or helical motions, Planck constant h is likely to be replaced by another macro-world $H \gg h$ constant, respecting validity of similar matter-waves and particle-wave duality concepts and mathematics known for micro-world situations (see Chapter 2., " 2.3.3. Macro-Cosmological Matter-Waves and Gravitation").

The most significant (experimentally confirmed) examples supporting concepts elaborated here are the creation of an electron + positron couple from sufficiently high-energy photons, and total annihilation of an electron + positron creating only photons. Other supporting examples are related to familiar phenomenology of Compton and Photoelectric effects, secondary emissions, and atoms excitations by absorbing or emitting photons, presenting foundations and step stones of Wave-Particle Duality of matter.

For instance, in chapter 4.2 of this book ("4.2.2. Example 3: Elastic collision photon-particle (Compton Effect)"), we can find experimentally verifiable arguments ***why and how matter-wave energy is equal to a relevant particle kinetic energy***.

The process which is opposite to (or inverse of) Compton Effect is the continuous spectrum of x-rays (of photons) emission, caused by impacts of electrons (accelerated in the electrical field between two electrodes) with the anode as their target. The emission of x-ray photons starts when the electrons are abruptly stopped on the anode surface. If the final, impact electron speed is non-relativistic, $\mathbf{v} \ll \mathbf{c}$, the maximal frequency of the emitted x-rays is found from the relation: $hf_{max.} = \frac{1}{2}mv^2$, and in cases of relativistic electron velocities, we have $hf_{max.} = (\gamma - 1)\, mc^2$ (and both of them are experimentally confirmed to be correct, being also kinetic or motional energy, fully in compliance with other PWDC relations, as (10.1), (10.2) and 10.2-1). If we now consider electrons (before the impact happen) as matter waves, where the electron matter-wave energy corresponds only to a kinetic or motional energy, without rest-mass energy content, we will be able to find de Broglie, matter wave frequency of such electrons (just before their impact with the anode). With such impact execution, the electrons are fully stopped, and the energy content of their matter waves is fully transformed and radiated in the form of x-ray photons (or into another form of waves, or motional energy), whose frequency corresponds to the matter wave frequency of electrons in the moment of the impact. This equality of the frequencies of radiated x-ray photons and electron matter-waves (in the moment of impact) explains the essential nature of electron matter waves (eliminating the possibility that the rest mass belongs to matter-wave energy content, except in particle-antiparticle annihilations). Similar experimental observations (about the kinetic energy of incident electrons and energy of generated x-rays) are also known for elastic scatterings or bremsstrahlung x-rays. An excellent theoretical background that explains equivalency between kinetic and matter waves (motional) energy is presented in [105], Himanshu Chauhan, Swati Rawal, and R K Sinha. WAVE-PARTICLE DUALITY REVITALIZED, CONSEQUENCES, APPLICATIONS, AND RELATIVISTIC QUANTUM MECHANICS. *Contemporary Quantum Theory is dominantly supporting wave equations, matter waves, and wave-particle duality concepts based on Classical Mechanics, non-relativistic motions and assumptions, when v <<< C, and when involved rest mass is included in a matter wave content, what is producing bizarre and useless (or incorrect) result that phase velocity could be higher than speed of light C. See an explanation under (4.10-5) in chapter 4.3.*

Only after we establish stable and correct ***particle-matter-wave*** duality foundations, like summarized in (10.1) to (10.9), and based on using Analytic Signal model, we could give more freedom to our conceptual and mathematical imagination (regarding wave functions and wave equations) and create something what looks familiar to our contemporary Quantum Theory. **Unfortunately, generations of scientists started to understand and analyze microphysics from a cloudy (statistical and probability related) assumption of somewhat artificially, ad-hock established and unreasonably complicated Quantum theory step-stones, without having very stable and conceptually clear, elementary, and basic physics related foundations.** Of course, Quantum theory founders well mixed and hybridized whatever makes such theory self-standing and mathematically well operating. Anyway, the final product is far from what an easily understandable, realistic, natural tangible and intuitively manageable Physics theory should be (meaning that in such unique and extraordinary Physics, too many of ad-hoc rules, freestanding ad-hock parameters and assumptions should be introduced, postulated, and respected).

*We know that **de Broglie matter waves** wavelength, and other wave-particle duality concepts are the consequences of unity, couplings, synchronizing and complementing relations between linear and angular, or spinning motions in two or multi-body situations (see chapter 4.1).* In the time of Luis de Broglie, it was maybe unavoidable to postulate wave-particle-duality concepts based on intuition and specific experimental and mathematical-fitting verifications, but presently we can explain such phenomenology on a much more natural, deterministic, conceptually clear, and basic-physics and mathematics verifiable grounds using the Analytic Signal framework. Unfortunately, in most of Quantum theory books, the same problematic is still being introduced dogmatically and axiomatically, or like in a time of L. de Broglie.

*We now know that famous **Schrödinger equation** (and number of familiar wave equations, known from contemporary Quantum theory, such as Klein-Gordon and Dirac's equations, is just an easy, logical, smooth and causal, analogical extension of d 'Alembert's and Classical Wave Equation, where wave function is treated as the Complex Analytic Signal function (see much more in Chapter 4.3.).* In most of

Quantum theory books, Schrödinger equation is still being presented as a fundamental, quantum-mechanical starting step stone, and unique, divine inspiration and a magic invention of E. Schrödinger (and, of course, in the time of Schrödinger this could be the case). Even electromagnetic waves' equations as Classical Wave Equations are easy transformable to Schrödinger equation (see [86], Victor Christianto. "Review of Schrödinger Equation & Classical Wave Equation"). Many of mutually analogical (or almost identical) forms of Classical Wave Equations are also known in acoustics, fluid motions and oscillations of mechanical systems (see [87] Hubert & Lumbroso), indicating that all matter waves, oscillations, and wave equations applicable within different phenomenology known in our Universe (or Physics) have the same mathematical, material and Physics related grounds. We should also not forget that for all matter waves the same equations are describing group and phase velocity, as shown in (10.2) and (10.4), being directly involved in all Classical Wave Equations. Complex, Analytic Signal based Schrödinger wave equation is simply compatible (or integrated) with Classical Wave Equation and all concepts, results, and relations already summarized in this chapter (for instance, around (10.1) until (10.5), and in chapter 4.3 summarized by (4.25)), as follows,

$$\Delta\overline{\Psi} - \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = \left(\frac{L}{\hbar u}\right)^2\overline{\Psi} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = j\frac{L}{\hbar u^2}\frac{\partial\overline{\Psi}}{\partial t} + \frac{1}{u^2}\cdot\frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,$$
$$\frac{\hbar^2}{\tilde{m}}\left(\frac{u}{v}\right)\Delta\overline{\Psi} + L\overline{\Psi} = 0\ ;\ \Delta\overline{\Psi} = \frac{1}{u^2}\frac{\partial^2\overline{\Psi}}{\partial t^2},$$
$$\left(\frac{L}{\hbar}\right)^2\overline{\Psi} + \frac{\partial^2\overline{\Psi}}{\partial t^2} = 0,\ \frac{L}{\hbar}\overline{\Psi} = j\frac{\partial\overline{\Psi}}{\partial t},$$
$$L \in \left[\ldots\boxed{\tilde{E} - U_p}\ \text{or}\ \boxed{\tilde{E}}\ \text{or}\ \boxed{\tilde{E} + E_0 - U_p}\ \text{or}\ \boxed{\tilde{E} + E_0}\ \text{or}\ \boxed{\tilde{E} + E_0 + U_p}\ \text{or}\ \boxed{\tilde{E} + U_p}\ldots\right]$$
$$S = \int_{t_1}^{t_2} L(q_i, \dot{q}_i \ldots, t)dt = \text{extremum}\ . \tag{4.25}$$

Starting from (4.25) we can generate all other wave equations known in Quantum theory by simply translating basic Lagrangian function ($L = \tilde{E} - U_p$) for certain relatively fixed or stable energy amount (in both directions, inwards and outwards, depending on the energy structure we like to analyze). For more profound elaborations of the same problematic, see chapter 4.3.

[♣ COMMENTS AND FREE-THINKING CORNER (still working on):

Real Matter-Wave and masses formations explained by effects of Ultrasonic, Mechanical Excitation of Weakly Coupled Electron States in Electro-Conductive Materials in relation to Wave-Particle Duality.

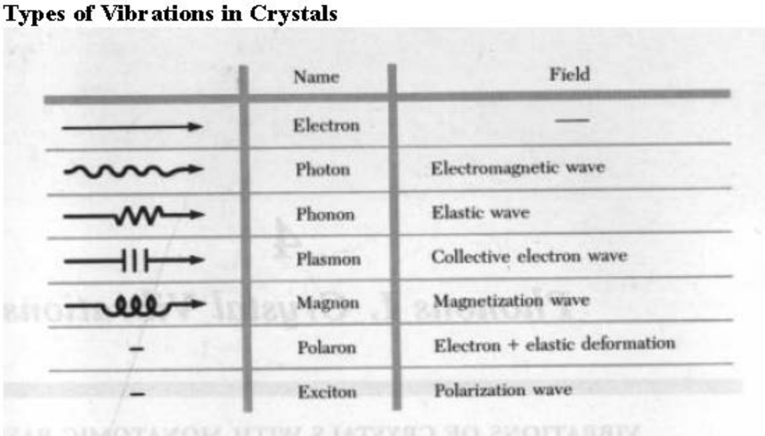
A working assumption (or hypothesis) here is that it is possible to influence, excite and increase the mobility of weakly coupled and relatively free electron states, through ultrasonic methods (meaning acoustically and mechanically). It is proposed that this can be achieved by vibrating certain electrically conductive material, electrode, galvanic or electrochemical cells and similar metal parts where electric charges, voltages, and currents are either being generated, consumed, charged and/or discharged.

As is known today, an ordinary and arbitrary mechanical, acoustic, or ultrasonic excitation would not produce mentioned results. Only the mechanical or electromechanical, ultrasonic excitation of specific natural, internal, parametric, and other resonant states of electro conductive materials (at the specific resonant frequencies) would increase free electrons mobility and produced electric current (including influence on associated voltages and charges).

We will start from the following citation (see below), which describes an approximated model of metals, crystals, electrically conductive materials, and similar solid matter structures ([119], http://grdelin.phy.hr/%7Eivo/Nastava/CvrstoStanje/arhiva/udzbenici/Upali_Siriwardane/586c4.htm):

Chapter 4 : Phonons and Crystal vibration

Types of Vibrations in Crystals



Name	Field
Electron	—
Photon	Electromagnetic wave
Phonon	Elastic wave
Plasmon	Collective electron wave
Magnon	Magnetization wave
Polaron	Electron + elastic deformation
Exciton	Polarization wave

Normal modes of crystal vibrations

Many solid materials, including all metals, are composed of atoms arranged in a lattice arrangement called crystals. There are a variety of crystal structures like cubic, hexagonal, cubic with an atom in the center of the cube, called body centered cubic, cubic with an atom in the center of each face of the cube, called face centered cubic, and others. The particular structure depends on the relative sizes of the atoms that are nestled together to form the crystal. The reason that materials take crystal form is that these neat geometrical structures represent the lowest energy configuration of the collection of atoms making up the material. To dislodge an atom from the crystal structure requires the addition of energy.

Theoretically, at a temperature of absolute zero, the atoms of a crystal lie at their lowest energy position without moving at all. As thermal energy is added to the crystal it is manifest by vibration of the atoms about this equilibrium location. Within the limits of fairly small vibrations the electric forces bonding the atoms together stretch or compress a bit to a higher energy configuration. Each atom acts as though it were connected to its neighbors by little springs. The added energy is stored in the crystal as the kinetic energy of the atoms in motion and the potential energy of the compressed or stretched springs.

Assumptions and Hypotheses Foundations

1. Electrons, Photons, Phonons, Plasmons, Magnons, Solitons etc. are all matter-wave groups, or wave-packets of a certain form of matter and fields, having motional energy like energy-moment properties of moving particle. Until a

certain level, the mentioned wave packets could be analogically treated as particles, having the group and phase velocity, wavelength, frequency, some spinning or angular moment properties, and linear moment properties (in contemporary physics described under Wave-Particle Duality of matter).

2. The mentioned matter-wave properties and parallelism (or analogy) of all matter-wave packets with corresponding equivalent particles, is manageable using the same mathematics (around Analytic Signal modeling). This is the part of the content or meaning of the particle-wave duality. There is no need or factual evidence forcing us to have different mathematical or physics-related theories, structures, and models specifically valid only for Electrons, or Photons, Phonons, Plasmons, Magnons, Solitons... Probabilistic and Statistics related approach here should not be an essential, exclusive, ontological, and best modeling strategy, except when it naturally applies (for quantitative and trends characterizations and modeling within big sets of mutually identical entities and events).

Many of the mentioned matter-wave packets (such as Electrons, Photons, Phonons, Plasmons, Magnons, Solitons etc.) can be in some way (more or less) coincidently or synchronously present and mutually electromechanically and electromagnetically coupled in cases of complex, real matter structures and motions. We could only have some mathematical, conceptual modeling preferences and facilities for treating them separately (like being mutually isolated and independent entities). For instance, mechanical and acoustical vibrations and waves in solid liquid and gaseous media are oscillatory motions of masses, molecules, and atoms. As we know, all atoms have very much essential electromagnetic structure or nature (at least having positive and negative electric charges with orbital and magnetic spinning moments). Consequently, mechanical, acoustic, and thermodynamic vibrations, motions and other energy-momentum interactions will always be electromechanically coupled with internal electromagnetic nature of atoms, by involving or creating Electrons, Photons, Phonons, Plasmons, Magnons, Solitons etc. Such interactive coupling exists also with external electromagnetic radiation that is influencing motional and electromechanical effects in real masses and atoms (when masses and atoms are exposed to an external electromagnetic radiation). For instance, here we could mention Photoelectric and Compton's effects, and photovoltaic electricity generation as being an external electromagnetic influence on the matter (or atoms), producing different (mutually coupled) matter-waves, masses, and electric charges motions. If external electromagnetic radiation is in a frequency domain of infrared and far below, towards microwave frequencies, corresponding heating, and thermodynamic effects in masses and around atoms will be produced (meaning atoms, molecules, and other particles will start oscillating randomly and proportionally to absorbed radiation). Such thermal and oscillatory motions are electromechanical perturbations of already existing, natural, parametric, and other mass-spring (or capacitance-inductance) internal resonant states of different masses and belonging atoms. Even many new and exotic (still partially unclear and ongoing) experiences regarding memory effects in liquids being agitated acoustically, mechanically, ultrasonically, electromechanically and with electromagnetic radiation, belong to the same family of electromechanical couplings (when structural content and shape of liquids can be on some recognizable way affected and experimentally verified).

Let us come back to the multiple, structural, and spatial matrix of mass-spring elements, representing metals, solid bodies, and crystals. Masses in such models are atoms, or molecules, or maybe some bigger agglomerations of atoms connected with attractive fields presented as equivalent springs. Springs here, are attractive, cohesion, adhesion and Van der Waal's forces based on electrostatic and magneto-static attractions and repulsions between atoms. For instance, involved particles (here meaning atom masses) with magnetic moments are mutually attracting, and the surrounding electron clouds are in some way mutually repulsing, this way keeping the spatial matrix of mass-spring elements relatively stable, what eventually creates liquids and solids. Such spatial mass-spring formations will have many natural, parametric, and resonant (electromechanically coupled) frequencies or resonant modes and states. Mentioned ***spatial matrix structure is a kind of (dominantly electromagnetic) standing waves formation with recognizable spatial periodicities, and such structuring naturally extends or propagates towards macro universe of planets and galaxies*** (somewhat like what Nikola Tesla and Rudjer Boskovic conceptualized long time ago. See the second chapter of this book regarding planetary spatial periodicities). See much more under [97] and [117].

3. The bottom line of an oversimplified modeling (of metals' mechanical structure) is that masses or atoms (within mentioned solid structures) are relatively stable and fixed particles (or nodes) mutually connected with springs, each of them being surrounded (or enveloped) with shells of relatively free electron clouds, or electron states that are weakly connected to mentioned atom masses. This is the background and simplified modeling situation for having electro-conductivity (as well as to have all other kinds of matter-wave and thermodynamic motions inside metals, realized when atoms are mutually communicating with Electrons, Photons, Phonons, Plasmons, Magnons, Solitons etc.). Most of mentioned matter wave-groups or wave-packets are mutually and coincidently present, connected, or electromagnetically coupled in the same solid body. Here, we should not forget that solutions of Classical Wave Equations are always present in pairs (like inwards and outwards traveling waves, propagating in mutually opposed spatial-temporal directions). "Quantum ***"extended-meaning-of-entanglement***" effects could connect mentioned matter-wave packet-pairs, (meaning Electrons, Photons, Phonons, Plasmons, Magnons, Solitons etc., are always being created as coupled and mutually communicating pairs). Unfortunately, contemporary Physics still did not make such generalizations or uniting concepts, but regardless of our theories, we know that Nature or our Universe is already united, without the need to fully respect all our, still evolving theories.

4. Let us now imagine that (in a certain way) we make an external, mechanical, ultrasonic excitation of the mass-spring spatial matrix (of a certain electro-conductive body). This external mechanical mass excitation could be produced with an attached piezoelectric transducer (or resonator) or by laser pulsing agitation (or by creating different oscillators and resonators based on oscillations and waves of Electrons, Photons, Phonons, Plasmons, Magnons, Solitons etc.). First, we need to ask ourselves about the most probable frequency-intervals or spectral domains where natural and parametric (mass-spring), mechanical and electromechanical resonant states of metal bodies exist. The reason is that we then have better chances to produce and detect new, exotic, challenging and still non-discovered effects in solid matter,

appearing around the existing natural resonant states of the relevant matter structure (here meaning spatial matrix of mass-spring oscillators).

Mechanical, thermodynamic, molecular, electro-mechanic and atomic micro mass's spectral domains (based on spring-mass, or inductance-capacitance conceptualization) are known to be in a deep infrared spectrum and in the MHz domain of frequencies. Of course, every different metal or crystal, or even liquid and plasma state, will have different (mechanical) resonant frequency modes and part of such resonant states will be in a MHz domain (see [118] and http://grdelin.phy.hr/%7Eivo/Nastava/CvrstoStanje/arhiva/udzbenici/Upali_Siriwardane/586c4.htm).

What will happen if we externally agitate metal masses with MHz resonators operating over the relevant parametric, mechanical, and natural resonant states of internal mass-spring matrix structure? Masses or atoms will start resonating (as particles) and will achieve large oscillating amplitudes, velocities, and forces (depending on resonant states conditions, like operating in series or parallel resonance). Since free electron clouds and weakly connected electron states surround the mentioned atom masses, then the applied stimulating mechanical resonance will also create increased mobility of such free electron states. This should have an influence on all related electric currents, voltages, charging and discharging effects (depending on the subject matter). The condition required to excite and produce these enhanced, or modified, electric-current and voltage properties would be to produce an external mechanical, resonant agitation of conductors, this way influencing and agitating specific internal mechanical resonant zones (meaning operating on specific MHz frequencies). Such external, vibrational excitation will also create some heating and self-organizing, stress-relief and restructuring of matter structures, affecting material properties.

As we can find in various references, it is already known that MHz domain ultrasonic excitation (at the correct natural, mass-spring, and parametric resonating frequencies) of electrodes in batteries significantly enhances the produced current and voltage, or accumulated power, and optimizes the charging and discharging properties. Something similar is also measured when an electrolyte is ultrasonically vibrated in electrochemical and galvanic cells. It is also known that laser or photonic excitation of liquids and solids produces similar acoustic, heat, and matter-wave effects as in the already mentioned situations.

Here, we consider mass-spring spatial matrix structure of matter as a standing waves structure, where atoms are in relatively stable nodal positions, and springs are analogically representing kind of balanced electromagnetic attraction between atoms (see [118] and [119]). In addition, we could conceptualize that distance between every of neighbors' atoms is equal to certain mechanical or acoustic (or ultrasonic) resonant half-wavelength. We also know that specific, resonant, external ultrasonic agitation will have a significant influence on the mentioned mass-spring structure. It is also clear that the mechanical oscillations of a mass-spring structure will generate a certain amount of heat energy. We also know that an external electromagnetic *radiation or radiative matter excitation (in infrared and microwaves domain of frequencies) will affect the same mass-spring structure and generate heating. Intuitively concluding (if we would like to produce vibrations and heating of mentioned mass-spring structure), it is clear that wavelength of an external electromagnetic radiation* λ_{em} *should be similar (or equal) to the relevant wavelength of natural mechanical resonance* λ_{us} *(since here we also have strong electromechanical coupling effects). This kind of thinking will give us (as an example) a rough estimation about the relation between relevant ultrasonic frequency* f_{us} *and frequency of external electromagnetic radiation* f_{em} *(necessary to produce the same heating effects, which are effects of mechanical resonance within mass-spring electromechanical resonators in question), as follows,*

$$\left\{\begin{array}{l} C_i = \lambda_i f_i \neq C_j\,,\ \ \lambda_i = C_i / f_i = C_j / f_j = \lambda_j \text{ (=) intuitive and initial assumption} \\ C_{em} = 3\cdot 10^8 \left[\frac{m}{s}\right] \text{ (=) speed of electromagnetic waves in vacuum} \\ C_i = C^*_{em} \in \left(10^8\,,\,3\cdot 10^8\right)\left[\frac{m}{s}\right] \text{ (=) range of estimated speeds of electromagnetic waves in} \\ \qquad\qquad\qquad\qquad \text{different materials} \\ C_j = C_{us} \in \left(10^3\,,\,5\cdot 10^3\right)\left[\frac{m}{s}\right] \text{ (=) estimated speed of ultrasonic waves in solids and liquids} \\ f_{em} \,(\approx) 10^{12}\ \text{Hz (=) some frequency between infrared and microwave radioation} \\ \lambda = \frac{C_{us}}{f_{us}} \cong \frac{C^*_{em}}{f_{em}} \approx \lambda_{em} \approx \lambda_{us} \end{array}\right\} \Rightarrow$$

$$\Rightarrow f_{us} \cong \frac{C_{us}}{C^*_{em}} f_{em} \in f_{em}\cdot(0.333\,,\,5)\cdot 10^{-5}\ \text{Hz} \ (\approx)\ 10^{12}\cdot 10^{-5}\cdot(0.333\,,\,5)\,\text{Hz}\ (\approx)$$

$$(\approx)\ (0.333\,,\,5)\cdot 10^7\,\text{Hz} \approx (3.33,\,50)\,\text{MHz}\,.$$

We already know that modal, mechanical, ultrasonic, electromechanical, electromagnetic, molecular, and atomic resonance effects in certain object will cower different frequency intervals. For instance, if we have a means to radiate an object with remarkably high frequency of electromagnetic waves, covering its molecular and atomic natural (and resonant) frequency range (with periodical frequency modulation within the same frequency interval), and if at the same time we apply low frequency, amplitude, or pulse-width modulation, covering much lower mechanical or ultrasonic resonant frequency range of the same body, we could expect to produce very unusual effects. Such effects will be stable under some low and moderate oscillating amplitudes, but after passing a certain amplitude threshold, multilevel resonance could become non-linear, destructive, and on some mechanical way it will decompose or soften a mass under such specific multifrequency treatment, as it was the case with John Hutchison effect (the same effect, if really and independently confirmed, could be exceptionally good supporting example here; -see [143]). Ideas just presented could initiate creation of many new applications and technologies, since acoustically and using electromagnetic waves (or combining both on different resonant frequencies) we can realize masses heating, stress relief and *transformations and modifications of electric and mechanical properties of solid objects.*

5. We should expect acoustically (or ultrasonically) enhanced electrical properties in applications related to ***batteries (or accumulators) charging and discharging, in optimizing electrochemical and galvanic or electrolytic reactions, in electrolysis, in maximizing Compton and Photoelectric effects, and in familiar situations related to lasers, semiconductors, and photovoltaic cells***. We can always expect certain electric resistance variations (like in NTC and PTC resistive elements) in electro-conductive metals (and in other matter states) under the relevant MHz, resonant excitation of internal and natural resonant modes. In other words, internal mechanical resonant states or modes should be (intrinsically and coincidently), coupled with similar resonant states of electromagnetic nature. Electrostrictive (piezoelectric) and magnetostrictive matter properties and effects are in close relation to here discussed electromechanical couplings and spatial mass-spring matter structures. For instance, in case of correct resonant excitation of photovoltaic (or solar) cells, we could (still hypothetically) expect that the incident photons will create an increased amount of electron flow (because electromechanical and parametric resonance effects will periodically reduce gaps between nonconductive and conductive zones, and current of electrons will be increased). This also means that the efficiency of solar cells would be significantly increased. Positive effects of ultrasonic resonant optimization and energy efficiency (or electrochemical activity) improvements on batteries, accumulators and galvanic electric sources are already known and largely, experimentally confirmed. There is also a certain similarity between thermal motions and ultrasonic matter agitation. For instance, for a thermal activity we can say that this is kind of chaotic and random (or omnidirectional) oscillatory motion, and for ultrasonic agitation, we could say that this can be closer to highly organized harmonic and periodical matter excitation. Anyway, the effects of thermal or ultrasonic activity (on matter or masses) could be similar and mutually interfering. By reducing temperature of certain body (including liquid and gaseous states), and/or by ultrasonically resonating it (of course, when at the same time we activate specific resonant states of matter under such treatment), we should be able to produce effects of superconductivity, superfluidity and Bose-Einstein condensates (what is really happening, since such effects are experimentally known).

Let us now demonstrate how we could compare or unify wave functions concept and Classical wave equation with electric circuit analysis. For instance, in chapter 1. (elaborating about analogies in physics), for the series connection R-L-C circuit, Kirchoff's Voltage Law states (see equation (1.1): "**The sum of all the *across element voltage differences* in a loop is equal to zero."** Based on the situation in Fig.1.1 a), we shall have:

$$\sum \mathbf{u}_i = 0,\ \mathbf{u} = \mathbf{u}_L + \mathbf{u}_R + \mathbf{u}_C = \mathbf{L}\frac{\mathbf{di}}{\mathbf{dt}} + \mathbf{Ri} + \frac{1}{\mathbf{C}}\int \mathbf{idt} = \mathbf{L}\frac{\mathbf{d}^2\mathbf{q}}{\mathbf{dt}^2} + \mathbf{R}\frac{\mathbf{dq}}{\mathbf{dt}} + \frac{\mathbf{q}}{\mathbf{C}},\quad (\mathbf{i} = \mathbf{i}_L = \mathbf{i}_R = \mathbf{i}_C = \frac{\mathbf{dq}}{\mathbf{dt}}), \tag{1.1}$$

To create deterministic and dimensional (power-related) wave function (as conceptualized in this book; -see equations (4.0.82) in chapter 4.0), let us multiply both sides of the equation (1.1) with the relevant current i. This way we will get,

$$[u]\cdot i = \psi^2 = \left[L\frac{di}{dt} + Ri + \frac{1}{C}\int idt\right]\cdot i = \left[L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}\right]\cdot i \Rightarrow$$

$$\psi^2 = Li\frac{di}{dt} + Ri^2 + \frac{i}{C}\int idt = Li\frac{d^2q}{dt^2} + Ri\frac{dq}{dt} + i\frac{q}{C} = L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}$$

$$\Rightarrow 2\psi\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial t}\left[L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}\right] \Rightarrow$$

$$\frac{\partial\psi}{\partial t} = \frac{1}{2\psi}\frac{\partial}{\partial t}\left[L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}\right] = \frac{1}{2\psi}\frac{\partial\psi^2}{\partial t}$$

$$\frac{\partial^2\psi}{\partial t^2} = \frac{1}{2}\frac{\partial}{\partial t}\left(\frac{1}{\psi}\right)\frac{\partial\psi^2}{\partial t} + \frac{\partial\psi}{\partial t} = -\frac{1}{2}\left(\frac{1}{\psi^2}\frac{\partial\psi}{\partial t}\right)\frac{\partial\psi^2}{\partial t} + \frac{\partial\psi}{\partial t} = \frac{\partial\psi}{\partial t} - \frac{1}{\psi}\left(\frac{\partial\psi}{\partial t}\right)^2$$

We already know that Classical wave equation (see more in chapter 4.3) has the following form,

$$\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial r^2}(=)\nabla^2\Psi,\ c = \omega/k\ (=)\ \text{phase velocity},$$

where c should not be mixed with the constant velocity of electromagnetic waves or photons. Usual symbol for a phase velocity in this book is u, but since here we are using u for voltage, we selected c for a phase velocity. Of course, again we will consider mutual relations between phase and group velocity as overwhelmingly present in this book, $c = \omega/k,\ v = d\omega/dk,\ v = c - \lambda dc/d\lambda$.

In fact, we know that temporal and spatial wave-function domains (and relevant spectral functions) of the same event are mutually coupled or related (as already elaborated in this chapter). This way, judging mostly by intention to establish an analogy, we will create an extension of Kirchoff's Voltage Law to certain (here relevant, but still not specifically described) spatial coordinate system. We also know that the same example of series connection of capacitive, inductive, and resistive elements (R-L-C), has its natural resonant frequency where relevant current and voltages are oscillating simple-harmonically (meaning that in a time domain we already have oscillations or matter waves). Corresponding, analogically produced Classical wave equation (for the same example) will be,

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{1}{c^2}\left[\frac{\partial\psi}{\partial t} - \frac{1}{\psi}\left(\frac{\partial\psi}{\partial t}\right)^2\right] = \frac{\partial^2\psi}{\partial r^2}(=)\nabla^2\psi = \Delta\psi = -k^2\psi,\ c = \omega/k,$$

where ψ describes kind of flow of relevant electric power, charge, or electric field, since here,

$$\psi^2 = Li\frac{di}{dt} + Ri^2 + \frac{i}{C}\int idt = L\frac{dq}{dt}\frac{d^2q}{dt^2} + R\left(\frac{dq}{dt}\right)^2 + \frac{q}{C}\frac{dq}{dt}.$$

Of course, additional elaborations will be necessary to develop, extend, and generalize here introduced ideas about wave functions and relations between spatial and temporal domains (since we will need to organize the same electric circuit as an electric waveguide line), but intuitive and analogical grounds for such projects are already paved.

Everything relevant in our physics is about networks and circuits of electric and mechanical active signals and waves, such as currents, voltages, forces, velocities, power, energy etc. This is the largest platform of relevance for analyzing and using wave functions and wave equations in physics. This way we can extend our conceptualizations in physics from simple and easy, perceptually tangible events and objects in motion, towards a higher level of mathematically constructed systems and models. ♣]

We now know that ***all kind of Uncertainty or Inequality Relations are related to ordinary mathematics and signal sampling, signal reconstructing and spectrum analysis, being universally valid and applicable to any signal or wave function (meaning to a micro and macro universe). We also know that Heisenberg Uncertainty relations are a simple mathematical consequence of well-known inequality relations between durations of original and transformation domains of certain wave function or signal, presenting natural unity and good mathematical modeling between involved Physics and Mathematics***. Contrary, in most Quantum Theory books, this is wrongly presented as a divine, independent, and unique enlightenment of W. Heisenberg, being exclusively applicable to a world of microphysics. For more profound elaborations of the same problematic, see Chapter 5.

Probability theory and Statistics in relation to contemporary Quantum Theory, ***is mostly related to the universal applicability of Parseval's theorem, and to a conceptual similarity, and non-contradictory compatibility between "normalized and averaged forms" of conservation laws of physics (like total energy and momentum conservations), and to the trivial fact that sum of all particular probabilities of certain event is equal to one, or to a total event probability.*** Of course, borrowed, invented, fitted, and postulated, surrounding mathematical environment (regarding probabilistic grounds of Quantum theory), has been tricky and demanding to make such mathematical conceptualization working well (see more in "4.3.3. Probability and Conservation Laws", from the Chapter 4.3). Anyway, Signal, Spectrum Analysis, Statistics, and Probability Theory have been perfectly and independently developed, self-standing and rich, ready-made theories, being very convenient to be used, slightly adjusted, hybridized, or absorbed by the contemporary statistical and probabilistic Quantum theory). When conditions for using statistics and probability theory on events with big numbers of similar (or identical) entities are naturally met, this could serve as a framework for very correct fittings, modeling, and conclusions in Physics (and in all other natural sciences). Parseval's theorem (see chapter 4.0) is also universally applicable in such situations and effectively mimics the energy conservation law (since relevant square of involved wave function can be normalized, becoming like a total probability law…). Another important and natural background is that most of stable molecular, atomic, and subatomic structures are formations of spatially structured standing matter waves with certain kind of intrinsic, quantized and relatively stable (or like fractals repeatable) periodicity and internal symmetry. Whenever mentioned natural and intrinsic periodicity and causality exists, we will be able to create mathematically operational, "Ptolemy type" theories (analogically to geocentric, or Ptolemaic system) by assembling certain conceptually wrong phenomenological picture, while inventing and associating rules and motions that are conveniently complementing, compensating, rectifying and eventually respecting involved, real, and natural, basic, and natural system periodicity. The good example of such Ptolemaic practices is Bohr's hydrogen atom model, which is conceptually wrong, but producing number of good and useful results (as well as many other, also well operating atom models; -see more in chapter 8.). All of that is still being a background of later stochastic and probabilistic upgrades and invented (or postulated) foundations of contemporary Quantum Theory.

Many of strange and abstract multi-reality or magic-reality consequences, understanding and popularizations of contemporary Quantum theory are products of invented, hybridized and postulated, also well operating, but partially not a natural mathematical framework. Of course, such abstract theory still works sufficiently or very well, but it should not be glorified as the divine, brilliant, and ultimate achievement of the contemporary Quantum theory or Physics as a natural science.

What founders of contemporary Quantum theory realized is successful subject and target manipulations, taking several of grandiose, independent, and self-standing mathematical theories like already proven and universally valid mathematical tools, and misrepresented such cocktail of cleverly combined concepts and tools as almost unique and brilliant product of Quantum theory. If by Quantum theory applied and combined mathematical tools are so powerful and non-doubtful (what is being presented as the fact in Quantum Theory literature), this is not giving an absolute guarantee, confidence, and certificate that builders and artists (who are using such tools) will really create eternally valid and perfect theory. Anyway, some of the artists will never create a masterpiece regardless of how rich they are with modern painting tools. What we too often read and hear is, that contemporary Quantum theory is being continuously and fully confirmed. This is, in fact, equivalent to the confirmations that Statistics and Probability theory (appropriately merged with Signal and Spectral Analysis) are really working perfectly well when conditions for such mathematical modeling and processing are met. This is an obvious and trivial statement, always valid in all scientific and non-scientific disciplines, where we correctly apply Statistics, Probability, and Signal Analysis (of course including conservation laws of Physics). *For more profound elaborations of the same problematic see the chapter 5., "4.3.3. Probability and Conservation Laws".*

Practically and briefly summarizing, we could say that it was not necessary to jump too early, too fast, and exclusively, to decisions, conceptualization, and postulations that everything in a micro world of physics has an essential and ontological, probabilistic and statistic nature. Such strategy and methodology unavoidably created an artificial, hybridized mathematical theory, like an intermediary link, interface or filtering mathematical toolbox, and later, to come back to real physics and relevant results, it was necessary to analogically apply or associate well known and always working laws of mechanics, Variational Principle of Least or Stationary Action, and Hamilton-Lagrange formalism to such artificial theory, to make it function very well. From signal analysis and particle-wave conceptualization, as elaborated in this book, we can see that healthy, natural, and almost ordinary mathematics, that is well and directly representing something tangible, what exists in the world of Physics, is already rich to give much more direct and explicit insights about wave-particle duality and matter waves than probability-based intermediary transformations, mapping, and concepts. Probability and Statistics toolbox is also mathematically powerful and rich. Anyway, it should be applied, primarily and dominantly, only where it naturally belongs, and where it is producing *exceptionally good results, like in Thermodynamics (to describe and model statistical sets with a big number of similar elements). Of course, when creating artificial mathematical concepts and theories based on statistics, we can always normalize involved wave functions by creating non-dimensional functions. Later, we can always apply probability and statistical methodology,* ***since conservation laws in Physics are on some way already represented by the probability law saying that the sum of all probabilities of a certain complex event is always equal to one (to 1), and this, when combined with Parseval theorem, is creating a particularly good theory, on some way isomorphic to classical Physics*** *...*

The differences and similarities between the Orthodox Quantum theory and concepts promoted in this book could be summarized as:

1. *Orthodox (by Copenhagen, consensus created) Quantum theory is dominantly based on statistics and probability concepts and modeling around non-dimensional wave functions (being conveniently merged with Fourier signal analysis). In this book is promoted mathematical modeling of the power-related wave function and energy-moments functions based on Analytic Signals and Kotelnikov-Whittaker-Nyquist-Shannon sampling theorem (see more in the Chapters 4.0, 4.1 and 4.3). Such mathematical framework is offering much clearer, richer, causal framework for wave-particle duality analyzes than specifically hybridized statistical and probability-based interpretations of Quantum theory.*

2. *Quantizing in Orthodox Quantum theory is on some intuitive and simplistic way assumed, conveniently postulated, and descriptively and superficially mathematically generalized, to fit always-valid Fourier signal analysis and statistics laws (of course, also based on experimentally supported grounds). In this book, Quantizing is dominantly linked to (by integers countable) standing-waves segments and energy-moments formations, and related interactions between such atomized, resonant, or standing waves structures, while respecting Kotelnikov-Whittaker-Nyquist-Shannon sampling and signals reconstruction or recovery mathematical concepts (see more in the chapter 4.0, and in [57, 58, 59, 118 and 119]).*

3. *Orthodox Quantum theory is vaguely and implicitly (often with some ambiguities) assuming or promoting that total particle energy (or its energy-moments, equivalent matter-wave state) presents its total matter-wave content. In this book, it is explicitly and precisely underlined that <u>only motional or kinetic energy (of any kind) presents matter-waves related content</u> (where standstill, stable rest-mass energy-content does not belong). Consequently, and wrongly, relations between group and phase velocity (of matter-wave packets) in Orthodox Quantum theory are often showing that phase speed could be higher than the speed of light (in cases when we consider that matter-wave energy covers the total particle energy). However, if we consider that only kinetic energy equals to matter-wave energy (as proven in this book), both group and phase speeds are never higher than the speed of light. Classical and Quantum Physics and Mechanics related to motional masses and particles, to associated moments, forces and velocities is still not exploiting such basic entities as complex Analytic signal functions, or* ***Phasors****, like it is the case in electric circuits theory, this way being "theoretically and conceptually blind", and non-selective regarding explaining complexity of matter structures, including mechanical and wave motions. See more about complex Phasors at the end of Chapter 4.0. In addition, to compensate missing concepts and items necessary to maintain universal validity of General Relativity theory (regarding macro-Universe), dark masses and dark energy needed to be postulated (still without any experimental verification but being mathematically useful). Since we know that a static mass is not the primary and unique source of gravitation (see more in the first and second chapter), all other concepts and theories developed on the grounds that static mass is the primary source of gravitation (like Special and General Relativity, Black holes, Dark energy and Dark masses, Big Bang hypothesis, Hubble constant related expansion of the Universe…) could have some seriously weak spots. Intrinsically vibrating masses and vibrating Universe is most probably the case explaining gravitation.*

4. *Wave-Particle Duality (in Orthodox Quantum theory) is mathematically merged with Statistics and Probability concepts, and with Heisenberg Uncertainty relations (see more in chapter 5). In this book, Wave-Particle duality is primarily related to PWDC (meaning Particle-Wave-Duality-Concept or Code), as largely elaborated in Chapter 4.1, essentially being formulated, and exploited without statistics and probability background. Instead of Heisenberg Uncertainty Relations, in this book is promoted that we should always start from (not statistical) mathematical uncertainty relations or inequalities between absolute and total, original and spectral signal durations (and later to deduce relevant Heisenberg Uncertainty relations by combining mathematical Uncertainty with mentioned PWDC; -see more in Chapter 5.). After establishing mathematical inequalities between mutually conjugate domains-durations, we will be able to relate such results to relevant statistical durations, and to sampling and total signal interval durations (in all relevant domains) in compliance with Kotelnikov-Whittaker-Nyquist-Shannon sampling theorem. In fact, real dimensions of an object, particle, or motional mass are not equal to what we see and consider mechanically and geometrically as dimensions and size of a solid body in question. Any motional mass usually has different effective, electromechanical, electromagnetic, and associated matter-wave size, shape, and dimensions, compared with mechanically stable, solid-body descriptions, and this is very much neglected when Uncertainty relations in Physics are being exploited. Here, we still miss good formulations and specifications in relation to temporary introduced names like "effective, electromechanical, electromagnetic and associated matter-wave size and dimensions", but the message about the proposed working items is sufficiently clear. Statistical modeling in Orthodox Quantum theory is on some way serving to compensate such hidden-variables situation (as missing phase, and unknown effective shape and dimensions), but this kind of stochastic mathematical modeling also has its limits regarding conceptual and qualitative clarity. However, we should not forget that Probability and Statistics are equally well applicable in "every kitchen", or in every of natural and other sciences, and everywhere else, where we can operate within big sets of similar or identical objects and events (when basic mathematical conditions for such mathematical processing are met). Such mathematical approach is like using exceptionally good and absolutely proven mathematical tools and nothing else regarding associated ontological meaning. It would tell a lot about events tendencies, distributions, main values, deviations, best fittings, etc., but the conceptual and qualitative picture about the phenomenology we analyze should be either, well known in advance, or it will not be discovered and sufficiently understood only by applying Statistics and Probability. This is the case of the present Quantum theory. If, instead of Probability and Statistics we consider, as dominant importance, to use Analytic Signal modeling, and associated signal analysis options, we will get much more natural, clearer, richer, and deterministic Quantum theory, or much better and natural understanding of particle-Wave Duality. Of course, the mathematical toolbox of Probability and Statistics will always remain the unavoidable and best mathematical support (whenever applicable).*

5. *Orthodox Quantum theory is postulating Schrödinger wave equation, as being kind of a divine and extraordinary enlightenment-creation of E. Schrödinger. In this book it is simply, deterministically, and explicitly, shown that Schrodinger's and familiar wave equations of Quantum Physics can be developed around universally applicable Classical Wave Equation when wave function is treated as a Complex, Analytic Signal (see more in Chapter 4.3).*

In number of publications related to Quantum theory, here mentioned similarities and conceptual and theoretical differences are often and arbitrarily, creatively, and imaginatively, exploited and mixed within different interpretations of Orthodox Quantum theory, creating a difficult situation to have clear and explicit differentiation between what is old and new, or what are classical and innovative concepts about Wave-Particle Duality. Most of traditional and innovative Quantum theory interpretations are applicable to the same reality, and only fine, systematic, and selective analysis could determine when and why concepts promoted in this book are different and better.

Self-organizing synchronizations of motions and matter states in Physics (or in our Universe), including living species, are noticeably clear and significant, empirical (experimentally verifiable) manifestations of matter waves and particle-wave duality.

Citation from Wikipedia: https://en.wikipedia.org/wiki/Self-organization .

Self-organization, also called (in the social sciences) spontaneous order, is a process where some form of overall order arises from local interactions between parts of an initially disordered system. The process is spontaneous, not needing control by an external agent. It is often triggered by random fluctuations, amplified by positive feedback. The resulting organization is wholly decentralized, distributed over all the components of the system. As such, the organization is typically robust and able to survive or self-repair substantial perturbation. Chaos theory discusses self-organization in terms of islands of predictability in a sea of chaotic unpredictability.

Self-organization occurs in many physical, chemical, biological, robotic, and cognitive systems. Examples of self-organization include crystallization, thermal convection of fluids, chemical oscillation, animal swarming, and artificial and biological neural networks.

Self-organization is realized[2] in the physics of non-equilibrium processes, and in chemical reactions, where it is often described as self-assembly. The concept has proven useful in biology,[3] from molecular to the ecosystem level.[4] Cited examples of self-organizing behavior also appear in the literature of many other disciplines, both in the natural sciences and in the social sciences such as economics or anthropology. Self-organization has also been observed in mathematical systems such as cellular automata.[5] Self-organization is not to be confused with the related concept of emergence.[6]

Self-organization relies on three basic ingredients:[7]

strong dynamical non-linearity, often though not necessarily involving positive and negative feedback

a balance of exploitation and exploration

multiple interactions

The cybernetician William Ross Ashby formulated the original principle of self-organization in 1947.[8][9] It states that any deterministic dynamic system automatically evolves towards a state of equilibrium that can be described in terms of an attractor in a basin of surrounding states. Once there, the further evolution of the system is constrained to remain in the attractor. This constraint implies a form of mutual dependency or coordination between its constituent components or subsystems. In Ashby's terms, each subsystem has adapted to the environment formed by all other subsystems.[8]

The cybernetician Heinz von Foerster formulated the principle of "order from noise" in 1960.[10] It notes that self-organization is facilitated by random perturbations ("noise") that let the system explore a variety of states in its state space. This increases the chance that the system will arrive into the basin of a "strong" or "deep" attractor, from which it then quickly enters the attractor itself. The thermodynamicist Ilya Prigogine formulated a similar principle as "order through fluctuations"[11] or "order out of chaos".[12] It is applied in the method of simulated annealing for problem-solving and machine learning.[13] The thermodynamicist Adrian Bejan formulated the constructal law as the law of physics of design emergence and evaluation in nature bio and non-bio.[14][15][16]

Citation from:** https://bettstetter.com/research/sync/ , from Christian Bettstetter, Professor, Networked and Embedded Systems, Klagenfurt; **Self-Organizing Synchronization:

Time synchronization algorithms that operate in a decentralized manner in large wireless networks are being developed and evaluated. It is demonstrated that the stochastic nature of interactions between devices is a key ingredient for convergence to synchrony. This convergence has been mathematically proven for arbitrarily connected network topologies and variably changing interaction delays. Implementation and experiments on programmable radios show synchronization precisions below one microsecond when applying automatic phase rate correction. The developed solutions were patented and expected to be employed in a resource-limited embedded sensor and robot networks in smart factories and other emerging applications.

There is a broad spectrum of work on pulse-coupled oscillators to model synchronization phenomena in biology, physics, and other basic sciences. A prominent example is swarms of fireflies that synchronize their blinking behavior. The beauty of these synchronization phenomena lies in the fact that system-wide synchrony emerges among the participating entities in a completely nonhierarchical manner without the need for central entities. Such synchronization is scalable with the number of entities and robust against complete failure of entities or appearance of new entities. It is considered as a prime example of self-organization in nature, like the flocking of birds or shoaling of fish—where simple rules in each entity and localized interactions between neighboring entities lead to pattern formation of the entire system.

There is an excellent, complementary, and brief resume, or bottom-line resume, how founders of modern Quantum Theory decided to take Probability and Statistics as the fundamental framework for introducing and explaining quantizing in Physics, and what they, really and mistakenly, created (see [89]).

***Citations (on the next pages) taken from [89]**:*
*"The Universal Force Volume 1, from **Mr. Charles W. Lucas Jr.***
(See below)

2.2 Quantum Mechanics

I think it is safe to say that no one understands Quantum Mechanics.
Richard Feynman [44]

Can the world possibly be as absurd as it appears to us in our scientific theories? Werner Heisenberg [45]

When quantum effects were first noticed in the emission spectra of the atom and black body radiation, physicists had a choice. The primary options were (1) the physical internal structures of the electron and the atom were the source of quantum effects or (2) all objects in nature are point particle quantum harmonic oscillators which can only be described by a mathematical quantum wave function which provides information about the probability amplitude of position, momentum, energy and other physical properties of the oscillator. The decision made by the leaders of the atomic physics community was to go with option (2).

In 1927 quantum mechanics was standardized on the Copenhagen interpretation formulated by Niels Bohr and Werner Heisenberg while collaborating in Copenhagen. Bohr and Heisenberg extended the probabilistic interpretation of the quantum mechanical wave function originally proposed by Max Born. In this interpretation questions like "Where was the particle before I measured its position?" are meaningless. The measurement process randomly picks out exactly one of the many possible states allowed by the quantum state's wave function in a manner consistent with the defined probabilities that are assigned to each possible state. According to this interpretation, the interaction of an observer or apparatus that is external to the quantum system is the cause of wave function collapse to a specific state. Thus, according to Heisenburg, "reality is in the observations, not in the structure (of the electron or the atom)". [46]

The Copenhagen interpretation of quantum mechanics departs from classical physics primarily at the atomic and subatomic scales where the dynamics of the systems can be described in terms of Planck's constant h. It provides a mathematical description of the dual particle and wave-like behavior and interactions of matter and energy. The name "quantum mechanics" is derived from the observation that some physical quantities can only change by discrete amounts, or quanta in Latin. The angular momentum of an electron bound to an atom is quantized. The energy of an atomic electron is quantized resulting in discrete atomic emission line spectra.

This version of quantum mechanics is significantly different from classical science in that there is no basis for the law of cause and effect. All of nature is based 100% on random statistical probabilities. Since no internal physical structure of quantum oscillators is considered in this approach, all elementary particles are treated as point oscillators with no internal structure. Within the Copenhagen version of quantum mechanics the wave-particle duality of energy and matter and the uncertainty principle attempt to provide a unified view of the behavior of photons, electrons and other atomic-scale objects.

The history of quantum mechanics [47] began with a number of different scientific experiments. In 1838 Michael Faraday discovered the existence of cathode rays later identified as beams of electrons by J. J. Thompson in 1897. Next in the winter of 1859-1860 Gustav Kirchoff made statements about black body radiation. In 1877 Ludwig Boltzmann suggested that the energy states of a physical system could be discrete instead of continuous. Then in 1887

48

Heinrich Hertz discovered the photoelectric effect. Finally in 1900 Max Planck made the quantum hypothesis that any radiating atomic system could be divided into a number of discrete energy elements such that the energy ε of each of these energy elements is proportional to the frequency ν with which each of them individually radiate energy, i.e. $\varepsilon = h\nu$ where h is called Planck's constant.

Then in 1905 Albert Einstein explained the photoelectric effect previously reported by Heinrich Heinz in 1887 by postulating that light itself is made of individual quantum particles in order to be consistent with Max Planck's quantum hypothesis. In 1926 these quantum particles were called photons by Gilbert N. Lewis. The phrase "quantum mechanics" was first used in Max Born's 1926 paper "Zur Quantenmechanik der Stoßvorgänge". [48]

In the field of physics quantum mechanics has made a significant impact on the science of the elementary particles, the atom, the nucleus and molecules. Some of the fundamental idealizations in the assumptions or postulates or axioms of the Copenhagen version of quantum Mechanics are

1. **The state of any system is determined by an idealized universal wave function $\Psi(x,t)$**
2. **$\Psi^* \Psi$ gives rise to an idealized statistical interpretation of the universe**
3. **There is no Law of Cause and Effect in the universe**
4. **The Heisenberg Uncertainty Principle $\sigma_x\sigma_p \geq h/4\pi$**
5. **All particles are point oscillators**
6. **There is no deformation or elasticity of particles**
7. **All processes obey the linear superposition principle**

These are all idealizations. Problems with these idealizations are reviewed below.

1. **Experiments have not been able to detect any sort of physical aether to support a universal wave. Thus the universal wave function has been declared non-physical and just mathematical, i.e. probabilistic in nature.**
2. **A non-physical mathematical universal wave function cannot give rise to anything but a statistical interpretation of the universe. The statistical interpretation of the universe is in disagreement with the well-established law of cause and effect.**
3. **The primary purpose of science is to explain effects in terms of causes.**
4. **The quantum uncertainty arises in quantum mechanics due to the idealized matter wave nature of all quantum objects. The uncertainty relationship between any pair of non-commuting self-adjoint operators such as position and momentum are subject to uncertainty limits. These are entirely mathematical in origin, since the universal wave function is not a physical wave. It allows violations of conservation of energy and momentum for a period of time.**
5. **Accelerator particle scattering experiments have shown that massive particles such as the proton and neutron have both a finite-size and an internal charge**

structure consisting of at least three primary structures. See Hofstadter's electron scattering data in Figure 2-1 above. [9]

6. **All experimentally observed massive physical particles have finite-size and are elastic and can deform. Only idealistic unphysical point particles are not deformable.**
7. **Many physical phenomena, such as that giving rise to the laser (Light Amplification by Stimulated Emission of Radiation), are based on non-linear electrodynamic processes. However idealistic point particles cannot participate in non-linear processes.**

Besides having problems with its fundamental axioms, Quantum Mechanics now has problems with the fundamental experiments upon which the theory was initially developed such as the photoelectric experiment, the Blackbody Radiation experiment and the atomic emission spectra. For the Photoelectric experiments see section 2.1.2 above. The Blackbody Radiation experiment and atomic emission spectra are reviewed in the next two sections.

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Reflections about matter-waves, thermal blackbody radiation, and Big Bang concept

Based on generally valid electromechanical and other analogies in physics (see the first chapter), all wave motions and oscillations, known in our universe are kinds of matter waves, being also mutually similar regarding universal mathematical models used to describe them. We only have or see, or measure different manifestations of matter waves, where dominant oscillating properties could be components of electric, magnetic, or electromagnetic fields, or mechanical displacements, velocities, and forces. For all matter waves is common and important existence of certain carrier fluid, or spatial matrix, or matter medium where matter waves will propagate, regardless our ignorance (in some cases) where we do not know what mentioned carrier or matter medium is.

Classical wave equation (second order, partial differential equation) is applicable in cases of all matter waves, including quantum mechanical matter waves, when the most general matter wave description is formulated as a complex Analytic Signal function (see more in chapter 4.3). Schrödinger equation can be easily and simply (causally, without any patchwork) developed starting from a wave function presented in a form of an analytic signal (what is also valid for other wave equations known in Quantum theory). Mentioned wave equations always have two sets of solutions, like waves propagating in mutually opposed directions, inwards and outwards. Property of quantum entanglement is closely related to mentioned solutions of wave equations, while satisfying Conservation laws of Physics.

Thermal radiation and Black Body radiation (Planck formula, Stefan-Boltzmann law...) are related to thermal motions of atoms and other particles, where excited electrons and related photons exchanges have the principal contribution to specific spectral distribution in the form of Planck blackbody radiation formula (see more in chapters 8 and 9). However, since atoms naturally have electron clouds or shells of electrons strongly electromagnetically coupled with positive charges of protons (inside atom cores), consequently black-body radiation curve for electrons should have kind of analogical, electromagnetic imaging, or picture of similar radiation curve related to what is happening inside an atom core. Among (or within) positive charges, we should also have similar frequency-shifted radiation, or spectral distribution curve, since whatever happens with electrons is immediately coupled, or sensed within a positively charged atom nucleus. Blackbody, relict, background cosmic radiation should be much more related to electromagnetically and electromechanically coupled, thermal or radiative events between excited and mutually coupled electrons and protons (since such events should coincidently exist). In other words, there is nothing to do with an over-simplistic assumption and hypothetical concept of a Big-Bang event. Of course, blackbody radiation related to atom core is frequency-shifted and much less intensive compared to similar radiation of electron clouds, because protons are almost 2000 times heavier, compared to electrons (see more in chapters 8 and 9). Also, atom core is too small compared to the spatial distribution of belonging electrons, and oscillating amplitudes within such (positively charged) core should be much smaller than what electrons are producing, but the shapes of the radiating curves in both cases are mutually similar (respecting the same Planck formula). Blackbody cosmic event, considered as being the absolute beginning and source of everything that we know as our universe, is very much simplistic and incomplete concept. All energy-moments events, motions, and transformations in our universe should respect the same conservation laws and have relevant front and last elements, or sources and loads (like in electric circuits theory; -see the first chapter of this *book). Flying, floating, self-standing, open-ends circuits (electrical, mechanical, electromagnetic, and of any kind) cannot exist. Big Bang hypothetical event (as presently assumed) is simply flying, floating, or hanging in an open space, missing its sources and loads. It is easy to imagine something like that, but to take it seriously should be much more problematic.*

Everything else regarding redshifts, Doppler effect, cosmic expansion etc. is universally (on the same way) valid or mathematically presentable for all matter waves and oscillatory motions... Wave-Particle Duality manifestations are, practically and theoretically manifesting the most generally valid unification platform within our universe (see more in the chapters 4.0., 4.1., 4.2., and 4.3.).

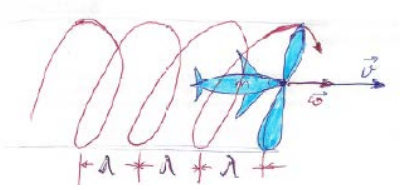
It is too early and prematurely to say (but most probably it is very much correct) that present four of fundamental natural forces would conceptually evolve in a direction that such forces will be explained as effects known among electromagnetic fields and resonant, standing waves structures (while understanding that our Universe is globally, macroscopically, and microscopically vibrating, spinning and rotating).

[♣ COMMENTS & FREE-THINKING CORNER**:* The remaining question to answer is ***what de Broglie, matter waves really are. Regarding matter waves, Contemporary Quantum theory is imaginatively and artificially diving in multi-possible combinations of virtual reality, “probability of possibilities”, abstract entities and assumptions... What we really know, see and measure around us (where matter-waves and particles duality has been observed) are mostly manifestations of electromagnetic energy as photons, light beams, heat waves, and interactions between electrons, protons, photons, and different particles with electromagnetic charges and spinning attributes… Of course, we can also measure macro matter-waves effects in different mechanical motions, oscillations, waves, and similar effects in fluids. In a deep, essential, or ontological background of all what we later specify as particles that are more complex, and matter waves, is anyway an electromagnetic phenomenology or combination and packing of electromagnetically charged entities… Several specific formatting or packing of electromagnetic entities is creating everything else (meaning electrons, protons, atoms, small and big masses…), and consequently, gravitation should be certain secondary, a macro effect of a hidden electromagnetic nature, which is behind. Most probably that we still do not know or master, or accurately describe all manifestations of electromagnetic reality, and some of such not-well-known elements of electromagnetic fields are creating what we presently specify as gravitation (see more in the second chapter of this book). For instance, ***all masses or atoms in our Universe have several elementary constituents (like electrons, protons, neutrons etc.) with spinning and associated angular and magnetic moments, this way always presenting agglomerations of some elementary magnets. In most cases such elementary magnets are chaotically and randomly distributed and being electromagnetically self-neutralized. Some residual, non-compensated and internal (spatially distributed) electromagnetic nature of real masses could be the source, or important ingredient, of gravitation (manifesting as Newton and Coulomb's laws are describing). In the second chapter of this book, we can find the number of striking analogies between planetary motions and N. Bohr atom model (see: “2.3.3. Macro-Cosmological Matter-Waves and Gravitation”), meaning that electromagnetic phenomenology is an essential part of Gravitation or vice versa.***

Electromagnetic fields and waves are on certain simplistic way elaborated or conceptualized in the third chapter of this book (as mutually and perpetually coupled, self-exciting and a fully symmetrical combination of involved electric and magnetic field vectors) … From such understanding of Maxwell equations, we can develop new, better, clearer and generalized Special Theory of Relativity… and probably we can go much further. Familiar and very original, symmetric reformulation of Maxwell equations we can find in publications of Prof. Jovan Djuric, explaining origins of gravitation based on electromagnetic dipoles creation, which is the consequence of center of mass and center of self-gravitation separation, [71].

If we need to imagine what matter waves really are, we could say that all waves and oscillations known in the world of Physics (including acoustic and electromagnetic waves) are matter waves (not only probability and possibility, wavelike functions, and different events distributions). Mathematics for describing and analyzing all matter waves is the same. Particles are also specific and stabilized matter waves formations. The opinion of the author of this book is that essential and unavoidable, necessary ingredient for a stable particle creation, starting from a certain combination of complex matter waves, is that some of the electromagnetic components of matter waves are also involved in such process. Electromagnetic matter waves are serving as a kind of "binding and gluing medium" connecting all other (non-electromagnetic) energy-momentum entities in a process of stable particles creation.

One of the oversimplified analogical ways to visualize de Broglie matter waves creation is to imagine that particle in a straight and uniform linear motion that has mass **m** and kinetic energy E_k presents a propeller-driven airplane (we could also take an example of a propeller-driven submarine; -see the picture below). Inside of an airplane is its engine that is responsible for supplying kinetic energy to its motion. More kinetic energy our airplane engine will produce will be directly proportional to the rotational (spinning) speed of its propeller. ***We can formulate the relation between rotational propeller speed*** ω ***and airplane kinetic energy*** E_k ***as,*** $\omega = k \cdot E_k, k = \text{constant.}$ ***(of course, naturally valid on average, for one full propeller revolution).***



Matter waves analogical visualization will be to imagine certain helix-line or helix-surface guided (or like a spiral mixer) waving motion behind a propeller that, in case of constant kinetic energy per one period (or constant spinning speed of the propeller, $\omega = k \cdot E_k, k = \text{constant.}$) will have elements of periodicity such as wavelength and frequency. Now we will exploit other necessary relations connecting de Broglie matter waves' properties with propeller-created waves (see chapter 4.1), such as,

$$\left\{\begin{array}{l} \omega = 2\pi f = k \cdot E_k, k = \text{constant.} \\ f = \frac{\omega}{2\pi} = \frac{k}{2\pi} E_k\ ,\ \lambda = \frac{h}{p}, E_k = \tilde{E} = hf \\ u = \lambda f = \frac{h}{p}\frac{k \cdot E_k}{2\pi} = \frac{E_k}{p} \Rightarrow hk = 2\pi,\ k = \frac{2\pi}{h} \end{array}\right\} \Rightarrow \omega = k \cdot E_k = \frac{2\pi}{h} E_k = \frac{2\pi}{h}\tilde{E} = \frac{2\pi}{h} hf = 2\pi f\,. \quad (10.10)$$

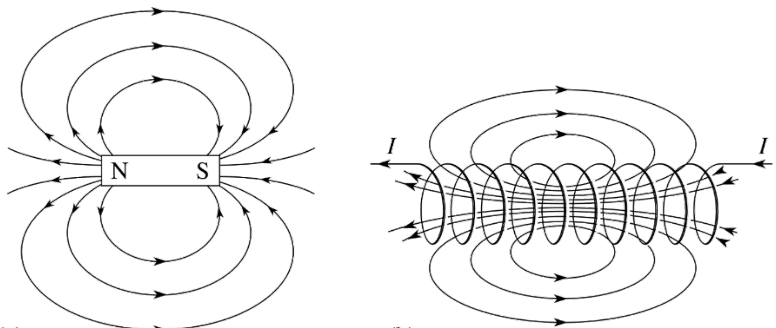
Obviously, certain analogical and mathematical complementarity and compatibility (between propeller rotation and relevant kinetic energy of the linear motion in question) exist in a **PWDC** framework of relations (10.10) applicable for generated matter waves. We could also imagine that our airplane model is imaginatively replaced with an analogic spinning bullet or gyroscope, where spinning is the stabilizing condition contributing to the stability of principal linear motion. ***Here is a place to notice that in such situations we are naturally operating with averaged parameters values since energy amount accumulation is a process that is taking a certain time to be quantified (by integration) until at least one revolution of a propeller***

rotation is achieved. In familiar, but more complicated cases with big numbers of similar participants, it is (in addition) natural to apply Statistics and Probability theory methods (like in modern Quantum theory).

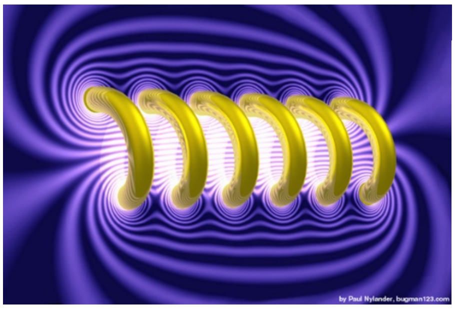
It would be an interesting project to analyze the rotational speed of a real airplane propeller in relation to airplane kinetic energy, considering realistic situation parameters under different conditions.

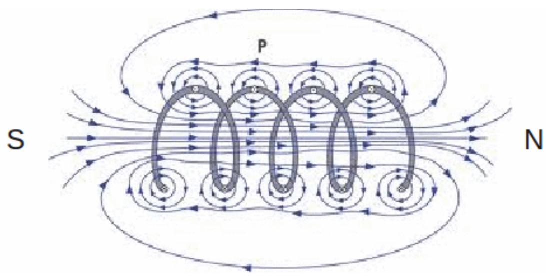
Now, we should ask ourselves what an analogical and exotic propeller should be in cases of real moving particles (which are not propeller-driven airplanes or spinning bullets) and de Broglie matter waves. The other important question (regarding analogical understanding) is what the nature of a surrounding fluid should be since we know that matter waves of micro and elementary particles (like electrons) are present even with motions in extremely high vacuum conditions. Maybe there are multiple answers, but the author of this paper is suggesting a direct involvement of spinning within electromagnetically charged and polarized dipoles (or multi-poles) coupled with corresponding electromagnetic and mechanical moments (already intrinsically incorporated in a moving particle structure), and electromagnetically coupled with a surrounding environment including vacuum conditions. ***Globally, our Universe is slowly moving and rotating (microscopically and macroscopically), and local centers of masses, centers of self-gravitation, as well as global centers of angular moments are not mutually overlapping (or not covering the same spot). This is effectively creating certain electromechanical tension, developing potentials and stresses, producing electric and magnetic dipoles (and maybe multi-poles), and manifesting as what we consider as gravitational phenomenology (see chapter 2 of this book and works of Prof. Dr. Jovan Djuric [71]). In other words, deep, essential, or ontological nature of de Broglie matter waves is linked to electromagnetic one.***

Another example that is visually and analogically implicating existence of a ***unity of linear and spiraling motions*** is related to the geometry of surrounding magnetic fields in a vicinity of conductive wires and solenoids when (helical) electric current (or helical spiral motions of charged particles) is circulating such wires. See self-explaining pictures below.



(a) (b)





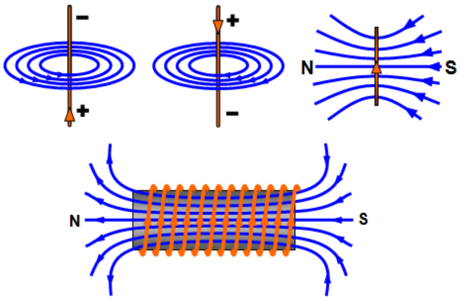
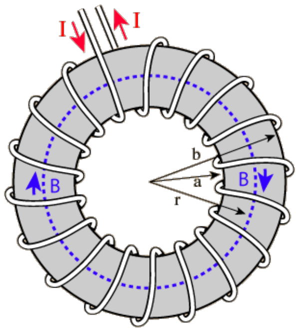
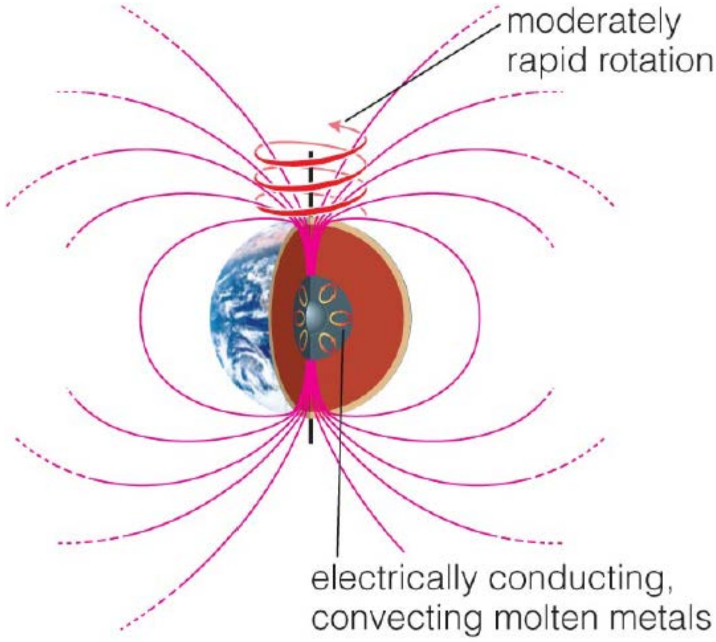


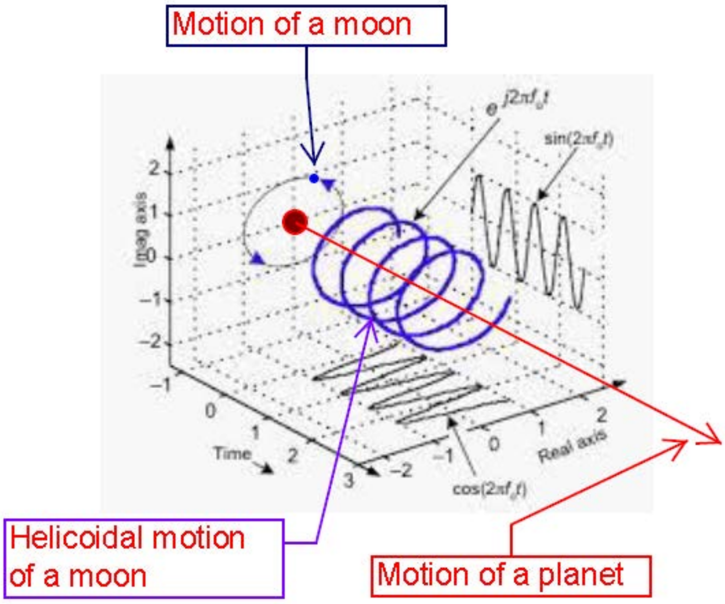
Figure 2





Citation and pictures are taken from the Internet (source or author were not mentioned): A planet can have a magnetic field if charged particles are moving helically inside. There are 3 requirements: 1° Molten interior (such as liquid core), 2° Convection (up/down), and 3° Moderately rapid rotation (around).

If we take into consideration an orbiting planet of a certain solar system that has its moon (or satellite orbiting the same planet), and if we place an observer in the center of such solar system, the moon's orbiting will look like a helical motion (see the picture, below). Of course, electric current and mass motion are not directly comparable, but in both cases, we have certain energy flow because electric charges and masses in motion effectively carry certain energy and power.



An excellent and familiar modeling of the helical electron (very much compatible to here conceptualized helical matter waves, associated to linear motions) can be found in [108]; -Oliver Consa. "g-factor and Helical Solenoidal Electron Model"

Helix and vortex spinning, rotations, waving and oscillations in fluids are also very natural phenomena where we can directly visualize manifestations of matter waves in all cases when we have certain relative motion between a solid body and surrounding fluid in an open space or in a tube. There is the spontaneous tendency of fluid (flow lines) to follow the matter wave's helix or spinning field structure. Vortex flowmeter is based on measuring the frequency of vortices in a passing liquid, and vortices frequency is directly and linearly proportional to a fluid speed, but this could also be represented (analogically and effectively) as kind of de Broglie or matter waves phenomenology (see much more in chapter 4.1). Following pictures are almost self-explicatory, intuitively supporting the idea that vortex phenomenology in fluids (and associated spinning and helix motion) is causally linked to matter waves manifestations (several of the pictures, below, are taken from Martin Roth, Automatic Extraction of Vortex Core. Lines and Other Line-Type Features for Scientific Visualization. ETH Zürich, Institute of Scientific Computing, Computer Graphics Group. Diss. ETH No. 13673, Hartung-Gorre Verlag Konstanz 2000. Other pictures are taken from different Internet sources).

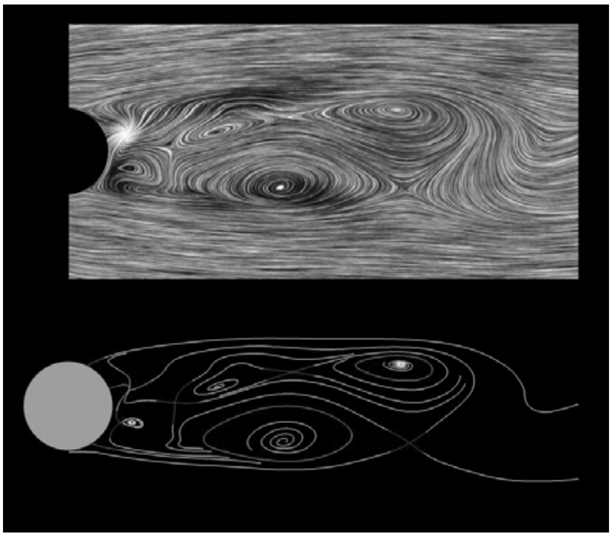


Fig. 2.13: Vector field topology (below) and LIC (above) of the flow past a cylinder (von Karman vortex street).

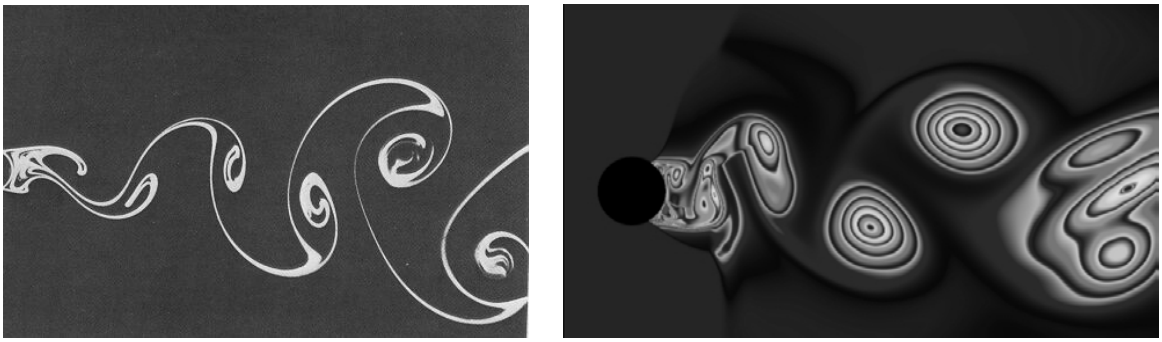


Fig. B.1: Flow behind a cylinder: von Karman vortex street (flow direction to the right).
Left: Photo of dye injected near the cylinder in an experiment
Right: Visualization of computer simulation: contours of vorticity magnitude.

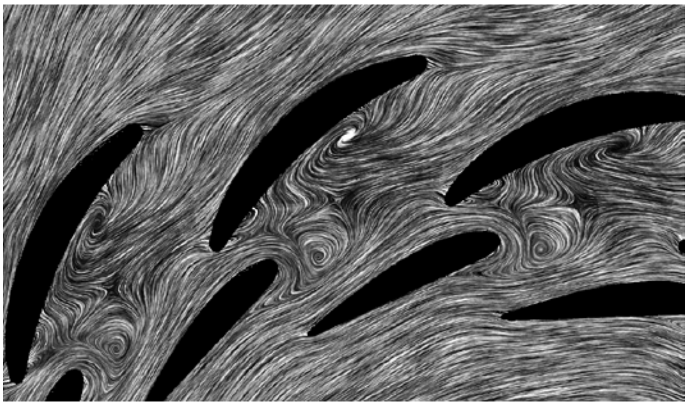
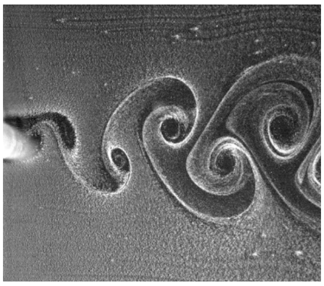
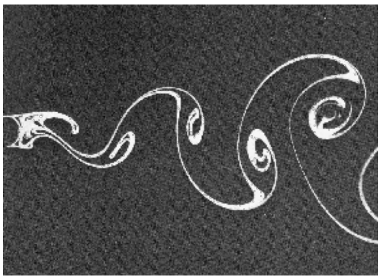
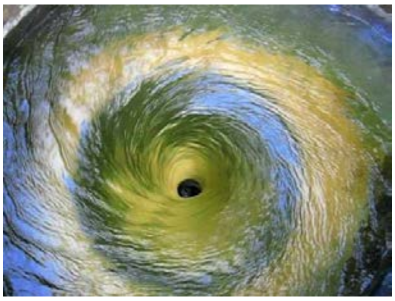


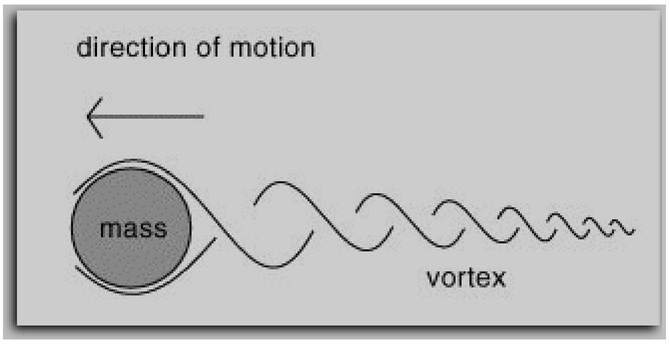
Fig. 2.9: Line Integral Convolution (LIC) image of 2D flow in the stator of a pump-turbine in pump mode (reversed); main flow direction is from lower left to upper right.







Another, significant support to micro and macro matter waves conceptualization is related to gyroscopic or spinning-disk motions. *We know that spinning disk tends to stabilize and keep its position and orientation, and if we could neglect or eliminate associated friction, such spinning motion could be considered as a form of an inertial motion (not at all to be mixed with linear inertial motions established by Newton). In addition, spinning disks can easily be part of certain stable, inertial, linear, Newtonian motion (somewhat like in cases of* *planetary systems). We also know that fast-moving bullets, satellites, and other objects in linear motion are getting increased stability and path control regarding keeping very stable linear motion if such objects are spinning (like gyroscopes).* ***Consequently, we should conclude that there is a natural tendency and coupling making that linear motion and associated spinning is mutually and naturally complementing and stabilizing. In addition, every inertial and linear motion, based on such background, could be coupled with a certain inertial spinning motion, like spontaneous vortex motion of water (or any other liquid) in a tank with an open sink (placed on the tank bottom). Water-sink draining is always creating vortex motion, and we should expect that fluid masses should be sensitive detectors for everything that is related to motions and structure of surrounding fields, matter waves, gravitational and inertial phenomenology.*** *In some cases of linear motions, we do not see, or we simply neglect the existence of associated spinning, but, such hidden spinning is stabilizing its linear-motion carrier, creating helix or spiraling matter waves (behind and around), and producing particle-wave duality effects as elaborated in this book. In reality, most of dualistic, particle-wave and matter waves experimental evidence is related to interactions involving electromagnetically charged particles (as electrons and protons) that also have intrinsic spin and magnetic moment attributes, like gyroscopes or spinning discs (including spinning toroidal forms). Since matter in form of masses and particles structurally has molecules, atoms, electrons, protons, neutrons, and other subatomic and elementary particles, and many of such entities have intrinsic angular and spin moments, we can already imagine what could be our "propellers and gyroscopes" necessary for supporting particle-wave duality and matter waves phenomenology as conceptualized in this book. In a state of rest, mentioned internal and intrinsic spin vectors are, randomly and omnidirectionally distributed. Externally, we do not see or measure resulting spinning effects (because of mutual cancellations). When certain mass is changing its state of linear or inertial motion, and accelerating, its ensemble of internally spinning elements* $\vec{L}_s$ *is somehow getting progressively aligned (as vectors), being in a direct proportionality with the mass velocity or its linear momentum* $\vec{p} = \frac{\omega_s}{v}\vec{L}_s, \vec{L}_s = \frac{v}{\omega_s}\vec{p}$ *and also valid vice versa (see more in the chapter 4.1; -equations (4.3-0)-(4.3-0)-h, and later in this chapter (10.1.3)-(10.1.5)). Practically, the kinetic energy of a moving mass* E_k *is being fully balanced (or followed) with an internal and intrinsic, resulting spinning (or vortex) energy* $\tilde{E} = E_s = hf_s = hf, \omega = \omega_s = 2\pi f_s = 2\pi f$ *being also valid in an opposite order, which here presents matter waves energy (see (10.11)). This way we can imagine or creatively visualize the formation of helically spinning matter waves, as the resulting "thrust force effect of many aligned, elementary spinning elements", each of them having magnetic moments (earlier imaginatively described as gyroscopes, spinning discs, spinning toroid and propellers), where basic matter-waves and particle-wave duality relations (10.1) and (10.2) are naturally applicable.*

$$E_k = \tilde{E} = E_s = hf_s = \begin{Bmatrix} \frac{1}{2}mv^2 \\ \frac{1}{2}J\omega_s^2 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{mv^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{J\omega_s^2}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{pv}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{L_s\omega_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} \frac{\vec{p}\vec{v}}{1+\sqrt{1-\frac{v^2}{c^2}}} \\ \frac{\vec{L}_s\vec{\omega}_s}{1+\sqrt{1-\frac{v^2}{c^2}}} \end{Bmatrix} \Rightarrow \quad (10.11)$$

$$\Rightarrow \boxed{pv = L_s\omega_s \Rightarrow \vec{p} = \frac{\omega_s}{v}\vec{L}_s, \vec{L}_s = \frac{v}{\omega_s}\vec{p}, \vec{F} = \frac{d\vec{p}}{dt}, \vec{\tau}_s = \frac{d\vec{L}_s}{dt} = \vec{p}\cdot\frac{d}{dt}\left(\frac{v}{\omega_s}\right) + \frac{v}{\omega_s}\cdot\vec{F}}$$

In other words, for cases of stable and complex (or multicomponent) inertial motions, it would be possible that all involved, linear and angular (or orbital) moments are on

some way mutually coupled and transformable, and application of laws of different moments conservation cannot be independently applied. Consequently, we need to consider existence of inertial motions that have dominantly conserved and stable angular (orbital and spinning) moments, or dominantly conserved linear moments, and mixed or combined situations, always accounting an associated magnetic field*. Most probably, that particles, atoms, and solar systems are being created in this fashion (thanks to specifically combined linear and angular motions with intrinsic electromagnetic entities).*

As we know, intrinsic spinning moments are coupled with corresponding magnetic moments, and because of certain hidden or holistic rotation within our universe, the matter gets slightly dipole and multi-pole electrically and magnetically polarized this way involving associated electromagnetic complexity and helix fields' structures in the same problematic.

Familiar concepts and ideas can also be extended to gravitation and planetary systems, as elaborated in the second chapter of this book (see chapter 2.3.3). We could also find the oldest and the best historical background regarding innovative understanding of Gravitation (different compared to Newton and A. Einstein theories, and familiar to concepts presented in this book) in the heritage and publications of Nikola Tesla and Rudjer Boskovic (see [6] , [97] and [117]).

In any case, we should have in mind that real frameworks, laboratories, or most significant matter and wave's interaction zones are primarily and dominantly related to mathematically manageable coordinate systems such as local centers of masses, or centers of inertia, centers of self-gravitation, centers of rotation, or centers of electromagnetic charges... Laboratory coordinate systems are only directly and easily accessible to our natural or sensorial perception and convenient for startup approximate and intuitive thinking, but not at all sufficiently well for analyzing and understanding the interaction between matter, particles, and waves. We know that wave-particle duality concepts and probabilistic Quantum Theory started from almost inexplicable experimental situations of wave-like interference and superposition effects in two-slit experiments with particles like electrons. Such experimental situations should have more of invisible (or effective) reaction participants, compared to what we see in a local laboratory system (to be deterministically explicable). Missing effective, virtual and invisible reaction participants here are mathematically manageable objects of local center of mass systems (such as center mass, reduced mass, etc.). ♣]

10.1 Hypercomplex **Analytic Signal functions and interpretation of energy-momentum 4-vectors in relation to matter-waves and particle-wave duality**

Since many of situations and practices in analyzing particles motions and interactions, using energy and momentum conservation laws are anyway theoretically known, mathematically manageable, and experimentally verifiable, here we will simply utilize such well operating energy-momentum, velocity dependent relativistic concepts and formulas until we rich the next, more general level of deeper understanding.

In this book, mass is not presented as the unique and primary source of gravitation (based on analogical conclusions drawn in the first chapter). Only energy-momentum properties of motional masses, with associated linear, angular, electromagnetic, and spinning moments are real and dominant sources or primary causes of gravitation (based on mentioned analogical conclusions). Motional masses also have matter waves properties and complex (or hyper-complex) nature. Consequently, we need to establish certain updated and richer mass conceptualization and quantification, compared to present concepts. Let we give the following intuitive, empirical, and already known theoretical foundations or guidelines from where to start when addressing masses (*as a good background regarding the same field it is recommendable to read "4.3.8. Mass, Particle-Wave Duality and Real Sources of Gravitation",* from Chapter 4.3):

1. All masses in our Universe are in relative motions and mutually connected (externally and internally). Connecting and intermediary items (or interfaces) are different natural fields, forces, and matter waves.

2. Globally and briefly approximating and summarizing, we could say that rest mass is a relatively stabilized and internally neutralized agglomeration of different spinning and electromagnetic states. For complex macro masses, this could be a united and mutually coupled set of atoms, molecules, and other particles and energy-momentum states; -of course, not necessarily all of them being present coincidently for simpler and elementary particles). Internal complex-mass constituents in most cases have spinning attributes, including magnetic and electric moments, and dipoles. If mentioned spinning states, moments, charges, and dipoles are mutually neutralized, or compensated and canceled (as vectors), mass is considered macroscopically and electromagnetically neutral, not charged, and/or not polarized (being also without externally measurable elements of spinning). We do not have such totally compensated and neutral masses in our universe, since all neutralized (or charge compensated) masses are still in relative motions, being mutually connected, having a certain internal temperature, and resulting spinning moments. Practically, in cases of complex macro masses, internal mass motions are presented by everything related to heat energy, to interatomic or intermolecular oscillations, vibrations and other chaotic, electromagnetic, and mechanical, or acoustic motions and mutual interactions of mass constituents. All such internal motions (often externally not very much detectable) are contributing to the rest mass amount, or to its total rest energy content. In cases of elementary and quite simple particles (or wave packets), mass is simply estimated by the amount of its relevant total energy, as $m_x = E_x / c^2$ (obviously being without additional heat energy or vibrating energy components). In some cases of matter-wave states, elementary particles, and quasi-particles, we are not able to find the presence of a stable rest mass, and such states naturally have only motional energy mass equivalents, but anyway, such matter states are also interacting with real particles with non-zero rest masses.

3. All masses are (mutually and holistically), connected within our Universe. Such connections are based on satisfying conservation laws of physics and can be additionally conceptualized by mutual radiant energy exchanges or mass fluctuations between masses. Known forms of mass-energy-moments fluctuations are electromagnetic waves, cosmic rays, radioactive emissions, annihilations and creations of new particles, or positive and negative mass defects based on fluid-flow, impacts and scatterings. Rest mass quantity based on such external, cosmic, and radiant matter wave's exchanges could be balanced and stable, or with a positive or negative trend regarding the total mass content. Of course, here we are dominantly addressing relatively small, or negligible masses fluctuations (when total masses are sufficiently stable and constant). In cases of fluids and plasma states, we have a kind of intermediary, flexible mass states, regarding mass fluctuations… Of course, here we could also introduce

some external, empirical mass qualifications such as solid, plastic, soft, fluidic, elastic, hard, granular, in a form of powders, as micro particles, or also as plasma states… Anyway, whatever can be characterized as having kind of rest mass, has some of the already mentioned properties.

4. Here we could also pay attention to Nikola Tesla's phenomenological and electromagnetic concepts about radiant energy and ether, as the finest state of matter, still existing in an empty, from gases and other particles totally evacuated, or close-to-absolute vacuum space. Even in such vacuum state and space, there is still certain fine, at least two-component fluid, with electro-conductive, and electrically dipole-polarizable masses composed of fine and small particles, mixed with an electrically isolating fluidic mass. Such ether, or fluidic mass, is everywhere around and inside us, including all other tangible masses in our Universe. Tesla performed experiments with DC and AC electrical discharges in highly evacuated vacuum tubes and discovered that remaining gas or detectable mass between two discharging electrodes can be, additionally and totally rarefied and evacuated by externally applied electric field, and by running electric discharge process (that is electromagnetically polarizing and rarefying electro-conductive fine matter fluid including remaining gas particles). What remains in such fully evacuated space is still kind of electro-conductive matter, mass, ether, or fluid since external electric current between electrodes is still measurable. Experiments conducted by Tesla are effectively supporting such ether conceptualization (although we still do not know what mentioned ether components really are). See much more under [97] and [117].

5. Presently, the state of the art in addressing masses or energy-moments complexity is essentially tested and confirmed being particularly useful and correct, as mathematical practices within Einstein-Minkowski 4-vectors concept, from his Relativity theory. Here, we will also use the same 4-vectors platform and implement still not explicitly considered matter-wave properties of mas-energy-moments states. Practically we will update and improve momentum-energy 4-vector and draw an additional, new insight, into more complete mass description and quantification, as follows.

$$\begin{aligned}
&\overline{P}_4 = (\vec{p}, \frac{E}{c}) = (\vec{p}, \frac{E_o + E_k}{c}),\\
&E_o = E_{oo} + E_{ov} + E_{os} = m_{oo}c^2 + m_{ov}c^2 + m_{os}c^2 = m_o c^2 = mc^2,\\
&E_k = (\gamma - 1)mc^2 = \frac{\vec{p}\vec{v}}{1+\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{\vec{L}\vec{\omega}}{1+\sqrt{1-\left(\frac{v}{c}\right)^2}},\\
&E = E_o + E_k = \gamma mc^2 \Leftrightarrow m_o = m_{oo} + m_{ov} + m_{os} = m, \vec{p} = \gamma m\vec{v}, \vec{L} = \gamma J\vec{\omega}, \vec{p}\vec{v} = \vec{L}\vec{\omega},
\end{aligned}$$

where the meaning of mass and energy components is:

$m_{oo} = \frac{E_{oo}}{c^2}$ (=) fully compensated part of the standstill rest mass (meaning neutral mass that has total zero charges and zero-vectors properties). In cases of complex macro masses (with many of atoms and/or molecules), this is the dominant total mass part.

m_{ov} (=) internal vibrational and heat-energy exited mass equivalent,

$E_{ov} = m_{ov}c^2 = \sum_{(i)} \frac{1}{2} m_{ov-i} v_i^2 = \sum_{(i)} \frac{1}{2} \vec{p}_{ov-i}\vec{v}_i$ (=) total, internal and active vibrational and heat energy,

m_{os} (=) total, not-compensated, active, and externally measurable ***spinning energy mass equivalent*** (or part of externally detectable spinning, where spinning moment vector is non-zero).

$E_{os} = m_{os}c^2 = \sum_{(i)} \frac{1}{2} J_{os-i}\omega_{s-i}^2 = \sum_{(i)} \frac{1}{2} \vec{L}_{os-i}\vec{\omega}_{s-i} = \frac{1}{2} J_{os}\omega_s^2 = \frac{1}{2}\vec{L}_{os}\vec{\omega}_s$, (=) total, active and externally detectable self-spinning energy.

Resulting linear mass (or particle) moment is,

$$\vec{p} = \gamma m\vec{v} = \gamma(m_{oo} + m_{ov} + m_{os})\vec{v} = \gamma(\frac{E_{oo}}{c^2} + \frac{E_{ov}}{c^2} + \frac{1}{2c^2}J_{os}\omega_s^2)\vec{v} .$$

Here we implicitly accept (or assume) that all masses or particles' motions are curvilinear, and locally like rotational and orbital motions (of course, apart from self-spinning), since,

$$E_k = (\gamma - 1)mc^2 = \frac{\vec{p}\vec{v}}{1+\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{\vec{L}\vec{\omega}}{1+\sqrt{1-\left(\frac{v}{c}\right)^2}} \Rightarrow$$

$$\vec{p}\,\vec{v} = \vec{L}\vec{\omega} = \gamma(m_{oo} + m_{ov} + m_{os})v^2 = \gamma(\frac{E_{oo}}{c^2} + \frac{E_{ov}}{c^2} + \frac{1}{2c^2}J_{os}\omega_s^2)v^2 = \gamma mv^2 .$$

Until here, we established or assumed the validity of certain facts, for instance by saying that all masses in our universe are in curvilinear (like orbital) motions, and that spinning motion is contributing to the rest mass amount (where linear mass motion is presentable as mentioned orbital motion). We also temporarily neglected the existence of possible external mass exchanges, mass flow, or radiant mass transfer between masses, practically focusing our attention mostly on relatively stable and mass-moments-balanced particles. In addition, we know (or assume) that motional masses are also being presentable and mutually communicating by de Broglie matter waves, where electromagnetic phenomenology could be dominant.

Let us now explore more profoundly motional mass-energy-moments formulas and relations, starting from Minkowski-Einstein 4-vectors. The fact is that A. Einstein Relativity theory, besides establishing a challenging and imaginative (maybe some of them not easy verifiable) concepts, also has few of very practical and useful building blocks and foundations. For instance, concepts related to "inertial frame time", "proper time", and Einstein-Riemann-Minkowski 4-vectors, which are very much productive in analyzing the world of mechanical interactions in physics. Regardless some objections and critics of Relativity Theory, we still cannot find a better replacement, or avoid using mentioned 4-vectors concepts and associated mathematical prescriptions, but we should not forget that much deeper and more general understanding of the same problematic is still in front of us.

The leading idea here is to show that rest mass is not the unique and primary source of gravitation, but mass with linear and orbital moments and electromagnetic charges is causally related to gravitation. Also, it is necessary to underline that motional mass is producing matter waves and/or being presentable as a matter-wave packet, where gravitation could be the simple consequence of attraction properties within standing matter waves field structure (around nodal zones of masses agglomerations). In addition, our cosmos or universe (with all masses inside) should be considered as a spatial (multidimensional) electromechanical, closed-circuit network with inputs and outputs, or with front-end sources and last-end loads, where naturally, all conservation laws of physics should be satisfied. Consequently, all masses in our Universe are on some way structured, mutually connected, and communicating. Such conceptualization is familiar with Nikola Tesla's Dynamic Gravity ideas, [97, 98, 99, 117], and with Rudjer Boskovic's universal natural force description, [6].

For instance (see in the second chapter of this book), we can analogically create the following table of matter attributes and properties, T.10.1 (here analogically and imaginatively named as 4-scalars, or complex numbers, based on known 4-vectors) that on some way mathematically mimic Minkowski-Einstein 4-vectors formalism,

T.10.1, (T.2.2-3)

	Ref. Frame Values	Symbolic 4-Vectors	Invariant Expressions
Time	$dt = \gamma d\tau$	$\overline{\tau} = (dt, \frac{dr}{c})$	$d\tau^2 = dt^2 - \frac{dr^2}{c^2} = inv.$
Mass	$M = \gamma M_0$	$\overline{M}_4 = (M, \frac{E/c}{c}) = (M, \frac{p}{c})$	$\overline{M}_4^2 = M^2 - \frac{p^2}{c^2} = M_0^2 - \frac{p_0^2}{c^2} = inv.$
Energy	$E = \gamma E_0$, $E_0 = M_0 c^2$	$\overline{\varepsilon} = (E, cp)$	$\overline{\varepsilon}^2 = E^2 - c^2p^2 = E_0^2 = inv.$
Momentum	$p = \gamma p_0$, $p_0 = Mv$	$\overline{P}_4 = (p, \frac{E}{c})$	$\overline{P}_4^2 = p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2} = inv.$
Velocity	$v = \frac{dr}{dt} = \frac{1}{\gamma^2}\frac{d\rho}{d\tau}$	$\overline{V} = (\gamma v, \gamma c)$	$\overline{V}^2 = \gamma^2 v^2 - \gamma^2 c^2 = -c^2 = inv.$
Distance (space interval)	$d\rho = \gamma v dt = \gamma dr$, $dr \cdot dt = d\rho \cdot d\tau$	$\overline{\tau} \cdot \overline{V}$	$\overline{\tau} \cdot \overline{V} = \gamma v dt - \gamma dr = 0 = inv.$

We could now start introducing and exercising new "Hyper-Complex Relativity", or ***"Hyper-Complex Analytic-Signal-Phasors" Relativity theory***, based on different mathematical options applied on 4-vectors and "4-scalar" functions from T.10.1 (see more about Complex Analytic Signals in chapter 4.0).

Let us first find what kind of complex and analytic signals (or **phasors**) functions are already present behind 4-vectors of the contemporary Relativity theory (regarding energies, masses, and linear and angular moments). We will effectively come close to the familiar situation that is already known in electronics and electro-technique regarding complex functions or **phasors** representation of currents, voltages, and electric impedances (see different elaborations regarding electromechanical analogies and symmetries in the first chapter). All such results (or complex phasors, developed from 4-vectors of Relativity theory) are briefly presented below:

$$\overline{E}_t = E_0 \pm I \cdot cp = E_t \cdot e^{\pm I\theta} = (E_0 + E_k) \cdot e^{\pm I\theta} = \sqrt{E_0^2 + c^2p^2} \cdot e^{\pm I\theta} = \gamma M c^2 \cdot e^{\pm I\theta} = \gamma \overline{M} c^2, \left|\overline{E}_t\right| = E_t = \gamma M c^2$$

$$\overline{E}_t = E_t \cdot \cos\theta \pm I \cdot E_t \cdot \sin\theta,\ E_t \cdot \sin\theta = H\left[E_t \cdot \cos\theta\right],\ H(=) \text{Hilbert transform},\ \overline{M} = M \cdot e^{\pm I\theta},$$

$$\overline{E}_t = \sum_{(i)} \overline{E}_i = \sum_{(i)} \left(\sqrt{E_{0i}^2 + E_{pi}^2} \cdot e^{\pm I\theta_i}\right) = \sqrt{\sum_{(i)} (E_{0i}^2 + E_{pi}^2)} \cdot e^{\pm I\theta} = \left|\overline{E}_t\right| \cdot e^{\pm I\theta}, \left|\overline{E}_t\right|^2 = \sum_{(i)} (E_{0i}^2 + E_{pi}^2),$$

$$E_k = E_t - E_0 = (\gamma - 1)E_0 = \frac{\gamma - 1}{\gamma} E_t, \quad \overline{E}_k = \frac{\gamma - 1}{\gamma} \overline{E}_t = (\gamma - 1) M c^2 \cdot e^{\pm I\theta} = (\gamma - 1) E_0 \cdot e^{\pm I\theta} = E_k \cdot e^{\pm I\theta},$$

$$\boxed{\begin{gathered} POWER = \frac{d\tilde{E}}{dt} = \frac{dE_k}{dt} = \left[\Psi(t)\right]^2 = \frac{1}{2}\left|\overline{\Psi}(t)\right|^2 = \frac{1}{\pi}\left|A(\omega)\right|^2 \frac{d\omega}{dt} = \frac{1}{2} a^2(t) = h\frac{df}{dt} = v\frac{dp}{dt} = c^2 \frac{d\tilde{m}}{dt} (=) \left[W\right], \\ \overline{\Psi}(x,t) = \begin{Bmatrix} \sum_{(n)} \overline{\psi}_n, \text{ or } \int_{[\Delta k]} \overline{\psi} dk, \\ \text{or summation of wave-groups} \\ \left|\overline{\psi}(x,t)\right| \frac{\sin(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)}{(\underline{\Delta\omega}\ t - \underline{\Delta k}\ x)} e^{I(\omega t \pm kx)} \end{Bmatrix} = \overline{\Psi}^+(kx - \omega t) + \overline{\Psi}^-(kx + \omega t) = \left|\overline{\Psi}\right| \cdot e^{I\Phi} . \end{gathered}}$$

$$\begin{Bmatrix} \overline{P}_4 = P(p, \frac{E_t}{c}) \Rightarrow p^2 - \frac{E_t^2}{c^2} = -\frac{E_0^2}{c^2} \Leftrightarrow E_t^2 = E_0^2 + c^2p^2 = E_0^2 + E_p^2 = (\gamma M c^2)^2 = \\ = (E_0 + E_k)^2 = E_0^2 + 2E_0E_k + E_k^2,\ E_p = cp = \gamma M v c = M c^2 \sqrt{\gamma^2 - 1} \end{Bmatrix} \Rightarrow$$

$$\left(E_0 = \gamma Mc^2 \cdot \cos\theta = Mc^2, pc = \gamma Mc^2 \cdot \sin\theta = \gamma Mcv\right) \Rightarrow v = c\cdot\sin\theta, 0 \le v \le c,\ \cos\theta = 1/\gamma = \sqrt{1-(\frac{v}{c})^2},$$

$$\gamma Mc^2 = \sqrt{E_0^2 + c^2p^2} \Leftrightarrow (\gamma M)^2 = (\frac{E_0}{c^2})^2 + (\frac{p}{c})^2, E_0 = mc^2 + E_s = Mc^2, \gamma = 1/\sqrt{1-(\frac{v}{c})^2},$$

$$\overline{M} = \overline{M} = M\cdot e^{\pm I\theta} = \left|\overline{M}\right|\cdot e^{\pm I\theta} = M_r \pm IM_i, M_r = M\cdot\cos\theta, M_i = M\cdot\sin\theta,$$

$$\overline{P} = \gamma\overline{M}v = p\cdot e^{\pm I\theta} = p\cdot\cos\theta \pm Ip\cdot\sin\theta, \left|\overline{P}\right| = p = \gamma Mv,$$

$$\theta = \theta(x,t) = \mathrm{arctg}\left(\frac{pc}{E_0}\right) = \mathrm{arctg}\left(\gamma\frac{v}{c}\right) = \mathrm{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}} = \theta(k,\omega) =$$

$$= \begin{cases} (0 \quad , v=0) \Rightarrow \overline{P} = p = \gamma M\vec{v}, \overline{M} = M, \overline{E}_t = E_t = E_0 = Mc^2 \\ (\pi/2, v=c) \Rightarrow \overline{P} = \pm Ip = \pm I\cdot\infty \\ (0 \le \theta \le \frac{\pi}{2}, 0 \le v \le c), 0 \le \left|\overline{P}\right| \le \left|p\cdot\cos\theta \pm Ip\cdot\sin\theta\right|. \end{cases},$$

$$\left.\begin{matrix} \omega = \omega_t = \left|\frac{\partial\theta}{\partial t}\right| = 2\pi f_t = \frac{2\pi}{T} = 2\pi f, \\ k = \omega_x = \left|\frac{\partial\theta}{\partial x}\right| = 2\pi f_x = \frac{2\pi}{\lambda} = 2\pi\frac{p}{h} \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} u = \frac{\omega}{k} = \frac{f_t}{f_x} = \frac{\lambda}{T} = \frac{\partial x}{\partial t} = \lambda f \\ v = \frac{d\omega}{dk} = \frac{dx}{dt} = u - \lambda\frac{du}{d\lambda} \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} dk\cdot dx = d\omega\cdot dt \\ \frac{2\pi}{h}dp\cdot dx = 2\pi df\cdot dt \end{matrix}\right\} \Rightarrow \quad (10.1.1)$$

$$\Rightarrow dp\cdot dx = hdf\cdot dt = dE\cdot dt \Leftrightarrow \Delta p\cdot\Delta x = \Delta E\cdot\Delta t,$$

$$(I^2 = -1, I = (i,j,k), i^2 = j^2 = k^2 = -1, ij = k, jk = i, ki = j...).$$

Analyzing similar case, when a motional particle will get an additional mechanical spinning moment, ($\vec{L}_{i-ms} \ne \vec{0}$) in certain interaction, we will be able to reproduce new results, like (10.1.1),

$$\left\{\begin{matrix} \left(\begin{matrix} (\vec{L} \to \vec{L} + \vec{L}_{ms} = \vec{L}^*) \Rightarrow (M \to M + m_{ms} = M^*) \Rightarrow \\ \vec{p} = \gamma M\vec{v} \to \gamma(M + m_{ms})\vec{v} = \vec{p} + \vec{p}_{ms} = \vec{p}^* = \gamma M^*\vec{v} \\ E \ = \gamma Mc^2 \to E + E_{ms} = E_0 + E_k + E_{ms} = E_0 + E_s + E_{ms} = E^* = \gamma M^*c^2 \\ E_0 = Mc^2 \to E_0 + E_{ms} = (M + m_{ms})c^2 = E_0^* = M^*c^2 \\ E_k \to E_k^* = (\gamma - 1)M^*c^2, E_{ms} = \text{mechanical spinning energy} \end{matrix}\right) \Rightarrow \\ \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \to \left(\vec{p}^*, \frac{E^*}{c}\right) \Leftrightarrow \left(\vec{p}^*, I\frac{E^*}{c}\right) \Rightarrow (\vec{p}^*)^2 - (\frac{E^*}{c})^2 = -(\frac{E_0^*}{c^2})^2 \Leftrightarrow (E_0^*)^2 + (\vec{p}^*)^2c^2 = (E^*)^2 = (E_0^* + E_k^*)^2 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \overline{E}^* = E^*\cdot e^{\pm I\theta^*} = (E_0^* + E_k^*)\cdot e^{\pm I\theta^*} = \sqrt{(E_0^*)^2 + (\vec{p}^*)^2c^2}\cdot e^{\pm I\cdot\mathrm{arctg}\frac{p^*c}{E_0^*}} = \gamma M^*c^2\cdot e^{\pm I\theta^*} = \gamma\overline{M}^*c^2, \\ E_0^* = M^*c^2,\ \overline{M}^* = M^*\cdot e^{\pm I\theta},\ I^2 = -1, \\ \gamma M^*c^2 = \sqrt{(E_0^*)^2 + (\vec{p}^*)^2c^2} = E_0^* + E_k^* = E^*, \\ \theta^* = \mathrm{arctg}\frac{p^*c}{E_0^*} = \mathrm{arctg}(\gamma\frac{v}{c}) = \mathrm{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}} = \theta,\ 0 \le (\theta = \theta^*) \le \frac{\pi}{2} \end{matrix}\right] \Rightarrow$$

$$\Rightarrow \overline{P}^* = \gamma\overline{M}^*\vec{v} = \gamma M^*\vec{v}\cdot e^{\pm I\theta^*} = \vec{p}^*\cdot e^{\pm I\theta^*} = \vec{p}^*\cdot e^{\pm I\theta}$$

(10.1.2)

What we can also extract from (10.1.1) is that (relativistic) motional mass is a complex function $\overline{M}$ that has its real part **M_r**, imaginary part **M_i**, and absolute value or the total mass amount $\left|\overline{M}\right| = \sqrt{M_r^2 + M_i^2} = M$. Complex, Hypercomplex and Analytic Signal function interpretation of energy-momentum vectors and scalars (like in (10.1), (10.1.1) and (10.1.3)) is revealing energy-momentum phase function "θ", which is the new quality parameter, still not exploited (or known) within the Minkowski 4-vectors concept, and in the contemporary theoretical physics. For instance, a complex mass function $\overline{M} = M_r \pm IM_i = |\overline{M}| \cdot e^{\pm I\theta} = M \cdot e^{\pm I\theta}$ has its (velocity dependent) phase angle, $\theta = \text{arctg}\left(\frac{pc}{E_0}\right) = \text{arctg}\left(\gamma\frac{v}{c}\right) = \text{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}}$. The same is valid for linear momentum $\overline{P} = \gamma\overline{M}v = p \cdot e^{\pm I\theta}$ and energy (since all of them are presentable by similar Complex or Hypercomplex and Analytic Signal functions, like **Phasors and complex impedances** in electric or electromagnetic sciences).

Here is the place to address de Broglie or matter waves. Matter-wave and particle-wave duality concept is on a certain specific way connected to a moving particle having a linear moment $\vec{p}$. Moving particle is also presentable as a wave-packet or wave group, which has an angular moment $\vec{L}_s$, spinning frequency $f_s = \omega_s / 2\pi = \tilde{E}/h$, (or for macro objects $\tilde{E}/H$), and helical wave nature. We could simply postulate (or exercise) that mentioned phase angle θ should have all essential properties of de Broglie matter waves, as for instance, $\theta = \text{arctg}\left(\frac{pc}{E_0}\right) = \text{arctg}\left(\gamma\frac{v}{c}\right) = \text{arctg}\frac{\frac{v}{c}}{\sqrt{1-(\frac{v}{c})^2}} = \omega_s t \pm kx + \theta_0,\ \theta_0 = \text{const.}$, presentable as,

$$\begin{bmatrix} \vec{p} = \frac{\omega_s}{v}\vec{L}_s = \gamma M\vec{v}, \vec{L}_s = \frac{v}{\omega_s}\vec{p} = \frac{v}{\omega_s}\gamma M\vec{v} \\ \overline{P} = \gamma\overline{M}\vec{v} = \gamma M\vec{v}\cdot e^{\pm I\theta} = \vec{p}\cdot e^{\pm I\theta} \\ \lambda = \frac{H}{p}, u = \lambda f_s = \frac{\tilde{E}}{p}, v = \frac{d\tilde{E}}{dp}, k = \frac{2\pi}{\lambda} = \frac{2\pi}{H}p \\ \omega_s = 2\pi f_s, \Psi^2(t) = \frac{d\tilde{E}}{dt} = \frac{dE_k}{dt} \end{bmatrix} \Rightarrow \begin{bmatrix} \vec{L}_s = \frac{v}{\omega_s}\vec{p} = J_s\vec{\omega}_s, \\ \overline{L}_s = \frac{v}{\omega_s}\overline{P} = \frac{v}{\omega_s}\vec{p}\cdot e^{\pm I\theta} = \vec{L}_s \cdot e^{\pm I\theta} = J_s\vec{\omega}_s\cdot e^{\pm I\theta} = \overline{J}_s\vec{\omega}_s, \\ \tilde{E} = \int_{-\infty}^{+\infty}\Psi^2(t)dt = \int_{-\infty}^{+\infty}\hat{\Psi}^2(t)dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{\Psi}(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{+\infty}\left[\frac{a(t)}{\sqrt{2}}\right]^2 dt = \int_{-\infty}^{+\infty}\left|\frac{\overline{U}(\omega)}{\sqrt{2\pi}}\right|^2 = E_k \end{bmatrix}. \tag{10.1.3}$$

(See more in the chapters 4.1 and 4.3).

If we consider results and formulations from (10.1.1) - (10.1.3) as relevant, (of course, after appropriate theoretical "brushing"), complex linear momentum p, or its Phasor function $\overline{P}$, in relation to matter-waves' and particle-wave duality, should effectively present kind of replacement or complement for de Broglie matter waves concept. Phasor $\overline{P} = \gamma\overline{M}v = p\cdot e^{\pm I\theta}$, which is in the form of Complex Analytic Signal, is a mixed particle-wave, or "*particle and surrounding spinning-filed*" entity (since it is combining elements of a particle linear motion, and certain kind of associated spinning, like helix or solenoidal field-tail), like turbulent and waving motion behind a particle moving on a fluid surface. See (4.3-0), in Chapter 4.1, and (4.41-1) - (4.41-4), in Chapter 4.3).

It looks like we will come closer to understanding natural or proper time flow (time scale, or time dimension) in relation to certain mass motion and associated helix matter-wave, if we could find a connection between energy-momentum phase "θ" and local time "t". Apparently, certain (local and dominant) matter-wave, spinning frequency ω_s, associated to linear momentum of a dominant particle, could serve as the time clock, or proper time-reference signal, for registering (measuring or representing) real-time flow (of a motional particle in question). This way, we can represent instantaneous time-dependent, matter waves functions (of linear and angular moments) as cosines and sinusoidal functions with a phase equal to, $\theta = \omega_s t \pm kx + \theta_0$,

$$\begin{Bmatrix} \overline{P}(t) = \gamma M\vec{v}\cdot e^{\pm I(\omega_s t \pm kx + \theta_0)} = \vec{p}\cdot e^{\pm I(\omega_s t \pm kx + \theta_0)} = \frac{\omega_s}{v}\vec{L}_s\cdot e^{\pm I(\omega_s t \pm kx + \theta_0)} = \frac{\omega_s}{v}\overline{L}_s(t) \\ \overline{L}_s(t) = \vec{L}_s\cdot e^{\pm I(\omega_s t \pm kx + \theta_0)} = \frac{v}{\omega_s}\vec{p}\cdot e^{\pm I(\omega_s t \pm kx + \theta_0)} = \frac{v}{\omega_s}\overline{P}(t), \quad \theta_0 = \text{const., or} \\ \overline{P}(t,x) = \vec{p}\cdot e^{+I\cdot\theta(t,x)} + \vec{p}\cdot e^{-I\cdot\theta(t,x)}, \overline{L}_s(t,x) = \vec{L}_s\cdot e^{+I\cdot\theta(t,x)} + \vec{L}_s\cdot e^{-I\cdot\theta(t,x)} \end{Bmatrix} \Rightarrow \gamma M = \frac{p}{v} = \frac{\omega_s}{v^2}L_s, \tag{10.1.4}$$

Or, by generalizing for cases of time-space propagating matter-waves in a direction r = r(x, y, z), we will analogically have the following linear and angular moments, or complex Phasors, including relevant wave function,

$$\overline{P}(r,t) = \gamma M\vec{v}\cdot e^{\pm I(\omega_s t \pm kr+\theta)} = \vec{p}\cdot e^{\pm I(\omega_s t \pm kr+\theta)},\ \overline{L}_s(r,t) = \vec{L}_s\cdot e^{\pm I(\omega_s t \pm kr+\theta)} = \frac{v}{\omega_s}\vec{p}\cdot e^{\pm I(\omega_s t \pm kr+\theta)}\ ,\ . \quad (10.1.5)$$

$$\text{POWER} (=) d\overline{E}/dt = v\cdot\left(d\overline{P}/dt\right) = \overline{\Psi}^2(r,t) = \text{Wave function.}$$

Regarding mentioned time-reference signal, as described in (10.1.3) - (10.1.5) and earlier in (4.3-0)-r, we could try to understand it on the following way (by giving an example in relation to our planetary system). For us, dominant and natural time flow is related to the orbital motion of our planet around our local Sun, or about the local and dominant center of mass. Planet Earth is at the same time self-rotating and has certain spin. This is creating associated, helix matter-wave, characterized by its spiraling frequency $\vec{\omega}_s$. Since our masses and sizes (related to humans and other living species on our planet Earth) are negligible compared to the local planetary mass (and local center of mass), it is logical that our real-time flow is dominated by the "signal carrier" time flow belonging to the orbiting mass of our planet. We also know that our SI unit of time is really extracted from the parameters of the orbital motion of our planet Earth. For the micro world, something similar is valid in a different scale of time-space relations.

From (10.1.4) and (10.1.5) we can also see that mass **M** has an explicit relation to its resulting or effective and intrinsic, angular or spin moment $\gamma M = \frac{p}{v} = \frac{\omega_s}{v^2} L_s$. Now is becoming much clearer the meaning of Mobility type analogies (elaborated in the first chapter of this book; -see T 1.8), from where it is evident that real charges or sources of gravity (based on electromechanical analogies) should be relevant linear and angular momenta. For instance, Newton force of gravitation between two masses (see the second chapter of this book) should be more correctly written as,

$$F_g = G\frac{M_1\cdot M_2}{R^2} \Leftrightarrow G\frac{\gamma_1 M_1\cdot\gamma_2 M_2}{R^2} = \left[\frac{G}{v_1 v_2}\right]\frac{p_1\cdot p_2}{R^2} = \left[\frac{G\omega_{s1}\omega_{s2}}{v_1^2 v_2^2}\right]\frac{L_{s1}\cdot L_{s2}}{R^2}\,. \quad (10.1.6)$$

Linear and angular moments and corresponding velocities, $p_1, p_2, v_1, v_2, L_{s1}, L_{s2}, \omega_{s1}, \omega_{s2}$, we can also consider as mutually collinear vectors in relation to stable inertial motions of involved participants.

It is also clear that every mass (even in a relative state of rest) should have certain resulting intrinsic spin, and certain minimal rest mass,

$$\left[\gamma M = \frac{p}{v} = \frac{\omega_s}{v^2}L_s\right]_{v\to 0} \Rightarrow M = M_0 = \left(\frac{\omega_{s0}}{v_0^2}\right)L_s = \left(\frac{\omega_{s0}}{v_0^2}\right)L_{s0} \Rightarrow$$
$$\Rightarrow F_g = G\frac{\gamma_1\left(\frac{\omega_{s0}}{v_0^2}\right)L_{s01}\cdot\gamma_2\left(\frac{\omega_{s0}}{v_0^2}\right)L_{s02}}{R^2} = \left(\frac{G\omega_{s0}^2}{v_0^4}\right)\frac{\gamma_1 L_{s01}\cdot\gamma_2 L_{s02}}{R^2} \quad (10.1.7)$$
$$\Rightarrow\left(F_g\right)_{v_{1,2}<<c} \cong \left(\frac{G\omega_{s0}^2}{v_0^4}\right)\frac{L_{s01}\cdot L_{s02}}{R^2} = G\frac{M_1\cdot M_2}{R^2}\ .$$

Consequently, our universe should have certain intrinsic or hidden and holistic, linear, and angular velocity parameters (v_0 and ω_{s0}, probably not always and everywhere the same), because gravitational attraction exists also between two masses in relative states of rest. In fact, here we need to admit that masses are not primary and only sources of gravitation, but masses in motion (having linear and angular moments, with associated and properly oriented electric and magnetic dipoles and moments) are most relevant for gravitational attraction (see the second chapter of this book, "2.

GRAVITATION", where similar problematic is analyzed more profoundly). Similar intuitive conclusions 0we could draw if we expand our creative freedom and exercise what will be the consequences if we present Newton law of "*complex gravitational force*" in terms of complex masses, such as,

$$\overline{F}_g = G\frac{\overline{M}_1 \cdot \overline{M}_2}{R^2} = G\frac{\gamma_1 M_1 \cdot \gamma_2 M_2}{R^2} \cdot e^{\pm I(\theta_1+\theta_2)} = G\frac{\gamma_1 M_1 \cdot \gamma_2 M_2}{R^2} \cdot e^{\pm I[(\omega_{s1}+\omega_{s2})t+\theta_0]}. \quad (10.1.8)$$

Since angular moments (and intrinsic spin parameters) are also causally linked to, and proportional with associated magnetic field moments, it should be possible to relate origins of gravitation based on such intrinsic magnetic properties. We also know that the motion of elements with magnetic flux properties is producing electric currents and this way (systematically) we should be able to establish natural relations between Gravitation and Electromagnetism. The unusual and extraordinarily striking insight from mass, momentum, and energy expressions (10.1.1) - (10.1.5) is also related to the complex character of all of them (because we succeeded to present mass, linear and angular moments, and energy using complex mathematical functions with real and imaginary parts, like in Complex and Analytic Signal functions). Here, we started with relevant 4-vectors and Einstein-Minkowski energy-momentum invariance of such vectors (as established in general Relativity Theory; see T.10.1 or T.2.2-3). Correctness and applicability of such mathematical practices is extremely productive, verified and well supported in analyzing and predicting experimental results of all interactions in macro and microphysics, we should admit that the complex nature of masses, moments, and energies (as in (10.1), (10.1.1) - (10.1.5)) is something very natural, exact and unavoidable in Physics. The remaining problem is how to make our best interpretation of such complex nature of masses, moments, and energy, or how to understand what imaginary and apparent components of masses, moments and energies are. Maybe, challenging discussions and somewhat useful concepts about missing cosmic masses, dark energy, dark or invisible matter and masses, and similar items (including stochastic and exotic energy of "zero point" vacuum states) known from our contemporary Physics have something common to do with Analytic, Complex, Real and Imaginary values found in (10.1.1) - (10.1.5). Such complex (and Hypercomplex) functions background could support existence of Dark Matter & Energy by presenting mentioned items as kind of imaginary or apparent mass components with amplitude and phase functions, since mass of our Universe is "Mass in permanent motion", where linear and angular motions are specifically united, respecting Particle Wave Duality Concepts, as widely elaborated in this book.

Presently, still the best ideas and concepts about gravitation are coming from Rudjer Boskovic's universal Natural Force theory, [6] and from complementing ideas of N. Tesla about Dynamic Theory of Gravity, [97]. Mentioned references are creatively indicating that Gravitation should be certain specific manifestation of universal electromagnetic force associated to resonant oscillations of our Universe (creating attractive forces in nodal zones where gravitational masses are). The opinion of the author of this book is that in our Universe (microscopically and macroscopically) we have only two families of acting forces (not four as speculated in the contemporary Physics), such as electromagnetic forces, and forces related to standing waves structured matter waves. If all matter-waves are related to certain electromagnetic nature or origin (essentially or ontologically), in that case, our Universe has only one acting family of forces, meaning only electromagnetic forces. See much more under [117].

*In (10.1.1) - (10.1.5), and earlier in (4.3-0)-p,q,r, we can recognize kind of combination between complex and vector functions. Circular spinning frequency (or a matter wave frequency) $\vec{\omega}_s$ is a vector collinear with the particle linear moment $\vec{p}$, and hyper-complex imaginary unit **I** also has the structure like vectors, since it is composed of three, mutually orthogonal, more elementary imaginary units **i, j, k** (see more in the chapter 6. around equations (6.8) – (6.13)). Ordinary vectors can naturally be related (or* *fixed) to certain observer system of reference, but hyper-complex imaginary units, here naturally coupled with particle linear momentum and associated spinning, could also get other dynamic and structural meanings, and have specific and rich relations with observer's system of reference.*

In parallel to contemporary 4-vectors of Relativity theory, we can try to formulate analogical and equivalent Hypercomplex Analytic functions or Hypercomplex 4-vectors (see chapter 4.0 where Hypercomplex Analytic Signal functions are introduced). On the following brainstorming example (see below) in relation to linear momentum 4-vector (from Relativity Theory), it is possible to demonstrate (or propose) how we could analogically create Hypercomplex 4-vectors, especially when certain micro-world entity is composed of three simpler energy-momentum 4-vectors (like quarks, antiquarks, and familiar triadic structures and combinations).

[♣ COMMENTS & FREE-THINKING CORNER:

Particularly challenging opportunity or project *(combined with the facts under (10.1), (10.1.1) - (10.1.5)) will be to test, develop, or exercise the idea that Minkowski-Einstein 4-vectors are remarkably successful concept (or mathematical model) because of being essentially compatible and complementary to corresponding, **Complex Analytic Signal Phasors and relevant Wavefunction, as well as with de Broglie matter-waves framework**. This will create strong foundations of Wave-Particle Duality and Matter-Waves concept (after certain mathematical makeup), as follows,*

$$\left\{\left(\begin{array}{l} \overline{P}_4 = P(p, \frac{E}{c}), \left(p, H[p]\right) = (p, \hat{p}) \\ \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right) \\ \vec{p} = -H\left[\frac{E}{c}\right] = -H\left[\frac{E_k}{c}\right] \\ \frac{\overline{E}}{c} = H[p] = \frac{E_0}{c} + \frac{E_k}{c},\ H[\text{const.}] \cong 0 \end{array}\right) \Rightarrow p^2 - \frac{E^2}{c^2} = -\frac{E_0^2}{c^2}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} \overline{P} = p + I\hat{p} = \sqrt{p^2 + \hat{p}^2} \cdot e^{I\theta} = \gamma\overline{M}v = p \cdot e^{\pm I\theta} = p \cdot \cos\theta \pm Ip \cdot \sin\theta,\ \left|\overline{P}\right| = p = \gamma M v, \\ \overline{M} = \overline{M} = M \cdot e^{\pm I\theta} = \left|\overline{M}\right| \cdot e^{\pm I\theta} = M_r \pm IM_i, M_r = M \cdot \cos\theta,\ M_i = M \cdot \sin\theta, \\ \overline{E} = E_t = E + I\hat{E} = \sqrt{E^2 + \hat{E}^2} \cdot e^{I\theta} = E_0 \pm I \cdot cp = E_t \cdot e^{\pm I\theta} = (E_0 + E_k) \cdot e^{\pm I\theta} = \\ = \sqrt{E_0^2 + c^2 p^2} \cdot e^{\pm I\theta} = \gamma M c^2 \cdot e^{\pm I\theta} = \gamma \overline{M} c^2, \left|\overline{E}_t\right| = E_t = \gamma M c^2,\ I^2 = -1, \\ \theta = \operatorname{arctg}\left(\frac{\hat{p}}{p}\right) (\Leftrightarrow) \left[\omega t \mp kx = 2\pi(f \cdot t \mp \frac{x}{\lambda})\right], \\ \left.\begin{array}{l} \omega = \omega_t = \left|\frac{\partial\theta}{\partial t}\right| = 2\pi f_t = \frac{2\pi}{T} = 2\pi f, \\ k = \omega_x = \left|\frac{\partial\theta}{\partial x}\right| = 2\pi f_x = \frac{2\pi}{\lambda} = 2\pi\frac{p}{h} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} u = \frac{\omega}{k} = \frac{f_t}{f_x} = \frac{\lambda}{T} = \frac{\partial x}{\partial t} = \frac{v}{1 + \sqrt{1 - v^2/c^2}} \\ v = \frac{d\omega}{dk} = \frac{dx}{dt} = u - \lambda\frac{du}{d\lambda},\ \lambda = \frac{h}{p} = \frac{2\pi}{k} \end{array}\right\}, \\ E = E_0 + E_k = E_0 + \frac{pv}{1 + \sqrt{1 - v^2/c^2}} = E = E_t,\ E_k = \tilde{E} = \frac{pv}{1 + \sqrt{1 - v^2/c^2}} = hf \\ \hat{E} = H[E], p = -H[\hat{p}],\ I^2 = -1,\ (\text{here}, H\,(=)\ \text{Hilbert transform}), \\ \Rightarrow \\ \boxed{dE = vdp \Rightarrow \text{Power} = dE/dt \Rightarrow \left(d\overline{E}/dt\right) = v\left(d\overline{P}/dt\right) = \overline{\Psi}^2\ (=)\ \text{wavefunction squared}} \end{array}\right.$$

Obviously, very promising evolution of energy-momentum 4-vectors, known from Relativity theory, is to extend analogically the same concept towards Hypercomplex and "Analytic Signal Phasor" and wave functions, and at the same time to profit from amazing applicability of 4-vectors, as well as to create very stable grounds of Matter-Waves, and Wave-Particle Duality theory.

Since we already know all analogies between properties of linear and rotational motions, compared with electromagnetic phenomenology (see the first chapter of this book), and having an excellent, and successful mathematical processing related to complex phasor functions in electromagnetic science (related to voltages, currents, and impedances), we can analogically apply the same Phasors methodology to linear motion and rotation. We know how currents and voltages are being phase-shifted or transformed by different electric loads, we operate with different meanings of power (such as active, reactive, apparent, complex, reflected...), we define different electric impedances, and we can analyze coherence and correlations between different electric signals or Phasors. Consequently, we can analogically apply all of that to mechanics of linear and angular motions. In addition, wave-functions concept presented in this book is widely extended to everything what presents temporal-spatial signals, voltages, currents, power, velocities, forces, and different moments (see Chapter 4.0, and (4.0.82)), where phasors notation naturally belongs. ***Black or dark, invisible mass and energy could be related to imaginary parts of masses and energies, when presented as complex functions and Phasors.***

This way doing, we will enormously enrich understanding of interactions in Mechanics and Wave-Particle Duality theory, but the reality is that we are still not taking such steps, and we are diving into insufficiently clear and incomplete, artificial, unnatural, and conceptually limited probability and assumptions related strategies. Of course, Probability and Statistics will always have their significant place in any mass-data and associated processes descriptions, quantifications, and modeling, in any field of science and life (not only or dominantly in the Quantum theory).

A. Einstein General Relativity theory is explaining that space and time are curved, or specifically deformed around big masses, and such curved geometry is giving an illusion about existence of Gravitational force... The same concept about curved space could be analogically applied on any "*electromagnetically charged*" entity, and with convenient mathematical modeling, we will again have very correct modeling regarding Gravitation, Wave-Particle Duality concepts, Electromagnetic forces, fields, and interactions with electrically and magnetically charged particles.

Here is the example how to extend Einstein-Minkowski 4-vectors toward Hypercomplex 4-vectors:

$$\left\{\begin{array}{l} \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right) = \left(\vec{p}_i, i\frac{E_i}{c_i}\right) + \left(\vec{p}_j, j\frac{E_j}{c_j}\right) + \left(\vec{p}_k, k\frac{E_k}{c_k}\right) \Leftrightarrow \\ \Leftrightarrow \left(\vec{p}, I\frac{E}{c}\right) = \left(\vec{p}_i + \vec{p}_j + \vec{p}_k, \frac{iE_i + jE_j + kE_k}{c_i}\right) = \left(\vec{p}_i + \vec{p}_j + \vec{p}_k, I\frac{E}{c}\right) \\ \overline{P}_4 = \left(\vec{p}, \frac{E}{c}\right) = \text{invariant} \Rightarrow \vec{p}^2 - (\frac{E}{c})^2 = -(\frac{E_0}{c^2})^2 \Leftrightarrow E_0^2 + p^2c^2 = E^2 = (E_0 + E_k)^2, \\ \vec{p} = \vec{p}_i + \vec{p}_j + \vec{p}_k = \vec{p}_t, E = E_{0t} + E_{kt}, E_0 = E_{0t}, E = E_t, \\ E_i = E_{0i} + E_{ki}, E_j = E_{0j} + E_{kj}, E_k = E_{0k} + E_{kk}, \\ I^2 = i^2 = j^2 = k^2 = -1, ij = k, \ jk = i, ki = j, ji = -k, kj = -i, ik = -j \end{array}\right\} \Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} I\dfrac{E}{c}=i\dfrac{E_i}{c_i}+j\dfrac{E_j}{c_j}+k\dfrac{E_k}{c_k}, \\ (\vec{p}_i+\vec{p}_j+\vec{p}_k)^2-(\dfrac{E}{c})^2=-(\dfrac{E_0}{c^2})^2, \\ \left(\dfrac{E_{0t}+E_{kt}}{c}\right)^2=\left(\dfrac{E_{0i}+E_{ki}}{c_i}\right)^2+\left(\dfrac{E_{0j}+E_{kj}}{c_j}\right)^2+\left(\dfrac{E_{0k}+E_{kk}}{c_k}\right)^2 \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left\{\begin{array}{l} \left(\dfrac{E}{c}\right)^2=\left(\dfrac{E_i}{c_i}\right)^2+\left(\dfrac{E_j}{c_j}\right)^2+\left(\dfrac{E_k}{c_k}\right)^2 \\ \left(\dfrac{E_0}{c}\right)^2=\left(\dfrac{E_{0i}}{c_i}\right)^2+\left(\dfrac{E_{0j}}{c_j}\right)^2+\left(\dfrac{E_{0k}}{c_k}\right)^2 \\ \left(\dfrac{E_{kt}}{c}\right)^2=\left(\dfrac{E_{ki}}{c_i}\right)^2+\left(\dfrac{E_{kj}}{c_j}\right)^2+\left(\dfrac{E_{kk}}{c_k}\right)^2 \\ \dfrac{E_{0t}E_{kt}}{c^2}=\dfrac{E_{0i}E_{ki}}{c_i^2}+\dfrac{E_{0j}E_{kj}}{c_j^2}+\dfrac{E_{0k}E_{kk}}{c_k^2} \end{array}\right\}\Rightarrow\left\{\begin{array}{l} p^2=\dfrac{E^2-E_0^2}{c^2}=\dfrac{E_i^2-E_{0i}^2}{c_i^2}+\dfrac{E_j^2-E_{0j}^2}{c_j^2}+\dfrac{E_k^2-E_{0k}^2}{c_k^2}=\vec{p}_i^2+\vec{p}_j^2+\vec{p}_k^2 \\ \vec{p}_i^2=\dfrac{E_i^2-E_{0i}^2}{c_i^2},\vec{p}_j^2=\dfrac{E_j^2-E_{0j}^2}{c_j^2},\vec{p}_k^2=\dfrac{E_k^2-E_{0k}^2}{c_k^2} \\ \qquad\text{or, if}\quad c=c_i=c_j=c_k\rightarrow \\ p^2=E^2-E_0^2=E_i^2-E_{0i}^2+E_j^2-E_{0j}^2+E_k^2-E_{0k}^2=\vec{p}_i^2+\vec{p}_j^2+\vec{p}_k^2 \\ \vec{p}_i^2=\dfrac{E_i^2-E_{0i}^2}{c^2},\vec{p}_j^2=\dfrac{E_j^2-E_{0j}^2}{c^2},\vec{p}_k^2=\dfrac{E_k^2-E_{0k}^2}{c^2} \\ E^2=E_i^2+E_j^2+E_k^2,E_0^2=E_{0i}^2+E_{0j}^2+E_{0k}^2 \end{array}\right\}$$

(4.3-0)-s, (10.1.6)

In (10.1.6), constants c_i, c_j, c_k that have dimensions of speed (like the universal speed constant $c \cong 3\cdot 10^8 m/s$), could be equal to $c = c_i = c_j = c_k$ (but here involved mathematics is also giving chances for other options). Energies indexed with 0 and k in (10.1.6) are effectively paving the way to explain creations of big numbers of products in impact reactions (zeroes are indicating particles with rest masses, and k-indexing stands for kinetic energy states. Here we have a small indexing problem, since k is the imaginary unit in the Hypercomplex number definition, and such confusing indexing should be (one day) corrected, but here it was easier and faster to neglect such small problem since main intention has been to present the idea about Hypercomplex 4-Vectors). Such extended energy-momentum framework can be later merged with universal Complex and Hypercomplex Analytic Signal representation of wave functions (leading to all famous wave equations of Quantum Theory), and with novel foundations of multidimensional Universe (see Chapter 4.3 and chapter 6, equations (6.10) - (6.13)). Presence of three imaginary units in (10.3.6) is intuitively igniting ideas about mutually coupled energy triplets such as three quarks, three anti-quarks etc., what could create another, more general and more precise concept of Super-Symmetry in the world of microphysics (and significantly or essentially enrich and simplify the Standard Model).

Another challenging project *could be to show, or make it presentable, that mutually coupled electric and magnetic fields of the same event will behave like a couple of an original and its Hilbert transform function pair (eventually creating a corresponding, complex Analytic signal functions; -see more in chapter 3. of this book). The starting steps in describing such objective will be,*

$$\mathrm{E},\mathrm{H}\ (=)\ \text{electric and magnetic fields},\ \mathcal{H}[\ ]\ (=)\ \text{Hilbert transform},$$
$$\overline{\mathrm{E}}_4 = \overline{\mathrm{E}}(\mathrm{E},\hat{\mathrm{E}}) = (\mathrm{E}, \mathrm{I}\hat{\mathrm{E}}) = (\mathrm{E}, \alpha\mathrm{H}),\ \alpha\mathrm{H} = \alpha(\mathrm{v})\cdot\mathrm{H} = \mathcal{H}[\mathrm{E}] = \hat{\mathrm{E}}$$
$$\overline{\mathrm{H}}_4 = \overline{\mathrm{H}}(\mathrm{H},\hat{\mathrm{H}}) = (\mathrm{H}, \mathrm{I}\hat{\mathrm{H}}) = (\mathrm{H}, \beta\mathrm{E}),\ \beta\mathrm{E} = \beta(\mathrm{v})\cdot\mathrm{E} = \mathcal{H}[\mathrm{H}] = \hat{\mathrm{H}}\ ,\mathrm{I}^2 = -1.$$

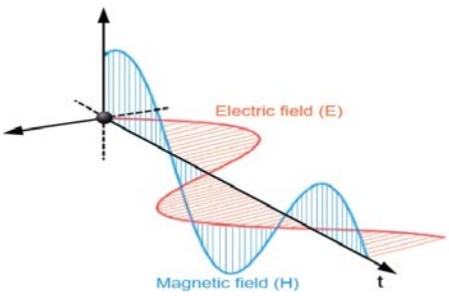
Since products between 4-vectors are also 4-vectors, we could define (and later develop) corresponding scalar and vectors' products, such as,

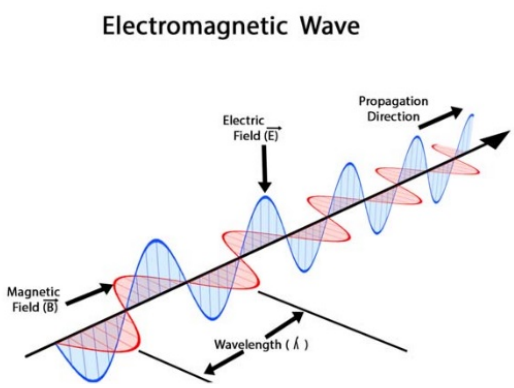
$$\overline{\mathrm{E}}\cdot\overline{\mathrm{H}} = (\mathrm{E}, \mathrm{I}\hat{\mathrm{E}})\cdot(\mathrm{H}, \mathrm{I}\hat{\mathrm{H}}) = (\mathrm{E}, \mathrm{I}\alpha\mathrm{H})\cdot(\mathrm{H}, \mathrm{I}\beta\mathrm{E}) = \ldots$$

and / or

$$\overline{\mathrm{E}}\times\overline{\mathrm{H}} = (\vec{\mathrm{E}}, \mathrm{I}\hat{\vec{\mathrm{E}}})\times(\vec{\mathrm{H}}, \mathrm{I}\hat{\vec{\mathrm{H}}}) = (\vec{\mathrm{E}}, \mathrm{I}\alpha\vec{\mathrm{H}})\times(\vec{\mathrm{H}}, \mathrm{I}\beta\vec{\mathrm{E}}) = \ldots,$$

(where $\alpha(\mathrm{v}) = \vec{\alpha}$ *and* $\beta(\mathrm{v}) = \vec{\beta}$ *could also be vectors). This should (later, united with some creative imagination) produce kind of Poynting vectors, and we will be closer to creation of relevant electromagnetic, energy-momentum 4-vectors and their complex (or hypercomplex) Phasor functions. This could lead us towards an innovative modeling of associated electromagnetic waves... See pictures below with proposed, phase-shifted, and mutually orthogonal electric and magnetic fields (on the left side), what could or should be an electromagnetic-wave or photon. A situation on the left-side of illustrations found below, obviously is not the same as always presented in standard literature about electromagnetic waves and photons (as what we find on the right picture side), because electromagnetic signal-power cannot naturally fluctuate between zero power and certain maximal power. Power can only be continuously balanced, and in an oscillatory manner transformable (or redistributed) between electric and magnetic field energy content (since conservation laws should be satisfied in any moment; -not statistically, and not "in-average"). Here we also need to take care about the phase difference between relevant electric and magnetic field components in relation to "complex impedance loads", or to resistive, capacitive, inductive, and combined impedance loads, since this is influencing the optimal energy transfer from its source towards its load. From such point of view, electromagnetic (or any other) power (and energy flow) could be Active, Reactive, Apparent, and presented as an average, effective or RMS power, reflected power etc. (see much more in Chapter 4.0, under "4.0.11. Generalized Wave Functions and Unified Field Theory", until the end of the same chapter). The far-reaching and an essential situation here is that spatial, temporal, source and load-related properties of involved matter-waves or wave functions are mutually related. If we compare such wave function and signal-power characterization (as presented in Chapter 4.0, and in familiar relations (3.7-1) and (3.7-2) from the third chapter of this book) with what contemporary Quantum theory is offering, conclusions are obvious and clear (no need to be more specific and more explicit).*





Minkowski space foundations, 4-vectors, and Hypercomplex Analytic Signal

*Citation from: "https://en.wikipedia.org/wiki/Minkowski_space: In mathematical physics, **Minkowski space** (or **Minkowski spacetime**) is a combination of three-dimensional Euclidean space and time into a four-dimensional manifold where the spacetime interval between any two events is independent of the inertial frame of reference in which they are recorded. Although initially developed by mathematician Hermann Minkowski for Maxwell's equations of electromagnetism, the mathematical structure of Minkowski spacetime was shown to be an immediate consequence of the postulates of special relativity.[1]*

Minkowski space is closely associated with Einstein's theory of special relativity and is the most common mathematical structure on which special relativity is formulated. While the individual components in Euclidean space and time may differ due to length contraction and time dilation, in Minkowski spacetime, all frames of reference will agree on the total distance in spacetime between events.[nb 1] Because it treats time differently than it treats the 3 spatial dimensions, Minkowski space differs from four-dimensional Euclidean space.

In 3-dimensional Euclidean space (e.g., simply space in Galilean relativity), the isometry group (the maps preserving the regular Euclidean distance) is the Euclidean group. It is generated by rotations, reflections and translations. When time is amended as a fourth dimension, the further transformations of translations in time and Galilean boosts are added, and the group of all these transformations is called the Galilean group. All Galilean transformations preserve the 3-dimensional Euclidean distance. This distance is purely spatial. Time differences are separately preserved as well. This changes in the spacetime of special relativity, where space and time are interwoven.

Spacetime is equipped with an indefinite non-degenerate bilinear form, variously called the Minkowski metric,[2] the Minkowski norm squared or Minkowski inner product depending on the context.[nb 2] The Minkowski inner product is defined so as to yield the spacetime interval between two events when given their coordinate difference vector as argument.[3] Equipped with this inner product, the mathematical model of spacetime is called Minkowski space. The analogue of the Galilean group for Minkowski space, preserving the spacetime interval (as opposed to the spatial Euclidean distance) is the Poincaré group.

In summary, Galilean spacetime and Minkowski spacetime are, when viewed as manifolds, actually the same. They differ in what further structures are defined on them. The former has the Euclidean distance function and time (separately) together with inertial frames whose coordinates are related by Galilean transformations, while the latter has the Minkowski metric together with inertial frames whose coordinates are related by Poincaré transformations".

....

We know that Minkowski space and 4-vectors concept is extremely productive and useful mathematical modeling in Physics and Relativity theory (proven as working without problems). Let us reestablish and explain it (again) on a quite simple, elementary, and step by step way, as follows.

1° We consider that everything what is happening in the world of Physics or Nature belongs to the united spatial-temporal domain. We start from the concept (or assumption) that spatial and temporal domains are coexistent, and on some direct and natural way mutually linked, related, replaceable and transformable (or mutually directly proportional, $\mathbf{r} \rightleftarrows \mathbf{t}$ *).*

2° The second assumption is that spatial and temporal domain are mutually orthogonal (meaning mutually phase shifted for $\pi/2$ or for $90°$ *). This is giving us a chance to consider that the Time Domain is the Hilbert transform of the Space Domain, or vice versa (since we do not have direct destructive interferences and overlapping between them). Of course, here we only conceptually and intuitively describe how spatial and temporal domain could be mutually orthogonal and phase shifted, and how to manage this mathematically using Hilbert transform. More rigorous and much better mathematical elaboration about mentioned concept is still missing here, being an ongoing objective and project. For instance, if we describe certain motion with its (spatial) radius vector* $r = r(x, y, z)$, $x = x(t), y = y(t), z = z(t)$*, and with its temporal position* $t = t(t_x, t_y, t_z)$*, then we will be able to assemble four-dimensional, or Minkowski-space 4-vector situation, such as,*

$$R\left[r(x, y, z), t(t_x, t_y, t_z)\right] = \mathbf{R}(\mathbf{r}, \mathbf{t}),$$
$$\mathbf{r} = \mathbf{u}\cdot\mathbf{t},\ \mathbf{u} = \lambda\mathbf{f} \leq \mathbf{c} = \mathbf{const.},$$
$$H[r] = H[r(x, y, z)] = \mathbf{u}\cdot t(t_x, t_y, t_z) = \mathbf{u}\cdot\mathbf{t}\ (\cong \mathbf{c}\cdot\mathbf{t}\text{ , for photons}),$$
$$R_4 = R\left[r(x, y, z), t(t_x, t_y, t_z)\right] = \mathbf{R}(\mathbf{r}, \mathbf{Ict}),\ \mathbf{I}^2 = -\mathbf{1}.$$

3° Here, spatial $\mathbf{r}$ *, and temporal* $\mathbf{Ict}$ *positions (dimensions, coordinates, lengths, or domains) are mutually orthogonal, like in some right-angle triangle, and we can simply apply Pythagoras' Theorem in order to get relevant hypotenuse length, which is A. Einstein, relativistic space-time interval,* ΔS *(see more here:* *https://www.sciencedirect.com/topics/physics-and-astronomy/minkowski-space),*

$$S_4 = S(r, Ict),\ S^2 = (r)^2 + (\mathbf{Ict})^2 = \mathbf{r}^2 - \mathbf{c}^2\mathbf{t}^2,\ \mathbf{I}^2 = -\mathbf{1},$$
$$\Delta\mathbf{S}_4 = \Delta\mathbf{S}(\Delta r, Ic\Delta t),\ (\Delta S)^2 = (\Delta\mathbf{r})^2 - \mathbf{c}^2(\Delta\mathbf{t})^2,$$
$$(\Delta\mathbf{r})^2 = (\Delta\mathbf{x})^2 + (\Delta\mathbf{y})^2 + (\Delta\mathbf{z})^2.$$

The idea about spatial-temporal proportionality here is intrinsically embedded. If we imagine that in different referential systems, spatial and temporal lengths $\Delta\mathbf{r},\ \Delta\mathbf{t}$ *are different, but mutually linked, orthogonal and proportional on the same way, making that in a higher dimensional (spatial-temporal) framework this always presents the same hypotenuse, length or duration* $\Delta S = \text{const.}$*, we will have validity of the following invariance relation (which is carrying the meaning of Special Relativity theory),*

$$\Delta\mathbf{S}_4 = \Delta\mathbf{S}(\Delta r, Ic\Delta t) \Rightarrow \Delta\bar{S}_4 = \left|\Delta\bar{S}\right|\cdot\mathbf{e}^{\mathbf{I}\cdot\mathrm{arctg}\frac{c}{v}} \Rightarrow$$
$$(\Delta S)^2 = \left|\Delta\bar{S}\right|^2 = (\Delta\mathbf{r})^2 - \mathbf{c}^2(\Delta\mathbf{t})^2 = (\Delta\mathbf{r}_i)^2 - \mathbf{c}^2(\Delta\mathbf{t}_i)^2 = \mathbf{const.},\ \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots$$

4° Now (just for exercising, to see how much something like that would be useful) we can define complex or hypercomplex function of certain space-time interval $\Delta R\left[r(x, y, z), t(t_x, t_y, t_z)\right] = \Delta\mathbf{R}(\mathbf{r}, \mathbf{t})$ *as,*

$$\Delta\bar{R} = \Delta\mathbf{r} + \mathbf{Iu}\Delta\mathbf{t} = \sqrt{(\Delta\mathbf{r})^2 + \mathbf{u}^2(\Delta\mathbf{t})^2}\cdot\mathbf{e}^{\mathbf{I}\cdot\mathrm{arctg}\frac{\mathbf{u}\Delta\mathbf{t}}{\Delta\mathbf{r}}} = \left|\Delta\bar{R}\right|\cdot\mathbf{e}^{\mathbf{I}\cdot\mathrm{arctg}\frac{\mathbf{u}}{\mathbf{v}}} =$$
$$= \left|\Delta\bar{R}\right|\cdot\cos\varphi + \mathbf{I}\left|\Delta\bar{R}\right|\cdot\sin\varphi,\ \varphi = \mathbf{artctg}\frac{\mathbf{u}}{\mathbf{v}} = \mathbf{artctg}\frac{\mathbf{1}}{\mathbf{1} + \sqrt{\mathbf{1} - \mathbf{v}^2/\mathbf{c}^2}} = \begin{cases} 0.46364761, \text{ for } v = 0 \\ 0.78539816\ (= 45°), \text{ for } v \to c \end{cases}$$
$$(\mathbf{u} = \mathbf{c}) \Rightarrow \Delta\bar{R} = \left|\Delta\bar{R}\right|\cdot\mathbf{e}^{\mathbf{I}\cdot\mathrm{arctg}\frac{c}{v}} = \left|\Delta\bar{R}\right|\cdot\cos\varphi + \mathbf{I}\left|\bar{R}\right|\cdot\sin\varphi,\ \varphi = \mathbf{artctg}\frac{\mathbf{c}}{\mathbf{v}} = \begin{cases} 1.57079633\ (= \pi/2 = 90°), \text{ for } v = 0 \\ 0.78539816\ (= 45°), \text{ for } v \to c \end{cases}$$

Here, it is obvious that $\Delta S = (\Delta\mathbf{r})^2 - \mathbf{c}^2(\Delta\mathbf{t})^2$ *and* $\left|\Delta\bar{R}\right| = (\Delta\mathbf{r})^2 + \mathbf{c}^2(\Delta\mathbf{t})^2$ *are very much different intervals, and that* ΔS *has much stronger or more relevant meaning, since it can be the same or constant interval in different (higher level, spatial-temporal) referential frames, if as a phase velocity of associated wave motion we always consider a photon velocity* $\mathbf{u} = \lambda\mathbf{f} = \mathbf{c} = \mathbf{const.},\ \mathbf{r} = \mathbf{c}\cdot\mathbf{t}$. *In addition, we could say that* $\Delta\bar{R} = \left|\Delta\bar{R}\right|\cdot\mathbf{e}^{\mathbf{I}\cdot\mathrm{arctg}\frac{c}{v}}$ *is the* ***complex or Analytic signal function (or Phasor) that belongs to Euclidean-Descartes space****, and that* $\Delta\bar{S} = \left|\Delta\bar{S}\right|\cdot\mathbf{e}^{\mathbf{I}\cdot\mathrm{arctg}\frac{c}{v}}$ *is the* ***complex or Analytic signal function (or Phasor) that belongs to Minkowski space****. We already see that Analytic signals established in the Minkowski space are very much and more relevant for the world of Physics, than signals from Euclidean-Descartes space.*

5° The reference system invariant structure of the complex 4-vector, which describes certain spatial-temporal length is on some intuitive way comprehensible as,

$$\Delta S_4 = \Delta S(\boxed{\Delta r}, I\boxed{c\Delta t}) = \Delta S\left(\boxed{\begin{matrix}\text{spatial interval that}\\ \text{motional particle passed}\end{matrix}}, \boxed{\begin{matrix}\text{spatial or time interval, which a}\\ \text{photon would cover}\end{matrix}}\right).$$

By analogy, we can now formulate similar 4-vector for certain particle that has linear momentum p,

$$\mathbf{P_4 = P(\boxed{p}, I\boxed{\frac{E_{tot}}{c}}) = P}\left(\boxed{\begin{matrix}\text{moment}\\ \text{of a particle}\end{matrix}}, \boxed{\begin{matrix}\text{moment of the same particle}\\ \text{which is effectively transformed}\\ \text{in a "photonic wave-group"}\end{matrix}}\right) \mathbf{= P(p, I\frac{E_{tot}}{c})}.$$

Again, by analogically applying Pythagoras' Theorem, we can get the following, reference systems invariance relation for the higher space-time dimensional framework,

$$\mathbf{P_4 = P(p,\ I\frac{E_{tot}}{c}) \Rightarrow p^2 - \left(\frac{E_{tot}}{c}\right)^2 = p_i^2 - \left(\frac{E_{tot-i}}{c}\right)^2 = ... = p_0^2 - \left(\frac{E_{tot-0}}{c}\right)^2 = -\left(\frac{E_{tot-0}}{c}\right)^2 = const.,}$$

$$\mathbf{p_0 = mv = p(v=0) = 0,\ i = 1, 2, 3, ...}$$

$$\mathbf{\Delta\overline{P}_4 = P(\Delta p,\ I\frac{\Delta E_{tot}}{c}) = \left|\Delta P_4\right| \cdot e^{-Iarctg\frac{\Delta E_{tot}}{c\Delta p}} = \left|\Delta P_4\right| \cdot e^{-I\cdot arctg\frac{v}{c}} = \sqrt{(\Delta p)^2 - \left(\frac{\Delta E_{tot}}{c}\right)^2} \cdot e^{-I\cdot arctg\frac{v}{c}} =}$$

$$\mathbf{= I\cdot\frac{E_0}{c}\cdot e^{-I\cdot arctg\frac{v}{c}} = \frac{E_0}{c}\cdot e^{I(\frac{\pi}{2} - arctg\frac{v}{c})},\ v = \frac{\Delta E_{tot}}{\Delta p} = \frac{\Delta E_k}{\Delta p}}.$$

The (just described) and already self-standing, or self-sufficient Minkowski concept of 4-vectors has been demonstrating such a great success and applicability in Physics, that it was easily assigned and associated to the essential meaning of Special Relativity theory. It is almost obvious that Mileva Maric and Albert Einstein devotionally accepted and assimilated such mathematical framework to serve as the foundation of Special Relativity theory (like majority of others who became admirers of the Relativity theory). In fact, Special Relativity theory without Minkowski-space 4-vectors would be mostly as somewhat confusing and not well-founded, or as only imaginatively and incompletely described concept. Minkowski did not make any objection or claims (concerning his far-reaching intellectual property rights), and it is maybe a time to recognize his grandiose creation and contribution additionally and more-completely to Physics.

*Here, we can easily extend Minkowski, **Complex 4-dimensional space-time (with only one imaginary unit $I^2 = -1$)**, to a higher level of **Hypercomplex space-time (which has at least three imaginary units; - see much more in the chapter 6)**. This would develop fruitful imaginative thinking and new revelations about possible multidimensional nature of a time domain. Hypercomplex and Analytic Phasors modeling, Minkowski space concept, and Hilbert transform are obviously serving as a window or good mathematical framework towards realistic, multidimensional insights. Since we already know that temporal and spatial domain or dimensions are mutually proportional, phase shifted, mutually orthogonal, mutually transformable and coupled, it would be logical that every multidimensional universe, at least theoretically, has a chance to have equal number of spatial and temporal dimensions. Since every imaginary unit of certain hypercomplex space-time is also mathematically presentable as being composed of three (or many) new imaginary units, this way we can imagine structural evolution or penetration towards always higher-dimensional spatial-temporal worlds. This is becoming complicated and endless process, but at least, it is mathematically explicable and structured.*

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Signals representation with Hypercomplex functions and relevant Hyper Complex Analytic Signals or Phasors has rich and challenging structure, and can be developed to represent many of complex and elementary matter states, as we can imaginatively and creatively judge from the following exercises,

$$\overline{Z} = \overline{Z}_{si} + \overline{Z}_{sj} + \overline{Z}_{sk} = \overline{Z}_{pi}\cdot\overline{Z}_{pj}\cdot\overline{Z}_{pk} = \left|\overline{Z}\right|\cdot e^{I\cdot\Theta},$$

$$i^2 = j^2 = k^2 = I^2 = -1,\ ij = k,\ jk = i,\ ki = j,\ ik = -j,\ kj = -i,\ ji = -k,$$

$$\begin{bmatrix} \overline{Z}_{si} = A_{si}\cdot e^{i\cdot\theta_{si}} = a_{si} + i\cdot b_{si} = \sqrt{a_{si}^2 + b_{si}^2}\cdot e^{i\cdot arctg(\frac{b_{si}}{a_{si}})}, \\ \overline{Z}_{sj} = A_{sj}\cdot e^{j\cdot\theta_{sj}} = a_{sj} + j\cdot b_{sj} = \sqrt{a_{sj}^2 + b_{sj}^2}\cdot e^{j\cdot arctg(\frac{b_{sj}}{a_{sj}})}, \\ \overline{Z}_{sk} = A_{sk}\cdot e^{k\cdot\theta_{sk}} = a_{sk} + k\cdot b_{sk} = \sqrt{a_{sk}^2 + b_{sk}^2}\cdot e^{k\cdot arctg(\frac{b_{sk}}{a_{sk}})} \end{bmatrix},$$

$$\begin{bmatrix} \overline{Z}_{pi} = A_{pi}\cdot e^{i\cdot\theta_{pi}} = a_{pi} + i\cdot b_{pi} = \sqrt{a_{pi}^2 + b_{pi}^2}\cdot e^{i\cdot arctg(\frac{b_{pi}}{a_{pi}})}, \\ \overline{Z}_{pj} = A_{pj}\cdot e^{j\cdot\theta_{pj}} = a_{pj} + j\cdot b_{pj} = \sqrt{a_{pj}^2 + b_{pj}^2}\cdot e^{j\cdot arctg(\frac{b_{pj}}{a_{pj}})}, \\ \overline{Z}_{pk} = A_{pk}\cdot e^{k\cdot\theta_{pk}} = a_{pk} + k\cdot b_{pk} = \sqrt{a_{pk}^2 + b_{pk}^2}\cdot e^{k\cdot arctg(\frac{b_{pk}}{a_{pk}})} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \overline{Z} = \overline{Z}_{si} + \overline{Z}_{sj} + \overline{Z}_{sk} = (a_{si} + i\cdot b_{si}) + (a_{sj} + j\cdot b_{sj}) + (a_{sk} + k\cdot b_{sk}) = (a_{pi} + i\cdot b_{pi})\cdot(a_{pj} + j\cdot b_{pj})\cdot(a_{pk} + k\cdot b_{pk}) = \\ = A_{pi}\cdot A_{pj}\cdot A_{pk}\cdot e^{i\cdot\theta_{pi} + j\cdot\theta_{pj} + k\cdot\theta_{pk}} = \left|\overline{Z}\right|\cdot e^{I\cdot\Theta} = \left|\overline{Z}\right|\cdot\cos\Theta + I\cdot\left|\overline{Z}\right|\cdot\sin\Theta = \\ = (a_{si} + a_{sj} + a_{sk}) + i\cdot b_{si} + j\cdot b_{sj} + k\cdot b_{sk} = A_s + I\cdot B_s,\ A_s = \left|\overline{Z}\right|\cdot\cos\Theta,\ B_s = \left|\overline{Z}\right|\cdot\sin\Theta \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} I\cdot\Theta = i\cdot\theta_{pi} + j\cdot\theta_{pj} + k\cdot\theta_{pk},\ I = i\cdot\frac{\theta_{pi}}{\Theta} + j\cdot\frac{\theta_{pj}}{\Theta} + k\cdot\frac{\theta_{pk}}{\Theta}, \\ \left|\overline{Z}\right| = A_{pi}\cdot A_{pj}\cdot A_{pk} = \frac{a_{si} + a_{sj} + a_{sk}}{\cos\Theta} = \sqrt{A_s^2 + B_s^2}, \\ \\ A_s = a_{si} + a_{sj} + a_{sk} = \left|\overline{Z}\right|\cdot\cos\Theta,\ I\cdot B_s = i\cdot b_{si} + j\cdot b_{sj} + k\cdot b_{sk} = I\cdot\left|\overline{Z}\right|\cdot\sin\Theta, \\ I\cdot\left|\overline{Z}\right|\cdot\sin\Theta = i\cdot b_{si} + j\cdot b_{sj} + k\cdot b_{sk},\ B_s^2 = (i\cdot b_{si} + j\cdot b_{sj} + k\cdot b_{sk})^2 = (b_{si}^2 + b_{sj}^2 + b_{sk}^2), \\ i\cdot j\cdot b_{si}\cdot b_{sj} + i\cdot k\cdot b_{si}\cdot b_{sk} + j\cdot k\cdot b_{sj}\cdot b_{sk} = k\cdot b_{si}\cdot b_{sj} - j\cdot b_{si}\cdot b_{sk} + i\cdot b_{sj}\cdot b_{sk} = 0, \\ I = i\cdot\frac{b_{si}}{\left|\overline{Z}\right|\cdot\sin\Theta} + j\cdot\frac{b_{sj}}{\left|\overline{Z}\right|\cdot\sin\Theta} + k\cdot\frac{b_{sk}}{\left|\overline{Z}\right|\cdot\sin\Theta} = i\cdot\frac{\theta_{pi}}{\Theta} + j\cdot\frac{\theta_{pj}}{\Theta} + k\cdot\frac{\theta_{pk}}{\Theta} = i\cdot\frac{b_{si}}{B_s} + j\cdot\frac{b_{sj}}{B_s} + k\cdot\frac{b_{sk}}{B_s}, \\ \frac{\theta_{pi}}{\Theta} = \frac{b_{si}}{\left|\overline{Z}\right|\cdot\sin\Theta},\ \frac{\theta_{pj}}{\Theta} = \frac{b_{sj}}{\left|\overline{Z}\right|\cdot\sin\Theta},\ \frac{\theta_{pk}}{\Theta} = \frac{b_{sk}}{\left|\overline{Z}\right|\cdot\sin\Theta}, \\ \frac{\sin\Theta}{\Theta} = \frac{b_{si}}{\left|\overline{Z}\right|\cdot\theta_{pi}} = \frac{b_{sj}}{\left|\overline{Z}\right|\cdot\theta_{pj}} = \frac{b_{sk}}{\left|\overline{Z}\right|\cdot\theta_{pk}},\ \frac{\theta_{pi}}{\Theta} = \frac{b_{si}}{B_s},\ \frac{\theta_{pj}}{\Theta} = \frac{b_{sj}}{B_s},\ \frac{\theta_{pk}}{\Theta} = \frac{b_{sk}}{B_s}, \\ \Theta = \frac{\theta_{pi}}{b_{si}}B_s = \frac{\theta_{pj}}{b_{sj}}B_s = \frac{\theta_{pk}}{b_{sk}}B_s,\ B_s = \frac{b_{si}}{\theta_{pi}}\Theta = \frac{b_{sj}}{\theta_{pj}}\Theta = \frac{b_{sk}}{\theta_{pk}}\Theta = \sqrt{b_{si}^2 + b_{sj}^2 + b_{sk}^2} = \frac{1}{3}\left(\frac{b_{si}}{\theta_{pi}} + \frac{b_{sj}}{\theta_{pj}} + \frac{b_{sk}}{\theta_{pk}}\right)\cdot\Theta \end{bmatrix}$$

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